

2014

New Insights on Emergence from the Perspective of Weak Values and Dynamical Non-Locality

Jeff Tollaksen

Chapman University, tollakse@chapman.edu

Follow this and additional works at: http://digitalcommons.chapman.edu/scs_articles



Part of the [Quantum Physics Commons](#)

Recommended Citation

Tollaksen, Jeff. "New insights on emergence from the perspective of weak values and dynamical non-locality." *Journal of Physics: Conference Series*. Vol. 504. No. 1. IOP Publishing, 2014. doi: 10.1088/1742-6596/504/1/012029

This Article is brought to you for free and open access by the Science and Technology Faculty Articles and Research at Chapman University Digital Commons. It has been accepted for inclusion in Mathematics, Physics, and Computer Science Faculty Articles and Research by an authorized administrator of Chapman University Digital Commons. For more information, please contact laughtin@chapman.edu.

New Insights on Emergence from the Perspective of Weak Values and Dynamical Non-Locality

Comments

This article was originally published under license in *Journal of Physics: Conference Series*, volume 504, issue 1, in 2014. Article is available online at DOI: [10.1088/1742-6596/504/1/012029](https://doi.org/10.1088/1742-6596/504/1/012029)

Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.

Creative Commons License



This work is licensed under a [Creative Commons Attribution 3.0 License](https://creativecommons.org/licenses/by/3.0/).

Copyright

The author

New insights on emergence from the perspective of weak values and dynamical non-locality

Jeff Tollaksen

Institute for Quantum Studies and Schmid College of Science and Technology, Chapman University, 1 University Drive, Orange, California USA 92866

E-mail: tollaksen@chapman.edu and quantum.chapman.edu

Abstract. In this article, we will examine new fundamental aspects of “emergence” and “information” using novel approaches to quantum mechanics which originated from the group around Aharonov. The two-state vector formalism provides a complete description of pre- and post-selected quantum systems and has uncovered a host of new quantum phenomena which were previously hidden. The most important feature is that any weak coupling to a pre- and post-selected system is effectively a coupling to a “weak value” which is given by a simple expression depending on the two-state vector. In particular, weak values, are the outcomes of so called “weak measurements” which have recently become a very powerful tool for ultra-sensitive measurements. Using weak values, we will show how to separate a particle from its properties, not unlike the Cheshire cat story: “Well! I’ve often seen a cat without a grin,” thought Alice; “but a grin without a cat! It’s the most curious thing I ever saw in all my life!” Next, we address the question whether the physics on different scales “emerges” from quantum mechanics or whether the laws of physics at those scales are fundamental. We show that the classical limit of quantum mechanics is a far more complicated issue; it is in fact dramatically more involved and it requires a complete revision of all our intuitions. The revised intuitions can then serve as a guide to finding novel quantum effects. Next we show that novel experimental aspects of contextuality can be demonstrated with weak measurements and these suggest new restrictions on hidden variable approaches. Next we emphasize that the most important implication of the Aharonov-Bohm effect is the existence of non-local interactions which do not violate causality. Finally, we review some generalizations of quantum mechanics and their implications for “emergence” and “information.” First, we review an alternative approach to quantum evolution in which each moment of time is viewed as a new “universe” and time evolution is given by correlations between different moments. Next, we present a new solution to the measurement problem involving future boundary conditions placed on the universe as a whole. Finally, we introduce another fundamental approach to quantum evolution which allows for tremendous richness in the types of allowable Hamiltonians.

1. Introduction

The conference’s title “Emergent Quantum Mechanics” (EmQM) was defined as “...a possible *deeper-level theory* that interconnects three fields of knowledge: emergence, the quantum, and information.” In this article, we will examine this from a number of novel approaches originating from the group around Aharonov. We will also try to address several ways in which EmQM conference participants interpreted the meaning of EmQM. In particular: 1) **Quantum Mechanics (QM) emerges from a deeper underlying theory:** accepting that the predictions of non-relativistic QM are correct, perhaps QM could emerge as a statistical theory



from a deeper, underlying theory (potentially involving hidden-variables); 2) **Generalizations of QM:** assuming that someday non-relativistic QM is disproven (although there is as yet no experimental evidence to support this), what are possible generalizations of QM, 3) **Whole-part relations:** as Gerhard Grössing best put it: “Emergence through the co-evolution of ‘microscopic,’ local processes and of ‘macroscopic’ boundary conditions,” this could involve novel arrows of causation, the traditional “bottom-up” arrow combined with another “top-down” arrow; and finally 4) **Emergence and the origin of laws:** i.e. whether the physics on different scales “emerges” from QM or whether the laws of physics at those scales are fundamental.

Often these inquiries are approached by seeking new physics beyond the core of standard non-relativistic QM. While we too will examine such an approach, first in §2 we ask what novel perspectives can be gleaned just from standard non-relativistic QM. Then, in §3, we go beyond existing QM and suggest several speculative generalizations of QM and their potential implications for EmQM.

2. Novel aspects of emergence based on standard non-relativistic QM

In this section, we assume that QM, which to-date has been vindicated by all experiments, is correct and need not be recast with hidden variables or classical/stochastic elements. There are still some logical shortcomings such as the apparently non-unitary evolution which occurs during measurements with the ensuing “collapse” of the wave function. Nevertheless, in this section, we are not espousing any of the existing approaches which try to explain such issues. Instead, we find it most beneficial to keep prying and searching for new effects within, and new re-formulations of, the existing quantum theory. Rather than impose new rules, in this section, we utilize the established rules, trying to better understand the underlying logic and their potential relevance to the above mentioned issues. Cognizant of the popular saying “if your only tool is a hammer, then you tend to treat everything as if it were a nail,” we have used new measurement paradigms, such as weak measurements, protective measurement, and the measurement of deterministic operators, to investigate these issues.

Entanglement and nonlocality are now considered to be some of the most basic aspects of QM. In particular they are at the core of quantum information science, and, more generally, in all applications of quantum coherence. As such, entanglement has been studied extensively, and it is one of the most active fields of research in theoretical physics. To date, QM indicates non-locality in two fundamental, empirically proven ways: kinematic (e.g. Einstein-Podolsky-Rosen a/k/a EPR) and dynamic (e.g. the Aharonov-Bohm effect a/k/a AB). The AB effect demonstrated that an electromagnetic field inside a confined region can have a measureable impact on the interference pattern of a charged particle which never traveled inside that region. The EPR non-locality is evidenced when the various parts of a quantum system are spatially and/or temporally separated. Bell’s theorem shows us that the correlations between these parts cannot be explained by any local-realistic theory. Some have argued that the non-factorizability of a many body state into a product of one body states implies a kind of holism. As Shimony elegantly stated: [42]: “The parts-wholes problem has an ontological aspect, which concerns the properties of the components and the composite system without explicit consideration of how knowledge of them is obtained... are there properties of composite systems which are radically different from those of the components, and which might properly be characterized as ‘emergent’?”

We consider novel aspects of emergence using 2 different approaches to standard QM originated by the group around Aharonov. First, in §2.1, we review a time-symmetric re-formulation of quantum mechanics which led to weak measurements performed on pre- and post-selected ensembles. It also led to the “weak value” [29]. Secondly in §2.2 we apply weak values to EmQM. Next in §2.3 we use weak values to introduce new restrictions on hidden variables and then, in §2.4 we discuss a new approach to information, which we call “weak

information,” and in §2.6 we introduce a new “internal” quantum frame-of-reference. In the final sub-section in this chapter concerning standard QM, namely §2.5, we consider dynamical non-locality originating from the AB effect as described by modular variables which are particularly useful when considering quantum states composed of several lumps involving interference.

2.1. Review of time-symmetry, weak values and weak measurements

The “time-asymmetry” in QM involves the assumption that measurements only have consequences **after** they are performed, i.e. towards the future. Nevertheless, a positive spin was placed on QM’s non-trivial relationship between initial and final conditions by Aharonov, Bergmann and Lebowitz (ABL) [1] who showed that the new information obtained from subsequent measurements were also relevant for the **past** of every quantum-system and not just the future. This inspired ABL to re-formulate QM in terms of *Pre- and Post-Selected ensembles* (PPS). The traditional paradigm for ensembles is to simply prepare systems in a particular state and thereafter subject them to a variety of experiments. These are “pre-selected-only-ensembles.” For **pre-and-post-selected**-ensembles, we add one more step, a subsequent measurement or post-selection. By collecting only a subset of the outcomes for this later measurement, we see that the “pre-selected-only-ensemble” can be divided into sub-ensembles according to the results of this subsequent “post-selection-measurement.” Because PPS ensembles are the most refined quantum ensemble, they are of fundamental importance and subsequently led to the *two-vector* or *Time-Symmetric re-formulation of Quantum Mechanics* (TSQM) [9,11]. TSQM provides a complete description of a quantum-system at a given moment by using two-wavefunctions, one evolving from the past towards the future (the one utilized in the standard paradigm) and a second one, evolving from the future towards the past.

While TSQM is a new conceptual point-of-view that has predicted novel, verified effects which **seem** impossible according to standard QM, TSQM is in fact a *re-formulation* of QM. Therefore, experiments cannot prove TSQM over QM (or vice-versa). The motivation to pursue such re-formulations, then, depends on their usefulness. TSQM fulfills several criterion in order to be useful and interesting:

- TSQM is consistent with all the predictions made by standard QM (§2.1.1),
- TSQM has revealed new features of QM that were missed before (§2.1.2.1),
- TSQM has led to new mathematics, simplifications in calculations, and stimulated discoveries in other fields (§2.1.3),
- TSQM suggests generalizations of QM (§3).

2.1.1. The main idea behind this re-formulation of QM TSQM contemplates measurements which occur at the present time t while the state is known both at $t_{\text{in}} < t$ (past) and at $t_{\text{fin}} > t$ (future). More precisely, we start at $t = t_{\text{in}}$ with a measurement of a nondegenerate operator \hat{O}_{in} . This yields as one potential outcome the state $|\Psi_{\text{in}}\rangle$, i.e. we prepared the “pre-selected” state $|\Psi_{\text{in}}\rangle$. At the later time t_{fin} , we perform another measurement of a nondegenerate operator \hat{O}_{fin} which yields a number of possible outcomes and we select one of them, the post-selected state $|\Psi_{\text{fin}}\rangle$. At an intermediate time $t \in [t_{\text{in}}, t_{\text{fin}}]$, we measure a non-degenerate (for simplicity) observable \hat{A} , with eigenvectors $\{|a_j\rangle\}$. We wish to determine the conditional probability of a_j , given that we have both boundary conditions, $|\Psi_{\text{in}}\rangle$ and $\langle\Psi_{\text{fin}}|$.

¹ To answer this, we use the time displacement operator: $U_{t_{\text{in}} \rightarrow t} = \exp\{-iH(t - t_{\text{in}})\}$ where H

¹ Such an arrangement has long been considered in actual experiments such as a bubble chamber/scattering experiment. The incoming particle, $|\Psi_{\text{in}}\rangle$, interacts with a target and then evolves into various outgoing states, $|\Psi_{\text{fin}}\rangle_1, |\Psi_{\text{fin}}\rangle_2$, etc. Typically, photographs are not taken for every target-interaction, but only for certain ones that were triggered by subsequently interacting with detectors. In classical mechanics, there is (in principle) a

is the Hamiltonian for the free system. For simplicity, we assume H is time independent and set $\hbar = 1$. The standard theory of collapse states that the system collapses into an eigenstate $|a_j\rangle$ after the measurement at t with an amplitude $\langle a_j|U_{t_{\text{in}} \rightarrow t}|\Psi_{\text{in}}\rangle$. The amplitude for our series of events is $\alpha_j \equiv \langle \Psi_{\text{fin}}|U_{t \rightarrow t_{\text{fin}}}|a_j\rangle\langle a_j|U_{t_{\text{in}} \rightarrow t}|\Psi_{\text{in}}\rangle$ which is illustrated in figure 1.a. This means that the conditional probability to measure a_j given that $|\Psi_{\text{in}}\rangle$ is pre-selected and $|\Psi_{\text{fin}}\rangle$ will be post-selected is given by the ABL formula [1]:

$$Pr(a_j, t|\Psi_{\text{in}}, t_{\text{in}}; \Psi_{\text{fin}}, t_{\text{fin}}) = \frac{|\langle \Psi_{\text{fin}}|U_{t \rightarrow t_{\text{fin}}}|a_j\rangle\langle a_j|U_{t_{\text{in}} \rightarrow t}|\Psi_{\text{in}}\rangle|^2}{\sum_n |\langle \Psi_{\text{fin}}|U_{t \rightarrow t_{\text{fin}}}|a_n\rangle\langle a_n|U_{t_{\text{in}} \rightarrow t}|\Psi_{\text{in}}\rangle|^2} \quad (2.1)$$

As a first step toward understanding the underlying time-symmetry in the ABL formula, we consider the time-reverse of the numerator of Eq. (2.1) and consequently the time reverse of figure 1.a. First we apply $U_{t \rightarrow t_{\text{fin}}}$ on $\langle \Psi_{\text{fin}}|$ instead of on $\langle a_j|$. We note that $\langle \Psi_{\text{fin}}|U_{t \rightarrow t_{\text{fin}}} = \langle U_{t \rightarrow t_{\text{fin}}}^\dagger \Psi_{\text{fin}}|$ by using the well-known QM symmetry $U_{t \rightarrow t_{\text{fin}}}^\dagger = \{e^{-iH(t_{\text{fin}}-t)}\}^\dagger = e^{iH(t_{\text{fin}}-t)} = e^{-iH(t-t_{\text{fin}})} = U_{t_{\text{fin}} \rightarrow t}$. We also apply $U_{t_{\text{in}} \rightarrow t}$ on $\langle a_j|$ instead of on $|\Psi_{\text{in}}\rangle$ which yields the time-reverse reformulation of the numerator of Eq. (2.1), $\langle U_{t_{\text{fin}} \rightarrow t} \Psi_{\text{fin}}|a_j\rangle\langle U_{t \rightarrow t_{\text{in}}} a_j|\Psi_{\text{in}}\rangle$ as depicted in Fig. 1.b. Further work is needed to formulate what we mean by the 2-vectors in TSQM. E.g. if we

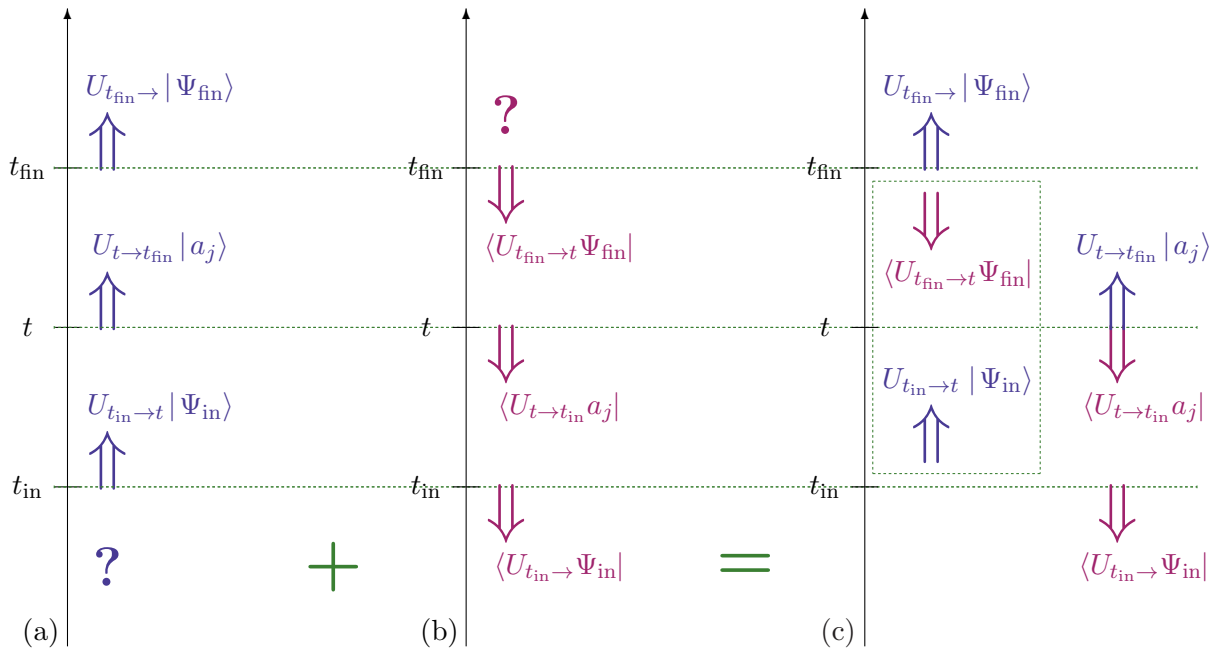


Figure 1. Time-reversal symmetry in probability amplitudes

are interested in the probability for possible outcomes of a_j at time t , we must consider both $U_{t_{\text{in}} \rightarrow t}|\Psi_{\text{in}}\rangle$ and $\langle U_{t_{\text{fin}} \rightarrow t} \Psi_{\text{fin}}|$, since these expressions propagate the pre- and post-selection to the present time t (see the conjunction of both figures 1.a and 1.b giving 1.c; Note: these 2-vectors are not just the time-reverse of each other). This represents the basic idea behind TSQM:

$$Pr(a_j, t|\Psi_{\text{in}}, t_{\text{in}}; \Psi_{\text{fin}}, t_{\text{fin}}) = \frac{|\langle U_{t_{\text{fin}} \rightarrow t} \Psi_{\text{fin}}|a_j\rangle\langle a_j|U_{t_{\text{in}} \rightarrow t}|\Psi_{\text{in}}\rangle|^2}{\sum_n |\langle U_{t_{\text{fin}} \rightarrow t} \Psi_{\text{fin}}|a_n\rangle\langle a_n|U_{t_{\text{in}} \rightarrow t}|\Psi_{\text{in}}\rangle|^2} \quad (2.2)$$

one-to-one mapping between incoming states and outgoing states, whereas in QM, it is one-to-many. By selecting a single outcome for the post-selection-measurement, we define the pre- and post-selected-ensemble that has no classical analog.

While this mathematical manipulation clearly proves that the predictions of TSQM is consistent with standard QM, it nevertheless suggests a very different interpretation. For example, the action of $U_{t_{\text{fin}} \rightarrow t}$ on $\langle \Psi_{\text{fin}} |$ (i.e. $\langle U_{t_{\text{fin}} \rightarrow t} \Psi_{\text{fin}} |$) can be interpreted as sending $\langle \Psi_{\text{fin}} |$ back in time from t_{fin} to the present, t . In summary, ABL clarified a number of issues in QM. E.g. in this formulation, both the probability and the amplitude are symmetric under the exchange of $|\Psi_{\text{in}}\rangle$ and $|\Psi_{\text{fin}}\rangle$. Therefore, the possibility of wavefunction collapse in QM does not necessarily imply irreversibility of an arrow of time at the QM level. TSQM suggested a number of new experimentally observable effects, one important example of which are *weak measurements* (§2.1.2.1), which we now begin to motivate by considering strange pre- and post-selection effects.

2.1.2. Pre- and post-selection, weak measurements and weak values of spin-1/2 systems One of the simplest, surprising, examples of pre- and post-selection is to pre-select a spin-1/2 system with $|\Psi_{\text{in}}\rangle = |\hat{\sigma}_x = +1\rangle = |\uparrow_x\rangle$ at time t_{in} . After the pre-selection, spin measurements in the direction perpendicular to x yields complete uncertainty so if we post-select at time t_{fin} in the y -direction, we obtain $|\Psi_{\text{fin}}\rangle = |\hat{\sigma}_y = +1\rangle = |\uparrow_y\rangle$ one-half of the time. Since the particle is free, the spin is conserved in time and thus for any $t \in [t_{\text{in}}, t_{\text{fin}}]$, an ideal-measurement of either $\hat{\sigma}_x$ or $\hat{\sigma}_y$, yields +1 for this pre- and post-selection. This by itself, two non-commuting observables known with certainty, is a most surprising property which no pre-selected-only-ensemble could possess.

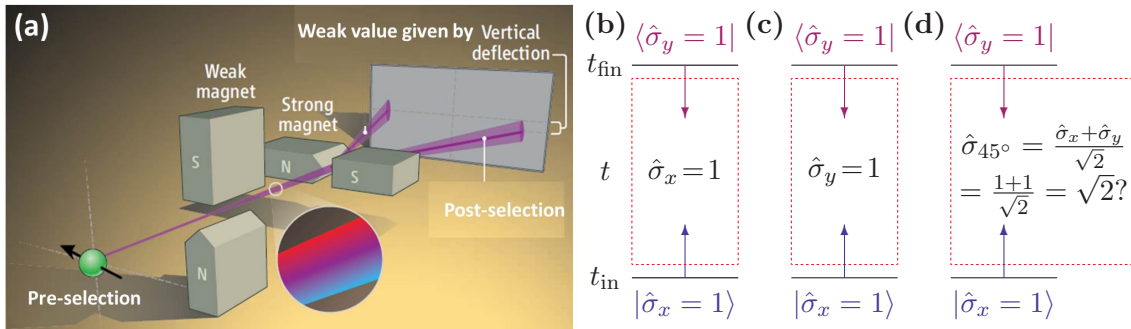


Figure 2. Stern-Gerlach apparatus both prepares (pre-selects) a spin-1/2 particle at time t_{in} to be $|\hat{\sigma}_x = 1\rangle$, and post-selects at t_{fin} to be $\langle \hat{\sigma}_y = 1 |$ and weakly measures the spin (from [43]; (b) During the intermediate time $t \in [t_{\text{in}}, t_{\text{fin}}]$, ABL tells us that an ideal-measurement of either $\hat{\sigma}_x$ or (c) $\hat{\sigma}_y$, yields +1 with certainty, suggesting that such a particle has well-defined values of the two non-commuting spin components (d) It would seem to follow that the spin component $\hat{\sigma}_{45^\circ}$ would have to be $\sqrt{2}$ which is not an allowed eigenvalue.

We now ask a slightly more complicated question about the spin in a direction $\xi = 45^\circ$ relative to the $x - y$ axis. This yields:

$$\hat{\sigma}_\xi = \hat{\sigma}_x \cos 45^\circ + \hat{\sigma}_y \sin 45^\circ = \frac{\hat{\sigma}_x + \hat{\sigma}_y}{\sqrt{2}} \quad (2.3)$$

From the results $Pr(\hat{\sigma}_x = +1) = 1$ and $Pr(\hat{\sigma}_y = +1) = 1$, one might wonder why we couldn't insert both values, $\hat{\sigma}_x = +1$ and $\hat{\sigma}_y = +1$ into Eq. (2.3) and obtain $\hat{\sigma}_\xi = \frac{1+1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$ (see figure 2). Such a result is incorrect for an ideal-measurement because the eigenvalues of any spin operator, including $\hat{\sigma}_\xi$, must be ± 1 . Performing this step of replacing $\hat{\sigma}_x = +1$ and $\hat{\sigma}_y = +1$ in Eq. (2.3) can only be done if $\hat{\sigma}_x$ and $\hat{\sigma}_y$ commute, which would allow both values simultaneously to be definite. The ABL statements, namely $Pr(\hat{\sigma}_x = +1) = 1$ and $Pr(\hat{\sigma}_y = +1) = 1$, are said to be “counterfactuals.” Although it appears we have reached the

end-of-the-line with this argument, nevertheless, it still seems that there should be some sense in which both $Pr(\hat{\sigma}_x = +1) = 1$ and $Pr(\hat{\sigma}_y = +1) = 1$ manifest themselves simultaneously to produce $\hat{\sigma}_\xi = \sqrt{2}$.

2.1.2.1. Counterfactuals: There is a widespread tendency to “resolve” many quantum mechanical paradoxes by pointing out that there is an element of counter-factual reasoning, just as we pointed out in the previous example. I.e., the contradictions arise only because inferences are made that do not refer to actual experiments. Had the experiment actually been performed, then standard measurement theory predicts that the system would have been disrupted so that no paradoxical implications arises.

We have proven [12, 19] that one shouldn’t be so quick in throwing away counter-factual reasoning; though indeed counter-factual statements have no observational meaning, such reasoning is actually a very good pointer towards interesting physical situations. *Without invoking counter-factual reasoning*, we have shown that the apparently paradoxical reality implied counter-factually has new, *experimentally accessible* consequences. These observable consequences become evident in terms of *weak measurements*, which allow us to test – to some extent – assertions that have been otherwise regarded as counter-factual.

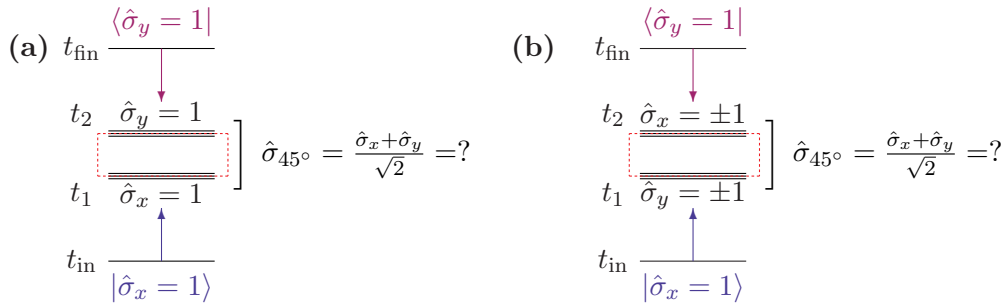


Figure 3. Measurement of $\hat{\sigma}_{45^\circ}$ is effectively a simultaneous measurement of both $\hat{\sigma}_x$ and $\hat{\sigma}_y$. In fig 2, we argued that if we measured either one separately, then we would obtain +1 with certainty. It is more complicated if we try to measure **both** of them together as required for a measurement of $\hat{\sigma}_{45^\circ}$. (a) E.g., if the first measurement performed during the intermediate time $t \in [t_{\text{in}}, t_{\text{fin}}]$ is in the same direction as the pre-selection (i.e. $\hat{\sigma}_x$) and the second measurement during the intermediate time $t \in [t_{\text{in}}, t_{\text{fin}}]$ is in the same direction as the post-selection (i.e. $\hat{\sigma}_y$) then both measurements will be determined with certainty (i.e. +1) as a result of the pre- and post-selection; (b) However, if we reverse the order of the measurements performed during the intermediate time $t \in [t_{\text{in}}, t_{\text{fin}}]$ then that certainty is destroyed. I.e. the first measurement, that of $\hat{\sigma}_y$ can result in either +1 or -1. The second measurement, that of $\hat{\sigma}_x$ can also result in either +1 or -1. In other words, $\hat{\sigma}_x$ is not determined by the pre-selection because the previous measurement performed during the intermediate time $t \in [t_{\text{in}}, t_{\text{fin}}]$ **disturbs** it.

For our spin-1/2 example, the counter-factual argument works as follows: if we verify $\hat{\sigma}_x$ at $t = t_1$ and $\hat{\sigma}_y$ at $t = t_2$, $t_{\text{in}} < t_1 < t_2 < t_{\text{fin}}$, then $Pr(\hat{\sigma}_x = +1) = 1$ and $Pr(\hat{\sigma}_y = +1) = 1$ are simultaneously true. But if we switch the order and perform $\hat{\sigma}_y$ before $\hat{\sigma}_x$, then $Pr(\hat{\sigma}_x = +1) = 1$ and $Pr(\hat{\sigma}_y = +1) = 1$ are not simultaneously true, since measuring $\hat{\sigma}_y$ at time $t = t_1$ would not allow the information from the earlier ($t_{\text{in}} < t$) pre-selection of $\hat{\sigma}_x = +1$ to propagate to the later time ($t_2 > t_1 > t_{\text{in}}$) of the $\hat{\sigma}_x$ measurement. As a consequence, the $\hat{\sigma}_x$ measurement at time t_2 would yield both outcomes $\hat{\sigma}_x = \pm 1$. So, in general, the finding that $\hat{\sigma}_x = +1$ with certainty or $\hat{\sigma}_y = +1$ with certainty in the pre- and post-selected ensemble only held when *one* of these two measurements was performed in the intermediate time, not both. Therefore, we should not expect both $\hat{\sigma}_y = +1$ and $\hat{\sigma}_x = +1$ when measured simultaneously through $\hat{\sigma}_{\xi=45^\circ}$.

Since we have understood the reason why both statements are not simultaneously true as a result of disturbance, we can now see the “sense” in which the definite ABL assignments can be simultaneously relevant. Our main argument is that if one doesn’t perform absolutely precise (ideal) measurements but is willing to accept some finite accuracy, then one can bound the disturbance on the system. For example, according to Heisenberg’s uncertainty relations, a precise measurement of position reduces the uncertainty in the position of the measuring device to zero $\Delta Q_{\text{md}} = 0$ but produces an infinite uncertainty in momentum $\Delta P_{\text{md}} = \infty$. On the other hand, if we measure the position only up to some finite precision $\Delta Q_{\text{md}} = \Delta$ we can limit the disturbance of momentum to a finite amount $\Delta P_{\text{md}} \geq \hbar/\Delta$. By replacing precise measurements with a bounded-measurement paradigm, we produce limited-disturbance measurements. With this, there **is** a sense in which both $Pr(\hat{\sigma}_x = +1) = 1$ and $Pr(\hat{\sigma}_y = +1) = 1$ are simultaneously relevant because measurement of one does *not* disturb the other. With limited-disturbance measurements, we can simultaneously use both $\hat{\sigma}_x = +1$ and $\hat{\sigma}_y = +1$ to

$$\text{obtain } (\hat{\sigma}_{\xi=45^\circ})_{\text{w}} = \frac{\langle \uparrow_y | \frac{\hat{\sigma}_y + \hat{\sigma}_x}{\sqrt{2}} | \uparrow_x \rangle}{\langle \uparrow_y | \uparrow_x \rangle} = \frac{\{\langle \uparrow_y | \hat{\sigma}_y \rangle + \{\langle \hat{\sigma}_x | \uparrow_x \rangle\}}{\sqrt{2} \langle \uparrow_y | \uparrow_x \rangle} = \frac{\langle \uparrow_y | 1+1 | \uparrow_x \rangle}{\sqrt{2} \langle \uparrow_y | \uparrow_x \rangle} = \sqrt{2}.$$

2.1.2.2. Quantum Measurements: Weak-measurements [11] originally grew out of the quantum measurement theory developed by von Neumann though weak measurements and their outcome, weak values, can be derived in all approaches to quantum measurement theory. E.g. the usual projective measurement typically utilized in quantum experiments is a special case of these weak-measurements [21]. Using an interaction Hamiltonian H_{int} of the form $H_{\text{int}} = -g(t)\hat{Q}_{\text{md}}\hat{A}$ with $\int_0^T g(t)dt = g_o$, we note that to make a more precise determination of \hat{A} , that is, to make an ideal measurement, requires that the shift in P_{md} , i.e. $\delta P_{\text{md}} = P_{\text{md}}(T) - P_{\text{md}}(0)$, be distinguishable from its uncertainty, ΔP_{md} . This occurs, e.g., if $P_{\text{md}}(0)$ and $P_{\text{md}}(T)$ are more precisely defined and/or if g_o is sufficiently large (see figure 4.a). However, under these conditions (e.g. if the measuring device approaches a delta function in P_{md}), then the disturbance or back-reaction on the system is increased due to a larger H_{int} , the result of the larger ΔQ_{md} ($\Delta Q_{\text{md}} \geq \frac{1}{\Delta P_{\text{md}}}$). When \hat{A} is measured in this way, then any operator \hat{O} ($[\hat{A}, \hat{O}] \neq 0$) is disturbed because it evolved according to $\frac{d}{dt}\hat{O} = ig(t)[\hat{A}, \hat{O}]\hat{Q}_{\text{md}}$, and since $g_o\Delta Q_{\text{md}}$ is not zero, \hat{O} changes in an uncertain way proportional to $g_o\Delta Q_{\text{md}}$. In the spin-1/2 example, the conditions for an ideal-measurement $\delta P_{\text{md}}^\xi = g_o\hat{\sigma}_\xi \gg \Delta P_{\text{md}}^\xi$ will also necessitate $\Delta Q_{\text{md}}^\xi \gg \frac{1}{g_o\hat{\sigma}_\xi}$ which will thereby create a back-reaction causing a precession in the spin such that $\Delta\Theta \gg 1$ (i.e. more than one revolution), thereby destroying (i.e. making completely uncertain) the information that in the past we had $\hat{\sigma}_x = +1$, and in the future we will have $\hat{\sigma}_y = +1$. In the Schrödinger picture, the time evolution operator for the complete system from $t = t_0 - \varepsilon$ to $t = t_0 + \varepsilon$ is $\exp\{-i \int_{t_0-\varepsilon}^{t_0+\varepsilon} H(t)dt\} = \exp\{-ig_o\hat{Q}_{\text{md}}\hat{A}\}$. This shifts P_{md} (see figure 4.a). If before the measurement the system was in a superposition of eigenstates of \hat{A} , then the measuring device will also be superposed in direct correspondence with the system. This leads to the “quantum measurement problem.” A conventional solution to this problem is to argue that because the measuring device is macroscopic, it cannot be in a superposition, and so it will “collapse” into one of these states and the system will collapse with it.

2.1.2.3. Weakening the interaction between system and measuring device: Following the intuition gained in Fig. 3, we now perform measurements which do not disturb either the pre- or post-selections. The interaction $H_{\text{int}} = -g(t)\hat{Q}_{\text{md}}\hat{A}$ is weakened by minimizing $g_o\Delta Q_{\text{md}}$. For simplicity, we consider $g_o \ll 1$ (assuming without lack of generality that the state of the measuring device is a Gaussian with spreads $\Delta P_{\text{md}} = \Delta Q_{\text{md}} = 1$). We may then set

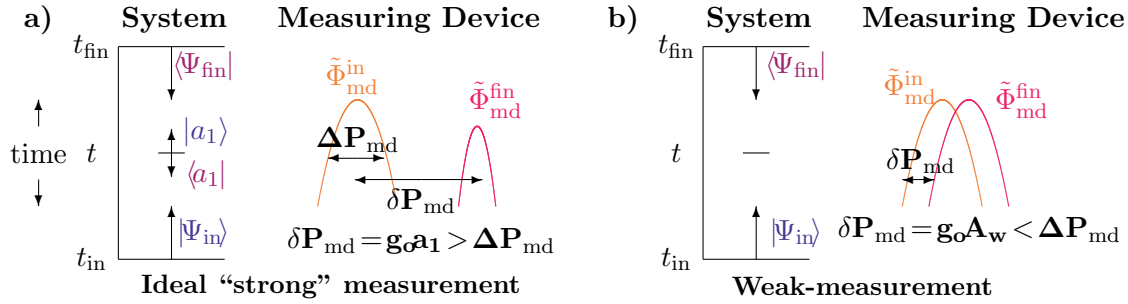


Figure 4. a) with an ideal or “strong” measurement at t (characterized e.g. by $\delta P_{\text{md}} = g_o a_1 \gg \Delta P_{\text{md}}$), then ABL gives the probability to obtain a collapse onto eigenstate a_1 by propagating $\langle \Psi_{\text{fin}} |$ backwards in time from t_{fin} to t and $|\Psi_{\text{in}}\rangle$ forwards in time from t_{in} to t ; in addition, the collapse caused by ideal-measurement at t creates a new boundary condition $|a_1\rangle \langle a_1|$ at time $t \in [t_{\text{in}}, t_{\text{fin}}]$; b) if a weak-measurement is performed at t (characterized e.g. by $\delta P_{\text{md}} = g_o A_w \ll \Delta P_{\text{md}}$), then the outcome of the weak-measurement, the weak-value, can be calculated by propagating the state $\langle \Psi_{\text{fin}} |$ backwards in time from t_{fin} to t and the state $|\Psi_{\text{in}}\rangle$ forwards in time from t_{in} to t ; the weak-measurement does not cause a collapse and thus no new boundary condition is created at time t .

$e^{-ig_o \hat{Q}_{\text{md}} \hat{A}} \approx 1 - ig_o \hat{Q}_{\text{md}} \hat{A}$ and use a theorem²:

$$\hat{A}|\Psi\rangle = \langle \hat{A} \rangle |\Psi\rangle + \Delta A |\Psi_{\perp}\rangle, \quad (2.4)$$

to show that before the post-selection, the system state is:

$$e^{-ig_o \hat{Q}_{\text{md}} \hat{A}} |\Psi_{\text{in}}\rangle = (1 - ig_o \hat{Q}_{\text{md}} \hat{A}) |\Psi_{\text{in}}\rangle = (1 - ig_o \hat{Q}_{\text{md}} \langle \hat{A} \rangle) |\Psi_{\text{in}}\rangle - ig_o \hat{Q}_{\text{md}} \Delta \hat{A} |\Psi_{\text{in}\perp}\rangle. \quad (2.5)$$

Using the norm of this state $\|(1 - ig_o \hat{Q}_{\text{md}} \hat{A}) |\Psi_{\text{in}}\rangle\|^2 = 1 + g_o^2 \hat{Q}_{\text{md}}^2 \langle \hat{A}^2 \rangle$, the probability to leave $|\Psi_{\text{in}}\rangle$ un-changed after the measurement is:

$$\frac{1 + g_o^2 \hat{Q}_{\text{md}}^2 \langle \hat{A} \rangle^2}{1 + g_o^2 \hat{Q}_{\text{md}}^2 \langle \hat{A}^2 \rangle} \rightarrow 1 \quad (g_o \rightarrow 0) \quad (2.6)$$

while the probability to disturb the state (i.e. to obtain $|\Psi_{\text{in}\perp}\rangle$) is:

$$\frac{g_o^2 \hat{Q}_{\text{md}}^2 \Delta \hat{A}^2}{1 + g_o^2 \hat{Q}_{\text{md}}^2 \langle \hat{A}^2 \rangle} \rightarrow 0 \quad (g_o \rightarrow 0). \quad (2.7)$$

The final state of the measuring device is now a superposition of many substantially overlapping Gaussians with probability distribution given by $Pr(P_{\text{md}}) = \sum_i |\langle a_i | \Psi_{\text{in}} \rangle|^2 \exp \left\{ -\frac{(P_{\text{md}} - g_o a_i)^2}{2 \Delta P_{\text{md}}^2} \right\}$. This sum is a Gaussian mixture, so it can be approximated by a single Gaussian $\tilde{\Phi}_{\text{md}}^{\text{fin}}(P_{\text{md}}) \approx \langle P_{\text{md}} | e^{-ig_o \hat{Q}_{\text{md}} \langle \hat{A} \rangle} | \Phi_{\text{md}}^{\text{in}} \rangle \approx \exp \left\{ -\frac{(P_{\text{md}} - g_o \langle \hat{A} \rangle)^2}{\Delta P_{\text{md}}^2} \right\}$ centered on $g_o \langle \hat{A} \rangle$.

² where $\langle \hat{A} \rangle = \langle \Psi | \hat{A} | \Psi \rangle$, $|\Psi\rangle$ is any vector in Hilbert space, $\Delta A^2 = \langle \Psi | (\hat{A} - \langle \hat{A} \rangle)^2 | \Psi \rangle$, and $|\Psi_{\perp}\rangle$ is a state such that $\langle \Psi | \Psi_{\perp} \rangle = 0$.

2.1.2.4. Information gain without disturbance – safety in numbers: It follows from Eq. (2.7) that the probability for a collapse decreases as $O(g_o^2)$, but the measuring device's shift grows linearly $O(g_o)$, so $\delta P_{\text{md}} = g_o a_i$ [22]. For a sufficiently weak interaction (e.g. $g_o \ll 1$), the probability for a collapse can be made arbitrarily small, while the measurement still yields information but becomes less precise because the shift in the measuring device is much smaller than its uncertainty $\delta P_{\text{md}} \ll \Delta P_{\text{md}}$ (figure 4.b). Nevertheless, if a large ($N \geq \frac{N'}{g_o}$) ensemble of particles is used, then the shift of all the measuring devices ($\delta P_{\text{md}}^{\text{tot}} \approx g_o \langle \hat{A} \rangle \frac{N'}{g_o} = N' \langle \hat{A} \rangle$) becomes distinguishable because of repeated integrations, while the collapse probability still goes to zero.

Using these observations, we now emphasize that the average of any operator \hat{A} , i.e. $\langle \hat{A} \rangle \equiv \langle \Psi | \hat{A} | \Psi \rangle$, can be obtained in three distinct cases [18, 22]:

- (i) **Statistical method with disturbance:** the traditional approach is to perform ideal-measurements of \hat{A} on each particle, obtaining a variety of different eigenvalues, and then manually calculate the usual statistical average to obtain $\langle \hat{A} \rangle$.
- (ii) **Statistical method without disturbance** as demonstrated by using $\hat{A}|\Psi\rangle = \langle \hat{A} \rangle |\Psi\rangle + \Delta A |\Psi_\perp\rangle$. Using an ensemble, we can verify there was no disturbance.
- (iii) **Non-statistical method without disturbance** is the case where $\langle \Psi | \hat{A} | \Psi \rangle$ is the “eigenvalue” of a single “collective operator,” $\hat{A}^{(N)} \equiv \frac{1}{N} \sum_{i=1}^N \hat{A}_i$ (with \hat{A}_i the same operator \hat{A} acting on the i -th particle). Using this, we are able to obtain information about $\langle \Psi | \hat{A} | \Psi \rangle$ without causing disturbance (or a collapse) and without using a statistical approach because any product state $|\Psi^{(N)}\rangle$ becomes an eigenstate of the operator $\hat{A}^{(N)}$. To see this, we apply the theorem $\hat{A}|\Psi\rangle = \langle \hat{A} \rangle |\Psi\rangle + \Delta A |\Psi_\perp\rangle$ to $\hat{A}^{(N)}|\Psi^{(N)}\rangle$, i.e.:

$$\hat{A}^{(N)}|\Psi^{(N)}\rangle = \frac{1}{N} \left[N \langle \hat{A} \rangle |\Psi^{(N)}\rangle + \Delta A \sum_i |\Psi_\perp^{(N)}(i)\rangle \right]. \quad (2.8)$$

where $\langle \hat{A} \rangle$ is the average for any one particle and the states $|\Psi_\perp^{(N)}(i)\rangle$ are mutually orthogonal and are given by $|\Psi_\perp^{(N)}(i)\rangle = |\Psi\rangle_1 |\Psi\rangle_2 \dots |\Psi_\perp\rangle_i \dots |\Psi\rangle_N$. That is, the i th state has particle i changed to an orthogonal state and all the other particles remain in the same state. If we further define a normalized state $|\Psi_\perp^{(N)}\rangle = \sum_i \frac{1}{\sqrt{N}} |\Psi_\perp^{(N)}(i)\rangle$ then the last term of Eq. (2.8) is $\frac{\Delta A}{\sqrt{N}} |\Psi_\perp^{(N)}\rangle$ and it's size is $|\frac{\Delta A}{\sqrt{N}} |\Psi_\perp^{(N)}\rangle|^2 \propto \frac{1}{N} \rightarrow 0$. Therefore, $|\Psi^{(N)}\rangle$ becomes an eigenstate of $\hat{A}^{(N)}$, with the value $\langle \hat{A} \rangle$ and not even a single particle has been disturbed (as $N \rightarrow \infty$).

In the last case, the average for a single particle becomes a robust property over the entire ensemble, so a single experiment is sufficient to determine the average with great precision [22]. There is no longer any need to average over results obtained in multiple experiments. Tradition has dictated that when measurement interactions are limited so there is no disturbance on the system, then no information can be gained. However, we have now shown that when considered as a limiting process, the disturbance goes to zero more quickly than the shift in the measuring device, which means for a large enough ensemble, information (e.g. the expectation value) can be obtained even though not even a single particle is disturbed. This viewpoint thereby shifts the standard perspective on two fundamental postulates of QM.

2.1.2.5. Adding a post-selection to the weakened interaction – Weak Values and Weak Measurements: Having established a new measurement paradigm -information gain without disturbance – it is fruitful to inquire whether this type of measurement reveals new values or properties. With weak-measurements (which involve adding a post-selection to this

ordinary – but weakened – von Neumann measurement), the measuring device registers a new value, the weak-value. As an indication of this, we insert a complete set of states $\{|\Psi_{\text{fin}}\rangle_j\}$ into the outcome of the weak interaction of §2.1.2.3 (i.e. the expectation value $\langle\hat{A}\rangle$):

$$\langle\hat{A}\rangle = \langle\Psi_{\text{in}}|\left[\sum_j |\Psi_{\text{fin}}\rangle_j \langle\Psi_{\text{fin}}|_j\right] \hat{A} |\Psi_{\text{in}}\rangle = \sum_j |\langle\Psi_{\text{fin}}|_j \Psi_{\text{in}}\rangle|^2 \frac{\langle\Psi_{\text{fin}}|_j \hat{A} |\Psi_{\text{in}}\rangle}{\langle\Psi_{\text{fin}}|_j \Psi_{\text{in}}\rangle} \quad (2.9)$$

If we interpret the states $|\Psi_{\text{fin}}\rangle_j$ as the outcomes of a final ideal-measurement on the system (i.e. a post-selection) then performing a weak-measurement (e.g. with $g_o \Delta Q_{\text{md}} \rightarrow 0$) during the intermediate time $t \in [t_{\text{in}}, t_{\text{fin}}]$, provides the coefficients for $|\langle\Psi_{\text{fin}}|_j \Psi_{\text{in}}\rangle|^2$ which gives the probabilities $Pr(j)$ for obtaining a pre-selection of $\langle\Psi_{\text{in}}|$ and a post-selection of $|\Psi_{\text{fin}}\rangle_j$. The intermediate weak-measurement does not disturb these states and the quantity $A_w(j) \equiv \frac{\langle\Psi_{\text{fin}}|_j \hat{A} |\Psi_{\text{in}}\rangle}{\langle\Psi_{\text{fin}}|_j \Psi_{\text{in}}\rangle}$ is the weak-value of \hat{A} given a particular final post-selection $\langle\Psi_{\text{fin}}|_j$. From $\langle\hat{A}\rangle = \sum_j Pr(j) A_w(j)$, one can think of $\langle\hat{A}\rangle$ for the whole ensemble as being constructed out of sub-ensembles of pre- and post-selected-states in which the weak-value is multiplied by a probability for a post-selected-state.

The weak-value arises naturally from a weakened measurement with post-selection: the final state of measuring device in the momentum representation becomes:

$$\begin{aligned} \langle P_{\text{md}} | \langle \Psi_{\text{fin}} | e^{-ig_o \hat{Q}_{\text{md}} \hat{A}} | \Psi_{\text{in}} \rangle | \Phi_{\text{in}}^{\text{MD}} \rangle &\approx \langle P_{\text{md}} | \langle \Psi_{\text{fin}} | 1 + ig_o \hat{Q}_{\text{md}} \hat{A} | \Psi_{\text{in}} \rangle | \Phi_{\text{in}}^{\text{MD}} \rangle \\ &\approx \langle P_{\text{md}} | \langle \Psi_{\text{fin}} | \Psi_{\text{in}} \rangle \{1 + ig_o \hat{Q} \frac{\langle \Psi_{\text{fin}} | \hat{A} | \Psi_{\text{in}} \rangle}{\langle \Psi_{\text{fin}} | \Psi_{\text{in}} \rangle}\} | \Phi_{\text{in}}^{\text{MD}} \rangle \\ &\approx \langle \Psi_{\text{fin}} | \Psi_{\text{in}} \rangle \langle P_{\text{md}} | e^{-ig_o \hat{Q} A_w} | \Phi_{\text{in}}^{\text{MD}} \rangle \\ &\rightarrow \langle \Psi_{\text{fin}} | \Psi_{\text{in}} \rangle \exp \left\{ -(P_{\text{md}} - g_o A_w)^2 \right\} \end{aligned} \quad (2.10)$$

where $A_w = \frac{\langle \Psi_{\text{fin}} | \hat{A} | \Psi_{\text{in}} \rangle}{\langle \Psi_{\text{fin}} | \Psi_{\text{in}} \rangle}$

The final state of the measuring device is almost un-entangled with the system; it is shifted by a very unusual quantity, the weak-value, A_w , which is not in general an eigenvalue of \hat{A} .

2.1.2.6. How the weak-value of a spin-1/2 can be 100: The weak-value for the spin-1/2 considered in §2.1.2 (which was confirmed experimentally for an analogous observable, the polarization [6]) is:

$$(\hat{\sigma}_{\xi=45^\circ})_w = \frac{\langle \uparrow_y | \frac{\hat{\sigma}_y + \hat{\sigma}_x}{\sqrt{2}} | \uparrow_x \rangle}{\langle \uparrow_y | \uparrow_x \rangle} = \frac{\{\langle \uparrow_y | \hat{\sigma}_y \rangle + \langle \hat{\sigma}_x | \uparrow_x \rangle\}}{\sqrt{2} \langle \uparrow_y | \uparrow_x \rangle} = \frac{\langle \uparrow_y | 1 + 1 | \uparrow_x \rangle}{\sqrt{2} \langle \uparrow_y | \uparrow_x \rangle} = \sqrt{2} \quad (2.11)$$

With a strong measurement, the component of spin $\hat{\sigma}_{\xi}$ is an eigenvalue, ± 1 , but the weak-value $(\hat{\sigma}_{\xi})_w = \sqrt{2}$ (outside the range of eigenvalues of $\hat{\sigma} \cdot \mathbf{n}$) Weak values further outside the spectrum can be obtained by post-selecting states which are more anti-parallel to the pre-selection: e.g. if we post-select the $+1$ eigenstate of $(\cos \alpha) \sigma_x + (\sin \alpha) \sigma_z$, then $(\hat{\sigma}_z)_w = g_o \tan \frac{\alpha}{2}$, yielding arbitrarily large values such as spin-100.

We have used such limited disturbance measurements to explore many paradoxes (see, e.g. [12, 19, 20]). A number of experiments have been performed to test the predictions made by weak-measurements and results have proven to be in very good agreement with theoretical predictions [4–8]. Since eigenvalues or expectation values can be **derived** from weak-values [3], we believe that the weak-value is indeed of fundamental importance in QM. In addition, the

weak-value is the relevant quantity for all generalized weak interactions with an environment, not just measurement interactions. The only requirement being that the 2-vectors, i.e. the pre- and post-selection, are not significantly disturbed by the environment.

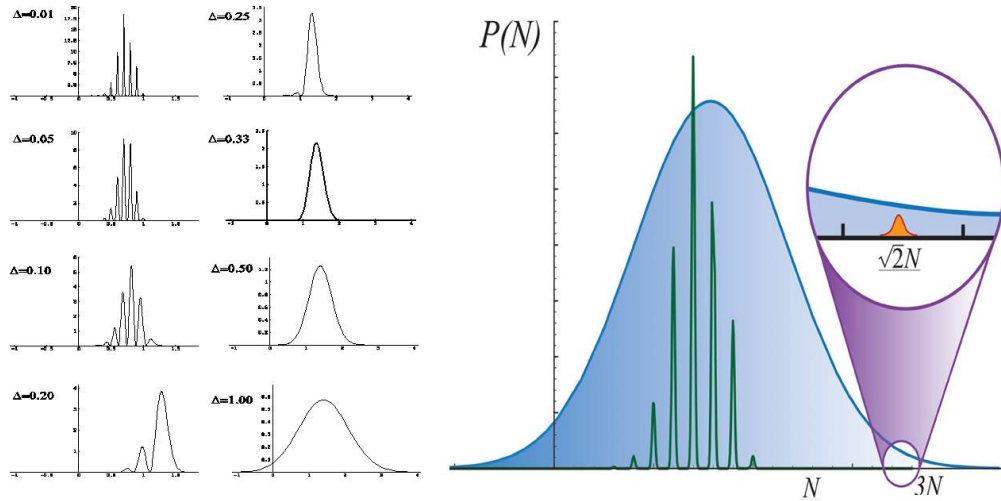


Figure 5. **Left figure:** “Measurement on a single system. Probability distribution of the pointer variable for the measurement of $A = (\sum_{i=1}^{20} (\sigma_i)_\xi)/20$ when the system of 20 spin- $\frac{1}{2}$ particles is pre-selected in the state $|\Psi_1\rangle = \prod_{i=1}^{20} |\uparrow_x\rangle_i$ and post-selected in the state $|\Psi_2\rangle = \prod_{i=1}^{20} |\uparrow_y\rangle_i$. While in the very strong measurements, $\Delta = 0.01 - 0.05$, the peaks of the distribution located at the eigenvalues, starting from $\Delta = 0.25$ there is essentially a single peak at the location of the weak value, $A_w = \sqrt{2}$.” from [17]; **Right figure:** Combining the core results from the left figure, we draw here the probability distributions for strong and weak measurements. Before the post-selection is performed, the spikes in the distribution (colored in green) represent the possible measurement outcomes (which are eigenvalues) for an ideal measurement. The wider curve (colored in blue) represents the probabilities for a weak measurement. After the post-selection is performed, a single peak is left (colored in red) way out in the tails at the “impossible” location of the weak value, $A_w = N\sqrt{2}$.

2.1.2.7. New approach to axiomatic structure of QM: A central theme is that the future can only be relevant to the present if there is room to write-off its “influence” as a mistake. As can be seen in figure 5, the probability to obtain the weak value as an error of the measuring device is greater than the probability to actually obtain the weak value. This is essential to preserve both causality and free will: following a measurement of \hat{A} , we can choose to either post-select the system, or measure \hat{A} again. If we post-select, we may interpret the result of the measurement of \hat{A} as a weak-value A_w ; if we re-measure \hat{A} , we may interpret the same result as an error. These two interpretations are consistent, for they apply to different ensembles. How we interpret a measured value depends on what else we choose to measure. We emphasize however that the weak value is not a random error. It is given by a simple heuristic expression; it is a highly predictable property; it always occurs whenever we obtain a given post-selection and any weakened interaction, and not just measurements, are sensitive to the weak value.

Traditionally, the uncertainty of quantum mechanics was interpreted to mean that nature is “capricious.” Using weak measurement we can derive the uncertainty from two axioms: 1) the future can be relevant to the present and 2) causality is maintained. That is, uncertainty is derived as a consequence of the consistency between causality and weak values: the amount of

uncertainty is exactly what is needed in order that the future can be relevant for the present, without violating causality, thus providing a new answer to the question “Why does God play dice?” In order to enrich nature with temporal non-locality, and yet preserve cause-effect relations, we must have indeterminacy.

2.1.3. TSQM led to new mathematics: Superoscillations We might ask the question “How is it that the measuring device indicates this answer?” As we mentioned previously, during the process of the measurement, the pointer and the system get entangled. Upon performing the post-selection on the system, the entanglement is eliminated. This leaves the measuring device pointer in a superposition, i.e. shifted by eigenvalues of the measured operator. There is constructive interference around the weak value $\sqrt{2}N$ and destructive interference everywhere else.

TSQM is a **re-formulation** of QM, and therefore it must be possible to view the novel effects from the traditional single-vector perspective. This is precisely what super-oscillations teach us. In summary, there are 2 ways to understand weak values:

- the measuring device is registering the weak value as a property of the **system** as characterized by TSQM
- the weak value is a result of a complex interference effect in the **measuring device**, i.e. a superoscillation; the system continues to be described with a single-vector pursuant to the standard approach to QM

A universally accepted truth in spectral analysis is that signals, be they space dependent, as in optical imaging, or time dependent, cannot have details on a scale shorter than the shortest wavelength or shortest time period of their Fourier components. The new subject called *superoscillations* seems to violate this principle [23–27]. This is one of many examples in which TSQM has led to new mathematics, simplifications in calculations, and stimulated discoveries in other fields (see, e.g. [30]).

2.2. Emergence and weak values

In §2.2.1, we shall see that when properly analyzed, what seemed to be a “whole inseparable system” can in fact be physically separated into distinct parts residing at different locations. This opens a new door on the relationship between wholes and parts. In §2.2.2, we review a new category of Gedanken-experiment which uncovers a loophole in the correspondence principle and provides a new perspective on the transition from quantum to classical. With respect to item 4, i.e. “origin of laws”, we show the transition is in fact dramatically more involved and it requires a complete revision of all our intuitions. Finally, in §2.2.3, because weak values follow a very different logical structure (e.g. the weak value of a product of observables is not equal to the product of their weak-values), we introduce even more dramatic categories of emergence.

2.2.1. Quantum Cheshire cat Another surprising effect originating from weak values is the ability to separate a system from its properties [12, 40, 41], as suggested by the Cheshire cat story: “Well! I’ve often seen a cat without a grin,” thought Alice; “but a grin without a cat! It’s the most curious thing I ever saw in all my life!”

The essential property of a quantum Cheshire Cat is that the cat itself is located in one beam path, while its grin is located in the other one: i.e. the object and its property are spatially separated (see figure 6). In our experiment, the neutron plays the role of the cat and the cat’s grin is represented by the neutron’s spin, and, we have indeed successfully performed the first experimental confirmation of the quantum Cheshire Cat. [41]

To be explicit, we will first calculate a variety of operators such as $\langle \hat{\Pi}_j \rangle \equiv |j\rangle\langle j|$, with $j = I, II$, which are the projections on the path which the neutron takes, namely $|I\rangle$, stands for path I

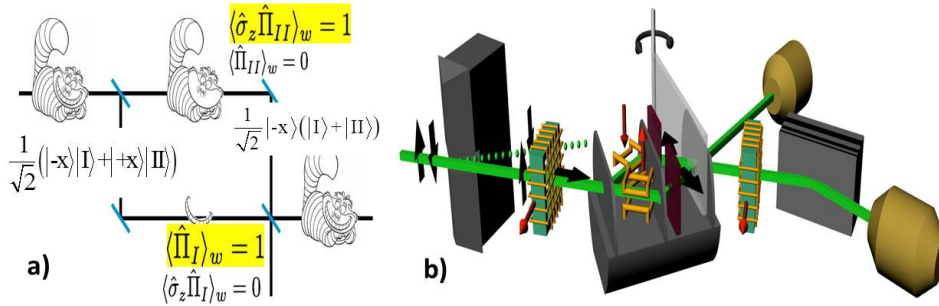


Figure 6. a) Artistic depiction of the quantum cheshire cat paradox: Inside the interferometer the cat travels along the upper beam path (path I), while its grin is located in the lower beampath (path II); b) Illustration of the experimental setup for the observation of a quantum Cheshire Cat in a neutron interferometer. [41]

and $|II\rangle$ for path II . We next consider operators corresponding to the cat's grin states which are given by the spin in the z -direction, namely $\hat{\Pi}_\uparrow$ the projector corresponding to $|\sigma_z = +1\rangle$ (grinning) and $\hat{\Pi}_\downarrow$, the projector corresponding to $|\sigma_z = -1\rangle$ (frowning). The pre-selected wave function for an individual particle is a superposition of the particle being in path I with a spin $+1$ in the x -direction superposed with the particle being in path II with a spin -1 in the x -direction:

$$|\Psi_{\text{in}}\rangle = \frac{1}{\sqrt{2}}|\sigma_x = +1\rangle|I\rangle + \frac{1}{\sqrt{2}}|\sigma_x = -1\rangle|II\rangle. \quad (2.12)$$

To make calculations easier and more intuitive, we re-write the pre-selection Eq. (2.12) in the z -basis as Eq. (2.13):

$$|\Psi_{\text{in}}\rangle = \frac{1}{\sqrt{2}}\left[|\sigma_z = +1\rangle + |\sigma_z = -1\rangle\right]|I\rangle + \frac{1}{\sqrt{2}}\left[|\sigma_z = +1\rangle - |\sigma_z = -1\rangle\right]|II\rangle. \quad (2.13)$$

In order to measure the Cheshire Cat, after we pre-select the ensemble, we will next perform weak measurements on both the number of neutrons in a given path along with the value of the spin in a given path. After these weak measurements, the ensemble is then post-selected in the final state:

$$|\Psi_{\text{fin}}\rangle = \frac{1}{\sqrt{2}}|\sigma_x = -1\rangle\left[|I\rangle + |II\rangle\right]. \quad (2.14)$$

That is, it is post-selected in a superposition of both paths with a -1 spin in the x -direction. Again, it will be convenient to re-express this in the z -basis:

$$\begin{aligned} |\Psi_{\text{fin}}\rangle &= \frac{1}{\sqrt{2}}\left[|\sigma_z = +1\rangle - |\sigma_z = -1\rangle\right]\left[|I\rangle + |II\rangle\right] \\ &= \frac{1}{\sqrt{2}}\left[|\sigma_z = +1\rangle - |\sigma_z = -1\rangle\right]|I\rangle + \frac{1}{\sqrt{2}}\left[|\sigma_z = +1\rangle - |\sigma_z = -1\rangle\right]|II\rangle. \end{aligned} \quad (2.15)$$

The ABL formula (2.2) tells us that if an ideal measurement is performed on Path I during the intermediate time, then, with certainty, the particle will not be found there. This can be seen intuitively: suppose we find the particle in Path I . The state is then $|\sigma_x = +1\rangle|I\rangle$ which is

orthogonal to the post-selection. Because this is a contradiction, we conclude that the particle must be found in Path II . Calculating with ABL (ignoring unitary evolutions) confirms this:

$$|\langle \Psi_{\text{fin}} | \hat{\Pi}_I | \Psi_{\text{in}} \rangle|^2 = \frac{1}{2} \left[\langle \sigma_z = +1 | + \langle \sigma_z = -1 | \right] \frac{1}{2} \left[| \sigma_z = +1 \rangle - | \sigma_z = -1 \rangle \right] = 0 \quad (2.16)$$

and

$$|\langle \Psi_{\text{fin}} | \hat{\Pi}_{II} | \Psi_{\text{in}} \rangle|^2 = \frac{1}{2} \left[\langle \sigma_z = +1 | - \langle \sigma_z = -1 | \right] \frac{1}{2} \left[| \sigma_z = +1 \rangle - | \sigma_z = -1 \rangle \right] = \frac{1}{2}. \quad (2.17)$$

Therefore, the probability that the particle will be found in path II is:

$$\frac{|\langle \Psi_{\text{fin}} | \hat{\Pi}_{II} | \Psi_{\text{in}} \rangle|^2}{|\langle \Psi_{\text{fin}} | \hat{\Pi}_I | \Psi_{\text{in}} \rangle|^2 + |\langle \Psi_{\text{fin}} | \hat{\Pi}_{II} | \Psi_{\text{in}} \rangle|^2} = \frac{\frac{1}{2}}{(0 + \frac{1}{2})} = 1 \quad (2.18)$$

and the probability that the particle will be found in path I is:

$$\frac{|\langle \Psi_{\text{fin}} | \hat{\Pi}_I | \Psi_{\text{in}} \rangle|^2}{|\langle \Psi_{\text{fin}} | \hat{\Pi}_I | \Psi_{\text{in}} \rangle|^2 + |\langle \Psi_{\text{fin}} | \hat{\Pi}_{II} | \Psi_{\text{in}} \rangle|^2} = \frac{0}{(0 + \frac{1}{2})} = 0. \quad (2.19)$$

For the spin, we can again use ABL to ascertain its state:

$$|\langle \Psi_{\text{fin}} | \hat{\Pi}_{\uparrow} | \Psi_{\text{in}} \rangle|^2 = \frac{1}{2} \left[\langle I | + \langle II | \right] \frac{1}{2} \left[| I \rangle + | II \rangle \right] = \frac{1}{2} \quad (2.20)$$

and

$$|\langle \Psi_{\text{fin}} | \hat{\Pi}_{\downarrow} | \Psi_{\text{in}} \rangle|^2 = \frac{1}{2} \left[\langle I | + \langle II | \right] \frac{1}{2} \left[| I \rangle - | II \rangle \right] = 0. \quad (2.21)$$

Therefore, the probability that the spin is $\sigma_z = +1$ is:

$$\frac{|\langle \Psi_{\text{fin}} | \hat{\Pi}_{\uparrow} | \Psi_{\text{in}} \rangle|^2}{|\langle \Psi_{\text{fin}} | \hat{\Pi}_{\uparrow} | \Psi_{\text{in}} \rangle|^2 + |\langle \Psi_{\text{fin}} | \hat{\Pi}_{\downarrow} | \Psi_{\text{in}} \rangle|^2} = \frac{\frac{1}{2}}{(\frac{1}{2} + 0)} = 1. \quad (2.22)$$

A theorem [9, 19, 32]³ states that if a single ideal measurement of an observable is performed between the pre- and post-selection, then if ABL asserts that the outcome is definite, then the weak value will be equal to this eigenvalue. Furthermore and more importantly, we may perform many weak measurements simultaneously on the same particle since they do not disturb each other. Therefore, all the counterfactual ABL statements can be measured (weakly) on the same particle and thereby simultaneously confirmed (to a certain extent). Using this theorem, we can **simultaneously** confirm Eq. (2.19), i.e. $\langle \hat{\Pi}_I \rangle_w = 0$ and Eq. (2.29), i.e. $\langle \hat{\Pi}_{II} \rangle_w = 1$. We can also simultaneously confirm Eq. (2.22), i.e. $\langle \sigma_z \rangle_w = \langle \hat{\Pi}_{\uparrow} - \hat{\Pi}_{\downarrow} \rangle_w = 1$ for all t . However, it is not the case that $\langle \sigma_z \rangle_w = 1$ along path II , i.e. where the particle is located. That is, the weak value of the spin in path II is

$$\langle \hat{\sigma}_z \hat{\Pi}_{II} \rangle_w = \langle \hat{\Pi}_{\uparrow} \hat{\Pi}_{II} \rangle_w - \langle \hat{\Pi}_{\downarrow} \hat{\Pi}_{II} \rangle_w = 0. \quad (2.23)$$

³ This also provides a direct link to the counterfactual statements (§2.1.2.1) because all counterfactual statements which claim that something occurs with certainty, and which can actually be experimentally verified by **separate** ideal-measurements, continue to remain true when tested by weak-measurements. However, given that weak-measurements do not disturb each other, all these statements can be measured **simultaneously**.

The reason for this is that in general, the weak value of a product of observables is not equal to the product of their weak values (even when the observables commute). Therefore, to more fully understand what is going on, we will be interested in the weak values of all products of projection operators. Using Fig. 7, we demonstrate how these weak values, Eq. (2.9), can be easily ascertained. First, we can neglect normalization since the normalization of the numerator and denominator of Eq. (2.9) cancel. The denominator of Eq. (2.9), i.e. the scalar product $\langle \Psi_{\text{fin}} | \Psi_{\text{in}} \rangle$, is 2. The numerator of Eq. (2.9), $\langle \Psi_{\text{fin}} | \hat{A} | \Psi_{\text{in}} \rangle$, is then calculated by referring to figure 7, wherein each “column” yields ± 1 :

$$\langle \hat{\Pi}_{\uparrow} \hat{\Pi}_I \rangle_w = \frac{1}{2} \quad (\text{column 1}) \quad (2.24)$$

$$\langle \hat{\Pi}_{\downarrow} \hat{\Pi}_I \rangle_w = -\frac{1}{2} \quad (\text{column 2}) \quad (2.25)$$

$$\langle \hat{\Pi}_{\uparrow} \hat{\Pi}_{II} \rangle_w = \frac{1}{2} \quad (\text{column 3}) \quad (2.26)$$

$$\langle \hat{\Pi}_{\downarrow} \hat{\Pi}_{II} \rangle_w = \frac{1}{2} \quad (\text{column 4}) \quad (2.27)$$

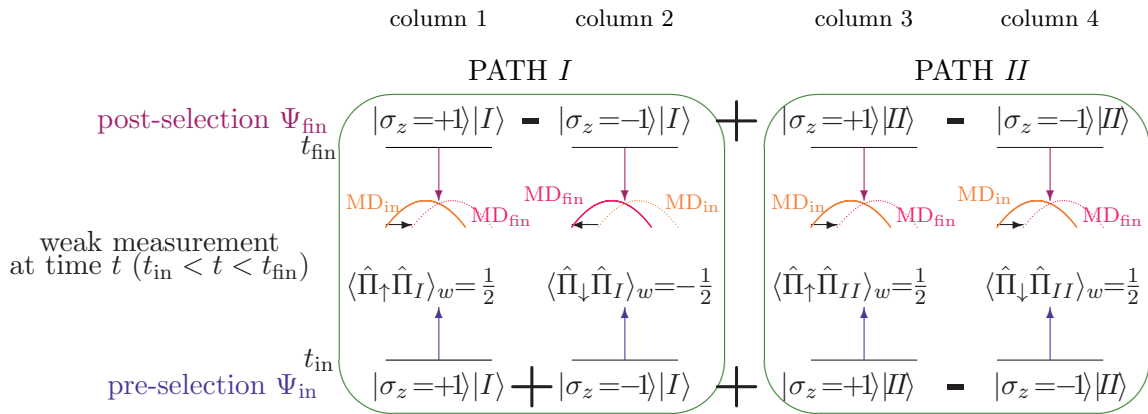


Figure 7. Graphical depiction of the Cheshire Cat calculations

We can now prove our main point, i.e. that the weak value of the spin in path I is

$$\langle \hat{\sigma}_z \hat{\Pi}_I \rangle_w = \langle \hat{\Pi}_{\uparrow} \hat{\Pi}_I \rangle_w - \langle \hat{\Pi}_{\downarrow} \hat{\Pi}_I \rangle_w = 1. \quad (2.28)$$

And if we now use an ensemble of N particles, we get the same results, only now with high precision: all N particles are (weakly) in path II , but there is no field there; there are no particles in path I , but there is a field equal to N spins there. Alice would say “Curiouser and curiouser.”

Lastly, we see that $\langle \hat{\Pi}_{\downarrow} \hat{\Pi}_I \rangle_w = -\frac{1}{2}$ is not only outside the spectrum, it is even a seemingly impossible negative number. Such predictions have been made before [12, 19, 32] and confirmed experimentally [41]. More importantly, though strange, these weak values form a logical and consistent pattern, and can provide intuition for the discovery of new phenomena. For example, instead of performing the full calculation using Eq. (2.9) to give eqs. 2.24-2.27, let us derive the results using this “weak” logic. The weak value of the occupation in path II , $\langle \hat{\Pi}_{II} \rangle_w$, i.e. not caring what is the value of the spin, is just the sum of Eq. (2.26) and Eq. (2.27),

$$\langle \hat{\Pi}_{\uparrow} \hat{\Pi}_{II} \rangle_w + \langle \hat{\Pi}_{\downarrow} \hat{\Pi}_{II} \rangle_w = \langle \hat{\Pi}_{II} \rangle_w = 1 \quad (2.29)$$

Using Eq. (2.23) and Eq. (2.29) we easily derive Eq. (2.26) and Eq. (2.27). Furthermore, the weak value of the occupation in path I , $\langle \hat{\Pi}_I \rangle_w$, i.e. not caring what is the value of the spin, is just the sum of Eq. (2.24) and Eq. (2.25):

$$\langle \hat{\Pi}_\uparrow \hat{\Pi}_I \rangle_w + \langle \hat{\Pi}_\downarrow \hat{\Pi}_I \rangle_w = \langle \hat{\Pi}_I \rangle_w = 0. \quad (2.30)$$

and therefore, we can deduce that $\langle \hat{\Pi}_\uparrow \hat{\Pi}_I \rangle_w = -\langle \hat{\Pi}_\downarrow \hat{\Pi}_I \rangle_w$. Finally, since there is just one particle, we know:

$$1 = \langle \sigma_z \rangle_w = \langle \hat{\Pi}_\uparrow - \hat{\Pi}_\downarrow \rangle_w = \langle \hat{\Pi}_\uparrow \hat{\Pi}_I \rangle_w + \langle \hat{\Pi}_\uparrow \hat{\Pi}_{II} \rangle_w - \langle \hat{\Pi}_\downarrow \hat{\Pi}_I \rangle_w - \langle \hat{\Pi}_\downarrow \hat{\Pi}_{II} \rangle_w \quad (2.31)$$

from which we can easily derive Eq. (2.24) and Eq. (2.25).

2.2.2. New perspectives on emergence and the correspondence principle It is generally believed that, similar to the way in which relativistic mechanics reduces to Newtonian mechanics when c becomes infinite, also QM “reduces” to classical mechanics in a limit specified by the correspondence principle (i.e. in the classical limit of $\hbar \rightarrow 0$). Similarly, the reduction of quantum optics to wave optics has been considered to be relatively simple. It is not so. As we show in [35], the classical limit of quantum optics is dramatically more involved and requires a fundamental revision of our intuitions. The revised intuitions can serve as a guide to finding novel quantum effects.

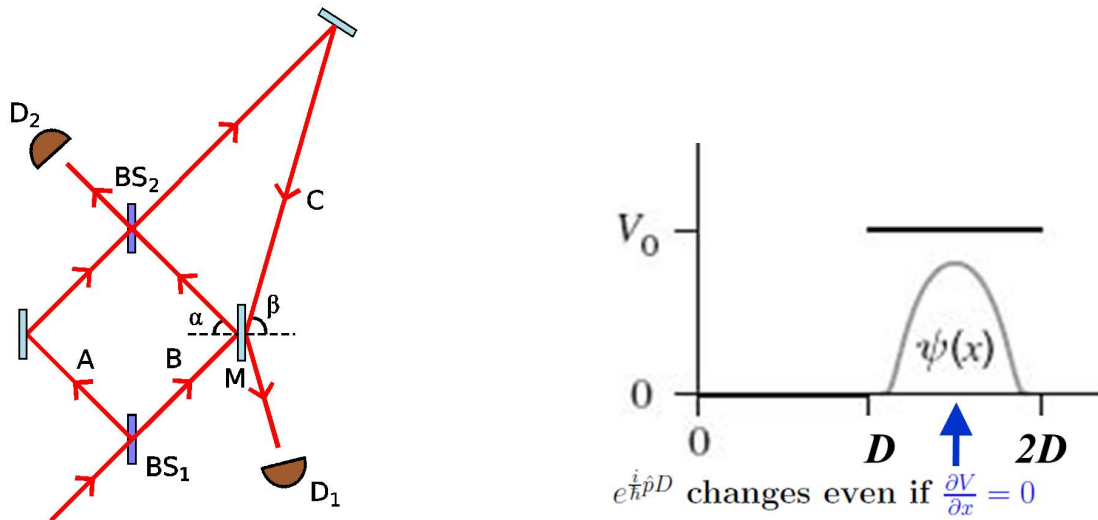


Figure 8. Left figure: Mach-Zehnder interferometer with one output beam reflected back onto the exterior side of mirror M . [35]; **Right figure:** A potential with 2 values and a wave-packet with support only in the interval $D < x < DL$.

Consider the interferometric experiment in Fig. 8. The three mirrors used in the experiment are perfectly reflecting, with mirror M being silvered on both sides. One of the (macroscopic) beams of light emerging from the interferometer is first reflected by a supplementary mirror onto the mirror M which then reflects it towards the detector. Mirror M receives therefore two momentum kicks, one from the light inside the interferometer and one from the beam that reflects on it from the outside. The quantum and classical calculations of course lead to the same result. The issue however is with the story each theory has to tell. Although the external beam has a shallower incidence angle than the inside beam, its intensity is much higher and the

momentum kick given by it is larger, hence the mirror is pushed inwards. Classically, the external beam plays the central role – one would be tempted to assume that quantum mechanically the photons that constitute this beam are the ones responsible for the inward push. Remarkably, this is not so! According to QM, it is the photons that end up in the other detector, D_2 , that kick M inside. Astonishingly, although they collide with the mirror only from the inside of the interferometer, they do not push the mirror outwards; rather they pull it in!

Since the story told by quantum mechanics is dramatically different from the classical one, this will have implications for our understanding of the relationship between different physical scales. It also serves as guide for intuition. E.g., suppose that, by a quantum fluctuation, we receive more than the average number of photons at detector D_1 . The classical intuition will lead us to expect that now the mirror will receive an even larger momentum inwards. Our QM analysis tells us differently since the effect is due to photons going towards D_2 and now there are fewer of them, the inward momentum will be smaller.

2.2.3. New aspects of non-locality and weak values We recently introduced [14] a family of new weak value effects which have a number of implications for EmQM. First of all, up until now, it was generally thought that any unusual connection between particles must be due to the fact that the particles are in an entangled state. In order to create the entangled state, originally the particles needed to be near each other. However, we have proposed situations in which the particles could have been prepared at space-like separated intervals, i.e. they simply knew nothing about each other. Nevertheless, they all exhibit non-classical anti-correlation, even though the particles are in a product state, i.e. there is no entanglement involved. This is the first such example that we know of.

2.3. Weak values and hidden variables

In [32, 33], we analyzed contextuality in terms of pre- and post-selection, and showed that it is possible to assign definite values to observables in a new and surprising way. Novel experimental aspects of contextuality can be demonstrated with weak measurements. We also proved that every PPS-paradox with definite predictions directly implies ‘quantum contextuality’ which is introduced as the analogue of contextuality at the level of quantum mechanics rather than at the level of hidden variable theories. Finally, we argued that certain results of these measurements (e.g. eccentric weak values outside the eigenvalue spectrum) cannot be explained by a ‘classical-like’ hidden variable theory and introduce new restrictions on hidden variable approaches.

2.4. Weak information

We recently introduced [15] a novel approach to information called “weak information.” We believe it is quintessentially unique to quantum mechanics and therefore a potential precursor and/or resource for quantum applications. Weak information has aided us in exploring the implications for our conventional measures and concepts of randomness. We applied weak information to quantum communication systems and were able to prove that it can be used to significantly enhance the security of practical quantum key distribution systems.

2.5. Dynamical nonlocality and the whole-part dialogue

Dynamical nonlocality [37] impacts the dialogue concerning the relationship between parts and wholes. Motivated by the AB non-locality and by weak measurements, we look for new manifestations of the dynamics of QM which are not predicted by the dynamics of classical mechanics. The key difference is that the equation of motion of QM exhibits a new kind of non-locality, which is best described by using modular variables. This is in contrast with Bell-inequality violations which follow from the Hilbert-space structure of QM (i.e. they are purely kinematic).

In [31] we applied this to the double-slit using single localized particles. From the perspective of a single particle, the central mystery is how does the localized particle passing through one of the slits sense whether or not the distant slit is open (closed), causing it to scatter (or not scatter) into a region of destructive interference? Feynman said “Nobody knows how it can be like that.” We showed that the Heisenberg picture leads us to a **physical** explanation for the different behavior of a *single* particle when the distant slit is open or closed. Instead of having a quantum wave that passes through all slits, we have a localized particle with *non-local* interactions with the other slits.

Interference of course depends on the relative phase α between different lumps: $\Psi_\alpha = \psi_L + e^{i\alpha}\psi_R$. Until $\psi_L(x, t)$ later overlaps with $\psi_R(x, t)$ (during interference), neither the current nor the probability densities depend on α . Observables which are simple functions of position and momentum are not sensitive to the relative phase between the “lumps,” suggesting that these dynamical variables are not the most appropriate to describe quantum interference phenomena. Operators that *are* sensitive to the relative phase are exponentials of the position and momentum. We call these “modular variables.” These operators translate the different “lumps” so that they overlap. Using $H = \frac{p^2}{2m} + V(x)$ and $e^{\frac{i}{\hbar}\hat{p}D}V(x)e^{-\frac{i}{\hbar}\hat{p}D} = V(x + D)$, we find **non-local** [2, 10] Heisenberg equations of motion for modular variables:

$$\frac{d}{dt}e^{\frac{i}{\hbar}\hat{p}D} = \frac{i}{\hbar}[H, e^{\frac{i}{\hbar}\hat{p}D}] = \frac{i}{\hbar}[V(x) - V(x + D)]e^{\frac{i}{\hbar}\hat{p}D} \quad (2.32)$$

with $e^{\frac{i}{\hbar}\hat{p}D}$ changing even when $\frac{\partial V}{\partial x} = 0$. This, essentially quantum phenomenon, has no classical counterpart (see Fig. 8). The classical equations of motion for any function $f(p)$ derives from the Poisson bracket:

$$\frac{df(p)}{dt} = \{f(p), H\}_{PB} = -\frac{\partial f}{\partial p} \frac{\partial H}{\partial x} + \underbrace{\frac{\partial f}{\partial x} \frac{\partial H}{\partial p}}_{=0} = 0 \quad (2.33)$$

i.e. $f(p)$ changes only if $\frac{\partial V}{\partial x} \neq 0$ at the particle’s location. We first proposed a way to experimentally observe dynamical non-locality (a method which requires weak measurements) [31] and this experiment has been successfully carried out [45].

2.6. Internal and external quantum frames-of-reference

Relevant to EmQM is our recent introduction of a new “internal” quantum frame-of-reference. E.g. ordinarily we consider a charged particle moving in the presence of fixed EM potentials. If however the charge is strong enough, it can influence these potentials. It will then see an effective, “private” potential, which rather than being determined by external charges only, is critically influenced by the particle under consideration- whereas other infinitesimal test charges, even if put at the very same location, see only the original, predetermined external “public” potential. The public potential can even vanish while the private potential remains non-zero. [36]

3. Generalizations of QM

While the alternative formulations of QM discussed in the previous section (§2) are equivalent to standard QM (i.e. they yield identical predictions), the underlying concepts are dramatically different. If new physics someday requires us to change QM, then these alternative formulations may provide a jumping-off point. We now present several generalizations of QM based on these fundamental conceptual shifts and discuss their implications for EmQM.

3.1. Reformulation of dynamics: Each moment a new universe

We review a generalization of QM suggested by TSQM [12,13,44] which addresses the “artificial” separation in all areas of theoretical physics between the kinematic and dynamical descriptions. We note [12,13,44] that the description of the time evolution given by QM does not appropriately represent multi-time-correlations (similar to EPR entanglement but instead of being between two particles in space, they are correlations for a single particle between two different *times*). Multi-time-correlations, however, can be represented by using TSQM. As a consequence, the general notion of time in QM is changed from the current conceptual framework which was inherited from classical mechanics, i.e.:

- 1) the universe is viewed as unique, and the objects which inhabit it just change their state in time. In this view, time is “empty,” it just propagates a state forward; the operators of the theory create the time evolution;

to a new conceptual framework in which:

- 2) each instant corresponds to a new pair of Hilbert spaces, (i.e., each instant is a new degree of freedom; in a sense, a new universe); instead of the operators creating the time evolution as in the previous approach, an entangled state (in time) “creates” the propagation: a whole new set of structures within time is able to “propagate” a quantum state forward in time.

This new approach has a number of useful qualities, e.g.: 1) the dynamics and kinematics can both be represented simultaneously in the same language, a single entangled vector (in many Hilbert spaces), and 2) a new, more fundamental complementarity between dynamics and kinematics is naturally introduced. This approach also leads to a new solution to the measurement problem which we model by uncertain Hamiltonians.

While we leave all details to other publications [12,13,44]), in brief, consider a spin-1/2 particle, initially polarized “up” along the z axis, and having the Hamiltonian $H = 0$. In this case the time evolution of the particle is trivial,

$$|\Psi(t)\rangle = \text{constant} = |\sigma_z = 1\rangle. \quad (3.34)$$

To see the deficiency in representing multi-time-correlations, we will consider an isomorphism between the correlations for a **single** particle at **multiple** instants of time and the correlations between **multiple** particles at a **single** instant of time. Therefore, we ask if we could prepare N spin-1/2 particles such that if we perform measurements on them at some time t_0 we would obtain the same information as we would obtain by measuring the state of the original particle at N different time moments, $t_1, t_2 \dots t_N$? Since the state of the original particle at all these moments is $|\sigma_z = 1\rangle$, one would suppose that this task can be accomplished by preparing the N particles each polarized “up” along the z axis, that is:

$$|\sigma_z = 1\rangle_1 |\sigma_z = 1\rangle_2 \dots |\sigma_z = 1\rangle_N. \quad (3.35)$$

But this mapping is not appropriate for many reasons. One reason is that the time evolution (3.34) contains subtle correlations (i.e. multi-time-correlations) and which do not appear in the state (3.35) but which can actually be measured. It is generally believed that since the particle is at every moment in a definite state of the z -spin component, the z -spin component is the only thing we know with certainty about the particle – all other spin components do not commute with σ_z and cannot thus be well-defined. However, there are *multi-time* variables whose values are known with certainty, given the evolution (3.34). For example, although the x spin component is not well defined when the spin is in the $|\sigma_z = 1\rangle$ state, we know that it

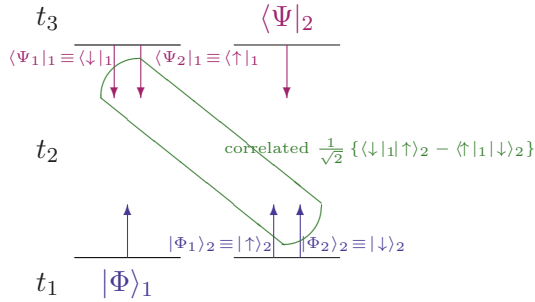


Figure 9. Particle 1 is correlated to the pre-selected state of particle 2.

is constant in time, since the Hamiltonian is zero. Thus, for example, the two-time observable $\sigma_x(t_4) - \sigma_x(t_2) = 0$ is definite ($t_2 < t_4$). However, there is no state of N spins such that

$$\hat{\sigma}_{\hat{n}}^1 = \hat{\sigma}_{\hat{n}}^2 = \dots = \hat{\sigma}_{\hat{n}}^N \quad (3.36)$$

for every direction \hat{n} as would be required for all the multi-time-correlations. At best, one may find a **two-particle state** (the EPR or Bell state) for which the spins are *anti-correlated* instead of correlated i.e. $\hat{\sigma}_{\hat{n}}^1 = -\hat{\sigma}_{\hat{n}}^2$. However, e.g., for 3 particles, only 2 of them can be completely anti-correlated, thus it cannot be extended to N particles.

Although a state of N spin 1/2 particles with complete correlations among all their spin components as required by Eq. (3.36) doesn't exist in the usual sense, there are pre-and-post-selected states with this property given by TSQM. By way of example (see figure 9), the post-selected state of particle 1 can be completely correlated with the pre-selected state of particle 2 as described by the state $\Phi = \frac{1}{\sqrt{2}} \{ \langle \downarrow |_1 | \uparrow \rangle_2 - \langle \uparrow |_1 | \downarrow \rangle_2 \}$. We are now able to preserve the single particle's multi-time-correlations by simply “stacking” the N spin-1/2 particles “one on top of the other” along the time axis (Fig. 10). As a result of the correlations between the pre-and-post-selected states, a verification measurement of $\hat{\sigma}_x(t_4) - \hat{\sigma}_x(t_2)$ (see left part of Fig. 10), will yield 0, i.e. perfect multi-time correlations because $\hat{\sigma}_x(t_2, \text{particle 2}) - \hat{\sigma}_x(t_2, \text{particle 1}) = 0$ (see right part of Fig. 10). When “stacked” onto the time axis, these correlations act like the identity operator and thus evolve the state forward, handing-off or effectively propagating a state from one moment to the next (although nothing is “really” propagating in this picture).

3.2. Destiny states: New solution to measurement problem

Up until now we have limited ourselves to the possibility of 2 boundary conditions which obtain their assignment due to selections made before and after a measurement. It is feasible and even suggestive to consider an extension of QM to include both a wavefunction arriving from the past and a second “destiny” wavefunction coming from the future which are determined by 2 boundary conditions, rather than a measurement and selection. This proposal could solve the issue of the “collapse” of the wavefunction in a new and more natural way: every time a measurement takes place and the possible measurement outcomes decohere, then the future boundary condition simply selects one out of many possible outcomes [12, 34].

3.3. Emergence and origin of laws

Usually, whole-part interaction is considered only to be more *methodologically* efficient. However, the participants of the EmQM conference inquired about the controversial notion of the *ontological* status of top-down or whole-part interactions. Unlike the more familiar bottom-up form of causality, the idea of top-down causality is in general unstable with respect

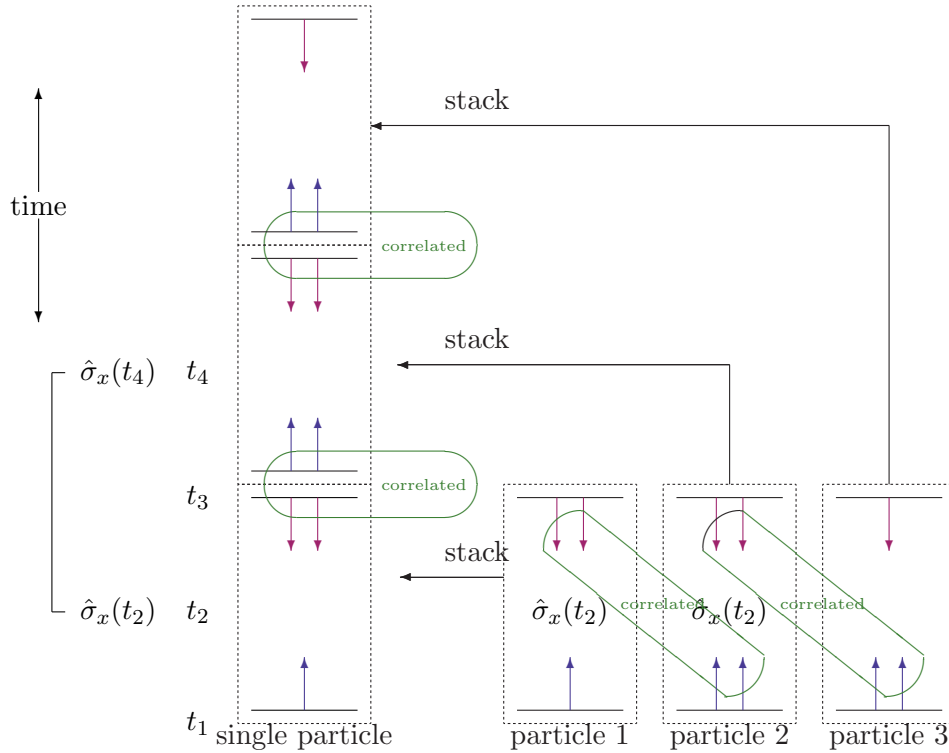


Figure 10. Measuring $\hat{\sigma}_x(t_4) - \hat{\sigma}_x(t_2)$ for the single spin-1/2 particle on the left ensures perfect multi-time-correlations because $\hat{\sigma}_x(t_2, \text{particle } 2) - \hat{\sigma}_x(t_2, \text{particle } 1) = 0$.

to fluctuations. Suggestions that quantum entanglement implies whole-part (top-down) interactions in atomic physics have been criticized by Shimony [42]:

Atoms thus exhibit form in Aristotle's sense, and even have the tendency to maintain this form, which phenomenologically is like his final cause. But the Aristotelian form is achieved by Democritean means – by interactions among the electrons and the nucleus, which leave these building blocks intact.

To conclude this article, we mention some very speculative ways to address this. First, a “destiny-state” suggests a form of top-down causality which is stable to fluctuations because post-selections are performed on the entire Universe and by definition no fluctuation exists outside the Universe.

Secondly, the dynamics-kinematics generalization (§3.1) [12, 13, 44] suggests a novel way to think about dynamical laws. One implication is the fact that although we may know the dynamics on a particular time-scale T , this doesn't mean that we know anything about the dynamics on a smaller time-scale: consider a superposition of unitary evolutions (using $e^{-iHT} = \{e^{-\frac{iHT}{N}}\}^N$):

$$\int g(\nu) e^{-iH(\nu)t} d\nu \rightarrow \int g(\nu) \{1 + iH(\nu)t\} d\nu \xrightarrow[\text{if } \int g(\nu) d\nu = 1]{\underbrace{\quad}} 1 + i \int g(\nu) H(\nu) t d\nu. \quad (3.37)$$

This theory is the same as the usual theory but with an effective Hamiltonian $H_{eff} = \int g(\nu) H(\nu) t d\nu$. The finer grained Hamiltonian can be expressed as a superposition of evolutions $e^{-\frac{iHT}{N}} = \sum_n \alpha_n e^{-\frac{i\beta_n HT}{N}}$, i.e. the Hamiltonian can be represented as a superposition of different laws given by pre- and post-selection [12, 34]. This would allow new types of non-locality in which

patterns at higher levels can set boundary conditions for lower levels. E.g. new potentials which become non-zero only when a certain number of particles are in a particular configuration so that new forms of non-locality come into being at different levels of complexity. Such new phenomena would not be inconsistent with existing experiments because at physical scales previously tested, per Eq. (3.37), these non-localities would destructively interfere with each other.

References

- [1] Aharonov Y, Bergmann P G and Lebowitz J L 1964 *Phys. Rev.* **134** B1410
- [2] Aharonov Y and Rohrlich D 2005 *Quantum paradoxes*, Weinheim; Cambridge: Wiley-VCH
- [3] Aharonov Y and Bohm D 1961 *Phys. Rev.* **122** 1649
- [4] Parks A D, Cullin D W and Stoudt D C 1979 *Proc. R. Soc. London, Ser. A* **454** 2997-3008
- [5] Pryde G J, O'Brien J L, White A G, Ralph T C and Wiseman H M 2005 *Phys. Rev. Lett.* **94** (22) 220405
- [6] Ritchie N W M, Story J G and Hulet R G 1991 *Phys. Rev. Lett.* **66** 1107
- [7] Ahnert S E and Payne M C 2004 *Phys. Rev. A* **70** (4) 042102
- [8] Wiseman H M 2002 *Phys. Rev. A* **65** (3) 032111 Part A
- [9] Aharonov Y and Vaidman L 1991 *J. Phys. A* **24** 2315
- [10] Aharonov Y, Pendelton H and Petersen A 1969 *Int. J. Theor. Phys.* **2** 213
- [11] Aharonov Y and Vaidman L 1990 *Phys. Rev. A* **41** 11
- [12] Tollaksen J 2001 *Quantum Reality and Nonlocal Aspects of Time* PhD thesis, Boston University
- [13] Aharonov Y, Popescu S, Tollaksen J and Vaidman L 2009 *Phys. Rev. A* **79** 052110
- [14] Aharonov Y, Colombo F, Popescu S, Sabadini I, Struppa D C and Tollaksen J, *Quantum violation of the pigeonhole principle*, forthcoming
- [15] Aharonov Y, Colombo F, Popescu S, Sabadini I, Struppa D C and Tollaksen J, *Weak Information*, forthcoming.
- [16] Nussinov S and Tollaksen J 2012 *Found. Phys.* **42** (9) 1186-1199
- [17] Aharonov Y and Vaidman L 2001 *Preprint* arXiv:quant-ph/0105101
- [18] Tollaksen J 2007 *J. Phys.: Conf. Series* **70** 012015 *Preprint* quant-ph/0703038
- [19] Aharonov Y, Botero A, Popescu S, Reznik B and Tollaksen J 2002 *Phys. Lett. A* **301** (3-4) 130-138
- [20] Brout R, Massar S, Parentani R, Popescu S and Spindel Ph 1995 *Phys. Rev. D* **52** 1119; Nussinov S and Tollaksen J 2008 *Phys. Rev. D* **78** 036007
- [21] Oreshkov and Brun 2005 *Phys. Rev. Lett.* **95** 110409
- [22] Tollaksen J 2007 *Non-statistical weak measurements* in: Quantum Information and Computation V, Ed E Donkor, A Pirich and H Brandt, *Proc of SPIE* **6573** (SPIE, Bellingham, WA), CID 6573-33
- [23] Aharonov Y, Colombo F, Sabadini I, Struppa D C and Tollaksen J 2011 *J. Phys. A* **44** 365304
- [24] Aharonov Y, Colombo F, Sabadini I, Struppa D C and Tollaksen J 2013 *Complex Anal. Oper. Theory* **7** 1299-1310
- [25] Aharonov Y, Colombo F, Sabadini I, Struppa D C and Tollaksen J 2013 *J. Math.: Pure Appl.* **99** (9) 165-173
- [26] Aharonov Y, Colombo F, Nussinov S, Sabadini I, Struppa D C and Tollaksen J 2012 *Proc. R. Soc. London, Ser. A* **468** 3587-3600
- [27] Aharonov Y, Colombo F, Sabadini I, Struppa D C and Tollaksen J 2013 *On superoscillations longevity: a windowed Fourier transform approach* in: Quantum Theory: A Two-Time Success Story, Springer, 313-325
- [28] Aharonov Y, Colombo F, Sabadini I, Struppa D C and Tollaksen J 2013 *Evolution of superoscillatory data*, *Preprint*
- [29] Aharonov Y, Popescu S and Tollaksen J 2010 *Physics Today* November, p.27
- [30] Nussinov S and Tollaksen J 2008 *Phys. Rev. D* **78** 036007
- [31] Tollaksen J, Aharonov Y, Casher A, Kaufherr T and Nussinov S 2010 *New J. Phys.* **12** 013023
- [32] Tollaksen J 2007 *J. Phys. A: Math. Gen.* **40** 9033-9066; *Preprint* quant-ph/0602226
- [33] Tollaksen J 2007 *J. Phys.: Conf. Ser.* **70** 012014
- [34] Aharonov Y and Tollaksen J 2009 *New insights on Time-Symmetry in Quantum Mechanics* in: Visions of Discovery, Ed. R Y Chiao, M L Cohen, A J Leggett, W D Phillips and C L Harper Jr, Cambridge: Cambridge University Press
- [35] Aharonov Y, Botero A, Nussinov S, Popescu S and Tollaksen J 2013 *New J. Phys.* **15** 093006
- [36] Kaufherr T, Aharonov Y, Nussinov S, Popescu S and Tollaksen J 2011 *Phys. Rev. A* **83** 052127
- [37] Tollaksen J and Aharonov Y 2009 *J. Phys.: Conf. Ser.* **196** 012006
- [38] Aharonov Y, Botero A, Popescu S, Reznik B and Tollaksen J 2002 *Phys. Lett. A* **301** (3-4) 130-138
- [39] Aharonov Y, Massar S, Popescu S, Tollaksen J and Vaidman L 1996 *Phys. Rev. Lett.* **77** 983-987
- [40] Aharonov Y, Popescu S, Rohrlich D and Skrzypczyk P 2013 *New J. Phys.* **15** 113015; *Preprint* arXiv:1202.0631

- [41] Denkmayr T, Geppert H, Sponar S, Lemmel H, Matzkin A, Tollaksen J and Hasegawa Y 2013 *Preprint* arXiv:1312.3775
- [42] Shimony A 1993 *The methodology of synthesis: parts and wholes in low-energy physics* in: Kelvin's Baltimore Lectures and Modern Theoretical Physics, Eds R Kargon and P Achinstein, 1987, M.I.T. Press, re-printed in *Search for a Naturalistic Worldview* by A Shimony, vol 2, Cambridge Univ. Press
- [43] Cho A 2011 *Science* **333** 690-693
- [44] Aharonov Y, Popescu S and Tollaksen J 2013 *Each instant of time a new universe* in: Quantum Theory: a Two-Time Success Story. Yakir Aharonov Festschrift. Eds D Struppa and J Tollaksen, Springer, Milan
- [45] Spence S E and Parks A D 2012 *Found. Phys.* **42** (6) 803-815; *Preprint* arXiv:1010.3289