1-2021

Culture, Institutions & the *Long Divergence*

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Abstract

During the medieval and early modern periods the Middle East lost its economic advantage relative to the West. Recent explanations of this historical phenomenon—called the *Long Divergence*—focus on these regions' distinct political economy choices regarding religious legitimacy and limited governance. We study these features in a political economy model of the interactions between rulers, secular and clerical elites, and civil society. The model induces a joint evolution of culture and political institutions converging to one of two distinct stationary states: a *religious* and a *secular* regime. We then map qualitatively parameters and initial conditions characterizing the West and the Middle East into the implied model dynamics to show that they are consistent with the *Long Divergence* as well as with several key stylized political and economic facts. Most notably, this mapping suggests non-monotonic political economy dynamics in both regions, in terms of legitimacy and limited governance, which indeed characterize their history.

This draft: March 2023; *First draft*: February 2018

[*]Thanks to Timur Kuran for excellent and detailed comments on an earlier draft of this paper. We also thank Oded Galor, four anonymous referees, Vladimir Asriyan, Marcello D'Amato, James Feigenbaum, Martin Fiszbein, Jordi Gali, Roger Lagunoff, Gilat Levy, Dilip Mookherjee, Andy Newman, Debraj Ray, Jaume Ventura and participants in seminars at Boston University, Brown University, George Mason University, Georgetown University, Iowa State University, LSE-NYU Conference in Political Science and Political Economy, Oxford University, Universitat Pompeu Fabra, UC Riverdale, Toulouse University, Vancouver School of Economics, CEPR Workshop on the Economics of Religion, ASSA, ASREC, Summer School for Quantitative History in Shanghai, DIAL, ThRed, and SIOE. Avner Seror acknowledges funding from the French government under the "France 2030" investment plan managed by the French National Research Agency (reference: ANR-17-EURE-0020) and from Excellence Initiative of Aix-Marseille University - A*MIDEX. All errors are our own.

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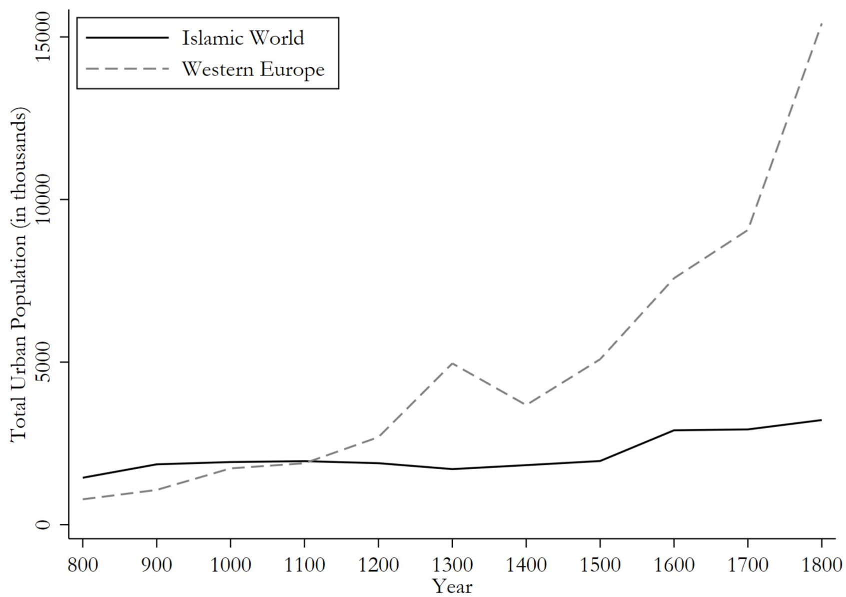
JEL Classification Numbers: O10, P16, P48, N34, N35, Z12, O33.
Keywords: Long Divergence; cultural transmission; institutions; legitimacy; religion; political economy.

1 Introduction

Around the year 1000 C.E., the Muslim Middle East was far ahead of Christian Western Europe in terms of socio-economic development. By the dawn of the industrial period (circa 1750), however, the Middle East severely lagged behind along several dimensions, including technology, innovation, literacy, wages, and financial development (Bosker, Buringh and Van Zanden 2013, Kuran 2011, Mokyr 1990, Özmucur and Pamuk 2002, Rubin 2017). This is what Timur Kuran (2011) calls the *Long Divergence.* Urban population is one metric illustrating the socio-economic divergence, as seen in Figure 1.[1]

Figure 1: Urban Population, 800–1800



Data source: Bosker, Buringh and Van Zanden (2013).

The historical narratives in the literature consistently interpret the economic divergence between Western Europe and the Middle East as the outcome of institutional and technological progress brought about or hindered by different strategies political authorities adopted to sustain their political support and to enlarge fiscal capacity in the medieval and early modern periods. Specifically, Kuran (2011) identifies the root cause of Middle

[1]Note that the *timing* of the reversal of fortunes cannot be inferred from this figure. First of all, pre-modern population data are subject to significant measurement error, perhaps mis-dating the precise point of reversal by centuries. Second, urban population is just one of many metrics social scientists employ as an indicator of socio-economic development. Levels of trade, science, technology, and architecture almost certainly diverged at different times.

East stagnation in Islamic law or Sharia—especially its inheritance system and partnership law—that governed most economic activities. Rubin (2017) argues that the persistence of Islamic law is at least partly a consequence of the role of the political power ceded to Muslim religious authorities due to their ability to provide legitimacy. This power was in turn used to block important technological and economic advancements, a leading example being the printing press. In Europe, on the other hand, the Catholic Church had a much weaker legitimating role, and economic elites in parliaments developed laws and policies that favored economic development. Blaydes and Chaney (2013) posit that Western European rulers had to rely on feudal institutions for tax collection and military recruitment. This led to a balance of power more favorable to local economic elites, which promoted economic growth in the long run. Muslim sultans, on the other hand, were not constrained by secular economic elites, in large part due to their access to slave soldiers, who satisfied both fiscal and military needs.

Motivated by these narratives, we propose a political economy model which aims at elucidating the historical mechanisms possibly responsible for the Long Divergence while mapping qualitatively into relevant historical facts.[2] Specifically, the model centers on the interactions between political authorities, secular and clerical elites, and civil society. It captures three fundamental elements of the socio-economic environment under study. The first concerns the role of religious legitimacy. Religious elites provide services which can shape the moral beliefs of the religious component of civil society. Political authorities can leverage this ability of religious elites to legitimate rule by delegating political power to them. The second element is a trade-off between religious legitimacy and religious proscriptions. These proscriptions may often end up dampening economic activity as, arguably, in the case of Islamic law in business affairs. The third element concerns the role of secular elites in enhancing the state's fiscal capacity. The delegation of political power from rulers to secular elites results in limited governance, which increases tax revenues by ceding tax collection power to those with greater capacity to collect it.

At the heart of the model is a complementarity between religious legitimacy and the profile of religious values in civil society. Religious agents see taxation as more legitimate

[2] We focus our explanatory analysis of the Long Divergence on historical forces that arose when the divergence took place during the medieval and early modern periods. Arguably, and importantly, these forces act upon—and interact with—more deeply-rooted dynamics at a slower frequency, like those stressed in Ashraf and Galor (2013), Bockstette, Chanda and Putterman (2002), Galor and Moav (2002), Galor and Özak (2016) and Galor (2022). Our analysis hence necessarily relies on a sort of adiabatic assumption that the political economy and cultural processes we study are rapid enough to be the ones which mostly matter at the time-scale under consideration.

than the non-religious portion of the population. A higher fraction of religious agents in the population therefore augments the political incentives for the ruler to delegate power to clerics to increase legitimacy; and in turn a more religious institutional set-up reinforces the incentives of religious individuals to transmit their values across generations, increasing their relative share in the population.

The reinforcement of religious cultural values and the political power of religious elites fundamentally affects socio-economic dynamics. The dynamics display two types of stationary states: i) a *religious regime* where clerics have substantial political power, they legitimate the ruler, and religious cultural values are predominant in the population; and ii) a *secular regime* in which clerics have little political power and secular beliefs are predominant. Allowing for limited governance induces a further characterization of the *secular regime* in which rulers delegate political power to secular elites at the expense of religious clerics.

The structural parameters of the socio-economic environment (e.g., the legitimating capacity of religious elites) and the initial conditions (e.g., the initial share of religious individuals in civil society) determine both the characteristics of the transitory dynamics of society as well as whether these dynamics converge to the religious or the secular stationary states. Importantly, these dynamics are not necessarily monotonic. In a subset of the basin of attraction of the religious state, for instance, and specifically when religious values are not predominant initially, rulers will not seek legitimacy from religious authorities for some time, only to change strategies after religious values are spread enough in the population. Conversely, when religious values are initially predominant, non-monotonic dynamics in which rulers delegate power to clerical elites for some time before delegating power to secular elites occur in the basin of attraction of the secular stationary state. In both cases, the dynamics are characterized by a "horse race" between cultural and institutional change.

We argue that this model provides a unitary account of the historical mechanisms which might have contributed to the Long Divergence. To this end, we first map various historical stylized facts into their structural parameters and initial conditions. We then show that the implied dynamics of the model are not only consistent with the Long Divergence, but also produce convergence paths with qualitative characteristics which can be historically identified in the growth paths of Western Europe and the Middle East.[3]

[3] More precisely, we focus on the period starting from the end of the Western Roman Empire in the West and the emergence of the Umayyad Caliphate in the Middle East until the onset of the Reformation in Europe and the capture of the Egyptian Mamluk Empire by the Ottoman Empire. Beginning the historical narratives from the end of the Roman Empire in the West and the Umayyad Caliphate in the Middle East

The main structural parameter of interest is the legitimating ability of religious elites. As discussed in detail in Section 4.2, we posit that—due to the contexts in which these religions were born—Christianity was relatively weak at legitimating rule, while the opposite was true for Islam in the Middle East (Feldman 1997, Rubin 2011, 2017).[4] The most relevant initial conditions in the model are the initial religious cultural values in the population. Christianity was widespread in the former Roman lands (i.e., religious cultural beliefs were widespread), while this was not the case for Islam in the Middle East, at least at the beginning of the period under consideration.[5] Under this mapping, the dynamics of the model are consistent with the Middle East and the West converging, respectively, to the religious and secular stationary states and with the historical narratives regarding the *Long Divergence*. The Middle East, in a religious stationary state, is expected to be less economically vibrant in the long-run due to the effects of religious proscriptions on economic activity. The main mechanisms driving the convergence to the distinct stationary states are i) the persistent use of religious legitimacy in the Middle East but not in Western Europe; and ii) the lack of limited governance in the Middle East relative to the West.

Furthermore, our mapping of historical facts into parameters and initial conditions suggests a tension between the structural ability of religious elites to provide legitimacy and the initial fraction of the population with religious beliefs—for both the Middle East and the West. This tension gave rise to the non-monotonic convergence dynamics the model allows for: the incentives to seek religious legitimacy were initially high in the Christian West, to be overtaken because of the limited legitimating ability of Christianity; while the opposite was the case in the Islamic Middle East. This non-monotonicity of

is appropriate because both represent "initial conditions" far from either a secular or a religious stationary state.

[4] Christianity was born in the Roman Empire, and its followers were a persecuted minority. It was hence in no position to legitimate the emperor. Meanwhile, Islam formed conterminously with an expanding empire, and numerous important Islamic dictates specify the righteousness of following leaders who act in accordance with Islam (Hallaq 2005, Rubin 2011, 2017). There is a historical literature, discussed further in Section 4.2, which disputes how much early Islamic empires, especially the Umayyad Empire (661–750), relied on religious legitimacy. For the time being, we note the distinction between the exogenous legitimating technology and its endogenous use by the state. Our model attempts to understand the latter conditional on the former, and in fact predicts that early Islamic empires should have initially sought to reduce reliance on religious legitimacy despite the relatively high (exogenous) capacity of Muslim religious authorities to legitimate rule.

[5] Islamic political power spread rapidly—spanning the Iberian Peninsula to South Asia within a century of Muhammad—but the population living under Islamic regimes were largely non-Muslim for the first few Islamic centuries (Bessard 2020, Saleh 2018).

the dynamic paths is consistent with the historical political economy patterns in the two regions. As discussed in much more detail in Section 4.3, in Western Europe, following the fall of the Roman Empire, rulers of the Germanic "follower kingdoms" either converted to Christianity or promoted it, as for instance was the case of the Frankish king Clovis (r. 481–509). These strategies characterized Western Europe until the 11th century, when the re-birth of commerce gave rise to independent cities and increased tensions between the religious and secular elite. In the Middle East, early rulers established law and order, administered the state, and encouraged loyalty to the empire by sending "proto-kadis" (religious judges) to the provinces. After the religious establishment consolidated in the ninth century, and especially after the rise of the madrasa system in the 11th century, religious authorities were the primary agents capable of determining whether rulers acted in accordance with Islam.

The paper proceeds as follows. In Section 2 we lay out the basic socio-economic environment in terms of preferences and technologies of the ruler, clerics, and civil society. We also describe the space of available policy interventions. In Section 3 we study the societal equilibrium for each generation t (Section 3.1) and the processes of institutional and cultural change across generations (Sections 3.2 and 3.3, respectively). In Section 4 we map the model into historical facts and narratives. In Section 5 we extend the model to study equilibria and dynamics when we allow for political decentralization to secular elites. Section 6 concludes.[6]

2 Ruler, Clerics, and Civil Society

We consider a political economy model of the distribution of power between three types of agents: a ruler, religious clerics, and civil society.[7] Religious legitimacy is an equilibrium phenomenon. It results from an institutional process of delegation of power and it depends

[6]In the Appendix, we further extend the model to consider the role of religion and religious legitimacy in inhibiting innovation and technological change (Bénabou, Ticchi and Vindigni 2015, 2020, Coşgel, Miceli and Rubin 2012, Davids 2013, Mokyr 1990, 2010, 2016, Squicciarini 2020, White 1972, 1978). More generally, it is certainly not the case that religion as a whole always has a negative impact on economic development; see Barro and McCleary (2003) and McCleary and Barro (2019) for an overview of the literature and a theory of the positive associations between religion and economic development.

[7]In Section 5 we extend the model to study the equilibrium relationship between religious legitimacy and limited governance.

on the profile of religious values in the population, the efficiency of the clerics' "legitimating technology", and the degree of restrictiveness of religious proscriptions imposed by clerics.[8]

Let $t = 0, 1, \ldots$ index generations. All agents only live for one generation. As a consequence, the game played between the ruler, clerics, and civil society is a series of one-shot games in which behavior is not forward-looking with respect to institutional or cultural evolution.[9]

Civil Society. Each generation consists of a continuum $[0, 1]$ of citizens. Civil society is composed of two types i of citizens: religious individuals ($i = Re$) in proportion q_t in generation t, and secular individuals ($i = S$) in proportion $1 - q_t$. Citizens employ effort in production activities. Total production is $E_t = q_t e_{Re,t} + (1 - q_t)e_{S,t}$, where $e_{i,t},\ i = Re, S$ is the per-capita work effort employed by an individual of type i in generation t.

Ruler and Clerics. The ruler lives off taxing civil society at a tax rate τ_t. The tax base to which the ruler has access is the total production of citizens, E_t. The ruler also contributes to building and maintaining religious infrastructure, $m_t \geq 0$, for the clerics to provide religious services. The total religious services provided for society are $m_t \cdot \alpha_{c,t}$, where $\alpha_{c,t} \geq 0$ is the effort of the (representative) cleric at time t. The building of religious infrastructure has cost $C(m_t)$ that the ruler pays for. Meanwhile, clerics pay for the daily maintenance costs $F(m_t)$ of religious infrastructure.[10]

[8] The study of political legitimacy has a long history in the social sciences. Perhaps most famously, Weber (1947) defined political legitimacy as either charismatic, traditional, or legal-rational. Our definition follows more closely in the footsteps of the definition of political legitimacy employed by Lipset (1959, p. 86): "the capacity of a political system to engender and maintain the belief that existing political institutions are the most appropriate or proper ones for the society." For similar definitions of political legitimacy, see Hurd (1999), Tyler (2006), Gilley (2006), Levi, Sacks and Tyler (2009), Greif and Tadelis (2010), Rubin (2017), and Greif and Rubin (2023*a*,*b*). More specifically, in our context, see also Auriol and Platteau (2017), Auriol, Platteau and Verdier (2022), Coşgel, Miceli and Rubin (2012), Coşgel and Miceli (2009), Lewis (1974, 2002), Platteau (2017), Rubin (2011), and Kuru (2019). In our context, legitimacy takes the form of a religious justification, provided by religious elites, supporting the ruler's right to rule and have her demands obeyed (Greif and Rubin 2023*b*).

[9] This is in line with the conceptualization of institutional change proposed in Greif and Laitin (2004) and Greif (2006), in which institutions are exogenous to the players at any given point in time but evolve over time in response to the actions taken by the players at that time in response to institutional and cultural incentives. A fully forward-looking model of institutional change is analytically intractable when joined with cultural dynamics; see Bisin and Verdier (2017) for a discussion and Lagunoff (2009) and Acemoglu, Egorov and Sonin (2015) for forward-looking institutional change. Some historical motivation for myopic institutional change in the study of the emergence of democracy is found in Treisman (2020).

[10] These costs are assumed to be increasing in m_t and sufficiently convex to satisfy a regularity condition, needed to ensure that when religious clerics have a high political weight λ_t, the policy problem associated with institutional design is well behaved and provides a finite equilibrium provision of m.

Legitimacy. Clerics can provide the ruler with *legitimacy* through religious services which facilitate governance and obedience for religious individuals. We focus on the role legitimacy plays in *tax collection* (e.g., Coşgel and Miceli 2009, Levi and Sacks 2009, Wintrobe 1998). In particular, citizens are more likely to defer to tax authorities when governance is viewed as legitimate, and they likewise may feel better about paying taxes to a divinely sanctioned political authority.[11] This is a source of political power for religious authorities. However, this power is limited by the fact that religious legitimacy only operates on the religious component of civil society. In our formulation, religious individuals, when taxed by the ruler, subjectively perceive a tax rate $\tau^e_{Re,t}$ smaller than the actual τ chosen by the ruler and decreasing in the religious effort of clerics, $\alpha_{c,t}$:

$$\tau^e_{Re,t} = \tau_t(1 - \theta\alpha_{c,t}). \tag{1}$$

For secular individuals, $\tau^e_{S,t} = \tau$.[12]

As a consequence, the total level of taxes collected is increasing in the cleric's effort, $\alpha_{c,t}$, and the efficiency of the legitimating technology. We denote the exogenous component of the legitimating technology by $\theta \in [0, 1]$, and we interpret it as the efficiency of religious legitimacy in encouraging compliance with authority.

Proscriptions. Religious services have an indirect cost, in that they require the imposition of various *proscriptions* (i.e., regulations and constraints) on individual behavior. These proscriptions are imposed on both religious and secular individuals.[13] Examples of these types of proscriptions are inheritance laws, prohibitions on technologies such as printing, and usury restrictions on the entire credit market. We capture the effect of religious

[11] This is just one of the several dimensions of the ruler's governance ability which are affected by legitimacy. Importantly, for instance, legitimacy lowers the likelihood of revolt (Bentzen and Gokmen 2022, Chaney 2013, Gill 1998, Gilley 2008, Greif and Rubin 2023*a*,*b*, Guo 2003, Hechter 2009, Hurd 1999, Tyler 2006).

[12] Alternatively, we could assume that when clerical effort and the legitimating technology are greater, fewer religious citizens evade taxes (Coşgel and Miceli 2009, Greif and Rubin 2023*a*). Another interpretation is that religious individuals work in the public sector for lower pay.

[13] These are the types of proscriptions that typically have the largest effect on economic growth. We are not concerned with other types of prohibitions that only affect religious believers, such as certain dietary restrictions or marriage or divorce restrictions (Freidenreich 2013, 2015, Tolan 2019). The model could be amended to allow secular individuals to be less affected than religious individuals by the cost of religious proscriptions. In such a case, it can be shown that this increases the likelihood of a long-run theocratic state compared to a secular state. See footnote 26 for a discussion.

proscriptions by assuming that the cost of individual production effort is

$$c(\alpha_{c,t})\Phi(e_{i,t}),\ \text{with}\ \Phi(e_{i,t}) = \frac{e_{i,t}^2}{2}\ \text{and}\ c(\alpha_{c,t}) = 1 + \phi\alpha_{c,t},\ i = Re, S. \tag{2}$$

The parameter $\phi > 0$ represents the degree of restrictiveness of religious proscriptions on economic activities.[14]

Preferences. Preferences of the agents in this society in any generation t are as follows. The ruler has utility

$$U_r(m_t) = \tau_t E_t - C\left(m_t\right). \tag{3}$$

Clerics derive utility $m_t \cdot \alpha_{c,t}$ from religious services, at effort cost $\Psi(\alpha_{c,t})$.[15] The utility of the clerics therefore is

$$U_c(m_t, \alpha_{c,t}) = m_t \cdot \alpha_{c,t} - \Psi(\alpha_{c,t}) - F(m_t). \tag{4}$$

Finally, the utility of agents of type $i = Re, S$ in civil society is

$$U_i(e_{i,t}) = e_{i,t}(1 - \tau_{i,t}^e) - c(\alpha_{c,t})\Phi(e_{i,t}), \quad i = Re, S. \tag{5}$$

We assume the cost functions $C(.)$, $F(.)$ and $\Psi(.)$ are increasing and convex in their argument.[16]

This setup establishes—somewhat starkly—one of the model's fundamental building blocks: the trade-off between religious legitimacy and religious proscriptions with respect to the size of the taxable surplus. Legitimacy increases the incentive to provide effort for

[14] The parameter ϕ is held as exogenous in the model, even though there are clearly endogenous elements of religious proscriptions (Rubin 2011, Seror 2018). In fact, both Islamic law and Christian (canon) law changed over time to address economic exigencies (Berman 1983, Hallaq 1984, 2005, Noonan 1957, 2005). Nonetheless, note that the effective cost of economically-inhibitive religious proscriptions $c(\alpha_{c,t}) = 1+\phi\alpha_{c,t}$, is an *outcome* of the "religious" political-economy equilibrium. Consequently the *effective* impact of the restrictiveness of religious proscriptions on economic development depends on the relative weight of religious authorities in political decision-making, which is endogenous in the model.

[15] In various times and places, such as Golden Age Islam or medieval Europe, religious authorities were also directly involved in economic activities. Although we do not explicitly model this possibility here, it follows from our setup that religious authorities can benefit from a greater economic surplus since it provides more revenue for expenditure on religious services.

[16] We also assume that $F'(m) < C'(m)$ for all $m > 0$; i.e., that the marginal cost of infrastructure maintenance is smaller than the marginal cost of building infrastructure.

the religious (or alternatively, lowers their incentive to evade taxation), but comes at the cost of lowered productivity due to proscriptions.

Policy. Policy choices are not necessarily the sole responsibility of the ruler. They are, in general, the outcome of a collective choice problem in any given generation t, reflecting the political power and preferences of the three groups, and representing indirectly the political economy process in society (Bisin and Verdier 2017, Paniagua and Vogler 2022).[17] In other words, policies are the outcome of a "bargain" implicit in the institutional structure of society. More specifically, this is how the choice of religious infrastructure m_t, over which both religious clerics and civil society have a say, is made in our model.[18]

The relative political power of the groups is captured by their respective weight in the social welfare function W_t, which is the objective of policy choices.[19] Specifically, the social welfare function W_t to be maximized by the policy choice m_t is:

$$W_t = \frac{1}{2}U_r(m_t) + \frac{\lambda_t}{2}U_c(m_t, \alpha_{c,t}) + \frac{1-\lambda_t}{2}\left[q_t U_{Re,t}(e_{Re,t}) + (1-q_t)U_S\left(e_{S,t}\right)\right]. \tag{6}$$

Fixing the relative power of the ruler (to $\frac{1}{2}$),[20] the power of clerics and civil society is, respectively, $\frac{\lambda_t}{2}$ and $\frac{1-\lambda_t}{2}$ with $\lambda_t \in [0,1]$.

Each generation's *societal equilibrium* will obtain as the ruler, clerics, and agents in civil society choose $\tau_t(\leq \overline{\tau})$,[21] $\alpha_{c,t}$, and $e_{i,t}$ (for $i = Re, S,$) to maximize their utility given by (3), (4), and (5), respectively. The policy choice m_t is determined by the institutional bargaining process to maximize (6). At a *societal equilibrium* in each generation t, the ruler, policy-maker, clerics, and civil society, take as given i) the distribution of power between the groups in society, λ_t; as well as ii) the distribution of religious and secular

[17] In turn, the relative political power of the groups is endogenously determined in the model; see Section 3.2.

[18] In the logic of our model, religious infrastructure represents all those policies which are the outcome of political economy factors and whose effects are not fully internalized by the political economy process (and over which the political economy process does not have full commitment). With respect to these policies, the institutional forces identified in our analysis are salient.

[19] In accordance with our interpretation of the political economy process, the social welfare function W_t can be thought as the objective of a "fictitious policy-maker," who makes decisions based on the political weight of each segment of society.

[20] This is just for simplicity and concreteness: all that is needed is that the ruler has large enough power with respect to the other members of society.

[21] $\overline{\tau} < 1$ is associated with the fiscal capacity of the ruler (i.e., the maximum tax rate implementable in this economy).

types in civil society, q_t. But both the distribution of power and the distribution of types in civil society are endogenously determined. In the next section, we study first the *societal equilibrium* for any t, and then the dynamics of λ_t and q_t in the model.

3 Societal Equilibrium and Dynamics

At any time t, for a given institutional power structure and population profile of religious and secular individuals, the *societal equilibrium* is a Nash equilibrium of the simultaneous game between the ruler, policy-maker, clerics, and civil society. The non-cooperative nature of choices captures the idea of a public choice environment plagued by externalities and lack of commitment, whereby policy-makers and agents do not internalize the full impact of their behavior on society.

Institutional change arises as a mechanism to internalize the externalities associated with the political process, given the changing cultural composition of society (Acemoglu and Robinson 2019, Bisin and Verdier 2017, Iyigun, Rubin and Seror 2021). Cultural dynamics derive from purposeful inter-generational transmission, emanating from parental socialization and imitation of society at large (Bisin and Verdier 2001, 2017).

3.1 Societal Equilibrium

At a *societal equilibrium* for generation t, the choices of τ_t, $\alpha_{c,t}$, $e_{i,t}$ $(i = Re, S)$, and m_t constitute a Nash equilibrium, denoted by $\{\tau_t(\lambda_t), m_t(\lambda_t), \alpha_{c,t}(\lambda_t), e_{S,t}(\lambda_t), e_{Re,t}(\lambda_t)\}$.[22]

It is easy to see that the equilibrium tax rate $\tau_t(\lambda_t)$ is equal to its maximum possible value $\bar{\tau}$, indicating fully extractive taxation.[23] In order to simplify notation, we write τ instead of $\bar{\tau} = \tau_t(\lambda_t)$ in the remainder of the paper. The comparative statics at equilibrium in any period t are summarized in the following Lemma. For notational convenience, we suppress the time subscript t in the rest of this section.

[22] The equilibrium is fully characterized in the Appendix. Since there is a complementarity between the provision of the religious good m_t and the investments of the clerics in religious infrastructure $\alpha_{c,t}$, the uniqueness of the equilibrium is not guaranteed. Under mild conditions, however, the equilibrium is uniquely determined.

[23] In the societal equilibrium, the ruler takes as given citizens' efforts and does not internalize the negative effect of taxation on the tax base. Therefore he chooses the maximum possible tax rate $\bar{\tau}$.

Lemma 1 ***Religious infrastructure:*** *The equilibrium investment in religious infrastructure, $m(\lambda)$, and the equilibrium effort of the clerics, $\alpha_c(\lambda)$, are increasing in λ and independent of θ and ϕ.*

When the weight of the clerics in social choice increases, so does the marginal benefit of provisioning the religious infrastructure m. In turn, clerics increase their own effort in provisioning religious services $\alpha_c(\lambda)$. Since the weight of the clerics in social choice is $\frac{\lambda}{2}$, both $\alpha_c(\lambda)$ and $m(\lambda)$ increase with λ.

In the model, clerics do not derive utility from imposing proscriptions on economic activity nor from legitimating the ruler. Hence, the investment in religious infrastructure $m(\lambda)$ and the provision of the religious services $\alpha_c(\lambda)$ are independent from θ and ϕ.

Lemma 2 ***Labor effort:*** *The equilibrium effort of secular individuals $e_S(\lambda)$ is decreasing in λ and ϕ and is independent of θ. On the other hand, as long as $\theta \geq \frac{\phi(1-\tau)}{\tau}$, the equilibrium effort of religious individuals $e_{Re}(\lambda)$ is increasing in λ and θ, and is decreasing in ϕ.*

When the efficiency of the clerics to legitimate the ruler θ increases, so does the effort of religious individuals who subjectively perceive a lower tax rate. By contrast, the efficiency of the legitimating technology has no effect on the effort of secular individuals. An increase in the degree of restrictiveness of religious proscriptions, ϕ, leads to lower efforts from both religious and secular individuals, as harsher proscriptions decrease individuals' labor productivity.

The political weight of the clerics affects labor efforts through $\alpha_c(\lambda)$, their equilibrium effort. While more effort from the clerics makes secular individuals reduce their own labor effort—through costly regulations and prohibitions ϕ—when $\theta \geq \frac{\phi(1-\tau)}{\tau}$, clerics have the opposite effect on the labor effort of religious individuals e_{Re}. This is because when clerics provide more effort, religious individuals perceive a lower tax rate. Despite costly religious regulations, they increase their effort due to higher investments in religious infrastructure. In order to make this key difference between secular and religious individuals stark, we make the following Assumption:

Assumption 1 $\theta \geq \frac{\phi(1-\tau)}{\tau}$.

We denote the tax base as $E(\lambda) = qe_{Re}(\lambda) + (1-q)e_S(\lambda)$. From the two previous Lemmas, we deduce the following result:

Lemma 3 ***Tax base:*** *Under Assumption 1, the tax base is increasing in q and θ and it is decreasing in ϕ. It increases with λ as long as $q \geq \frac{\phi(1-\tau)}{\tau\theta}$.*

While religious infrastructure increases the scope of religious proscriptions, it also positively affects the effort of the religious individuals under Assumption 1. Hence, when religious individuals are sufficiently numerous, the latter effect dominates, and the tax base $E(\lambda)$ increases with the effort of the clerics $\alpha_c(\lambda)$, so it increases with λ. Similarly, since θ positively affects the labor effort of religious individuals, it also positively affects the tax base. Religious proscriptions ϕ negatively affect the tax base, as they decrease labor efforts. The tax base increases with the fraction of religious q, who provide greater effort than their secular counterparts.

3.2 Institutional Dynamics

Each generation brings about institutional change in the relative power delegated to clerics and civil society in the future. That is, at the end of any generation t, λ_{t+1} is chosen from the point of view of the social welfare function with weight λ_t.[24] In other words, institutions are exogenous from the perspective of all players at any point in time but change over time to reduce externalities associated with the decisions made by policymakers.[25] More formally, at any time t, given institutions λ_t, future institutions λ_{t+1} are designed as the solution to:

$$\max_{\lambda_{t+1}} \frac{1}{2}U_r(m_t(\lambda_{t+1})) + \frac{\lambda_t}{2}U_c(m_t(\lambda_{t+1}), \alpha_{c,t}(\lambda_{t+1}))+ \\ \frac{1-\lambda_t}{2}\left[q_t U_{Re}(e_{Re,t}(\lambda_{t+1})) + (1-q_t)U_S\left(e_{S,t}(\lambda_{t+1})\right)\right]. \quad (7)$$

Institutional change between periods t and $t+1$ therefore internalizes two externalities that are not taken into account by the optimal decisions characterizing the Nash equilibrium

[24]We assume that institutional design is myopic, anticipating only socio-economic outcomes one generation ahead. This implies that the institutional structure does not internalize institutional "slippery slopes," whereby moving to a different structure of decision rights may in turn trigger subsequent institutional changes leading to undesirable outcomes from the point of view of the initial structure. See Bisin and Verdier (2017) for a discussion of how this issue can be accounted for in this kind of framework.

[25]In this sense, our conception of institutional change follows in the spirit of Greif and Laitin (2004), Greif (2006), and Bisin and Verdier (2017) in that institutions change over time in response to the actions taken by the relevant players at a point in time given the incentives they face at that time. As in our conception of λ_t, such "quasi-parameters" (to use the term coined in Greif and Laitin (2004)) are exogenous to all players in period t but change over time in response to their actions.

of period t. The first one relates to the fact that the provision of religious infrastructure m grants *legitimacy* to the ruler, reducing the subjectively perceived tax rate for religious individuals. The second is the fact that it also has a depressing effect on labor productivity via proscriptions. Hence, increased provision of the religious good m not only affects the utility of the clerics, but also feeds back into the utility of both the ruler and the citizens. Solving the optimization problem (7), we obtain the following result:

Proposition 1 *The solution $\lambda_{t+1} \in [0,1]$ to optimization problem (7) is unique. The solution is characterized by a threshold $\overline{q}(\lambda_t) \in [0,1]$ such that,*

$$\lambda_{t+1} > \lambda_t \text{ (resp. } \leq \text{), if } q_t > \overline{q}(\lambda_t) \text{(resp. } \leq \text{).}$$

Furthermore, the threshold $\overline{q}(\lambda_t)$ is decreasing in θ and increasing in ϕ.

The uniqueness result follows from the convexity of the optimization problem. Whether more power is delegated to the clerics over time depends on the fraction of religious individuals q_t. A larger weight to clerics $\lambda_{t+1} > \lambda_t$ increases their effort $\alpha_c(\lambda_{t+1})$. This in turn increases the utility of the ruler U_r, who benefits from a larger tax base (Lemma 3). When the religious are sufficiently numerous, this also increases the total welfare of the citizens $q_t U_{Re} + (1 - q_t) U_S$. In such a case, while secular individuals suffer from religious proscriptions, civil society as a whole can still benefit from higher effort from the clerics. Religious individuals are better off when they perceive a lower tax rate and they comprise a large enough share of the population.[26]

When the severity of religious proscriptions ϕ increases, so does the cost to the ruler of using religious legitimacy as a means of extracting resources from the population. When clerics are efficient at legitimating the ruler, i.e. when θ increases, delegating power to the clerics enables the ruler to extract more resources and lowers the perceived cost of effort of the religious. As a result, the parameter space over which λ_t increases expands, so $\overline{q}$ decreases.

[26] Note that if secular individuals suffer less than religious individuals from religious proscriptions, an increase in the clerics' weight λ_t is more likely to happen as civil society as a whole is less affected by the economic cost of such religious proscriptions. Formally, the threshold $\overline{q}(\lambda_t)$ becomes smaller when religious proscriptions are less satisfied by secular individuals than by religious individuals.

3.3 Cultural Dynamics

Cultural dynamics are modeled as purposeful inter-generational transmission via parental socialization and imitation of society at large (Bisin and Verdier 2001, 2017). *Direct vertical* socialization to the parent's trait $i \in \{Re, S\}$ occurs with probability d_i. If a child from a family with trait i is not directly socialized, which occurs with probability $1 - d_i$, he/she is *horizontally/obliquely* socialized by picking the trait of a role model chosen randomly in the population.[27] The probability P_{ij} that a child in group i is socialized to trait j writes as:

$$\begin{aligned} P_{ii} &= d_i + (1 - d_i)q_i \\ P_{ij} &= (1 - d_i)q_j; \end{aligned} \tag{8}$$

with $q_{Re} = q$ and $q_S = 1 - q$. We assume that the probability of direct socialization d_i is the solution of a parental socialization problem[28] in which: a) parents are paternalistic (i.e., imperfectly altuistic) and have a bias for children sharing their own cultural trait; b) such paternalistic bias writes as $\Delta V_i(\lambda_t) = V_{ii}(\lambda_t) - V_{ij}(\lambda_t)$, where $V_{ij}(\lambda_t) = U_i(e_j(\lambda_t))$ is the utility perceived by a type i parent of having a type j child, for $i, j \in \{Re, S\}$ and $j \neq i$; c) parents of type $i \in \{Re, S\}$ have socialization costs that are increasing and convex in d_i; d) religious infrastructure m_t may act as a complementary input to the transmission effort d_{Re} of religious families in the socialization of children to the religious trait.

More specifically, denote $h_{Re}(d_{Re}, m_t)$ the socialization cost of religious families and $h_S(d_S)$ the socialization cost of secular families. Then religious parents solve the following socialization problem:

$$\max_{d_{Re}} -h_{Re}(d_{Re}, m_t) + P_{ReRe} \cdot V_{ReRe}(\lambda_t) + P_{ReS} \cdot V_{ReS}(\lambda_t), \tag{9}$$

while secular parents solve the following socialization problem:

$$\max_{d_S} -h_S(d_S) + P_{SS} \cdot V_{SS}(\lambda_t) + P_{SRe} \cdot V_{SRe}(\lambda_t). \tag{10}$$

As shown in the appendix, the solution to (9) provides the equilibrium socialization effort of religious families $d^*_{Re,t} = D_{Re}\left[(1 - q_t)\Delta V_{Re}(\lambda_t), m(\lambda_t)\right]$, which is an increasing function of both $(1 - q_t)\Delta V_{Re}(\lambda_t)$ and $m(\lambda_t)$. Similarly, the solution of (10) defines the

[27] Vertical, horizontal, and oblique transmission are the core mechanisms in the dual-inheritance theory of cultural evolution. For more, see Cavalli-Sforza and Feldman (1981) and Boyd and Richerson (1985).

[28] See Bisin and Verdier (1998, 2000, 2001) for a similar approach in different contexts and Bisin and Verdier (2011, 2022) for surveys of the economic literature on cultural transmission.

equilibrium socialization effort of secular families $d^*_{S,t} = D_S\left[q_t \Delta V_S(\lambda_t)\right]$, which is an increasing function of $q_t \Delta V_S(\lambda_t)$. In addition, the dynamics of the proportion of the population with the religious trait is characterized by the following "cultural replicator" dynamics:

$$q_{t+1} - q_t = q_t(1 - q_t)\{d^*_{Re,t} - d^*_{S,t}\}. \tag{11}$$

In equation (11), the term

$$D(q_t, \lambda_t) = d^*_{Re,t} - d^*_{S,t} = D_{Re}\left[(1 - q_t)\Delta V_{Re}(\lambda_t), m(\lambda_t)\right] - D_S\left[q_t \Delta V_S(\lambda_t)\right],$$

can be interpreted as the relative "cultural fitness" of the religious trait in the population. This term is frequency dependent (i.e., it depends on the state of the population q_t). It is also affected by the institutional environment λ_t, as this variable interacts with the process of parental cultural transmission both through paternalistic motivations $\Delta V_i(\lambda_t)$, and through the provision of religious infrastructure $m_t = m(\lambda_t)$ as a complementary input to religious family socialization.

In other words, there is a complementarity between religious legitimacy and the profile of religious values in the population. We deduce the following result:

Proposition 2 *There exists a threshold $q^*(\lambda_t)$ such that*

$$q_{t+1} < q_t \text{ (resp. } \geq \text{) if } q_t > q^*(\lambda_t) \text{ (resp. } \leq \text{).}$$

Furthermore, the threshold $q^*(\lambda_t)$ is increasing in θ and λ_t and decreasing in ϕ.

Because the process of cultural transmission (8) is characterized by cultural substitution between vertical and oblique transmission, the relative "cultural fitness" of the religious trait $D(q_t, \lambda_t)$ is decreasing in the frequency q_t of religious individuals in the population (Bisin and Verdier 2001). Consequently, the proportion $q^*(\lambda_t)$ such that $D(q^*(\lambda_t), \lambda_t) = 0$ is the unique attractor of the cultural dynamics in (11). When the fraction of religious individuals q_t is above (resp. below) $q^*(\lambda_t)$, then it decreases (resp. increases) in order to converge in the direction of $q^*(\lambda_t)$.

An increase in the political weight of the clerics λ_t affects cultural transmission in two ways, through its effect on socialization incentives $\Delta V_{Re}(\lambda_t)$ and $\Delta V_S(\lambda_t)$ and through its effect on religious infrastructure, $m = m(\lambda_t)$. On the one hand, an increase in λ_t promotes the clerics' effort $\alpha_c(\lambda_t)$ and consequently leads to a lower perceived tax rate τ^e_{Re} by religious individuals. The labor effort choice of religious and secular individuals is, therefore, further apart and, consequently, the incentives of parents to socialize their children to their own cultural trait, $\Delta V_{Re}(\lambda_t)$ and $\Delta V_S(\lambda_t)$, are larger in both groups.[29] However, when the socialization effort of religious parents is more sensitive to these incentives than the effort of secular parents, the religious trait is relatively more successfully transmitted than the secular trait, and $D(q_t, \lambda_t)$ is shifted up with an increase in λ_t. An increase in λ_t also increases the amount of religious infrastructure $m = m(\lambda_t)$. When such infrastructure enters as a complementary input in the socialization process of the religious trait, then again religious parents tend to socialize more intensively than secular ones when m increases. The religious trait has consequently higher cultural fitness than the secular trait and again $D(q_t, \lambda_t)$ is shifted up with λ_t. In either situation, the diffusion of the religious trait is favored by an increase in λ_t, and $q^*(\lambda_t)$ becomes larger.

A change in the other parameters θ and ϕ affects the relative cultural fitness of the religious trait only through their induced changes on the paternalistic motives $\Delta V_{Re}(\lambda_t)$ and $\Delta V_S(\lambda_t)$. For instance, a higher efficiency of the clerics θ tends to widen the gap between the optimal work effort of a religious individual compared to that of a secular individual. As a consequence, an increase in θ shifts up both $\Delta V_{Re}(\lambda_t)$ and $\Delta V_S(\lambda_t)$. As mentioned above, when religious parents are more sensitive to paternalistic motives than secular parents, these shifts lead religious parents to socialize more intensively than secular parents, and religious values are passed from generation to generation with a higher intensity. This results in a higher value of $q^*(\lambda_t)$. Conversely, a higher value of religious proscriptions ϕ dampens the impact of work effort on economic outcomes. Consequently, behavioral differences induced by cultural traits are less relevant from a utility point of view. This in turn reduces the paternalistic motives $\Delta V_{Re}(\lambda_t)$ and $\Delta V_S(\lambda_t)$ of religious and secular parents. The effect of a change in proscriptions ϕ on cultural evolution is then qualitatively the opposite of that of a change in θ.

4 Model Dynamics and Historical Narrative

In this section we draw out the implications of the model with regards to the joint dynamics of culture and institutions and match them with various elements of the historical narrative

[29] Given the quadratic specification of the utility function $U_i(e_i)$, and substituting the optimal labor efforts in the utility of the citizens, we find that $\Delta V_{Re}(\lambda_t) = \Delta V_S(\lambda_t) = \frac{(\tau\theta\alpha_c(\lambda_t))^2}{2(1+\phi\alpha_c(\lambda_t))}$, which is increasing in λ_t.

regarding Middle Eastern and Western European political economy during the medieval and early modern periods.

In Section 4.1 we represent the dynamics of the model by a phase diagram. To this end, we exploit the characterization we obtained in the previous section of the dynamics' stationary states, their stability properties, and their basins of attraction, as a function of structural parameters and initial conditions. In Section 4.2 we lay out relevant historical information to draw a qualitative mapping of structural parameters and initial conditions for the Middle East and the West into the basins of attraction of the different dynamics identified by the model. Finally, in Section 4.3 we match the model's implied dynamics for these two regions to the historical narrative regarding the Long Divergence as well as other characteristics of the political economy patterns of the history of these regions.[30]

4.1 The Joint Dynamics of Culture and Institutions

Under the conditions of Propositions 1 and 2, we can represent the joint cultural and institutional dynamics in the phase diagram of Figure 2. The solid black line represents the threshold of the institutional dynamics $\overline{q}(\lambda_t)$. The dotted line represents the threshold $q^*(\lambda)$ associated with the cultural dynamics.[31] The arrows in Figure 2 depict the joint dynamics of culture and institutions, given our results in Propositions 1 and 2.

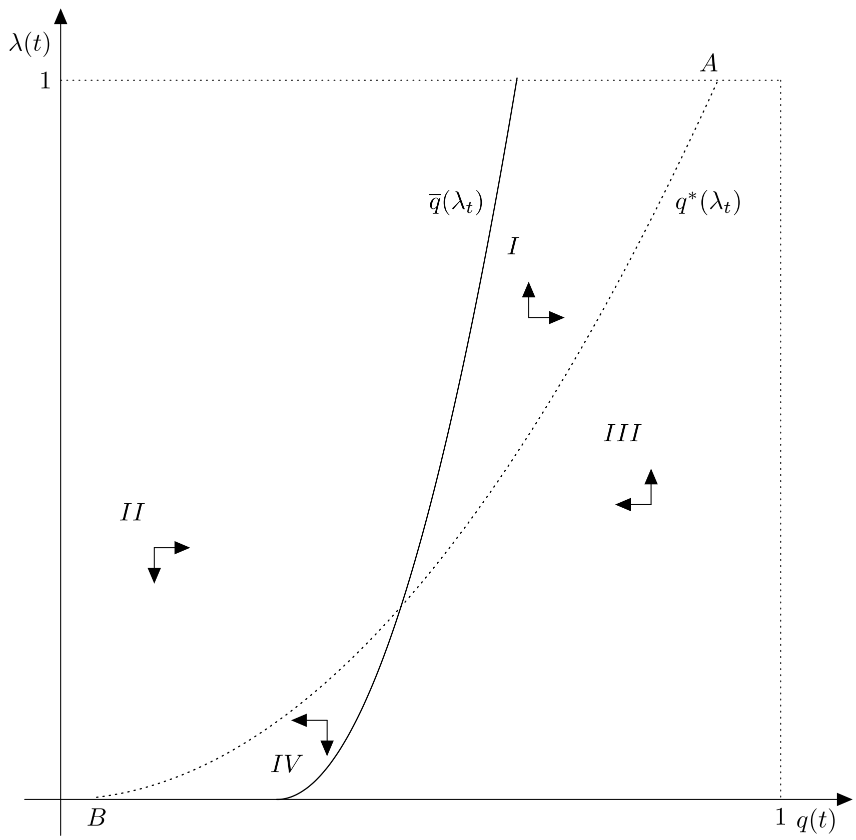
Stationary states. As described in the figure, the joint dynamics of culture and institutions in this society display two stable steady states and one saddle point steady state.[32] The first stable steady state could be characterized as a *religious regime* represented by point A in Figure 2, where the ruler is legitimated by religion, clerics have significant political power (λ_t is high), taxation is high (the tax rate τ is maximal and the tax base

[30] Random economic shocks or uncertainty regarding the parameters would help provide a closer map with historical narratives. For instance, the re-emergence of European commerce around 1000CE could be construed as one such shock, as could the Mongol invasions of the Middle East or the Black Death. We stick to a deterministic model, however, since allowing for such stochastic structure should not change the qualitative insights of the model, while the analytical complexity would increase by orders of magnitude.

[31] It can be shown that $q^*(0) = 0$, and that $\overline{q}(0) > 0$ with $\overline{q}'(0) > 0$. Under parametric conditions ensuring that $\overline{q}(1) < q^*(1)$, continuity of $\overline{q}(\lambda)$ and $q^*(\lambda)$ implies that $\overline{q}(\lambda)$ necessarily cuts from below $q^*(\lambda)$ characterizing an interior steady state point (q^*, λ^*) as shown in Figure 2. Such a point can be shown to be a saddle point steady state of the joint dynamics of culture and institutions, leading formally to the possibility of institutional divergence away from (q^*, λ^*). See Appendix A.6 for details.

[32] $\overline{q}(\lambda)$ and $q^*(\lambda)$ may intersect more than once at some interior point. This would provide other steady states whose dynamic stability will alternate between saddle points and stable points. The qualitative discussion of our analysis about institutional and cultural divergence between secular and a religious steady states are not affected by these possibilities.

Figure 2: Joint Dynamics of Culture and Institutions



E is high), and the share of religious individuals in civil society is high (q is high). The second stable steady state, point B in Figure 2, could be characterized as a *secular regime* where the ruler is not legitimated by religion, clerics have little political power (λ_t is zero), taxation is limited (the tax rate τ is maximal but the tax base E is small), and civil society is secular (q is small). Two mechanisms characterize the dynamics.

Monotonic convergence paths. In regions I and IV of Figure 2, the ruler's option to rely on religious legitimacy to increase tax capacity induces a fundamental *complementarity* between religious legitimacy and the profile of religious values in the population. On the one hand, religious elites provide services to the religious component of civil society, which shape civil society's moral beliefs that support an obligation to obey the ruler, which in turn lowers the subjective tax rate for the religious. Institutions delegating power to clerics (i.e., high λ_t) therefore reinforce the incentives of religious individuals to transmit their values. This in turn increases the relative share of the religious in the population. In addition, a higher fraction of religious individuals in the population augments the political incentives for the ruler to delegate power to clerics to increase legitimacy. This complementarity operates to produce dynamics converging to the *religious regime*, as represented by point A in Figure 2 or to the *secular regime*, as represented by point B. In these regions, the complementarity between culture and institutions *locks-in* society to one of the two stable equilibria.

Non-monotonic convergence paths. In regions II and III of Figure 2, the dynamics are not characterized by complementarity. In these regions of the phase diagram, a "horse race" arises between cultural and institutional change. The "winner" of the horse race determines which stable equilibrium—religious or secular—emerges in the long run. In region II, religious individuals are insufficiently numerous and λ_t decreases over time. At the same time, religious values grow: as the religious trait is not widespread, religious individuals invest more in direct socialization. Depending on the speed of institutional change relative to cultural change, the joint dynamics can either reach region I or region IV.

Region II may give rise to a transitory path to the religious equilibrium when the religious population grows fast despite the political weight of the clerics decreasing over time. This might occur because, being in the minority, religious parents have higher incentives to exert effort transmitting their cultural trait to their child. In this case, religious individuals become sufficiently numerous at some point that the course of institutional change is reversed, and the political power of religious clerics starts to grow after a transitory period. In region III, religious individuals are sufficiently numerous for the political power of the religious clerics to increase over time. But the religious population is too large, so secular individuals invest more in direct socialization. Again, depending on the speed of institutional change relative to cultural change, either region I or region IV could be reached by the joint dynamics. If the religious population decreases faster than religious institutions grow, we can expect the joint dynamics to reach region IV. In this case, the religious population becomes so low after a transitory period that the political weight of the clerics decreases over time and equilibrium B is reached in the long-run.

Comparative dynamics. The basin of attraction of each stationary state—the subset of initial conditions from which the dynamical system converges to this state in the phase diagram in Figure 2—depends on the parameters of the society. Since the size of each basin of attraction can be interpreted as a likelihood of reaching that stationary state, it is important for our analysis to characterize their dependence on the efficiency of the legitimating technology of the clerics, θ, and the degree of restrictiveness of the religious proscriptions imposed by the clerics, ϕ:

Proposition 3 *The size of the basin of attraction of the religious (resp. secular) stationary state is increasing (resp. decreasing) in religious legitimacy θ and decreasing (resp. increasing) in the restrictiveness of religious proscriptions ϕ.*

As an illustration, consider the basin of the religious state. A higher efficiency of the clerics θ—by definition—decreases the subjectively perceived tax rate of the religious. As a consequence, religious parents have a higher willingness to transmit their cultural values inter-generationally. At the same time, clerics become more important in the institutional apparatus, as they increase social welfare by (i) lowering the perceived cost of effort and (ii) increasing the rents extracted by the ruler. Therefore, the complementarity between the spread of religious values and institutional changes delegating power to the clerics is reinforced when θ is higher; and the size of the basin of attraction of the religious state is enlarged.

On the other hand, when the degree of religious proscriptions ϕ increases, the cost for the ruler from using religious legitimacy as a means of extraction also increases. The threshold $\overline{q}(\lambda_t)$ consequently increases. Similarly, greater religious proscriptions dampen the impact of work effort on economic outcomes. As a result, behavioral differences induced by cultural traits are less relevant. To the extent that religious parents are more sensitive to paternalistic motives than secular parents, these shifts lead religious parents to socialize less intensively than secular parents, so the threshold $q^*(\lambda_t)$ associated with the cultural dynamics decreases. As a consequence, the complementarity between the spread of religious values and institutional changes delegating power to the clerics is weakened; and the size of the basin of attraction of the religious state is reduced.

4.2 Historical Parameters and Initial Conditions

In the historical context we study—Western Europe and the Middle East over the period starting from the end of the Western Roman Empire in the West and the emergence of Umayyad Caliphate in the Middle East until the onset of the Reformation in Europe and the capture of the Egyptian Mamluk Empire by the Ottoman Empire—the historical literature has identified several key differences between the regions.

Parameters θ and ϕ. We contend, for reasons given below, that Muslim religious authorities had greater *exogenous* capacity to legitimate (θ) than their Christian counterparts. It is worth noting that there is dispute among historians regarding the degree to which early Muslim rulers, especially the Umayyad Empire (661–750) employed religious legitimacy.[33] For instance, Rubin (2003, p. 87–99) argues that the Umayyads based their legitimacy on their right of succession, not specifically their religious credentials. Bessard (2020, ch. 1, 9) shows that the Umayyads and Abbasids sponsored markets to bolster their legitimacy among merchants. Yet, these insights do not undermine our claim. The key distinction made in the model is between the *exogenous* legitimating technology (θ) and the *endogenous* political power (λ) devolved to religious authorities. The dispute in the literature primarily concerns the latter. For reasons we discuss below, the view that early Muslim empires limited their use of religious legitimacy is consistent with our model.

The primary reason provided in the literature why the *exogenous* legitimating technology of Islam was relatively greater than in Christianity stemmed from the environment in which the religions were born. Christianity was born in the Roman Empire and was in no position to legitimate the emperor. Early Christian doctrine is reflective of the low legitimating capacity of Christianity (Feldman 1997, Rubin 2011). For instance, Jesus famously said "Render unto Caesar the things which are Caesar's, and unto God the things that are God's" (Matthew 22:21). Meanwhile, Islam formed conterminously with expanding empire, and there are numerous important Islamic dictates specifying the righteousness of following leaders who act in accordance with Islam (Hallaq 2005, Rubin 2011, 2017). There are several Qur'anic passages and hadiths (reports of the teachings of Muhammad, which are among the most important sources of authority in Islam) supporting this idea. Among the most explicit is Qur'an passage 4:59: "O you who have believed, obey Allah and obey the Messenger and those in authority among you. And if you disagree over anything, refer it to Allah and the Messenger, if you should believe in Allah and the Last Day. That is the best [way] and best in result." This passage suggests that one should follow those in authority, but only if they rule in accordance with Allah. In short, the growing corpus of Islamic doctrine motivated rulers to employ religious authorities for all sorts of functions, including legitimating the state. This legitimating relationship became codified as

[33] Part of the reason for the dispute is the difficulty in interpreting the sources. The Abbasid Empire (750–1258), who followed the Umayyads, attempted to undermine the legitimacy and religious credentials of the Umayyads in order to justify their own rule. Historians have been forced to read between the lines to determine the degree to which the Umayyads (and early Abbasids) actually employed religious legitimacy. For more on this debate, see Donner (2010, 2020), El-Hibri (2002), and Anthony (2020).

the corpus of Islamic doctrine, including the most trusted hadiths, was formulated in the first Islamic centuries. We denote this as the "exogenous component" of the legitimating technology, or θ. In the context of our model, these historical differences are mapped into a higher θ for the Islamic Middle East.

Secondly, economically-inhibitive religious proscriptions existed—and in fact abounded—in both Christianity and Islam. Although it is not clear whether they were initially more restrictive in Western Europe or the Middle East, they *persisted* for much longer in the latter. For instance, Kuran (2005, 2011) cites how Islamic law regarding partnerships and inheritance combined to discourage long-lived or large business ventures. More generally, Islamic law, as formulated in the first few centuries of Islam, covers numerous aspects of commercial life. Another well-known set of proscriptions are those related to usury, which persisted for over a millennium in both Islam and Christianity (Noonan 1957, Rubin 2011, 2017). For now, we note that proscriptions typically lasted for much longer in Islam. We do not claim that proscriptions were initially more severe in one religion or the other.

Initial conditions q and λ. At the starting point of our analysis of the Middle East, the beginning of the Umayyad Caliphate in 661CE, the "Islamic world" was not thoroughly Muslim. In fact, it was not so for at least a few centuries after the onset of Islam, which first spread along trade routes before spreading into other Muslim-controlled territory (Ensminger 1997, Michalopoulos, Naghavi and Prarolo 2016, 2018). Though Islamic political authority spread quickly, reaching the Iberian Peninsula in the west and the Indian subcontinent in the east within its first century under the Umayyad Caliphate (661–750), "Muslims still formed a small part of the populace... [Umayyad] authorities, who realized that this would deprive them of much-needed tax revenue, did not encourage conversion" (Bessard 2020, p. 18).[34] In the context of our model, this suggests a "low q" initial condition in the Middle East.

Moreover, as we already noted, Islam was born conterminously with empire, to the point that in its first few decades (through the end of the first Caliphate in 661CE), political and religious authority was concentrated in the ruler. The first four Muslim caliphs (632–661CE), who were all companions of Muhammad, claimed to have religious authority vested in themselves. As noted above, there is dispute in the historical literature regarding the extent to which their successors, the Umayyad Caliphate (661–750CE), attempted to make similar claims. Some argue that the Umayyads attempted to do so, although less

[34]For more on the role that tax revenue, particularly the *jizya* tax on non-Muslim subjects, played in conversion goals, see Saleh and Tirole (2021).

successfully given their distance from the Prophet (Crone and Hinds 1986, Donner 2010, 2020). Others argue that other sources of legitimacy were also employed, such as claims to hereditary rule and supporting market activity (Bessard 2020, Rubin 2003). While it is certainly true that several Umayyad leaders were not personally pious, they did play a significant role in defining Islamic rituals—including the daily prayer, Friday prayer, and the hajj—and their coins featured statements of faith and were written in the Arabic script, which at the time was closely associated with the Qur'an (Donner 2010, p. 193–205). Regardless of how subsequent Umayyad (and Abbasid) rulers ultimately employed religious legitimacy, at the onset of the period under study (i.e., 661CE), we interpret this history (as argued by the work of historians of the period) as mapping directly into a high initial λ.

In summary, despite the population largely being non-Muslim, initially at least, the legitimating relationship between rulers and religious authorities was clearly codified in the Islamic Middle East during the early Middle Ages. These historical characteristics can be mapped, in the context of our model, into "low q, high λ" initial conditions.

The historical characteristics of Western Europe, following the fall of the Roman Empire, were somewhat opposite to those we identified for the Middle East. First of all, the Roman population had largely become Christianized in the fourth and fifth centuries, so that Christianity was predominant in the Germanic "follower kingdoms." On the other hand, again as a consequence of the environment in which Christianity was born, the political power of the church was relatively small, to the point that the Germanic "follower kingdoms" were not initially ruled by Christians. We map therefore these historical characteristics of Western Europe into "high q, low λ" initial conditions in the model.

4.3 Matching Model Dynamics and Historical Trajectories

Qualitatively, the parameters and the initial conditions we identified in the historical narratives in the previous section suggest a mapping into region II of Figure 2 for the Islamic Middle East and into region III for the Christian West. We consider the two regions in turn, providing a narrative match between the dynamics implied by the model starting from these regions and the documented historical trajectories.

Christian West. Our mapping of the Christian West into region III of Figure 2 following the fall of the Western Roman Empire implies that the West could have converged to either the *secular* or the *religious* stationary state in the long-run. The implied dynamics from this region are sensitive to slight variations in initial conditions and they depend on the relative speeds of cultural and institutional change. Since the exogenous component of the legitimating technology, θ, was relatively low in the Christian West, Proposition 3 indicates that the basin of attraction should be larger for the "secular" stationary state than it was for the Muslim Middle East. Importantly, however, the paths to this basin of attraction, should these paths reach the basin, are *not monotonic*: they allow for historical trajectories characterized by early institutional changes whereby rulers delegated power to religious clerics to gain religious legitimacy in the face of a largely religious civil society, before turning back to secular institutional structures.

These transitory, non-monotonic dynamics of institutions characterized Western Europe until the 11th century (although not in Northern Europe, which was Christianized between the 8th and 12th centuries). We begin the analysis after the fall of the Western Roman Empire in 476CE. As noted above, the Christian West was in a "high q, low λ" state at this starting off point. The model's dynamics (see region III of Figure 2) suggest that the institutionalized use of religious legitimacy (λ) should increase initially, while the population should become less religious. At some point, depending on which of these effects occurs more rapidly, a basin of attraction will be reached whereby either a "secular" or "religious" equilibrium emerges.

Following the fall of the Roman Empire, the majority-Christian civil society provided a strong incentive for Germanic rulers to either convert to Christianity or promote Christianity. For instance, the Frankish king Clovis (r. 481–509) converted and employed Christianity to legitimate his Frankish expansion into new territory (Tierney 1970, Rubin 2017, pp. 62–63). Likewise, the Visigoths converted to Christianity under Recared (r. 586–601), with the Church serving as an important source of legitimacy until they were overrun by Muslim invaders in 711. Germanic rulers ultimately became among the leading defenders of Christianity, with Charlemagne's crowning by the pope in 800CE the most visible manifestation.

Around 1000 CE, the re-birth of commerce gave rise to independent cities and increased tensions between religious and secular elites (Angelucci, Meraglia and Voigtländer 2022, Rubin 2011). Although we do not model the re-emergence of trade endogenously—indeed, it can be viewed as an exogenous shock relative to the political economy environment we model—it had clear implications for the institutional and cultural dynamics at the heart of the model. The rebirth of commerce entailed that religious proscriptions (ϕ in our model), such as the ban on usury, were more economically harmful. In the absence of widespread trade prior to the Commercial Revolution, such proscriptions had little dampening effect on the economy. Yet, they became increasingly harmful as trade flourished (Rubin 2011). Using the terminology of our model, the increase in ϕ combined with the relatively low θ increased the basin of attraction of the "secular equilibrium," encouraging rulers to break with the Church as a primary means of legitimation.

The most important event in this break was the Investiture Controversy (1075–1122), a conflict between various secular rulers and the papacy over the role of the former in religious affairs. The Investiture Controversy took place in part due to the political economy dynamics noted above. In response to growing secular power over religious affairs, Pope Gregory VII (r. 1073–85) issued a series of reforms regarding the role of secular rulers in Church affairs, including investiture. Although there was back and forth between rulers and the Church, by this point the value of religious legitimation was on the decline, and a movement towards the basin of attraction of the "secular equilibrium" had commenced. The Investiture Controversy culminated with the Concordat of Worms in 1122. In the following two centuries, the Church sought to impose its own set of laws (canon law) across Europe, but to no avail. Rulers, lords, merchants and other elites increasingly turned to other forms of law that covered manorial relations, merchant activity, urban codes, and royal jurisprudence (Berman 1983). With respect to legitimating arrangements, European rulers increasingly sought alternative justifications for their rule (i.e., further lowering λ) (Tierney 1988, pp. 33–95). They found these alternative justifications in the universities, where leading scholars provided justification for secular rule based on Aristotelian thought, while others helped codify various branches of secular law such as merchant law, feudal law, and manorial law (Berman 1983, Cantoni and Yuchtman 2014, Hollenbach and Pierskalla 2020). By the 14th century, the papacy was under the thumb of the French king. The entire papal court was moved to Avignon from 1309–76. This transition can be seen in the type of advice given to monarchs on the "art of ruling." Blaydes, Grimmer and McQueen (2018) find that it was precisely in this period that European political advice texts began to de-emphasize religious appeals.

As a whole, these events helped place much of Western Europe on a path towards the more "secular" equilibrium described in our model. Institutional change in the direction of more political power to the Church did not arise fast enough, especially after the Investiture Controversy gave local rulers greater suzerainty over their lands. In the context of the model, Western Europe thus ultimately ended up in region IV of Figure 2—the basin of attraction that results in a "secular equilibrium". In this region, the declining political power of religious clerics reinforced cultural changes that placed less emphasis on religious values. These reinforcing mechanisms ultimately resulted in lock-in, whereby there was little role for religious authorities in legitimating political rule, and more political power rested in civil society.

The Reformation played a key role in further secularizing civil society. In the context of the model, such secularization is necessary for a society to reach region IV of Figure 2. In England, Greif and Rubin (2023*a*) argue that following the Reformation, the political power of religious authorities dropped significantly and the law (as formed in Parliament) became a key source of royal legitimacy. In Germany, Cantoni, Dittmar and Yuchtman (2018) find that, following the Reformation, there was a massive reallocation of resources and education from religious to secular purposes. In other words, where the Reformation undermined the political power of the Church (i.e., lowered λ), less cultural capital was invested in religious pursuits. This is precisely the type of lock-in the model predicts will arise in a society in region IV.

Islamic Middle East. Our qualitatively mapping of the Middle East initially (i.e., after 661CE) into region II of Figure 2 suggests historical trajectories somewhat specular with respect to those of the West: convergence to the *religious* stationary state in the long-run but through historical trajectories characterized by institutional changes whereby rulers limited the power of religious clerics early on, before turning back to a strategy of delegation in exchange for legitimacy which led society to a religious stationary state. This insight helps resolve—or at the very least, shines a new light on—the debate in the historical literature regarding the use of religious legitimacy by early Islamic empires. While there is much reason to believe that early Islamic empires sought sources of legitimacy outside of Islam (though not to its exclusion), this is *precisely* what our model predicts should happen, initially at least, in a "high λ, low q" society.

Following the rapid political spread of Islam in its first few decades under the First Four Caliphs (632–61), institutional change transpired favoring economic—not religious—elites. The merchant class saw a rise in its economic and political power in the first few centuries of Islam (Bessard 2020, ch. 9). A common currency and political institutions facilitated a massive expansion of trade. The Umayyad and Abbasid states sponsored markets and provided privileges for leading merchants, directly involving themselves in urban retailing to "establish their power and legitimacy from the first decades of the eighth century" (Bessard 2020, p. 5). This was not just a period of economic growth; it was also the

"Golden Age" of rationalist Islamic thought. Islamic science, technology, mathematics, architecture, and medicine were the envy of Western Eurasia. Hence, there were forces pushing against the political power of religious elites (i.e., lower λ, as is predicted in region II).

Yet, these forces did not move fast enough to reach the basin of attraction in which a "secular" equilibrium emerged in the long run. Religious authorities provided administrative services to a largely non-Muslim population throughout the Middle East and North Africa. After 661, in the Sunni successor empires (the Umayyad and Abbasid Caliphates), religious authorities served a central role in administering the state although the population was not yet Islamized. Most important was their role in providing legal services and overseeing various aspects of state administration. With respect to the early Abbasid Empire (8th century), Hallaq (2005, p. 182–83) writes:

> [T]he government was in dire need of legitimization, which it found in the circles of the legal profession. The legists served the rulers as an effective tool for reaching the masses, from whose rank they emerged and represented ... Jurists and judges emerged as the civil leaders who, though themselves products of the masses, found themselves involved in the day-to-day running of their affairs ... [T]he judges were not only justices of the court, but the guardians and protectors of the disadvantaged, the supervisors of charitable trusts, the tax-collectors and the foremen of public works. They resolved disputes, both in the court and outside it, and established themselves as the intercessors between the populace and the rulers.

As a result, the Umayyads and their successors, the Abbasid Caliphate (750–1258CE), relied on legitimacy supplied by religious authorities. Especially after the religious establishment consolidated in the ninth century (Coşgel, Miceli and Ahmed 2009, Hallaq 2005, Rubin 2017), religious authorities were the primary agents capable of determining whether rulers acted in accordance with Islam (i.e., whether secular authorities and Allah "disagreed over anything", to quote the Qur'anic passage cited above). This relationship was formally institutionalized with the rise of the madrasa system in the 11th century and the diversion of resources away from secular intellectual pursuits (e.g., science, mathematics) and into religious learning (Chaney 2016, Kuru 2019).

Importantly, as posited in our model, the Middle East became Islamicized *prior to* an unraveling of political power for religious clerics. In the context of Figure 2, this placed much of the Muslim Middle East in the basin of attraction of a "religious equilibrium" (region I). In the model, as in Bisin and Verdier (2001), the dominant cultural group (initially, non-Muslims) had less incentive to pass down their cultural traits, *especially* when the institutional structure was not aligned with their cultural (religious) beliefs. Institutional pressures favoring the minority culture can incentivize conversion to that culture. In the Islamic context, such institutionalized incentives were provided via taxes on non-Muslims (*jizya*).[35] In Egypt, for example, Saleh (2018) finds evidence of massive conversions of lower socio-economic status Copts into Islam: by 1200, Muslims were 80% of the Egyptian population, and by 1500 they were over 90% of the population. Saleh (2018) argues that negative selection among Copts was due to the poll tax that non-Muslims had to pay; those that could not afford it simply converted to Islam.

This history is consistent with the dynamics predicted in our model. As a society approaches the basin of attraction of the "religious equilibrium," religious culture reinforces clerical political power, and a religious stationary state becomes locked-in in the long run. In the Middle East and North Africa, this equilibrium was characterized by a massive expansion in madrasas (Chaney 2016, Kuru 2019), less frequent "rationalist" interpretation of Islam in favor of traditionalist interpretation (i.e., the "closing of the gate of *ijtihād*" (Coulson 1969, Hallaq 1984, 2001, Schacht 1964, Weiss 1978)), and little political bargaining power for the economic elite (Pamuk 2004*a*,*b*).

Two examples from two different periods and regions highlight the reinforcement of Muslim institutions and culture in a "high q, high λ" world. First, Chaney (2013) finds that medieval Egyptian religious authorities were more secure in their rule (e.g., higher λ) when the Nile flooded or there was a drought. This is precisely when a ruler would most need religious legitimacy, both because the tax base would be lower and because there was a greater threat of revolt. Moreover, as noted above, this was a period of increasing Islamization of the Egyptian population (i.e., q was increasing). This suggests the presence of a "high q, high λ" equilibrium, with cultural and institutional forces reinforcing each other.

A second example comes from the Arab provinces of the Ottoman Empire, where the population had largely converted to Islam centuries prior to Ottoman expansion (i.e., q was high). In the late 15th century, the Ottomans brought the religious establishment into

[35]While we do not explicitly model discriminatory taxes, religious legitimacy works as such a tax in the model, given the presence of religious proscriptions. Formally including an explicit discriminatory tax would strengthen the results.

the state, establishing the office of the Grand Mufti (chief religious jurist). This gave the Ottomans significant power to formulate controversial decisions in a manner consistent with Islam (Imber 1997). Meanwhile, the reinforcement of institutions and culture strengthened after the Ottomans conquered the Egyptian Mamluk Empire (in 1517) and took control over Mecca and Medina, the two holy cities of Islam. This further enhanced the capacity of clerics to confer legitimacy by associating the sultan with Islamic piety (e.g., mentioning his name in each Friday sermon or supporting obedience to him in judicial rulings) (Hallaq 2005, ch. 8). Thus, the high level of religious legitimacy (θ) provided by Muslim clerics resulted in a "high q, high λ" equilibrium for much of Ottoman history.

4.4 The Long Divergence through Lens of the Model

Our model squares two of the leading theories of the "Long Divergence," and in doing so directly addresses one stylized fact highlighted in the literature: the persistence of religious legitimacy in the Middle East and the secularization of politics in Western Europe. The model suggests that the diverging long-run paths of the economies of these two regions—"high q, high λ" in the Middle East and "low q, low λ" in Western Europe—were in part a result of the relatively high efficacy of religious legitimacy (θ) in the Islamic world. This meant that the two regions had different responses to religious proscriptions (ϕ), which were not necessarily stronger in either region. In Western Europe, once commerce revived in the 11th and 12th centuries, religious proscriptions were sufficiently economically damaging to push society towards the basin of attraction that ultimately resulted in a low q, low λ equilibrium. On the other hand, in the Islamic world such religious proscriptions may have been even more economically damaging initially, given that the Islamic world was ahead of Europe. However, the relatively high θ in Middle Eastern societies helps account for the presence (and persistence) of strict religious proscriptions in a "high q, high λ" equilibrium. Although proscriptions diminish the attractiveness of religious legitimacy to rulers and of passing down religious traits to one's child, proscriptions are mitigated for the ruler if religious legitimacy is effective enough (i.e., θ is high) and enough of the population is religious (i.e., q is high). Hence, supporting economically-inhibitive religious doctrine is more than worth it for a ruler in a high-q society when θ is also large.

These insights therefore unify Kuran's theory emphasizing religious proscriptions with theories emphasizing religious legitimacy (Kuru 2019, Platteau 2017, Rubin 2017). Kuran's theory centers not just on the fact that religious proscriptions existed in Islamic law, but that they *persisted* for so long after they were useful. Our theory sheds light on the how religious culture reinforced clerical political power, and vice versa, which resulted in the persistence of religious proscriptions. Meanwhile, an emphasis on religious proscriptions reveals why legitimating arrangements changed over time in Europe.

These insights also shed light on a second stylized fact central to the literature: the long-run economic vibrancy of Western Europe relative to the Middle East. Even though there are welfare-enhancing properties of religious legitimacy (as highlighted in the model), these welfare gains can be overwhelmed by religious proscriptions. As Kuran (2011) points out, such proscriptions can have unforeseeable, path dependent consequences for economic growth. For instance, Islamic partnership law and inheritance law jointly discouraged larger enterprises, which ultimately stifled the creation of anything remotely resembling the corporate form (Kuran 2005, 2011). Meanwhile, the persistent dominance of Islamic law over commercial transactions entailed the slow (or non-) adoption of new organizational forms and financial instruments from abroad, which itself had numerous unforeseeable economic consequences (Kuran and Rubin 2018, Rubin 2010, 2017).

So far, our model does not account for the third major theory of the Long Divergence: Middle Eastern rulers had more unconstrained power relative to other elites (i.e., European governance was more limited). As such, it cannot account for an important stylized fact mentioned in the introduction: the growth in limited governance in Western Europe but not the Middle East. Blaydes and Chaney (2013) ascribe the relatively greater power of Middle Eastern rulers to their access to slave soldiers, which gave rulers access to coercive power without ceding political power. Meanwhile, weaker European rulers had greater incentive to negotiate with their economic (i.e., feudal) elites for revenue and military power, since they had little capacity to rule otherwise (Duby 1982). Throughout Europe, rulers also ceded power to urban burghers, who had relative freedom from imperial rule (Angelucci, Meraglia and Voigtländer 2022, Mann 1986, Putnam, Leonardi and Nanetti 1994, Schulz 2022). More generally, this meant that Muslim rulers had fewer constraints on their power, which a large literature suggests is harmful for economic growth (Acemoglu and Robinson 2012, Acemoglu, Johnson and Robinson 2005*b*, North and Weingast 1989, North, Wallis and Weingast 2009, van Zanden, Buringh and Bosker 2012). Our model currently does not permit the ruler to share power with other (secular) elites that may constrain her, so it cannot speak to the conditions under which this occurs. In the next section, we extend the model to consider how the devolution of political power interacts with the various parameters of importance in our model (namely, θ and ϕ).

5 Religious Legitimacy and Limited Governance

In this section we extend and enrich the model introduced in Section 2 to consider the emergence of limited governance.[36] Pre-modern states tended to have little fiscal capacity or capacity to provide law and order to regions far away from the capital. Administrative capacity tended to be quite weak in most parts of the world, meaning that rulers could not easily implement their desired policies (Besley and Persson 2014, González de Lara, Greif and Jha 2008, Greif 2008, Karaman and Pamuk 2013, Ma and Rubin 2019). As such, there was a limit to the potential tax revenue available to rulers that was well below the optima on a Laffer curve (Besley and Persson 2009, 2010, Dincecco 2009, Johnson and Koyama 2017). This issue is (implicitly) central to the framework proposed by Blaydes and Chaney (2013). Without the capacity to collect revenue on their own, pre-modern rulers had to delegate tax collection to powerful agents. Such powerful agents could deter tax evasion via force and more easily assess taxable surpluses. More importantly, these powerful agents could *limit* what the ruler could do because they held the power of the purse.

The degree to which rulers had to delegate tax collection (and, more generally, the administrative functions of the state) depended on their own power vis-à-vis other elites. According to Blaydes and Chaney (2013), Muslim rulers had to delegate less because they had access to slave soldiers. This meant they did not need local elites for military service or, oftentimes, tax collection. Meanwhile, feudal arrangements in medieval Europe were such that local taxes were collected by powerful local elites, and in return rulers received military service and, occasionally, tax revenue.

We study the interactions between rulers and local elites in a political economy model where political power is divided between three groups: the ruler, religious clerics, and a secular elite (e.g., feudal lords, parliament, or the military). This allows us to incorporate into the model a fundamental element of the socio-economic environment under study, as discussed in the Introduction: a tradeoff between religious legitimacy and limited governance with respect to the state's fiscal capacity. This, in turn, allows us to study the

[36] This notion of limited governance is akin to "inclusive political institutions" (Acemoglu and Robinson 2012) or a broad-based ruling coalition (North, Wallis and Weingast 2009). Limited governance is distinct from fiscal decentralization (Dincecco 2009, Gennaioli and Voth 2015, Gennaioli and Rainer 2007). Fiscal decentralization is typically associated with lower tax revenue. Dincecco (2015) calls states that had both fiscal centralization and limited governance "effective states." For more on the connection between fiscal capacity and executive constraint, see Acemoglu and Robinson (2012, 2019), Acemoglu, Johnson and Robinson (2005*b*), Besley and Persson (2009, 2010), Bisin and Verdier (2017), Dincecco (2009), Mann (1986), North and Weingast (1989), Tilly (1990), and Johnson and Koyama (2017).

conditions under which the ruler shares political power with the secular elite, who have the capacity to collect taxes.

We treat secular elites as representatives of the citizenry. In terms of the distribution of power between groups, we assign the "ruling coalition" the combined weight of the ruler and the secular elites, $\frac{1}{2} + \frac{1-\lambda}{2} = 1 - \frac{\lambda}{2}$, in social welfare. This is similar to the baseline model, with the citizenry being replaced by the secular elites. In other words, if the ruler and the secular elites are the "ruling coalition" (as in North, Wallis and Weingast 2009), then $1 - \frac{\lambda}{2}$ is the total weight of the coalition. Clerics have weight $\frac{\lambda}{2}$ and citizens have no political power (i.e., zero weight).[37]

Secular elites enforce tax compliance and share with the ruler the tax surplus. The share of this surplus accruing to the ruler vis-a-vis the secular elites is $\beta \in [0, 1]$.[38] As a simple illustration, a regime where $\lambda = 1$ can be interpreted as a theocracy, while $\lambda = 0$ is a dictatorship when $\beta = 1$ and a republic when $\beta = 0$, as the ruler does not benefit from tax revenue in the latter case. It is therefore the tradeoff between β and λ that determines the state's fiscal capacity.

We denote $\alpha_l \in [0, \overline{\alpha}_l]$ the enforcement effort of the secular elites, with $\overline{\alpha}_l > 0$. Let $\mu\frac{\alpha_l^2}{2}$, with $\mu > 0$, be a quadratic cost associated with this effort. The utility of the secular elites can be expressed as:

$$U_l(m, \alpha_l) = (1 - \beta)[\tau E - C(m)] - \mu\frac{\alpha_l^2}{2}. \tag{12}$$

Consider now the utility of the ruler. We assume the ruler faces a cost $\rho\alpha_l$ when letting the secular elite enforce tax compliance α_l. For instance, medieval European rulers provided feudal lords with lands to administer. Tax enforcement was accompanied with the hiring and building of a force capable of violence by these lords. These elements suggest that the more the ruler cedes to lords the power of tax enforcement, the larger is the military power of the lords, which may eventually be turned against the ruler herself. The cost $\rho\alpha_l$ is a simple way to capture such threats. We maintain the assumption that the maintenance

[37] This is a simplification to reduce the dimensionality of the dynamics of institutions while expanding the qualitative features of the narrative of the interactions between ruler, clerics, and citizens analyzed in Section 3.

[38] This setup captures the idea that there is an implicit bargaining process within the "secular ruling coalition" (ruler and secular elite) that is related to the institutional governance structure and which determines how the two parties share the rents extracted in society. This institutional structure implies that the equilibrium level of religious infrastructure only depends on the weight of the clerics relative to the secular ruling coalition, independent of the structure of power within the coalition.

cost of religious infrastructure paid by the clerics is $F(m)$. The utility of the ruler is then

$$U_r(m) = \beta(\tau E - C(m)) - \rho\alpha_l,$$

and the utility of the clerics is:

$$U_c(m, \alpha_c) = m\alpha_c - \Psi(\alpha_c) - F(m).$$

In order to focus on the institutional implications of endogenous tax enforcement, we also simplify the production structure of the economy. More precisely, we assume that all citizens are now endowed with one unit of resource out of which they produce $\frac{1}{1+\phi\alpha_{c,t}}$ of the consumption good. They then face the dichotomous choice of complying or not with tax collection. When an individual of type $i \in \{Re, S\}$ complies with taxation, he pays the effective tax rate τ on his output, while enjoying from a welfare point of view, a "perceived" tax rate $\tau_{i,t}$, with as before $\tau_{Re,t} = \tau(1 - \theta\alpha_{c,t})$ and $\tau_{S,t} = \tau_t$. When the individual decides to evade tax collection, he faces an expected consumption penalty which depends on two factors: i) the capacity of tax enforcement on the part of the elites, and ii) the capacity of that individual to escape taxation. More precisely, denote by $\epsilon(\alpha_{l,t})$ a measure of the capacity of tax enforcement by the elites, increasing in the elite's tax collection effort $\alpha_{l,t}$.[39] Assume as well that each individual has an idiosyncratic (inverse) capacity to evade taxes c drawn from a uniform distribution on a segment $[0, \overline{c}]$, with $\overline{c} > 0$. An individual with characteristic c who does not comply with tax collection incurs an expected consumption penalty $c\epsilon(\alpha_{l,t})$.[40] In this modified version of the model, the expected utility of an individual belonging to type $i \in \{Re, S\}$ with an (inverse) evasion capacity c is:[41]

$$U_i = \begin{cases} \frac{1-\tau_{i,t}}{1+\phi\alpha_{c,t}} & \text{if the individual complies} \\ \frac{1-c\epsilon(\alpha_{l,t})}{1+\phi\alpha_{c,t}} & \text{otherwise} \end{cases}. \tag{13}$$

[39] For analytical convenience, we assume $\epsilon(\alpha_{l,t}) = \frac{\epsilon_0}{1-\alpha_{l,t}}$, so that $\epsilon_0 \in (0, 1)$ is the enforcement level when the secular elites are not providing an effort ($\alpha_{l,t} = 0$). For simplicity, we also assume that the maximum enforcement level that the secular elite can undertake $\overline{\alpha}_{l,t}$ is less than $1 - \epsilon_0$, so that $\epsilon(\alpha_{l,t})$ always lies in the interval $[\epsilon_0, 1]$.

[40] This consumption penalty is "burned out" and not recovered by tax collectors.

[41] With this production specification, we highlight the distortions associated with the extensive margin of taxation, rather than the intensive margins of labor effort as in the base model. Introducing the intensive margin of production effort does not change the qualitative conclusions of this section, at the cost of increased analytical complexity.

5.1 Societal Equilibrium and Dynamics

The *societal equilibrium* in generation t is a Nash equilibrium of the game between the ruler, clerics, secular elite, and civil society. In this equilibrium, religious infrastructure m is chosen to maximize social welfare,

$$\left(1-\frac{\lambda_t}{2}\right)[U_r(m_t)+U_l(m_t,\alpha_{l,t})]+\frac{\lambda_t}{2}U_c(m_t,\alpha_{c,t}). \tag{14}$$

The clerics and secular elite choose, respectively, $\alpha_{c,t}$ and $\alpha_{l,t}$. We denote $\{m_t(\lambda_t)$, $\alpha_{c,t}(\lambda_t)$, $\alpha_{l,t}(\lambda_t,\beta_t)\}$ the equilibrium. In the rest of this section, we omit the time indices when not necessary. Solving the equilibrium in any period t, we obtain the following results:

Lemma 4 ***Religious infrastructure:*** *The equilibrium investment in religious infrastructure $m(\lambda)$ and the optimal effort of the clerics $\alpha_c(\lambda)$ are increasing in λ, and independent of β, θ, and ϕ.*

Lemma 5 ***Tax enforcement:*** *The equilibrium enforcement effort of the secular elite $\alpha_l(\lambda,\beta)$ is decreasing in β, λ, q, θ, and ϕ.*

Lemma 4 is similar to Lemma 1 in the previous model and has the same intuition. Lemma 5 highlights several results. First, when the ruler receives a larger share of the tax revenue β, the secular elite invests less in enforcing tax collection. Second, since individuals subjectively perceive a lower tax rate when clerics provide more effort, they also comply more with taxation, reducing the need for the secular elite to supply their own enforcement effort. Additionally, more effort from the clerics implies more religious proscriptions, which depress citizens' labor productivity, and decreases the proceeds of the tax collection. This also decreases the effort provided by the secular elite in enforcing the tax collection. Hence for both reasons, the clerics' legitimizing effort α_c, and the secular elite tax enforcement effort α_l are *strategic substitutes* with respect to building up the tax base. Consequently, given that clerics provide more effort when they are more powerful (i.e., when λ is higher), the secular elite is conversely less willing to enforce the tax collection in such a case: (i.e., $\alpha_l(\lambda,\beta)$ decreases with λ).

The same intuition explains both the effect of a higher frequency q of religious individuals and of more efficient clerics θ on the effort of the secular elite α_l. Finally, when the degree of religious proscriptions ϕ is greater, the proceeds of the tax collection are reduced, so secular elites provide less tax enforcement effort.

We now turn to the analysis of institutional change, i.e., the change in the structure of political weights. The ruler can delegate power to clerics (λ) and also constrain herself to share more revenues with secular elites by decreasing her own fraction β of fiscal revenues.

Institutional change internalizes two types of externalities that are not taken into account by equilibrium individual decisions. First, as in the previous model, the religious provision m grants *legitimacy* to the ruler, reducing the subjectively perceived tax rate of religious individuals while at the same time depressing labor productivity because of religious proscriptions. Second, institutions now also respond to the externality implied by the enforcement effort α_l of the secular elite on the fiscal revenue received by the ruler. By committing to share the proceeds of tax collection, the ruler can indirectly induce greater fiscal capacity for her own benefit. This is the trade-off at the heart of this extension of the model.

Hence, given the current institutional structure (λ_t, β_t), future institutions $(\lambda_{t+1}, \beta_{t+1})$ are designed as the solution to:

$$\max_{\lambda_{t+1},\beta_{t+1}} \left(1 - \frac{\lambda_t}{2}\right) [U_r(m_t(\lambda_{t+1}), \alpha_{l,t}(\lambda_{t+1}, \beta_{t+1})) + U_l(m_t(\lambda_{t+1}), \alpha_l(\lambda_{t+1}, \beta_{t+1}))] + \frac{\lambda_t}{2} U_c(m_t(\lambda_{t+1}), \alpha_{c,t}(\lambda_{t+1})), \tag{15}$$

with $\{m_t(\lambda_{t+1}), \alpha_{c,t}(\lambda_{t+1}), \alpha_{l,t}(\lambda_{t+1}, \beta_{t+1})\}$ denoting the Nash equilibrium of period t, as evaluated under an institutional set-up $(\lambda_{t+1}, \beta_{t+1})$. Solving this optimization problem, we deduce the following results which characterize the institutional dynamics:

Proposition 4 *When $C(m)$ and $F(m)$ are sufficiently convex, the optimization problem (15) admits a unique solution $(\lambda_{t+1}, \beta_{t+1}) \in [0, 1]^2$ and:*

there exists a threshold $\overline{q}_d(\lambda_t) \in [0, 1]$ such that if $q_t > \overline{q}_d(\lambda_t)$, then $\lambda_{t+1} > \lambda_t$. Otherwise, $\lambda_{t+1} \leq \lambda_t$. Moreover $\overline{q}_d(\lambda_t)$ is decreasing in λ_t;

there exists a threshold $\tilde{q}_d(\lambda_t, \beta_t) \in [0, 1]$ with $\tilde{q}_d(\lambda_t, 1) = 1$ such that if $q_t > \tilde{q}_d(\lambda_t, \beta_t)$, then $\beta_{t+1} > \beta_t$. Otherwise, $\beta_{t+1} \leq \beta_t$. Moreover, the threshold $\tilde{q}_d(\lambda_t, \beta_t)$ is decreasing in λ_t and increasing in β_t.

The uniqueness result follows from the convexity and the separability of the two dimensions of the optimization problem (15). This result highlights the trade-off between religious legitimacy and limited governance with respect to the state's fiscal capacity as well as the role that the cultural profile (q_t) plays in tipping the balance of this trade-off. As before, whether the ruler delegates more power to clerics over time depends on the fraction of religious individuals q_t. If the religious are sufficiently numerous, then more weight to the clerics $\lambda_{t+1} > \lambda_t$ increases their effort $\alpha_{c,t}(\lambda_{t+1})$. This will increase the utility of the ruler, who benefits from a larger tax base (Lemma 4). Second, when the religious are sufficiently numerous, the political weight of the secular elite relative to the ruler tends to decrease, $\beta_{t+1} > \beta_t$. As the ruler becomes more reliant on religious legitimacy to raise revenues, he also faces weaker incentives to delegate power to the secular elite and to build fiscal capacity.

Cultural evolution of the religious and secular traits is driven by some process of intergenerational transmission emanating from paternalistic parents and oblique social role models. The formal features of the cultural dynamics need, however, to be amended to the new specification of production and taxation as outlined above.[42] Again one may compute the paternalistic motives ΔV_{Re} and ΔV_S to transmit the religious and the secular trait in this context. As shown in the appendix, due to the quadratic specification of the expected payoff functions, these paternalistic motives simply write as functions of the state variables λ_t, β_t, and q_t such that $\Delta V_S = \Delta V_{Re} = \Delta V(\lambda_t, \beta_t, q_t)$.[43] The dynamics of the frequency of the religious trait is again characterized by the following "cultural replicator" dynamics:

$$q_{t+1} - q_t = q_t(1 - q_t)D(q_t, \lambda_t, \beta_t). \tag{16}$$

where again

$$\begin{aligned} D(\lambda_t, \beta_t, q_t) &= d^*_{Re,t} - d^*_{S,t} \\ &= D_{Re}\left[(1 - q_t)\Delta V(\lambda_t, \beta_t, q_t), m_t(\lambda_t)\right] - D_S\left[q_t \Delta V(\lambda_t, \beta_t, q_t)\right] \end{aligned}$$

is the relative "cultural fitness" of the religious trait in the population, and in general depends on the three state variables λ_t, β_t, and q_t. When the cultural substitutability between vertical and oblique transmission is strong enough, the relative "cultural fitness" of the religious trait $D(\lambda_t, \beta_t, q_t)$ is decreasing in the frequency q_t of religious individuals in the population and we deduce the following result:

[42] When deciding on their optimal socialization effort, parents take into account that their children will draw in their adult life an idiosyncratic evasion capacity c, which matters for their decision to comply or not with taxation.

[43] Because the equilibrium tax collection effort $\alpha_l(\lambda, \beta, q)$ of the secular elite enters into the paternalistic motives, we note that $\Delta V(\lambda_t, \beta_t, q_t)$ is an increasing function of q_t (see the appendix).

Proposition 5 *With strong enough cultural substitution between vertical and horizontal cultural transmission, there exists a unique threshold $q_d^*(\lambda_t, \beta_t)$ such that*

$$q_{t+1} < q_t \text{ (resp. } \geq \text{) if } q_t > q_d^*(\lambda_t, \beta_t) \text{ (resp. } \leq \text{).}$$

As before, the threshold $q_d^*(\lambda_t, \beta_t)$ is the unique attractor of the cultural dynamics (16). Hence, when the fraction of religious individuals q_t is above (resp. below) the threshold $q_d^*(\lambda_t, \beta_t)$, it tends to decrease (resp. increase).

5.2 Model Dynamics and Historical Trajectories

The joint dynamics of culture and institutions in this society are now three dimensional: the two institutional parameters, λ_t and β_t, and the cultural component q_t evolve jointly, as characterized in Propositions 4 and 5. A full characterization of this dynamic system is difficult. Still we can derive insight on the forces behind the joint dynamics by investigating how the thresholds $\overline{q}_d(\lambda_t)$, $\tilde{q}_d(\lambda_t, \beta_t)$, and $q_d^*(\lambda_t, \beta_t)$, which characterize respectively the dynamics of λ_t, β_t, and q_t, are themselves affected by the state variables.

As in the benchmark model, there is a fundamental complementarity between the dynamics of culture and institutions. To see this, note first that because $\overline{q}_d(\lambda_t)$ is decreasing in λ_t, from Proposition 4, the political weight of religious clerics λ_t keeps increasing (resp. decreasing) over time as soon as it is above (resp. below) a threshold $\overline{\lambda}(q_t)$ defined by $\overline{q}_d(\lambda) = q_t$. A strong (resp. weak) institutional representation for clerics is reinforced (resp. weakened) over time. This feature creates a force towards an institutional steady state characterized as a *religious institutional regime* with $\lambda = 1$, or on the contrary a *secular institutional regime* with $\lambda = 0$. Also, given that the threshold $\overline{\lambda}(q_t)$ is decreasing in q_t, the reinforcing dynamics of the religious institutional regime are facilitated (resp. weakened) when the religious (resp. secular) trait is already well disseminated in society.

Conversely, from Proposition 5, $q_d^*(\lambda_t, \beta_t)$ is increasing in the institutional weight λ_t of the clerics. As before, a religious institutional regime with a high value of λ_t stimulates more religious infrastructure and reinforces the incentive of religious individuals to pass their values inter-generationally. Religious values are more widely diffused within a religious institutional regime, while secular values widely prevail under a secular institutional regime.

With respect to the dynamics of limited governance β_t, Proposition 4 reveals that β_t is more likely to increase as q_t and λ_t become larger. Indeed, as the threshold $\tilde{q}_d(\lambda_t, \beta_t)$

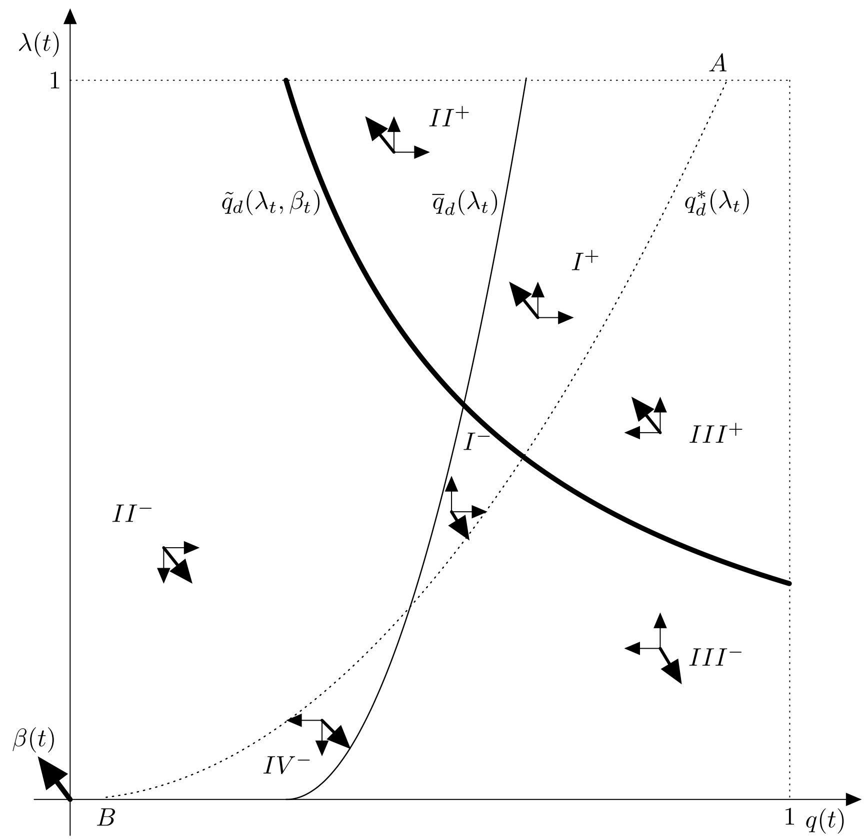
is decreasing in λ_t and increasing in β_t, the condition for $\beta_{t+1} - \beta_t \gtrless 0$ rewrites as $\beta_t \lessgtr \widetilde{\beta}_d(\lambda_t, q_t)$ with $\widetilde{\beta}_d(\lambda_t, q_t)$ increasing both in λ_t and q_t. This feature underscores why the system moves in the direction of a steady state level of limited governance $\widetilde{\beta}_d^*$ that is increasing both in the level of institutional power λ of the clerics, and the extent q of religious values prevailing in the society. The more religious the society and the more diffused the religious values in the population, the larger the religious legitimacy enjoyed by the ruler, and the lower the need to empower the secular elite.

Qualitatively, the previous discussion indicates that the joint dynamics of culture and institutions entails the possibility of two stable steady states. The first is a *religious regime with unlimited governance*, where the ruler has a strong say on fiscal revenues (β is high) and is legitimated by religion, while the clerics have significant political power ($\lambda = 1$). Fiscal capacity is low, as the secular elite have minimal incentives to enforce tax collection. The share of religious individuals in civil society is high (q is high). The second steady state is a *secular regime with limited governance*. The ruler is fiscally weak while the secular elite is strong (β is low). Clerics have little political power ($\lambda = 0$), while fiscal capacity is high given that secular elites have strong incentive to enforce tax collection. At the same time, the share of religious individuals is low (q is low).

In the appendix, we show that the previous discussion can be made analytically more precise in the case where the threshold of the cultural dynamics $q_d^*(\lambda_t, \beta_t)$ does not depend on β_t. The dynamics of λ_t and q_t are then decoupled from the dynamics of β_t and follow the same pattern as in the benchmark model. Depending on the initial conditions (λ_0, q_0), (λ_t, q_t) converge towards *a religious regime* $(1, q_d^*(1))$ or a *secular regime* $(0, q_d^*(0))$. Associated with these dynamics, the society converges towards unlimited governance with $\beta_1^* = \widetilde{\beta}_d(1, q_d^*(1))$, or limited governance $\beta_0^* = \widetilde{\beta}_d(0, q_d^*(0)) < \beta_1^*$. This case is depicted in Figure 3, where the threshold $\tilde{q}_d(\lambda_t, \beta_t)$ for a given value of β_t is represented by a thick black line in the space $\{q_t, \lambda_t\}$. The arrows indicate the joint dynamics of culture and institutions, given our results in Propositions 4 and 5. Moreover the direction of change of β_t is summarily indicated, decreasing towards limited governance β_0^* or increasing towards unlimited governance β_1^*.

Monotonic convergence paths. As in the benchmark model, a ruler's option to rely on religious legitimacy induces a fundamental complementarity between the dynamics of culture and institutions. When a ruler relies more on religious legitimacy to raise revenues, she also faces increasingly weaker incentives to delegate power to the secular elite and to

Figure 3: Joint Dynamics of Culture and Institutions with Limited Governance



consolidate fiscal capacity. As she becomes fiscally stronger relative to the secular elite, she also commits to an institutional set-up delegating more power to the clerics, leading to increased diffusion of religious values in the society. In turn, the predominance of religious individuals augments the political incentives to bias the institutional structure towards both the clerics and the ruler. This dynamic complementarity between institutions and culture operates in region I^+ of Figure 3. It produces a process converging towards a *religious regime with unlimited governance*, as represented by point A.

Alternatively, when a ruler relies less on religious legitimacy to raise revenues, she also faces stronger incentives to delegate power to the secular elite, who consequently consolidate fiscal capacity. As the ruler becomes more reliant on her secular elite to collect taxes, she accordingly faces lower incentives to commit to an institutional set-up where religious clerics are powerful. Both the political weight of the clerics and the value of passing religious values inter-generationally decrease. A lower predominance of religious individuals in society and a lower legitimacy to directly raise taxes further augments the political incentives to consolidate fiscal capacity by empowering the secular elite. This dynamic complementarity between institutions and culture operates in region IV^- of Figure 3. It produces a process converging towards a *secular regime with limited governance*, as represented by point B.

Non-monotonic convergence paths. In all the regions of Figure 3 except I^+ and IV^-, the dynamics are not characterized by complementarity and hence by monotonicity. Similar to Section 4, a "horse race" arises between cultural and institutional change in these regions of the phase diagram. In the case where the threshold of the cultural dynamics $q_d^*(\lambda_t, \beta_t)$ does not depend on β_t, these transitory paths are essentially similar to those described in Section 4.

5.3 Matching Model Dynamics and Historical Trajectories

This extension allows us to unify the three main theories of the "Long Divergence." It takes seriously the idea that rulers can be constrained by other powerful elites in society and searches for the conditions under which this is likely to happen. Importantly, it does so in the context of the previously-established framework in which religious legitimacy and religious proscriptions play a role in determining the joint evolution of institutions and culture.

We first consider the relationship between limited governance and fiscal capacity. This relationship is central to the extension proposed in Section 5. Recall that Western Europe became more limited politically (via parliaments and other organizations that constrained executive power) in the medieval and early modern periods but the Middle East did not.

There is a large literature claiming that states in which fiscal capacity and the "power of the purse" are held by groups outside of the central executive are able to collect more taxes due to greater constraints on executive power (Besley and Persson 2009, Dincecco 2009, Karaman and Pamuk 2013, Ma and Rubin 2019, North and Weingast 1989, Stasavage 2011, 2020). Our model adds additional insight to this literature by shedding light on the process through which limited governance, as we define it, engenders cultural change (i.e., secularization) that reinforces the state's fiscal capacity. One of our primary insights is that rulers will only devolve political authority when the returns from religious legitimacy are sufficiently low. This in turn triggers cultural change to a more secular society. On the contrary, when society is religious, the returns from religious legitimacy may be high even when religious proscriptions impinge on productive effort. In this case, culture and institutions evolve in tandem and society becomes more religious over time.

Section 5 highlights multiple reasons why European political institutions became more limited in the medieval period. First, following the fall of the Western Roman Empire, European rulers had little fiscal power relative to other elites. In the terms of our model, their initial level of β was low. This also follows from the framework of Blaydes and Chaney (2013), who argue that European rulers were weak relative to other elites because they lacked access to independent sources of military power, unlike Muslim rulers who could employ slave soldiers.

However, an explanation relying solely on limits on governance placed by secular elites leaves a major question unanswered. If Muslim rulers were so strong relative to other elites, why should they have feared devolving some of their power to those secular elites, which could have yielded more tax revenue? Even as late as the early modern period, Ottoman tax collection was notoriously low (Karaman and Pamuk 2013). Why did the Ottomans not give more power to local notables, who would have almost certainly had more capacity to collect taxes? These elites should not have been a threat to Muslim rulers. After all, rulers had slave soldiers and local elites did not.

Our model provides insights which help resolve this puzzle. It suggests the possibility that a Muslim ruler's fiscal power relative to other elites (β) *interacted* with the greater legitimating capacity of religious authorities. Muslim rulers failed to devolve political power not because they feared that other elites would become too strong. They did so because devolution of power to secular elites would have resulted in a weakening of the efficacy of religious legitimacy. Granting more power to secular authorities would have encouraged a cultural shift to a more secular state, yielding religious legitimacy less effective. Given the relative efficacy of religious legitimacy, this would not have been an optimal strategy for a Muslim ruler. This was exacerbated by access to slave soldiers, which gave rulers more power vis-à-vis other elites. However, as the model indicates, this relative power (β) changes endogenously over time. Just because Muslim rulers had an initial advantage vis-à-vis other elites does not explain why it persisted.

The opposite was true in medieval Western Europe. The relatively weak initial power of rulers combined with the relatively weak legitimating capacity of the Church incentivized rulers to devolve political power. This ultimately yielded a secular equilibrium in which religious proscriptions barely impinged on economic development.

These insights accord well with the historical record. Medieval European feudal institutions gave lords—secular lords as well as powerful bishops—great power over their local domains, and in return they provided military service and tax revenue to their sovereign (Duby 1982). Over the course of the late medieval and early modern periods, parliaments became the primary institution which bargained with European rulers (Angelucci, Meraglia and Voigtländer 2022, van Zanden, Buringh and Bosker 2012). Parliaments allowed the economic elite to gain representation at the political bargaining table, and they generally included three classes: the landed nobility, powerful churchmen, and commercial/urban elite. As warfare became more expensive, European rulers ceded more to these elites, who could provide them with revenue (Gennaioli and Voth 2015, North and Weingast 1989, Stasavage 2011, Tilly 1990). Ultimately, parliaments became the main tool for constraining rulers, which resulted in a massive increase in fiscal capacity (Dincecco 2009, Johnson and Koyama 2017, North and Weingast 1989, Tilly 1990, van Zanden, Buringh and Bosker 2012).[44]

On the other hand, in the Middle East economic power was decentralized but *political power* remained relatively unconstrained (Coşgel and Miceli 2005, Karaman and Pamuk 2013, Karaman 2009). In early Islam, under the Umayyad Caliphate, regional governors subject to imperial control administered and collected taxes. This differed both from feudal European as well as the pre-existing Byzantine systems in that these governors were *not* locally dominant aristocratic families subject to little discretion from the center. They were not as powerful and had relatively little fiscal independence (Bessard 2020, p. 37–38). Centuries later this was still the case. At the height of Ottoman power in the fifteenth and early sixteenth centuries, the sultan derived two-thirds to three-quarters of his revenue through the *tımar* system, a military lease contract whereby the provincial cavalry collected agricultural taxes directly from the peasantry as remuneration for their military services to the state (Coşgel and Miceli 2005). The *tımar* system was similar to the tax collection system of feudal Europe, where local feudal lords controlled revenues in return for military service. However, a key difference between the two is that European feudal lords also had political power: their families ruled over their domains for generations, providing local law and order, collecting taxes, and representing them in parliament. On the other hand, *tımar* holders were rotated every few years *precisely* so that they would not acquire local political power. All political power remained vested in the sultan and key religious authorities, not

[44] For theoretical treatments of the rise of state capacity and its affect on economic development, see Acemoglu (2005) and Besley and Persson (2009, 2010, 2014).

tımar holders. Unlike European elites, who were ultimately able to constrain their rulers and receive concessions in return for revenue, *tımar* holders never organized collectively in any manner close to resembling a parliament, and Ottoman rulers remained relatively unconstrained (Balla and Johnson 2009). As a result, the economic elite rarely had any real political power in the Ottoman Empire (Pamuk 2004*a*,*b*). Meanwhile, religious legitimacy remained important (as discussed in Section 4.3), and as a result sultans ceded purview over commercial law to religious authorities, and the associated proscriptions dampening economic activity lasted for centuries (Kuran 2011).

These insights help account for another stylized fact of the Long Divergence: Middle Eastern fiscal capacity was much greater than in Western Europe in the centuries following the spread of Islam, but there was ultimately a reversal of fortunes, with Western European fiscal capacity well-outpacing that of the Ottoman Empire in the early modern period. According to Stasavage (2020, p. 12), the Abbasid Empire was able to extract around 7% of GDP in tax revenues in 850 CE, whereas centuries later England and France were only about to extract about 1% of GDP (in 1300). However, by 1700, the leading economies of Western Europe (England, the Dutch Republic, and France) were able to extract many times more of per capita GDP than the Ottoman Empire (Karaman and Pamuk 2013). Our model highlights one reason for this reversal of fortunes. In the early medieval period, prior to the rise of European parliaments and the reduction in sovereign political power that came with it, European states received little revenue from feudal tax collection, much of which remained in the pockets of local feudal lords. Meanwhile, Middle Eastern states benefited from religious legitimacy, which increased the tax base and thus the revenues taken in by the central state. Indeed, religious authorities and institutions were employed to facilitate tax collection in many cities, including Basra, in the Umayyad period, with mosques playing a central role (although this role ultimately came under the purview of military and economic elites) (Bessard 2020, pp. 205–06, 256). After the rise of European parliaments and the reduction of clerical influence in politics, economic elites gained significant political power (i.e., λ and β were low), in the process placing constraints on the power of the sovereign. In this setting, there was much incentive for the economic elite (i.e., parliaments) to raise taxes because those taxes were spent on their policy preferences. In other words, the benefits of limited governance outweighed the benefits of religious legitimation with respect to tax revenue collected by the state.

6 Conclusion

In this paper we provide an explanation for an important historical phenomenon: the *Long Divergence* between Middle Eastern and Western European economies during the medieval and early modern periods. We provide an explanation with a model of institutional and cultural change. In doing so, we unify prevailing theories based on religious legitimacy, religious proscriptions, and decentralization of political power. In the process, our model resolves many puzzles left unaddressed in the literature.

The model centers on the power dynamics of rulers, clerics, and secular elites and the related process of institutional change. It highlights three central historical features of these power dynamics: rulers derive *legitimacy* from the religious elites, religious authorities impose *proscriptions* that impinge on economic development, and constraints on executive power have a fundamental role in inducing economic growth. Most importantly, the model highlights how the institutions resulting from the power dynamics of rulers, clerics, and secular elites interact with the spread of culture (religious beliefs) in civil society. Limited governance interacts with religious legitimacy and religious proscriptions to determine long-run economic and political paths. Citizens remain religious or not in the face of religious proscriptions, depending on the feedback between religious institutions and cultural evolution. The religious legitimacy of the political system depends crucially on the prominence of religious values in society.

Our analysis concentrates on the role of religious proscriptions, legitimacy, and limited governance as the main components of the Long Divergence between the Middle East and the West. In the appendix, we show how our framework can also accommodate the role of innovation and technological change as another key driver, interacting with religion and religious legitimacy in the process of institutional and cultural divergence. In particular, we discuss how our model is consistent with recent theories which argue that culture (Davids 2013, Mokyr 1990, 2010, 2016, White 1972, 1978), and religious proscriptions in particular (Bénabou, Ticchi and Vindigni 2015, 2020, Coşgel, Miceli and Rubin 2012, Squicciarini 2020) can inhibit technological change.

More generally, our approach can be seen as an illustration of the explanatory power of a class of models centered on some simple, general, and yet minimal components: i) institutions as reflective of the relative political power of different groups in society to affect policy decisions, ii) institutional change as a mechanism to internalize externalities and other distortions characterizing the equilibrium, iii) the cultural profile of values and preferences in society as evolving according to socioeconomic incentives.[45] In this type of set-up, the interdependence between institutions and culture is a fundamental factor, along with technology, driving socio-economic change and long-term institutional development. We hope that this methodology is a stepping stone for further theoretical and empirical analyses in economic history, projecting along those lines historical processes of the evolution of power and social structures across groups and individuals.

[45]See Acemoglu, Johnson and Robinson (2005*a*), Acemoglu, Egorov and Sonin (2021), Bisin and Verdier (2021), and Persson and Tabellini (2021) for surveys of this class of models.

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Appendices

A.1 Extension: Religious Legitimacy and Technological Progress

Although not highlighted in the central theories of the Long Divergence, nearly every theory of Britain's (and eventually Europe's) industrialization asks why Britain eventually became technologically advanced beginning of the 18th century (Allen 2009, Mokyr 1990, 2010, 2016). While not all the advancements of the Industrial Revolution were science based—especially inventions in textile production—many were, including the quintessential invention of the period, the steam engine. That Europe pulled ahead in science and technology is puzzling: for centuries after the spread of Islam, the Middle East had a massive technological and scientific lead on Western Europe (Chaney 2016). What happened? Why was there a reversal of scientific and technological fortunes between the two regions?

In this appendix, we consider an extended version of our framework to sketch and discuss another potentially important driver of the Long Divergence between the Middle East and the West, namely technological and scientific progress. As in the main text, we first sketch the formal model and then discuss the historical stylized patterns. Proofs of the propositions and mathematical derivations are provided afterwards.

A.1.1 A model of institutional and cultural divergence with technological progress

Again, we consider an extended version of the model where political power is divided between religious clerics and the ruler. But now we study the conditions under which the ruler allows an endogenous technological choice or adoption of a scientific innovation, which is a source of productivity gains although it sometimes erodes religious beliefs.

More specifically, let the ruler and the clerics have political weights $1-\lambda$ and λ respectively. Let also the parameter $\alpha_I \in [0, \alpha_{\max}]$ denote a variable characterizing the technology level of the society. We assume that the level of technology is a policy instrument bounded by the knowledge frontier $\alpha_{\max}$.

Given that our primary interest is to study the joint evolution of culture, institutions, and technology, we consider again a reduced form model where the political power of the citizens is set to zero. The ruler now has utility

$$U_r = \tau E - C(m);$$

and religious clerics have utility

$$U_c(m, \alpha_c) = m\alpha_c - \Psi(\alpha_c) - F(m).$$

We now consider religious legitimacy as a function of technology. Specifically, the religious legitimacy of the ruler, $\theta(\alpha_I) = \theta_0 - k\alpha_I$, is a decreasing function of the level of technology α_I.[46] In other words, adoption of innovative and sophisticated technologies erodes traditional religious beliefs where the ruler is seen as legitimate. This can be inherent to the process of innovative or scientific discoveries, which question the relationship between people and the natural world (Bénabou, Ticchi and Vindigni 2020, Mokyr 1990, Squicciarini 2020).[47] Finally, we assume that labor productivity is proportional to the technology level: $a = \alpha_I$.

As in Section 5, citizens do not necessarily comply with tax collection and differ in their (inverse) evasion capacity c. We fix now the taxation enforcement measure to $\epsilon_0 < 1$.

Equilibrium: At any time t, society reaches an equilibrium of the game between the ruler, the clerics, and civil society. Following the same line of reasoning as in Section 3 in the main text, the tax base of the economy is:

$$E = E(\alpha_I, \alpha_c, q_t) = \frac{\alpha_I}{1 + \phi\alpha_c}\{1 - \frac{\tau(1 - q_t\theta(\alpha_I) \cdot \alpha_c)}{\epsilon_0 \bar{c}}\}$$

The policy choices, that is the religious infrastructure m and the technology level α_I are collectively chosen so as to maximize social welfare:

$$W = (1 - \lambda_t)U_r(m, \alpha_I, \alpha_c, q_t) + \lambda_t U_c(m, \alpha_c); \tag{A.1}$$

[46] To avoid some cumbersome taxonomy, we assume that $k\alpha_{\max} < \theta_0 < 2k\alpha_{\max}$. The first inequality ensures that religious legitimacy can always be produced at any potential technological level. The second inequality ensures that maximum knowledge $\alpha_{\max}$ is sufficiently large not to always constrain the equilibrium technology choice by society.

[47] Religious precepts are not always antithetical to scientific advancement. Indeed, White (1972, 1978) and Davids (2013) argue that certain medieval European technologies were complementary to the Church's interest. For the sake of this extension, we focus on technologies that are antithetical to the interests of religious authorities. Mokyr (1990) argues that this more often than not the case with new and disruptive technologies.

while the clerics choose α_c. Solving the equilibrium:

$$\alpha_c = m, \ -C'(m) + \lambda\alpha_c = 0, \tag{A.2}$$

$$\alpha_I\left(\alpha_c, q_t\right) = \min\left[\frac{\epsilon_0\overline{c} - \tau(1 - q_t\theta_0\alpha_c)}{2\tau q_t k\alpha_c}, \alpha_{\max}\right]. \tag{A.3}$$

The equilibrium choice of technology reflects the trade-off with respect to the tax base of an increase in labor productivity and the erosion of religious legitimacy provided by the clerics. It can also be seen that the optimal level of technology $\alpha_I\left(\alpha_c, q_t\right)$ is decreasing in q_t and in α_c. When the religious are more numerous and/or clerics undertake higher religious efforts, the ruler is more reliant on religious legitimacy to raise revenues. Consequently, he is also more reluctant to adopt innovative activities that may erode such legitimacy.

The solution to (A.2) and (A.3) provides the equilibrium values $m\left(\lambda_t\right)$, such that $C'(m) = \lambda_t m$, $\alpha_c\left(\lambda_t\right) = m\left(\lambda_t\right)$, and $\alpha_I\left(\lambda_t, q_t\right) = \alpha_I\left(m\left(\lambda\right), q_t\right)$.

Institutional Dynamics. We allow the ruler to delegate power to the clerics λ. Institutional change again internalizes the externality that is not taken into account by individual decisions in equilibrium. As in the benchmark model, the provision of religious infrastructures m grants legitimacy to the ruler, reducing the subjectively perceived tax rate of religious individuals, while at the same time depressing labor productivity because of increased religious proscriptions. As will be clear below, this interacts with the choice of the optimal technology level adopted by society.

More specifically, given institutions λ_t, future institutions λ_{t+1} are designed as the solution to:

$$\max_{\lambda_{t+1}}(1 - \lambda_t)\left[U_r(m(\lambda_{t+1}), \alpha_I(\lambda_{t+1}), \alpha_c\left(\lambda_{t+1}\right), q_t)\right] + \lambda_t U_c(m\left(\lambda_{t+1}\right), \alpha_c(\lambda_{t+1})), \tag{A.4}$$

with $\{m(\lambda_{t+1}), \alpha_c(\lambda_{t+1}), \alpha_I(\lambda_{t+1})\}$ the equilibrium of period $t+1$, as evaluated under the institutional set-up λ_t. Solving this optimization problem, we deduce that:

Proposition 6 *The optimization problem (A.4) admits a unique solution* $(\lambda_{t+1}) \in [0, 1]$. *Furthermore, there exists a threshold* $\overline{q}_I(\lambda_t)$ *such that*

$$\lambda_{t+1} > \lambda_t \text{ (resp. } \leq\text{) if } q_t > \overline{q}_I(\lambda_t) \text{ (resp. } \leq\text{).}$$

The uniqueness result follows from the convexity of the optimization problem (A.4). Whether the ruler delegates more power to clerics over time depends again on the fraction of religious individuals q_t. If the religious are sufficiently numerous, then religious legitimacy matters relatively more than technology for the ruler's tax base. Consequently, more weight to the clerics $\lambda_{t+1} > \lambda_t$ is provided, as this increases their effort $\alpha_c(\lambda_{t+1})$. The ruler consequently benefits from a larger tax base.

Cultural Dynamics. As in the main text, cultural dynamics are driven by inter-generational transmission decisions from the citizens, and we have the following result:

Proposition 7 *There exists a unique threshold $q_I^*(\lambda_t)$ such that*

$$q_{t+1} < q_t \text{ (resp. } \geq \text{) if } q_t > q_I^*(\lambda_t) \text{ (resp. } \leq \text{).}$$

Furthermore, the threshold $q_I^*(\lambda_t)$ is increasing in λ_t.

The cultural dynamics are still as in (11) and the threshold value $q_I^*(\lambda_t)$ is their unique attractor. Hence, when the fraction of religious individuals q_t is above (resp. below) $q_I^*(\lambda_t)$, it tends to decrease (resp. increase).

Joint Dynamics. There are two steady states. In the *religious regime* equilibrium, the ruler is legitimated by religion. The clerics have significant power (λ is high) and religious beliefs are widespread (q is high). For both reasons, the technology level implemented in society is low, as this threatens the religious legitimacy generated in this religious state. Because, innovation adoption and scientific activity is limited, labor productivity is low, as are fiscal revenues despite extractive taxation. The second steady state is a *secular innovative regime* where a high level of technology close to the knowledge frontier is adopted. Clerics are weak, given that innovations limit their capacity to legitimate the ruler (λ is zero) and the share of religious individuals is low (q is low). Fiscal revenues can be substantial, given that a process of scientific innovation leads to an overall increase in labor productivity.

Complementarity. Again, a ruler's option to rely on religious legitimacy induces a fundamental complementarity of the dynamics of culture and institutions. Along the path towards a religious steady state, the ruler relies more on religious legitimacy to raise revenues. She also faces increasingly lower incentives to adopt efficient innovations that erode her legitimacy. The ruler then commits to an institutional set-up delegating an increasingly large share of power to the clerics, reinforcing the incentive of religious individuals to pass their values inter-generationally. In turn, this further decreases the incentive of the ruler to adopt innovative technologies. Labor productivity stays low, given that technology is limited. Finally, taxes are increasingly more extractive given that the population becomes more religious but labor productivity remains low.

On the other hand, as a ruler relies less on religious legitimacy to raise revenues, she also faces stronger incentives to adopt innovations that increase labor productivity and consequently the fiscal base. As the ruler becomes more reliant on innovative activities to raise revenues, her religious legitimacy erodes, so she faces less incentive to commit to an institutional set-up where the religious clerics are powerful. Both the political weight of the clerics and the value of passing religious values inter-generationally decrease. A lower predominance of religious individuals further augments the political incentives to commit and change the institutional set-up so as to adopt more efficient technologies, leading to a substantial increase over time in labor productivity and fiscal revenues. Eventually, the joint dynamics of culture and institutions converge to a *secular regime* where the implemented technology is not constrained by political forces, but only by the existing knowledge frontier.

A.1.2 The Historical Stylized Pattern

One of the great mysteries of the Long Divergence is the reversal of fortunes between Middle Eastern and Western European science and technology. Data presented in Chaney (2016) reveal that not only were scientific topics among the most ubiquitous in the corpus of Islamic writings up through the 11th century, but up to that point the Islamic world well out-paced Europe in scientific output. At some point in the 11th and 12th centuries, however, a reversal of fortunes occurred. Islamic scientific production began to wane around the 12th century. This was not simply a matter of the Islamic world falling behind relative to Europe; it fell behind in *absolute* terms relative to what had once been. At the same time, scientific works became much more prevalent in Western Europe. By the end of the medieval period, Western Europe had a technological and scientific lead, and this would only grow in subsequent centuries. Can this reversal of fortunes be explained by our model?

Our model, along with the history overviewed in Section 4, suggests that the reversal of technological and scientific fortunes was a consequences of a changing equilibrium in which Muslim religious authorities became increasingly important for legitimating the state while European rulers sought alternative forms of legitimacy. In the Middle East, the 11th century saw the rise of the madrasa system (Chaney 2016, Kuru 2019). This institutionalized the political role that had increasingly been played by religious authorities since their consolidation under the Abbasids in the 9th and 10th centuries (Coşgel, Miceli and Ahmed 2009, Rubin 2017). In this equilibrium, as we describe in Section A.1, religion played an important role in legitimating rule (λ was large), society was largely religious (q was large), and science and technology were impeded. As in Bénabou, Ticchi and Vindigni (2020), technological stagnation mutually benefited religious authorities and the state: the former lost power when alternative means of discovering truths or interpreting the world were present, and the latter was harmed when one of its key sources of legitimacy was undermined.

In the context of Middle Eastern history, this logic sheds light on both why madrasas were allowed to thrive in spite of their negative effects on scientific production and why rulers throughout the Muslim world banned one of the most important technologies of the late medieval period: the printing press. Coşgel, Miceli and Rubin (2012) argues that the Ottomans banned the press for over 240 years after first hearing of it precisely because it threatened the religious establishment. By the 15th century, religious authorities across the Islamic world (not just in the Ottoman Empire) had set up high barriers to entry. The largest of these barriers was the years of training required to know various religious texts and interpretations of those texts. These barriers raised the status of the religious elite, further entrenching the "high-λ, high-q" equilibrium. The printing press threatened to undermine these barriers and the equilibrium they helped uphold. Had printing become widespread, a much larger share of the population would have had access to the great religious and non-religious texts of the Islamic world (and beyond). This would have undermined one of the very features that gave Muslim religious authorities the power to legitimate in the first place. Hence, as our model predicts, heavy restrictions were placed on this vastly important technology.

Muslim religious authorities had good reason to fear the spread of printing. They only needed to look to Europe, where the press helped facilitate one of the great movements against Church power in the history of Christianity: the Protestant Reformation (Boerner, Rubin and Severgnini 2021, Dittmar and Seabold 2020, Rubin 2014). Unlike Ottoman religious authorities, the Church was not able to stop the spread of the printing press. The reason why this was the case follows from the logic of the model. As noted in Section 4.3, the Church had already lost much of its legitimating power in Europe prior to the spread of printing. Alternative sources of legitimacy had emerged in the form of universities (which provided a theoretical justification for monarchical rule) and parliaments (which brought together elites who could legitimate rule in return for a seat at the political bargaining table). By 1200 or so, religious authorities had lost their monopoly over the printed word as well; book demand and supply was increasingly found in university towns and urban centers (Buringh and Van Zanden 2009). As a result, there was little the Church could have done to stop the spread of printing had it wanted to. By the mid-15th century, Europe was in a "low-λ, low-q" equilibrium. Our model suggests that this should also entail few restrictions on technology—at least those technologies that damage the capacity of religious authorities to legitimate. The history of printing suggests that this was the case.

The Christian world was hardly uniform in the degree to which religious legitimacy was part of the broader political equilibrium. This was especially true after the Reformation, which fundamentally undermined the role of religious authorities in the ruling coalition (Rubin 2017). This had consequences for the spread of science and technology. Bénabou, Ticchi and Vindigni (2020) summarize many of the scientific and technological advances blocked or suppressed by the Church, including the works of Galileo, the Copernican Revolution, Newtonism, the Scientific Revolution, and technical education in schools. These restrictions were much more widely applied in Catholic areas than Protestant ones. According to Mokyr (2016), it was the "culture of growth" supported by the Republic of Letters that permitted the spread of the new, rational thinking of those like Bacon and Newton. While the Republic of Letters was a pan-European phenomenon, there was little resistance in the leading Protestant lands (England and the Dutch Republic). Meanwhile, even after the first wave of industrialization, the Church attempted to limit secular education and curriculum in schools (Squicciarini 2020).

In short, this extension helps explain both the technological and scientific reversal of fortunes between Western Europe and the Middle East as well as the the divergence within Europe. In Protestant Europe, new inventions and scientific ideas were allowed to spread relatively unimpeded. This is what the model predicts would be the case in a "low-λ, low-q" equilibrium. The equilibrium in Catholic Europe was one of higher λ and q, and as a result some (though certainly not all) scientific and technological advances were suppressed. In the "high-λ, high-q" equilibrium that pervaded most of the medieval and early modern Middle East (at least, after the 11th century), scientific and technological advancements were even more restricted. Our model explains these outcomes not solely as reflecting the desires of religious authorities, but also their place in their society's broader political-economy and cultural equilibria.

A.1.3 Proofs of Extension A.1

- **Proof of Proposition 6**

We consider that the policymaker chooses the amount of religious infrastructures m, and level of technology $\alpha_I \in [0, \alpha_{\max}]$ to maximize

$$W(m, \alpha_I, \alpha_c, \lambda, q) = (1-\lambda)\left[U_r(m, \alpha_I, q)\right] + \lambda U_c(m, \alpha_c)$$

while the cleric maximizes $U_c(m, \alpha_c)$ with respect to α_c with

$$\begin{aligned} U_r(m, \alpha_I, q) &= \tau E(\alpha_I, \alpha_c, q) - C(m) \\ U_c(\alpha_c, m) &= m\alpha_c - \frac{\alpha_c^2}{2} - C(m) \end{aligned}$$

(we assume for convenience that the cost of the religious infrastructures $C(m)$ is paid as a lump-sum cost by all segments of society) with

$$E(\alpha_I, \alpha_c, q) = \frac{\alpha_I}{1+\phi\alpha_c}\{1 - \frac{\tau(1-q_t\theta\alpha_c)}{\epsilon_0\bar{c}}\}$$

where religious legitimacy is decreasing in the innovation effort: $\theta = \theta(\alpha_I) = \theta_0 - k\alpha_I$. We assume $k\alpha_{\max} < \theta_0 < 2k\alpha_{\max}$ Given the institutional framework λ, one immediately gets

$$\alpha_c = m, \ -C'(m) + \lambda\alpha_c = 0$$

and α_I determined by the FOC:

$$\alpha_I(\alpha_c, q) = \min\left[\frac{1 - \frac{\tau(1-q\theta_0\alpha_c)}{\epsilon_0\bar{c}}}{\frac{2\tau qk\alpha_c}{\epsilon_0\bar{c}}}, \alpha_{\max}\right]$$

This gives the equilibrium values $m(\lambda)$, such that $C'(m) = \lambda m$ and $\alpha_c(\lambda) = \alpha_c(\lambda) = m(\lambda)$ and $\alpha_I(\lambda, q) = \alpha_I(m(\lambda), q)$. (We assume that $C'(0) = 0$ and $C''(m) > 1$ to ensure the existence of a unique equilibrium for all $\lambda \in [0,1]$. This provides also $\alpha_c(\lambda) = m(\lambda)$, As in the related proofs of Propositions 1 and 4, we first demonstrate that the optimization problem (A.4) admits a unique solution $\lambda_{t+1} \in [0, 1]$:

$$\max_{\lambda_{t+1}} (1 - \lambda_t) \left[U_r(m(\lambda_{t+1}), \alpha_I(\lambda_{t+1}), q_t) \right] + \lambda_t U_c(m\left(\lambda_{t+1}\right), \alpha_c(\lambda_{t+1}))) \tag{A.5}$$

In order to solve this maximization problem, we solve the following related optimization problem:

$$\max_{m, \alpha_I} \widetilde{W}\left(m, \alpha_I, \lambda_t, q_t\right) = (1 - \lambda_t) \left[U_r(m, \alpha_I, q_t) \right] + \lambda_t U_c(m), \tag{A.6}$$

where the solution, denoted $(\tilde{m}(\lambda_t, q_t), \widetilde{\alpha}_I(\lambda_t, q_t))$ maximizes the social welfare when the externalities are internalized, so given that $U_c(m) = U_c(m, \alpha_c(m)) = \frac{1}{2}m^2 - C(m)$, as $\alpha_c(m) = m$. $U_r(m, \alpha_I, q_t) = \tau E(m, \alpha_I, q_t) - C(m)$, with

$$E(m, \alpha_I, q_t) = \frac{\alpha_I}{1 + \phi m} \{1 - \frac{\tau(1 - q_t \left[\theta_0 - k\alpha_I\right] m)}{\epsilon_0 \overline{c}}\}. \tag{A.7}$$

We also assume that in the previous optimization problem, the choices of both the religious provision m and of the effort of the innovators α_I are made by a ruler who has a policy commitment capacity, internalizing the externalities associated with the policy choice problem described in the main text. We find that $(\tilde{m}(\lambda_t, q_t), \widetilde{\alpha}_I(\lambda_t, q_t))$ solves the following equations:

$$\begin{cases} \frac{\partial \widetilde{W}}{\partial m} = \lambda_t m - C'(m) + (1 - \lambda_t) \frac{\alpha_I}{1+\phi m} \left[\frac{-\phi}{1+\phi m} \left[1 - \frac{\tau(1 - q[[\theta_0 - k\alpha_I]]m)}{\epsilon_0 \overline{c}} \right] + \frac{\tau q [\theta_0 - k\alpha_I]}{\epsilon_0 \overline{c}} \right] = 0, \\ \frac{\partial \widetilde{W}}{\partial \alpha_I} = \frac{(1-\lambda_t)}{1+\phi m} \{ \alpha_I \left[1 - \frac{\tau(1 - q_t \theta m)}{\epsilon_0 \overline{c}} \right] - \frac{k\alpha_I \tau q_t m}{\epsilon_0 \overline{c}} \} = 0. \end{cases} \tag{A.8}$$

From the second FOC equation we again get the optimal level of technology:

$$\alpha_I\left(m, q_t\right) = \min \left[\frac{1 - \frac{\tau(1 - q_t \theta_0 m)}{\epsilon_0 \overline{c}}}{\frac{2\tau q_t k m}{\epsilon_0 \overline{c}}}, \alpha_{\max} \right]$$

which rewrites as

$$\begin{aligned} \alpha_I\left(m, q_t\right) &= \frac{\frac{\epsilon_0 \overline{c}}{\tau} - 1}{2k q_t m} + \frac{\theta_0}{2k} = \alpha_I^{op}\left(m, q_t\right) \quad \text{when} \quad \frac{A}{q_t} \leq m \\ &= \alpha_{\max} \quad \text{when} \quad \frac{A}{q_t} \geq m \end{aligned}$$

with

$$A = \frac{\frac{\epsilon_0 \overline{c}}{\tau} - 1}{2k\alpha_{\max} - \theta_0} > 0$$

Note that $\alpha_I(m, q)$ is decreasing in q_t and m. Now the characterization of $\tilde{m}(\lambda_t, q_t)$ is obtained from

$$\Theta(m) = \frac{\partial \widetilde{W}}{\partial m}(m, \alpha_I(m, q_t), \lambda_t, q_t) \leq 0 \quad \text{and} \quad m \geq 0$$

When $C(m)$ is sufficiently convex, $\Theta(m)$ is decreasing in m. Moreover given that

$$\Theta(0) = (1-\lambda_t)\alpha_{\max}\left[-\phi\left[1 - \frac{\tau}{\epsilon_0 \overline{c}}\right] + \frac{\tau q_t[\theta_0 - k\alpha_{\max}]}{\epsilon_0 \overline{c}}\right]$$

we have $\Theta(0) > 0$ when

$$q_t > \overline{q} = \frac{\phi}{[\theta_0 - k\alpha_{\max}]}\left[\frac{\epsilon_0 \overline{c}}{\tau} - 1\right]$$

Thus $\tilde{m}(\lambda_t, q_t) = 0$ for $q_t \leq \overline{q}$ and $\tilde{m}(\lambda_t, q_t) > 0$ for $q_t > \overline{q}$. Substitution provides $\widetilde{\alpha}_I(\lambda_t, q_t) = \alpha_I(\tilde{m}(\lambda_t, q_t), q_t)$.

Moreover as

$$\begin{aligned} \frac{\partial^2 \widetilde{W}}{\partial m \partial q} &= (1-\lambda_t)\frac{\alpha_I}{1+\phi m}\left[\frac{-\phi}{1+\phi m}\left[\frac{\tau[[\theta_0 - k\alpha_I]]m}{\epsilon_0 \overline{c}}\right] + \frac{\tau[\theta_0 - k\alpha_I]}{\epsilon_0 \overline{c}}\right] \\ &= (1-\lambda_t)\frac{\alpha_I}{[1+\phi m]^2}\frac{\tau[\theta_0 - k\alpha_I]}{\epsilon_0 \overline{c}} > 0 \end{aligned}$$

Then $\tilde{m}(\lambda_t, q_t)$ is increasing in q_t. As well $\tilde{m}(\lambda_t, q_t) \geq m(\lambda_t)$ if and only if

$$\begin{array}{c} \frac{-\phi}{1+\phi m(\lambda_t)}\left[1 - \frac{\tau(1-q_t[[\theta_0 - k\alpha_I(m(\lambda_t), q_t)]]m)}{\epsilon_0 \overline{c}}\right] \\ + \frac{\tau q_t[\theta_0 - k\alpha_I(m(\lambda_t), q_t)]}{\epsilon_0 \overline{c}} \end{array} \geq 0$$

or

$$\phi\left[\frac{\epsilon_0 \overline{c}}{\tau} - 1\right] \leq q_t[\theta_0 - k\alpha_I(m(\lambda_t), q_t)] \tag{A.9}$$

$q_t[\theta_0 - k\alpha_I(m(\lambda_t), q_t)]$ is an increasing function of q_t and decreasing function of λ_t. Condition (A.9) can be rewritten as a threshold condition $q_t \geq \overline{q}_I(\lambda_t)$ for $\overline{q}_I(\lambda_t) \in (0, 1]$ with $\overline{q}_I(\lambda_t)$ is a decreasing function of λ_t.

Summarizing we get $\tilde{m}(\lambda_t, q_t) \geq m(\lambda_t)$ if and only if $q_t \geq \overline{q}_I(\lambda_t)$ for $\overline{q}_I(\lambda_t) \in (0, 1]$.

Since $(\tilde{m}(\lambda_t, q_t), \tilde{\alpha_I}(\lambda_t, q_t))$ maximizes the social welfare when the externalities are internalized, λ_{t+1} solves the optimization problem (A.4) when:

$$\begin{cases} \tilde{m}(\lambda_t, q_t) = m(\lambda_{t+1}), \text{ and} \\ \tilde{\alpha}_I(\lambda_t, q_t) = \alpha_I\left(m(\lambda_{t+1}), q_t\right) \end{cases} \tag{A.10}$$

Given the first equality, it is immediate to see that the second equality is automatically satisfied from the definition of $\alpha_I\left(m, q_t\right)$. Given this the institutional dynamics of λ_t is uniquely determined. Observe as well that $\tilde{m}(\lambda_t, q_t) \geq m\left(\lambda_t\right)$ if and only if $q_t \geq \overline{q}_I(\lambda_t)$. This can be rewritten as $m(\lambda_{t+1}) \geq m\left(\lambda_t\right)$ if and only if $q_t \geq \overline{q}_I(\lambda_t)$. Given the fact that $m\left(\lambda\right)$ is increasing in λ, we deduce the following result:

$$\lambda_{t+1} \geq \lambda_t \text{ if and only if } q_t \geq \overline{q}_I(\lambda_t)$$

This concludes the proof of Proposition 6.

- **Proof of Proposition 7**

The paternalistic motives have to be amended to take into account the fact that productivity is optimally determined by the endogenous choice of technology: More precisely we have:

$$\begin{cases} V_{ReRe}(\lambda, q) = \frac{(1-\tau_{Re})\alpha_I(\lambda,q)}{1+\phi\alpha_c(\lambda)} \int_{\tau_{Re}/\epsilon_0}^{\overline{c}} \frac{dc}{\overline{c}} + \int_0^{\tau_{Re}/\epsilon_0} \frac{(1-c\epsilon_0)}{1+\phi\alpha_c(\lambda)} \frac{dc}{\overline{c}} \\ V_{Re\ S}(\lambda, q) = \frac{(1-\tau_{Re})\alpha_I(\lambda,q)}{(1+\phi\alpha_c(\lambda))} \int_{\tau/\epsilon_0}^{\overline{c}} \frac{dc}{\overline{c}} + \int_0^{\tau/\epsilon_0} \frac{(1-c\epsilon_0)}{1+\phi\alpha_c(\lambda)} \frac{dc}{\overline{c}}, \end{cases} \tag{A.11}$$

Hence,

$$\Delta V_{Re}(\lambda, \beta, q) = \frac{(\tau\theta\alpha_c(\lambda))^2\alpha_I\left(\lambda, q\right)}{2\overline{c}\epsilon_0(1+\phi\alpha_c(\lambda))}. \tag{A.12}$$

Similarly, we find that

$$\Delta V_S(\lambda, \beta, q) = \Delta V_{Re}(\lambda, \beta, q) = \Delta V(\lambda, \beta, q) = \frac{(\tau\theta\alpha_c(\lambda))^2\alpha_I\left(\lambda, q\right)}{2\overline{c}\epsilon_0(1+\phi\alpha_c(\lambda))}. \tag{A.13}$$

Again the result that $\Delta V_s(\lambda, \beta, q) = \Delta V_{re}(\lambda, \beta, q)$ follows from the quadratic specification of the expected payoff functions. Note as well that because $\alpha_I\left(\lambda, q\right))$ depends on q (ie. is a decreasing function in q), $\Delta V(\lambda, \beta, q)$ also depends on q and is decreasing function of q

Now, the cultural dynamics write as

$$q_{t+1} - q_t = q_t(1 - q_t)D(\lambda_t, q_t). \quad \text{(A.14)}$$

with

$$D(\lambda_t, q_t) = d^*_{Re} - d^*_S = D_{Re}\left[(1 - q_t)\Delta V(\lambda_t, q_t), m(\lambda_t)\right] - D_S\left[q_t \Delta V(\lambda_t, q_t)\right]$$

can be interpreted as the relative "cultural fitness" of the religious trait in the population. Again simple inspection shows

$$D(\lambda_t, 0) = D_{Re}\left[\Delta V(\lambda_t, 0), m(\lambda_t)\right] > 0$$

and

$$D(\lambda_t, 1) = -D_S\left[\Delta V(\lambda_t, 1)\right] < 0$$

From this it follows that there exists a threshold $q^*_I(\lambda_t) \in (0, 1)$ such that

$$D(\lambda_t, q^*_I(\lambda_t)) = 0 \quad \text{(A.15)}$$

Compared to the benchmark model, $D(\lambda_t, q_t)$ may not be always decreasing function in q_t, as $\Delta V(\lambda_t, q_t)$ is decreasing in q_t and the uniqueness of the threshold $q^*_d(\lambda_t)$ is not necessarily ensured. When however $q\Delta V(\lambda, q)$ is increasing function of q,[48] simple inspection shows that $D(\lambda_t, q_t)$ is a decreasing function of q_t and that $q_{t+1} < q_t$ if and only if $q_t > q^*_I(\lambda_t, \beta_t)$, as stated in proposition 7. **QED.**

[48] This is ensured when $1 > \frac{\tau^2}{\bar{c}\epsilon_0} \max\left(\frac{\theta}{\phi}, 1\right)$

A.2 Proofs of Lemmas 1, 2 and 3

In order to prove the three Lemmas of the main text, we solve the equilibrium, where the amount of religious infrastructures m is determined by the institutional system so as to maximize the social welfare W,

$$W = \frac{1}{2}U_R(m) + \frac{\lambda}{2}U_c(m, \alpha_c) + \frac{1-\lambda}{2}\left[qU_{Re}(e_{Re}) + (1-q)U_S\left(e_S\right)\right]. \tag{A.16}$$

while the ruler, the clerics and the individuals choose, respectively, τ, α_c and e_i, $i = Re, S$ to maximize their utility (taking as given what the other segments of society do, as well as the policy variable m). The Nash equilibrium of this policy game is denoted $\{\tau(\lambda), m(\lambda), \alpha_c(\lambda), e_S(\lambda), e_{Re}(\lambda)\}$. It is clear that $\tau(\lambda)$ is equal to $\overline{\tau} \equiv \tau$ and the remaining first-order conditions are:

$$\begin{cases} -C'(m) - \lambda F'(m) + \lambda \cdot \alpha_c = 0 \\ m - \Psi'(\alpha_c) = 0 \\ (1-\tau_{Re}) - (1+\phi\alpha_c)e_{Re} = 0 \\ (1-\tau) - (1+\phi\alpha_c)e_S = 0, \end{cases} \tag{A.17}$$

or after substitution:

$$\begin{cases} C'(m) + \lambda F'(m) = \lambda\alpha_c \\ \Psi'(\alpha_c) = m \\ e_{Re} = \frac{1-\tau+\tau\theta\alpha_c}{1+\phi\alpha_c} \\ e_S = \frac{1-\tau}{1+\phi\alpha_c} \end{cases} \tag{A.18}$$

Assuming that the marginal cost functions $C'(.)$, $F'(.)$ and $\Psi'(.)$ are increasing convex functions (ie. $C'''(.) \geq 0, F'''(.) \geq 0$ and $\Psi'''(.) \geq 0$) with at least one of these cost derivatives strictly convex), and the limit condition $\lim_{x\to\infty} F''(x) > 1$, and $F''(0)\Psi''(0) < 1$, then the first two equations of (A.18) simply characterize a unique equilibrium couple $m(\lambda) > 0$ and $\alpha_c(\lambda) > 0$ when $\frac{C''(0)\Psi''(0)}{1-F''(0)\Psi''(0)} < \lambda$, while $m(\lambda) = \alpha_c(\lambda) = 0$ for $\lambda \leq \frac{C''(0)\Psi''(0)}{1-F''(0)\Psi''(0)}$.

Lemma 1: Differentiating the previous first-order conditions, it is easy to note that the optimal provision of religious infrastructure $m(\lambda) > 0$ and the effort of the clerics

$\alpha_c(\lambda) > 0$ are both increasing in λ and independent from θ and ϕ. This concludes the proof of Lemma 1.

Lemma 2: The equilibrium production efforts are obtained as

$$\begin{cases} e_{Re}(\lambda) = \frac{1-\tau+\tau\theta\alpha_c(\lambda)}{1+\phi\alpha_c(\lambda)} \\ e_S(\lambda) = \frac{1-\tau}{1+\phi\alpha_c(\lambda)} \end{cases} \tag{A.19}$$

The equilibrium secular effort $e^*_S(\lambda)$ is decreasing in clerics activities α^*_c and thus, it is decreasing in λ. It is independent from ϕ and θ

Additionally, from the equation above, $e_{Re}(\lambda)$ increases with θ and decreases with ϕ. The effect of $\alpha_c(\lambda)$ on $e_{Re}(\beta, \lambda)$ is ambiguous. By deriving $e_{Re}(\lambda)$ with respect to α_c, we find that when $\theta > \frac{1-\tau}{\tau}\phi$, then $e_{Re}(\lambda)$ increases with $\alpha_c(\lambda)$, in which case $e_{Re}(\lambda)$ increases with λ. This concludes the proof of Lemma 2.

Lemma 3: The equilibrium tax base of the ruler writes as

$$E(\lambda) = q \cdot e_{Re}(\lambda) + (1-q) \cdot e_s(\lambda), \tag{A.20}$$

so

$$E(\lambda) = \frac{1-\tau+\tau\theta q \cdot \alpha_c(\lambda)}{1+\phi\alpha_c(\lambda)}. \tag{A.21}$$

By deriving the previous expression with respect to $\alpha_c(\lambda)$, we find that the tax base is increasing in the clerics' effort if and only if $q \geq \frac{1-\tau}{\tau\theta}\phi$. Hence, when the previous condition is satisfied, $E(\lambda)$ is increasing in λ. Finally, from (A.21), $E(\lambda)$ is increasing in q and θ, and decreasing in ϕ. This concludes the proof of Lemma 3.

A.3 Proof of Proposition 1

- First, we demonstrate that the optimization problem (7) rewritten below admits a unique solution $\lambda_{t+1} \in [0,1]$:

$$\max_{\lambda_{t+1}} \frac{1}{2}U_r(m(\lambda_{t+1})) + \frac{\lambda_t}{2}U_c(m(\lambda_{t+1}), \alpha_c(\lambda_{t+1}))+ \\ \frac{1-\lambda_t}{2}\left[q_t U_{re}(e_{Re}(\lambda_{t+1})) + (1-q_t)U_s\left(e_s(\lambda_{t+1})\right)\right]. \tag{A.22}$$

In order to solve this maximization problem, we consider the following related optimization problem:

$$\max_{m} W(m, q_t) = \frac{1}{2}\{U_r(m) + \lambda_t \widetilde{U}_c(m) + (1-\lambda_t)\left[q_t \widetilde{U}_{re}(m) + (1-q_t)\widetilde{U}_s(m)\right]\}, \quad \text{(A.23)}$$

with

$$\begin{cases} \widetilde{\alpha}_c(m) = \psi'^{-1}(m) \\ E(m) = \frac{1-\tau+\tau\theta q \widetilde{\alpha}_c(m)}{1+\phi\widetilde{\alpha}_c(m)} \\ U_r(m) = \tau E(m) - C(m) \\ \widetilde{U}_c(m) = \widetilde{\alpha}_c(m) m - \psi(\widetilde{\alpha}_c(m)) - F(m) \\ \widetilde{U}_{Re}(m) = \frac{[1-\tau+\tau\theta\widetilde{\alpha}_c(m)]^2}{2(1+\phi\widetilde{\alpha}_c(m))} \\ \widetilde{U}_S(m) = \frac{(1-\tau)^2}{2(1+\phi\widetilde{\alpha}_c(m))}. \end{cases} \quad \text{(A.24)}$$

In the optimization problem (A.23), the choice of the religious infrastructure m is made by a ruler able to commit to the provision of m, and therefore internalizing the two externalities detailed in the main text. We find that:

$$2\frac{\partial W}{\partial m} = \lambda_t\left[\widetilde{\alpha}_c(m) - F'(m)\right] - C'(m) + \tau E'(m) + (1-\lambda_t)[q_t \widetilde{U}'_{Re}(m) + (1-q_t)\widetilde{U}'_S(m)]. \quad \text{(A.25)}$$

When $C(.)$ and $F(.)$ are sufficiently convex, the function W is concave in m, and the previous optimization admits a unique solution $\tilde{m}(\lambda_t, q_t) \geq 0$.

Note that $\alpha_c(\lambda) = \widetilde{\alpha}_c(m(\lambda))$, $U_i(e_i(\lambda)) = \widetilde{U}_i(m(\lambda))$ for $i = \{Re, S\}$, and $U_c(m(\lambda), \alpha_c(\lambda)) = \widetilde{U}_c(m(\lambda))$. Given that $\tilde{m}(\lambda_t, q_t)$ maximizes the social welfare when the externalities are internalized, the solution λ_{t+1} of the optimization problem (7), should be such as to induce an equilibrium choice $m(\lambda_{t+1})$ as close to $\tilde{m}(\lambda_t, q_t)$ as possible:

$$\lambda_{t+1} = \begin{cases} \lambda \text{ s.t } m(\lambda) = \tilde{m}(\lambda_t, q_t) & \text{if } \tilde{m}(\lambda_t, q_t) \in (m(0), m(1)) \\ 1 & \text{if } \tilde{m}(\lambda_t, q_t) > m(1) \\ 0 & \text{if } \tilde{m}(\lambda_t, q_t) < m(0). \end{cases} \quad \text{(A.26)}$$

When the clerics have power λ_{t+1} given by (A.26), institutions are designed for $t+1$ so as to induce a choice $m(\lambda_{t+1})$ in that period that maximizes the social welfare of period t. Given that $m(\lambda)$ is increasing in λ,this solution λ_{t+1} of problem (7) is unique and the institutional dynamics are well defined.

Note that $\frac{\partial \tilde{m}}{\partial q} = -\frac{\partial^2 W}{\partial m \partial q} / \frac{\partial^2 W}{\partial m^2}$ and has the sign of $\frac{\partial^2 W}{\partial m \partial q}$ (as W is concave in m). As

$$2\frac{\partial^2 W}{\partial m \partial q} = (1-\lambda_t)[\widetilde{U}'_{Re}(m) - \widetilde{U}'_S(m)]$$

and

$$\widetilde{U}_{Re}(m) - \widetilde{U}_S(m) = \frac{\tau\theta\widetilde{\alpha}_c(m)\left[2(1-\tau)+\tau\theta\widetilde{\alpha}_c(m)\right]}{2(1+\phi\widetilde{\alpha}_c(m))}$$

is an increasing of $\widetilde{\alpha}_c(m)$ and therefore an increasing function of m. It follows that $\widetilde{U}'_{Re}(m) - \widetilde{U}'_S(m) > 0$ and $\frac{\partial^2 W}{\partial m \partial q} > 0$, from which we conclude that $\tilde{m}(\lambda_t, q_t)$ is increasing in q_t.

- In the second step of the proof, we demonstrate that there exists a threshold $\overline{q}(\lambda_t)$ such that if $q_t > \overline{q}$, then $\lambda_{t+1} > \lambda_t$. Otherwise, $\lambda_{t+1} \leq \lambda_t$.

In order to demonstrate this claim, we first show the following intermediary result:

Lemma 6 *$\lambda_{t+1} > \lambda_t$ if and only if $\tilde{m}(\lambda_t, q_t) > m(\lambda_t)$.*

Proof: Indeed, $\tilde{m}(\lambda_t, q_t) > m(\lambda_t)$ means that if the ruler had the capacity to commit, in period t, to provide religious infrastructures m, then he would chooses a level $\tilde{m}(\lambda_t, q_t)$ strictly above what he actually provides in equilibrium. Since $m(.)$ is an increasing function (Lemma 1), we deduce that λ_{t+1} is such that $\lambda_{t+1} > \lambda_t$.[49] **QED.**

Lemma 7 *$\tilde{m}(\lambda_t, q_t) > m(\lambda_t)$ if and only if $q_t > \overline{q}(\lambda_t)$, with:*

$$\overline{q}(\lambda_t) = \frac{1}{\tau\theta} \frac{\phi(1-\tau)\left[\tau + (1-\lambda_t)\frac{1-\tau}{2}\right]}{\tau + (1-\lambda_t)\left[1-\tau+\tau\theta\alpha_c^*(\lambda_t)\left(1+\frac{\phi}{2}\alpha_c^*(\lambda_t)\right)\right]} \tag{A.27}$$

Proof: From the proof of Lemma 1 above, the first-order condition associated with the determination of $m(\lambda)$ is:

$$\lambda_t\left[\widetilde{\alpha}_c(m) - F'(m)\right] - C'(m) = 0, \tag{A.28}$$

given that $\widetilde{\alpha}_c(m) = \psi'^{-1}(m)$.

The first order condition for the determination of $\tilde{m}(\lambda_t, q_t)$ writes as $\frac{dW}{dm} = 0$, with

$$\frac{dW}{dm} = \frac{1}{2}\left[\lambda_t\left[\widetilde{\alpha}_c(m) - F'(m)\right] - C'(m) + \tau E'(m) + (1-\lambda_t)[q_t\widetilde{U}'_{Re}(m) + (1-q_t)\widetilde{U}'_S(m)]\right]. \tag{A.29}$$

[49] When an interior solution exists, λ_{t+1} solves $\tilde{m}(\lambda_t) = m(\lambda_{t+1})$. Hence, if $\tilde{m}(\lambda_t) > m(\lambda_t)$ then $\lambda_{t+1} > \lambda_t$.

Consider the expression

$$H(m) = \tau \cdot E'(m) + (1-\lambda)[q_t \widetilde{U}'_{Re}(m) + (1-q_t)\widetilde{U}'_S(m)]\}.$$

Given the two FOCs above, we deduce that $\tilde{m}(\lambda_t, q_t) > m(\lambda_t)$ if and only if $H(m(\lambda_t)) > 0$. We show that condition $H(m(\lambda_t)) > 0$ is equivalent to a condition over the possible values of q.

$$\begin{aligned} E'(m) &= q_t \cdot \frac{de_{Re}}{dm} + (1-q_t) \cdot \frac{de_S}{dm} && \text{(A.30)} \\ &= \frac{q_t \tau\theta - (1-\tau)\phi}{[1+\phi\widetilde{\alpha}_c(m)]^2} \frac{d\widetilde{\alpha}_c(m))}{dm}, && \text{(A.31)} \end{aligned}$$

$$\begin{aligned} U'_{Re}(m) &= e_{Re}(m)\left[\theta\tau - \phi \cdot \frac{e_{Re}(m)}{2}\right]\frac{d\widetilde{\alpha}_c(m)}{dm} && \text{(A.32)} \\ &= \frac{1-\tau+\tau\theta\widetilde{\alpha}_c(m)}{1+\phi\widetilde{\alpha}_c(m)}\left[\theta\tau - \phi \cdot \frac{1}{2}\frac{1-\tau+\tau\theta\widetilde{\alpha}_c(m)}{1+\phi\widetilde{\alpha}_c(m)}\right]\frac{d\widetilde{\alpha}_c(m)}{dm} && \text{(A.33)} \end{aligned}$$

and

$$\begin{aligned} U'_s(m) &= -\frac{\phi\,(e_S(m))^2}{2}\frac{d\widetilde{\alpha}_c(m)}{dm} && \text{(A.34)} \\ &= -\phi \cdot \frac{1}{2}\left[\frac{1-\tau}{1+\phi\widetilde{\alpha}_c(m)}\right]^2 \cdot \frac{d\widetilde{\alpha}_c(m)}{dm} && \text{(A.35)} \end{aligned}$$

Thus,

$$2\frac{dW}{dm} = \lambda\widetilde{\alpha}_c(m) - C'(m) - \lambda F'(m) + H(m),$$

with

$$[1+\phi\widetilde{\alpha}_c(m)]^2 \frac{H(m)}{\frac{d\widetilde{\alpha}_c(m)}{dm}} = \tau \cdot (q_t\tau\theta - (1-\tau)\phi) + (1-\lambda)G(m)$$

and

$$
\begin{aligned}
G(m) &= q_t(1-\tau+\tau\theta\widetilde{\alpha}_c(m))\left[\theta\tau\left(1+\phi\widetilde{\alpha}_c(m)\right)-\frac{\phi}{2}(1-\tau+\tau\theta\widetilde{\alpha}_c(m))\right] \\
&\quad -(1-q_t)\frac{\phi}{2}\left[1-\tau\right]^2 \\
&= q_t\tau\theta\left[(1-\tau)+\tau\theta\widetilde{\alpha}_c(m)\left(1+\frac{\phi}{2}\widetilde{\alpha}_c(m)\right)\right]-\frac{\phi}{2}\left[1-\tau\right]^2
\end{aligned}
$$

Then the condition $H(m(\lambda_t)) > 0$ writes as

$$
\tau\cdot(q_t\tau\theta-(1-\tau)\phi)+(1-\lambda)\left[\begin{array}{c} q_t\tau\theta\left[(1-\tau)+\tau\theta\widetilde{\alpha}_c(m)\left(1+\frac{\phi}{2}\widetilde{\alpha}_c(m)\right)\right] \\ -\frac{\phi}{2}\left[1-\tau\right]^2 \end{array}\right]\geq 0
$$

or using $\alpha_c(\lambda) = \widetilde{\alpha}_c(m(\lambda)) = \Psi'^{-1}(m(\lambda))$ and rearranging terms $H(m(\lambda_t)) > 0$ if and only if $q_t > \overline{q}(\lambda_t)$ with

$$
\overline{q}\left(\lambda\right)=\frac{1}{\tau\theta}\frac{\phi\left(1-\tau\right)\left[\tau+(1-\lambda)\frac{1-\tau}{2}\right]}{\tau+(1-\lambda)\left[1-\tau+\tau\theta\alpha_c(\lambda)\left(1+\frac{\phi}{2}\alpha_c(\lambda)\right)\right]}, \tag{A.36}
$$

and $\alpha_c(\lambda)$ is an increasing function of λ. We conclude that $\tilde{m}(\lambda_t, q_t) > m(\lambda_t)$ if and only if $q_t > \overline{q}(\lambda_t)$.**QED.**

Combining the results established in Lemmas 6 and 7, it follows that $\lambda_{t+1} > \lambda_t$ if and only if $q > \overline{q}(\lambda_t)$.

Finally, from (A.36), we deduce that $\overline{q}(\lambda_t)$ is decreasing in θ and ϕ. This concludes the proof of the first point of Proposition 1.

A.4 Proof of Proposition 2

As mentioned in the main text, cultural dynamics are modeled as purposeful inter-generational transmission (Bisin and Verdier (2001), Bisin and Verdier (2017)), through parental socialization and imitation of society at large. *Direct vertical* socialization to the parent's trait $i \in \{Re, S\}$ occurs with probability d_i. If a child from a family with trait i is not directly socialized, which occurs with probability $1 - d_i$, he/she is *horizontally/obliquely* socialized by picking the trait of a role model chosen randomly in the population. The probability

P_{ij} that a child in group i is socialized to trait j writes as:

$$\begin{cases} P_{ii} = d_i + (1 - d_i)q_i \\ P_{ij} = (1 - d_i)q_j \end{cases} \tag{A.37}$$

with $q_{Re} = q_t$ and $q_S = 1 - q_t$. Let $V_{ij}(\lambda_t) = U_i(e_j(\lambda_t))$ denote the utility to a cultural trait i parent of a type j child, with $i, j \in \{Re, S\}$. We denote the paternalistic bias of a parent of type i as $\Delta V_i(\lambda_t) = V_{ii}(\lambda_t) - V_{ij}(\lambda_t)$, for $j \neq i$. The socialization cost $h_{Re}(d_{Re}, m)$ of a parent of type Re (respectively S) is assumed to be a smooth function with $\frac{\partial h_{Re}(d_{Re},m)}{\partial d_{Re}} \geq 0$; $\frac{\partial^2 h_{Re}(d_{Re},m)}{\partial d_{Re}^2} > 0$ (ie. $h_{Re}(d_{Re}, m)$ is increasing convex in d_{Re}) and the Inada conditions $h_{Re}(0, m) = \frac{\partial h_{Re}(0,m)}{\partial d_{Re}} = 0$, $\lim_{d \to 1} h_{Re}(d, m) = \lim_{d \to 1} \frac{\partial h_{Re}(d,m)}{\partial d_{Re}} = +\infty$ Similarly, the socialization cost $h_S(d_S)$ of a parent of type S satifies $h'_S(d_S) \geq 0$; $h''_S(d_S) > 0$ (ie. $h_S(d_S)$ is increasing convex in d_S (ie.), and $h_S(0) = h'_S(0) = 0$, $\lim_{d \to 1} h_S(d) = \lim_{d \to 1} h'_S(d) = +\infty$.

Furthermore, to reflect the fact religious infrastructures may enter as a complementary input to parental effort for transmission of the religious trait, we assume that $\frac{\partial h_{Re}(d_{Re},m)}{\partial m} \leq 0$ and $\frac{\partial^2 h_{Re}(d_{Re},m)}{\partial d_{Re} \partial m} \leq 0$, (ie. m affects negatively the cost and the marginal cost of socialization of religious parents). Following Bisin and Verdier (2001), direct socialization d^*_{Re} of religious parents is the solution to the following socialization problem:

$$\max_{d_{Re}} -h_{Re}(d_{Re}, m_t) + P_{ReRe} \cdot V_{ReRe}(\lambda_t) + P_{ReS} \cdot V_{Re\ S}(\lambda_t), \tag{A.38}$$

while direct socialization d^*_S of secular parents is the solution to the following socialization problem:

$$\max_{d_S} -h_S(d_S) + P_{SS} \cdot V_{SS}(\lambda_t) + P_{SRe} \cdot V_{SRe}(\lambda_t), \tag{A.39}$$

The FOCs of the previous programs determine the optimal socialization efforts as:

$$\frac{\partial h_{Re}(d^*_{Re}, m_t)}{\partial d_{Re}} = (1 - q_t)\Delta V_{Re}(\lambda_t) \quad \text{and } h'_S(d^*_S) = q_t \Delta V_s(\lambda_t)$$

which can be rewritten as $d^*_{Re}(q_t, \lambda_t) = D_{Re}((1 - q_t)\Delta V_{Re}(\lambda_t), m(\lambda_t))$ and $d^*_S(q_t, \lambda_t) = D_S(q_t \Delta V_S(\lambda_t))$.

Note that by the Inada conditions on $h_{Re}(\cdot,\cdot)$, $d^*_{Re} \in [0,1]$, and $D_{Re}(0,m) = 0$. As well $D_{Re}(\cdot,\cdot)$ is an increasing function of both arguments $(1-q_t)\Delta V_{Re}(\lambda_t)$ and m, as we have:

$$\frac{\partial d^*_{Re}}{\partial(1-q_t)\Delta V_{Re}(\lambda_t))} = \frac{1}{\frac{\partial^2 h_{Re}}{\partial d^2_{Re}}} > 0 \text{ and } \frac{\partial d^*_{Re}}{\partial m_t} = -\frac{\frac{\partial^2 h_{Re}}{\partial d_{Re}\partial m}}{\frac{\partial^2 h_{Re}}{\partial d^2_{Re}}} > 0$$

Similarly the Inada conditions on $h_S(\cdot)$ ensure that $d^*_S \in [0,1]$, $D_S(0) = 0$. As well $d^*_S = D_S(q_t\Delta V_S(\lambda_t))$ is an increasing function of $q_t\Delta V_S(\lambda_t)$ as

$$\frac{\partial d^*_S}{\partial(q_t\Delta V_S(\lambda_t))} = \frac{1}{h''_S} > 0$$

Using the Law of Large Numbers, one easily obtains the intergenerational evolution of the frequency of the religious trait q_t in the population as

$$q_{t+1} = q_t \cdot P_{ReRe} + (1-q_t) \cdot P_{SRe}$$

or after substitution of (A.37) and the values of d^*_{Re} and d^*_S,

$$q_{t+1} - q_t = q_t(1-q_t)\{d^*_{Re}(q_t,\lambda_t) - d^*_S(q_t,\lambda_t)\}. \tag{A.40}$$

As mentioned in the main text, in equation (A.40), the term

$$\begin{aligned} D(q_t,\lambda_t) &= d^*_{Re}(q_t,\lambda_t) - d^*_S(q_t,\lambda_t) \\ &= D_{Re}\left[(1-q_t)\Delta V_{Re}(\lambda_t), m(\lambda_t)\right] - D_S\left[q_t\Delta V_S(\lambda_t)\right] \end{aligned}$$

can be interpreted as the relative "cultural fitness" of the religious trait in the population. This term is frequency dependent (ie. depends on the state of the population q_t). Moreover simple inspection shows that $D(q_t,\lambda_t)$ is a decreasing function of q_t, with $D(0,\lambda_t) = D_{Re}\left[\Delta V_{Re}(\lambda_t), m(\lambda_t)\right] > 0$ and $D(1,\lambda_t) = -D_S\left[\Delta V_S(\lambda_t)\right] < 0$. From this it follows that there exists a unique threshold $q^*(\lambda_t) \in (0,1)$ such that

$$D(q^*(\lambda_t),\lambda_t) = 0 \tag{A.41}$$

Inspection of equation (A.40) and the fact that $D(q_t,\lambda_t)$ is a decreasing function of q_t provides immediately that $q_{t+1} < q_t$ if and only if $q_t > q^*(\lambda_t)$, proving therefore proposition in the main text. **QED.**

A.5 Comparative statics on the cultural threshold $q^*(\lambda_t)$

The relative "cultural fitness" of the religious trait $D(q_t, \lambda_t)$ is affected by the institutional environment λ_t, as this variable interacts with the process of parental cultural transmission both through paternalistic motivations $\Delta V_i(\lambda_t)$, and through the provision of religious infrastructures $m_t = m(\lambda_t)$ as a complementary input to religious family socialization. Therefore the dependence of the threshold $q^*(\lambda_t)$ on the institutional environment λ_t and the comparative statics on the parameters θ and ϕ depends on how the relative "cultural fitness" $D(q_t, \lambda_t)$ of the religious trait is affected by changes in such features.

It is first useful to note that with the quadratic specification for the utility functions $U_i(.)$ of workers, the paternalistic motives .$\Delta V_{Re}(\lambda_t)$ and $\Delta V_S(\lambda_t)$ are equal and take a simple form. Indeed we have:

$$\begin{cases} V_{ReRe}(\lambda) = \frac{(1-\tau_{Re})^2}{2(1+\phi\alpha_c(\lambda))} \\ V_{Re\ S}(\lambda) = (1-\tau_{Re})\frac{1-\tau}{1+\phi\alpha_c(\lambda)} - \frac{1}{2}(1+\phi\alpha_c(\lambda))\frac{(1-\tau)^2}{(1+\phi\alpha_c(\lambda))^2}. \end{cases} \tag{A.42}$$

Hence,

$$\Delta V_{Re}(\lambda) = V_{ReRe}(\lambda) - V_{Re\ S}(\lambda) = \frac{(\tau\theta\alpha_c(\lambda))^2}{2(1+\phi\alpha_c(\lambda))}. \tag{A.43}$$

Similarly, we find that

$$\begin{cases} V_{SS}(\lambda) = \frac{(1-\tau)^2}{2(1+\phi\alpha_c(\lambda))} \\ V_{SRe}(\lambda) = (1-\tau)\frac{1-\tau_{Re}}{1+\phi\alpha_c(\lambda)} - \frac{1}{2}(1+\phi\alpha_c(\lambda))\frac{(1-\tau_{Re})^2}{(1+\phi\alpha_c(\lambda))^2}. \end{cases}$$

and

$$\Delta V_S(\lambda) = V_{SS}(\lambda) - V_{SRe}(\lambda) = \frac{(\tau\theta\alpha_c(\lambda))^2}{2(1+\phi\alpha_c(\lambda))} \tag{A.44}$$

Thus posing $\Delta V(\lambda) = \frac{(\tau\theta\alpha_c(\lambda))^2}{2(1+\phi\alpha_c(\lambda))}$, we get $\Delta V_{Re}(\lambda) = \Delta V_S(\lambda) = \Delta V(\lambda)$ and the relative "cultural fitness" of the religious trait $D(q_t, \lambda_t)$ rewrites as:

$$D(q_t, \lambda_t) = D_{Re}\left[(1-q_t)\Delta V(\lambda_t), m(\lambda_t)\right] - D_S\left[q_t\Delta V(\lambda_t)\right]$$

Now, considering the functions $D_{Re}(x, y)$ and $D_S(z)$ that respectively characterize the optimal socialization behavior of religious parents as

$$d^*_{Re}(q_t, \lambda_t) = D_{Re}\left[(1-q_t)\Delta V(\lambda_t), m(\lambda_t)\right], \text{and} \;\; d^*_S(q_t, \lambda_t) = D_S(q_t\Delta V_S(\lambda_t))$$

define the sensitivity of parents' socialization to paternalistic motives by the following elasticities:

$$\epsilon_{Re}(q,\lambda) = \frac{\partial D_{Re}(x,y)}{\partial x} \cdot \frac{x}{D_{Re}} \text{ and } \epsilon_S(q,\lambda) = \frac{\partial D_S}{\partial z} \cdot \frac{z}{D_{Re}}\Bigg|$$

evaluated respectively at $x = (1-q)\Delta V(\lambda)$ and $y = m(\lambda)$, and $z = q\Delta V_S(\lambda)$.

Differentiation of (A.41) then provides with $d^*(\lambda_t) = d^*_{Re}(q^*(\lambda_t), \lambda_t) = d^*_S(q^*(\lambda_t), \lambda_t)$

$$q^{*\prime}(\lambda_t) = \frac{[\epsilon_{Re}(q^*,\lambda_t) - \epsilon_S(q^*,\lambda_t)]\, d^*(\lambda_t) \cdot \frac{\Delta V'(\lambda_t)}{\Delta V(\lambda_t)} + \frac{\partial D_{Re}}{\partial m} \cdot m'(\lambda_t)}{-\frac{\partial D}{\partial q}(q^*(\lambda_t), \lambda_t)} \tag{A.45}$$

Given that $\frac{\partial D}{\partial q}(q^*(\lambda_t), \lambda_t) < 0$, $\frac{\partial q^*}{\partial \lambda_t}$ has the sign of the numerator. This numerator is composed of two terms reflecting the two channels through which the institutional environment λ_t affects cultural transmission. The first term $K(\lambda_t) = [\epsilon_{Re}(q^*,\lambda_t) - \epsilon_S(q^*,\lambda_t)]\, d^*(\lambda_t) \cdot \frac{\Delta V'(\lambda_t)}{\Delta V(\lambda_t)}$ is the paternalistic motive channel. As $\Delta V'(\lambda_t) > 0$, both types of parents increase the intensity of socialization to their own traits. The sign of $K(\lambda_t)$ depends on the relative sensitivity of parents' socialization to paternalistic motives. It is positive when $\epsilon_{Re}(q^*,\lambda_t) > \epsilon_S(q^*,\lambda_t)$, namely when the socialization rate of religious parents d^*_{Re} is more sensitive to paternalistic motives than the one of secular parents d^*_S.

The second term $\frac{\partial D_{Re}}{\partial m} \cdot m'(\lambda_t)$ is positive. It reflects the fact that by promoting religious infrastructures that enter as complementary inputs in the socialization process of the religious trait, an increase in the clerics weight λ_t makes the religious trait to be relatively more successfully transmitted than the secular trait.

From this discussion it follows that when religious parents' socialization efforts are more sensitive to paternalistic motives than secular parents (ie. $\epsilon_{Re}(q,\lambda_t) > \epsilon_S(q,\lambda_t)$) , and (or) when religious infrastructures are strong enough complementary inputs to socialization to the religious trait, then the numerator of (A.88) is positive and $q^*(\lambda_t)$ is increasing in λ_t.

As can be seen from (A.43) and (A.44), a change in the other parameters θ (the efficiency of the clerics) and ϕ (the restrictiveness of religious proscriptions) affects the relative cultural fitness of the religious trait only through their induced changes on the paternalistic motive $\Delta V(\lambda_t)$, with $\Delta V(\lambda)$ increasing in θ,and decreasing ϕ. It follows that

$$\frac{\partial q^*(\lambda_t)}{\partial \theta} = \frac{K(\lambda_t) \cdot \frac{\partial \Delta V(\lambda_t)}{\partial \theta}}{-\frac{\partial D}{\partial q}(q^*(\lambda_t), \lambda_t)} \text{ and } \frac{\partial q^*(\lambda_t)}{\partial \theta} = \frac{K(\lambda_t) \cdot \frac{\partial \Delta V(\lambda_t)}{\partial \phi}}{-\frac{\partial D}{\partial q}(q^*(\lambda_t), \lambda_t)}$$

When religious parents are more sensitive to paternalistic motives than secular parents, one has $K(\lambda_t) > 0$ and a positive shift in θ (negative shift of ϕ) leads to a higher value of $q^*(\lambda_t)$. This provides the comparative statics discussion on $q^*(\lambda_t)$ in the main text. **QED**.

- **Example with constant elasticity socialization cost functions:**

Consider the following socialization cost functions:

$$\begin{cases} h_{Re}(d) = \frac{d^{1+\eta_{re}}}{1+\eta_{re}} \cdot \frac{1}{m^{\gamma}} \text{ and} \\ h_s(d) = \frac{d^{1+\eta_s}}{1+\eta_s}, \end{cases} \tag{A.46}$$

with $\eta_s \geq \eta_{re} > 0$ and $\gamma > 0$**.** The optimal socialization efforts are such that:

$$\begin{cases} d^*_{Re}(q_t, \lambda_t) = ((1-q_t)\Delta V(\lambda_t))^{\frac{1}{\eta_{re}}} \cdot m\,(\lambda_t)^{\frac{\gamma}{\eta_{re}}} \\ d^*_S(q_t, \lambda_t) = (q_t \Delta V(\lambda_t))^{\frac{1}{\eta_s}}. \end{cases} \tag{A.47}$$

and in this constant elasticity specification $\epsilon_{Re}(q, \lambda) - \epsilon_S(q, \lambda) = \frac{1}{\eta_{re}} - \frac{1}{\eta_s} \geq 0$. Rewriting the cultural dynamics equation (11), we deduce that:

$$q_{t+1} - q_t = q_t(1-q_t)\{((1-q_t)\Delta V(\lambda_t))^{\frac{1}{\eta_{re}}} \cdot m\,(\lambda_t)^{\frac{\gamma}{\eta_{re}}} - (q_t \Delta V(\lambda_t))^{\frac{1}{\eta_s}}\}, \tag{A.48}$$

which admits two unstable steady states $q = 0$ and $q = 1$, and a unique interior attractor, which we denote $q^*(\lambda_t)$ such that:

$$\frac{q^*(\lambda_t)^{\frac{1}{\eta_s}}}{(1-q^*(\lambda_t))^{\frac{1}{\eta_{re}}}} = \Delta V(\lambda_t)^{\frac{\eta_S - \eta_{re}}{\eta_S \eta_{re}}} \cdot m\,(\lambda_t)^{\frac{\gamma}{\eta_{re}}} \tag{A.49}$$

given that $\eta_S \geq \eta_{re}$, we deduce that $q^*(\lambda_t)$ is increasing in θ, λ_t, and decreasing in ϕ.

A.6 Existence and Stability Analysis of interior steady states

Let Γ, The set of interior steady states of the joint dynamics of culture and institutions:

$$\Gamma = \left\{(\lambda, q) \in (0,1)^2 \mid q = \overline{q}\,(\lambda) \text{ and } q = q^*(\lambda)\right\}$$

namely the set of interior intersection points of the institutional and cultural manifolds $q = \overline{q}(\lambda)$ and $q = q^*(\lambda)$.

- *When $\overline{q}(1) < q^*(1)$, the set Γ is not empty.*

Proof: First note that $q^*(0) = 0$. Indeed the thresholds $q^*(\lambda)$ is the solution of (A.48):

$$D(q, \lambda) = d^*_{Re}(q, \lambda) - d^*_S(q, \lambda) = 0$$

Now given that religious infrastructures m are an essential input in the socialization of religious individuals and that $m(0) = 0$,

$$d^*_{Re}(q, 0) = D_{Re}[(1-q)\Delta V_{Re}(0), m(0)] = D_{Re}[(1-q)\Delta V_{Re}(0), 0] = 0$$

while $d^*_S(0, \lambda) = D_S[0] = 0$, Thus $D(0,0) = 0$ and therefore $q^*(0) = 0$.

The thresholds $\overline{q}(\lambda)$ is characterized by (A.36):

$$\overline{q}(\lambda) = \frac{\phi(1-\tau)}{\tau\theta} \frac{\tau + (1-\lambda)\frac{1-\tau}{2}}{\tau + (1-\lambda)\left[1 - \tau + \tau\theta\alpha_c(\lambda)\left(1 + \frac{\phi}{2}\alpha_c(\lambda)\right)\right]}$$

Hence $\overline{q}(0) = \frac{\phi\left(1-\tau^2\right)}{2\tau\theta} \in (0,1)$ under assumption 1. As well differentiation of $\overline{q}(\lambda)$ provides

$$\begin{aligned} \overline{q}'(0) &= \frac{\phi(1-\tau)}{\tau\theta}\left\{-\frac{1-\tau}{2} - \left(\frac{1+\tau}{2}\right)\cdot\left(-[1-\tau] + \tau\theta\cdot\alpha'_c(0)\right)\right\} \\ &= \frac{\phi(1-\tau)\tau}{2\tau\theta}\left\{(1-\tau) - (1+\tau)\theta\cdot\alpha'_c(0)\right\} \end{aligned}$$

the function $\Lambda(\lambda) = q^*(\lambda) - \overline{q}(\lambda)$ is continuous and such that $\Lambda(0) = -\overline{q}(0) < 0$, and $\Lambda(1) = q^*(1) - \overline{q}(1) > 0$. Thus there is a $\lambda^* \in (0,1)$ such that $\Lambda(\lambda^*) = 0$ and given that $q^* = q^*(\lambda^*) < 1$, the point $(\lambda^*, q^*) \in \Gamma$ and the set Γ is non empty.**QED**

- *Condition for $\overline{q}(1) < q^*(1)$*

Note that $\overline{q}(1) = \frac{\phi(1-\tau)}{\tau\theta} > \frac{\phi\left(1-\tau^2\right)}{2\tau\theta} = \overline{q}(0)$. Moreover the condition $\overline{q}(1) < q^*(1)$ is equivalent to $D(\overline{q}(1), 1) > 0$, or

$$D_{Re}[(1 - \overline{q}(1))\Delta V(1), m(1)] > D_S[\overline{q}(1)\Delta V(1)]$$

with $\Delta V(1) = \frac{(\tau\theta\alpha_c(1))^2}{2(1+\phi\alpha_c(1))}$. We know that when religious parents are more sensitive to paternalistic motives than secular parents, $q^*(\lambda_t)$ is increasing in θ and decreasing in ϕ. This hold in particular for $q^*(1)$. Given that $\overline{q}\,(1)$ is a decreasing function of θ and an increasing function of ϕ, it follows that the condition $\overline{q}(1) < q^*(1)$ is more likely to be satisfied when θ is large enough and ϕ small enough. In the parametrization with constant elasticity socialization cost functions, the condition for $\overline{q}(1) < q^*(1)$ writes as:

$$\frac{\overline{q}(1)^{\frac{1}{\eta_s}}}{(1-\overline{q}(1))^{\frac{1}{\eta_{re}}}} < \left[\frac{(\tau\theta\alpha_c(1))^2}{2(1+\phi\alpha_c(1))}\right]^{\frac{\eta_S-\eta_{re}}{\eta_S\eta_{re}}} \cdot m\,(1)^{\frac{\gamma}{\eta_{re}}}$$

which will hold when $m\,(1)$ is large enough.

- *Saddle node steady state in the joint dynamics of culture and institutions:*

- Let denote the interior steady state $(\lambda_E^*, q_E^*) \in \Gamma$ such that $\lambda_E^* = \min\{\lambda \in (0,1) \mid \overline{q}\,(\lambda) = q^*(\lambda)\}$ and $q_E^* = \overline{q}\,(\lambda_E) = q^*(\lambda_E)$. (λ_E^*, q_E^*) is the "lowest" interior steady state of the system. It is clear that because of the smoothness of the function $\Lambda(\lambda) = q^*(\lambda) - \overline{q}\,(\lambda)$, one should have $\Lambda'(\lambda_E^*) > 0$ or $q^{*\prime}(\lambda_E^*) > \overline{q}'\,(\lambda_E^*)$.

Consider now the local dynamics around the interior steady state (λ_E^*, q_E^*). Inside the interior of $[0,1]^2$, the joint dynamics of institutions and culture write as a discrete time dynamic system:

$$\begin{aligned}\lambda_{t+1} &= m^{-1}\left[\tilde{m}(\lambda_t, q_t)\right] \\ q_{t+1} &= q_t + q_t(1-q_t)D(q_t, \lambda_t).\end{aligned} \tag{A.50}$$

Note first that the threshold $q = \overline{q}\,(\lambda)$ is obtained from the relationship $\tilde{m}(\lambda, q) = m(\lambda)$, while the threshold $q^*(\lambda)$ is obtained from the relationship $D(q,\lambda) = 0$. From this we obtain that the slopes of the manifolds:

$$\overline{q}'\,(\lambda_E^*) = \frac{m'(\lambda_E^*) - \tilde{m}'_\lambda(\lambda_E^*, q_E^*)}{\tilde{m}'_q(\lambda_E^*, q_E^*)} \text{ and } q^{*\prime}(\lambda_E) = \frac{D_\lambda(q_E^*, \lambda_E^*)}{-D_q(q_E^*, \lambda_E^*)}$$

To analyze the stability properties of the steady state (λ_E^*, q_E^*), we can consider the jacobian matrix of the system (A.50) at the steady state (λ_E^*, q_E^*) which is given by

$$J = \begin{pmatrix} \frac{\tilde{m}_\lambda(\lambda_E^*, q_E^*)}{m'(\lambda_E^*)} & \frac{\tilde{m}_q(\lambda_E^*, q_E^*)}{m'(\lambda_E^*)} \\ q_E^*(1-q_E^*)D_\lambda(q_E^*, \lambda_E^*) & 1 + q_E^*(1-q_E^*)D_q(q_E^*, \lambda_E^*) \end{pmatrix}$$

The two eigenvalues $\mu_{1,2}$ of such matrix J are the roots of the characteristic polynomial

$$\mu^2 - (TrJ) \cdot \mu + \det J = 0$$

First, note that the eigenvalues $\mu_{1,2}$ are real and distinct. Indeed simple computations give:

$$\begin{aligned}(TrJ)^2 - 4\det J &= \left[\frac{\tilde{m}_\lambda}{m'_\lambda} + 1 + q^*_E(1-q^*_E)D_q\right]^2 - 4\left[\frac{\tilde{m}_\lambda}{m'_\lambda}(1 + q^*_E(1-q^*_E)D_q) - q^*_E(1-q^*_E)D_\lambda \frac{\tilde{m}_q}{m'_\lambda}\right] \\ &= \left[\frac{\tilde{m}_\lambda}{m'_\lambda} - 1 - q^*_E(1-q^*_E)D_q\right]^2 + 4q^*_E(1-q^*_E)D_\lambda \frac{\tilde{m}_q}{m'_\lambda} > 0\end{aligned}$$

as D_λ, $\tilde{m}_q$ and m'_λ are positive at λ^*_E, q^*_E. This implies that the eigenvalues $\mu_{1,2}$ are real and distinct (Galor (2007)).

For historical meaningful dynamic non oscillatory trajectories, the eigenvalues $\mu_{1,2}$ have to be positive. This will be the case if and only if $\det J > 0$; which is equivalent to the following condition:

$$\tilde{m}_\lambda(1 + q^*_E(1-q^*_E)D_q) - q^*_E(1-q^*_E)D_\lambda \tilde{m}_q > 0.$$

Sufficient conditions for this to be satisfied are $|D_q| < 4$ and $\tilde{m}_\lambda/\tilde{m}_q > D_\lambda/(4 - |D_q|)$. This will be satisfied when cultural substituability in the cultural transmission process is not too strong (ie. $0 > D_q > -4$) and that the relative sensitivity of the optimal social commitment policy $\tilde{m}$ to institutions λ compared to culture q is larger than the relative sensitivity of cultural transmission process to these same variables.

Assuming such conditions to be satisfied, (λ^*_E, q^*_E) is then a saddle if and only if the eigenvalues satisfy $0 < \mu_1 < 1 < \mu_2$. The condition for this is given by (Galor (2007)):

$$\det J < TrJ - 1$$

This rewrites as

$$\frac{\tilde{m}'_\lambda}{m'_\lambda}(1 + q^*_E(1-q^*_E)D_q) - q^*_E(1-q^*_E)D_\lambda \frac{\tilde{m}'_q}{m'_\lambda} < \frac{\tilde{m}'_\lambda}{m'_\lambda} + q^*_E(1-q^*_E)D_q$$

or after simplification

$$\tilde{m}'_\lambda D_q - D_\lambda \tilde{m}'_q < m'_\lambda D_q \tag{A.51}$$

Now at the steady state λ_E^*, the condition $q^{*\prime}(\lambda_E^*) > \overline{q}'(\lambda_E^*)$ rewrites as

$$\frac{D_\lambda}{-D_q} > \frac{m'_\lambda - \tilde{m}'_\lambda}{\tilde{m}'_q}$$

or (given that $D_q < 0$ and $\tilde{m}'_q > 0$, as $\tilde{m}(\lambda, q)$ is increasing in q), one gets

$$\tilde{m}'_q D_\lambda > [\tilde{m}_\lambda - m'_\lambda]\, D_q$$

which after rearrangement is the same as condition (A.51). This means that the jacobian J at the steady state (λ_E^*, q_E^*) satisfies condition $\det J < TrJ - 1$, and consequently that (λ_E^*, q_E^*) is a saddle of the joint dynamics of culture and institutions.

- *Many steady states and stability*

Assume that $\overline{q}(1) < q^*(1)$ and therefore $\Lambda(1) > 0$. When the set Γ includes more than one point (say N), one may order the various steady states by increasing order of their institutional values λ_i^* for $i \in [1, N]$. Moreover λ_i^* for $i \in [1, N]$ are the zeros of the smooth function $\Lambda(\lambda)$ in $[0, 1]$ with $\Lambda(0) < 0 < \Lambda(1)$. Therefore N is necessarily odd and $N = 2K + 1$. Recall that in such a case the steady state associated to $\lambda_1^* = \lambda_E^*$ is a saddle and $\Lambda'(\lambda_{2k+1}^*) > 0$ for $k \in [0, K]$ odd and $\Lambda'(\lambda_{2k}^*) < 0$ for $k \in [1, K]$.

We assume as before that sufficient conditions for non oscillatory trajectories are satisfied (ie. $|D_q| < 4$ and $\tilde{m}_\lambda / \tilde{m}_q > D_\lambda / (4 - |D_q|)$. Then we have:

- For $k \in [1, K]$, the steady states $\left(\lambda_{2k+1}^*, q_{2k+1}^*\right)$ are saddle, and the steady state $(\lambda_{2k}^*, q_{2k}^*)$ are locally stable.

Proof: The jacobian matrix at a steady state λ_i^*, q_i^* $i \in [1, N]$ is

$$J_i = \begin{pmatrix} \frac{\tilde{m}_\lambda(\lambda_i^*, q_i^*)}{m'(\lambda_i^*)} & \frac{\tilde{m}_q(\lambda_i^*, q_i^*)}{m'(\lambda_i^*)} \\ q_i^*(1 - q_i^*) D_\lambda(q_i^*, \lambda_i^*) & 1 + q_i^*(1 - q_i^*) D_q(q_i^*, \lambda_i^*) \end{pmatrix}$$

As before the condition $(TrJ_i)^2 - 4 \det J_i > 0$ is satisfied and consequently the eigenvalues $\mu_{1,2}^i$ are real and distinct. As well the conditions for non oscillatory dynamics ensure that $\det J_j > 0$ is satisfied, which imply that both eigenvalues $\mu_{1,2}^i$ are positive. (λ_i^*, q_i^*) is then a saddle steady state (resp. a stable steady state) when the eigenvalues satisfy

$0 < \mu_1^i < 1 < \mu_2^i$ (resp. $0 < \mu_1^i < \mu_2^i < 1$). Thus (λ_i^*, q_i^*) is a saddle when $\det J_i < TrJ_i - 1$ and a stable steady state when $\det J_i > TrJ_i - 1$ (Galor 2007)

Now given that $m'_\lambda(\lambda_i^*) > 0$, the sign of $\det J_i - (TrJ_i - 1)$ is the same as the sign of

$$\overline{\Delta}_i = [\tilde{m}_\lambda(\lambda_i^*, q_i^*) - m'(\lambda_i^*)]\, D_q(q_i^*, \lambda_i^*) - \tilde{m}_q(\lambda_i^*, q_i^*) D_\lambda(q_i^*, \lambda_i^*)$$

Recalling the fact that at an interior steady state λ_i^* :

$$\Lambda'(\lambda_i^*) = q^{*\prime}(\lambda_i^*) - \overline{q}'\,(\lambda_i^*) = \frac{D_\lambda(q_i^*, \lambda_i^*)}{-D_q(q_i^*, \lambda_i^*)} - \frac{m'(\lambda_i^*) - \tilde{m}'_\lambda(\lambda_i^*, q_i^*)}{\tilde{m}'_q(\lambda_i^*, q_i^*)} = \frac{-\overline{\Delta}_i}{-D_q(q_i^*, \lambda_i^*)\tilde{m}'_q(\lambda_i^*, q_i^*)}$$

and because $\tilde{m}'_q > 0$, and $D_q < 0$, it follows that the sign of $\overline{\Delta}_i$ at a steady state λ_i^* is the opposite to the sign of $\Lambda'(\lambda_i^*)$.

From this we conclude that:

i) for $k \in [1, K]$ as $\Lambda'(\lambda_{2k+1}^*) = q^{*\prime}(\lambda_{2k+1}^*) - \overline{q}'\left(\lambda_{2k+1}^*\right) > 0$, the sign of $\overline{\Delta}_{2k+1}$ is negative, thus $\det J_{2k+1} < TrJ_{2k+1} - 1$ and the steady state $\left(\lambda_{2k+1}^*, q_{2k+1}^*\right)$ is a saddle node.

ii) For $k \in [1, K]$ as $\Lambda'(\lambda_{2k}^*) = q^{*\prime}(\lambda_{2k}^*) - \overline{q}'\,(\lambda_{2k}^*) < 0$, the sign of $\overline{\Delta}_{2k}$ is positive. Thus $\det J_{2k} > TrJ_{2k} - 1$the steady state $\left(\lambda_{2k+1}^*, q_{2k+1}^*\right)$ is a local stable steady state. **QED.**

A.7 Proof of Proposition 3

The likelihood of reaching the religious equilibrium is increasing in θ: From Proposition 1, $\overline{q}(.)$ is decreasing in θ. From Proposition 2, $q^*(.)$ is increasing in θ. Hence, the measure of parameters for which there is a complementarity between the spread of religious values and an increase in the political weight of the clerics is larger. This explains why the likelihood of reaching the religious equilibrium increases.

The likelihood of reaching the religious equilibrium is decreasing in ϕ: From Proposition 1, $\overline{q}(.)$ is increasing in ϕ. From Proposition 2, $q^*(.)$ is decreasing in ϕ. Hence, the measure of parameters for which there is a complementarity between the spread of religious values and an increase in the political weight of the clerics is lower.

A.8 Proof of Lemmas 4 and 5:

In order to prove the two Lemmas, we first derive the tax base E. Since an individual of type $i \in \{re, s\}$ complies only when

$$\frac{1-\tau_i}{1+\phi\alpha_c} > \frac{1-c\epsilon}{1+\phi\alpha_c}, \tag{A.52}$$

with $\epsilon = \frac{\epsilon_0}{1-\alpha_l}$, the fraction of individuals of type i that comply is:

$$\int_{\tau/\epsilon}^{\overline{c}} \frac{dc}{\overline{c}} = 1 - \frac{\tau_i(1-\alpha_l)}{\epsilon_0\overline{c}}. \tag{A.53}$$

Summing the taxes that are collected in the two cultural groups, we find that the tax base is:

$$E = \frac{1}{1+\phi\alpha_c}\{1 - \frac{\tau(1-q\theta\alpha_c)(1-\alpha_l)}{\epsilon_0\overline{c}}\}. \tag{A.54}$$

We are now able to solve the equilibrium. As a matter of simplification, we assume throughout the extension that $\psi(\alpha_c)$ is quadratic with $\psi(\alpha_c) = \alpha_c^2/2$.

The first-order conditions associated with the determination of $m(\lambda)$, $\alpha_l(\lambda, \beta, q)$, and $\alpha_c(\lambda)$ are respectively:

$$\begin{cases} -(1-\frac{\lambda}{2})C'(m) + \frac{\lambda}{2}\left(\alpha_c - F'(m)\right) = 0, \\ -\alpha_l + (1-\beta)\tau\frac{\partial E}{\partial \alpha_l} \leq 0, \text{ and} \\ m - \alpha_c = 0. \end{cases} \tag{A.55}$$

The equilibrium is unique, when the marginal cost functions $F'(.)$ and $C'(.)$ are strictly increasing convex functions and $\lim_{m\to\infty} F''(m) > 1 > F''(0) + C''(0)$. Typically $m(\lambda) = \alpha_c(\lambda) = 0$ when $\lambda \leq 2\frac{C''(0)}{C''(0)+1-F''(0)}$, and $m(\lambda) = \alpha_c(\lambda) > 0$ is the positive solution of

$$(1-\frac{\lambda}{2})C'(m) + \frac{\lambda}{2}F'(m) = \frac{\lambda}{2}m, \tag{A.56}$$

when $\lambda > 2\frac{C''(0)}{C''(0)+1-F''(0)}$. From this, we deduce that $m(\lambda)$ and $\alpha_c(\lambda)$ are increasing in λ, when $F'(m) < C'(m)$ and is independent from β, θ, and ϕ. This concludes the proof of Lemma 4.

Substituting (A.54) in the the second FOC above, we find

$$\alpha_l(\lambda,\beta,q) = \begin{cases} (1-\beta)\frac{\tau^2(1-q\theta\alpha_c(\lambda))}{(1+\phi\alpha_c(\lambda))\bar{c}\epsilon_0} \text{ if } (1-\beta)\frac{\tau^2(1-q\theta\alpha_c(\lambda))}{(1+\phi\alpha_c(\lambda))\bar{c}\epsilon_0} < \overline{\alpha}_l \text{ and} \\ \overline{\alpha}_l \text{ otherwise.} \end{cases} \tag{A.57}$$

We deduce that $\alpha_l(\lambda,\beta,q)$ is decreasing in β, λ q, θ and ϕ. This concludes the proof of Lemma 5.

A.9 Proof of Proposition 4

As in the related proof of Proposition 1, we first demonstrate that the optimization, problem (15) – rewritten below – admits a unique solution $(\lambda_{t+1},\beta_{t+1}) \in [0,1]^2$:

$$\max_{(\lambda_{t+1},\beta_{t+1})}(1-\frac{\lambda_t}{2})\{U_r(m(\lambda_{t+1}),\alpha_l(\lambda_{t+1},\beta_{t+1},q_t)) + U_l(m(\lambda_{t+1}),\alpha_l(\lambda_{t+1},\beta_{t+1},q_t))\}$$
$$+\frac{\lambda_t}{2}U_c(m(\lambda_{t+1}),\alpha_c(\lambda_{t+1})),$$

In order to solve this maximization problem, we solve the following related optimization problem:

$$\max_{m,\alpha_l} W(m,\alpha_l,\lambda_t) = (1-\frac{\lambda_t}{2})\{U_r(m,\alpha_l)+U_l(m,\alpha_l)\} + \frac{\lambda_t}{2}U_c(m). \tag{A.58}$$

The solution, denoted $(\tilde{m}(\lambda_t,q_t),\tilde{\alpha}_l(\lambda_t,q_t))$,[50] maximizes the social welfare when the externalities are internalized, with

$$\begin{aligned} U_c(m) &= m\alpha_c(m) - \psi(\alpha_c(m)) - F(m) = \frac{1}{2}m^2 - F(m) \\ U_r(m,\alpha_l) &= \beta_t(\tau E(m,\alpha_l,q_t) - C(m)) - \rho\alpha_l \\ U_l(m,\alpha_l) &= (1-\beta_t)(\tau E(m,\alpha_l,q_t) - C(m)) - \frac{\alpha_l^2}{2} \end{aligned}$$

and

$$E(m,\alpha_l,q_t) = \frac{1}{1+\phi m}\{1 - \frac{\tau(1-q_t\theta m)(1-\alpha_l)}{\epsilon_0\bar{c}}\}. \tag{A.59}$$

[50] making now explicit the dependence on the state variables (λ_t,q_t).

The previous optimization problem can be rewritten:

$$\max_{m,\alpha_l} W(m,\alpha_l,\lambda_t) = (1-\frac{\lambda_t}{2})\{\tau E(m,\alpha_l,q_t)-C(m)-\rho\alpha_l-\frac{\alpha_l^2}{2}\}+\frac{\lambda_t}{2}\{\frac{1}{2}m^2-F(m)\}, \quad (A.60)$$

In this optimization problem, the choices of both the religious provision m and of the effort of the secular elite α_l are made by a ruler who can commit, and hence that internalizes the externalities detailed in the main text. We find that the solution $(\tilde{m}(\lambda_t,q_t),\tilde{\alpha}_l(\lambda_t,q_t))$ of (A.58) solves the following equations:

$$\begin{cases} \frac{\partial W}{\partial m} = \frac{\lambda_t}{2}(m-F'(m)) - (1-\frac{\lambda_t}{2})C'(m) + (1-\frac{\lambda_t}{2})\{\frac{-\phi}{(1+m\phi)^2}[1-\frac{\tau(1-q\theta m)(1-\alpha_l)}{\epsilon_0\bar{c}}] \\ \qquad +\frac{1}{1+m\phi}\frac{\tau q\theta(1-\alpha_l)}{\epsilon_0\bar{c}}\} = 0, \\ \text{and} \\ \frac{\partial W}{\partial \alpha_l} = -\alpha_l - \rho + \frac{\tau^2(1-q_t\theta m)}{\bar{c}\epsilon_0(1+\phi m)} = 0. \end{cases} \quad (A.61)$$

We deduce the following lemma which characterizes the solution $(\tilde{m}(\lambda_t,q_t),\tilde{\alpha}_l(\lambda_t,q_t))$ of (A.58)

Lemma 8 *the solution $(\tilde{\alpha}_l(\lambda_t,q_t),\tilde{m}(\lambda_t,q_t))$ is uniquely determined when $C(.)$, and $F(m)$ are sufficiently convex (ie $W(m,\alpha_l,\lambda_t)$ is concave in m,α_l).*

Proof: Specifically, it is a simple matter to see that

$$\begin{aligned} \frac{\partial^2 W}{\partial m^2} &= \frac{\lambda_t}{2}(1-F''(m)) - (1-\frac{\lambda_t}{2})C''(m) \\ &\quad +(1-\frac{\lambda_t}{2})\frac{2\phi}{(1+m\phi)^2}\begin{bmatrix} -[\frac{\tau q\theta(1-\alpha_l)}{\epsilon_0\bar{c}}] \\ +\frac{\phi}{(1+m\phi)}[1-\frac{\tau(1-q\theta m)(1-\alpha_l)}{\epsilon_0\bar{c}}] \end{bmatrix} \\ &< \frac{\lambda_t}{2}(1-F''(m)) - (1-\frac{\lambda_t}{2})C''(m) + (1-\frac{\lambda_t}{2})\frac{2\phi^2}{(1+m\phi)^3} < 0 \end{aligned}$$

when $F''(m) > 1$ and $C''(m) > 2\phi^2$, while:

$$\frac{\partial^2 W}{\partial \alpha_l^2} = -1 < 0 \text{ and } \frac{\partial^2 W}{\partial m \partial \alpha_l} = -\frac{\tau^2}{\bar{c}\epsilon_0}\frac{q_t\theta+\phi}{(\phi m+1)^2} < 0 \quad (A.62)$$

Therefore the Hessian of $W(m,\alpha_l,\lambda_t)$ is given by:

$$
\begin{aligned}
\Delta &= \frac{\partial^2 W}{\partial m^2}\cdot\frac{\partial^2 W}{\partial \alpha_l^2} - \left(\frac{\partial^2 W}{\partial m \partial \alpha_l}\right)^2 \\
&= \left[\frac{\lambda_t}{2}(F''(m)-1) + (1-\frac{\lambda_t}{2})\left[\begin{array}{c} C''(m) + \frac{2\phi}{(1+m\phi)^2}\{[\frac{\tau q\theta(1-\alpha_l)}{\epsilon_0 \bar{c}}] \\ -\frac{\phi}{(1+m\phi)}[1-\frac{\tau(1-q\theta m)(1-\alpha_l)}{\epsilon_0\bar{c}}]\} \end{array}\right]\right] \\
&\quad -\frac{\tau^4}{(\bar{c}\epsilon_0)^2}\frac{(q_t\theta+\phi)^2}{(\phi m+1)^4} \\
&> \left[\frac{\lambda_t}{2}(F''(m)-1) + (1-\frac{\lambda_t}{2})\left[C''(m) - \frac{2\phi^2}{(1+m\phi)^3}\right]\right] \\
&\quad -\frac{\tau^4}{(\bar{c}\epsilon_0)^2}\frac{(q_t\theta+\phi)^2}{(\phi m+1)^4}
\end{aligned}
$$

and $\Delta > 0$ when $F''(m) > 1 + \frac{(\theta+\phi)^2}{(\bar{c}\epsilon_0)^2}$ and $C''(m) > 2\phi^2 + \frac{(\theta+\phi)^2}{(\bar{c}\epsilon_0)^2}$. Therefore $W(m, \alpha_l, \lambda_t)$ is concave in m, α_l when $C(.)$, and $F(m)$ are sufficiently convex. (ie. when $F''(m) > 1+\frac{(\theta+\phi)^2}{(\bar{c}\epsilon_0)^2}$ and $C''(m) > 2\phi^2 + \frac{(\theta+\phi)^2}{(\bar{c}\epsilon_0)^2}$) **QED**.

Now consider $(\widetilde{m}^0(q_t), \widetilde{\alpha}_l^0(q_t)) = \arg\max_{m,\alpha_l} W(m, \alpha_l, 0)$ and $\widetilde{m}^1 = \arg\max_{m,\alpha_l} W(m,\alpha_l,1)$. $\widetilde{m}^0$ respectively the optimal level of religious infrastructure of (A.58)when the secular elite (and the ruler) have full political power (ie. $\lambda = 0$), and when the society is in a religious state (the religious clerics weight is $\lambda = 1$). It is reasonable to make the following assumption:[51]

$$\text{Assumption } \mathbf{M}\text{:}\ \ \widetilde{m}^0(q_t) < \widetilde{m}^1 \ \text{ for all }\ q_t \in [0,1]$$

namely that the clerics group always wish to have a higher level of religious infrastructures than the secular fraction of society (ruler and secular elite). We have then the following result:

Lemma 9 *Under assumption M, $\tilde{m}(\lambda_t, q_t)$ is increasing in λ_t and q_t.and $\widetilde{\alpha}_l(\lambda_t, q_t)$ is decreasing in λ_t and q_t.*

[51] A sufficient condition for assumption M to be satisfied is :

$$\frac{\tau\theta}{\epsilon_0\bar{c}} < C'(\widetilde{m}^1)$$

where $\widetilde{m}^1$ is determined by the condition $\widetilde{m}^1 = \Phi'(\widetilde{m}^1)$.

Proof: Partial differentiation yields:

$$\frac{\partial W}{\partial m\partial\lambda} = \frac{m-F'(m)}{2}+\frac{C'(m)}{2} \tag{A.63}$$

$$-\frac{1}{2}\left[\begin{array}{c}\frac{-\phi}{(1+m\phi)^2}[1-\frac{\tau(1-q\theta m)(1-\alpha_l)}{\epsilon_0\overline{c}}] \\ +\frac{1}{1+m\phi}\frac{\tau q\theta(1-\alpha_l)}{\epsilon_0\overline{c}}\end{array}\right] \tag{A.64}$$

$$\frac{\partial W}{\partial m\partial q} = (1-\frac{\lambda_t}{2})\frac{1}{1+m\phi}\frac{\tau\theta(1-\alpha_l)}{\epsilon_0\overline{c}}\{\frac{1}{(1+m\phi)}\}>0$$

and

$$\frac{\partial^2 W}{\partial\alpha_l\partial\lambda}=0 \text{ and } \frac{\partial^2 W}{\partial\alpha_l\partial q}=-\frac{\tau^2\theta m}{\overline{c}\epsilon_0(1+\phi m)}<0 \tag{A.65}$$

Substitution of the FOC (A.61) into (A.63),one obtains when evaluated at the optimal point $\tilde{m},\tilde{\alpha}_l$:

$$\left(\frac{\partial W}{\partial m\partial\lambda}\right)=\frac{1}{(1-\frac{\lambda_t}{2})}(\tilde{m}-F'(\tilde{m})) \tag{A.66}$$

which is positive as long as $\tilde{m}\left(\lambda_t,q_t\right)\leq\widetilde{m}^1$. Moreover differentiation of the FOC in (A.61), provides

$$\begin{aligned}\begin{pmatrix} d\tilde{m} \\ d\tilde{\alpha}_l\end{pmatrix} &= \frac{1}{\Delta}\begin{pmatrix}\frac{\partial^2 W}{\partial\alpha_l^2} & -\frac{\partial^2 W}{\partial m\partial\alpha_l} \\ -\frac{\partial^2 W}{\partial m\partial\alpha_l} & \frac{\partial^2 W}{\partial m^2}\end{pmatrix}\begin{pmatrix}-\frac{\partial^2 W}{\partial m\partial\lambda}d\lambda_t-\frac{\partial^2 W}{\partial m\partial q}dq_t \\ -\frac{\partial^2 W}{\partial\alpha_l\partial\lambda}d\lambda_t-\frac{\partial^2 W}{\partial\alpha_l\partial q}dq_t\end{pmatrix} \\ &= \frac{1}{\Delta}\begin{pmatrix}-\frac{\partial^2 W}{\partial\alpha_l^2}\frac{\partial^2 W}{\partial m\partial\lambda}d\lambda_t+\left(-\frac{\partial^2 W}{\partial\alpha_l^2}\frac{\partial^2 W}{\partial m\partial q}+\frac{\partial^2 W}{\partial m\partial\alpha_l}\frac{\partial^2 W}{\partial\alpha_l\partial q}\right)dq_t \\ \frac{\partial^2 W}{\partial m\partial\alpha_l}\frac{\partial^2 W}{\partial m\partial\lambda}d\lambda_t+\left(-\frac{\partial^2 W}{\partial m^2}\frac{\partial^2 W}{\partial\alpha_l\partial q}+\frac{\partial^2 W}{\partial m\partial\alpha_l}\frac{\partial^2 W}{\partial m\partial q}\right)dq_t\end{pmatrix}\end{aligned}$$

with all derivatives evaluated at $\tilde{m},\tilde{\alpha}_l$. Hence using (A.62), (A.63) and (A.65), one gets

$$\frac{\partial\tilde{m}}{\partial\lambda_t}=\frac{1}{\Delta}\cdot\frac{\partial^2 W}{\partial m\partial\lambda}$$

the sign of which is the same as the sign of $\frac{\partial^2 W}{\partial m\partial\lambda}$. Now under assumption M, one can see from (A.66) that $\tilde{m}\left(\lambda_t,q_t\right)$ is increasing in λ_t as long as $\tilde{m}\left(\lambda_t,q_t\right)<\widetilde{m}^1$. Note first that $\tilde{m}\left(1,q_t\right)=\widetilde{m}^1$. Suppose then that there exists a value $\lambda<1$ such that $\tilde{m}\left(\lambda,q_t\right)=\widetilde{m}^1$. From (A.61), and noting that

$$W\left(m,\alpha_l,\lambda\right)=\lambda W\left(m,\alpha_l,1\right)+(1-\lambda)W\left(m,\alpha_l,0\right)$$

at this point $\tilde{m}(\lambda, q_t)$, $\widetilde{\alpha}_l(\lambda, q_t)$, one should have

$$\left(\frac{\partial W}{\partial m}\right)_{\tilde{m}^1, \tilde{\alpha}_l^1} = \lambda \frac{\partial W(m, \alpha_l, 1)}{\partial m} + (1-\lambda) \frac{\partial W(m, \alpha_l, 0)}{\partial m} = 0$$

But $\tilde{m}(\lambda, q_t) = \widetilde{m}^1 = \arg\max_{m,\alpha_l} W(m, \alpha_l, 1)$, implies that $\left(\frac{\partial W(m,\alpha_l,1)}{\partial m}\right) = 0$ at such point. Hence to satisfy the previous equation, we should also have $\frac{\partial W(m,\alpha_l,0)}{\partial m} = 0$, which in turn implies that $\tilde{m}(\lambda, q_t) = \widetilde{m}^0(q_t)$, a contradiction with assumption M. From this we conclude that $\tilde{m}(\lambda, q_t) < \widetilde{m}^1$ for all $\lambda < 1$ or $\tilde{m}(\lambda, q_t) > \widetilde{m}^1$ for all $\lambda < 1$. The only case consistent with assumption M is obviously that $\tilde{m}(\lambda, q_t) < \widetilde{m}^1$ for all $\lambda < 1$. From this we conclude that under assumption M, $\frac{\partial^2 W}{\partial m \partial \lambda}$ evaluated at $\tilde{m}(\lambda, q_t)$, $\widetilde{\alpha}_l(\lambda, q_t)$ is positive and therefore $\frac{\partial \tilde{m}}{\partial \lambda_t} > 0$ (ie. religious infrastructures $\tilde{m}(\lambda_t, q_t)$ is increasing in the clerics' political weight λ_t).

Similarly, using (A.62), (A.63) and (A.65), we have:

$$\frac{\partial \tilde{\alpha}_l}{\partial \lambda_t} = \frac{1}{\Delta} \cdot \underset{-}{\underleftrightarrow{\frac{\partial^2 W}{\partial m \partial \alpha_l}}} \frac{\partial^2 W}{\partial m \partial \lambda}$$

Hence $\frac{\partial \tilde{\alpha}_l}{\partial \lambda_t} < 0$ under assumption M (ie. the tax enforcement effort of the secular elite $\widetilde{\alpha}_l(\lambda_t, q_t)$ is decreasing in the clerics' weight λ_t).

Finally, substituting (A.62), (A.63) and (A.65),we obtain

$$\frac{\partial \tilde{m}}{\partial q_t} = \frac{1}{\Delta} \left(\underset{+}{\underleftrightarrow{\frac{\partial^2 W}{\partial m \partial q}}} + \underset{-}{\underleftrightarrow{\frac{\partial^2 W}{\partial m \partial \alpha_l}}} \cdot \underset{-}{\underleftrightarrow{\frac{\partial^2 W}{\partial \alpha_l \partial q}}} \right) > 0$$

$$\frac{\partial \tilde{\alpha}_l}{\partial q_t} = \frac{1}{\Delta} \left(-\underset{+}{\underleftrightarrow{\frac{\partial^2 W}{\partial m^2}}} \cdot \underset{\text{-}}{\underleftrightarrow{\frac{\partial^2 W}{\partial \alpha_l \partial q}}} + \underset{-}{\underleftrightarrow{\frac{\partial^2 W}{\partial m \partial \alpha_l}}} \cdot \underset{+}{\underleftrightarrow{\frac{\partial^2 W}{\partial m \partial q}}} \right) < 0$$

QED.

In order to simplify the problem, we make the following assumption on the higher bound $\overline{\alpha}_l$:

$$\textbf{Assumption A: } \overline{\alpha}_l < \frac{\tau^2}{1+\phi m(1)} \frac{1-\theta m(1)}{\epsilon_0 \overline{c}}$$

Before going further with the proof, we establish this intermediary result:

Lemma 10 *Under Assumption A,* $\alpha_l(\lambda, \beta = 0) = \overline{\alpha}_l$ *for any* $(\lambda, q) \in [0, 1]^2$.

Proof: In order to prove Lemma 10, we need to write the first-order derivative of the utility of the secular elites with respect to α_l is:

$$\frac{\partial U_l}{\partial \alpha_l} = -\alpha_l + (1-\beta) \frac{\tau^2(1-q\theta m)}{(1+\phi m)\epsilon_0 \overline{c}}. \tag{A.67}$$

Hence, when $\beta = 0$, under Assumption A, $\frac{\partial U_l}{\partial \alpha_l} > 0$ for any $\alpha_l \in [0, \overline{\alpha}_l]$ and for any $(\lambda, q) \in [0, 1]^2$, so $\alpha_l(\lambda, \beta = 0, q) = \overline{\alpha}_l$ for any $(\lambda, q) \in [0, 1]^2$. This concludes the proof of the Lemma. **QED.**

Since $(\tilde{\alpha}_l(\lambda_t, q_t), \tilde{m}(\lambda_t, q_t))$ maximizes the social welfare when the externalities are internalized, $(\lambda_{t+1}, \beta_{t+1})$ solves the optimization problem (15) when:

$$\begin{cases} \tilde{m}(\lambda_t, q_t) = m(\lambda_{t+1}), \text{ and} \\ \tilde{\alpha}_l(\lambda_t, q_t) = \alpha_l(\lambda_{t+1}, \beta_{t+1}, q_t) \end{cases} \tag{A.68}$$

Indeed, when the clerics and the ruler have power λ_{t+1} and β_{t+1}, institutions are designed for $t+1$ so as to induce a choice $m(\lambda_{t+1})$ and $\alpha_l(\lambda_{t+1}, \beta_{t+1}, q_t)$ in that period that maximizes the social welfare under the institutional framework of period t. It remains to be proven that the solution $(\lambda_{t+1}, \beta_{t+1})$ of the system (A.68) is unique. Consider the following system with two unknown variables x and y:

$$\begin{cases} \tilde{m}(\lambda_t, q_t) = m(x), \text{ and} \\ \tilde{\alpha}_l(\lambda_t, q_t) = \alpha_l(x, y, q_t), \end{cases} \tag{A.69}$$

Consider first the case where an interior solution exists. Since the function $m(.)$ is increasing in its argument, from Lemma 4, there exists a unique value $x(\lambda_t, q_t) \in [0, 1]$ such that $\tilde{m}(\lambda_t, q_t) = m(x)$. Substituting $x(\lambda_t, q_t)$ in the second equation, we find:

$$\tilde{\alpha}_l(\lambda_t, q_t) = \alpha_l(x(\lambda_t, q_t), y, q_t), \tag{A.70}$$

By definition, $\tilde{\alpha}_l(\lambda_t, q_t) \in [0, \overline{\alpha}_l]$. Furthermore, as $\alpha_l(x(\lambda_t, q_t), y, q_t)$ is decreasing in y from Lemma 5, under Assumption A, $\alpha_l(x(\lambda_t, q_t), 1, q_t) = 0 \leq \alpha_l(x(\lambda_t, q_t), y, q_t) \leq \alpha_l(x(\lambda_t, q_t), 0, q_t) = \overline{\alpha}_l$. Hence, applying the theorem of intermediate values, there exists a single vector $(x(\lambda_t, q_t), y(\lambda_t, q_t)) \in [0, 1]^2$ such that (A.68) holds. We have demonstrated that the system (A.68) admits a unique interior solution, when this solution exists.

An interior solution does not always exists, as it can be that $\tilde{m}(\lambda_t, q_t) > m(\lambda_{t+1})$ or $\tilde{m}(\lambda_t, q_t) < m(\lambda_{t+1})$ for any $\lambda_{t+1} \in [0, 1]$. In these two cases, there is a single solution $(\lambda_{t+1}, \beta_{t+1})$ to the optimization problem (15), which is the unique vector such that $(m(\lambda_{t+1}), \alpha_l(\lambda_{t+1}, \beta_{t+1}, q_t))$ maximizes (A.58). Indeed, when $\tilde{m}(\lambda_t, q_t) > m(\lambda_{t+1})$, then $\lambda_{t+1} = 1$, and β_{t+1} solves

$$\tilde{\alpha}_l(\lambda_t, q_t) = \alpha_l(1, \beta_{t+1}, q_t) \tag{A.71}$$

for $\beta_{t+1} \in [0, 1]$. As $\alpha_l(1, \beta_{t+1}, q_t)$ is decreasing in β_{t+1} from Lemma 5, under Assumption **??**, $\alpha_l(1, 1, q_t) = 0 \leq \alpha_l(1, \beta_{t+1}, q_t) \leq \alpha_l(1, 0, q_t) = \overline{\alpha}_l$. Applying the theorem of intermediate values, there exists a single $\beta_{t+1} \in [0, 1]$ such that $\tilde{\alpha}_l(\lambda_t, q_t) = \alpha_l(1, \beta_{t+1}, q_t)$

The reasoning is similar when $\tilde{m}(\lambda_t, q_t) < m(\lambda_{t+1})$ for any $\lambda_{t+1} \in [0, 1]$: $\lambda_{t+1} = 0$ and there is a unique solution $\beta_{t+1} \in [0, 1]$ to the equation $\tilde{\alpha}_l(\lambda_t, q_t) = \alpha_l(0, \beta_{t+1}, q_t)$. From this we conclude that the optimization problem (15) admits a unique solution $(\lambda_{t+1}, \beta_{t+1})$.

We are now going to demonstrate that there exists a threshold $\overline{q}_d(\lambda_t)$ such that if $q_t > \overline{q}_d(\lambda_t)$, then $\lambda_{t+1} > \lambda_t$. Otherwise, $\lambda_{t+1} \leq \lambda_t$. In order to demonstrate this claim, we will show the following intermediary result:

Lemma 11 *$\lambda_{t+1} > \lambda_t$ if and only if $\tilde{m}(\lambda_t, q_t) > m(\lambda_t)$.*

Proof: Indeed, $\tilde{m}(\lambda_t, q_t) > m(\lambda_t)$ means that in (A.58), the ruler would want to commit to a provision level $\tilde{m}(\lambda_t, q_t)$ strictly above what is provided in equilibrium. Since $m(.)$ is increasing in λ (Lemma 4), we deduce that when the political weight λ_{t+1}, that decentralizes $\tilde{m}(\lambda_t, q_t)$ is such that $\tilde{m}(\lambda_t, q_t) = m(\lambda_{t+1})$, one has that $\lambda_{t+1} > \lambda_t$. A similar reasoning can be applied for the corners when $\lambda_{t+1} = 1$ when $\tilde{m}(\lambda_t, q_t) > m(1)$ or $\lambda_{t+1} = 0$ when $\tilde{m}(\lambda_t, q_t) < m(0)$. **QED.**

Lemma 12 *$\tilde{m}(\lambda_t, q_t) > m(\lambda_t)$ if and only if $q > \overline{q}_d(\lambda_t)$, with $\overline{q_d}(\lambda_t)$ is defined as the threshold the value of $q \in [0, 1]$ such that*

$$q = \max\left[\min\left[\frac{\phi}{\theta}\{\frac{1}{\tau(1 - \tilde{\alpha}_l(\lambda_t, q))} - \frac{1}{\epsilon_0 \overline{c}}\}, 1\right], 0\right]. \tag{A.72}$$

Proof: Given that $\tilde{m}(\lambda_t, q_t)$ is increasing in q_t, the condition $\tilde{m}(\lambda_t, q_t) > m(\lambda_t)$ is equivalent to $q_t > \overline{q}_d(\lambda_t) \in [0,1]$ with $\overline{q}_d(\lambda_t)$ defined such

$$\begin{aligned}
\tilde{m}(\lambda_t, \overline{q}_d(\lambda_t)) &= m(\lambda_t) \text{ when } \tilde{m}(\lambda_t, 0) \leq m(\lambda_t) \leq \tilde{m}(\lambda_t, 1) \\
\overline{q}_d(\lambda_t) &= 0 \quad \text{when } \tilde{m}(\lambda_t, 0) > m(\lambda_t) \\
\overline{q}_d(\lambda_t) &= 1 \quad \text{when } \tilde{m}(\lambda_t, 1) < m(\lambda_t)
\end{aligned}$$

More specifically, the first-order condition associated with the determination of $m(\lambda)$ is:

$$\frac{\lambda}{2}(m - F'(m)) - (1 - \frac{\lambda}{2})C'(m) = 0. \tag{A.73}$$

The first-order condition for the determination of $\tilde{m}(\lambda, q)$ writes as

$$\frac{\lambda}{2}(m - F'(m)) - (1 - \frac{\lambda}{2})C'(m) + (1 - \frac{\lambda}{2})\left[\begin{array}{c} \frac{-\phi}{(1+m\phi)^2}[1 - \frac{\tau(1-q\theta m)(1-\tilde{\alpha}_l(\lambda,q))}{\epsilon_0 \overline{c}}] \\ +\frac{1}{1+m\phi}\frac{\tau q\theta(1-\tilde{\alpha}_l(\lambda,q))}{\epsilon_0 \overline{c}} \end{array}\right] = 0. \tag{A.74}$$

Given the two FOCs above, we deduce that $\tilde{m}(\lambda_t, q_t) > m(\lambda_t)$ if and only if:

$$(1 - \frac{\lambda_t}{2})\{\frac{-\phi}{(1+m\phi)^2}[1 - \frac{\tau(1 - q_t\theta m)(1 - \tilde{\alpha}_l(\lambda_t, q_t))}{\epsilon_0 \overline{c}}] + \frac{1}{1+m\phi}\frac{\tau q_t\theta(1 - \tilde{\alpha}_l(\lambda_t, q_t))}{\epsilon_0 \overline{c}}\} > 0, \tag{A.75}$$

or

$$\begin{aligned}
\phi[1 - \frac{\tau(1 - q_t\theta m)(1 - \tilde{\alpha}_l(\lambda_t, q_t))}{\epsilon_0 \overline{c}}] &< (1 + m\phi)\frac{\tau q_t\theta(1 - \tilde{\alpha}_l(\lambda_t, q_t))}{\epsilon_0 \overline{c}} \\
\phi\left[1 - \frac{\tau(1 - \tilde{\alpha}_l(\lambda_t, q_t))}{\epsilon_0 \overline{c}}\right] &< \frac{\tau q_t\theta(1 - \tilde{\alpha}_l(\lambda_t, q_t))}{\epsilon_0 \overline{c}} \\
\frac{\phi}{\theta}\left[\frac{\epsilon_0 \overline{c}}{\tau(1 - \tilde{\alpha}_l(\lambda_t, q_t))} - 1\right] &< q_t
\end{aligned}$$

which rewrites

$$q_t > \frac{\phi}{\theta}\{\frac{\epsilon_0 \overline{c}}{\tau(1 - \tilde{\alpha}_l(\lambda_t, q_t))} - 1\}. \tag{A.76}$$

Denote $\Sigma(\lambda, q)$ the function

$$\Sigma(\lambda, q) = q - \frac{\phi}{\theta}\{\frac{\epsilon_0 \overline{c}}{\tau(1 - \tilde{\alpha}_l(\lambda, q))} - 1\}$$

Given that $\tilde{\alpha}_l(\lambda, q)$ is a decreasing function of q, $\Sigma(\lambda, q)$ is an increasing function of q. Now condition (A.76)is equivalent to $q_t > \overline{q_d}(\lambda_t)$ with

$$\begin{aligned}
\overline{q_d}(\lambda_t) &= 0 \text{ when } \Sigma(\lambda_t, 0) = -\frac{\phi}{\theta}\{\frac{\epsilon_0 \overline{c}}{\tau(1-\tilde{\alpha}_l(\lambda_t, 0))} - 1\} > 0 \\
\overline{q_d}(\lambda_t) &= 1 \text{ when } \Sigma(\lambda_t, 1) = 1 - \frac{\phi}{\theta}\{\frac{\epsilon_0 \overline{c}}{\tau(1-\tilde{\alpha}_l(\lambda_t, 1)} - 1\} < 0 \\
\overline{q_d}(\lambda_t) &= q \in (0,1) \text{ such that } \Sigma(\lambda_t, q) = 0 \text{ otherwise}
\end{aligned}$$

Compactly, $\overline{q_d}(\lambda_t)$ is defined as the threshold the value of $q \in [0,1]$ such that

$$q = \max\left[\min\left[\frac{\phi}{\theta}\{\frac{\epsilon_0 \overline{c}}{\tau(1-\tilde{\alpha}_l(\lambda_t, q))} - 1\}, 1\right], 0\right]. \tag{A.77}$$

and $\tilde{m}(\lambda_t) > m(\lambda_t)$ if and only if $q > \overline{q}_d(\lambda_t)$. We deduce that $\overline{q}_d(\lambda_t)$ is increasing in ϕ and decreasing in θ and λ_t. Combining the results established in Lemma 11 and Lemma 12, we get that $\lambda_{t+1} > \lambda_t$ if and only if $q > \overline{q}_d(\lambda_t)$. **QED.**

Finally, we demonstrate that there exists a threshold $\tilde{q}_d(\lambda_t, \beta_t)$ such that if $q_t > \tilde{q}_d(\lambda_t, \beta_t)$, then $\beta_{t+1} > \beta_t$. Otherwise, $\beta_{t+1} \leq \beta_t$. In order to demonstrate this claim, we proceed in two steps. First, we show the following result:

Lemma 13 *$\beta_{t+1} > \beta_t$ if and only if $\tilde{\alpha}_l(\lambda_t, \beta_t) < \alpha_l(\lambda_{t+1}, \beta_t)$, with*

$$\lambda_{t+1} = \begin{cases} \lambda \ s.t\ m(\lambda) = \tilde{m}(\lambda_t) & \text{if } \tilde{m}(\lambda_t) \in (m(0), m(1)) \\ 1 & \text{if } \tilde{m}(\lambda_t) > m(1) \\ 0 & \text{if } \tilde{m}(\lambda_t) < m(0). \end{cases} \tag{A.78}$$

Proof: Indeed, $\tilde{\alpha}_l(\lambda_t, q_t) < \alpha_l(\lambda_{t+1}, \beta_t, q_t)$ means that – given that the clerics have an optimal weight λ_{t+1} – if the ruler could, he would wish the secular elite to provide a lower enforcement effort. Since $\alpha_l(\lambda_{t+1}, q_t, .)$ is a decreasing function of β_t, the ruler increases his own political weight β_t, so that the secular elite provides less effort: $\beta_{t+1} > \beta_t$. **QED.**

Lemma 14 *There exists a threshold $\tilde{q}_d(\lambda_t, \beta_t) \in [0,1]$ such that $\tilde{\alpha}_l(\lambda_t, q_t) < \alpha_l(\lambda_{t+1}, \beta_t, q_t)$ if and only if $q > \tilde{q}_d(\lambda_t, \beta_t)$, with $\tilde{q}_d(\lambda_t, 1) = 1$ and λ_{t+1} given in (A.78).*

Proof: The first-order condition associated with the determination of $\tilde{\alpha}_l(\lambda_t, q_t)$ is:

$$-\tilde{\alpha}_l(\lambda_t, q_t) - \rho + \frac{\tau^2(1 - q_t\theta\alpha_c(\ \tilde{m}(\lambda_t, q_t)))}{\bar{c}\epsilon_0(1 + \phi\alpha_c(\ \tilde{m}(\lambda_t, q_t))} = 0 \tag{A.79}$$

Given that $\tilde{m}(\lambda_t, q_t) = m(\lambda_{t+1})$, this rewrites as

$$-\tilde{\alpha}_l(\lambda_t, q_t) - \rho + \frac{\tau^2(1 - q_t\theta\alpha_c(m(\lambda_{t+1}))}{\bar{c}\epsilon_0(1 + \phi\alpha_c(m(\lambda_{t+1}))} = 0 \tag{A.80}$$

The first-order condition associated with the determination of $\alpha_l(\lambda_{t+1}, \beta_t, q_t)$ is:

$$-\alpha_l(\lambda_{t+1}, \beta_t, q_t) + (1 - \beta_t)\frac{\tau^2(1 - q_t\theta\alpha_c(m(\lambda_{t+1}))}{\bar{c}\epsilon_0(1 + \phi\alpha_c(m(\lambda_{t+1}))} = 0 \tag{A.81}$$

Hence, the inequality $\tilde{\alpha}_l(\lambda_t, q_t) < \alpha_l(\lambda_{t+1}, \beta_t, q_t)$ is verified when

$$\rho > \beta_t\frac{\tau^2(1 - q_t\theta\alpha_c(\tilde{m}(\lambda_t, q_t))}{\bar{c}\epsilon_0(1 + \phi\alpha_c(\tilde{m}(\lambda_t, q_t))}, \tag{A.82}$$

Now the RHS of (A.82) is decreasing in q_t as $\tilde{m}(\lambda_t, q_t)$ is an increasing function of q_t so there exists a unique threshold $\tilde{q}_d(\lambda_t, \beta_t)$ such that if $q > \tilde{q}_d(\lambda_t, \beta_t)$, then (A.82) is satisfied. Otherwise, it is not satisfied. Moreover given that the RHS of (A.82) is decreasing in λ_t (as $\tilde{m}(\lambda_t, q_t)$ and $\alpha_c(\tilde{m}(\lambda_t, q_t)$ are increasing in λ_t), and increasing in β_t, it follows that the threshold $\tilde{q}_d(\lambda_t, \beta_t)$ is decreasing in λ_t and increasing in β_t. **QED.**

Combining the results established in Lemmas 12 and 14, we have demonstrated that $\beta_{t+1} > \beta_t$ if and only if $q > \tilde{q}_d(\lambda_t, \beta_t)$.

Summarizing, we have demonstrated the followings:

- The optimization problem (15) admits a unique solution $(\lambda_{t+1}, \beta_{t+1}) \in [0, 1]^2$.
- there exists a threshold $\tilde{q}_d(\lambda_t, \beta_t)$ such that if $q > \tilde{q}_d(\lambda_t, \beta_t)$ then $\beta_{t+1} > \beta_t$. Otherwise, $\beta_{t+1} \leq \beta_t$.
- There exists a threshold $\overline{q}_d(\lambda_t)$ such that if $q_t > \overline{q}_d(\lambda_t)$, then $\lambda_{t+1} > \lambda_t$. Otherwise, $\lambda_{t+1} \leq \lambda_t$.
- $\tilde{q}_d(\lambda_t, \beta_t)$ is decreasing in λ_t and increasing in β_t. and $\overline{q}_d(\lambda_t)$ is decreasing in λ_t.

Finally, $\tilde{q}_d(\lambda_t, 1) = 1$ because in equilibrium, the secular elite provides no effort, $\alpha_l(\lambda_t, 0, q_t) = 0$ and have zero utility. Hence, an epsilon increase in their political weight $1 - \beta_t$ will increase the social welfare by increasing both the utility of the ruler, and of the secular elite. This concludes the proof of Proposition 4. **QED.**

A.10 Proof of Proposition 5

As in the proof of Proposition 2, we first deduce from the maximization program (9) that $d^*_{Re} = D_{Re}((1-q_t)\Delta V_{Re}, m)$ with $D_{Re}(0, m) = 0$, and $D_{Re}(\cdot, \cdot)$ an increasing function of both arguments $(1-q_t)\Delta V_{Re}$ and m. Also from (10) $d^*_S = D_S(q_t \Delta V_S)$ is an increasing function of $q_t \Delta V_S$.

Parents do not know the realization of their children's capacity c to escape taxation when cultural transmission occurs. Consequently, the paternalistic motives have to be amended to involve expectations of the induced utilities with respect such capacity c. More precisely we have:

$$\begin{cases} V_{ReRe}(\lambda, \beta, q) = \frac{(1-\tau_{Re})}{1+\phi\alpha_c(\lambda)} \int_{\tau_{Re}/\epsilon}^{\overline{c}} \frac{dc}{\overline{c}} + \int_0^{\tau_{Re}/\epsilon} \frac{(1-c\epsilon)}{1+\phi\alpha_c(\lambda)} \frac{dc}{\overline{c}} \\ V_{Re\ S}(\lambda, \beta, q) = \frac{(1-\tau_{Re})}{(1+\phi\alpha_c(\lambda))} \int_{\tau/\epsilon}^{\overline{c}} \frac{dc}{\overline{c}} + \int_0^{\tau/\epsilon} \frac{(1-c\epsilon)}{1+\phi\alpha_c(\lambda)} \frac{dc}{\overline{c}}, \end{cases} \tag{A.83}$$

with $\epsilon = \epsilon_0/(1 - \alpha_l(\lambda, \beta, q))$. Hence,

$$\Delta V_{Re}(\lambda, \beta, q) = \frac{(\tau\theta\alpha_c(\lambda))^2(1 - \alpha_l(\lambda, q, \beta))}{2\overline{c}\epsilon_0(1 + \phi\alpha_c(\lambda))}. \tag{A.84}$$

Similarly, we find that

$$\Delta V_S(\lambda, \beta, q) = \Delta V_{Re}(\lambda, \beta, q) = \Delta V(\lambda, \beta, q) = \frac{(\tau\theta\alpha_c(\lambda))^2(1 - \alpha_l(\lambda, \beta, q))}{2\overline{c}\epsilon_0(1 + \phi\alpha_c(\lambda))}. \tag{A.85}$$

Again the result that $\Delta V_s(\lambda, \beta, q) = \Delta V_{re}(\lambda, \beta, q)$ follows from the quadratic specification of the expected payoff functions. Note as well that because $\alpha_l(\lambda, \beta, q)$ depends on q (ie. is a decreasing function in q), $\Delta V(\lambda, \beta, q)$ also depends on q and is an increasing function of q

Now, the cultural dynamics write as

$$q_{t+1} - q_t = q_t(1 - q_t)D(\lambda_t, \beta_t, q_t). \tag{A.86}$$

with

$$D(\lambda_t, \beta_t, q_t) = d^*_{Re} - d^*_S = D_{Re}\left[(1-q_t)\Delta V(\lambda_t, \beta_t, q_t), m(\lambda_t)\right] - D_S\left[q_t \Delta V(\lambda_t, \beta_t, q_t)\right]$$

can be interpreted as the relative "cultural fitness" of the religious trait in the population. Again simple inspection shows

$$D(\lambda_t, \beta_t, 0) = D_{Re}\left[\Delta V(\lambda_t, \beta_t, 0), m(\lambda_t)\right] > 0$$

and

$$D(\lambda_t, \beta_t, 1) = -D_S\left[\Delta V(\lambda_t, \beta_t, 1)\right] < 0$$

From this it follows that there exists a threshold $q^*_d(\lambda_t, \beta_t) \in (0, 1)$ such that

$$D(\lambda_t, \beta_t, q^*_d(\lambda_t, \beta_t)) = 0 \tag{A.87}$$

Compared to the benchmark model, $D(\lambda_t, \beta_t, q_t)$ may not be always decreasing function in q_t, as $\Delta V(\lambda_t, \beta_t, q_t)$ is increasing in q_t and the uniqueness of the threshold $q^*_d(\lambda_t, \beta_t)$ is not necessarily ensured. When however $(1-q)\Delta V(\lambda, \beta, q)$ is a decreasing function of q,[52] simple inspection shows that $D(\lambda_t, \beta_t, q_t)$ is a decreasing function of q_t and that $q_{t+1} < q_t$ if and only if $q_t > q^*_d(\lambda_t, \beta_t)$, as stated in proposition 5.

In such a case, defining again the sensitivity of parents' socialization to paternalistic motives by the following elasticities:

$$\epsilon_{Re} = \frac{\partial D_{Re}(x, y)}{\partial x} \cdot \frac{x}{D_{Re}} \text{ and } \epsilon_S = \frac{\partial D_S}{\partial z} \cdot \frac{z}{D_{Re}}\Bigg|$$

evaluated respectively at $x = (1-q)\Delta V(\lambda, \beta, q)$ and $y = m(\lambda)$, and $z = q\Delta V(\lambda, \beta, q)$, we obtain

$$\frac{\partial q^*_d(\lambda_t, \beta_t)}{\partial \lambda} = \frac{\left[\epsilon_{Re} - \epsilon_S\right] d^*\left(\lambda_t, \beta_t\right) \cdot \frac{\Delta V'_\lambda(\lambda_t, \beta_t)}{\Delta V(\lambda_t, \beta_t)} + \frac{\partial D_{Re}}{\partial m} \cdot m'(\lambda_t)}{-\frac{\partial D}{\partial q}(\lambda_t, \beta_t, q^*_d(\lambda_t, \beta_t))} \tag{A.88}$$

with $d^*\left(\lambda_t, \beta_t\right) = d^*_{Re}\left((\lambda_t, \beta_t, q^*_d(\lambda_t, \beta_t)\right) = d^*_S\left((\lambda_t, \beta_t, q^*_d(\lambda_t, \beta_t)\right)$, the equilibrium commun socialization rate at the threshold $q^*_d(\lambda_t, \beta_t)$. Again the numerator is composed of two terms reflecting the two channels through which the institutional environment λ_t affects cultural

[52] This is ensured when $1 > \frac{\tau^2}{\bar{c}\epsilon_0} \max\left(\frac{\theta}{\phi}, 1\right)$

transmission. The first term $K(\lambda_t) = [\epsilon_{Re} - \epsilon_S]\, d^*(\lambda_t, \beta_t) \cdot \frac{\Delta V'_\lambda(\lambda_t,\beta_t)}{\Delta V(\lambda_t,\beta_t)}$ is the paternalistic motive channel. As $\Delta V'_\lambda(\lambda_t, \beta_t) > 0$, the sign of $K(\lambda_t)$ depends on the relative sensitivity of parents' socialization to paternalistic motives. It is positive when $\epsilon_{Re} > \epsilon_S$, namely when the socialization rate of religious parents d^*_{Re} is more sensitive to paternalistic motives than the one of secular parents d^*_S. The second positive term $\frac{\partial D_{Re}}{\partial m} \cdot m'(\lambda_t)$ reflects the positive effect of promoting religious infrastructures as complementary inputs in the transmission process of the religious trait.

As in the benchmark model, it follows again that when religious parents' socialization efforts are more sensitive to paternalistic motives than secular parents (ie. $\epsilon_{Re} > \epsilon_S$) , and (or) when religious infrastructures are strong enough complementary inputs to socialization to the religious trait, then the numerator of (A.88) is positive and $q^*_d(\lambda_t, \beta_t)$ is increasing in λ_t.

- **Example with constant elasticity socialization cost functions**

Consider the following socialization cost functions:

$$\begin{cases} h_{Re}(d) = \frac{d^{1+\eta_{re}}}{1+\eta_{re}} \cdot \frac{1}{\chi(m)} \text{ and} \\ h_s(d) = \frac{d^{1+\eta_s}}{1+\eta_s}, \end{cases} \tag{A.89}$$

with $\eta_s \geq \eta_{re} > 0$ and $\chi'(m) > 0$**.** The optimal socialization efforts are such that:

$$\begin{cases} d^*_{Re}(q_t, \lambda_t) = ((1-q_t)\Delta V(\lambda_t, \beta_t, q_t))^{\frac{1}{\eta_{re}}} \cdot [\chi(\lambda_t)]^{\frac{1}{\eta_{re}}} \\ d^*_S(q_t, \lambda_t) = (q_t \Delta V(\lambda_t, \beta_t, q_t))^{\frac{1}{\eta_s}}. \end{cases} \tag{A.90}$$

and in this constant elasticity specification $\epsilon_{Re} - \epsilon_S = \frac{1}{\eta_{re}} - \frac{1}{\eta_s} \geq 0$. Cultural dynamics are described as:

$$q_{t+1} - q_t = q_t(1-q_t)\{((1-q_t)\Delta V(\lambda_t, \beta_t, q_t))^{\frac{1}{\eta_{re}}} \cdot [\chi(\lambda_t)]^{\frac{1}{\eta_{re}}} - (q_t \Delta V(\lambda_t, \beta_t, q_t))^{\frac{1}{\eta_s}}\}, \tag{A.91}$$

which admits two unstable steady states $q = 0$ and $q = 1$, and in general a unique interior attractor, which we denote $q^*_d(\lambda_t, \beta_t)$ such that:

$$\frac{q^*_d(\lambda_t, \beta_t)^{\frac{1}{\eta_s}}}{(1 - q^*_d(\lambda_t, \beta_t))^{\frac{1}{\eta_{re}}}} = [\frac{(\tau\theta\alpha_c(\lambda_t))^2(1 - \alpha_l(\lambda_t, \beta_t, q^*_d(\lambda_t, \beta_t))}{2\overline{c}\epsilon_0(1 + \phi\alpha_c(\lambda_t))}]^{\frac{\eta_S - \eta_{re}}{\eta_S \eta_{re}}} \cdot [\chi(\lambda_t)]^{\frac{1}{\eta_{re}}} \tag{A.92}$$

From the last equation, and given that $\eta_S > \eta_{re}$, we deduce that $q_d^*(\lambda_t, \beta_t)$ is increasing in θ, λ_t and β_t and decreasing in ϕ. This concludes the proof of Proposition 5.

- **Joint dynamics with $q_d^*(\lambda_t, \beta_t)$ independent from β_t.**

Consider the case where the socialization cost functions of religious and secular parents are given by the following form

$$h_{Re}(d, m) = \frac{d^{1+\eta}}{1+\eta} \frac{1}{\chi(m)},\ h_s(d) = \frac{d^{1+\eta}}{1+\eta} \quad \text{with } \eta > 0$$

from (A.92), it is immediate that the threshold $q_d^*(\lambda_t, \beta_t)$ is given by:

$$q_d^*(\lambda_t, \beta_t) = q_d^*(\lambda_t) = \frac{[\chi(\lambda_t)]}{1 + [\chi(\lambda_t)]}$$

and is therefore independent from β_t. In such a case the dynamics of λ_t and q_t are such that: $\lambda_{t+1} > \lambda_t$ if and only if $q_t > \overline{q}_d(\lambda_t)$, and $q_{t+1} > q_t$ if and only if $q_t < q_d^*(\lambda_t)$ They are then decoupled from the dynamics of β_t and follow the same pattern as in the benchmark model. Consequently, depending on the initial conditions (λ_0, q_0), (λ_t, q_t) converge towards *a religious regime* $(1, q_d^*(1))$ or a *secular regime* $(0, q_d^*(0))$. Associated to these dynamics, the dynamics of political centralization then converges towards strong state centralization with $\beta_1^* = \widetilde{\beta}_d(1, q_d^*(1))$, or weak state centralization $\beta_0^* = \widetilde{\beta}_d(0, q_d^*(0)) < \beta_1^*$. **QED.**