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Andrew N. Jordan
University of Rochester

Areeya Chantasri
University of Rochester

Kater Murch
Washington University in St. Louis

Justin Dressel
Chapman University, dressel@chapman.edu

Alexander N. Korotkov
Chapman University

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Janus Sequences of Quantum Measurements and the Arrow of Time

Andrew N. Jordan[1,2,3], Areeya Chantasri[1,2], Kater Murch[4], Justin Dressel[3,5] and Alexander N. Korotkov[6]

[1]*Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627, USA*
[2]*Center for Coherence and Quantum Optics, University of Rochester, Rochester, New York 14627, USA*
[3]*Institute for Quantum Studies, Chapman University, 1 University Drive, Orange, CA 92866, USA*
[4]*Department of Physics, Washington University, St. Louis, Missouri 63130, USA*
[5]*Department of Electrical and Computer Engineering, University of California, Riverside, California 92521, USA*
[6]*Schmid College of Science and Technology, Chapman University, 1 University Drive, Orange, CA 92866, USA*

Abstract. We examine the time reversal symmetry of quantum measurement sequences by introducing a forward and backward Janus sequence of measurements. If the forward sequence of measurements creates a sequence of quantum states in time, starting from an initial state and ending in a final state, then the backward sequence begins with the time-reversed final state, exactly retraces the intermediate states, and ends with the time-reversed initial state. We prove that such a sequence can always be constructed, showing that unless the measurements are ideal projections, it is impossible to tell if a given sequence of measurements is progressing forward or backward in time. A statistical arrow of time emerges only because typically the forward sequence is more probable than the backward sequence.

INTRODUCTION

The question of the arrow of time is fundamental to many areas of theoretical physics [1, 2, 3]. Any possibility of retrocausality must first confront the question of the seeming irreversibility of quantum measurement. The collapse of the wavefunction gives an arrow of time to quantum physics, and an outstanding question is whether this is a fundamental arrow of time or not. In this conference proceeding, we will show that even with quantum measurements included in the analysis, the quantum state dynamics may still be time-reversal invariant, provided the measurements are not ideal projections.

In classical physics, it is well known that the dynamical laws are time-reversal invariant. Any collection of particles, described with their position and momentum coordinates $\{x_i, p_i\}$, has a dynamics that can be time-reversed (in the absence of friction) simply by reversing the momentum of each particle. This transformation gives rise to a fully legitimate forward dynamical evolution that corresponds to the time-reversed dynamics. This is most easily visualized by considering a movie of - for example - the opening break of a game of pool, where the cue ball disperses the rest of the pool balls around the table (Fig. 1). By playing the movie backwards (effectively reversing the momentum of each ball while keeping the position the same), the balls converge together, coming to a stop in their triangular shape, and jettisoning the cue ball out with high velocity. This example both illustrates the valid time-reversal of the equations of motion, as well as the "statistical arrow of time": we can infer the arrow of time because of the statistical likelihood that the system will progress from ordered to disordered.

It is also the case in quantum mechanics that a closed quantum system is time reversible. By applying an anti-unitary time reversal operator to the quantum state, further forward propagation results in the time-reversed dynamics [4]. One exception to this is if there is an external magnetic field - in that case, the sign of the field must also be reversed as in classical physics. However, the introduction of quantum measurement is generally thought to break the time-symmetry of the system. Quantum measurement introduces wavefunction collapse that is irreversible, thus giving a clear time-arrow. However, it was shown that in the case where measurements are generalized to POVMs, that such

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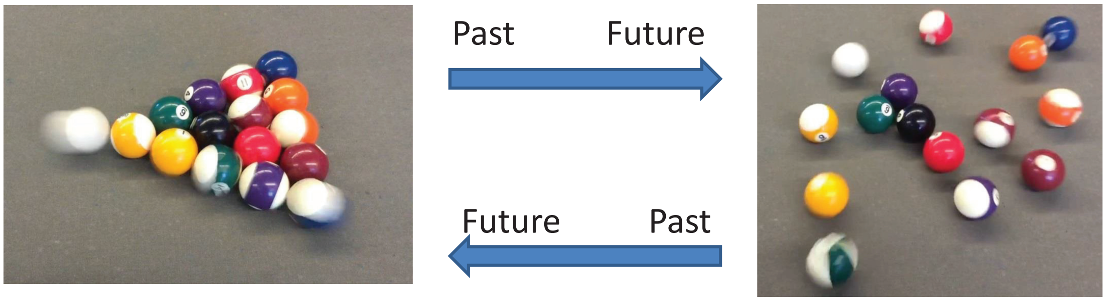

FIGURE 1. In classical physics, the dynamics is time-reversal invariant. The starting break of a game of pool (left-to-right) results in a disordered arrangement of pool balls. However, if we time reverse the momenta of the balls, then the dynamics (in the absence of friction) will reverse itself (right-to-left). Therefore the dynamics of the system along cannot distinguish the arrow of time. Only a likelihood argument (it is more likely that the dynamics is from left-to-right, or greater to lesser order) enables the inference of the statistical arrow of time. We note that the initial condition is the special one.

a measurement may be stochastically reversed, unless the measurement is a perfect projection operator [5, 6]. This suggests that for any physical measurement process, a dynamical time symmetry may be restored. Indeed, we will show in the remainder of this article that is the case. Other approaches to restoring time symmetry rely on imposing boundary conditions on the quantum state, such as Aharonov, Bergmann, and Lebowitz's "two-time formalism" [7]. We note that the phenomenon of "wavefunction uncollapse" by a reversing measurement has strong experimental support [8, 9, 10].

TIME REVERSAL SYMMETRY FOR JANUS MEASUREMENT SEQUENCES

We now show how it is possible to restore time reversal symmetry for a time sequence of generalized measurements. We begin with a systematic treatment of the time reversal operation. We introduce the anti-unitary time-reversal operator Θ that operationally reverses the direction of time [11]. The anti-unitary action of the operator gives the correct time-reversed actions on familiar operators such as position, $\Theta\mathbf{x}\Theta^{-1} = \mathbf{x}$, momentum $\Theta\mathbf{p}\Theta^{-1} = -\mathbf{p}$, and spin $\Theta\mathbf{S}\Theta^{-1} = -\mathbf{S}$. We consider dynamics whose system Hamiltonian is time-reversal invariant, $\Theta\mathbf{H}\Theta^{-1} = \mathbf{H}$.[1] When applied to the quantum state $|\psi(t)\rangle$, the time-reversal operator Θ, together with forward evolution, will "rewind" the dynamics back to $\Theta|\psi(0)\rangle$. In the case of position-space wavefunctions, the time-reversal operator is just the complex conjugate.

We now consider the possibility of extending this arrangement to include time sequences of generalized measurements in addition to unitary dynamics. To do so, consider two such time sequences, named the *forward Janus sequence* and the *backward Janus sequence*, for reasons that will become clear. The forward Janus sequence is a series of measurements in time, $A, B, C, \ldots$, with possible measurement results $a, b, c, \ldots$, with unitary evolution included in the generalized measurements. Here, the discussion is quite general, so the measurement results may be discrete or continuous variables. We illustrate the forward and backward Janus sequences in Fig. 2. Let us consider an initial state $|\Psi\rangle$, so each generalized measurement creates an effect on the quantum system, described by a sequence of measurement operators (or Kraus operators), $M_a^A, M_b^B, M_c^C, \ldots$, such that the final state $|\Phi\rangle$ is proportional to

$$|\Phi\rangle \propto \ldots M_c^C M_b^B M_a^A |\Psi\rangle. \tag{1}$$

We now introduce the backwards Janus sequence, which is another series of (in general) different measurements $\ldots, C', B', A'$ applied sequentially to the system in "initial" state $\Theta|\Phi\rangle$, but in the reverse order. Let the results of said measurements be $\ldots, c', b', a'$, such that for some *possible* measurements $m = A, B, C, \ldots; m' = \ldots, C', B', A'$ with results $j = a, b, c, \ldots; j' = \ldots, c', b', a'$ of both sequences, the system state rewinds its path, and is restored to its initial (time-reversed) state:

$$M_{a'}^{A'} M_{b'}^{B'} M_{c'}^{C'} \ldots \Theta|\Phi\rangle \propto \Theta|\Psi\rangle. \tag{2}$$

FIGURE 2. The forward and backward Janus sequences: Starting from an initial state, a forward Janus sequence of generalized measurements $A, B, C, \ldots$ creates a time-sequence of quantum states, ending with a final state. A backward Janus sequence starts with the final states, and applies a generalized measurement sequence $\ldots, C', B', A'$ to "rewind" the movie. The state now starts in the previous final state, and a sequence of measurements creates the time-reversed sequence of state disturbances.

We can find the condition for this to happen by inserting $\mathbf{1} = \Theta^{-1}\Theta$ between every pair of operators, to find in addition to the usual time-reversal invariance of the Hamiltonian, the condition

$$M_{j'}^{m'} \propto (\Theta M_j^m \Theta^{-1})^{-1}. \tag{3}$$

That is, each measurement operator of the backward Janus sequence must be proportional to the *inverse* time-reversed measurement operator of the forward Janus sequence. This condition may be understood as an application of quantum measurement uncollapse.

We now prove that we can always construct such a measurement operator from another POVM set. If M_j^m is the forward measurement operator, then $M_{j'}^{m'} = \sqrt{c_j^m}\Theta\left(M_j^m\right)^{-1}\Theta^{-1}$ is a possible backward operator, where c_j^m is an undetermined proportionality constant. Let us decompose the operator into a unitary part and a non-unitary part, $M_j^m = U_j^m\sqrt{E_j^m}$. The unitary part can always be time-reversed in the usual way, so we focus on the non-unitary part. The inverse of the operator $\sqrt{E_j^m}$ cannot be a physical operation in general because the eigenvalues of E_j^m, named λ_j^m, are less than or equal to 1. This must be so to satisfy the POVM relation $\sum_j E_j^m = \mathbf{1}$, where we sum over all possible results of measurement m, and guarantee that the probability of measuring the particular result j (given by $\mathrm{Tr}\rho E_j^m$) is a true probability. The inverse operation will therefore have eigenvalues $(\lambda_j^m)^{-1} \geq 1$, which cannot correspond to a valid POVM. We can fix this by adjusting the proportionality constant c_j^m, so the inverse POVM elements are given by $F_{j'}^{m'} = c_j^m(E_j^m)^{-1}$, where c_j^m is chosen so that the new POVM elements satisfy $\sum_{j'} F_{j'}^{m'} = \mathbf{1}$, where we sum over all possible results of measurement m', and every eigenvalue of $F_{j'}^{m'}$ lies between 0 and 1. In particular, the last constraint demands that $c_j^m/\min_j \lambda_j^m \leq 1$, which bounds c_j^m. The bound on c_j^m also bounds the probability of reversing the measurement [6]. This proves that it is always possible to construct another POVM element which corresponds to the inverse of the first POVM element. While there are an infinite number of choices for the inverse operation, we give here a 2-outcome POVM example, defined by the elements $\{F_a^{m'}, F_b^{m'}\}$ at sequence step m'. The first element, F_a, optimally satisfies our requirements above, $F_a^{m'} = (\min_j \lambda_j^m)(E_j^m)^{-1}$, while the second element, $F_b^{m'}$, completes the set, $F_b^{m'} = \mathbf{1} - (\min_j \lambda_j^m)(E_j^m)^{-1}$. The measurement operators $M_{j'}^{m'}$ are given by the square root of the two POVM elements, times the appropriate adjoint unitaries to reverse U_j^m.

As alluded to above, there is no guarantee that such sequences will happen; however, what is important for our argument, is that such a pair of sequences is *possible*. For such pairs of sequences, the time reversal symmetry is restored for the system's quantum state. Now consider a game, where a given sequence of measurements with their results is presented to us, together with a movie of the dynamics of the quantum state. We are not told if the sequence of measurements is $(A, B, C, \ldots)$, corresponding to the forward movie, or is instead $(\ldots, C', B', A')$, corresponding to the backward movie. We must then guess whether the movie is running backward or forward in time, based on the results of the measurements. There is no way to tell for certain, since either sequence of forward or backward measurement results is very unlikely for a long sequence of measurements, and each step in the quantum state movie, in either direction, corresponds to a possible forward evolution. In absence of special constraints (such as postselection), we can still statistically discern an arrow of time, by finding which sequence is more probable, relative to the other sequence, in close analogy to arguments in statistical mechanics. Define the collective forward Janus measurement operator as $M_F = \cdots M_c^C M_b^B M_a^A$, and the collective backward Janus measurement operator as $M_B = M_{a'}^{A'} M_{b'}^{B'} M_{c'}^{C'} \cdots$, where the product is reversed in time. The probability of all of the measurement results, given (known) forward or reverse Janus sequences is $P_F(a, b, c, \ldots) = \|M_F|\Psi\rangle\|^2$, or $P_B(\ldots, c', b', a') = \|M_B\Theta|\Phi\rangle\|^2$. We can now use these as likelihood functions to test the two hypotheses (forward or backward), given only the measurement results. The ratio of the likelihood functions, $\mathcal{R} = P_F/P_B$ for a given sequence will serve as our arrow of time estimator. We note that if the number of outcomes of the forward and backward measurements is the same (the same type of measurement), then on average, the forward sequence will be more probable than the backward sequence. However, if the forward sequence is composed of many-outcome measurements, then we can construct (for example) a two-outcome backward sequence that will be more probable on average.

CONTINUOUS TIME SYMMETRY

For the special case of observables with only two eigenvalues, we can readily construct such a Janus sequence for the diffusive continuous measurement case. To see why this is so, we consider the combination of unitary dynamics followed by partial measurement collapse, such that a density operator ρ changes after a time-step δt (up to normalization) as $\rho \to M_r U \rho U^\dagger M_r^\dagger$, where U is a unitary operator, and M_r is a measurement operator indexed by a suitably normalized result r that we take to be a continuous variable. For a diffusive measurement to have a sensible continuum limit, it must come from a valid Gaussian POVM $E_r = M_r^\dagger M_r \propto \exp(-\delta t(r - A_h)^2/2\tau)$, where A_h is the Hermitian observable being monitored, and τ is a characteristic measurement timescale (inverse strength). In this limit as $\delta t \to 0$, a succession of independent Gaussian timesteps then produces a readout $r(t)$ that is a stochastic process $r(t) = \bar{A}_h(t) + \sqrt{\tau}\,\xi(t)$, where $\bar{A}_h = \mathrm{Tr}[\rho A_h]$ is the moving average of A_h.

In the same continuum limit, the unitary dynamics may be written to first order as $U \approx 1 - i\delta t H/\hbar$, where H is the Hamiltonian. The Gaussian POVM $E_r = M_r^\dagger M_r$ naturally factors as $M_r \propto \exp(i\delta t\, r A_{ah}/2\tau - \delta t(r - A_h)^2/4\tau)$, where iA_{ah} is an anti-Hermitian operator that describes additional phase backaction. To first order this yields $M_r \propto 1 + \delta t(r/2\tau)A + \delta t A_h^2/4\tau$, where $A \equiv A_h + iA_{ah}$ contains both Hermitian and anti-Hermitian parts. The r-independent term with A_h^2 is not simply reversible while retaining the record r; however, this term may be easily reversed by a Gaussian POVM for any observable whose square is a constant c^2 (implying A_h has eigenvalues of only $\pm c$). As such, in what follows we will assume the form $A = \alpha \cdot \sigma$ of an effective qubit with Pauli matrix vector σ, so $A_h = \mathrm{Re}(\alpha) \cdot \sigma$ and $A_{ah} = \mathrm{Im}(\alpha) \cdot \sigma$. Similarly, we assume a general qubit Hamiltonian $H = \hbar\mathbf{\Omega} \cdot \sigma/2$. These considerations then lead to a (Markovian) stochastic differential equation for the normalized qubit state ρ

$$\frac{d\rho}{dt} = \frac{1}{i\hbar}[H, \rho] + \frac{r}{\tau}\left[\frac{A\rho + \rho A^\dagger}{2} - \mathrm{Tr}\left[\frac{A + A^\dagger}{2}\rho\right]\rho\right], \tag{4}$$

expressed in the time-symmetric (Stratonovich) picture [12, 13] where $d\rho/dt \equiv \lim_{\delta t \to 0}[\rho(t + \delta t) - \rho(t - \delta t)]/2\delta t$.

We now examine the requirements for time-reversal symmetry of Eq. (4). The time reversed solution, $\tilde{\rho}(t) = \Theta\rho(T - t)\Theta^{-1}$, must satisfy the same equation of motion (4). Direct calculation indicates that is true, provided we transform to (time-reversed) operators, $\tilde{H} = \Theta H \Theta^{-1}$, and $\tilde{r}(t)\tilde{A} = -r(T - t)\Theta A \Theta^{-1}$. This transformation is a special case of our general Janus criterion (3). On physical grounds for a spin, we take the Pauli matrix vector to flip sign under time reversal, $\Theta\sigma\Theta^{-1} = -\sigma$, but it is straightforward to generalize this to flip the sign of only one of the Pauli matricies for a general pseudo-spin [15]. The full inversion gives the time-reversed quantities, $\tilde{\mathbf{\Omega}} \cdot \tilde{\sigma} = -\mathbf{\Omega} \cdot \sigma$, and $\tilde{\alpha} \cdot \tilde{\sigma} = -\alpha^* \cdot \sigma$. We can define time reversed quantities in one of two ways. The first is an active transformation, which keeps the reference frame the same ($\tilde{\sigma} = \sigma$), and the second is a passive transformation, which inverts the reference frame into a left-handed system, ($\tilde{\sigma} = -\sigma$), thus changing the commutator structure. The active transformation (a) dictates the mappings, $\tilde{r}_a(t) = r(T - t)$, $\tilde{\mathbf{\Omega}}_a = -\mathbf{\Omega}$ (analogous to inverting an external magnetic field), and $\tilde{\alpha}_a = -\alpha^*$ (measuring the negated observable and reversing phase backaction), together with an inversion of the components of the Bloch coordinates, $\tilde{x}_a(t) = -x(T - t), \tilde{y}_a(t) = -y(T - t), \tilde{z}_a(t) = -z(T - t)$, which actively flips the spin. On the other hand, the passive transformation (p) inverts the sign of the measurement readout, $\tilde{r}_p(t) = -r(T - t)$, keeps the energy definitions the same $\tilde{\mathbf{\Omega}}_p = \mathbf{\Omega}$, and measures the same observable with reversed phase backaction, $\tilde{\alpha}_p = \alpha^*$, while preserving the coordinates in this frame, $\tilde{x}_p(t) = x(T - t), \tilde{y}_p(t) = y(T - t), \tilde{z}_p(t) = z(T - t)$. We give an illustration of the passive time-reversal transformation in Fig. 3, for the case where $A = \sigma_z$, so the observable is Hermitian, and

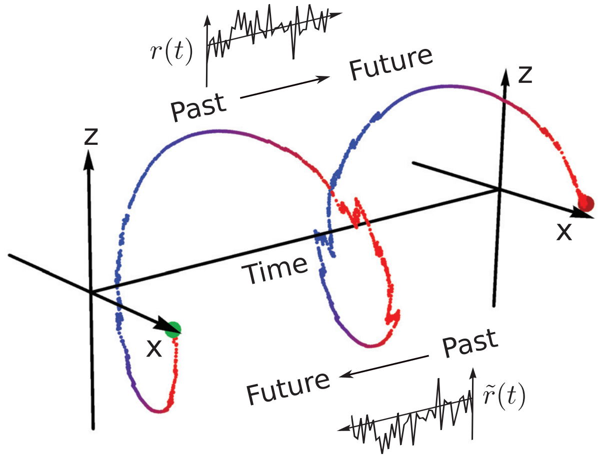

FIGURE 3. (After Ref. [14]). Example of time-reversal invariance for continuous qubit measurements: Red corresponds to positive values of the x Bloch sphere coordinate; blue corresponds to negative values of x. Is the movie being shown forward in time with continuous measurement result $r(t)$, or backward in time with flipped measurement result $\tilde{r}(t)$? In the quantum measurement example, it is the final state of (forward) measurement sequence that is the statistically "special" boundary state.

$H = \hbar\Omega\sigma_y/2$, which drives Rabi oscillations in the $x-z$ plane of the Bloch sphere. In this case, the Bloch coordinates are invariant, while the measurement record flips sign under the time-reversal transformation.

CONCLUSIONS

We have demonstrated that it is possible to restore time reversal symmetry to a sequence of quantum operations that includes unitary operations, as well as generalized measurements. For any given sequence of such measurements, called the forward Janus sequence, we can construct another sequence of measurements, the backward Janus sequence, where the quantum state dynamics is time-reversed. It then follows that we cannot know for certain whether a given temporal sequence of measurements is progressing forward in time, or backward in time. However, based on the relative likelihood of observing one sequence or the other, we may infer a statistical arrow of time: the sequence which is the most likely defines our best estimate of the arrow of time. Continuous measurements have been given as an illustration of this time reversal invariance for the especially simple qubit measurement case, where the time-reversed measurement has the same form as the forward measurement, but with an inverted measurement result.

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