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Learning to Teach Mathematics and to Analyze Teaching Effectiveness: Evidence from a Video- and Practice-Based Pre-Service Course

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Running Head: LEARNING TO TEACH AND ANALYZE TEACHING EFFECTIVENESS

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Abstract

Although emerging consensus exists that practice -based approaches to teacher preparation assist in closing the distance between university coursework and fieldwork experiences and in assuring that future teachers learn to implement innovative research-based instructional strategies, little empirical research has investigated teacher learning from this approach. This study examines the impact of a video and practice-based course on pre-service teachers' mathematics classroom practices and analysis of their own teaching. Two groups of elementary pre-service teachers participated in the study—one attended the course and one did not. Findings reveal that the course assisted participants in making student thinking visible and in pursuing it further during instruction and in conducting evidence-based analyses of their own teaching. Conclusions discuss the importance of teaching these skills systematically during teacher preparation.

Learning to Teach Mathematics and to Analyze Teaching Effectiveness:

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1. Introduction

This study investigates elementary pre-service teachers' (PSTs) learning from a video- and practice-based course. Under investigation is the development of PSTs' abilities to teach mathematics in ways that make student thinking visible and to analyze productively their own teaching. This work is exploratory in nature, but provides initial evidence of the potential impact of video- and practice-based teacher preparation on PSTs' learning.

Most teacher preparation programs include two main components: university coursework and fieldwork experiences. Often the latter have a greater impact on PSTs' learning because they occur in the settings where future teachers will exercise their profession and thus are perceived by them as more meaningful (Feiman-Nemser, 2001). Teacher preparation programs often find it challenging to build strong connections between coursework content and the practice of teaching. Many share the challenge of preparing teachers who both know about the most innovative, research-based approaches to teaching and learning and how to implement them effectively in the classroom.

We, like others, conceive of teaching as a cultural practice that is ingrained in the system of beliefs, value and practices of each culture, and is thus hard to change (Gallimore, 1996; Stigler & Hiebert, 1999, Stigler & Thompson, 2009; Author et al., 2005). Although fieldwork experiences are thought of as the settings where most teacher learning occurs, they are not always the places where the most up-to-date knowledge is situated (Feiman-Nemser, 2001; Feiman-Nemser & Buchmann, 1985). In addition, because teaching is represented in all its complexity in the school setting, PSTs may become overwhelmed and get distracted by features or elements of teaching that are not as important as others for their learning progress (Author, et al., 2007; Grossman, et al., 2009).

1.1 Practice-Based Teacher Preparation

For the reasons delineated above, teacher educators are exploring ways to optimize PST learning in university coursework to make it more practice-based. Recent efforts have focused on designing settings, representations of teaching, and activities that approximate practice and decompose it into parts that are more manageable for learning (Ball, Sleep, Boerst, & Bass, 2009; Grossman, et al., 2009). For example, instructional conversations during which teacher and students co-construct explanations are considered at the center of reform mathematics teaching and essential to develop students' mathematical proficiency. Yet, instructional conversation is a practice extremely hard to learn for PSTs because not everything can be planned in advance. The core of this practice is the ability to respond to particular students and to particular mathematical ideas in the moment (Leinhardt & Steele, 2005). Unpacking instructional conversations into more manageable routines thus become essential to make this practice accessible to novices. These routines include ways of questioning students to elicit their thinking or probing their thinking further, or ways of using students' mathematical ideas to highlight important conceptual connections. Other routines mathematics teacher educators have focused on are choral counting, strategy sharing, and problem solving (Hunter & Anthony, 2012; Lampert, Beasley, Ghousseini, Kazemi, & Franke, 2010).

University courses are ideal places to unpack complex teaching practices into routines that can be studied and analyzed. In these settings, PSTs can experiment with these practices outside the confines of real-time teaching, through cycles of planning, enactment, and reflection that include assisted rehearsals and role playing (Kazemi & Hubbard, 2008; Lampert & Graziani, 2009; Lampert, 2010). After this initial learning phase, PSTs can take these practices with them into the field and try them out with actual school children.

The project summarized here builds on this line of work. The course under investigation includes structured activities that provide opportunities for PSTs to study representations of mathematics teaching in the form of videos of teacher-student interactions and classroom lessons, transcripts, and sample of student work. These representations of teaching are used to decompose complex practices (Grossman, et al., 2009). The teacher educator guides PSTs' analyses by providing what Ghousseini & Sleep (2010) call "lenses for viewing." She unpacks complex practices to highlight and name their components. The course also includes opportunities for PSTs to role-play specific strategies with colleagues in the university setting and then practice these strategies with students in classrooms. Thus PSTs participating in the course are exposed to what Grossman et al. (2009) call "approximations of practice."

The course takes advantage of the parallel fieldwork experience PSTs complete during the same quarter of instruction, which is supervised by a different instructor (i.e., the coordinator of the program). This fieldwork placement provides PSTs with access to a classroom where they can complete the teaching assignments. Table 1 summarizes the course objectives, activities, and resources. As shown in the table, the course is structured into two parts. During the first part, activities focus on individual students and engage PSTs in the analysis of student mathematical thinking and then of strategies that make student thinking visible (such as effective questioning and the design and implementation of mathematical tasks that allow for students to express their mathematical reasoning (Stein, Smith, Henningsen, & Silver, 2009), and finally PSTs enact what they have learned in the context of an interview with a student. In the second part of the course, PSTs engage in the same sequence of activities, but this time, they focus on classroom lessons and enact strategies in the context of a mathematics lesson they teach to a small group of students.

INSERT TABLE 1 APPROX. HERE

1.2 Focus on Analysis

What distinguishes our approach to the preparation of mathematics teachers to that of the teacher educators cited above is perhaps a stronger emphasis on analysis, echoed in the project name: Learning to Learn from Mathematics Teaching (LLMT). In this project, we build on previous work on lesson analysis (Author et al., 2007) and on the idea of treating lessons like experiments (Hiebert, Morris, & Glass, 2003). PSTs learn to reason about the effectiveness of teaching by engaging in a process of analysis guided by a framework (i.e., the Lesson Analysis framework) that includes the following questions: (1) What is the main learning goal of this lesson (or lesson segment)? (2) Are the students making progress towards the learning goal? What evidence do you have that students are making progress? What evidence is missing? (3) Which specific activities and instructional strategies assisted students in making progress towards the learning goal? Why? How do you know? Which did not assist students? Why? How do you know? And (4) What alternative strategies do you think the teacher could use? And how would those strategies better assist students to make progress towards the learning goal?

By using this framework to analyze various artifacts of practice, PSTs learn to integrate teaching elements (i.e., the teacher, the students, and the content) and to reason about the effects of teaching on student learning (Author et al., 2007; Author et al., 2010). Specifically, the course emphasizes questions 2 and 3 of the Lesson Analysis Framework. Various course activities were designed to teach PSTs to attend carefully to student thinking and learning of specific mathematical ideas and identify evidence of student progress towards specific mathematical goals. Once evidence of progress or difficulties is identified, PSTs are asked to reason about strategies the teacher used that might have assisted students in learning or might not have been very successful.

The image of effective mathematics teaching that PSTs in this project are presented with and asked to implement in their classrooms is driven by the idea of creating opportunities during teaching that allow for teacher learning. These opportunities include multiple ways for students to make their thinking visible and instructional strategies that deliberately build on students' current understanding to move learning forward. An important distinction is made between knowing what final solution students arrived at and knowing the "how" and "why" they arrived at that final solution (Hiebert, et al., 1997; Carpenter, et al., 1999). PSTs are encouraged to go beyond superficial knowledge of students to investigate in deep ways students' understanding, difficulties, misconceptions, and dispositions. A strong emphasis is also placed on the importance of teaching mathematics for understanding to all children and on providing opportunities for students to engage in rigorous mathematical reasoning through cognitive effort (Hiebert & Grouws, 2007).

We consider the emphasis on analysis particularly important in order for PSTs to come to learn and understand mathematics teaching practices that are so common in their cultures that they become transparent and hard to see. We use videotapes of teaching episodes (see Table 1 for a list of video resources we use) to slow down the teaching process, highlight student thinking, mathematical ideas, and the interrelation between teachers' decisions and student learning. Such analyses often reveal the limits of teacher-centered practices or practices relying solely on procedural knowledge. When analyzing student-focused, conceptually-driven practices, the process of analysis develops PSTs' abilities to reason about these practices in terms of their impact on student learning and to question their effectiveness. This process allows PSTs to appropriate these practices with understanding as opposed to emulating them mechanically without thinking (Authors, 2012).

In sum, our course combines activities that allow PSTs to study and analyze teaching and activities that provide opportunities to practice important routines for teaching mathematics for understanding. The emphasis on analysis is evident in the multiple opportunities we have designed for PSTs to analyze both their own teaching and the teaching performed by others (See Table 1). Previous work has outlined more in detail the orientations and skills future teachers need in order to develop into practitioners who see themselves as learners and possess beginning analysis abilities (Author et al., 2010).

Through this process of analysis and reflection, PSTs are also given opportunities to work collaboratively (Author et al, in press). They co-construct analyses of student learning, discuss effectiveness of teaching strategies, plan lessons, review student work together, and share artifacts of their teaching (including transcripts and videos). We designed these opportunities with the goal of developing future teachers' abilities to talk about teaching with their colleagues and engage in the disciplined analysis of the work of teaching. This goal is important if we agree that preparing teachers involves helping them not only to acquire content and pedagogical knowledge, but also to see their teaching as something that can be studied and questioned and to develop an inquiry stance (Cochran-Smith & Lytle, 2001; Jaworski, 2006). The knowledge gained through that inquiry process can be shared with colleagues and contribute to the knowledge base of the teaching profession (Stigler & Thompson, 2009).

Existing Evidence on Practice-Based and Analysis-Focused Teacher Preparation

Most existing work on practice-based and analysis-focused teacher preparation has concentrated on developing conceptual frameworks (Ball & Cohen, 1999; Ball, et. al., 2009; Feinam-Nemser, 2001; Grossman, et al., 2009; Hiebert, Morris, & Glass, 2003; Hiebert, Morris, Berk, & Jansen, 2007; Kazemi , Lampert, & Ghousseini, 2007; Lampert, 2010; Rowland, 2008) and on describing and detailing the activities, strategies, and roles teacher educators have designed to engage PSTs in learning in, from, and for practice (Ghousseini, 2009; Ghousseini & Sleep, 2010; Lampert, et. al, 2010).

Little is known about what PSTs learn from these experiences. A few studies have reported on PSTs' ability to consider evidence of student learning when reflecting on the success of a lesson. For example, Spitzer, Phelps, Beyers, Johnson, and Sieminski (2010) studied the effects of a course designed to improve PSTs' ability to identify and evaluate evidence of student-learning. Study findings showed that the course, using lesson transcripts and sample student responses to mathematical tasks to develop PSTs' analysis skills, supported PSTs' ability to consider evidence of student learning to assess the effectiveness of a lesson and to discount student responses that were irrelevant to a specified learning goal. Similarly, in a study conducted by Bartell, Webel, Bowen, and Dyson (2012), PSTs' ability to analyze evidence of student learning improved as a result of a video-enhanced intervention. Specifically, PSTs learned to discount superficial conceptual features and procedural knowledge as evidence of student conceptual understanding. Data on PSTs' content knowledge also revealed that content knowledge in itself is not sufficient to carry out evidence-based analyses of learning; rather interventions targeting specifically this ability are necessary.

Other studies have described interventions (mostly video-based and/or on-line) that were successful at facilitating pre-service teacher learning to notice important elements of instruction, such as student thinking. These were carried out in various countries demonstrating the wide-spread focus on analysis as an important component of teacher preparation (Alsawaie and Alghazo, 2010; Fernandez, Llinares, and Valls, 2012; Huppertz, Massler, and Ploetzner, 2005; Star and Strickland, 2008; Stockero, 2008; van Es and Sherin, 2002). Rowland and colleagues have taken a slightly different approach by using the lens of mathematics content knowledge to guide teachers' systematic reflection on practice. They developed a tool, the Knowledge Quartet, that observers such as mentor teachers or supervisors can use to review and discuss lessons with PSTs. The framework assists PSTs and teacher educators in identifying mathematical content knowledge in the act of teaching and in developing such knowledge through reflection on teaching (Rowland, 2008; Turner & Rowland, 2008). Similarly, Pang successfully used video-based cases that structured PSTs' learning through analysis focused on the mathematical aspects of the lessons (Pang, 2011).

1.3 Contributions of this Study and its Larger Context

While the studies mentioned above highlight interventions that have successfully facilitated PSTs' analysis skills, little is known about the relation between PSTs' abilities to analyze teaching performed by *others* and their abilities to analyze their *own* teaching. Even less is known about the relationship between PSTs' ability to analyze teaching and the ability to teach in ways that make student thinking visible and thus make learning from one's own teaching possible.

This study is part of a larger project that investigates the impact of the LLMT approach on PSTs' abilities to analyze teaching and teach in ways that allow for systematic analysis. This larger project includes three phases: (1) a beginning phase during which PST learning was investigated through the study of PSTs' group conversations during video-based course activities (Authors at el., 2012) and through a pre/post test study of PSTs' abilities to reflect on a videotaped lesson taught by another teacher (Author et al., 2011); (2) an intermediate phase, in which PST learning is investigated through the study of PSTs' abilities to teach in ways that allow for systematic analysis of teaching and to analyze their practices; and, (3) a final phase, funded by the National Science Foundation, during which PST learning is investigated by experimentally studying the impact of the LLMT approach both short-term, on analysis abilities and beginning practices, and long-term (for three years after graduation) on PSTs' teaching and professional practices once they find a job. This manuscript reports on an exploratory study conducted during the intermediate phase of the project. The study focuses on 12 PSTs, who participated in the Learning to Learn from Mathematics Teaching (LLMT course. These are compared to a group of 12 PSTs who did not participate in the course (i.e., the non-LLMT participants). During the first phase of the larger project, we investigated LLMT participants' ability to analyze a lesson taught by others through a video-based analysis task they completed prior to and at completion of the course and found statistically significant improvements over time (Author et al., 2011). The study we summarize here seeks to investigate PSTs' ability to a) teach in ways that make student thinking visible and builds on and to b) analyze their own teaching. We will first describe the study research questions and then discuss the study methods and findings.

1.4 Research Questions

This study answers the following research questions:

(1) Can a video-based course, aimed at developing knowledge and skills for learning from mathematics teaching, impact PSTs' initial classroom practice? Specifically, can PSTs who took part in the course adopt teaching practices that make student thinking visible and build on student thinking, to a greater extent than PSTs who did not participate in the course?

(2) Can this course impact PSTs' analyses of their own classroom practices? Specifically, can PSTs, who took part in the course, use evidence of student thinking and learning to analyze the effectiveness of their own teaching to a greater extent than PSTs who did not participate in the course? And

(3) What is the relationship between the ability to teach in ways that make student thinking visible and the ability to use evidence of student thinking and learning to assess the effectiveness of teaching? Specifically, can PSTs who make student thinking visible during instruction also use evidence of student thinking during analysis or are these two separate skills? Are there group differences in this respect?

2. Methods

2.1 Participants and Data Sources

This study investigates the teaching practices and analysis abilities of two groups of PSTs enrolled in a 5th-year post-bachelor elementary teacher preparation program at a public university in the United States. The first group was enrolled in the program in 2006/07 and constitutes the comparison group for this study. The second group was enrolled in the program in 2008/09 and serves as the treatment group. The program and the characteristics of the PSTs' background remained virtually the same from 06/07 to 08/09 (i.e., the large majority of PSTs has a BA in a discipline from the social sciences or humanities, an undergraduate GPA of at least 3.0, and very limited teaching experience). The only difference was that in 08/09, the course intended to support the fieldwork component of the program, targeting general topics such as how to establish a good relationship with the mentor teacher, classroom management, and multiple intelligences, was substituted with the LLMT course. The program coordinator continued to support PSTs in their fieldwork by managing their schedules, communicating with the mentor teachers, and guiding PSTs in developing productive relationships at their school site, but her contact with the PSTs was greatly reduced to accommodate the LLMT course, which was taught by a different instructor. The courses were offered during the first quarter of the teacher preparation program during both years. During both years, PSTs also completed at the beginning of their second quarter (approximately 4-5 months into the program and after they attended the course in their first quarter) the Performance Assessment for California Teachers – Teaching Event (PACT-TE). The PACT-TE consists of a portfolio that includes five tasks: (1) context for learning; (2) planning instruction and assessment; (3) instructing students and supporting learning; (4) assessing student learning; and (5) reflecting on teaching and learning. For the purpose of this study, we selected material PSTs turned in for task 3. Specifically, we reviewed video clips of a lesson PSTs taught and guided commentaries they wrote about the clip(s).

Although the specific wording of the directions varied slightly, both years, PSTs were given the following prompt about videotaping their instruction:

> Provide one or two video clips of no more than fifteen minutes total. Select clip(s) that demonstrate how you engage students in understanding mathematical concepts and in participating in mathematical discourse. (You may select conceptual understanding either as the primary focus of instruction or integrate it with the development of your students' understanding of a computation or procedure.) The clip(s) should include

interactions among you and your students and your responses to student comments, questions, and needs.

Although the video clip(s) only allow us access to approximately 15 minutes of a lesson, we felt that was sufficient to uncover discourse routines we were interested in. The prompt clearly pointed attention to student participation in discourse and teacher-student interactions. In addition, these 15 minute clip(s) were chosen by PSTs to represent their best practices, thus we felt they exemplified well what PSTs considered their best efforts to include student participation in classroom discourse. PACT also includes a reflection task, but prompts changed from 06/07 to 08/09 so we were not able to use that task to compare groups; instead we used commentaries PSTs wrote about the lesson they turned in with the video clip(s). Prompts for the commentary remained the same across years. In addition, prompts that guided the commentary referred specifically to the video clip(s), thus encouraging PSTs to discuss evidence that the PACT scorer could see in the video. This also made it easier for us as researchers to consider PSTs' commentaries in the context of the teaching shown in the clip(s). Two prompts that focused on the effects of teaching on student learning and on future instructional decisions were chosen for analysis as they directly linked to the knowledge and abilities in which we were interested:

1. Describe the strategies you used to monitor student learning during the learning task shown on the video clip(s). Cite one or two examples of what students said and/or did in the video clip(s) or in assessments related to the lesson that indicated their progress toward accomplishing the lesson's learning objectives.
2. Reflect on the learning that resulted from the experiences featured in the video clip(s). Explain how, in your subsequent planning and teaching, successes were built upon and missed opportunities were addressed.

Each year, 80-90 students enroll in the elementary teacher preparation program at the university at which we conducted the study. One cohort of 30 students from 08/09 participated in the LLMT study. Using PACT total scores, we selected 12 participants from this cohort, and 12 participants from students enrolled in the program in 06/07. PACT includes a series of 4-point rubrics that are used to score PSTs' PACT portfolios. Three rubrics are used to score planning, two to score instruction, three to score assessment, two to score reflection, and two to score academic language. The maximum total number of points that each PST can obtain is 48, and in order to pass, a score of at least 2 must be obtained on each rubric. Detailed information about PACT and its scoring can be found at http://www.pacttpa.org/. To study teacher learning across various levels of knowledge and abilities, we used PACT total scores from each group to randomly select 4 participants who performed at 1 SD above the mean (scores ranged from 33 to 41), 4 with average performance (scores from 26 to 29), and 4 with performance at 1 SD below the mean (scores ranged from 16 to 22).

2.2 Coding PSTs' Teaching Videos

Because PSTs at our institution videotape themselves for PACT during the first month of their student teaching experience, it is fair to say that what we studied is the very beginning of their teaching careers. While one could argue that it is too early to measure their practice, we believe that initial practice has the potential to place PSTs on different learning trajectories. Others have argued that where teachers are in their practices at a certain point in time impacts where they will go next (Blanton, Berenson, & Norwood, 2001; Franke, Carpenter, Levi, & Fennema, 2001; Steinberg, Empson, & Carpenter, 2004).

To develop our coding system, we drew on existing frameworks that focus on classroom discourse or include it as one of the coding dimensions. Specifically, we reviewed the Math-

Talk Community framework developed by Hufferd-Ackles, Fuson, & Sherin (2004), the Quality of Instruction instrument of the Learning Mathematics for Teaching project (Learning Mathematics for Teaching, 2006) and the codes for quality of instruction developed as part of the Capturing Teacher Knowledge project by Kersting and colleagues (Kersting, et al., 2012). In addition, we reviewed a coding scheme developed by Sherin & van Es (2009) to capture teacher learning from participation in a video club. Drawing from these frameworks and in line with our course and our research questions, we developed codes that capture the extent to which student thinking was made visible during instruction and pursued by the teacher. Our classroom discourse code include three levels of sophistication: (1) Low sophistication: Student thinking only minimally visible, (2) Medium sophistication: Student thinking made visible, and (3) High sophistication: Student thinking made visible and pursued. Table 2 includes definitions of each level.

INSERT TABLE 2 APPROX. HERE

All 24 videos were rated by two independent observers, one of which was blind to group membership. Each video was rated as belonging to one of the coding categories described above. Inter-rater reliability, calculated as percentage of agreement between the two scorers, was equal to 83.3%. Disagreements were resolved through discussion.

2.3 Coding PSTs' Instruction Commentaries

To investigate PST's analyses of their own teaching, we reviewed their responses to the prompts reported above and used both a top-down and bottom-up approach (Strauss & Corbin, 1998; Miles & Huberman, 1994) to develop coding categories that describe the variations we observed in instruction commentaries. Because of the focus of the LLMT course, we were particularly interested in the extent to which PSTs utilize evidence of student learning to assess the effectiveness of their teaching and to identify missed opportunities. A prior study by Spitzer and colleagues found this skill particularly challenging for PSTs. PSTs have difficulty selecting evidence of student progress towards a specific learning goal and using that evidence to assess the effectiveness of a teacher's instructional strategies (Spitzer, et. al., 2010). Thus in our project, taking into consideration the learning goal that our participants stated for the lesson they videotaped, we looked at whether PSTs selected evidence linked to students' progress toward the lesson learning goal to discuss successes and missed opportunities. We found that some PSTs cited students' progress toward the lesson learning goals as evidence of their teaching effectiveness while others, as in Spitzer and colleagues' data, cited decision they made and instructional strategies they used effectively as evidence that students had made progress towards the learning goal of the lesson. Thus, these PSTs used what they thought they did well as evidence that their students learned. We also drew from the work of van Es & Sherin (2008) to look at the level of specificity of the evidence cited. Again, we found differences among commentaries in the extent to which PSTs referred to specific mathematical ideas or procedures made visible during instruction that provided evidence that students had made progress toward the learning goals. Our final coding scheme captured both the focus of the evidence and the level of specificity according to the following three levels of sophistication: (1) Low sophistication: teacher focused, (2) Medium sophistication: student focused, general, weakly or not linked to learning goals, and (3) High sophistication: student focused, specific, and linked to lesson learning goals.

We coded the use of evidence to discuss successes and missed opportunities separately for each participant. While missed opportunities were discussed in response to prompt 2 only, PSTs discussed successes both in response to prompt 1 and 2. We thus reviewed both responses to assess the evidence related to teaching successes. Two research assistants, blind to group membership, scored all commentaries and reached a satisfactory level of reliability (=.85). Disagreements were resolved through discussion. Table 3 includes a description of the coding categories for both successes and missed opportunities.

INSERT TABLE 3 APPROX. HERE

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3. Findings

3.1 Teaching Videos: Can PSTs Make Student Thinking Visible and Pursue Student Thinking during Instruction?

Table 4 summarizes findings related to the extent to which PSTs made student thinking visible and built on it during instruction. PSTs who attended the LLMT teaching course on average made student thinking more visible during instruction and began to pursue and build on student thinking in the midst of instruction. All LLMT participants, with the exception of one, were able to make student thinking visible during their lessons, and eight of them were also able to probe student thinking further and make it an important part of their lessons. The following sample interaction is drawn from a fourth-grade lesson on the addition of fractions with unlike denominators. The students provide solution steps as the PST writes on the board. This video was coded as medium in sophistication because although students are asked to explain their solution process, the PST is looking for a pre-determined answer and does not probe student thinking further.

PST: So we have 3/4 and 4/12. Are these denominators the same? No, so we have to change them to like fractions. So we're going to find the multiples of 4 and 12. What are the multiples of 4? 4 - What are the multiples of 4 everyone?

SS: 4, 8, 12.

PST: Can we stop there?

S: Yes.

PST: So we take 12, and what do we do with the 12?

S: You make 3/4 into a fraction that has 12 as the denominator, and this is how you do it. You go, "what times 4 equals 12?" That's 3. You multiply the top by 3 and you get 9/12.

PST: Okay, so here's your 9/12. Can we add 9/12 and 4/12 then?

SS: Yes.

PST: So what would that give us?

S: 13/12.

PST: So we have 13/12. What kind of fraction is this? Tina?

S: Improper.

PST: Improper. So what do we have to do?

SS: Change it to a mixed number

PST: Change it to a mixed number. So how do we change it to a mixed number?

S: You divide 12 into 13. You do 1, minus 12 is 1.

PST: And how do we write the remainder as a fraction? 1/12. So thumbs up if you got 1 and 1/12.

This type of interaction is different from that of lessons in which student thinking is not only made visible, but it is also built upon. The following excerpt is drawn from a third-grade lesson on developing division concepts. The class is working on partitioning using the overhead projector. A student uses base-ten blocks to show the division problem 60÷3 = 20. Prior to the interaction reported below, a classmate working at the overhead made two groups of base-ten blocks instead of three (see figure 1, part a). Nicholas was called to the front of the room by the teacher to fix the error displayed on the overhead. Nicholas approaches the overhead and takes one ten from the group of three tens on the right side and another ten from the group of three tens on the left side to form three groups of twenty (see figure 1, part b).

PST: Very good, we do have three groups in here. Now Nicholas, I noticed that you took one of your tens from this group (*PST points to the group of tens on the left)* and one of your tens from this group (*PST points to the group of tens on the right).* Can you tell me why you did that?

Nicholas: 'Cause... it was in three groups?

PST: Ok, um... if you started off this way (*PST repositions the three groups of two tens back into two groups of three tens)*, why didn't you just take two from here? *(PST moves two tens from the group of thirty on the right to form a center group. So on the overhead, there is one group of ten, one group of two tens, and one group of three tens displayed).*

Nicholas: 'Cause, then, I'd have to take this one all the way, too, *Nicholas moves one ten from the group of thirty on the right to the group of only one ten on the right to make three equal groups of twenty.*

PST: Why would you have to do that?

Nicholas: 'Cause then, this would belong over here and this would belong over here (*Nicholas is moving the base ten blocks on the overhead but the image is not visible in the video),* so I put it right there.

PST: Ok, so if we had left them and taken them like this (*PST again moves the base ten blocks into three unequal groups: one group of ten, one group of two tens, and a group of three tens),* are all of our groups equal right now? Thumbs up if yes, thumbs down if no. (*Camera zooms out and students visible have their thumbs down.)* Are they all equal right now? They're not equal right now, right? So how could we make them equal? Nicholas, go ahead and show them again.

In this lesson, the PST builds on student thinking as the lesson progresses. When a student error occurs, she publicly draws attention to the error to clarify an important mathematical concept in division for all of the students in the class. In addition, even when correct solution strategies are shared, she continues to probe students' thinking to make explicit the reasoning behind the solution strategy. On the contrary, almost half of the non-LLMT participants (N=5) made student thinking only minimally visible in their lessons, and only two were able to probe student thinking further. The following excerpt, drawn from a fifth-grade lesson, exemplifies teacher-student interactions that made student thinking only minimally visible. In this lesson, students are asked to provide a rule to explain the relationship between input and output values within a function table.

PST: 2, so we put in 4 and we got 2. Can you please graph those under input and output. Can anybody make a prediction already?

SS: I already did.

PST: Hold up your white boards if you already have a prediction. Hold up your white board really high if you already have a prediction. (*Students write their answer on individual white boards and show their answers to the PST. PST checks answers on white boards.)*

S: What if we already did it?

PST: I already saw yours; it's fine. I already saw it.

SS: You already saw mine.

PST: Yes, I already saw yours too.

SS: You saw mine.

PST: Okay, you know, we have a lot of guesses on our rule. Let's see if they’re right. Everybody, what is your hypothesis? What is your prediction?

SS: N-2

PST: Function box, can you please send out your rule, and let us see if we are right? N-2. Ding!

SS: Yea, WHOOO!

PST: Thank you function box.

In this lesson, students are only asked to provide the final solution to problems. The teacher response is limited to an assessment of the accuracy of the answer with no further discussion. The majority of the interaction is directed from the PST to the students, while students have little opportunity to contribute to the lesson content.

INSERT TABLE 4 APPROX. HERE

3.2 Instructional Commentaries: Can PSTs Use Evidence of Student Thinking and Learning to Analyze the Effectiveness of Their Own Teaching?

The tables below summarize findings related to PSTs' ability to use evidence of student thinking and learning to analyze the effectiveness of their teaching. Overall, LLMT participants demonstrated more sophisticated skills. Table 5 reports findings for commentaries on teaching successes. Eight of the 12 LLMT participants gave specific examples of student mathematics thinking or learning as evidence of their teaching success. The following excerpt exemplifies this type of commentary:

> There were a few comments that reflected a more complex understanding. The first comment, by student 2, came about while we were reviewing the first slide of how to count the square units to find the area. The class counted out the 12 square units as a class and I posed the question: "Then what is the area of the figure?" When I called on student 2 she did not say twelve but rather responded with, "Well you could do something similar like 4 x 3 and then that would give you 12." She had come up with the formula for finding area by recognizing the fact family of 3, 4, and 12. Student 2 is a student who generally struggles in math, and has trouble conceptualizing new ideas, so for her to come up with this somewhat abstract thought was a pivotal point in the lesson. This showed me that the material was in fact being presented in a way that made student understanding possible.

Four LLMT participants provided evidence that was focused on student learning, but was general and not, or only weakly, linked to the mathematical learning goals of the lesson, as in the following sample excerpt:

> The same students always had their hands raised to answer. I called on a couple people who did not have their hand raised to see if they understood the concept. Students gave articulate rules and answers that some thought were too long to write. At the end of the lesson, I asked the students to show me whether they understood the lesson or not by giving me a thumbs up, thumbs down, or shaky.

On the contrary, only two non-LLMT participants were able to provide math specific evidence of their students' learning, and seven of them focused their comments on strategies they used as teachers to reflect on their teaching success (see table 5). The following excerpt exemplifies this type of commentary:

> Although choral answering allows for me to hear the entire class at once, it makes it difficult to know who exactly understands what. I found calling on students to be much more successful because it helped me monitor individual student learning and also deepened all of the students' understanding as we discussed an answer.

Table 6 reports findings related to PSTs' ability to use evidence of students' difficulties to identify missed opportunities in their teaching. Group differences are similar to those identified for instruction commentaries focused on teaching successes. All LLMT participants used evidence of student difficulties to identify missed opportunities in their teaching, and nine of them cited evidence that was mathematics specific. The following excerpt exemplifies commentaries focused on students, but not math specific:

> Since this was the first time the students participated in a project such as this, there was a lot of confusion with the process of how to fill out the Budget Sheet.

The next excerpt exemplifies comments that were both focused on students and math specific.

> One of the things that I missed during this lesson, and tried to correct in another, was when one of the students came up to the board and split the tens blocks into two groups of three, instead of three groups of two. While I did not pick up on this different interpretation at the time, I did use the review from the next lesson to be able to try to recreate and address that misconception.

On the contrary, only half of the non-LLMT participants were able to cite evidence of student difficulties and, among them, only three cited evidence that was math specific. Half of the non-LLMT participants discussed things they did not do well as teachers as missed opportunities without indicating whether those missed opportunities impacted student learning (see Table 6). The following excerpt exemplifies this type of commentary:

> I saw that I made many mistakes in teaching. First of all, I wrote a word problem with incorrect grammar. I wrote, "How many do each person get?" I should have written "How many does each person get?" I was really mad at myself for making that mistake because students read it. I want to be a good model for them by learning proper grammar myself.

INSERT TABLES 5 & 6 APPROX. HERE

3.4 What is the Relationship between PSTs' Ability to Make Student Thinking Visible during Instruction and their Ability to Use Evidence of Student Thinking to Assess the Effectiveness of their Teaching?

To answer this question, within each group, we looked separately at PSTs' use of evidence to comment on teaching successes and on missed opportunities and investigated the relationship with making student thinking visible during instruction. Tables 7 and 8 report these findings. First we will review findings related to instruction commentaries focused on successes in teaching beginning with the LLMT participants (see Table 7). All but one PSTs in the high sophistication group, whose teaching made student thinking visible and built on it, also cited mathematics specific evidence of student learning to comment on their teaching success. Among the three whose teaching was coded at a medium level of sophistication, two were able to use specific evidence of student learning in their analysis. The one PST, who made student thinking only minimally visible during teaching, was able to use general evidence of student learning to reason about her teaching successes. None of the PSTs' commentaries in this group were coded at a low level of sophistication.

For PSTs in the non-LLMT group (see Table 8), the relationship between the two skills is not as clear. Those PSTs who made student thinking only minimally visible during teaching, with the exception of one, did not use evidence of student learning in their analysis. This relationship was expected. To be able to draw on evidence of student thinking and learning to reason about the effectiveness of teaching, teachers need to create opportunities in their lessons for students to express their thinking. Among those PSTs who made student thinking visible during their teaching (without pursuing it further), two only mentioned evidence of things they did as teachers to comment on the success of their teaching, and three cited evidence of student learning (one general evidence and two math specific evidence). Finally, the two PSTs, who made student thinking visible and pursued it during their teaching, only cited general evidence of student learning in their commentary. In sum, in the non-LLMT group, the two skills—making student thinking visible during teaching and using evidence of student thinking to reason about the successes in teaching—are not clearly linked. It appears that some PSTs have acquired one skill or another, but none have acquired them both at a high level of sophistication. In addition, making student thinking visible during teaching does not assure that these PSTs will use evidence of student thinking to assess the effectiveness of teaching. We think these findings provide evidence that a purposeful and structured approach to the teaching of these skills to PSTs is necessary.

INSERT TABLE 7 & 8 APPROX. HERE

Findings related to the relationship between making student thinking visible and citing evidence of students' difficulty to reason about missed opportunities in teaching follow a similar pattern with one exception. As shown in Table 9, most LLMT participants (i.e., 7 out of 12) both made visible and pursued student thinking during teaching and cited mathematics-specific evidence of student thinking to reason about missed opportunities. Also, a few PSTs used math-specific evidence of student thinking in their analysis even though they did not pursue student thinking in their teaching, both in the case of teaching successes and missed opportunities. This might indicate that high sophistication in analysis abilities are easier to acquire than teaching that actively pursue and build on student thinking; although given the small sample size, this remains a hypothesis that needs to be further explored.

Contrary to the analysis of teaching successes, in which none of the LLMT participants cited things they did as teachers to reason about teaching successes, three LLMT participants cited their shortcoming when asked to reason about missed opportunities. A reason for this might be that it is harder for PSTs to learn to use evidence of student difficulty to identify missed opportunities than to reason about successes in teaching. Another hypothesis is that the phrasing of the prompt may have impacted PST's commentaries. While for the teaching successes, they were specifically asked to provide one or two examples of what the students did or said as examples, this was not the case for missed opportunities. Nonetheless, as a group, LLMT participants did better than their counterparts.

Among the non-LLMT participants (see Table 10), only one was coded at a high level of sophistication for both skills. Almost half were coded at a low level of sophistication (i.e., 5 out of 12) in both skills, and the remaining PSTs are scattered across the table. This provides additional evidence that PSTs might acquire these skills during teacher preparation, but only a structured course can assist in the development of both skills and reach a larger number of PSTs.

INSERT TABLES 9 & 10 APPROX. HERE

3.4 Learning from Teaching through Practice and Analysis Focused on Student Thinking

To better illustrate the potential that making student thinking visible during instruction coupled with a reflection on one's own teaching that focuses on evidence of student thinking, creates for teacher learning, we provide an example of a PST in the LLMT group who received high scores in both skills.

Claudia student-taught in a second grade classroom when she was working on her PACT-TE. The video clip she turned in included an excerpt from a lesson on subtraction. In the lesson plan, Claudia states the following as the lesson learning goal: “Students will investigate similarities between ten and hundred partners to exemplify that they understand different methods to solve 100 - n problems.” At the beginning of the lesson, the students were given the problem in Figure 2 to solve.

The video begins with students presenting their invented solution methods to the rest of the class. The PST is then seen eliciting responses from students who used different strategies and probing students to provide additional explanations by connecting concepts and procedures. Although the PST led the discussion and students directed their answers mainly to her, the lesson clearly built on students’ input. Once solutions to the first problem were discussed, the PST posed additional problems keeping as minuend 100 and asking students to subtract different numbers. Multiple solution methods were made visible and discussed throughout the video. The following excerpt exemplifies the kind of teacher-student interactions that was captured:

> PST: Please solve for me one hundred minus fifty-one. Show me one hundred minus fifty-one. Show me one hundred minus fifty-one … Ok, what is the answer on the count of three?
>
> S: Forty-nine!
>
> Teacher: One, two, three!
>
> SS: Forty-nine.
>
> PST: How did you know, Ellen? Show me your thinking. How did you know it was forty-nine? Come up and show me. She's going to solve the problem for us. (*Ellen approaches the board.*) Pencils up, eyes up here. Ok, they're ready for you. (*Ellen draws on the board.)*
>
> Ellen: Well, I found out there was forty-nine...(inaudible)

PST: How did you know it was forty-nine? First, you colored in what?

Ellen: Because...I colored, I colored in five tens, and I colored in one dot and I could see that there were three- four tens left and … it's nine ones.

PST: So you knew that one hundred minus fifty-one had to be what?

Ellen: It had to be forty-nine because forty plus nine equals forty-nine.

PST: Oh, I love your thinking. Very good, give her a round of applause. Ok, one more, and then it's recess time.

When asked to provide examples of students' progress towards the learning goal, Claudia provided several examples. Using evidence from the video excerpt above, she wrote:

> … one example of student interaction shown in the video clip that indicated student progress toward accomplishing the lesson's learning objective of getting students to subtract numbers up to three digits is toward the end of the clip where students came up and verbally explained their thinking. One student counted the number that she subtracted from one hundred then explained that she knew what left uncounted was the solution to the problem. She then counted the remaining lines of dots by tens, and counted the remaining values of ones by one. This showed me that she understood the meaning of subtraction, as evidenced by values being subtracted and the numbers left over. This exercise showed me that this student understood what the values behind the physical layout of a subtraction problem mean; the number being subtracted is the initial sequence counted, and the remaining values stands for the solution.

In addition, she commented on what she learned as a teacher about her students' understanding of subtraction:

> When all the students in the class solved this problem, and four specific students came up to explain their thinking on the board, I was able to learn a lot about different problem solving methods that made sense to the students. By having them verbally explain their solutions, their underlying conceptual, procedural and reasoning skills were revealed, which allowed me to understand if they really understood the principles behind two (and soon to be three) digit subtraction; in essence, this activity highlighted their understanding and did achieve the lesson objective.

This commentary indicates that Claudia was deliberate in asking students to present their methods on the board. Making student thinking visible served two purposes: it allowed Claudia to build on students' ways of reasoning about subtraction during the lesson and provided evidence of student progress towards the lesson learning goal.

In sum, both Claudia's teaching and reflections indicate that she is well positioned for developing into a teacher who deliberately plans to make student thinking and learning visible in her teaching and systematically reflects on her teaching by looking for evidence of student learning. This process of analysis allows this PST to learn from her practice two important aspects of teaching: student thinking and instructional strategies to further support student understanding.

Conclusions

We conducted this study to contribute to an emerging body of literature in mathematics education on video- and practice-based teacher preparation. Specifically, we investigated the potential of video- and practice-based teacher preparation for the development of PSTs' abilities to teach in ways that makes student thinking visible and to use evidence of student thinking to assess the effectiveness of teaching. In addition, we have explored the relationship between these two skills.

Overall, we believe that this study provides promising evidence of the kind of learning that video- and practice-based teacher preparation might create for future teachers. PSTs in our project not only improved in their analyses of teaching performed by others (Author et al., 2011), but also learned to analyze their own teaching and to teach in ways that make analysis, particularly of student thinking, possible. The effects on teaching practices are particularly noteworthy given the overall limited teaching experience of our participants and the fact that changing teaching (from teacher-centered approaches common in US classrooms to student-centered approaches) is not easy (Authors et al, 2005). PSTs in the LLMT course often lamented the fact that they had never experienced as students the kind of teaching we were promoting. Analyses of their group conversations around video-based tasks during the course revealed their need to discuss the features of this "new" approach to teaching and contrast them to a more traditional approach in order to appropriate it (Authors et al., 2012). This highlights for us the affordances of video-based discussions in teacher preparation, particularly when, as we did in the LLMT course, PSTs are exposed to images of teaching that build on student thinking and are encouraged to use evidence-based reasoning to assess its effectiveness.

PSTs who participated in the LLMT course were also able to transfer skills they learned in a university course to their classroom teaching and to the analysis of their own teaching. These are not trivial outcomes. As discussed in the introduction, because of the brevity of teacher preparation programs, it is important to place PSTs on the right trajectory to continue to learn from their practices overtime. We believe that PSTs like Claudia, who enter the teaching career with the ability both to teach in ways that make student thinking accessible and to analyze their teaching effectiveness by identifying strategies that support and do not support student learning will be able to continue to grow professionally once they have left their teacher preparation program. We think that the enactment activities of the LLMT course enabled PSTs to implement in their classrooms strategies they had observed through video and to transfer their analysis skills from the analysis of the teaching of others to their own. Specifically, role playing activities and planning, teaching, and analysis assignments of increasing difficulty provided opportunities for PSTs to practice these skills with the necessary support and feedback to be able to implement them more independently later in the PACT-TE lessons.

Although limited in scope by the small number of participants, this study also suggests that the ability to focus on students during both teaching and analysis is not something PST teachers can develop by simply completing fieldwork experiences (as evidenced by the outcomes of non-LLMT participants). Structured opportunities for developing these abilities in systematics ways need to be embedded in teacher preparation programs. This study thus serves as the basis for future investigations, already underway, that will include larger samples, will integrate the LLMT course into the mathematics methods course, and will be longitudinal in nature.

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