

Supplement for:

Portable polarimetric fiber stress sensor system for visco-elastic and biomimetic material analysis

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1 Data Acquisition and Analysis

1.1 Fitting Algorithm

There are four variables which are always unknown every time the fiber sensor system is set up: $(\alpha, \beta, \gamma, \delta)$. In order to calibrate the sensor, these variables must be solved for.

1.1.1 Stokes Parameters

By multiplying out the matrices in equation 4 and plugging in values from equations 1-3, we can find equations for E_x and E_y as a function of α, β, γ , and δ . Using these equations in conjunction with the equations below, we can generate four equations for the Stokes parameters:

$$s_0 = |E_x|^2 + |E_y|^2 \quad (S1)$$

$$s_1 = |E_x|^2 - |E_y|^2 \quad (S2)$$

$$s_2 = 2Re\{E_x E_y^*\} \quad (S3)$$

$$s_3 = -2Im\{E_x E_y^*\} \quad (S4)$$

However, the polarimeter measures normalized stokes parameters. This has some important implications. Specifically, $s_0 = 1$ and the vector $\langle s_1, s_2, s_3 \rangle$ will have magnitude 1 and lie on the surface of the Poincaré sphere. Even using the value of E_x and E_y obtained from equation 4, $s_0 = 1$ because the system we consider only allows for fully polarized light. Therefore we have a system of three equations (s_1, s_2, s_3) with four unknowns ($\alpha, \beta, \gamma, \delta$) which we cannot solve directly. Fortunately, by using some knowledge of how the four angles affect the polarization trace generated by the sensor and knowledge of how much force was applied, we can fit our system of equations to experimental data using an algorithm and solve for the four angles. In the present work, this fit was performed using a Matlab script and the reference data measured in parallel with our sensor data.

1.1.2 Solving for β and γ

When stress is applied to the PM fiber, the changing polarization state (s_1, s_2, s_3) will trace out a line on the Poincaré sphere. There are two key features about this process that are consistent given our theoretical analysis: 1) the line traced out will always be circular, and 2) the circle-trace will always be centered on the equator of the sphere ($s_3=0$, or linear polarization). To accelerate analysis, we leverage this behavior and solve for β and γ first.

The angle β affects the radius of the circle trace (Fig. S1a). Because the circle-trace is always on the surface of the sphere, changing the radius of the circle will also change its horizontal position on the sphere. The first step to solve for β is to convert the stokes parameter vector $\langle s_1, s_2, s_3 \rangle$, which represents x, y, and z in Cartesian coordinates, to spherical coordinates. Since $\langle s_1, s_2, s_3 \rangle$ always lies on the surface of a sphere of radius 1, converting them to spherical coordinates results in $r=1$, leaving only θ (polar angle) and ϕ (azimuthal angle) as the free variables. Using θ and ϕ like an x and y coordinate in a 2D plane, the circle-trace is fit to a circle to obtain a radius. This radius will be equal to the maximum value of ϕ $[-\frac{\pi}{2}, \frac{\pi}{2}]$ (azimuthal). The angle β is equal to radius/2.

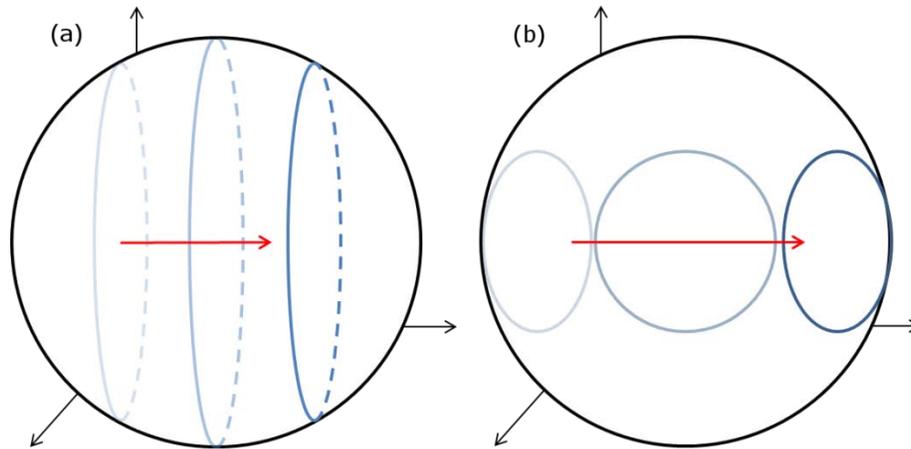


Fig. S1: Diagrams showing the effects on the circle-trace of angles a) β and b) γ . a) For different angles of β , the circle-trace will have a different horizontal position on the Poincaré sphere. b) For different values of γ , the center of the circle-trace will rotate around the Poincaré sphere.

The angle γ affects the center of the circle-trace on the equator of the Poincaré sphere (Fig S1b). Taking the circle fit generated from θ and ϕ previously, notice that the center y (ϕ) coordinate will always be zero and the center x (θ) coordinate corresponds to the center of the sphere. The angle γ is equal to $-x_{\text{Center}}/2$. Fitting both β and γ is performed with the Matlab script.

1.1.3 Solving for δ

The angle δ affects the starting point of the circle-trace on the Poincaré sphere (Fig S2a,b). The starting point will always lie somewhere on the circle defined by β and γ , but it will change position according to δ . Accordingly, fitting for δ is straightforward, but cannot be done until β and γ are already fit. All that needs to be done is to take the initial experimental value for $\langle s_1, s_2, s_3 \rangle$ and sweep through theoretical values of $\langle s_1, s_2, s_3 \rangle$ with varying δ and with applied force, f , equal to zero. The angle δ can have a value from 0 to 2π before it starts repeating values of $\langle s_1, s_2, s_3 \rangle$. After the sweep, the value of δ that produced a theoretical $\langle s_1, s_2, s_3 \rangle$ closest to the experimental $\langle s_1, s_2, s_3 \rangle$ is chosen. This best fit sweep is accomplished using the Matlab script.

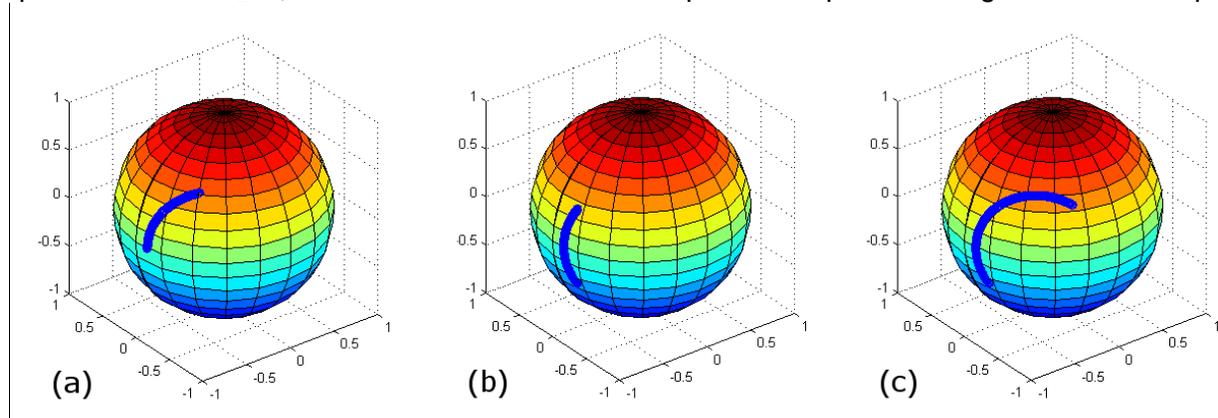


Fig. S2: Diagrams showing the effects on the circle-trace of angles δ and α . a) A circle trace with a given value for δ and α is shown. b) When δ is changed, the circle-trace rotates, and the starting point changes. Notice that the whole trace moves together. In this graph, α has not yet been changed. c) When α is changed, the circle-trace expands. Notice that the starting point of the circle-trace remains fixed to the same point as in b), but the end-point changes. Changing α can also cause the circle-trace to contract.

1.1.4 Solving for α

The angle α is the most important of the four angles and the trickiest angle to solve for. This is because α controls the rate at which the polarization-state traces out the circle-trace when force is applied to the fiber. For different values of α , the polarization-state will trace out more or less of the circle-trace (Fig S2b,c). In order to fit for α , we need to know the stress applied to the fiber at the end-point of the circle-trace. Additionally, α cannot be fit until β , γ , and δ have been fit. We obtain the final applied stress from the reference data in our experiments, but it could easily be obtained from other means, such as placing a weight on top of the fiber in a controlled manner. Once this stress is determined, it is multiplied by the diameter of the fiber used for testing to obtain the applied force, f , in [N/m] at the final polarization state of the circle-trace. After the force is obtained, α is fit in a manner similar to δ . Taking the final experimental value for $\langle s_1, s_2, s_3 \rangle$, theoretical values are swept through with varying α and with the f equal to the final applied force. The range of α is 0 to $\frac{\pi}{2}$ before it starts repeating. After the sweep, the value of α that produced a theoretical polarization state closest to the experimental polarization state is chosen. This best fit sweep is also performed in Matlab.

1.2 Data Processing

The raw data from the polarimeter is in the form of three stokes parameters (s_1, s_2, s_3) and must be converted to a single variable representing the change in polarization (ΔPol). This should be done in a way that preserves any information contained in the polarization state (s_1, s_2, s_3), without simply throwing out any of the variables.

The raw data from the load-frame is in the form of an extension from zero compression and force on the sample from a load-cell. These values must be converted to strain and stress, respectively, in order to generate an appropriate stress-strain curve.

1.2.1 Phase-angle of circle-trace

With the knowledge that the polarization-state of the light always traces out a circle on the Poincaré sphere, there is a simple measure of the circle which can serve as ΔPol without losing any of the information contained in the polarization state (s_1, s_2, s_3). The phase angle that the circle-trace arcs through as stress is applied (Fig S3a) can be calculated without losing the information contained in the polarization state. This will become apparent as the process for calculating it is described.

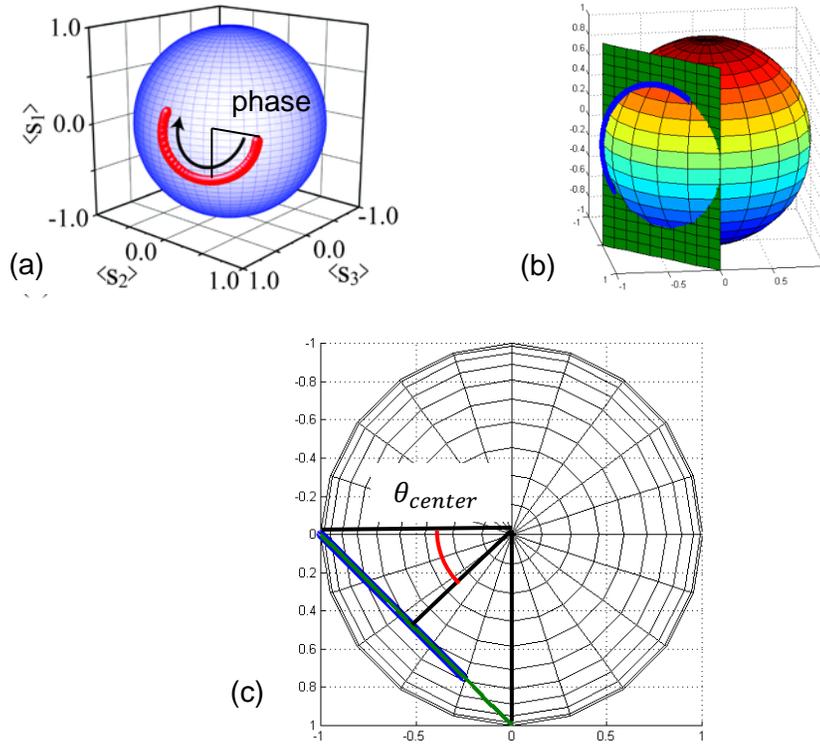


Fig. S3: When the fiber is stressed, the polarization state will trace out a circle arc on the Poincaré sphere. a) The phase angle of the circle-trace arc can be calculated to represent the change in polarization state. b) The circle-trace will always be an intersection of a vertical plane and the Poincaré sphere. The circle-trace lies on both the sphere and the intersecting plane. c) The Stokes parameters s_1 and s_2 can be converted to a single coordinate that lies on the intersecting plane. The sphere is viewed from above, with the intersecting plane indicated in green and the polarization-state data indicated in blue.

Because the center of the circle-trace is always on the equator of the Poincaré sphere ($s_3=0$), the circle-trace will always be an intersection of a vertical plane parallel to s_3 -axis with the sphere (Fig S3b). Finding the x-y coordinates of the circle-trace on this plane will allow us to get the phase angle we desire. Since the plane is always vertical and parallel to the s_3 -axis, s_3 of the polarization state will be the y-coordinate of the circle-trace in the intersecting plane.

The x-coordinate of the circle-trace on the intersecting plane can be found by rotating the s_1 and s_2 points to the reference frame of the intersecting plane (Fig S3c). In section 1.2, we found the center of the circle-trace in terms of θ in spherical coordinates. The center in angle θ is the angle of rotation of the reference frame that is needed. Using this angle, θ_{center} , the x-coordinate of the circle-trace on the intersecting plane is found by

$$x - coordinate = -s_1 \times \sin(\theta_{center}) + s_2 \times \cos(\theta_{center}) \quad (S5)$$

We now have x-y coordinates of the circle-trace on its intersecting plane. These coordinates are found using s_1 , s_2 and s_3 . Therefore, all of the information about the polarization state contained in those parameters remains embedded in the x-y coordinates. Finally, the phase of the circle-trace is simply

$$phase = \tan^{-1} \left(\frac{y-coordinate}{x-coordinate} \right) \quad (S6)$$

To determine ΔPol , the phase is normalized by subtracting the initial phase-angle from every phase value and dividing by π .

1.2.2 Stress and strain

The data measured from the Instron load-frame (extension and force) must be processed slightly to convert it to stress and strain. The extension is divided by the initial, measured thickness (height) of the sample to produce strain. Similarly, the force is divided by the initial cross-sectional area (width x depth) over which the force is applied to produce the stress. Technically, processing the data in this way results in engineering stress and strain, but it is common practice to measure the engineering stress and strain and report it as stress and strain.

The raw data from the fiber sensor, measured by the polarimeter, gives the polarization state (s_1, s_2, s_3) with respect to time. To align the fiber sensor data with that from the Instron loadframe, the time axis must be converted to strain. This is done by multiplying the recorded time by the crosshead velocity, 0.1 mm/s to produce extension, and dividing the extension by the initial measured thickness (height) of the sample. Once the time has been converted to strain, it is possible to match the fiber sensor data to the load-frame reference data.

1.3 System Calibration

After the raw data has been processed, the data from the fiber sensor must still be calibrated. This process combines results of both the fitting algorithm and data processing steps to produce a stress-strain curve from the fiber sensor data.

In order to run the fitting algorithm, the maximum stress from the Instron load-frame must be used. This value is taken straight from the processed load-frame reference data. Once it has been plugged into the fitting algorithm script, it is used to generate a theoretical circle-trace that closely matches the experimental data. This theoretical circle-trace relates polarization state to applied force (f , [N/m]), but the polarization state is in the form of stokes parameters (s_1, s_2, s_3).

The theoretical value of $\langle s_1, s_2, s_3 \rangle$ is converted to ΔPol using the same data processing method as the experimental data, described above. After the theoretical polarization state is converted to ΔPol , we have a relationship between ΔPol and applied force, f . The relationship between ΔPol and f is plotted and fit to a polynomial curve (usually 2nd or 3rd order) (Fig S4). This function is our calibration curve. By plugging in the experimental values for ΔPol , we can calculate the experimentally measured f and convert it to stress by dividing by the width of the fiber used in the sensor. This calibration process only needs to be performed once for each sample. Additional measurements on the same sample (and with the experimental setup unchanged) can be calibrated using the same calibration curve.

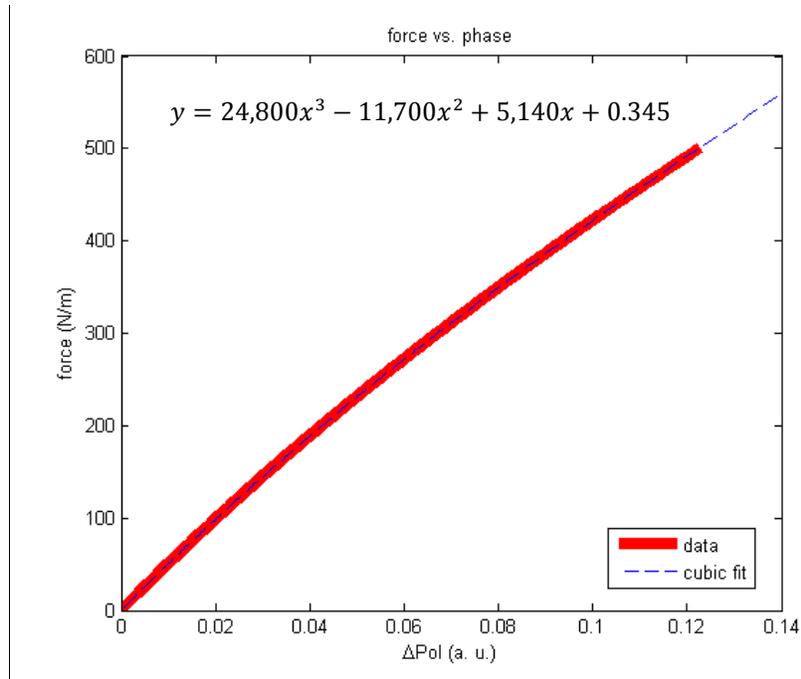


Fig. S4: Calibration curve generated from a Matlab script. The red solid line shows the relationship between applied force (f) and polarization state (ΔPol) derived from the theoretical fit of the circle-trace. The blue dashed line shows a cubic fit of the plotted relationship. The cubic fit equation is shown and is used as a calibration curve to calibrate the raw data in the form of ΔPol and convert it first to applied force (f) and finally applied stress.

2 System Configuration

The sensor described in this work is portable in the sense that it may be moved easily without excessive calibration or alignment each time. It is important to note that portable is distinct from handheld. Throughout the process of performing the present series of experiments, the system was transported in a backpack to and from the materials analysis lab in a different building by one person repeatedly and was set-up within minutes. The main components of the sensor easily fit within a standard backpack and may be packed up without any damage to them. Additionally, minimal set-up time is needed due to the lack of free space optics which require alignment. A photo of the experimental setup with a sample is shown in Fig. S5. It is shown with a four wavelength laser. Clearly, this component could be further reduced in size.

Another requirement of a portable sensor is that it may operate in several environments. As indicated by the noise measurements and sensitivity calculations, the sensor operation is affected by noise. That being said, it shows an acceptably low noise level (low enough to measure the Young's modulus in biomaterials) in all the environments in which we measured the noise. This includes a materials analysis lab which had other experiments going on in parallel, such as stress/strain and fracture measurements. These measurements cause extremely high mechanical vibrations, significantly higher than levels experienced in a normal lab setting. This level is clearly evident in the measurements on a normal lab countertop and in a laminar flowhood, which are below the materials analysis lab values.



Fig. S5: Photo of the components of the fiber stress sensor system on a table. The laptop to the right is used to control the polarimeter and log data and gives a sense of the scale of the other components.

3 Noise and sensitivity

3.1 Noise

In order to evaluate the robustness of our sensor system in different environments, noise measurements were performed in four locations: the mechanical materials analysis lab where the primary experiments were performed, in an optical device characterization lab on a vibration isolating optical table, on a countertop in a chemistry lab, and inside a laminar flowhood inside a chemistry lab.

In the materials analysis lab where the primary experiments were performed, the fiber sensor was set up on the load frame in exactly the same configuration as during other experiments. However, the measurements were taken with no sample on top of the fiber. These measurements had very high noise levels due to environmental vibrations. Therefore, to more rigorously establish a baseline noise threshold, we performed additional measurements on an optical table, on a countertop, and inside a laminar flowhood. In these offsets of measurements, the fiber sensor was set up in a similar manner to the primary experiments with a similar length of fiber, and the PM fiber was taped down to the each surface (optical table, countertop, flowhood) where the measurement was performed.

In all sets of measurements, several sets of data were measured at both wavelengths and the polarization state was analyzed as described in section 1. After the data was analyzed, we evaluated the noise distribution, which can be seen in histograms in Fig. S6. By fitting the distribution to a gaussian or normal function, we can determine the noise threshold for each set of experimental conditions.

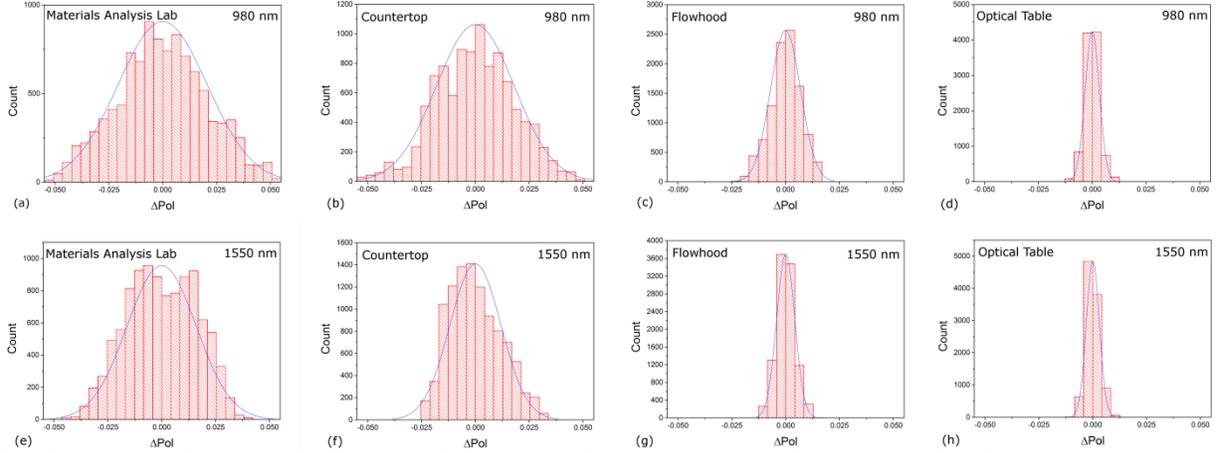


Fig. S6: Noise histograms characterizing the noise level of our fiber sensor in different environments and at different testing wavelengths of light, along with their normal distribution curves. a-d) Histograms for 980 nm light taken a) in the materials analysis lab with the sensor set up on the load-frame, b) in a chemistry lab on a countertop, c) inside of a laminar flowhood, and d) in an optical device characterization lab with the sensor set up on a vibration isolation table. e-h) Histograms for 1550 nm light taken e) in the materials analysis lab with the sensor set up on the load-frame, f) in a chemistry lab on a countertop, g) inside of a laminar flowhood, and h) in an optical device characterization lab with the sensor set up on a vibration isolation table.

3.2 Sensitivity

Sensitivity was calculated using the noise measurements. First, Matlab was used to calculate a relationship between ΔPol , applied stress (σ), and interaction length (l) assuming the optimum value for alpha. This relationship was plotted and fit to a function (with $R^2=1$) to generate an ideal operating curve that predicts the polarization change (ΔPol) for a given applied stress (σ) and interaction length (l). In practice, the operating curve may vary as alpha will not always have the optimum value in the experimental setup. The ideal operating curves for 980 nm (equation S7) and 1550 nm (equation S8) are given below:

$$\Delta Pol_{980} = (3.785 \times 10^{-5})\sigma l \quad (S7)$$

$$\Delta Pol_{1550} = (3.907 \times 10^{-5})\sigma l \quad (S8)$$

Once these ideal operating curves were obtained, sensitivity was calculated. Using the standard deviation of the measured noise for ΔPol and 18 mm for l (the approximate length of our samples), we calculated the applied stress. This stress corresponds to the standard level of noise in the system, so anything above this noise should be detectable using our sensor and data analysis method.