Review of *The Mathematical Language of Quantum Theory: From Uncertainty to Entanglement*

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Comments
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BOOK REVIEW

The Mathematical Language of Quantum Theory: From Uncertainty to Entanglement
by Teiko Heinosaari and Mario Ziman
Cambridge University Press, 2012, **$85.00**
ISBN-13: 9780521195836 (hardcover)

Back in the dark days of early 2000, when I was first starting my Ph.D., it was not very easy to learn about the generalized quantum formalism of density operators, Positive Operator Valued Measures, and Completely Positive maps. Difficult as it may be to believe in this post Nielsen and Chuang age, most books on these topics were long out of print and university libraries were not well stocked in them. Most of us had to rely on John Preskill's lecture notes and, excellent though these are, the generalized formalism is covered in a bit of a hurry in order to move on to all the exciting new results in quantum information and computation.

When Nielsen and Chuang appeared at the end of 2000, it was a revelation. For the first time, the formalism was explained cleanly, simply and accessibly. Still, Nielsen and Chuang's treatment is not unproblematic. For one thing, it is restricted to finite dimensional Hilbert spaces. This is not ideal for those who want a bit more mathematical sophistication, or those who are working primarily with continuous variable systems. It is also fairly odd that we have to rely on a quantum information textbook to learn this material, since the generalized formalism is applicable to a wide variety of other subjects, such as the theory of open quantum systems, quantum statistical mechanics, algebraic quantum field theory, and the foundations of quantum theory. What is needed then is a book that covers the generalized formalism with more care and attention to detail, that is not also burdened with the task of covering all of quantum information and computation, and that is accessible to novices. The short version of this review is that Heinosaari and Ziman have written this book, and it would have made my life much easier had it been available when I was a Ph.D. student. I will be recommending it to any research student that works with me in the future as the first thing that they should read.

Heinosaari and Ziman have struck the right note with regard to mathematical rigour. They present most of the material in a way that is applicable to infinite dimensional Hilbert spaces, but do not shy away from specializing to finite dimensions or to discrete observables when the general proofs would involve heavy functional analysis. In this way, the book is very readable for people from a variety of backgrounds, provided they have a reasonable understanding of the basics of quantum theory. The book is well referenced, so it is very easy to look up the more general proofs as and when necessary. This is a significant plus point for the book, as most books on this topic either require a sophisticated knowledge of functional analysis or they restrict themselves entirely to the finite dimensional case.

The structure of the first five chapters of the book is coherent and logical, starting with a review of Hilbert space and then building up the formalism in increasing generality, from states and effects (yes/no measurements) to general observables to quantum channels and finally quantum instruments, which describe the most general way of updating a quantum system after a measurement. I particularly liked the chapter on quantum instruments and its careful treatment of how these are related to the description of measurements in terms of unitary evolution and POVMs. The text provides proofs of several fundamental theorems that are usually just stated or omitted in most quantum information texts, such as the proof that all mixture-preserving automorphisms of the set of density operators are unitary or anti-unitary. This level of thoroughness is very useful for those interested in the foundations of quantum theory. The

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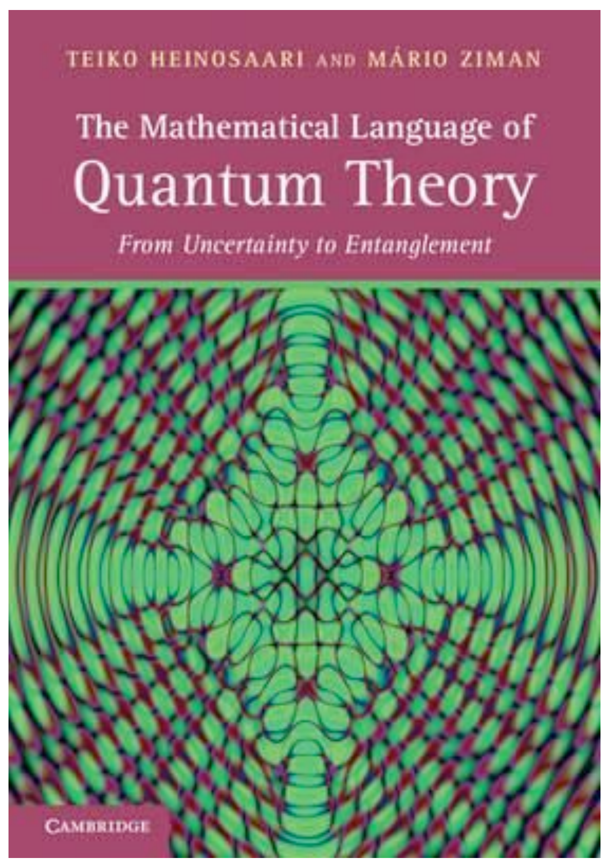

book also touches on topics of recent research that are not covered in other texts, such as the coexistence of POVMs and the existence of Symmetric-Informationally-Complete POVMs. Given the scope of the book, the treatment of these topics is necessarily introductory, but it does serve to give the reader a flavor of current research.

Although this is a very good book, it is not completely perfect, and there are three main areas in which it could be improved. Firstly, there are many exercises for the student within the text, and these usually come with helpful hints of how to go about solving them. However, in many cases reading the hints makes the exercises completely trivial to solve. This would not be so bad if the hints were collected at the end of the book, but in fact they appear immediately after the exercises themselves, so it is very difficult to stop yourself from reading them before thinking about the problem. Hopefully these can be moved to the back in future editions, or at least made a little more cryptic. The book's website states that a collection of problems and solutions are in preparation, which may also help with this problem.

Secondly, whilst there is obviously a subjective element to the selection of material for a textbook, there are some topics that I would definitely add to it if I were teaching a course on this subject. The first is the proof of the Stinespring dilation theorem, which is used, for example, to prove that all quantum channels can be represented as unitary evolutions on a larger Hilbert space. The book states the theorem, but does not provide a proof. This strikes me as an odd omission given the central importance of the theorem and the fact that it is not too hard to prove it in finite dimensions (perhaps it was omitted because the general version is rather more difficult). Secondly, there is some discussion of the fixed point sets of some specific quantum channels, e.g. phase damping channels, but the general characterization of the fixed-point sets of quantum channels is not given. This is a shame as it is a very useful theorem in many applications, such as quantum error correction. It also allows for a simple proof of the no-broadcasting theorem, which is another significant omission. Although the no-cloning theorem is discussed, it would have been good to have a proof of no-broadcasting given that the book emphasizes mixed states.

Finally, the book ends with what the blurb describes as "a separate chapter on entanglement". It really is quite separate, in the sense that it does not really follow the logical development of the rest of the book. One has to wonder whether it was included simply because entanglement it is a trendy topic (and whether it will be replaced by a chapter on quantum discord in the second edition). This would not be so bad if it was a good review of entanglement, but unfortunately I do not think it is up to the excellent standard of the rest of the book. First of all, it does not do a good job of explaining why we should care about entanglement in the first place. Teleportation is not covered until the end of the chapter in a section entitled "Additional topics in entanglement theory". This belies its status as perhaps the key example of using entanglement as a resource for quantum communication. Similarly, the discussion of Bell's theorem is relegated to almost a side-note after the discussion of entanglement witnesses. Both of these things should be discussed up front as motivation for the more detailed discussion of entanglement. Secondly, I think that any review of entanglement really ought to include the asymptotic theory of conversion between bipartite pure states, and the consequent identification of the entanglement entropy as the unique measure of entanglement. It is hard to understand why ersatz facts about isolated classes of mixed entangled states, LOCC channels, and separability criteria are considered more important than this. In fact, I consider most of the material covered in the main body of the chapter to be "additional topics",

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Review, continued

of interest mainly to specialists, whilst the main points of entanglement theory have been entirely missed or skirted over very quickly.

Given that I have ended with the negatives, let me remind you that the first five chapters of this book are excellent. Beginning graduate students should definitely read it and it provides an essential reference for those teaching courses that involve the generalized quantum formalism. Just maybe tear out chapter six before lending it to your students.

***Matt Leifer** is one of the two people behind the innovative new online quantum information and foundations seminars known as Q+ (the other is Daniel Burgarth). He is also on the Editorial Board of the very publication you are now reading.*

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