

Chapman University

Chapman University Digital Commons

ESI Working Papers

Economic Science Institute

3-2-2023

Failed Secular Revolutions: Religious Belief, Competition, and Extremism

Jean-Paul Carvalho
University of Oxford

Jared Rubin
Chapman University, jrubin@chapman.edu

Michael Sacks
Clarkson University

Follow this and additional works at: https://digitalcommons.chapman.edu/esi_working_papers



Part of the [Econometrics Commons](#), [Economic Theory Commons](#), and the [Other Economics Commons](#)

Recommended Citation

Carvalho, J.-P., Rubin, J., & Sacks, M. (2023). Failed secular revolutions: Religious belief, competition, and extremism. *ESI Working Paper 23-04*. https://digitalcommons.chapman.edu/esi_working_papers/384/

This Article is brought to you for free and open access by the Economic Science Institute at Chapman University Digital Commons. It has been accepted for inclusion in ESI Working Papers by an authorized administrator of Chapman University Digital Commons. For more information, please contact laughtin@chapman.edu.

Failed Secular Revolutions: Religious Belief, Competition, and Extremism

Comments

ESI Working Paper 23-04

Failed Secular Revolutions: Religious Belief, Competition, and Extremism*

Jean-Paul Carvalho Jared Rubin Michael Sacks
University of Oxford Chapman University Clarkson University

Abstract

All advanced economies have undergone secular revolutions in which religious belief and institutions have been subordinated to secular forms of authority. There are, however, numerous examples of failed secular transitions. To understand these failures, we present a religious club model with endogenous entry and cultural transmission of religious beliefs. A spike in the demand for religious belief, due for example to a negative economic shock, induces a new and more extreme organization to enter the religious market and exploit the dissatisfaction of highly religious types with the religious incumbent. The effect is larger where institutional secularization is more advanced, for example where the religious establishment has moderated itself or has been moderated by the political authority. The greater the moderation of the religious incumbent, the more extreme is the position chosen by the religious entrant, and the larger is the rise in religious participation. Hence, unanticipated shifts in religious demand can lead to the emergence of new and more extreme religious organizations, and reverse previous trends toward secularization. Our model sheds light on the causes and consequences of failed secular revolutions and religious revivals in Latin America and Egypt.

February 28, 2023

JEL Classification: Z12, L1, O5

Keywords: Secularization, religious organizations, co-optation, extremism

* *Carvalho*: Department of Economics, University of Oxford, Manor Road Building, Manor Road, Oxford OX1 3UQ, UK, jean-paul.carvalho@economics.ox.ac.uk. *Rubin*: George L. Argyros School of Business and Economics, Chapman, CA 92866, jrubin@chapman.edu. *Sacks*: School of Business, Clarkson University, Potsdam, New York 13699, msacks@clarkson.edu. This paper replaces an earlier working paper titled 'Faith-based Organizations'.

1 Introduction

All advanced economies have gone through a secular transition marked by the drastic reduction of the power and privilege of the clerical elite, the institutional importance of religion, and demands for religious sacrifice by the population (Rubin, 2017; Auriol et al., 2022). There are, however, numerous examples of failed secular revolutions in which a backlash has occurred and the religious organizations have ended up with greater political power (Auriol and Platteau, 2017). This paper aims to shed new light on these failures by developing a religious club model with endogenous entry and cultural transmission of religious beliefs (Bisin and Verdier, 2000, 2001). We use this model to examine the conditions under which secular transitions succeed and where they fail.

Following the seminal work of Iannaccone (1992), religious organizations are modeled as clubs that produce religious goods from the inputs of members (see also Berman, 2000; Aimone et al., 2013; Carvalho and Sacks, 2021). Seemingly bizarre religious restrictions have a purpose in this setting. By imposing restrictions on members' outside activity, religious organizations induce members to substitute resources toward group activity and screen out uncommitted types. These restrictions mitigate the free-rider problem in the joint production of religious club goods. The model we present in this paper adds the production and cultural transmission of religious beliefs to the standard club good model (see also Verdier and Zenou, 2015; Carvalho, 2016).

In our model, individuals join a religious club to cultivate desired religious beliefs/traits that provide them, for example, with meaning and belonging (Carvalho, 2022). Religious beliefs can be acquired by contact with believers and undermined by contact with nonbelievers. The transmission of beliefs thus generates a free-rider problem. Religious clubs are susceptible to free-riders who acquire religious beliefs from other members of the group without themselves investing in religious beliefs. Thus, religious clubs form to regulate the production and cultural transmission of religious beliefs. To induce investment in religious beliefs and screen out nonbelievers, religious organizations demand a minimum level of religious participation, which we refer to as *strictness*. More generally, this measures how strict the rules are that members of the religious club must follow to enjoy the fruits of membership. Individuals vary in their demand for religious belief. In setting its strictness, each religious organization faces a tradeoff between forming a large club at low strictness or a small (highly religious) club at high strictness. We characterize the size of the active religious population as a function of aggregate religious demand and the organizational preferences for group size over participation intensity. We then use this framework to study the consequences of shocks to religious

demand and religious co-optation.

When the demand for religious belief is high, which can be the result of negative economic shocks, natural disasters, and other factors that affect the shadow price of religious participation, we find that a rival organization enters the religious market, exploiting the dissatisfaction of highly religious types with the religious incumbent. The effects of entry are more pronounced where institutional secularization is more advanced, for example, where the religious establishment has moderated itself or where it has been moderated by the political authority. Religious organizations often moderate themselves as their leaders seek status in the broader society (Stark and Bainbridge, 1985). In addition, the political authority might want to moderate the religious establishment because reduced strictness means lower aggregate religious participation. It also means that the religious establishment attracts more moderate followers, who would otherwise remain inactive. When religious demand turns out to be sufficiently high, however, the reduction in strictness by the religious establishment induces entry by a competing organization where it would otherwise not occur. The more the religious establishment has been moderated, the more extreme the position chosen by the religious entrant, and the larger the increase in aggregate religious participation. Hence, unanticipated shocks to religious demand can create religious competition and extremism, reversing previous trends toward secularization.

We apply our model to two case studies of failed secular revolutions in Latin America and Egypt. The common themes across the case studies are (i) increased demand for religion following negative economic shocks, and (ii) moderation of the predominant religion prior to the failed secular revolution. In Latin America (and, to a similar extent, sub-Saharan Africa), high-strictness Pentecostal churches have become the fastest growing religious groups since the 1980s. This followed a series of macroeconomic shocks and moderation of the dominant Catholic Church. In Egypt, the Islamic Revival Movement—which reached its peak with the electoral success of the Muslim Brotherhood in the 2012 presidential election—followed a similar pattern. A combination of decades of moderation of the Islamic religious establishment and negative shocks that left the economic aspirations of the educated middle class unfulfilled preceded the Revival. These case studies illustrate the conditions under which secular transitions can fail, and indicate that there is nothing inherent in Islam or Christianity which precipitates such failure.

This paper is related to several strands of literature. First, there is a growing literature in economics on co-optation of religious authorities (Rubin, 2017; Auriol and Platteau, 2017; Auriol et al., 2022).

In particular, Auriol and Platteau (2017) show how secular reforms can be blocked by conservative clerics and how the risk of this occurring is greater in decentralized religions. Our focus is instead on shocks to religious demand and the role of (endogenously formed) religious clubs. In particular, we show that spikes in religious demand induce entry by more extreme religious organizations, and that the effects of this are more pronounced where institutional secularization is more advanced. Second, our analysis of failed secular transitions has obvious applications to the economics of Islam and Muslim societies (e.g., Kuran, 2006, 2012). For example, Chaney (2013) presents historical evidence that Egyptian rulers granted greater power to the clerical elite after deviant Nile floods to induce them to suppress revolt. We uncover a new reason for why political authorities might ease religious co-optation in response to economic shocks and natural disasters, that is, to avoid the entry of new and more extreme religious organizations. Third, we contribute an analysis of failed religious co-optation to the growing literature on backlash to cultural policies, including Carvalho (2013), Fouka (2020), Bazzi et al. (2020), Iyigun et al. (2021), Carvalho et al. (2022), and Carvalho and Sacks (2022). Finally, our model adds to a small, but growing literature on the regulation of cultural transmission by religious clubs (e.g. Verdier and Zenou, 2015; Carvalho, 2016; Carvalho, Koyama and Sacks, 2017; Carvalho and Sacks, 2022). Unlike prior work, our model has a religious incumbent (establishment) and focuses on the entry of more radical organizations.

The remainder of the paper is structured as follows. Section 2 introduces and analyzes our model of club formation and transmission of religious beliefs. Section 3 presents two case studies of failed secular revolutions in Latin America and Egypt. Section 4 concludes.

2 The Model

We begin with a single religious organization that recruits from a population $I = [0, 1]$ endowed with Lebesgue measure μ . Each individual $i \in I$ is born with a level of religiosity λ_i (her type), which we assume is an i.i.d. draw from the distribution $U(0, \bar{\lambda})$, where $\bar{\lambda} \leq 1$. At the individual level, λ_i is the payoff i gets when acquiring religious beliefs. At the population level, a society is more religious if it has a higher $\bar{\lambda}$. We refer to $\bar{\lambda}$ as the *demand for religious belief*.

The religious organization recruits individuals to form a religious club. We introduce competition between religious organizations for members in Section 2.3.

Timing. Interactions take place as follows:

DATE 0. The religious organization announces its strictness $s \in [0, 1]$, which is the minimum level of participation required of club members. Once announced, the organization is committed to s .

DATE 1. Each individual chooses whether to become a member of the club ($m_i = 1$) or not ($m_i = 0$). Denote the set of club members by M and the set of nonmembers by N .

DATE 2. Each agent chooses participation $x_i \in [0, 1]$ at cost x_i^2 . Club members are required to spend at least proportion s of their time on participation in the club: if $m_i = 1$, then $x_i \geq s$. Nonmembers cannot participate in the club: if $m_i = 0$, then $x_i = 0$. Participation converts an individual into a ‘carrier’ of religious beliefs with probability x_i .

DATE 3. For each individual $i \in I$, the following process of social transmission determines their final beliefs. If i is a club member, they are matched uniformly at random with another club member j . If j is a carrier of religious beliefs, then i ends the period as a believer. Otherwise, i ends as a nonbeliever. The process is the same if i is a nonmember except that she is randomly matched with a nonmember.¹

2.1 Equilibrium Religious Membership and Participation

Beliefs in this model are a form of consumption; individuals deliberately (but not necessarily consciously) cultivate certain kinds of beliefs. Again, we assume that individual i receives a payoff of λ_i if they end the period with religious beliefs, and zero otherwise.

Individual i ’s expected payoff when choosing religious activity x_i is then

$$p_i \lambda_i - x_i^2, \tag{1}$$

where p_i is i ’s probability of ending up with religious beliefs, which is the average participation among club members:

$$p_i = \int_M \frac{x_i}{\mu(M)} d\mu. \tag{2}$$

To simplify statements, we impose the tie-breaking rule that an individual joins the club if indifferent. An *equilibrium* of this game is a subgame perfect equilibrium that respects this tie-breaking rule.

Individual membership and participation choices are characterized as follows:

¹Hence, the club is like the restricted pool in Bisin and Verdier’s model of homogamy (Bisin and Verdier, 2000); except here cultural transmission within the club depends on the distribution of types in the club, which is endogenous and heterogeneous.

Proposition 1. *Suppose that the religious organization chooses strictness s . In equilibrium, in the ensuing subgame:*

(i) $x_i = s$ for all club members $i \in M$.

(ii) All individuals i for which $\lambda_i \geq s$ join the club and the size of the club is

$$\mu(M) = \begin{cases} \frac{\bar{\lambda} - s}{\bar{\lambda}} & \text{if } 0 \leq s < \bar{\lambda} \\ 0 & \text{if } s \geq \bar{\lambda}. \end{cases}$$

In this setting, there is a severe free-rider problem in belief formation. The reasoning is that religious participation can convert an individual i into a *carrier* of religious beliefs and thereby increase the likelihood of other club members ending up with religious beliefs. However, it does not affect i 's own likelihood of ending up with religious beliefs, which is a product of social transmission and depends on the share of club members who are carriers of religious beliefs. This has two implications. First, no club member participates more than they are required to: $x_i = s$ for all $i \in M$. Second, in this absence of a religious organization, no individual would participate in the production of religious beliefs. A religious organization can alleviate the free-rider problem and attract members by imposing a minimum standard of participation on group members $s > 0$. This ensures that group members are likely to interact with other carriers of religious beliefs. Therefore, *religious belief is a club good* in our model and religious organizations are critical to religious participation and belief.

Part (ii) of the proposition states that as the organization becomes stricter, it screens out types with lower levels of religiosity λ_i . If the organization is too strict, $s \geq \bar{\lambda}$, it attracts no members. The larger is the demand for religious belief in the population $\bar{\lambda}$, the larger is the size of the club for a given level of strictness and the fewer members are lost from a given rise in strictness.

2.2 Equilibrium Religious Strictness

While individuals engage in religious participation to cultivate religious beliefs (via membership of a religious club), we assume that religious organizations also care about the more tangible products of religious participation, such as the provision of social services, political opposition, and rent seeking (Ekelund et al., 1989). In particular, we assume that the religious organization's objective is to maximize

$$\int_M x_i^\alpha d\mu, \tag{3}$$

where $\alpha \in (0, 1)$ measures the degree to which the organization cares about the *intensity* of religious participation relative to the size of the club. By choosing high strictness s^* the organization attracts fewer members, but induces more intensive participation by members. The larger is α , the more organization prefers a small group with high participation to a larger group with lower participation.

Proposition 2. *There exists a unique equilibrium. In this equilibrium, individual participation and membership choices are given by Proposition 1 and the religious organization sets strictness*

$$s^* = \frac{\alpha}{1 + \alpha} \bar{\lambda}.$$

In addition, aggregate religious participation is

$$\int_M s^* d\mu = \frac{\alpha}{(1 + \alpha)^2} \bar{\lambda},$$

which is strictly increasing in α and $\bar{\lambda}$.

Hence, equilibrium strictness and aggregate religious participation are strictly increasing in the demand for religious beliefs $\bar{\lambda}$ and the religious organization's preference for intensive participation α .

The proposition adds to key debates in the literature on religious clubs. First, Iannaccone (1994) argues that strict religious organizations prosper by limiting free-rider problems in club good provision. Rather than resting our argument on strictness (an endogenous variable in our model), we show how the deep parameter α affects religious participation in equilibrium. We can restate Iannaccone's proposition as follows: an increase in the disposition of religious organizations toward forming smaller, stricter clubs increases average religious participation in society.

Second, we can explain why religious groups are unusually effective providers of public goods and mobilizers of political opposition (e.g., Gruber and Hungerman, 2007; Berman and Laitin, 2008; Chen, 2010). There is a well known free-rider problem associated with these activities. We show how this problem might be mitigated by the cultivation of religious beliefs that provide meaning and belonging to members (see Carvalho, 2022). Individual actors in our model have no direct interest in collective action.² They are willing to participate in collective action only because it qualifies them for religious club membership, which in turn helps to cultivate religious beliefs by

²One reason why this would occur in our model is because individuals are non-atomic—individually, they make no difference, but bear the full cost of contributing.

facilitating interactions with like-minded people. Hence, *religious groups can overcome free-rider problems in public good provision and political opposition by providing access to a platform for acquiring religious beliefs.*

2.3 Religious Competition

Now suppose that after the incumbent organization forms club 1 and has committed itself to strictness s_1 , a rival organization can choose to enter at cost $c \geq 0$ and form club 2, announcing its strictness level s_2 . Individuals choose to remain unaffiliated ($m_i = 0$), join club 1 ($m_i = 1$) or join club 2 ($m_i = 2$). When indifferent, we assume that individuals join the incumbent's club.³ Denote the set of unaffiliated members by N , the set of club 1 members by M_1 , and the set of club 2 members by M_2 . Social transmission occurs *within* groups as before.

The incumbent maximizes

$$\int_{M_1} x_i^\alpha d\mu. \quad (4)$$

To simplify by ensuring the existence of a pure-strategy equilibrium, we assume $\alpha \in (0, \frac{1}{2})$.

The entrant organization has different preferences to the incumbent. We assume that it maximizes

$$\int_{M_2} x_i d\mu, \quad (5)$$

so that the entrant cares significantly more about intensive participation than the incumbent.⁴

Again, an *equilibrium* of the game is a subgame perfect equilibrium that respects the tie-breaking rule that when indifferent, individuals join the incumbent organization.

Proposition 3. *There is a unique equilibrium, which is defined by the cutoff values $\underline{c}(\alpha, \bar{\lambda})$ and $\bar{c}(\alpha, \bar{\lambda})$, where $\underline{c}(\alpha, \bar{\lambda}) < \bar{c}(\alpha, \bar{\lambda})$, as follows:*

- (i) *If $c < \underline{c}(\alpha, \bar{\lambda})$, then group 2 enters and the religious organizations choose strictness levels*

$$s_1^* = \frac{\alpha}{1 + \alpha} \bar{\lambda} \quad \text{and} \quad s_2^* = \frac{1}{2(1 + \alpha)} \bar{\lambda} > s_1^*,$$

respectively.

³In equilibrium, the set of indifferent individuals will have measure zero. Hence, the results are not substantially affected by the assumption.

⁴All subsequent results hold for the more general payoff function $\int_{i \in M_2} x(i)^\beta d\mu$ for all $\alpha \in (0, 1)$ and $\beta > \alpha/(1 - \alpha)$. As we do not study the effect of changing β , we set $\beta = 1$ and normalize $\alpha \in (0, 1/2)$.

The individuals with values $\lambda \in (s_1^*, s_1^* + s_2^*)$ join the incumbent club 1, those with values $\lambda \in [s_1^* + s_2^*, \bar{\lambda}]$ join the entrant club 2, and the remainder are unaffiliated.

- (ii) If $\bar{c}(\alpha, \bar{\lambda}) \leq c \leq \underline{c}(\alpha, \bar{\lambda})$, then the incumbent organization strategically deters entry by increasing its strictness level to

$$\bar{s}_1 = \bar{\lambda} - 2\sqrt{\bar{\lambda}c} > s_1^*.$$

Those individuals with values $\lambda \in (\bar{s}_1, \bar{\lambda}]$ join the incumbent club while the remainder are unaffiliated.

- (iii) If $c > \bar{c}(\alpha, \bar{\lambda})$, then entry is blockaded and the incumbent organization chooses strictness level s_1^* . Those individuals with values $\lambda \in (s_1^*, \bar{\lambda}]$ join the incumbent club while the remainder are unaffiliated.

Entry and accommodation operate as follows. When the entry cost is high ($c > \bar{c}(\alpha, \bar{\lambda})$), the incumbent can deter entry by simply choosing the monopoly strictness s_1^* . The cost of entry is so high that no distortion in the incumbent's choice is required to deter entry. As usual, we call this blockaded entry. When the entry cost is low ($c < \underline{c}(\alpha, \bar{\lambda})$), the incumbent can deter entry by raising strictness, but it has to distort strictness so much to do so that it would be better off accommodating. Hence, for a sufficiently low entry cost there is entry and accommodation. High-religiosity types join the entrant club, intermediate types join the incumbent, and low religiosity types are religiously unaffiliated. When the entry cost is intermediate, the incumbent is willing and able to deter entry by raising strictness to \bar{s}_1 .

Let us now turn to the strictness choices of the organizations. Remarkably, the incumbent's choice of strictness in the event of entry is the same as in the monopoly case of Proposition 2. In addition, there is an interesting relationship between the entrant's choice of strictness and the incumbent's preference for intensive participation α . Consider a decrease in α , i.e., a shift in the preference of the incumbent toward a larger club with less intensive participation (a feature of state religions). The optimal response of the entrant is not clear, *a priori*. It may want to decrease strictness to capture some of the incumbent's higher religiosity (λ) members. That turns out not to be the case, however. Instead, the entrant increases its level of strictness, because it can elicit higher levels of participation while losing fewer of its lower religiosity members. In this sense, the moderation of the religious establishment results in *religious polarization* in our model. Furthermore, an increase in the demand

for religious belief $\bar{\lambda}$ induces both organizations to raise strictness. However, the strictness of the entrant organization increases relative to that of the incumbent. Hence Proposition 3 predicts that smaller sects with intensive participation are more sensitive to changes in the aggregate demand for religious beliefs. We are unaware of prior work that makes this prediction.

The following is a straightforward implication of Proposition 3(i).

Corollary 1. *Aggregate religious participation in society is strictly higher in the event of entry by organization 2.*

By Proposition 3, all individuals with $\lambda_i \geq s_1^*$ join a religious club, as in the case without competition (see Proposition 1). However, some of these individuals now join the stricter club 2 at $s_2^* > s_1^*$. Therefore, aggregate participation increases. That is, competition raises religious participation as in the religious markets literature (Stark and Bainbridge, 1985; Iannaccone, 1991; Finke and Stark, 2005). Note that this contrasts with the religious club model of Carvalho and Sacks (2021) in which there is free entry, so that strictness choices are more closely tied to the preferences of individual members than they are here.

2.4 The Demand for Religion, Entry, and Extremism

We now examine how aggregate religious participation and entry by the more extreme religious organization varies with the demand for religious belief $\bar{\lambda}$. We find that an economic shock or natural disaster that raises the demand for religious belief, as in theories of religious coping (Binzel and Carvalho, 2017; Bentzen, 2019), increases religious participation and possibly induces entry where it would otherwise not occur.

Proposition 4. *Aggregate religious participation is strictly increasing in the demand for religious belief $\bar{\lambda}$.*

In addition, an increase in $\bar{\lambda}$ can induce entry, in the sense that the threshold cost $\underline{c}(\alpha, \bar{\lambda})$ is strictly increasing in $\bar{\lambda}$.

Hence, environmental changes that increase the demand for religious belief as a means of causal explanation and psychological well-being, including economic shocks and natural disasters, may lead to a rise in religious participation and the entry of new and more extreme religious organizations.

2.5 Miscalibrated Policy: Religious Co-optation and Extremism

We now show how active secularization attempts can fail, particularly when a political or religious authority with the power to affect preferences over the intensity of religious participation α underestimates the demand for religious belief. When the incumbent would otherwise deter entry, can a secularization policy that reduces the incumbent's preference for intensive participation α backfire by inducing entry, religious polarization, and greater religious participation? This is one way to characterize a *failed secular revolution*.

First, consider how religious participation varies with the incumbent's preference for club size over participation intensity, α .

Proposition 5. *Suppose $c > \frac{1}{9}\bar{\lambda}$, so that entry is blockaded for α close to $1/2$. Then there exist thresholds $\underline{\alpha}$ and $\bar{\alpha}$, where $0 < \underline{\alpha} < \bar{\alpha} < 1/2$, such that:*

- (i) *If $\alpha \in (0, \underline{\alpha})$, entry occurs and aggregate religious participation is strictly decreasing in α on $(0, \underline{\alpha})$.*
- (ii) *If $\alpha \in (\underline{\alpha}, \bar{\alpha})$, strategic entry deterrence occurs and aggregate religious participation is constant with respect to α on $(\underline{\alpha}, \bar{\alpha})$.*
- (iii) *If $\alpha \in (\bar{\alpha}, \frac{1}{2})$, entry is blockaded and aggregate religious participation is strictly increasing in α on $(\bar{\alpha}, \frac{1}{2})$.*

According to the proposition, aggregate religious participation is strictly increasing in α in the case of blockaded entry.⁵ Otherwise, aggregate religious participation is either constant (in the case of strategic entry deterrence) or strictly decreasing in α (in the case of accommodation). Hence, we can see why a political (or religious) authority might want to reduce α as a secularization measure, where there is a natural religious monopoly. However, where a reduction in α induces entry, the effect is the opposite. The further α is reduced, the larger is the rise in aggregate religious participation.

We shall now see that a reduction in α can inadvertently induce entry when the authority mistakenly believes that the demand for religious belief is lower than it actually is. In particular, suppose the authority holds religious belief $\lambda' \in [0, 1]$.

⁵When $c \leq \frac{1}{9}\bar{\lambda}$, the interval $(\bar{\alpha}, \frac{1}{2})$ does not exist, as there is no blockaded entry. In this case, aggregate religious participation is either constant or strictly decreasing in α .

Proposition 6. *Given the authority’s belief about religious demand $\lambda' \in [0, 1]$, suppose α is set to $\alpha(\lambda')$, which is the lowest α that does not induce entry.*

Suppose the authority underestimates religious demand, $\lambda' < \bar{\lambda}$, whereas the organizations know the true religious demand before choosing strategies. Then the policy of setting $\alpha = \alpha(\lambda')$ induces entry, whereas setting $\alpha = \alpha(\bar{\lambda})$ would not. In addition, it reduces the incumbent’s market share.

Hence a miscalibrated policy can induce entry where it would otherwise not occur and reduce the power of the religious establishment. We also know that the stronger the attempt at moderation (i.e., the reduction in α), the more extreme is the religious entrant (Proposition 3) and the larger is the rise in religious participation (Proposition 5). Putting our results together we can see how secularization policies can backfire as attempts to moderate the religious establishment induce the entry of more extreme religious organizations and increase religious participation and polarization. Therefore, bottom-up secularizing forces that reduce the demand for religious belief may be required for top-down secularization policies to work.

3 Case Studies

We now apply our model to two case studies of failed secular transitions.

3.1 The Rise of Pentecostalism

Our theory provides insight into the worldwide spread of Pentecostalism, a strict form of Protestantism that emerged in the US in the early 20th century as part of a broader religious revival. Prominent Pentecostal denominations include Assemblies of God, Four-Square Gospel, and the Church of God. Pentecostal denominations are among the strictest of the Abrahamic faiths. Congregants regularly speak in tongues and perform exorcisms, engage in intense proselytizing, have very high levels of attendance, read the Bible regularly, adhere to literal interpretations of the Bible, and in many countries refrain from certain actions such as drinking alcohol or consuming tobacco. Consistent with a ‘strict’ religion, there is a giant commitment gap between Pentecostals and mainstream Protestantism and Catholicism. For instance, in a recent survey of Latin America conducted by the Pew Research Center, there is a gap of at least 20 percentage points in almost every country between Protestants (most of whom are either Pentecostal or Charismatic) and Catholics regarding claims that religion is very important in their lives, they pray daily, and they attend service weekly (Pew, 2014).

Our model identifies factors that might make such a strict religion attractive. One factor is negative economic shocks that increase the demand for religious belief (Proposition 4). Low-intensity religions are relatively weak providers of social insurance due to the absence of strong mechanisms to screen out free-riders, which matters more in poor countries where public social insurance and private insurance markets may not be available (Chen, 2010; Ager and Ciccone, 2018). The desire for social insurance has been shown to be strong among Pentecostals in diverse settings including Ghana, Guatemala, and Brazil (Auriol et al., 2020; McCleary, 2018; Cavalcanti et al., 2022). Additionally, it has been shown across the world (and not just among Pentecostals) that negative economic shocks can increase religious intensity due to ‘religious coping’ (Binzel and Carvalho, 2017; Bentzen, 2019), where religion is a psychological mechanism used to overcome unexpected bad events.

This response is precisely what Costa, Marcantonio and Rocha (2023) find contributed to the rapid rise of Pentecostalism in Brazil. They provide empirical evidence that economic downturns lead directly to conversions into Pentecostal churches by people who were previously affiliated with other (typically more mainstream) Christian denominations. They show that a one standard deviation reduction in regional tariffs (which improve economic conditions for the poorest classes in the Brazilian context) is associated with 21% of a standard deviation increase in the growth of Pentecostals.

The recent growth in Pentecostal adherence has come largely at the expense of the more moderate Catholic church, which was the dominant traditional, ‘low strictness’ religion of the region for centuries. Around half of Brazilian Pentecostal survey respondents converted from Catholicism (Costa et al., 2023). Our model of competition explains why this is the case. The social transmission of high-strictness religious beliefs is more effective when the incumbent church has a low preference for strictness (Proposition 3). In such a setting, the incumbent church will have many members with low participation; indeed, within Latin America, participation rates (proxied by the percentage who pray daily and attend services weekly) is much lower in Catholic than in Protestant churches (Pew, 2014). As the recent surge in Pentecostalism suggests, such a strategy by the incumbent can leave the door open for a new entrant that requires high levels of strictness.

Furthermore, there is evidence that the mechanism proposed in our model—the social transmission of beliefs from believers to non-believers—has played an important role in the spread of Pentecostalism. In Guatemala, Pentecostalism spreads primarily along kinship lines, and one reason for

its success is that Pentecostal churches provide social insurance in the absence of provision by the state (McCleary, 2018). In Ghana, marriage market considerations play a key role in encouraging people to join Pentecostal churches (Seabright, 2023). This type of social transmission of beliefs is central to our theory. As noted in the literature, such belief transmission induces a free-rider problem. Our theory states that this allows more extreme religious organizations to enter the religious marketplace when there is either an increase in the demand for religious beliefs and services or there is a moderation of religious organizations.

These dynamics have led to a rapid rise in Pentecostal churches in many parts of the world. Despite the high costs associated with conversion, Pentecostal and related Charismatic Protestant churches now number around 500 million followers, and their numbers are rapidly growing (Pew, 2006). In Latin America, Pentecostalism is the fastest growing denomination. According to a 2014 Pew survey (Pew, 2014), 65% of Protestants in Latin America either identify as or attend a Pentecostal church, and Protestantism is the fastest growing religion in the region. Although only 9% of the population was raised Protestant, as of 2014 19% consider themselves Protestant (Pew, 2014). In Brazil, 2.78% of the population was Pentecostal in 1980; by 2010, 14.5% of the population identified as Pentecostal (Costa et al., 2023).

Pentecostalism is spreading rapidly elsewhere in the world, as well. For instance, in sub-Saharan Africa, the rise of Pentecostal churches over the last several decades has largely come at the expense of traditional religions. In 2015, around 40% of Christians in the region identified as Pentecostal or Charismatic (Auriol et al., 2020). In Ghana, for instance, 62.6% of the population was an ethno-religionist in 1950, while in 2020 only 10.4% are. In that period, the percentage of Christians has risen from 31.4% to 71.1% (The Association of Religion Data Archives, theARDA). Protestants make up a plurality of Christians, and Pentecostalism is the fastest growing religion in the country.

3.2 Egypt's Islamic Revival

In January 2012, Egypt's Arab Spring led to parliamentary elections that revealed the scale of support for Islamic political parties—the Muslim Brotherhood Freedom and Justice Party and the Salafist Al-Nour party attracted 47% and 28% of the vote, respectively. The Muslim Brotherhood candidate, Mohamed Morsi, was elected president (although he was ousted a year later). Along with the assassination of Anwar Sadat in 1981, their electoral success was the culmination of a wide-reaching Islamic movement beginning in the 1970s that reversed previous trends toward

secularization. The Muslim Brotherhood was a big winner in this reversal.

The Muslim Brotherhood is a transnational organization that originated in Egypt in 1928. It seeks the establishment of a state ruled by Sharia law. Forced into the shadows for decades after its founding, it began to become a social and political force in the 1970s. The Muslim Brotherhood is a ‘high strictness’ group, requiring significant sacrifice from its members. It advocates for strict separation of the sexes, high mosque attendance, frequent prayer and fasting, separate curricula for girls, the implementation of Sharia, and the prohibition of dancing. One consequence of these stated goals was that the rise of the Muslim Brotherhood in the 1980s contributed to a much slower decrease (relative to the Coptic Christian population) in ‘strict’ religious practices such as female genital mutilation (Blaydes and Platas, 2020) and an increase in female veiling (Patel, 2012; Carvalho, 2013). Our model can account for the widespread appeal of the Muslim Brotherhood and more generally of religious groups associated with Egypt’s Islamic Revival. The model suggests that a combination of religious competition and moderation by the incumbent religion (Islam) would have made stricter forms of Islam more attractive to Muslims. Both of these conditions arose during the course of the 20th century in Egypt.

We begin with the moderation of the prevailing religious establishment. By the 1930s, Islam had become an inherited culture rather than a source of practical guidance for a large part of the educated elite. They were living largely outside the bounds of the sharia; prayers and fasting were less frequently observed, and the consumption of alcohol was rising (Hourani, 2005, p. 345-6). In other words, the Egyptian middle and upper classes began to turn away from traditional Islamic values and practices. This was presaged by decades of state attempts to subjugate the religious establishment and repress religious organizations. Table 1 contains a selective list of relevant events since 1920. The governments of Gamal Abdel Nasser (1956-1970) and Anwar Sadat (1970-1981) co-opted the religious establishment, in particular the Al-Azhar scholars and clergymen, removing Al-Azhar’s authority over family law, appointing its Grand Sheikh, and adding secular subjects to its curricula. Al-Azhar was used as an organ of the state to legitimize policies that would otherwise be opposed on religious grounds (see Kepel, 1985; Hopwood, 1991; Barraclough, 1998; Moustafa, 2001). Most notably, the religious establishment moderated its stance toward religious obligations, practices such as female circumcision, and peace with Israel. Our model suggests that such moderation may produce endogenous religious polarization as smaller sects respond by increasing their strictness. This prediction is consistent with the subsequent growth of

Table 1: Religiopolitical Developments in Egypt: 1920-2013

Year	Event
1923	Voluntary de veiling movement led by Huda Shaarawi in Cairo
1924	Free secular education introduced for women
1928	Muslim Brotherhood founded by Hassan al-Banna; calls for Islamic state and <i>sharia</i>
1937	Al-Azhar's <i>fatwa</i> committee declares that veiling is not a religious obligation
1943–50	Muslim Brotherhood establishes branches in Jordan, Syria, Sudan, and elsewhere
1950	Sayyid Qutb returns from the United States, joins Muslim Brotherhood
1953	<i>Waqfs</i> (Islamic trusts) nationalized
1954	Muslim Brotherhood officially dissolved and members prosecuted
1956	Al-Azhar loses authority over family law which is incorporated into civil code
1955	<i>Sharia</i> courts abolished
1961	Nasser gains authority to appoint Grand Sheikh of Al-Azhar University
1961	Secular subjects and women's faculty introduced to Al-Azhar
1967	Arab defeat in Six-Day War
1973	Sadat initiates 'holy war' against Israel
1978	Al-Azhar issues support for peace with Israel, reversing previous <i>fatwa</i>
1975–79	Islamist student organizations flourish and gain control of student unions
1980	Constitutional amendment recognizing Islam as state religion, <i>sharia</i> primary source of legislation
1981	Sadat assassinated by religious extremists
1985	People's assembly votes to gradually amend existing laws to conform to the <i>sharia</i>
2005	Muslim Brotherhood wins 20% of parliamentary seats, despite electoral irregularities
2011	Revolution and fall of Mubarak
2012	Parliamentary elections: Muslim Brotherhood Freedom and Justice Party wins 47% of vote, Salafist Al-Nour Party wins 28% of vote; Mohamed Morsi elected President
2013	Mohamed Morsi ousted as President; Muslim Brotherhood banned and assets seized

strict religious groups such as the Muslim Brotherhood, which engaged intensively in public goods provision and political opposition, as well as more extreme militant groups such as Al-Gama'a al-Islamiyya, Islamic Jihad, and Takfir wal-Hijra.

The second condition highlighted by the model is negative economic shocks that result in an increase in the demand for both religious beliefs and for religion-provided social insurance. Economic conditions in Egypt deteriorated in the 1970s and 1980s, just as its Islamic Revival began to attract followers. In the 1970s, the first generation of Muslims who benefited from educational reforms implemented in 1952-53 (in the wake of a 1952 coup) entered the job market. The educational reforms attempted to 'modernize' Egypt's educational system by increasing the years of compulsory education and reforming the curriculum to better prepare students for secular secondary education. Prior to the reform, around five times as many Egyptian Muslim students enrolled in Islamic primary schools (*kuttabs*) compared with 'modern' elementary schools (Saleh, 2016). This limited their access to secondary education, resulting in lower overall levels of education and fewer white-collar jobs for Muslims relative to the minority of Coptic Christians. The reforms modernized the *kuttab* curriculum, better preparing students for secondary education. In the short run, the reform

worked: the education and employment gap between Muslims and Christians shrank by the 1980s (Saleh, 2016).

Yet, in the long run these reforms helped play a role in popularizing the Islamic Revival movement in Egypt, especially via the Muslim Brotherhood. The 1970s were a period of relative economic stagnation. This meant that many of this first generation of highly educated Egyptians, who expected high-paying (government) jobs, found themselves unemployed or underemployed. The social mobility they had been promised did not materialize. El Guindi (1981) claims that “[a] new form of inequality has emerged—wealth for the entrepreneur, unemployment for the college educated” [p. 481]. Several authors including Ayubi (1991), Amin (1995), and Wickham (2002) have suggested that this created feelings of relative deprivation, especially among educated youth, which fueled Egypt’s Islamic revival.

Binzel and Carvalho (2017) formalize this argument and provide empirical evidence of a sharp decline in social mobility in Egypt in the 1970s-80s. Their data indicate that university graduates born between 1949 and 1960 were 21 percentage points more likely to find a public sector job than those born between 1968 and 1977. Their model suggests that religious beliefs compensate for inequality and unfulfilled aspirations via a ‘coping’ mechanism, so that rising relative deprivation leads to greater demand for religious beliefs. This is one reason why Egypt’s Islamic Revival was most popular among the middle class. There were also extrinsic incentives to join (and vote for) Islamist groups; in the 1990s, Islamist organizations accounted for over half of all welfare organizations (Bayat, 2002). Our model adds to this insight by considering the role of religious competition in an environment in which the incumbent has moderated the required level of strictness. Our model predicts that new entrants will respond by increasing strictness and engaging more intensively in public good provision and political opposition. The model also predicts that this change should be most pronounced among sects with more intensive participation, such as the Muslim Brotherhood. These predictions are broadly consistent with the Egyptian experience since the 1970-90s.

4 Conclusion

This paper examines the forces stalling secular transitions and producing religious revivals in which new and more extreme religious organizations come to the fore. We develop a model in which religious organizations form clubs that compete for members and use the inputs of members to produce religious goods. In particular, each religious club acts as a platform for the production and

social transmission of religious beliefs. To overcome susceptibility to free-riding, each religious club sets its strictness, a minimum participation requirement. In setting strictness, each organization trades off club size with participation intensity. Lower strictness levels induce broader membership with lower participation per member, while higher strictness levels lead to a small, but highly committed pool of members. When faced with the threat of entry, we characterize the conditions under which the religious incumbent accommodates the entrant or strategically deters entry by raising its strictness beyond the monopoly level.

Moderation of the religious establishment, due either to internal or external forces, tends to induce entry of a more extreme organization. The more the religious establishment has been moderated, the more extreme is the entrant's choice of strictness and the larger is the rise in religious participation. Hence, the secularization process can stall by inducing entry, religious polarization, and revival. Economic shocks or natural disasters that increase the demand for religious belief increase religious participation and can induce religious entry, and its attendant effects, where it otherwise would not happen.

In addition to secularization due to the religious establishment moderating itself (as in the Latin America case), it can be the result of secularization policies by the political authority. If the incumbent religious organization is subordinate to the political authority but can use its position to grant the political authority legitimacy in the eyes of the population, then the political authority may have the incentive to moderate the religious organization to increase the share of the population participating (and thus the authority's legitimacy). If, however, the political authority underestimates the demand for religion, then it can unintentionally trigger entry by the more extreme organization, thereby undermining the authority's efforts and in the end, undermining the political authority itself. More extreme attempts at moderation lead to more extreme religious entrants. We can conclude that for top-down secularization policies to work, they must be coupled with bottom-up secularizing forces that reduce the demand for religious belief.

While we have applied our model to two case studies, the rapid rise of Pentecostalism in Latin America and the Islamic revival in Egypt, the model could potentially explain religious entry and extremism more broadly, especially the broader Islamic revival, e.g., the Iranian revolution and the rise of political Islam in Turkey from the 1990s.

References

- Ager, Philipp and Antonio Ciccone**, “Agricultural risk and the spread of religious communities,” *Journal of the European Economic Association*, 2018, 16 (4), 1021–1068.
- Aimone, Jason A, Laurence R Iannaccone, Michael D Makowsky, and Jared Rubin**, “Endogenous group formation via unproductive costs,” *Review of Economic Studies*, 2013, 80 (4), 1215–1236.
- Amin, Galal A.**, *Egypt’s Economic Predicament*, Leiden, Netherlands: E.J Brill, 1995.
- Auriol, Emmanuelle and Jean-Philippe Platteau**, “Religious co-option in autocracy: A theory inspired by history,” *Journal of Development Economics*, 2017, 127, 395–412.
- , —, and **Thierry Verdier**, “The Quran and the sword,” *Journal of the European Economic Association*, 2022, 20 (6).
- , **Julie Lassébie, Amma Panin, Eva Raiber, and Paul Seabright**, “God insures those who pay? Formal insurance and religious offerings in Ghana,” *Quarterly Journal of Economics*, 2020, 135 (4), 1799–1848.
- Yubi, Nazih**, *Political Islam: Religion and Politics in the Arab World*, London, UK: Routledge, 1991.
- Barraclough, Steven**, “Al-Azhar: Between the government and the Islamists,” *Middle East Journal*, 1998, 52 (2), 236–249.
- Bayat, Asef**, “Activism and social development in the Middle East,” *International Journal of Middle East Studies*, 2002, 34 (1), 1–28.
- Bazzi, Samuel, Masyhur Hilmy, and Benjamin Marx**, “Islam and the state: Religious education in the age of mass schooling,” NBER Working Papers 27073, National Bureau of Economic Research May 2020.
- Bentzen, Jeanet Sinding**, “Acts of God? Religiosity and natural disasters across subnational world districts,” *The Economic Journal*, 2019, 129 (622), 2295–2321.
- Berman, Eli**, “Sect, subsidy, and sacrifice: An economist’s view of Ultra-Orthodox Jews,” *Quarterly Journal of Economics*, 2000, 115 (3), 905–953.
- and **David D. Laitin**, “Religion, terrorism and public goods: Testing the club model,” *Journal of Public Economics*, 2008, 92 (10-11), 1942–1967.
- Binzel, Christine and Jean-Paul Carvalho**, “Education, social mobility, and religious movements: The Islamic revival in Egypt,” *The Economic Journal*, 2017, 127 (607), 2553–2580.
- Bisin, Alberto and Thierry Verdier**, “Beyond the melting pot: Cultural transmission, marriage, and the evolution of ethnic and religious traits,” *Quarterly Journal of Economics*, 2000, 115 (3), 955–988.
- and —, “The economics of cultural transmission and the dynamics of preferences,” *Journal of Economic Theory*, 2001, 97, 298–319.
- Blaydes, Lisa and Melina R Platas**, “Religion, family structure, and the perpetuation of female genital cutting in Egypt,” *Journal of Demographic Economics*, 2020, 86 (3), 305–328.
- Carvalho, Jean-Paul**, “Faith-based organizations,” *AALIMS Working Paper*, 2012.
- , “Veiling,” *Quarterly Journal of Economics*, 2013, 128 (1), 337–370.
- , “Identity-based organizations,” *American Economic Review: Papers and Proceedings*, 2016, 106, 410–414.
- , “Religion and Terrorism: The Religious Utility Hypothesis,” in A. Basuchoudhary and G. Schulze, eds., *Handbook of the Economics of Terrorism*, 2022.

- **and Michael Sacks**, “The economics of religious communities,” *Journal of Public Economics*, 2021, 201, 104481.
- **and** — , “Radicalization,” 2022.
- , **Mark Koyama**, **and Cole Williams**, “Resisting education,” 2022.
- , — , **and Michael Sacks**, “Education, identity, and community: Lessons from Jewish emancipation,” *Public Choice*, 2017, 171 (1-2), 119–143.
- Cavalcanti, Tiago, Sriya Iyer, Christopher Rauh, Christian Roerig, and Maryam Vaziri**, “A city of God: Afterlife beliefs and job support in Brazil,” *Working Paper*, 2022.
- Chaney, Eric**, “Revolt on the Nile: Economic shocks, religion, and political power,” *Econometrica*, 2013, 81 (5), 2033–2053.
- Chen, Daniel L.**, “Club goods and group identity: Evidence from Islamic resurgence during the Indonesian financial crisis,” *Journal of Political Economy*, 2010, 118 (2), 300–354.
- Costa, Francisco, Angelo Marcantonio, and Rudi Rocha**, “Stop suffering! Economic downturns and Pentecostal upsurge,” *Journal of the European Economic Association*, 2023, *Forthcoming*.
- Ekelund, Robert B, Robert F Hébert, and Robert D Tollison**, “An economic model of the medieval church: Usury as a form of rent seeking,” *Journal of Law, Economics, and Organization*, 1989, 5 (2), 307–331.
- El Guindi, Fadwa**, “Veiling Infitah with Muslim ethic: Egypt’s contemporary Islamic movement,” *Social Problems*, 1981, 28 (4), 465–485.
- Finke, Roger and Rodney Stark**, *The churching of America, 1776-2005: Winners and losers in our religious economy*, Rutgers University Press, 2005.
- Fouka, Vasiliki**, “Backlash: The unintended effects of language prohibition in US schools after World War I,” *Review of Economic Studies*, 2020, 87 (1), 204–239.
- Gruber, Jonathan and Daniel M. Hungerman**, “Faith-based charity and crowd-out during the great depression,” *Journal of Public Economics*, 2007, 91 (5-6), 1043–1069.
- Hopwood, Derek**, *Egypt 1945-1990: Politics and Society*, London: Routledge, 1991.
- Hourani, Albert**, *A History of the Arab Peoples*, London, UK: Faber and Faber, 2005.
- Iannaccone, Laurence R.**, “The consequences of religious market structure: Adam Smith and the economics of religion,” *Rationality and Society*, 1991, 3 (2), 156–177.
- Iannaccone, Laurence R.**, “Sacrifice and stigma: Reducing free-riding in cults, communes, and other collectives,” *Journal of Political Economy*, 1992, 100 (2), 271–291.
- , “Why strict churches are strong,” *American Journal of Sociology*, 1994, 99 (5), 1180–1211.
- Iyigun, Murat, Jared Rubin, and Avner Seror**, “A theory of cultural revivals,” *European Economic Review*, 2021, 135, 103734.
- Kepel, Giles**, *The Prophet and the Pharaoh*, London: Saqi Books, 1985.
- Kuran, Timur**, *Islam and Mammon*, New Jersey: Princeton University Press, 2006.
- , *The long divergence: How Islamic law held back the Middle East*, Princeton University Press, 2012.
- McCleary, Rachel M.**, “Pentecostals, kinship, and moral economy in Guatemala,” *Journal of Markets & Morality*, 2018, 21 (1), 167–189.
- Moustafa, T.**, “Conflict and cooperation between the state and religious institutions in contemporary Egypt,” *International Journal of Middle East Studies*, 2001, 32 (1), 3–22.

- Patel, David S**, “Concealing to reveal: The informational role of Islamic dress,” *Rationality and Society*, 2012, *24* (3), 295–323.
- Pew Research Center**, “Spirit and power: A 10-country survey of Pentecostals,” 2006. October 5, 2006. Available at <https://www.pewresearch.org/religion/2006/10/05/spirit-and-power/>.
- , “Religion in Latin America: Widespread change in a historically Catholic region,” 2014. November 13, 2014. Available at <https://www.pewresearch.org/religion/2014/11/13/religion-in-latin-america/>.
- Rubin, Jared**, *Rulers, Religion, and Riches: Why the West got rich and the Middle East did not*, Cambridge University Press, 2017.
- Saleh, Mohamed**, “Public mass modern education, religion, and human capital in twentieth-century Egypt,” *Journal of Economic History*, 2016, *76* (3), 697–735.
- Seabright, Paul**, “The origins of enchantment: How religions compete,” *Mimeo*, 2023.
- Stark, Rodney and William Sims Bainbridge**, *The future of religion: Secularization, revival, and culture formation*, University of California Press, 1985.
- The Association of Religion Data Archives (theARDA)**, “National/regional profiles: Ghana,” 2023. Accessed February 3, 2023.
- Verdier, Thierry and Yves Zenou**, “The role of cultural leaders in the transmission of preferences,” *Economics Letters*, 2015, *136*, 158–161.
- Wickham, Carrie Rosefsky**, *Mobilizing Islam: Religion, Activism and Social Change in Egypt*, New York, NY: Columbia University Press, 2002.

Appendix

Proof of Proposition 1.

Proof. A nonmember is matched with another nonmember at random and acquires that nonmember's beliefs. Thus, the probability of agent $i \in N$ ending with religious beliefs is

$$p_i = \int_N \frac{x_j}{\mu(N)} d\mu. \quad (6)$$

Notice that p_i is independent of x_i . Therefore, i 's expected payoff in (1) is strictly decreasing in x_i , which implies that $x_i = 0$ for all $i \in N$. Hence, $p_i = 0$ for each nonmember i . Substituting $p_i = x_i = 0$ into (1) yields a payoff of zero from being a nonmember.

Members are required to choose $x_i \geq s$. Because each individual i 's payoff is strictly decreasing in x_i , no individual will choose $x_i > s$. Thus, the probability of individual $i \in M$ ending up with religious beliefs is

$$p_i = \int_M \frac{s}{\mu(M)} d\mu = s. \quad (7)$$

Substituting (7) into (1), we get the payoff from being a club member

$$s\lambda_i - s^2. \quad (8)$$

Given the tie-breaking rule, individual i joins the group if (8) is greater than or equal to zero. This occurs if and only if $\lambda_i \geq s$. Given $\lambda_i \in U(0, \bar{\lambda})$, $\mu(M) = (\bar{\lambda} - s)/\bar{\lambda}$ for $s \in [0, \bar{\lambda})$ and zero otherwise. \square

Proof of Proposition 2.

Proof. We solve for the subgame perfect equilibrium via backward induction.

Membership and participation choices for each $s \in [0, 1]$ are given by Proposition 1. We know that if $s > \bar{\lambda}$, then $x_i = 0$ for all $i \in I$. In this case, the organization's payoff (3) is zero, a minimum of the function. Hence, it will never choose such an s .

Thus, consider strictness $s \in [0, \bar{\lambda}]$. In this case, we know $M = \{i : \lambda_i \geq s\}$ and $\mu(M) = (\bar{\lambda} - s)/\bar{\lambda}$.

We have also established that every group member chooses participation level s . Therefore, the organization's payoff from strictness s is

$$\begin{aligned} \int_M x_i^\alpha d\mu &= \int_M s^\alpha d\mu \\ &= \frac{(\bar{\lambda} - s)}{\bar{\lambda}} s^\alpha \end{aligned} \quad (9)$$

The first-order condition is

$$\alpha (\bar{\lambda} - s^*) (s^*)^{\alpha-1} - (s^*)^\alpha = 0. \quad (10)$$

It is straightforward to show that the second-order condition is satisfied for $\alpha \in (0, 1)$. Solving for s^* yields

$$s^* = \frac{\alpha}{1 + \alpha} \bar{\lambda}. \quad (11)$$

$$\int_I s^* d\mu = \frac{\bar{\lambda} - s^*}{\bar{\lambda}} s^* = \left(1 - \frac{\alpha}{1 + \alpha}\right) \frac{\alpha}{1 + \alpha} \bar{\lambda} = \frac{\alpha}{(1 + \alpha)^2} \bar{\lambda}. \quad (12)$$

By inspection, this is strictly increasing in $\bar{\lambda}$. Also, $\frac{d}{d\alpha} \left(\frac{\alpha}{(1+\alpha)^2} \bar{\lambda} \right) = \frac{1-\alpha^2}{(1+\alpha)^4} \bar{\lambda}$, which is positive because $\alpha \in (0, 1)$. \square

Proof of Proposition 3.

Proof. Working backwards, we first characterize what happens in the event of entry. Conditional on entry, we show that $s_2^* \geq s_1$ for any choice of $s_1 \in (0, \bar{\lambda})$. Consider the subgame following strictness s_1 and entry and suppose to the contrary that $s_2^* < s_1$. By the argument used in the proof of Proposition 1, $x_i = s_k$ for all i in club k and the probability a member of club k ends up holding religious beliefs is s_k for $k = 1, 2$. An individual i will join club 2 if the following incentive compatibility conditions are satisfied:

$$\begin{aligned} \lambda_i s_2^* - (s_2^*)^2 &> 0 \\ \lambda_i s_2^* - (s_2^*)^2 &\geq \lambda_i s_1 - s_1^2, \end{aligned} \quad (13)$$

which reduce to $\lambda_i \in (s_2^*, s_1 + s_2^*]$. Hence, we can write (5) as

$$\int_{s_2^*}^{s_1 + s_2^*} \bar{\lambda}^{-1} s_2^* d\lambda - c = \bar{\lambda}^{-1} s_1 s_2^* - c. \quad (14)$$

By inspection, (14) is strictly increasing in s_2 , so we cannot have $s_2^* < s_1$. Thus, we can restrict attention to $s_2^* \geq s_1$.

For $s_2 \geq s_1$, the incentive-compatibility conditions above reduce to $\lambda_i \in (s_1 + s_2, \bar{\lambda}]$. Again, by an earlier argument, $x_i = s_j$ for all individuals i in group j and the probability a member of group j ends up holding religious beliefs is s_j for $j = 1, 2$. Hence, we can write (5) as

$$\int_{s_1 + s_2}^{\bar{\lambda}} \bar{\lambda}^{-1} s_2 d\lambda - c = \bar{\lambda}^{-1} (\bar{\lambda} - s_1 - s_2) s_2 - c.$$

The corresponding first-order condition is

$$-s_2^* + (\bar{\lambda} - s_1 - s_2^*) = 0.$$

Clearly, the second-order condition for a maximum is satisfied. Solving for $s_2^*(s_1)$ yields a best response, conditional upon entry, of

$$s_2(s_1) = \frac{1}{2} \bar{\lambda} - \frac{1}{2} s_1. \quad (15)$$

Organization 2's payoff from entering is thus

$$\begin{aligned} &\bar{\lambda}^{-1} \left(\bar{\lambda} - s_1 - \frac{1}{2} \bar{\lambda} - \frac{1}{2} s_1 \right) \left(\frac{1}{2} \bar{\lambda} - \frac{1}{2} s_1 \right) - c \\ &= \bar{\lambda}^{-1} \left(\frac{\bar{\lambda} - s_1}{2} \right)^2 - c. \end{aligned} \quad (16)$$

Organization 2 will enter if and only if (16) is positive, so entry is deterred if

$$\bar{\lambda}^{-1} \left(\frac{\bar{\lambda} - s_1}{2} \right)^2 - c \leq 0,$$

or equivalently if

$$s_1 \geq \bar{\lambda} - 2\sqrt{\bar{\lambda}c} \equiv \bar{s}_1. \quad (17)$$

Thus, organization 2 enters and chooses the strictness characterized by (15) if $s_1 < \bar{s}_1$. If $s_1 \geq \bar{s}_1$, then there is no entry.

Anticipating club 2's entry and strictness decisions, club 1 chooses its strictness. If its chosen strictness is below \bar{s}_1 , then there is no entry and there is entry otherwise. Hence, the incumbent can choose to either accommodate or prevent entry.

Suppose for the moment that $s_1 < \bar{s}_1$, so there is entry. Given (13), organization 1's payoff is

$$\int_{s_1}^{s_1+s_2(s_1)} \bar{\lambda}^{-1} s_1^\alpha d\lambda = \bar{\lambda}^{-1} s_2(s_1) s_1^\alpha. \quad (18)$$

Substituting in (15) yields

$$\frac{1}{2\bar{\lambda}} (\bar{\lambda} - s_1) s_1^\alpha$$

The first-order condition is

$$-(s_1^*)^\alpha + \alpha (\bar{\lambda} - s_1^*) (s_1^*)^{\alpha-1} = 0$$

It is again straightforward to verify that the second-order condition is satisfied. Solving for s_1^* yields

$$s_1^* = \frac{\alpha}{1+\alpha} \bar{\lambda}. \quad (19)$$

Conditional on entry, organization 2's equilibrium strictness is

$$s_2(s_1^*) = \frac{1}{2} \left(\bar{\lambda} - \frac{\alpha}{1+\alpha} \bar{\lambda} \right), \quad (20)$$

which simplifies to

$$s_2^* = \frac{1}{2(1+\alpha)} \bar{\lambda}. \quad (21)$$

We now verify that $s_2^* > s_1^*$ and that $s_1^* + s_2^* < \bar{\lambda}$, so demand is nondegenerate. Summing the strictness levels yields

$$\frac{1+2\alpha}{2(1+\alpha)} \bar{\lambda} < \bar{\lambda},$$

so demand is nondegenerate. Organization 2's strictness exceeds 1's whenever

$$\frac{1}{2(1+\alpha)} \bar{\lambda} > \frac{\alpha}{1+\alpha} \bar{\lambda} \iff \alpha < \frac{1}{2},$$

which is true by assumption. Hence, conditional on entry, the equilibrium levels of strictness are given by (19) and (21).

Blockaded entry. Entry is blockaded if $s_1^* \geq \bar{s}_1$ or

$$\frac{\alpha}{1+\alpha}\bar{\lambda} \geq \bar{\lambda} - 2\sqrt{\bar{\lambda}c}. \quad (22)$$

Solving (22) for c yields

$$c \geq \left(\frac{1}{2(1+\alpha)}\right)^2 \bar{\lambda} \equiv \bar{c}(\alpha, \bar{\lambda}). \quad (23)$$

If $c \geq \bar{c}(\alpha, \bar{\lambda})$, then entry is blockaded and in equilibrium organization 1 selects strictness s_1^* , organization 2 does not enter, and all individuals with $\lambda \in (s_1^*, \bar{\lambda})$ join club 1.

Strategic entry deterrence. Now suppose that $c < \bar{c}(\alpha, \bar{\lambda})$.

Evaluate when organization 1 will strategically deter entry by increasing strictness from s_1^* to \bar{s}_1 . By (18), organization 1's payoff with entry is

$$\bar{\lambda}^{-1} s_2^* (s_1^*)^\alpha = \frac{\alpha^\alpha}{2(1+\alpha)^{1+\alpha}} \bar{\lambda}^\alpha. \quad (24)$$

When deterring entry by choosing $s_1 = \bar{s}_1$, organization 1's payoff is

$$\bar{\lambda}^{-1} (\bar{\lambda} - \bar{s}_1) \bar{s}_1^\alpha = 2(\bar{\lambda}c)^{\frac{1}{2}} (\bar{\lambda} - 2\sqrt{\bar{\lambda}c})^\alpha \bar{\lambda}^{-1} \quad (25)$$

Thus, organization 1 will strategically deter entry if and only if (25) is no less than (24):

$$2\sqrt{\bar{\lambda}c} (\bar{\lambda} - 2\sqrt{\bar{\lambda}c})^\alpha \bar{\lambda}^{-1} \geq \frac{\alpha^\alpha}{2(1+\alpha)^{1+\alpha}} \bar{\lambda}^\alpha. \quad (26)$$

Notice that the RHS of (26) is independent of c . We now show that the LHS is strictly increasing in c . Recall that the LHS is proportional to

$$(\bar{\lambda} - \bar{s}_1) \bar{s}_1^\alpha.$$

Differentiating the above with respect to c yields

$$\frac{\partial \bar{s}_1}{\partial c} (-\bar{s}_1^\alpha + \alpha (\bar{\lambda} - \bar{s}_1) \bar{s}_1^{\alpha-1}).$$

By inspection of (17), $\frac{\partial \bar{s}_1}{\partial c} < 0$ and as $\bar{s}_1 > s_1^*$ by hypothesis, $-\bar{s}_1^\alpha + \alpha (\bar{\lambda} - \bar{s}_1) \bar{s}_1^{\alpha-1} < 0$ by (10), so the LHS of (26) is strictly increasing in c .

At $c = 0$ the LHS of (26) is zero, so (26) is violated while at $c = \bar{c}(\alpha, \bar{\lambda})$, $\bar{s}_1 = s_1^*$ so (26) is

$$\frac{\alpha^\alpha}{(1+\alpha)^{1+\alpha}} \bar{\lambda}^\alpha > \frac{\alpha^\alpha}{2(1+\alpha)^{1+\alpha}} \bar{\lambda}^\alpha,$$

which holds. Hence, as the LHS of (26) is continuous and monotonic in c , there exists a unique threshold $\underline{c}(\alpha, \bar{\lambda})$, such that (26) is satisfied if and only if $c \geq \underline{c}(\alpha, \bar{\lambda})$.

Accommodation. When $c < \underline{c}(\alpha, \bar{\lambda})$ so entry is neither blockaded nor deterred, entry is accommodated.

Therefore, there is entry for $c < \underline{c}(\alpha, \bar{\lambda})$, entry is strategically deterred for $\underline{c}(\alpha, \bar{\lambda}) \leq c \leq \bar{c}(\alpha, \bar{\lambda})$, and entry is blockaded for $c > \bar{c}(\alpha, \bar{\lambda})$, as described in the proposition. \square

The proofs of Propositions 4, 5, and 6 rely on the following Lemma.

Lemma 1. $\underline{c}(\alpha, \bar{\lambda})$ is strictly decreasing in α and strictly increasing in $\bar{\lambda}$.

Proof. Recall that $\underline{c}(\alpha, \bar{\lambda})$ is given by the c that satisfies

$$\begin{aligned} 2\sqrt{\bar{\lambda}c} \left(\bar{\lambda} - 2\sqrt{\bar{\lambda}c} \right)^\alpha \bar{\lambda}^{-1} &= \frac{\alpha^\alpha}{2(1+\alpha)^{1+\alpha}} \bar{\lambda}^\alpha \\ 2\sqrt{\frac{c}{\bar{\lambda}}} \left(1 - 2\sqrt{\frac{c}{\bar{\lambda}}} \right)^\alpha &= \frac{\alpha^\alpha}{2(1+\alpha)^{1+\alpha}}. \end{aligned} \quad (27)$$

We first prove that $\underline{c}(\alpha, \bar{\lambda})$ is strictly decreasing in α . Recall (from the proof of Proposition 3) that the LHS of (27) is strictly increasing in c and that the LHS of (27) is $\bar{\lambda}^{-1} (\bar{\lambda} - \bar{s}_1) \bar{s}_1^\alpha$, which is positive and increasing in α (as \bar{s}_1 is independent of α).

Thus, it is sufficient to show that the RHS is strictly decreasing in α . Differentiating the RHS with respect to α yields

$$\frac{\alpha^\alpha (\ln(\alpha) - \ln(1+\alpha))}{2(1+\alpha)^{1+\alpha}},$$

which is negative. Therefore, $\underline{c}(\alpha, \bar{\lambda})$ is strictly decreasing in α .

We now show that $\underline{c}(\alpha, \bar{\lambda})$ is strictly increasing in $\bar{\lambda}$. The RHS of (27) is independent of $\bar{\lambda}$. Hence, by the implicit function theorem, it is sufficient to show that the LHS of (27) is strictly decreasing in $\bar{\lambda}$. Taking this derivative and simplifying yields

$$2(1+\alpha)\sqrt{c} < \sqrt{\bar{\lambda}}. \quad (28)$$

Thus, (28) is strictly negative if and only if

$$c < \frac{\bar{\lambda}}{4(1+\alpha)^2}.$$

Evaluating (26) at $c = \frac{\bar{\lambda}}{4(1+\alpha)^2}$:

$$2\sqrt{\frac{\bar{\lambda}^2}{4(1+\alpha)^2}} \left(\bar{\lambda} - 2\sqrt{\frac{\bar{\lambda}^2}{4(1+\alpha)^2}} \right)^\alpha \geq \frac{\alpha^\alpha}{2(1+\alpha)^{1+\alpha}} \bar{\lambda}^{1+\alpha},$$

which simplifies to $1 + \alpha \geq 0$. Therefore, $\underline{c}(\alpha, \bar{\lambda}) < \frac{\bar{\lambda}}{4(1+\alpha)^2}$, so $\underline{c}(\alpha, \bar{\lambda})$ is strictly increasing in $\bar{\lambda}$. \square

Proof of Proposition 4.

Proof. As in Proposition 3, we have three regimes: blockaded entry, strategic entry deterrence, and accommodation. We can compute the aggregate religious participation in each regime. Under blockaded entry, the incumbent organization sets strictness s_1^* and aggregate participation is given by $\frac{\alpha}{(1+\alpha)^2} \bar{\lambda}$ by

Proposition 2, which is strictly increasing in $\bar{\lambda}$. Under strategic entry deterrence, the incumbent chooses strictness \bar{s}_1 . Aggregate participation is given by

$$\begin{aligned}\mu([\bar{s}_1, \bar{\lambda}])\bar{s}_1 &= \bar{\lambda}^{-1}(\bar{\lambda} - \bar{s}_1)\bar{s}_1 \\ &= 2\sqrt{c\bar{\lambda}} - 4c,\end{aligned}$$

which is strictly increasing in $\bar{\lambda}$. Under accommodation, aggregate participation is given by

$$\mu([s_1^*, s_1^* + s_2^*])s_1^* + \mu([s_1^* + s_2^*, \bar{\lambda}])s_2^* = \frac{1 + 2\alpha}{4(1 + \alpha)^2}\bar{\lambda},$$

which is strictly increasing in $\bar{\lambda}$. Hence, aggregate participation is strictly increasing in $\bar{\lambda}$ under each regime.

Because $\underline{c}(\alpha, \bar{\lambda}) < \bar{c}(\alpha, \bar{\lambda})$ and both $\underline{c}(\alpha, \bar{\lambda})$ and $\bar{c}(\alpha, \bar{\lambda})$ are strictly increasing in $\bar{\lambda}$ by Lemma 1 and (23), respectively, there are two possible transitions as $\bar{\lambda}$ increases. We can either transition from blockaded entry to strategic deterrence or from strategic deterrence to accommodation. We now show that aggregate participation is increasing at each transition point.

Recall by (22) that $s_1^* = \bar{s}_1$ at $c = \bar{c}(\alpha, \bar{\lambda})$. Hence, aggregate participation is continuously increasing in $\bar{\lambda}$ as the equilibrium switches from blockaded entry to strategic deterrence. Now consider the strategic deterrence and accommodation regimes. We show that at the $\bar{\lambda}$ where the regimes transition from strategic deterrence to accommodation, aggregate participation is strictly greater under accommodation.

At $c = \underline{c}(\alpha, \bar{\lambda})$, accommodation yields greater aggregate participation than strategic deterrence if

$$\begin{aligned}\mu([s_1^*, s_1^* + s_2^*])s_1^* + \mu([s_1^* + s_2^*, \bar{\lambda}])s_2^* &\geq \mu([\bar{s}_1, \bar{\lambda}])\bar{s}_1 \\ \frac{1 + 2\alpha}{4(1 + \alpha)^2} &\geq 2\sqrt{\bar{\lambda}\underline{c}(\alpha, \bar{\lambda})} - 4\underline{c}(\alpha, \bar{\lambda}).\end{aligned}\tag{29}$$

The RHS of (29) is maximized at $\underline{c}(\alpha, \bar{\lambda}) = \frac{\bar{\lambda}}{16}$. Making this substitution into (29) and simplifying, we have

$$\bar{\lambda} \leq \frac{1 + 2\alpha}{(1 + \alpha)^2}.$$

Hence, the above expression is a sufficient condition for aggregate participation to be greater under accommodation than under strategic deterrence.

Now, suppose to the contrary that this expression is not satisfied. Then, aggregate participation is strictly greater under accommodation than under strategic deterrence only if $\underline{c}(\alpha, \bar{\lambda})$ is either below the smallest c that satisfies (29) with equality or above the largest c that satisfies (29) with equality (because the RHS of (29) is strictly concave, as depicted by Figure 1). We will show that $\underline{c}(\alpha, \bar{\lambda})$ is below the smallest such c , which implies that (29) holds strictly at $c = \underline{c}(\alpha, \bar{\lambda})$ and thus aggregate participation is greater under accommodation than strategic deterrence. The smallest c is given by

$$\tilde{c} \equiv \left(\frac{\sqrt{\bar{\lambda}} - \sqrt{\bar{\lambda} - \frac{1+2\alpha}{(1+\alpha)^2}}}{4} \right)^2.\tag{30}$$

Evaluating (26) at \tilde{c} and simplifying yields

$$\left(\frac{1}{2} - \frac{\sqrt{1 - \frac{1+2\alpha}{\bar{\lambda}(1+\alpha)^2}}}{2} \right) \left(\frac{1}{2} + \frac{\sqrt{1 - \frac{1+2\alpha}{\bar{\lambda}(1+\alpha)^2}}}{2} \right)^\alpha \geq \frac{\alpha^\alpha}{2(1 + \alpha)^{1+\alpha}}.\tag{31}$$

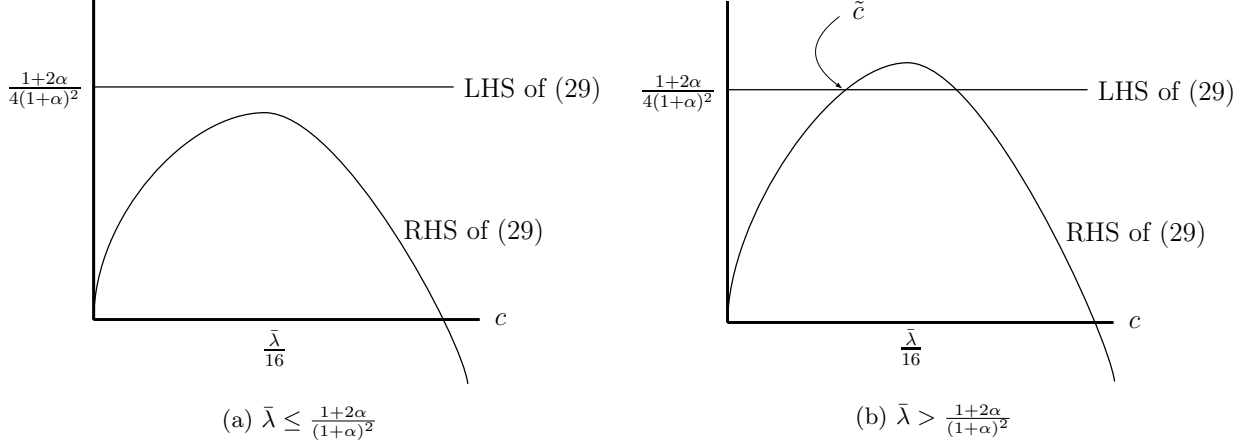


Figure 1: Visual representation of the LHS and RHS of (29) as a function of c .

Observe that (30) is strictly decreasing in $\bar{\lambda}$ and the LHS of (26) is strictly increasing in c and strictly decreasing in $\bar{\lambda}$ (by the proof of Lemma 1), which implies that the LHS of (31) is strictly decreasing in $\bar{\lambda}$. Therefore, it is sufficient to show that (31) is satisfied at $\bar{\lambda} = 1$. Evaluating (31) at $\bar{\lambda} = 1$:

$$\begin{aligned}
\left(\frac{1}{2} - \frac{\sqrt{1 - \frac{1+2\alpha}{(1+\alpha)^2}}}{2}\right) \left(\frac{1}{2} + \frac{\sqrt{1 - \frac{1+2\alpha}{(1+\alpha)^2}}}{2}\right)^\alpha &\geq \frac{\alpha^\alpha}{2(1+\alpha)^{1+\alpha}} \\
\left(\frac{1}{2} - \frac{\alpha}{2(1+\alpha)}\right) \left(\frac{1}{2} + \frac{\alpha}{2(1+\alpha)}\right)^\alpha &\geq \frac{\alpha^\alpha}{2(1+\alpha)^{1+\alpha}} \\
\left(\frac{1}{2(1+\alpha)}\right) \left(\frac{1+2\alpha}{2(1+\alpha)}\right)^\alpha &\geq \frac{\alpha^\alpha}{2(1+\alpha)^{1+\alpha}} \\
\left(\frac{1+2\alpha}{2}\right)^\alpha &\geq \alpha^\alpha \\
\frac{1+2\alpha}{2} &\geq \alpha \\
1 &\geq 0.
\end{aligned}$$

Thus, aggregate participation is strictly greater under accommodation.

The second part of the statement was established by Lemma 1. □

Proof of Proposition 5.

Proof. (i) Fix $c < \frac{1}{9}\bar{\lambda}$ and define $\underline{\alpha}$ as the solution to $c = \underline{c}(\underline{\alpha}, \bar{\lambda})$. Because $\underline{c}(\alpha, \bar{\lambda})$ is decreasing in α , $c < \underline{c}(\alpha, \bar{\lambda})$ for all $\alpha \in (0, \underline{\alpha})$. By Proposition 3, this implies that entry occurs and in the case of entry, aggregate participation is given by

$$\frac{(1+2\alpha)}{4(1+\alpha)^2} \bar{\lambda}. \tag{32}$$

By differentiation, (32) is strictly decreasing in α on $(0, \underline{\alpha})$, proving statement (i).

(ii) Define $\bar{\alpha}$ as the solution to $c = \bar{c}(\bar{\alpha}, \bar{\lambda})$.

First, we shall establish that $\bar{\alpha} < \frac{1}{2}$, as claimed in the proposition. At $\alpha = \frac{1}{2}$, $\bar{c}(\frac{1}{2}, \bar{\lambda}) = \frac{1}{9}\bar{\lambda}$ by (23). By hypothesis, $c > \frac{1}{9}\bar{\lambda} = \bar{c}(\frac{1}{2}, \bar{\lambda})$. Note that $\bar{c}(\frac{1}{2}, \bar{\lambda})$ is strictly decreasing in α by (23). Hence for $c = \bar{c}(\bar{\alpha}, \bar{\lambda})$, it must be that $\bar{\alpha} < \frac{1}{2}$, as claimed.

Because $\underline{c}(\alpha, \bar{\lambda}) < \bar{c}(\alpha, \bar{\lambda})$, $\underline{\alpha} < \bar{\alpha}$. For $\alpha \in (\underline{\alpha}, \bar{\alpha})$, $c \in (\underline{c}(\alpha, \bar{\lambda}), \bar{c}(\alpha, \bar{\lambda}))$, so entry is strategically deterred. Hence, average participation is given by

$$\mu([\bar{s}_1, \bar{\lambda}])\bar{s}_1 = \bar{\lambda}^{-1}(\bar{\lambda} - \bar{s}_1)\bar{s}_1 = 2\sqrt{\bar{\lambda}c} - 4c, \quad (33)$$

which is independent of α , so average participation is constant on $(\underline{\alpha}, \bar{\alpha})$, proving statement (ii).

(iii) Suppose that $\alpha \in (\bar{\alpha}, \frac{1}{2})$, so $c > \bar{c}(\alpha, \bar{\lambda})$. Then, entry is blockaded and average participation is given by $\frac{\alpha}{(1+\alpha)^2}\bar{\lambda}$ by Proposition 2, which we know is strictly increasing in α . \square

Proof of Proposition 6.

Proof. Define α' as the solution to $c = \underline{c}(\alpha', \lambda')$. Because $\underline{c}(\alpha, \lambda)$ is strictly increasing in λ by Lemma 1 and $\bar{\lambda} > \lambda'$ by hypothesis, $c < \underline{c}(\alpha', \bar{\lambda})$. Hence, there is entry at $(\alpha', \bar{\lambda})$.

Next, compare the incumbent's market share under strategic entry deterrence to the share under entry at $\alpha = \alpha'$:

$$\begin{aligned} \frac{\bar{\lambda} - \bar{s}_1}{\bar{\lambda}} &< \frac{\bar{\lambda} - s_1^*(\alpha')}{\bar{\lambda}} \\ \iff \frac{\bar{\lambda} - (\bar{\lambda} - 2\sqrt{\bar{\lambda}c})}{\bar{\lambda}} &< \frac{\bar{\lambda} - \frac{\alpha'}{1+\alpha'}\bar{\lambda}}{\bar{\lambda}} \\ \iff c &< \left(\frac{1}{2(1+\alpha')}\right)^2 \bar{\lambda} \\ &= \bar{c}(\alpha', \bar{\lambda}). \end{aligned}$$

This is satisfied because $c < \underline{c}(\alpha', \bar{\lambda}) < \bar{c}(\alpha', \bar{\lambda})$. \square