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Weakness of Weak Values: Incompatibility of Anomalous Pulse-spectrum Amplification and Optical Frequency Combs

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We probe the use of optical frequency combs to place lower bounds on anomalous amplification of a weak-value-measured pulse delay, potentially reaching a theoretical temporal resolution of better than 10^{-34} s. Owing to the interferometric behavior of weak values, we show that anomalous weak value amplification of a time delay is not equivalent to a temporal linear phase ramp. We show that the anomalous weak value is a rearrangement of amplitudes that generates an apparent shift that can be measured in direct detection, but does not change the actual frequency offset of a spectral distribution measurable in coherent detection. This implies that high-precision heterodyne beatnote frequency measurements of an optical frequency comb cannot be used in combination with the weak value amplification, at least in its currently conceived form. This unfortunate feature sheds light on the nature of anomalous weak values in shifting the pointer in a conjugate basis.

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Weak values, as conceived by Aharanov *et al.*, are a departure from the standard quantum mechanical measurement view [1–4]. A weak value is given by

$$A_w = \frac{\langle\phi_f|A|\phi_i\rangle}{\langle\phi_f|\phi_i\rangle}, \tag{1}$$

where $|\phi_i\rangle$ and $|\phi_f\rangle$ are pre- and postselection states. A weak value, as opposed to an expectation value, can be complex and, at least theoretically, unbounded. The enhanced nature of the weak value has led to a host of signal amplification experiments [5–10]. However, the complex nature of the weak value may be even more peculiar and remarkable than the raw amplification. Essentially, it allows the user to continuously tune the shift of a pointer between two conjugate domains, leading to real and anomalous weak values [11].

This paper explores the combination of the anomalous weak value amplification of a temporal pulse delay in conjunction with a high-precision optical frequency comb. If it were possible to combine these remarkable capabilities, unprecedented precision in the measurement of time delays, distance measurements, and phase could be measured. Brunner and Simon [12] (see, also, [10]) studied the effects of weak value projections for a differential time measurement τ for ultrashort pulses of time length σ. Remarkably, they showed that by creating an imaginary weak value, a short time differential could result in a frequency domain shift given by

$$\delta\omega \approx \frac{2\tau}{\sigma^2\phi}, \tag{2}$$

where $\phi \approx \langle\phi_f|\phi_i\rangle$ for small ϕ. The estimation of τ is then given by

$$\tau \approx \frac{\delta\omega\sigma^2\phi}{2}. \tag{3}$$

Using currently available systems, we can assume $\sigma = 10^{-15}$ and $\phi = 0.1$, leading to

$$\tau \approx 10^{-31}\delta\omega, \tag{4}$$

measured in seconds. Brunner and Simon then assumed that the best spectral resolution was given by spectrometers with a resolution of the order of 10 s of GHz. A spectrometer is a direct detection of the spectral intensity distribution of the pulse. The spectral resolution is a technical limitation of the spectrometer and not a fundamental limit. Using such a spectrometer resolution leads to subattosecond pulse delay measurements. After dividing by the speed of light to determine the physical displacement, the precision is of the order of 10 to 100 s of picometers. Such precision is routinely achieved by a standard cw interferometer.

We ask whether it is possible to find a fundamental lower bound to the time delay of a pulse using frequency combs [13,14], rather than using a spectrometer. Optical frequency combs have revolutionized time keeping by providing a stable and countable frequency reference to parts in 10^{19} [15]. They are realized in octave-spanning mode-locked pulse trains [13]. Optical frequency combs led to a number of important fundamental tests and applications [16–18]. One would hope that it is possible to combine the frequency resolution of an optical frequency comb with the anomalous weak value amplification of a short pulse spectral shift, yielding measurable pulse delays at or below 10^{-34} s.

Optical frequency combs can be understood from a few key parameters. The repetition rate f_r of the mode-locked pulse train sets the spectral spacing between the comb teeth. While the relative spacing between the teeth is fixed, the offset frequency f_0 is still a free parameter. The frequency of the nth tooth is spectrally defined by $f_n = f_0 + nf_r$. It is

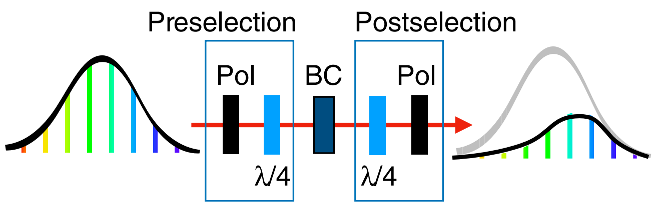

FIG. 1. Experimental schematic. An optical frequency comb spectrum is placed in an anomalous weak value configuration. A preselected state is chosen using a combination of a polarizer (Pol) and quarter wave plate (λ/4). A birefringent crystal (BC) imparts a relative phase time delay for each polarization. For simplicity, it is assumed that no transverse walk-off occurs in the crystal. The postselection is made with a second quarter wave plate and polarizer.

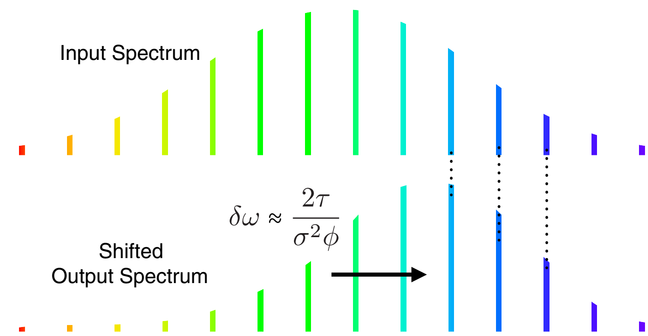

FIG. 2. Graphical depiction of input and output spectrum of the anomalous weak values interferometer. The comb teeth have not shifted, but the amplitudes have changed. It should be noted that the output spectrum will be considerably weaker than the input spectrum.

possible to determine f_0 by frequency doubling teeth from the long-wavelength portion of the spectrum and then heterodyne beating the frequency-doubled teeth with the teeth in the high-frequency portion of the spectrum, namely,

$$f_0 = 2(f_0 + nf_r) - (2nf_r + f_0). \tag{5}$$

Once determined, the frequency offset can be changed through modulation of the carrier envelope phase θ_e given by

$$f_0 = \frac{1}{2\pi}\frac{d\theta_e}{dt}. \tag{6}$$

This relationship is commonly understood through the shift theorem in Fourier transforms and is important for the final conclusion of this paper. Essentially, a linear ramp in the time domain has the effect of a frequency offset in the spectral domain. The ability to know and manipulate f_0 to very high accuracy allows the user to realize ultrahigh-precision frequency counting.

It should now be clear that the underlying parameters in a frequency comb are determined by coherent detection rather than direct detection. This is a departure from the typical direct detection techniques employed in weak value amplification experiments [5–9].

For the purposes of this paper, we assume a frequency comb with a Gaussian spectral envelope,

$$E(t) = A_0\Sigma_n^N \int d\omega e^{-(\omega-\omega_c)^2\sigma^2/4} e^{i\omega t}\delta(\omega - \omega_n), \tag{7}$$

where ω_c is the carrier frequency of the laser, $\omega_n = 2\pi(nf_r - f_0)$ is the frequency of the nth tooth, f_r is the spacing between frequency modes (comb teeth), and f_0 is the offset frequency [15].

We explore the effects of an anomalous weak value measurement on the spectral distribution of a frequency comb. Consider the experimental schematic in Fig. 1. Following Brunner and Simon, we use the preselected polarization state $|\phi_i\rangle = \frac{1}{\sqrt{2}}(ie^{i\phi}|H\rangle + e^{-i\phi}|V\rangle)$. This preselected state can be generated using a polarizer and quarter wave plate. The pulse is passed through a birefringent crystal, which, for demonstration purposes, we will assume has no transverse walk-off between the polarization modes. The crystal causes a pulse advance τ in the horizontal mode and pulse delay in the vertical mode. The postselection uses a second quarter wave plate and another polarizer creating the postselected state $|\phi_f\rangle = \frac{1}{\sqrt{2}}(|H\rangle + i|V\rangle)$. The output electric field is given by

$$E(t) = A_0\Sigma_n^N \int d\omega e^{-(\omega-\omega_c)^2\sigma^2/4} \sin(\omega\tau + \phi) e^{i\omega t}\delta(\omega - \omega_n). \tag{8}$$

We now make the approximation that $\omega\tau \ll \phi \ll 1$, which allows us to write $\sin(\omega\tau + \phi) \approx \phi(1 + \omega\tau/\phi) \approx \phi e^{\omega\tau/\phi}$, where one can see that ϕ now takes the form of a probability amplitude. The electric field is then

$$E(t) \approx \phi A_0\Sigma_n^N \int d\omega e^{-(\omega-\omega_c)^2\sigma^2/2} e^{\omega\tau/\phi} e^{i\omega t}\delta(\omega - \omega_n). \tag{9}$$

Upon completing the square, we obtain

$$E(t) \propto \phi A_0\Sigma_n^N \int d\omega e^{-(\omega-\omega_c+2\tau/\phi\sigma^2)^2\sigma^2/2} e^{i\omega t}\delta(\omega - \omega_n), \tag{10}$$

which is same spectral displacement as Brunner and Simon [see Eq. (2)]. Two effects should now be clear. First, the weak value shift of the pointer did not change the frequencies of the comb teeth. They are still set by the Dirac δ functions, which remain unchanged after the weak value operation. Second, the relative magnitude of the amplitudes of the comb teeth was reconfigured, giving the appearance of a spectral shift by enhancing the amplitude in some parts of the spectrum and decreasing the amplitudes in other parts of the spectrum. This is graphically shown in Fig. 2. The outcome could have been anticipated since weak values are an interferometric technique.

This feature points out a potential limitation of an anomalous weak value. As noted earlier, the frequency offset f_0 can be changed by a temporal linear ramp of the optical carrier envelope phase. Such linear phase ramps are commonly used to create frequency offsets in short pulses as they are a direct consequence of the Fourier shift theorem [19]. However, as we have just pointed out, a "frequency shift" using weak values does not actually result in a frequency shift of the comb teeth, but a rearrangement of the amplitudes of the spectrum to appear as a shift. Hence, a weak value anomalous amplification can mimic a shift in direct detection (e.g., a spectrometer), but not reproduce the coherent properties (e.g., heterodyne beating) of a linear phase ramp. This unfortunate result precludes the incredible precision possible by combining optical frequency combs and weak values.

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