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PBR, EPR, and all that jazz

Matt Leifer

In the past couple of months, the quantum foundations world has been abuzz about a new preprint entitled "The Quantum State Cannot be Interpreted Statistically" by Matt Pusey, Jon Barrett and Terry Rudolph (henceforth known as PBR). Since I wrote a blog post explaining the result, I have been inundated with more correspondence from scientists and more requests for comment from science journalists than at any other point in my career. Reaction to the result amongst quantum researchers has been mixed, with many people reacting negatively to the title, which can be misinterpreted as an attack on the Born rule. Others have managed to read past the title, but are still unsure whether to credit the result with any fundamental significance. In this article, I would like to explain why I think that the PBR result is the most significant constraint on hidden variable theories that has been proved to date. It provides a simple proof of many other known theorems, and it supercharges the EPR argument, converting it into a rigorous proof of nonlocality that has the same status as Bell's theorem. Before getting to this though, we need to understand the PBR result itself.

What are Quantum States?

One of the most debated issues in the foundations of quantum theory is the status of the quantum state. On the ontic view, quantum states represent a real property of quantum systems, somewhat akin to a physical field, albeit one with extremely bizarre properties like entanglement. The alternative to this is the epistemic view, which sees quantum states as states of knowledge, more akin to the probability distributions of statistical mechanics. A psi-ontologist (as supporters of the ontic view have been dubbed by Chris Granade) might point to the phenomenon of interference in support of their view, and also to the fact that pretty much all viable realist interpretations of quantum theory, such as many-worlds or Bohmian mechanics, include an ontic state. The key argument in favor of the epistemic view is that it dissolves the measurement problem, since the fact that states undergo a discontinuous change in the light of measurement results does not then imply the existence of any real physical process. Instead, the collapse of the wavefunction is more akin to the way that classical probability distributions get updated by Bayesian conditioning in the light of new data.

Many people who advocate a psi-epistemic view also adopt an anti-realist or neo-Copenhagen point of view on quantum theory in which the quantum state does not represent knowledge about some underlying reality, but rather it only represents knowledge about the consequences of measurements that we might make on the system. However, there remained the nagging question of whether it is possible in principle to construct a realist interpretation of quantum theory that is also psi-epistemic, or whether the realist is compelled to think that quantum states are real.

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PBR have answered this question in the negative, at least within the standard framework for hidden variable theories that we use for other no go results such as Bell's theorem. As with Bell's theorem, there are loopholes, so it is better to say that PBR have placed a strong constraint on realist psi-epistemic interpretations, rather than ruling them out entirely.

The PBR Result and its implications

To properly formulate the result, we need to know a bit about how quantum states are represented in a hidden variable theory. In such a theory, quantum systems are assumed to have real pre-existing properties that are responsible for determining what happens when we make a measurement. A full specification of these properties is what we mean by an ontic state of the system. In general, we don't have precise control over the ontic state so a quantum state corresponds to a probability distribution over the ontic states. This framework is illustrated in Figure 1.

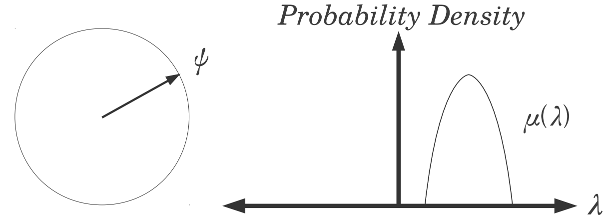

Figure 1. In a hidden variable theory, a quantum state (indicated heuristically on the left as a vector in the Bloch sphere) is represented by a probability distribution over ontic states, as indicated on the right.

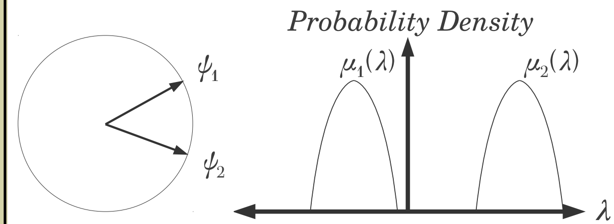

Figure 2. Representation of a pair of quantum states in a psi-ontic model.

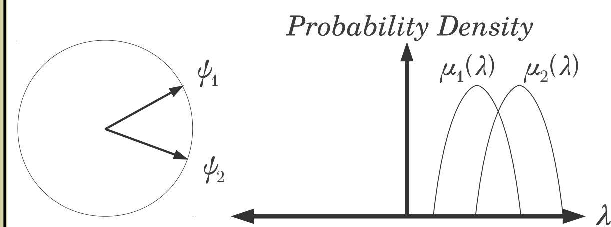

Figure 3. Representation of nonorthogonal states in a psi-epistemic model.

A hidden variable theory is psi-ontic if knowing the ontic state of the system allows you to determine the (pure) quantum state that was prepared uniquely. Equivalently, the probability distributions corresponding to two distinct pure states do not overlap. This is illustrated in Figure 2. A hidden variable theory is psi-epistemic if it is not psi-ontic, i.e. there must exist an ontic state that is possible for more than one pure state, or, in other words, there must exist two nonorthogonal pure states with corresponding distributions that overlap. This is illustrated in Figure 3.

These definitions of psi-ontology and psi-epistemicism may seem a little abstract, so a classical analogy may be helpful. In Newtonian mechanics the ontic state of a particle is a point in phase space, i.e. a specification of its position and momentum. Other ontic properties of the particle, such as its energy, are given by functions of the phase space point, i.e. they are uniquely determined by the ontic state. Likewise, in a hidden variable theory, anything that is a unique function of the ontic state should be regarded as an ontic property of the system, and this applies to the quantum state in a psi-ontic model. The definition of a psi-epistemic model as the negation of this is very weak, e.g. it could still be the case that most ontic states are only possible in one quantum state and just a few are compatible with more than one. Nonetheless, even this very weak notion is ruled out by PBR. The proof of the PBR result is quite simple, but I will not review it here. Rather, I refer the interested reader to the references below and, instead, focus on its implications.

A trivial consequence of the PBR result is that the cardinality of the ontic state space of any hidden variable theory, even for just a qubit, must be infinite, in fact continuously so. This is because there must be at least one ontic state for each quantum state, and there are a continuous infinity of the latter. The fact that there must be infinite ontic states was previously proved by Lucien Hardy under the name "Ontological Excess Baggage theorem", but we can now view it as a corollary of PBR. If you think about it, this property is quite surprising because we can only extract one or two bits from a qubit (depending on whether we count superdense coding) so it would be natural to assume that a hidden variable state could be specified by a finite amount of information.

Hidden variable theories provide one possible method of simulating a quantum computer on a classical computer by simply tracking the value of the ontic state at each stage in the computation. This enables us to sample from the probability distribution of any quantum measurement at any point during the computation. Another method is to simply store a representation of the quantum state at each point in time. This second method is clearly

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inefficient, as the number of parameters required to specify a quantum state grows exponentially with the number of qubits. The PBR theorem tells us that the hidden variable method cannot be any better, as it requires an ontic state space that is at least as big as the set of quantum states. This conclusion was previously drawn by Alberto Montina using different methods, but again it now becomes a corollary of PBR. This result falls short of saying that any classical simulation of a quantum computer must have exponential space complexity, since we usually only have to simulate the outcome of one fixed measurement at the end of the computation and our simulation does not have to track the slice-by-slice causal evolution of the quantum circuit. Indeed, pretty much the first nontrivial result in quantum computational complexity theory, proved by Bernstein and Vazirani, showed that quantum circuits can be simulated with polynomial memory resources. Nevertheless, this result does reaffirm that we need to go beyond slice-by-slice simulations of quantum circuits in looking for efficient classical algorithms.

As emphasized by Harrigan and Spekkens, a variant of the EPR argument favoured by Einstein shows that any psi-ontic hidden variable theory must be nonlocal. Thus, prior to Bell's theorem, the only open possibility for a local hidden variable theory was a psi-epistemic theory. Of course, Bell's theorem rules out all local hidden variable theories, regardless of the status of the quantum state within them. Nevertheless, the PBR result now gives an arguably simpler route to the same conclusion by ruling out psi-epistemic theories, allowing us to infer nonlocality directly from EPR.

A sketch of the argument runs as follows. Consider a pair of qubits in the singlet state. When one of the qubits is measured in an orthonormal basis, the other qubit collapses to one of two orthogonal pure states. By varying the basis that the first qubit is measured in, the second qubit can be made to collapse in any basis we like (a phenomenon that Schroedinger called "steering"). If we restrict attention to two possible choices of measurement basis, then there are four possible pure states that the second qubit might end up in. The PBR result implies that the sets of possible ontic states for the second system for each of these pure states must be disjoint. Consequently, the sets of possible ontic states corresponding to the two distinct choices of basis are also disjoint. Thus, the ontic state of the second system must depend on the choice of measurement made on the first system and this implies nonlocality because I can decide which measurement to perform on the first system at spacelike separation from the second.

PBR as a proto-theorem

We have seen that the PBR result can be used to establish some known constraints on hidden variable theories in a very straightforward way. There is more to this story that I can possibly fit into this article, and I suspect that every major no-go result for hidden variable theories may fall under the rubric of PBR. Thus, even if you don't care a fig about fancy distinctions between ontic and epistemic states, it is still worth devoting a few brain cells to the PBR result. I predict that it will become viewed as the basic result about hidden variable theories, and that we will end up teaching it to our students even before such stalwarts as Bell's theorem and Kochen-Specker.

Matt Leifer *is a postdoc at University College London. He obtained his Ph.D. in quantum information from the University of Bristol in 2004, and has since worked at the Perimeter Institute, the University of Waterloo, and the University of Cambridge. His research is focused on problems at the intersection of quantum foundations and quantum information. See* http://mattleifer.info *for more details.*

Further Reading

Blog post with discussion of proof:
http://mattleifer.info/2011/11/20/can-the-quantum-state-be-interpreted-statistically/

The PBR paper:
M. Pusey, J. Barrett, T. Rudolph, (2011). http://arxiv.org/abs/1111.3328

For constraints on the size of the ontic state space see:
L. Hardy, *Stud. Hist. Phil. Mod. Phys*. **35**:267-276 (2004).
A. Montina, *Phys. Rev. A* **77**, 022104 (2008). http://arxiv.org/abs/0711.4770

For the early quantum computational complexity results see:
E. Bernstein and U Vazirani, *SIAM J. Comput.* **26**:1141-1473 (1997). http://arxiv.org/abs/quant-ph/9701001

For a fully rigorous version of the PBR+EPR nonlocality argument see:
N. Harrigan and R. W. Spekkens, *Found. Phys*. **40**:125 (2010). http://arxiv.org/abs/0706.2661