Supplemental Appendices: Pricing and Quality Provision in a Supply Relationship: A Model of Efficient Relational Contracts

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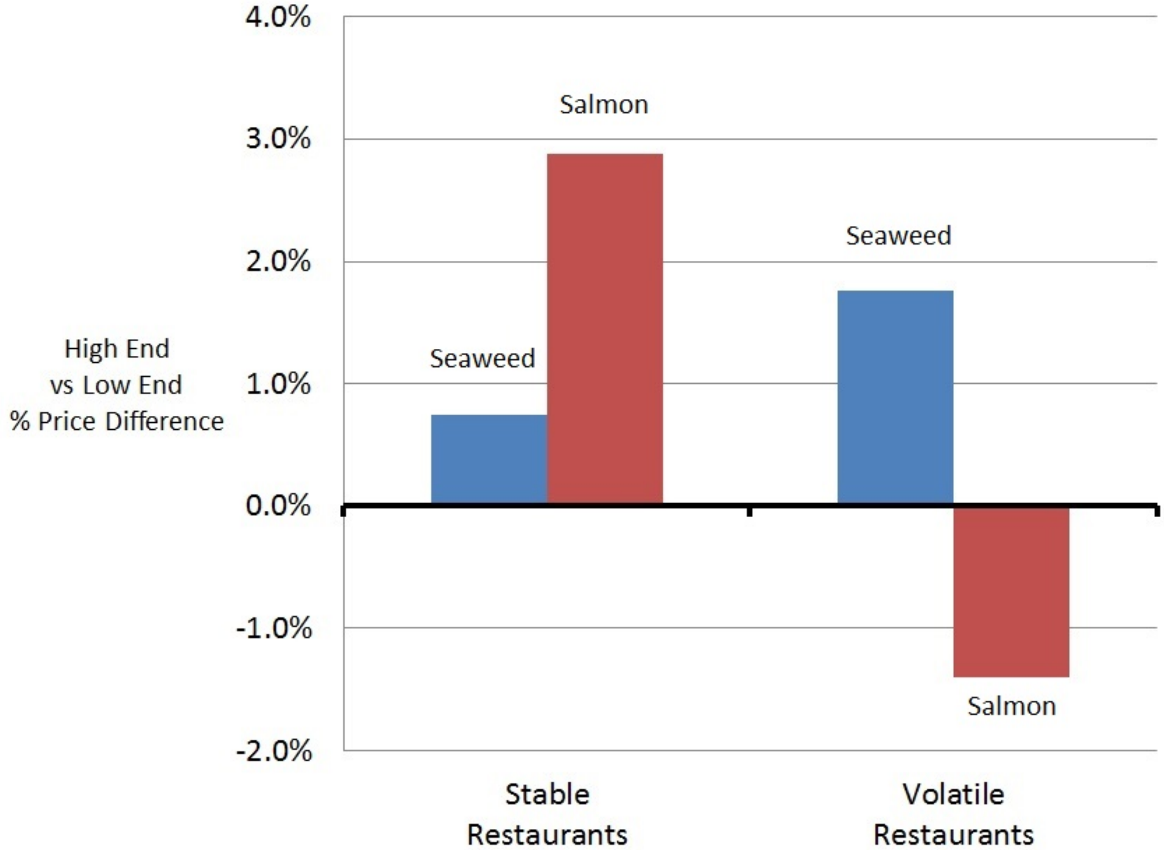
Abstract

Appendix A presents a pricing example based on a supplier to sushi restaurants. Appendix B contains formal proofs of all results.

Appendix A

Example: Salmon and Seaweed Price Premiums



Example is based on sales data and customization requests from a supplier of sushi restaurants in the Southeast US, from April 2010 to February 2011. Restaurants were divided into volatile and stable demand using a median split of sales variation. Low End restaurants are those with menu prices under $10 on Yelp.com or UrbanSpoon.com while High End restaurants are those with prices above that. 94% of all customization requests come from the high-end restaurants.

Appendix B

Proof of Lemma 1

For any prices P_u and P_b, if there is an equilibrium in which firms always trade with each other, we will show that the most efficient equilibrium of this type involves equilibrium effort e_t that depends only on the current quantity $q_{t,u}$.

If firms always trade with each other, then expected discounted premium payments in future periods, denoted by V_P, is given by:

$$V_P = \frac{\delta}{1-\delta} E[q_{T,u}(P_u - \bar{P}_u) + q_{T,b}(P_b - \bar{P}_b)] \tag{56}$$

We also let $V_{e,t}$ denote expected discounted value of future effort times quantity:

$$V_{e,t} \equiv \sum_{T=t+1}^{\infty} \delta^{(T-t)} E_t[e_T q_{T,u}]. \tag{57}$$

Surplus generated by the contract is $(\beta - \gamma)(e_t q_{t,u} + V_{e,t})$. An efficient contract sets effort levels that maximize this surplus subject to the constraints that the seller and buyer never want to deviate.

We will show that it is efficient to set effort $e_t = \min(1, \frac{C^*}{q_{t,u}})$ where C^* is defined as the greatest value of C that satisfies the following:

$$\gamma \left[C + \sum_{T=t+1}^{\infty} \delta^{(T-t)} E_t[\min(q_{T,u}, C)] \right] \leq V_P \tag{58}$$

Under the proposed policy, the seller sets its effort level so that its effort times quantity is C^* in each period, except when $q_{T,u} < C^*$, in which case it exerts the maximum possible effort, $e_t = 1$. The value C^* is chosen as the greatest possible value that always satisfies the seller's constraint not to deviate.

Consider any possible alternative effort policy, which can differ from the proposed policy at one or more time periods. We will show that, at any given time t, this alternative policy must generate weakly lower expected surplus than the proposed policy in order to be sustainable.

Suppose at a given time t, the seller sets a strictly higher effort level than under the proposed policy. In order for the seller's constraint to continue to hold, expected future effort must be reduced relative to the proposed policy, such that total surplus is weakly lower than under the proposed policy. Formally, at any time t, the seller will exert effort level e_t only if $\gamma e_t q_{t,u} \leq V_P - \gamma V_{e,t}$. Because this constraint holds with equality under the proposed policy when $e_t q_{t,u} = C^*$, in order sustain a higher effort level such that $e_t q_{t,u} = C^* + \epsilon$, where $\epsilon > 0$, the value of $V_{e,t}$ must be reduced by at least ϵ relative to the proposed policy. Thus, surplus, which given by $(\beta - \gamma)(e_t q_{t,u} + V_{e,t})$, cannot be increased by this deviation, and the incentives for the buyer to purchase from the seller in the current period also cannot be increased. Furthermore, future incentives for the buyer to remain in the relationship must be reduced by the seller's future reduction in effort.

On the other hand, if the seller ever sets effort *lower* than the proposed policy, then surplus and incentives for the buyer in the current period are strictly decreased. Furthermore, as shown above, any future deviation to an effort level greater than the proposed policy cannot make up for this reduction by increasing future surplus or future incentives for the buyer.

Because any alternative effort policy cannot generate greater surplus or stronger incentives that the proposed policy at any time t, such an alternative policy must generate weakly lower discounted surplus than the proposed policy. Thus, the proposed effort policy leads to the greatest possible surplus and strongest possible incentives for the buyer to buy from the seller in all periods, among the set of all effort policies that the seller is also willing to sustain in all periods. QED

Proof of Proposition 1

We show that a relationship in which firms always trade with each other with a price premium only on the customizable good cannot be sustained if condition (15) holds. The derivations in the body of the paper show that, given $P_u - \bar{P}_u > 0$ and $P_b - \bar{P}_b = 0$, if we combine the buyer's constraint, given by (8), with the seller's constraint, given by (11), together these constraints imply:

$$\gamma\Big(q_{t,u} + \frac{\delta}{1-\delta}E[q_{T,u}]\Big) \leq \beta\frac{\delta}{1-\delta}E[q_{T,u}] \tag{59}$$

Note the derivations of this condition do not depend on the particular effort levels specified by the contract. Therefore, this condition must hold for such a relationship (in which firms always trade with each other with a price premium only on the customizable good) to be sustainable for *any* effort levels. Rearranging terms, this condition is equivalent to:

$$\gamma q_{t,u} \leq (\beta - \gamma)\frac{\delta}{1-\delta}E\big[q_{T,u}\big] \tag{60}$$

If (15) holds, then the above condition does not hold when $q_{t,u} = \max(q_{t,u})$, which implies at least one firm must have an incentive to deviate during periods of peak demand for the customizable good, and such a relationship cannot be sustained.

On the other hand, if (15) is reversed, we show that a contract that sets $P_u - \bar{P}_u = \beta$ and $P_b - \bar{P}_b = 0$ can sustain effort $e(q_{t,u}) = 1$ in every period. Substituting these values into the buyer's constraint (8), we have:

$$\beta\Big(q_{t,u} + \frac{\delta}{1-\delta}E[q_{T,u}]\Big) \leq \beta\Big(q_{t,u} + \frac{\delta}{1-\delta}E[q_{T,u}]\Big) \tag{61}$$

Because this constraint always holds with equality, the buyer never has an incentive to deviate and go to the outside market.

Substituting the proposed prices and effort level into the supplier's constraint (11), we have:

$$\beta \frac{\delta}{1-\delta} E[q_{T,u}] \geq \gamma\Big(q_{t,u} + \frac{\delta}{1-\delta} E[q_{T,u}]\Big) \tag{62}$$

Rearranging terms, this condition is equivalent to:

$$\gamma q_{t,u} \leq (\beta - \gamma) \frac{\delta}{1-\delta} E[q_{T,u}] \tag{63}$$

If condition (15) is reversed, then the above constraint is always satisfied, so the supplier never has an incentive to deviate from setting $e(q_{t,u}) = 1$. Therefore, the proposed relationship is sustainable with optimal effort in all periods. QED

Proof of Proposition 2

Given the conditions of the proposition, we show that a relationship that sets $e_t = 1$ during low demand and $e_t = e_H^*$ from equation (21) during high demand, and prices according to (22) and (23) is sustainable and generates strictly greater surplus than any other sustainable relationship.

The total premium payment during *low* demand is $(P_u - \bar{P}_u)L + (P_b - \bar{P}_b)L$, which, given the proposed prices, equals βL. The total premium payment during *high* demand is $(P_u - \bar{P}_u)H_u + (P_b - \bar{P}_b)H_b$, which, given the proposed prices, equals $\beta H_u e_H^*$.

For these payment values and the proposed effort levels, the buyer's constraint (8) during low demand becomes

$$\beta L + \frac{\delta}{1-\delta}\beta\Big[(1-\omega)L + \omega H_u e_H^*\Big] \leq \beta L + \frac{\delta}{1-\delta}\beta\Big[(1-\omega)L + \omega H_u e_H^*\Big] \tag{64}$$

During high demand, the buyer's constraint becomes

$$\beta H_u e_H^* + \frac{\delta}{1-\delta}\beta\Big[(1-\omega)L + \omega H_u e_H^*\Big] \leq \beta H_u e_H^* + \frac{\delta}{1-\delta}\beta\Big[(1-\omega)L + \omega H_u e_H^*\Big] \tag{65}$$

Thus, the buyer's constraint always holds with equality, and so the buyer never has an incentive to deviate.

Inserting the proposed payment and effort levels into the seller's constraint, given by (11), during low demand, we have:

$$\beta \frac{\delta}{1-\delta}\Big[(1-\omega)L + \omega H_u e_H^*\Big] \geq \gamma\Big(L + \frac{\delta}{1-\delta}\Big[(1-\omega)L + \omega H_u e_H^*\Big]\Big) \tag{66}$$

During high demand, the seller's constraint becomes

$$\beta \frac{\delta}{1-\delta}\Big[(1-\omega)L + \omega H_u e_H^*\Big] \geq \gamma\Big(H_u e_H^* + \frac{\delta}{1-\delta}\Big[(1-\omega)L + \omega H_u e_H^*\Big]\Big) \tag{67}$$

Recall that e_H^* was chosen to solve equation (20), which implies (67) holds with equality. Furthermore, conditions (19) and (18) imply that e_H^* must lie in the set $(\frac{L}{H_u}, 1)$ to solve (20). Therefore, $H_u e_H^*$ is greater than L. Thus, if (67) holds with equality, then (66) holds strictly.

Because the constraints for the buyer and seller to stay in the relationship are always satisfied, the proposed relationship is sustainable. Furthermore, as shown in the discussion following (20), any effort level higher than e_H^* during high demand would imply that effort costs for the period exceed the ongoing value of the surplus generated by the relationship, which implies the relationship could not be sustainable. Formally, e_H^* was derived such that, for any $e_H > e_H^*$, the following inequality holds:

$$\gamma H_u e_H > (\beta - \gamma)\frac{\delta}{1-\delta}\Big[(1-\omega)L + \omega H_u e_H\Big] \tag{68}$$

Because this inequality implies effort cost during high demand exceeds future surplus from the relationship, it is not possible to provide the seller with strong enough incentives to sustain any effort level e_H during high demand that is strictly greater than e_H^*.

Thus, the proposed relationship sustains the greatest possible effort, with effort level 1 during low demand and e_H^* during high demand. Recall that surplus from the relationship is as follows:

$$E\left[\sum_{t=0}^{\infty}\delta^t(U_{R_t}+U_{S_t})\right]=E\left[\sum_{t=0}^{\infty}\delta^t(\beta-\gamma)(e_tq_{t,u})\right] \tag{69}$$

Therefore, any relationship with lower effort, or in which firms trade on the outside market, during either high or low demand would generate lower surplus than the proposed relationship.

Finally, we show that no other pair of prices can sustain these efficient effort levels. In order for the seller's constraint (11) to be satisfied during high demand, any alternative contract with the same effort levels would have to provide at least the same expected total premium payment as the proposed contract. In particular, the expected discounted value of future premium payments would have to be at least $\beta\frac{\delta}{1-\delta}\left[(1-\omega)L+\omega H_u e_H^*\right]$.

Any alternative prices that generate the same *expected* premium payment as the proposed contract would imply either higher total payments during *high* demand (and lower total payments during *low* demand) or higher total payments during *low* demand (and lower total payments during *high* demand). However, because the buyer's constraint (8) always holds with equality in the proposed equilibrium, any such alternative contract that resulted in a total premium payment greater than βL during low demand would result in (8) being violated during low demand, and similarly, any such contract that included a total premium payment higher than $\beta H_u e_H^*$ during high demand would result in (8) being violated during high demand. Thus, only the proposed prices can sustain efficient effort levels. QED

Proof of Corollary 1

If (24) holds, then the left side of (20) is greater than the right side if we set $e_H^*=\frac{H_b}{H_u}$. Therefore, optimal high demand effort must satisfy $e_H^*<\frac{H_b}{H_u}$, which implies the price premium on the customizable good given by (22) is negative, and the price premium on the basic good given by (23) is positive. QED

Proof of Corollary 2

Starting with values for which conditions (18) and (19) hold, these conditions continue to hold as H_u increases. Therefore, the results of Proposition 2 continue to hold as $H_u \to \infty$. From (21), we see that $H_u e_H^*$ remains constant as H_u grows, which implies (22) and (23) converge to the equilibrium prices stated in the corollary. QED

Proof of Proposition 3

Given the conditions of the proposition, we show that the proposed equilibrium is sustainable and generates strictly greater surplus than any other sustainable equilibrium. Most steps of this proof are similar to the proof of Proposition 2.

The proposed prices and payment adjustments imply the total premium payment is βL whenever demand for the customizable good is low and $\beta H_u e_H^*$ whenever demand for the customizable good is high. Therefore, as in the proof of the previous proposition, the buyer's constraint during low demand for the customizable good is given by (64), the buyer's constraint during high demand of this good is given by (65), and these constraints are always satisfied with equality.

The left side of (32) gives the seller's effort cost during high demand, and the right side of (32) gives the expected discounted value of future premium payments minus future effort costs and future adjustment costs, given the proposed relational contract. The effort level e_H^* was chosen so that the two sides of this equation are equal, which implies the seller's constraint to provide the required effort is satisfied with equality during high demand for the customizable good, and its constraint is satisfied strictly during low demand for this good.

For any period in which the seller needs to incur an adjustment cost and propose a total bonus payment lower than the one implied by the contract prices, the difference between the equilibrium bonus payment and effort cost for that particular period provides an incentive for the seller to incur this adjustment cost and to propose the equilibrium bonus. If the seller deviated from the equilibrium path and proposed any other bonus or stayed with the contract prices, the relationship would end, so the seller would not receive any bonus for that period. Therefore, as long as the adjustment cost is small enough, the seller would not deviate.

Because neither player ever has an incentive to deviate, the proposed relationship is sustainable. Lower effort levels or trading on the outside market would result in lower surplus. Furthermore, any effort level higher than e_H^* during high demand leads to effort costs during those periods that exceeds the ongoing value of the relationship, so such effort could not be sustained.

Any alternative total premium payments imply either that the seller's effort incentives are weaker (because the expected premium payment is smaller), or that the buyer's constraint to stay in the relationship is violated during periods when the total premium payment is higher than in the proposed contract. If d is sufficiently small, it is more efficient to generate an optimal payment in each period rather than to incur these consequences of suboptimal payments. Given that (H_u, L) and (L, H_b) are the two least likely demand states, the proposed equilibrium involves the lowest possible expected adjustment costs that generates the flexibility to set an optimal payment in each period. QED

Proof of Proposition 4

We show that the proposed contract is sustainable and generates greater surplus than any other contract.

The left side of (40) gives the value to the buyer of staying in the relationship during high demand periods, and prices specified by (41) ensure this value equals zero, so the buyer is willing to stay in the proposed relationship during high demand. The condition that (40) equals zero implies that $(\beta - P_u + \bar{P}_u)H_u F_{H,u} - f < 0$. Thus, the buyer derives strictly *negative* surplus from the relationship during high demand periods, which is offset by the future strictly *positive* surplus it receives during low demand periods, when it does not have order splitting costs. Therefore, the buyer is also willing to stay in the relationship during low demand periods.

The left side of (43) gives the seller's effort cost during high demand, and the right side of (43) gives the expected discounted value of future premium payments minus future effort costs. The order fraction $F_{H,u}$ was chosen so that the two sides of this equation are equal, so the seller is always willing to stay in the relationship.

By comparing (43) with (20), we see that, as $f \to 0$, the value of effort in each period in the proposed contract with order splitting approaches the same value of effort as in the optimal contract derived in Proposition 2. In particular, the order fraction $F_{H,u}$ during high demand in the current proposed contract converges to the effort level e_H^* during high demand derived in Proposition 2. Therefore, the same reasoning as the proof of Proposition 2 shows that, as $f \to 0$, an efficient contract must converge to total payment $\beta H_u e_H^*$ during states (H_u, L) and (H_u, H_b) and total payment βL during states (L, L) and (L, H_b). The proposed contract with order splitting sets prices to generate these optimal payments at states (L, L) and (L, H_b), and it uses order splitting to generate the optimal payments at states (H_u, L) and (H_u, H_b).

We will show that any other contract with order splitting either fails to generate these optimal payment levels or requires greater average order splitting costs than the proposed contract. To match the optimal payment levels during both states (H_u, H_b) and (L, L) without order splitting at these two states, a contract would need to set the same prices derived in Proposition 3. However, because (24) does not hold, both of these price premiums are positive, and order splitting would not allow the firms to increase the premium payment to the optimal level during state (H_u, L). Therefore, such a contract cannot be efficient for sufficiently small f. Furthermore, given $\omega < 0.5$, contracts that sets prices to generate the optimal payment at any other single state or pair of states, while using order splitting to generate optimal payments at the remaining states, require more frequent order splitting than the proposed contract. Such a contract cannot be efficient for sufficiently small f because it requires higher order splitting costs than the proposed contract. QED

Proof of Corollary 3

We first show that the right side of (43) is less than $\left(\frac{\delta}{1-\delta}\right)\left[(\beta-\gamma)[(1-\omega)L+\omega H_u e_H^*]-\omega f\right]$. This is equivalent to showing that $\phi[(1-\omega)L+\omega H_u F_{H,u}] > \omega$, where:

$$\phi \equiv \frac{1+\left(\frac{\delta}{1-\delta}\right)\omega}{H_u F_{H,u}+\left(\frac{\delta}{1-\delta}\right)(\omega H_u F_{H,u}+(1-\omega)L)} \tag{70}$$

Multiplying the denominator of ϕ by both sides, this is equivalent to:

$$\left[1+\left(\frac{\delta}{1-\delta}\right)\omega\right][(1-\omega)L+\omega H_u F_{H,u}] > \omega\left[H_u F_{H,u}+\left(\frac{\delta}{1-\delta}\right)(\omega H_u F_{H,u}+(1-\omega)L)\right] \tag{71}$$

Expanding terms, we have:

$$\begin{aligned}&(1-\omega)L+\omega H_u F_{H,u}+\left(\frac{\delta}{1-\delta}\right)\omega(1-\omega)L+\left(\frac{\delta}{1-\delta}\right)\omega^2 H_u F_{H,u} > \\ &\omega H_u F_{H,u}+\left(\frac{\delta}{1-\delta}\right)\omega^2 H_u F_{H,u}+\left(\frac{\delta}{1-\delta}\right)\omega(1-\omega)L\end{aligned} \tag{72}$$

Canceling terms, this is equivalent to $(1-\omega)L > 0$, which is true because $(1-\omega)$ and L are both positive.

We have shown the right side of (43) is less than $\left(\frac{\delta}{1-\delta}\right)\left[(\beta-\gamma)[(1-\omega)L+\omega H_u e_H^*]-\omega f\right]$, which implies that equilibrium effort with order splitting from solving (43) is less than equilibrium effort with payment adjustments from solving (32) under the condition of the corollary, that is, if $f > 2d(1-\rho)(1-\omega)$. Furthermore, the same condition implies that average order splitting costs in the equilibrium of Proposition 4 are greater than average payment adjustment costs in the equilibrium of Proposition 3. Therefore, this condition implies that payment adjustments involve greater equilibrium effort and lower adjustment costs than order splitting. QED

Proof of Proposition 5

The proof is similar to the proof of Proposition 2. The proposed equilibrium implies the total premium payment is βL_u when demand is low, βM_u when demand is medium, and $\beta H_u e_H^*$ when demand is high, and that the benefit to the buyer of effort in each period is always equal to these premium payments.

The left side of (51) gives the seller's effort cost during high demand, and the right side of (51) gives the expected discounted value of future premium payments minus future effort costs, given the proposed relational contract. The effort level e_H^* was chosen so that the two sides of this equation are equal.

Any alternative payments imply either that the buyer's constraint to stay in the relationship is sometimes violated or that the seller's effort incentives are weaker. QED