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Sabiou M. Inoua
Chapman University, inoua@chapman.edu

Vernon L. Smith
Chapman University, vsmith@chapman.edu

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Neoclassical Supply and Demand, Experiments, and the

Classical Theory of Price Formation

Sabiou M. Inoua[1] and Vernon L. Smith[2]
Chapman University

1 Introduction

The 1870s neoclassical marginal revolution in economics culminated a century later in a striking conclusion: The core utility maximization principle of neoclassical economics was shown to have no interesting implication for aggregate market behavior (Sonnenschein, 1972, 1973a, 1973b; Debreu, 1974; Mantel, 1974; Kirman, 1989; Shafer & Sonnenschein, 1993; Rizvi, 2006). We argue that neoclassical price theory was founded on two axioms—price-taking behavior and the law of one price in a market—that, if imposed on the theory, were logically inconsistent with a theory of market price formation. This logical gap in neoclassical theory was filled essentially with thought experiments: Jevons derives utility maximizing quantities, given prices, then postulates a 'theoretically perfect market' in which every trader has complete information on supply and demand and the consequent

[1] Economic Science Institute, Chapman University, 1 University Drive, Orange, CA 92866, USA; mahamaninoua@chapman.edu

[2] Economic Science Institute, Chapman University, 1 University Drive, Orange, CA 92866, USA; vsmith@chapman.edu

equilibrium price(s) (Jevons, [1871] 1888, p. 87);[3] Walras also derives utility maximizing quantities for given prices. (Walras, 1874, Lesson 8) Further, however, he proposed a mechanism whereby the price in each market might be determined by a trial-and-error (or tatonnement) process of adjustment (Walras, 1874, see, e.g., Lesson 48).[4] However, Bertrand (1883, p. 505) noted that Walras's process caused path-dependency problems that impacted the postulated equilibrium state. Careful analysis of this problem led Walras to realize increasingly the awesome difficulty of dealing with disequilibrium dynamics within the neoclassical framework.[5] Walras thus reformulated his original theory of

[3] Howey (1989, pp 16–18) reports that in September of 1862, W. S. Jevons recorded the transmission of the paper "Notice of a General Mathematical Theory of Political Economy" to the *British Association for the Advancement of Science.* The paper was read before the Association. However, only a short abstract was published in the Report of the Proceedings. This was the first articulation of the marginal utility and general equilibrium theories of economic equilibrium by Jevons launching the modern era of neoclassical equilibrium economics.

[4] From 1919 until its abandonment in 2015, due to recurrent charges of price manipulation, the London gold price was determined ("fixed") using a procedure that implemented Walras's *tâtonnement*—to our knowledge the only such market application, wherein it ultimately failed. Twice daily, a price was set by five gold dealers in London at meetings in which the chairman opened with a trial price, followed by each member reporting their net orders to buy or sell based on totals reported by their clients, plus a buy (sell) order for their own account. The chairman then raised (lowered) the price if there was an excess of buy (sell) orders. Each member signaled when the price range had narrowed to an interval in which they would no longer desire to adjust their order response. The process then stopped by unanimous consent when all members signaled that no change would be forthcoming. Jarecki (1976)

[5] The alteration of the tatonnement theory started in the second edition of Walras' *Elements* (1889, § 42), where he assumed that trade should be suspended at disequilibrium. The modification continued in the subsequent editions, particularly in the fourth (1926, §§ 207, 251). On this complex evolution of Walras's tatonnement theory away from its original realistic version, see Walras ([1874, 1896] 2014, Translators' introduction, notably p. xv-xix).

tatonnement, rephrasing it instead as a virtual trial-and-error disequilibrium price adjustment process executed while trade is suspended at disequilibrium, hence setting the stage for the modern concept of tatonnement as a virtual dynamics executed 'as if' it were effected by an auctioneer.[6] Although seriously incomplete, Jevons, Walras, and their general equilibrium followers, introduced the principle that rationality is a property of the individual, and indeed, rationality in the economy became identified with individual rationality throughout economics, game theory, and financial asset markets. The new tradition committed economic science to the proposition that markets and all economic interactions are rational if and only if their component individuals are rational.

Many attempts at remedying these gaps in neoclassical price theory were unsatisfactory (Hahn, 1982; Fisher, 2013). Thus, "we shall have to conclude that we still lack a satisfactory descriptive theory of the invisible hand." (Hahn, 1982, p. 746) More recently and self-critically: "we do not have an adequate theory of value, and there is an important lacuna in the center of microeconomic theory. Yet economists generally behave as though this problem did not exist.[7] (Fisher, 2013, p. 35)

[6] Martin Shubik was not one to leave unexpressed his distaste for these approaches to modelling markets. His Cowles Foundation Discussion Paper (no. 368, 1974), was entitled, "A Trading Model to Avoid Tatonnement Metaphysics." (cited in V. L. Smith, 1976, p 275, 279) Experimental theorists, however, have made extensive use of various implementations of the mechanism to study behavior in the laboratory. For example, Crockett, Friedman, and Oprea (2019).

[7] Theorists influenced by experimental markets studies, made progress by focusing on modeling the bid-ask double auction and other institution-specific processes, thereby implicitly breeching the constraints imposed by the neoclassical tradition. See for example one of the earliest such studies by Easley and Ledyard (1992).

Ironically, the classical school, which the marginal revolution overturned, contains quite fruitful foundations for a theory of market price formation. The old school, in regard to market "effectual demand", relies, not on an unobservable criterion like an individual utility function, but on the individual's willingness to sacrifice command over other goods, measured by an amount of monetary wealth, in order to acquire any given desired good.[8] Thus, as Adam Smith notes, if two people equally desire an antique book at auction, the one with the larger wealth will carry it. (A. Smith, 1978, p 358, 496) [9] Willingness to pay value directly measures opportunity cost, or foregone purchases. Hence, it is a reservation price as clarified by the French followers of A. Smith such as Jules Dupuit as a maximum willingness to pay value price, and the sellers' minimum willingness to accept value price.[10] (1844, p. 343)

[8] In Book I, Chapter VII and throughout *The Wealth of* Nations, reference is always to "effectual demand"; a poor man might like a coach and six but his demand is not effectual in supporting its being brought to market. (Smith, A. [1776] 1904, p 58)

[9] The analysis is incomplete from a modern perspective, but A. Smith recognizes that wants, as well as the capacity for paying, both matter; that the English auction procedure awards an item to the person willing to pay the most; that people are diverse in tastes, in capacity and in "effectual" demand.

[10] "Price" here is value per unit for the individual, a potential contract price in the market. The individual is modelled as comparing their maximum willingness to pay value of a unit consumed with forthcoming offers from sellers, or bids from buyers, and is motivated to buy cheap. If a stable contract price emerges from the market it is a consequence of the interaction of the collection of all buyers and sellers in the market. The "rationality" of the market price emerges from this collective interaction depending on the institution of the market, such as the rules of double auction trading on an exchange.

Section 2 justifies this reexamination of price formation in classical economics prior to its displacement in the 1870s. The market price of a good evolves through competition of buyers and sellers, which is by definition the process whereby firms undersell one another (seller-seller competition), or buyers outbid one another (buyer-buyer competition), or through the 'higgling and bargaining' of buyers and sellers (buyer-seller competition).

This old view of competition is familiar and often taken for granted; it reappears intuitively in most introductory textbooks. But it has little to do with the neoclassical axiom of price-taking behavior (the negation of competitive behavior), the law of one price, or utility maximization. Prominent neoclassical marginalists, who explain price formation in terms of the interactive behavior of the buyers and sellers, appeal revealingly to the old view: notably the Austrian marginalists and Alfred Marshall. Moreover, laboratory market experiments, starting in mid-twentieth century (V.L. Smith, 1962), established the stability, efficiency, and robustness of the market mechanism under privacy conditions in which neoclassical theory would predict 'market failures.' These markets typically involve a small number of buyers and sellers with private knowledge of reservation values who compete through double-auction trading rules (bids, asks and acceptance messages). The experimental findings corroborated the old view of the price mechanism, as argued in Section 2.7.

We revisit the history of market economics to extract the old conception of supply and demand before the marginalist revolution. It is not an exercise in historical scholarship for the sake of intellectual history; rather it seeks to extract and emphasize the implicit methodology at the foundation of the classical view of a market economy, and to show

that, despite any shortcomings such as a coherent mathematical representation, it offers an operational starting point for constructing a theory of price formation closely allied with observables that reflect the experience of people in markets.

2 Rediscovering classical price theory

2.1 The classical methodology

This old view on the price mechanism differs from the new one in two fundamental ways discussed in greater detail below:

Principle 1: "Realism." Market behavior is founded on concepts that are observable and operational. Supply and demand are classically defined by an observable, operational, monetary value: the reservation price—the buyer's maximum willingness to pay; the seller's minimum willingness to accept.[11]

Principle 2: Emergent rationality. Market interactions determine deep emergent properties that are the unintended consequences of people's actions, the results of human actions and not of human design.

[11] The operational grounding of these valuation concepts can be further clarified. Let R (Q) be the total amount that consumers are willing to pay for Q total units of an industry's product, then R (Q) is the potential total revenue of the industry. Hence, R' (Q)—the most the marginal buyer is willing to pay—is the potential (effectual) demand for the industry's product. Similarly, there is a total willingness to accept or potential expenditure by industry for inputs to produce Q, C (Q), and C'(Q) the industry minimum marginal willingness to accept. See our reexamination and critique of induced value theory in section 5.3 below.

2.2 Reservation price as core concept

Malthus also states in his *Principles* that "demand will be represented and measured by the sacrifice in money which the demanders are willing and able to make in order to satisfy their wants." (Malthus, [1820] 1836, p. 62) Similarly with J.-B. Say: an object's utility to a person, can be measured by the sacrifice this person makes to acquire the object. (See, e.g., Say, [1815] 1821b, ch. 2, pp. 5-6; [1815] 1821a, ch. 2, p. 7) Dupuit (1844, p. 343; 1849, p. 182), refining J.B. Say's intuition, emphasizes that use-value is measured by maximum willingness to pay, namely by the reservation price. J.S. Mill reached the same conclusion: "Value in use […] is the extreme limit of value in exchange", that is, price (Mill, [1848] 1965, bk. 3, ch. 1, § 2, p. 457). Or: "the utility of a thing in the estimation of the purchaser, is the extreme limit of its exchange value."(Mill, [1848] 1965, bk. 3, ch. 2, § 1, p. 462)

2.3 Supply and demand as distribution functions

Demand as a distribution function of individual maximum willingness to pay values is developed by a less-known French author, Germain Garnier, who translated into French the *Wealth of Nation*s, which inspired his *Abrégé élémentaire des principes de l'économie politique* ([1796] 1846).[12] In the second edition of this book, he derived the law of demand from the distribution of willingness to pay, expressed as a portion of wealth consumers are

[12] This French tradition on demand, and value more generally, is thoroughly covered in Ekelund Jr and Hébert (1999), although interpreted as an anticipation or even the origin of the marginal-utility basis of demand. In our view the classical writers were simply fleshing out demand in its role of defining the reservation values that enabled price formation, and any connection with utility was a distraction in developing that program. Indeed, the connection with wealth was lost.

willing to pay for a commodity, representing its distribution as a pyramid. J.-B. Say and Jules Dupuit also adopted explicitly this pyramidal representation of demand. (Garnier, 1846, pp. 195-196; Say, [1803] 2006, vol. II, bk. II, ch. 1, p. 607; [1828] 2010, vol. I, part III, ch. 4, p. 368-9; Dupuit, 1844, p. 368)

Cournot's treatment of demand is also distributional. Cournot is a pivotal figure in the transition from classical to neoclassical economics; although his view on demand is rigorously classical, he inspired much of the neo-classical theory of supply.[13] Thus, he observed that market demand can be assumed to be a smoothly decreasing function of price, even though individual demand is realistically discontinuous:

[13] Cournot's model of oligopoly supply was not, however, integrated with the demand side to articulate a price formation process. Realizing the need for a substitute, he put his equation expressing the law of demand, D = F (P), to work. His suppliers each chose profit maximizing quantities given price, yielding total output equal to D. He then closed the loop theoretically by inverting F (P) to obtain D* = F(P*). Jevons and Walras would later apply the same methodology. Each modelled demand (in the latter cases, max utility given prices) but could not articulate market price discovery. They closed the loop statically by imagining "black box" sources of prices, and imposed the law of one price in a market; Jevons' perfect information, or the metaphoric auctioneer, as theorists interpreted Walras. Cournot's "black box" contained his demand function, D = F (P), which is a distribution function of willingness to pay values that he truncates at P* such that all buyers with lower values are excluded from buying. A theoretical price is determined that is consistent with how the theorist thinks about a market end-result, not price(s) found by the theorist's model representing how buyers and sellers think and act in a market. Cournot is therefore the first to substitute ideal theoretical agent results for a direct modelling of agent behavior. Modern versions include models of agents that choose Nash best reply actions. This is different than modelling agent actions from the agents' postulated perspective and then showing that agent actions are also consistent with Nash best replies, either distribution-ally, or they converge to the same outcomes. For example, Williams, et al., (2000) report two-commodity market experiments showing that subjects in the aggregate converge to competitive equilibrium outcomes, but with individual deviations from the equilibrium "very common." (pp. 526-527)

> We will assume that the function F(p), which expresses the law of demand or of the market, is a *continuous function*, i.e. a function which does not pass suddenly from one value to another, but which takes in passing all intermediate values. It might be otherwise if the number of consumers were very limited: thus in a certain household the same quantity of firewood will possibly be used whether wood costs 10 francs or 15 francs the stere, and the consumption may suddenly be diminished if the price of the stere rises above the latter figure. But the wider the market extends, and the more the combinations of needs, of fortunes, or even of caprices, are varied among consumers, the closer the function F(p) will come to varying with p in a continuous manner. However little may be the variation of p, there will be some consumers so placed that the slight rise or fall of the article will affect their consumptions, and will lead them to deprive themselves in some way or to reduce their manufacturing output, or to substitute something else for the article that has grown dearer, as, for instance, coal for wood or anthracite for soft coal. (1838 [1897], pp. 49-50)

Individual demand is discontinuous for the obvious reason that goods are produced and consumed in discrete units.[14] Moreover, as Cournot emphasizes, a consumer responds to

[14] Items like cereals and liquids long have been prepackaged in discrete consumption bundles. We model demand as fundamentally discrete and expressed as a distribution function across individuals and obeying the law of demand as an order property of the distribution function. As we show below, classical price theory requires integration to get surplus, *V(P)*, and hence depends in no way on the continuity or smoothness of the integrand although these properties follow in the large market case by applying the law of large numbers. Importantly, the integral *V (P)* exists whether excess demand is discrete or continuous, and we get a far crisper, more satisfactory, model of price formation than artificially imposing neoclassical continuity on

price changes in a discontinuous manner, reacting only when the price exceeds a threshold, namely a reservation price (15 francs in Cournot's example). Yet demand can be a smoothly decreasing function of price on the aggregate by the law of large numbers, provided the distribution of reservation prices is sufficiently spread, which is the case if consumers are diverse in need and fortune. Cournot's law of demand is an order statistic property of the distribution of reservation values having nothing to do with continuous neoclassical diminishing marginal utility.

Cournot's intuition would resurface again and again in economics, starting from the early marginalists, many of whom invoke it to justify the treatment of demand and supply as smooth functions, but usually fall back on the average-agent simplification, and not by considering explicitly the distribution of agents' characteristics (Pareto, 1897, p. 9; Leon Walras, 1926, pp. 57-8; Jevons, [1871] 1888, pp. 89-90; Marshall, [1890] 1920, p. 83). For example, Jevons appeals to the concept of trading body, "Imagine that there is one trading body possessing only corn, and another possessing only beef." Jevons, [1871] 1888, p 95) Also Marshall's "representative" consumer (or producer), "Individual differences of character may be neglected when we consider the average of large numbers of people" (Marshall, [1890] 1920, p. xxi; also see p. 130 footnote 2)

The general principle of regularity by aggregation will reappear in various forms in economic theory. Some mathematical economists explored an abstract distributional view of

demand. Thus, a common methodology applies whether coke comes in prepackaged containers or is measured out by the ounce.

demand, as a remedy when the arbitrariness of neoclassical demand culminated in the Sonnenschein-Mantel-Debreu problem, deriving the law of demand by aggregation over a diverse population of consumers' preferences or wealth (Hildenbrand, 1983; Trockel, 1984; Grandmont, 1987).[15] This distributional approach retains, however, some of the strong neoclassical regularities whereby continuous individual preferences are defined on a continuous commodity space.

2.4 Competition; a fundamental principle

Interaction among buyers and sellers in the marketplace is simply competition, a confrontation which was so fundamental in classical price theory that J.S. Mill felt compelled to affirms that "only through the principle of competition has political economy any pretension to the character of a science" (Mill ([1848] 1909) Book I, Ch. IV, § 1). Yet competition lost its meaning in the passive price-taking axiom of the marginal revolution. In our formal expanded restatement, market competition takes two forms, which have precise distinguishing implications for both quantity allocation and price dynamics. Type 1 competition is that between the two sides of the market (buyer-seller confrontation, prominent in double auction exchange). Type 2 competition is that operating on each side of the market: on the demand side, when buyers compete to purchase a seller's unit of a good, as in an English auction, by the highest-value buyer outbidding all rival buyers and obtaining the unit to the exclusion of others. On the supply side: when firms compete, and

[15] For a review, see Rizvi (1997). For a recent investigation of regularity by aggregation in a general-equilibrium experimental context see Crockett et al. (2019).

the lower-cost firm undersells rival sellers and trades to the disadvantage of the latter. This competition involves quantity rationing among buyers or sellers. Units are allocated to buyers from highest to lowest priority ordering of their valuations, and to sellers, from lowest to highest costs. Both forms of competition yield directly the classical dynamic 'law of supply and demand'; that is, demanders and suppliers, through their 'higgling and bargaining' (Type 1 competition) or their outbidding and underselling (Type 2 competition), tend to offer or accept a higher price in case of an excess demand, and a lower price in case of an excess supply: *hence price change and excess demand have the same sign*, a key dynamic property of the classical theory because the integral of excess supply is a function V(P) that measures the overall distance between price and the distribution of reservation prices (values and costs), as in equations and below.

The relevance of the supply and demand diagram in illuminating competition has been enormously diminished by the modern tendency to follow Cournot's influential typology of markets based on the number of buyers and sellers. This trend substituted for the old unified view of competitive market price formation, a diversity of price theories based on an artificially static "given" number of sellers in a market. Thus, different theories are called for in the new school to deal with given static states of monopoly, duopoly, oligopoly, and so on; whereas the basic supply and demand diagram is believed to apply only in the vaguely defined limit of a large number of traders, or "perfect competition". The Austrian economists' perspective here is important and we shall illustrate graphically this unifying nature of the classical view on competition using Böhm-Bawerk's excellent example (see section 2.6). Experimental interpretations and findings also suggest a more general domain

of validity of competitive price theory than is usually assumed (Plott, 1982; Smith & Williams, 1990).

2.5 Market rationality as an emergent phenomenon; contrast with neoclassical theory's appeal to the socialist economists.

Adam Smith invoked his well-known invisible hand metaphor to convey the notion of an overall unintended "result of human action, but not the execution of any human design," a general theme of the Scottish Enlightenment. (Ferguson, 1782, p 205) Friedrich Hayek would rediscover and restate this classical view as a general theory of "spontaneous order." (Hayek, 1978) This intellectual tradition and its further development was lost in the Jevons-Walras neoclassical transition that Hayek would progressively realize amid the socialist calculation debate. Hayek insightfully identified emergent market rationality with the "central problem of economics as a social science", namely "how the spontaneous interaction of a number of people, each possessing only bits of knowledge, brings about a state of affairs in which prices correspond to costs, etc., and which could be brought about by deliberate direction only by somebody who possessed the combined knowledge of all those individuals." (1937, p. 49) The market mechanism brings about an overall order which, to come about by conscious and deliberate direction, would require a sum of knowledge that no single mind can possess (Hayek, 1945, 1948, 1980). That is, market prices synthetize a huge amount of information dispersed throughout the economy and hence coordinates economic activity vastly beyond the narrow scope of conscious individual rationality. To assume perfect, complete, or common knowledge of supply and demand, or rational expectations on the part of every individual, is to distract from, or renounce, an

explanation of this emergent order. Adam Smith never referred to this order as "rational" but deemed it central to accounting for the wealth of nations.

In contrast, the neoclassical focus on the utility maximizing individual required market rationality to depend on individual action without providing price discovery mechanisms showing how efficient prices, that maximized gains from trade, emerged out of a collective of such individuals. Appeals to the rationality of one idealized agent fail to achieve this, be it Robinson Crusoe in an island economy, or the Walrasian "auctioneer" responsible for finding equilibrium prices for all commodities.[16] Taken literally, these are clearly metaphors for a centrally planned economy, not a market economy the rationality of which is concentrated in no single individual, nor any special subset of individuals.

Indeed, neoclassical value theory makes perfect sense as a natural tool for socialist central planning, whereby the planner becomes the Walrasian auctioneer, fixing prices for the whole economy by trial-and-error adjustments (tatonnement). Then anonymous individuals, given these prices, choose optimal quantities. In the great socialist debate, Mises and Hayek had to confront this brilliant neoclassical case for socialism put forward formally by Barone ([1908] 1935), Lerner (1934), and Lange (1936, 1937), but already pointed out by Wieser ([1889] 1893, ch. VI) and Pareto (1897, pp. 364-371; 1909, pp. 362-

[16] Since Walras himself did not identify explicitly who is responsible for the tatonnement price adjustments in his revised formulation of tatonnement theory, the textbook appellation, "Walrasian auctioneer", might be misleading (we thank this Journal's editor for this reminder). The reference to an "auctioneer" was introduced later by authors filling in the gap of the Walrasian story. On this addition, perhaps suggested by Jaffe's famous translation, see Walras ([1874, 1896] 2014, Translators' introduction, p. xxxv).

364); the latter, without advocating socialism, sketched a rigorous neoclassical theory of its economy.

2.6 Adam Smith's sketch of market price theory

Adam Smith begins his narrative of price formation by describing the experience of producer-suppliers who, each knowing their own cost, bring corresponding quantities to market:

> "When the quantity of any commodity which is brought to market falls short of the effectual demand, all those who are willing to pay the whole value of the rent, wages and profit, which must be paid in order to bring it thither, cannot be supplied with the quantity which they want. Rather than want it altogether, some of them will be willing to give more. A competition will immediately begin among them, and the market price will rise more or less above the natural price, according as either the greatness of the deficiency, or the…eagerness of the competition." (A. Smith, 1776; 1904, Vol 1, p 58)

Notice from Adam Smith's careful choice of words that he is describing the interactive experience of sellers and buyers, and their responses in their shared context of interaction. Sellers know the "whole value" of their goods necessary to recover their costs. Buyers, whose wants are not all satisfied at that whole value price, are willing to pay more rather than want for it. Depending on the extent of the deficiency and their eagerness, competition among the buyers will raise the price.

A. Smith's language describes the experiences and actions of the actors in the market, as he observes, thinks about, and mentally models them. He is describing what a modern economist would say is excess demand, read off the supply curve and the demand curve as the economist visualizes them in governing the Walrasian movement of prices in response to excess demand. A. Smith, we infer, never uses this modern language because it is not part of the knowledge and experience of the actors. He describes behavior in its origins in human market behavior. For A. Smith there are outcomes involving the division of labor for the people in markets and for society—indeed, no less than the causes of the wealth of nations! —*but none of that is part of people's experience or intentions.* Adam Smith's thought process separates the description of trader behavior, in the context of market experience, from the market's larger ends achieved for society.

Finally, notice the motivation for our expansion and restatement of classical theory in terms of distinguishing Type 1 from Type 2 competition. In this case, Type I higgling quickly establishes that the quantity is short, and this unleashes a competition among buyers to ration that quantity among them much like an English auction, but with multiple units.

Similarly, and contrastingly to the above:

> "When the quantity brought to market exceeds the effectual demand, it cannot be all sold to those who are willing to pay the whole value of the rent, wages and profit, which must be paid in order to bring it thither. Some part must be sold to those who are willing to pay less, and the low price which they give for it must reduce the price of the whole. The market price will sink more or less below the natural price, according as the greatness of the excess increases more or less the competition of the sellers, or

according as it happens to be more or less important to them to get immediately rid of the commodity."[17] (A. Smith, 1776; 1904, Vol. 1, p. 59)

For A. Smith "competition" means buyers bidding higher—or sellers cutting—prices, and implicitly, sometimes simultaneously, sometimes one side or the other competing to ration a quantity temporarily given in the market price discovery process.

From Alfred Marshall's synthesis, and Böhm-Bawerk's, to experimental economics, important revivals of the classical paradigm have been part or the work of scholars who intuitively, if unintentionally, felt greater proximity with this abandoned paradigm.

Although Marshall and the Austrian economist Böhm-Bawerk were significant spokesmen for neoclassical marginal utility theory, their articulation of price formation in markets did not depend in any way on their championing of the marginalist framework. Marshall's excellent description of price discovery in "a corn-market in a country town" is an elaboration of the Smithian-classical process applying willingness to pay (willingness to accept) reservation values to buyer-seller "higgling and bargaining". (Marshall ([1890] 1920, pp 332-4) In this market context, marginal utility is neither applied nor mentioned by Marshall, because it is plainly irrelevant to his demonstration of price formation. This perspective, however, is not part of the established understanding of Marshall, nor of his

[17] A. Smith, ever testing his model against observation, distinguishes perishables from inventories of durables, going on to add: "The same excess in the importation of perishable, will occasion a much greater competition than in that of durable commodities; in the importation of oranges, for example, than in that of old iron." (A. Smith, 1776; 1904, Vol. 1, p. 59) A. Smith also makes clear that his thinking about market processes is not confined only to long run supply and the "natural price."

significant influence on early market experiments examining, ostensibly, neoclassical supply and demand theory. Similarly, in Böhm-Bawerk's peasant horse market, where each person is a buyer or seller of a single unit, first illustrated with one buyer and one seller, then one-sided competition (one seller, multiple buyers; one buyer, multiple sellers), and finally two-sided competition for multiple buyers and sellers.[18] (Böhm-Bawerk [1888] 1891, pp 198-208) He elegantly demonstrates how—as we would now be able to describe it —adding buyers and sellers to a market shrinks the trading gap between highest willingness to pay and lowest willingness to accept, thereby narrowing the distance between the center of value in the market and the trader's evaluations.

It is in the laboratory that this classical view of price formation was destined to be literally replicated, although the first market experimenters considered themselves entirely within the neoclassical marginal tradition as expounded by Marshall, whose influence they acknowledge, unaware that Marshall was following closely the non-utilitarian classical tradition in describing price dynamics. (Chamberlin, 1948, p 96; V. L. Smith, 1962, p 115, 121)

3 Rediscovering Classical Economics Through Experiments

[18] Any experimental economist who reads the pages cited will see designs for experiment in this narrative. The passages were not cited in V. L. Smith (1962) because he had not yet read them, and only later could they be read and fully appreciated.

3.1 Early Market Experiments were Rooted in Marshall and Considered Neoclassical.

The first competitive market experiments set out to examine price and allocation behavior by assigning private values to 10-20 buyers, and private costs to a similar number of sellers. These were "small" numbers by the vague thought standards of the time. Inspiration for this design came from four motivating sources: (1) Jevons (1871) provided the background utility-maximizing choice model, although the implementation was with discrete units having no explicit connection with utility maximization—an afterthought reconstruction in the form of the theory of induced value, on which we elaborate below. (V. L. Smith 1976) (2) The context was consciously in the tradition of Marshall ([1890] 1920) in that supply and demand operated as flows of nondurable goods into and out of the market over successive price making periods in the belief that if equilibrium is attainable it must somehow involve learning over time. (3) Chamberlin's (1948) experiments had pioneered the procedure used to represent supply and demand, later generalized as "the theory of induced value." (V. L. Smith, 1976) (4) Finally, prices formed endogenously, among the participant subjects, in a unifying collective search, by independent privately informed traders, for economic value via the bid-ask continuous "double auction" protocol long operating in the Chicago commodity markets and New York Stock Exchange. (Leffler, 1951) Buyers were each assigned a single private value, "which represents the maximum price he is willing to pay for one unit…" (V. L. Smith, 1962, p 112; the first reported experiments did not use cash payoffs, but by the end that was changed by learning from the observations) A unit bought below this maximum willingness to pay earns a profit in cash equal to the difference

between the private value and the price paid in the experiment. Each seller is assigned a private value (cost) representing their personal minimum willingness to accept for a unit and earns a profit in cash equal to the difference between the selling price and that value. Thus, Marshall and Leffler inspired the elements that distinguished the new experiments from those reported earlier by Chamberlin (1948) that had failed to yield the competitive equilibrium. However, there was no prior expectation that the results of the new experiments would differ qualitatively from those of Chamberlin, although the falsifying conclusion might be more powerful under conditions putatively more favorable to equilibrium emergence.

Chamberlin's experiments were also explicitly and firmly thought to be in the neoclassical tradition associated with Marshall. Thus, Chamberlin implemented supply and demand by giving each buyer (seller) a "ticket" representing their individual "Marshallian demand price or supply price…".[19] (Chamberlin, 1948, p 96) Marshall, and those following his example who thought they also were following the neoclassical tradition, had explicitly stated, and interpreted demand (supply) as representations of willingness to pay and willingness to accept reservation prices.[20] Hence, the reservation price language prevailed and was

[19] Chamberlin (1948, p 95) draws on the neoclassical tradition of both Jevons ([1871] 1888) and Edgeworth (1881), but without citation. His experiments were "designed to illuminate a particular problem….that of the effect of deviations from a perfectly and purely competitive equilibrium [Jevons' proposition] under conditions (as in real life) in which the actual prices involving such deviations are not subject to 'recontract' (thus perfecting the market) [Edgeworth's construction] but remain final.

[20] "This article reports on a series of experimental games designed to study some of the hypotheses of neoclassical competitive market theory." (V. L. Smith, 1962, p 111) Skepticism, reflecting prevailing beliefs, is

sufficient, without the trappings of utility theory: “These reservation prices generate a demand curve such as DD…” (V. L. Smith, 1962, p 112). Implicitly, they were “marginal” units, as were subsequent cases assigning more than one unit, each interpretable as marginal reservation willingness to pay (willingness to accept) prices (sic values).” Note, however, that if each buyer (seller) is a single unit trader, *marginal value is identical with total individual value*. If aggregated over all individuals in a large market, demand (supply) is a schedule of the maximum the market will pay (minimum it will accept), *rather than go without each unit*. This representation corresponds to that of the classical, not that of the neoclassical, paradigm. It generalizes if any or all have a demand for a second or third unit, etc., which, in the classical perspective might have lower (or equal) urgency but with the important measurable feature that no more would be paid for it in the hierarchical ordering of commodities in demand. Hence, in retrospect, the early experimental environments were classical to the core, where the double auction rules of interacting and trading governed the “higgling and bargaining” process.

Later, as experimental economics gained traction in research and teaching, experimentalists encountered the methodological objection that experiments are not about the application of economic theory to economic problems involving real goods. One

plain: “These schedules do nothing beyond setting extreme limits to the observable price-quantity behavior in that market. All we can say is that the area above the supply curve is a region in which sales are feasible, while the area below the demand curve is a region in which purchases are feasible…. We have no guarantee that the equilibrium defined by the intersection of these sets will prevail, even approximately, in the experimental market (or any real counterpart of it).” (V. L. Smith, p 114)

effective response to this objection was to demonstrate such a connection using Jevons' formalism showing how, through monetary rewards in the laboratory, we can induce a specified demand and supply on abstract items, or on the decisions in any economic environment familiar to economists. (V. L. Smith, 1976) Hence, "Induced value theory" formally relied on the commonly accepted neoclassical continuity of individual utility functions, while proposing discrete multiple unit implementations as reservation "prices" (values). Since US currency had value, or "utility" to all, money earned as a function of action induced monetary value on marginal successive decision actions. The intent and purpose of this rhetorical formalism was to shift the burden of proof to other neoclassicists as to why this was not what all economists were up to in their daily routines. Hence, the inference, that the laboratory microcosm was a recognizably familiar environment to all economists for the study of economic action. However, the continuous utility formalism obscured its classical observational foundations in people's revealed willingness to pay (willingness to accept), inadvertently justifying laboratory investigations dominated by decisions in continuous commodity spaces. That this was an essay in persuasion is plain in the following summary assertion:

> "The laboratory becomes a place where real people earn real money for making real decisions about abstract claims that are just as "real" as a share of General Motors." (V. L. Smith, 1976, p 275)

3.2 Experiment Results Were a Victory for Markets, A Failure for Jevons/Walras Theory. So how Do We Explain/Model the Observed Convergence?

In the early experiments none of the conditions believed to be strictly necessary by Jevons and the economics profession were satisfied. All value information was private, numbers were "small"; no participant in the double auction mechanism was a price-taker; each participant was a maker of prices, who entered bids or asks, as well as a taker of prices, in the sense that they accepted a standing best bid or ask entered by another person. [21] For these reasons, the strong expectation was that equilibrium would not emerge. Unaccountably, prices and exchange quantities converged to near-equilibrium levels in a few rounds of trading. The conjecture that this might be a non-confirming artifact of symmetry in buyer and seller surplus failed to find support, as new experiments with asymmetric designs showed empirically that the first results generalized. Experiments demonstrated convergence in both symmetric and asymmetric surplus designs. (Compare Test 1, p 113 and Test 7, p 119 in V. L. Smith, 1962) What, however, best explains the replicable observed dynamical equilibrating motion in these markets—the Walrasian, the Marshallian, or some other mechanism?

Early in the first reported experiments, the transactions in two experiments conflicted with the neoclassical "Walrasian hypothesis" that prices increase (decrease) in proportion to positive (negative) excess demand. (V. L. Smith, 1962, Test 2 and Test 3, pp 116-119) The

[21] At the start of trading in an experiment, and at the typical market opening on an Exchange, there is always a first bid or ask, and therefore public information volunteered non-rationally by a private participant. Such action, however, is a challenge to the theory of optimal action.

data also conflicted with the "Marshallian hypothesis" that the exchange quantity is an increasing function of the excess of the demand price over the supply price, "but this hypothesis would seem to be worth considering only in market processes in which some quantity-adjusting decision is made by the marketers." (V. L. Smith, 1962, p 119, footnote 7) In retrospect, this quotation reveals a failure to appreciate the short-side rationing principle fundamental to classical analysis, depending on whether initial prices begin below or above the unknown equilibrium level. Thus, at a lower price level, purchases are limited by supply, at the higher, sales are limited by demand. This short-side feature is central to our formal theory of classical markets, and critical to the classical characterization of the price formation process. This condition, reflecting the disequilibrium state of demand and supply, is foreign to trader understanding, but very much part of their experience and tacit knowledge-how to function. If product purchases are limited by supply, the corresponding demand price is temporarily above supply price and the shortage condition experienced by the traders leads naturally to price increases to "ration" the limited supply. If product sales are limited by demand, the corresponding supply price is temporarily above the demand price, and the losses experienced by the traders leads naturally to seller price cutting. Note that the experimental protocol does not literally follow Adam Smith's narrative in specifying what sellers "brought to market."[22] Rather, the market is for goods made to order; each

[22] That narrative, however, applies to one experiment reported in the Appendix by V. L. Smith (1962). Unlike the other experiments and the discussion in the text, in this experiment subject sellers were required to decide their production levels in advance, then enter or not the market with those inventories. The sunk-cost property of that experiment led to distress sales, low initial prices, and gradually increasing prices across

knows their capacity; people start trading, and the initial contract prices may be below or above equilibrium. In each case, however, the principles of short side rationing apply to the market trader, and to our modelling of the price adjustment process. Significantly, the adaptations to short-side rationing involves reducing loss or increasing gain, and therefore lives in the profit space of the actors who are focused on gain or loss for a next unit to be exchanged. This multilateral interaction process is obscured in the metaphor of the auctioneer who seeks a price that equates total units supplied with total units demanded—a mechanism that invites withholding to get an advantage in profit space. Evident in this metaphor is the Walrasian theorist, implementing their theory, in contrast to observing the experience and actions of those in the market and modeling them.

Marshall's effort to synthesize the classical and neoclassical traditions does not recognize classical short-side rationing principles. Rather, Marshall assumed that goods were in short-run inelastic supply (like Adam Smith's perishable oranges). (Marshall, [1890] 1920, Book V, ch. III, V) Marshall's demand price was above (below) the long run supply price, causing profit maximizing entry (loss minimizing exit). With personal services, and appliance or home maintenance services, and goods made to order (hamburgers), consumption (delivery) commonly occurs after market pricing. Hence, one simply does not make units that fail to be sold; supply is therefore elastic, and responses based on short side rationing are part of day-to-day short run price decision-making.

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successive trading periods, as sellers learned that buyers were willing to pay much higher prices than sellers were settling for initially. In effect, the product was perishable, as there is no provision in the experiment for allowing the carryover of unsold inventories into the next period.

The surprise finding in the first experiments, that prices tended strongly to converge, called for an explanation. The data and reasoning in V. L. Smith (1962) indicated that the equilibrating force of price adjustment in these markets is more strongly related to money being left on the table in the absence of adjustment—profit foregone—than to either the neo-classically dominant Walrasian excess demand hypothesis, or the Marshallian gap between demand price and supply price. Convergence was in price-profit space, which included quantities as part of finding prices that were mutually profitable in total.

Figure 3.2 illustrates the classical mathematical treatment in Section 4 and correction of the discoveries related to the similar figure 1 in V. L. Smith (1962, p 130). We use Figure 3.2 to explain the results established more formally below. We begin by identifying excess rent, and the empirical case favoring it in the first experiments; then we articulate our representation of the classical price adjustment process.

1) Excess rent; comparison with excess supply and excess demand price over supply price.

   At price P(t) < P* (market-clearing equilibrium), or by symmetrical construction at P' (t) > P*, the total market implied surplus, V(P), is defined as the area below the quantities demanded and supplied and illustrated in Figure 3.2 as the total area B+S +F +E, where B, S, F, and E, are each identified by the cross-hatched areas so labeled. In Section 4 it appears as the integral equation (3). Because it is not sustainable, it was named "virtual surplus" in V. L. Smith (1962; Figure 1). If we define EQ (P*) = B* + S* as the total Marshallian buyer plus seller equilibrium profit surplus achieved it the market, then excess rent is

$$E = V(P) - EQ(P^*).$$

Hence, E measures the total profit sacrificed if P fails to reach P*, the triangular area below supply and demand shown in Figure 3.2.

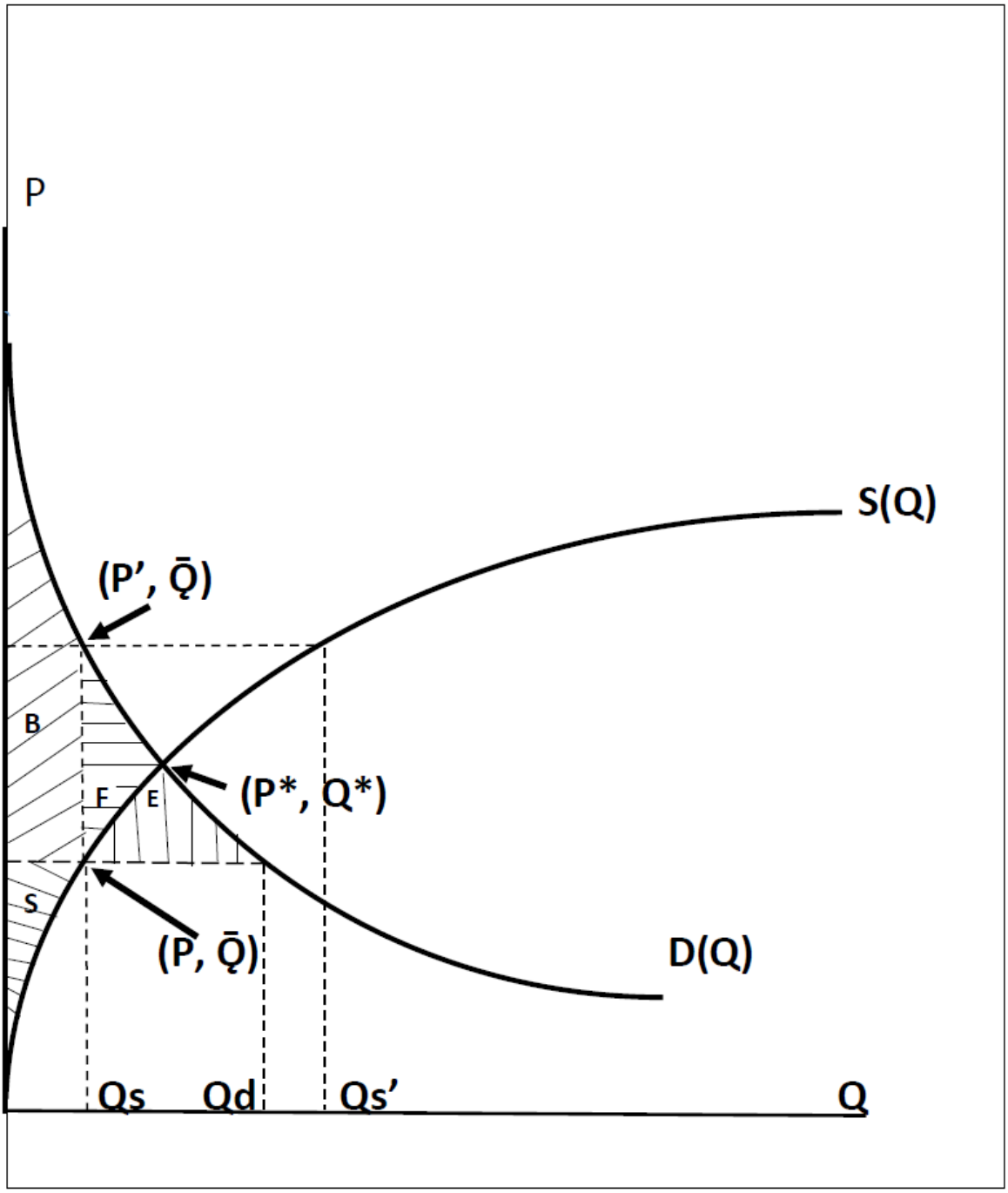

Figure 3.2. Short Side Rationing and Classical Price Adjustment.

B, F, S, and E surplus components identified by unique cross-hatching.

V(P) = B + F + S + E (Lyapunov function)

EQ(P*) = (B* + S*) (Marshall Surplus)

E (P) = V (P) – EQ (P*) (Excess Rent)

TS (P) = B (P, $\bar{Q}$) + S (P, $\bar{Q}$) (V (P) Constrained by Short-side rationing)
Min V (P) = Min E (P) = Max TS (P) = EQ (P*)

Empirically it was conjectured that price adjusts proportionately to E, the surplus profit that will fail to be captured unless there is an increase in the price, P(t) < P* (or lowering it if P'(t) > P*). (V. L. Smith, 1962, p 133) In comparison, according to the so-called Walrasian hypothesis, price adjusts proportionately to the excess supply, or rises in proportion to S(Qd) – D($\bar{Q}$), and falls at P'(t) in proportion to – [S(Qs') – D($\bar{Q}$)]. Marshall proposed that the quantity produced, $\bar{Q}$, increased in proportion the excess of demand price over supply price, P' – P.

Note, that virtual surplus, V (P), but also excess rent, E = V (P) – EQ (P*), is minimized at equilibrium. Thus teleologically, an efficient market *minimizes the profit reward necessary to evoke the supply necessary for satisfying demand*, a proposition on efficiency that exuded with intuitive appeal. Neither the excess supply, nor the excess of demand price over supply price, capture measures of profit foregone in the absence of price adjustment, features judged to be important and to characterize the results, if imperfectly understood in 1962.

The reported analysis of the data across all the experiments tended to support this "excess rent" hypothesis against the Walrasian (or Marshallian) hypothesis. (V. L. Smith, 1962, pp 127-132) However, none of the original experiments were specifically dsigned to perform a comparison test that would cleanly distingush these alternative prediction hypotheses.

In a subsequent study this problem was addressed using the "swastika" supply and demand exhibiting constant excess demand (supply) but declining excess rent; where excess rent =

price x excess supply. (V. L. Smith, 1965; illustrated in Figure 5.2 below.) Hence, under the excess demand hypothes, price decays linearly with transactions; under the excess rent hypothesis price decays exponentially with transactions. The new experiments offered stronger support for excess rent.

Missing in this modern theory-laden empirical description of price formation is a more precise specification of the "higgling" exchange process, conceptually involving the traders' experience, as described by the classical economists. As theorists we seek to read the theory content in actions; also desirable is to know how the actors read each other's actions. The latter task surely predominates in a fledgling science struggling to define itself. Experienced exchange provides the potential means of modelling the dynamics of agent price discovery.

2) Short side rationing and classical price dynamics.

Now modify the above argument to consider classical short side rationing and dynamic price adjustment, illustrated by further reference to Figure 3.2. We follow the argument in A. Smith, ([1776] 1904, Book I, Chapter VII), "Of the natural and market price of commodities." We read his argument as an observational description of trader experience, and how they respond, shorn of theoretical interpretation. By "natural price," A. Smith means supply price—the price that covers all unit costs including a unit profit necessary to bring forth and sustain the corresponding quantity in supply.[23] Although theory-laden, the

[23] Marshall importantly clarifies and extends the concept of "natural," or normal, supply price, which "is the real drift of that much quoted, and much-misunderstood doctrine of Adam Smith and other economists that

conditions he has in mind are not restricted only to the long run, as indicated by his example of perishable oranges, compared with scrap iron that can be carried over for tomorrow's selling opportunities. (A. Smith[1776] 1904, Vol. I, p. 59) When the quantity "brought to market ($Qs = \bar{Q}$ in Figure 3.2) falls short of the effectual demand, all those who are willing to pay the whole value" of the supply price P, in Figure 3.2, "which must be paid in order to bring it thither, cannot be supplied with the quantity they want" (Qd in Figure 3.2). A. Smith ([1776] 1904, p 58)[24] Similarly, "When the quantity brought to market (Qs' in Figure 3.2) exceeds the effectual demand ($Qd' = \bar{Q}$ in Figure 3.2), the market price falls below the natural; it cannot be all sold to those who are willing to pay the whole value (P' in Figure 3.2)….Some part must be sold to those who are willing to pay less, and the low price which they give for it must reduce the price of the whole."[25] (A. Smith [1776] 1904, p 59)

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the normal, or "natural," value of a commodity is that which economic forces tend to bring about in the long run." Marshall ([1890] 1920, p 289)

[24] Notice that in A. Smith's specification, sellers need have no perception, or understanding, that price is "below equilibrium." They simply experience the fact that when they bring $Qs = \bar{Q} < Q^*$ to market they find that buyers want to buy more (Qd) than the trucked amount, with each responding naturally in their own interest. Similarly, when they bring $Qs' > Q^*$ to market, they find that they cannot profitably dispose of it and cut prices. Each knows their own cost and, together with other sellers, know that market prices enable better or worse terms relative to those costs, and proceeds to adjust accordingly. Each may have beliefs, including conspiratorial beliefs that have no foundation, but each knows what to do—Ryle (1946) calls it knowledge-how as distinct from knowledge-that.

[25] Recall that in the experiments all goods (services) are non-durable and made to order (like sandwiches). Unlike A. Smith's narrative we do not have sellers with inventories "brought to market;" all sellers are present and eager to sell in the market.

If some trader in the market posts a bid (or ask), others may "enter" the competition for the unit demanded or offered. Hence, in general, at any temporary or trial price, $P(t) < P^*$, short side volume is $Q = \bar{Q}$ and we now write explicitly that if price is P, then

$$V(P) = B(P, \bar{Q}) + S(P, \bar{Q}) + F(P, \bar{Q}) + E(P),$$

where, as in Figure 3.2, B (S) is buyer (seller) realizable surplus and F is the disequilibrium surplus due to short side rationing and a source of lost profit if price fails to adjust. As "virtual surplus," V(P) measures, and serves as a collective summary signal of potential profit for all buyers and sellers. Each trader, experiencing a fragmentary part of the profit, V(P), is motivated to concede in the interest of capturing their individual part of the surplus. To concede in profit space is to out-bid a fellow buyer or under-cut a fellow seller but waiting for concessions from the other side is also part of the process. In the experiments think of P(t) as a temporary resting or trial price (bid or ask) at which the most urgent buyers receive profit up to $B(P, \bar{Q})$, and the most eager sellers earn up to $S(P, \bar{Q})$. We can infer that the traders contained in these sets (highest value and lowest cost) tend to contract with each other, because it explains why efficiency is so high in experiments although there are many disequilibrium prices. Hence, the total realizable surplus, at price P, is V(P) constrained by short-side rationing, or:

$$TS(P) = B(P, \bar{Q}) + S(P, \bar{Q}),$$

as shown in Figure 3.2. The set F defines contract-feasible prices greater than P(t) for closing the price-value gap. Buyers that are most profitable in F (at price P) easily outbid the lower

valued buyers. Hence, new contracts efficiently invade F, and the total surplus B+S+F is shared among the collective of all buyers and sellers.

A similar argument applies if the price P' is the temporary trial price, and the sellers most profitable in F (at price P') easily undercut their higher cost rivals The numerous experimental markets that achieve full efficiency are effectively narrowing the price-value gap through competition to capture contracts in the sets S and B, but also F. The "higgling" and contracting action by the collective of traders directly operates to shrink the set F. The set E and the price-value gap efficiently decline *because F is shrinking*—a (classical) regularity property of the experiments that is not part of the argument, not made plain, in V. L. Smith (1962; Figure 1); that we seek here to correct; nor of course is it plain in Marshall.[26] Similarly, excess demand, and Marshall's excess of demand price over supply price, both shrink derivatively because F is shrinking, and neither is causal in price determination. None of the three correlates of price adjustment relate directly to the individual trader's profit-loss motive for accepting a price change.

[26] Value (willingness to pay or willingness to accept) is the (potential price) dependent variable as in Marshall, and in the experimentalists' common representation of demand (supply). That is, quantity is the given independent variable, with value to be realized dependent on quantity, and in the classical model where buyers arrive with values, seller with costs. Total surplus, TS (P) imposes short side rationing for quantities that support $P \neq P^*$, and the dynamic market response distinguishes demand unit value from supply unit cost. If $P < P^*$, Q increases with entry as P increases; if $P' > P^*$, Q increases with entry as P decreases. Demand price exceeds supply price, and P *becomes the independent variable* in characterizing the market price adjustment effects, V (P) or TS (P).

Finally, referring to Figure 3.2, observe that Adam Smith states that if the market begins at a price too low, the price increases, and if it begins at a price too high, it falls. Mathematically, the price change always has the same sign as excess demand. This is equation (1) in Section 4.

Summary of the classical market process:

Buyers arrive at market with maximum willingness to pay values for given quantities of the good they desire, v = d (Q); symmetrically, sellers arrive with willingness to accept costs for given quantities they want to sell, c = s (Q).[27] Prices do not yet exist as they are found only in buyer-seller interactions in the market via the process described above. Price, P, negotiated in that process, becomes the independent variable associated with contract quantity, Q = D (P) = S (P).

4 Mathematical theory of classical price formation

Classical price theory leads naturally to characterizing market adjustments in profit opportunity space, and to the "principle of maximum information", which is more fundamental than the mechanistic market-clearing law of supply and demand: the principle of maximum information minimizes the distance between market price and the center of social-economic value. That center is more general than market clearing because it

[27] Marshall writes P for v and c in anticipation of a common price that emerges in a market such as the country corn exchange, but they exist as pre-market unit values and costs only, that serve to bound price. In the market process, some of them become contracts, and if a common price is reached, then conceptually, P determines the corresponding contract quantity, Q.

embraces short-side rationing, thereby including constant cost industry (and constant value demand) cases. The relevance of this generalization of competitive equilibrium is clearly illustrated in the swastika experimental design alluded to above. (Smith, 1965)

We provide here only a brief sketch of our classical mathematical theory of price discovery as it relates to the early experimental market literature. [See Inoua and Smith (2020b) for a more complete statement with proofs.] Competition of traders implies the classical dynamic law of supply and demand: consider a transaction price move from p_t to p_{t+1} resulting from competition of traders. If this price move involved more units demanded than supplied, then $p_{t+1} > p_t$, otherwise, $p_{t+1} \leq p_t$. The number of units buyers and sellers can profitably buy and sell respectively is by definition given by the distribution of values and costs $D = D(p)$ and $S(p)$. The law of supply and demand then reads:

$$\int_{p_t}^{p_{t+1}} [S(x) - D(x)]dx \leq 0.$$

Let

$$V(p) = V(0) + \int_0^p [S(x) - D(x)]dx,$$

which is in fact none other than the famous area below supply and demand linked to the Excess Rent Hypothesis referred to in Section 3.2:

$$V(p) = \int_0^p S(x)dx + \int_p^{\infty} D(x)dx.$$

then the law of supply and demand asserts that this function is decreasing in time (technically, it is a Lyapunov function)

$$V(p_{t+1}) \leq V(p_t).$$

This characterization of the law of supply and demand suggests a more general definition of competitive equilibrium than market clearing, which needs not hold, supply and demand being (discontinuous) step functions by construction. The competitive attractor is naturally the set of prices over which the function V is minimum, which we call the center of value:

$$C = \arg\min V.$$

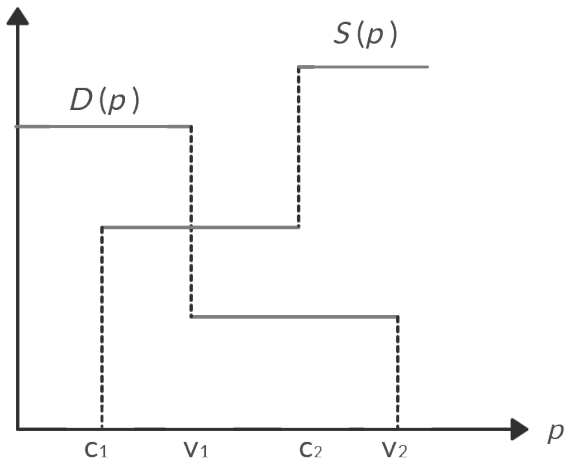

Figure 4: No market-clearing price exists for this market. The relevant equilibrium concept here is v_1, which can be shown to minimize market imbalance: we shall refer to this more general concept of competitive equilibrium, the center of value.

The fact that supply and demand are distribution functions has an important implication. It can be shown that V indicates the overall distance between price and the distribution of

reservation prices (values and costs), hence we shall call it the price-value gap, value meaning, more generally, all the reservation prices, costs included (cost is seller's value):

$$V(p) = \sum_{v \geq p} |v - p| + \sum_{c \leq p} |c - p|,$$

where the notation means summation of all values $v \geq p$ and all cost $c \leq p$, the qualification being due to that fact that no units will be traded at a loss. This equation establishes C as a generalized median of the distribution of values and costs. (Without the condition of mutually advantageous trade, we would have a simple median of the reservation prices, as is known from elementary statistics.) Thus, from the collective "higgling and bargaining" of traders each competing to extract surplus from trade, emerges a deep rationality of the aggregate: the traders are in fact unthinkingly causing the price to reflect in the best way their valuations of the good. Competitive price is a robust summary of value. This emergent informational optimization of the market can be unmasked by phrasing it in the language of Shannon information theory.

Consider again a move in the standing transaction price from p_t to p_{t+1} due to competition of the traders. This price move reveals public information about the traders' private underlying valuation that can be naturally quantified as a gain in information a la Shannon (1948):

$$I(p_{t+1}) - I(p_t) = \int_{p_t}^{p_{t+1}} \log \frac{D(x)}{S(x)} dx.$$

The total accumulated information gain from an initial price p_0 to the current price p is

$$I(p) = I(p_0) + \int_{p_0}^{p} \log \frac{D(x)}{S(x)} dx,$$

which is well-defined for transaction prices that live in the interval $[\min(c), \max(v)]$. [28] The law of supply and demand, as above formulated, can be phrased equivalently as

$$I(p_{t+1}) \geq I(p_t).$$

In other words, the competition of traders is such that price reveals more and more information about value, until maximum information is revealed, which is the case when price converges to the center of value C, and is also the set of prices for which I is maximum, as can be shown.

We refer to this informational characterization of competitive dynamics as the principle of maximum information.[29]

[28] The unit of information depends on the base of the logarithm, the standard choice being the base 2, corresponding to bits of information. The initial information $I(p_0)$ is of no major significance (since we are interested in the amount of information generated); so, we set it arbitrarily to 10 in the illustrations below.

[29] This characterization of classical competitive price dynamics is generic, qualitative, and "nonparametric": it does not depend on the specifics of the distribution of reservation prices, nor the speed of price adjustment, which involve parameters that likely depend on the specifics of the market institution. An extensive experimental literature developed on the rules of trading for a host of institutions, modern and ancient (Holt, in Roth and Kagel, 1995, pp. 360-377.) Among the market institutions, the continuous double auction is particularly interesting, in that it involves all forms of competition: buyer-buyer, seller-seller, and buyer-seller. Various models of this specific market institution have been offered in the experimental literature (Friedman, 1991; Cason & Friedman, 1996; Gjerstad & Dickhaut, 1998; Anufriev, Arifovic, Ledyard, & Panchenko, 2013; Friedman, 2018; Asparouhova, Bossaerts, & Ledyard, 2020).

5 Böhm-Bawerk and Marshall Invoke Classical Competition to Articulate Market Price Formation

5.1 Illustration of the PMI: Böhm-Bawerk Horse Market

Bohm-Bawerk's horse market illustrations offer one the best, detailed, expositions of competitive price formation in the neoclassical literature. (Bohm-Bawerk [1888] 1891 pp. 198-9) He begins with the simplest case of price formation, an isolated buyer-seller haggling, in which the final competitive price may end up anywhere in the range between the seller's willingness to accept valuation for the horse (10) and the buyer's willingness to pay valuation (30); the equilibrium bargaining price set, which, as can be easily derived graphically, coincides with what we call the value center: C = [10,30]. Then Bohm-Bawerk considers the other possible forms of buyer-seller competition (see our discussion of the two types of competition in section 2.4): one-sided (seller) competition (1 seller, 3 buyers); one-sided (buyer) competition (3 buyers, 1 seller); and finally, two-sided (buyer-seller) competition (10 buyers, 8 sellers: see Table 1 and Figure 7 below), showing how, as the number of competitors increases, the equilibrium bargaining price range shrinks toward a unique point: thus, in the final illustration, the equilibrium bargaining range is C=[20,20.1]. (Bohm-Bawerk [1888] 1891, p. 207)

Buyer Values	Seller Values
30	10
28	11
26	15

24	17
22	20
21	21.1
20	25
18	26
17	
15	

Table 1: Buyers' and sellers' values in a Bohm-Bawerk's illustration of Two-way Competition in a Horse Market 10 buyers, 8 sellers.

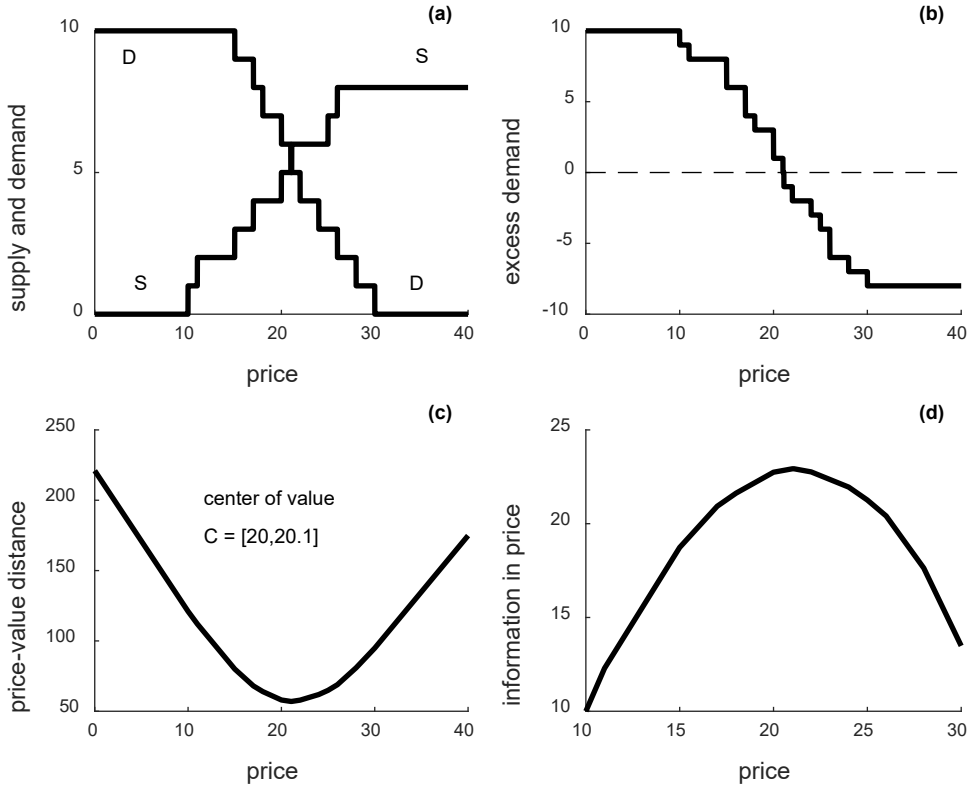

Figure 5.1: Two-sided (buyer-seller) competition (10 buyers, 8 seller).

Graphs of Bohm-Bawerk's: (a) piecewise constant supply and demand step functions; (b) piecewise constant excess demand step function; (c) continuous piecewise linear integral, V (p), of the excess supply function; (d) Shannon-like continuous piecewise linear measure of information revealed in price, with maximum information revealed at C.

5.2 **Example of a Market that Does Not Clear**

All markets clear in Böhm-Bawerk's discussion: hence the value center *C* refers merely to the set of clearing prices. His examples, however, nicely illustrate the general domain of validity of the supply and demand diagram (often narrowly interpreted as applying only to a hypothetical "perfect competition").

The PMI applies also to non-clearing markets, as illustrated in Figure 5.2.

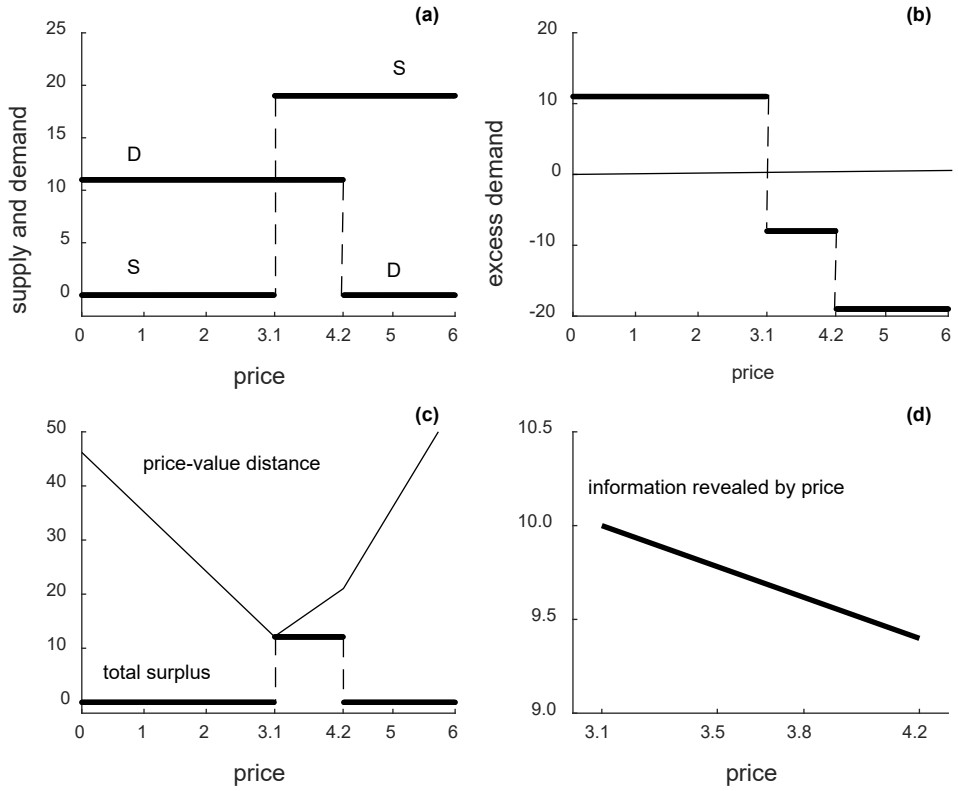

Figure 5.2: Illustration of a well-behaved non-clearing market. A swastika supply and demand configuration studied experimentally. (Smith, 1965). Graphs of: (a) the piecewise constant supply and demand functions; (b) the corresponding piecewise constant excess demand function; (c) the piecewise linear continuous integral of excess supply, V(p), in equation (3), minimum at center of value, C = 3.1; (d) Shannon-like continuous linear information revealed by hypothetical price change from an initial condition to a current position: the information revealed in price is maximum at C = 3.1.

These simple illustrations contrast sharply with neoclassical assumptions of the necessity of “perfect competition” and “market clearing”; of a “large number of traders”, and continuous individual preferences defined on infinitely divisible commodity spaces. The key characterizations of markets follows from the mathematical operation of integration, is

captured in the price value gap, V(p), and the Shannon-like measure of information in price, both continuous functions; based, however, on discrete distributions of willingness to pay unit values, and willingness to accept unit costs; where price change and excess demand have the same sign, and with short side rationing, at any standing price.

5.3 Alfred Marshall on Markets: More Classical than Neoclassical

Marshall's theory of supply and demand is expressed in classical language. However, it is based on neoclassical principles, seeking to derive an individual's quantity choices from their utilities. (Marshall [1890] 1920, Bk. 3, Ch. 3) This dichotomy stems from Marshall's intention to integrate classical market analysis, the relevance of which he greatly admired, with the neo-classical marginal utility calculus, which he believed brought important new insights and had been missing in the old framework. While his exposition was described in detail, Marshall chose not to reproduce explicitly its mathematical derivation. (Marshall [1890] 1920, Bk. V) This led to ambiguity in its meaning and interpretation because of *the inherent conflict between price formation as a process, and the derivation of optimal individual quantity responses to hypothetically given prices.* We will reconstruct Marshall's theory, while bearing this dichotomy in mind, and commenting thereon in context.

Marshall's statement of the theory goes as follows. "Suppose, for instance, that tea of a certain quality is to be had at 2s. per lb….a person might be willing to give 10s. for a single pound once a year rather than go without it altogether…But…he buys perhaps 10 lbs. in the year…the difference between the satisfaction which he gets from buying 9 lbs. and 10 lbs. is enough for him to be willing to pay 2s…it measures the marginal utility to him" of buying the 10^{th} pound. (Marshall [1890] 1920, pp. 94-5) In this example Marshall refers to 2s as the

person's *marginal demand price*. Let $P(x)$ be the amount the consumer is willing to pay for x units of the good; namely the maximum amount one is willing to pay for all x units thereof. Then $P'(x)$ is the marginal demand price of the x th unit. Notice that this function is the consumer's estimate of the value to him of a variable amount of the good, x. This value has the same dimensions as price, dollars per unit, but no actual prices are yet entering the theory. These values per unit are not hypothetical; literally, they are only estimates of value, and they reflect intentions subject to error. They are revealed as people drop out of the bidding with rising bid prices at an auction, the elements of which we have indicated that Adam Smith was fully aware. Although the values are uncertain in advance, they are real in intention and motivation. Obviously, in spite of the "tea" example, x is a discrete variable in practice so that $P(x)$ is trivially discontinuous. He further postulated a fictional 'representative consumer' whose demand-price would summarize or typify those of all consumers.

Postulating a large market, Marshall assumed $P(x)$ to be a smooth function. The principle of diminishing marginal utility in this context simply means that $P'' < 0.$ Classical economists, however, would have had no trouble with diminishing willingness to pay for successive units, which is quite explicit, for example with Dupuit (1844, p. 337). Rather, their focus was on the market and representing demand across individuals, not a quixotic continuously variable individual demand. The consumer 'surplus' is $P(x) - px,$ where p denotes not the market price in fact, but a price offered by some seller: Marshall was

imagining a 'haggling and bargaining' taking place in the market, even as he is illustrating the derivation of demand a la the neoclassicals. Demand *d* is defined by that quantity which makes for an 'efficient' consumption at a given price, namely that which maximizes the 'consumer's surplus', which is the case 'only when the price which he is willing to offer reaches that at which others are willing to sell'; that is, when $P'(x) = p$ so that the demand function, as in modern theory, is defined by $d(p) = P'^{-1}(p)$. Marshall ([1890] 1920, p. 95) Supply is identically derived. For Marshall, still wearing his classical hat, demand-price as a function of quantity is considered a primitive and intuitive concept in itself, and he did not judge it necessary to reverse the axes of his diagrams as he went from the marginal utility curve $[x, P'(x)]$ to the demand curve $[p, P'^{-1}(p)]$; and the same applies to supply. A sympathetic reading suggests that Marshal's intention was to preserve classical demand as representing willingness to pay value per unit as the dependent variable, given commodity quantity as the independent variable. This is precisely what people bring to the market in Marshall's description of corn-price formation in a country town. People are going to market with intentions—willingness to pay and willingness to accept as functionally dependent on quantities. Having arrived at the market, they proceed to turn this representation on its head. When offering quantities to buy or sell, they are quoting prices above their minimum willingness to accept, or below their maximum willingness to pay. To wit, in the higgling and bargaining, price estimates as intentions have been transformed into independent prices as contracts (if accepted) that determine realized quantities transacted. To Marshall's credit he wants to explicate that classical process right out of

Adam Smith. What survives, however, in the mind of every economist and student is that supply and demand is about quantities that maximize utility, given hypothetical prices, but demand is inverted in its representation in the Marshallian charts and in the math. *Missing is a clear indication that prices as unit value intentions become the independent variables in the price discovery process.* Hence, in failing to recognize this important transitional step in the process, Marshall failed to integrate the old and new traditions although both were part of his thinking.

Against this background reexamination of Marshall, we revisit the theory of induced value which was how experimentalists thought of connecting laboratory experiments with standard economic theory, and deriving demand in the form, $q = d(p)$. (V. L. Smith, 1976, p 275) Thus, subject i is paid cash, M_i, for q_i units where each unit is bought at some price, p_{ij}. Total payment is $M_i = R_i(q_i) - \sum_j^{q_i} p_{ij}$. [In Marshall above, total surplus from x units is $P(x) - px$.] If, in anticipation of what we expect (as theorists) to happen in the market, we impose the neoclassical hypothetical law of one price in the market, then $M_i = R_i(q_i) - pq_i$. If $U_i(M_i)$ is i's utility for money paid, $(U_i' > 0; U_i'' < 0)$, then Max $U_i[R_i(q_i) - pq_i]$ implies $U_i'[R_i'(q_i) - p] = 0$. Since $U_i' > 0$, $p = R_i'(q_i)$, where $R_i'(q_i)$ is i's classical willingness to pay demand function defined on units, qi. (V. L. Smith (1976, p 275), being thoroughly neoclassical, inverts this function noting that the demand is induced by the reward scheme. But the step inverting the demand is a neoclassical error—if inversion occurs it is in the market. $R_i'(q_i)$ is the willingness to pay demand function, corresponding

to Marshal's $P'(x)$ above. Of course, we do indeed have the math problem that if all, or some, come to market with the same willingness to pay, then only its inverse is well-defined, as in the piecewise-constant neoclassical demand experiments. (V. L. Smith (1965). But finding a lot of individuals who trade at the same price is in no sense problematic for a market. Now apply this procedure to Adam Smith and subsequent classical economists, including Marshall. They measured willingness to pay in terms of the maximum amount of money a person is willing to forgo for a unit of a good rather than do without it. Then attach utility, $U_i^{'}$ to that money measure. The above calculus follows, and we get the classical pre-market demand function.

So, Marshall's project was to classically reformulate the neoclassical view on value, namely by avoiding the notion of utility as a measure of 'pleasure', which is a problematic notion, quantitatively and observationally. Indeed no mention of 'pleasure' would appear anywhere in his theory, if not by his afterthought but his oft-debated clause of 'constant marginal utility of wealth', whose only purpose was to adjust the demand-price for differences in the utility of wealth, which is in reality not so much of a problem, as we saw in the text; it is only then that it erupted in his whole analysis, and in formal terms only in the Appendix, a pure neoclassical tone à la Walras, which can also be formally reproduced as follows. Let m be wealth (or 'money' as Marshall put it vaguely), $m(m)$ its utility for the consumer, u the pleasure he enjoys in consuming the good under study, and $u_1, \ldots, u_{n-1}$ the pleasures derived from consuming respectively $x_1, \ldots, x_{n-1}$ amounts of the remaining goods,

the demand-prices of which being $P_1(x_1), \ldots, P_{n-1}(x_{n-1})$. [30] Let the overall pleasure enjoyed by the consumer be additive, that is, $u(x) + u_1(x_1) + \ldots + u_{n-1}(x_{n-1})$; his budget constraint is $P(x) + P_1(x_1) + \ldots + P_{n-1}(x_{n-1}) \pounds m$. The first-order maximizing condition is $u'(x) = l\,P'(x)$, where l is a Lagrange multiplier, which, if identified with the marginal utility of wealth, leads to

$$\frac{du}{dx} = \frac{dm}{dm}\frac{dP}{dx},$$

which is the key equation of Marshall's *Mathematical Appendix.* (Marshall [1890] 1920, Note II). Assuming constant marginal utility of wealth, P becomes equivalent to u. Through this equation, Marshall merely wanted to show the general link between utility in the purely neoclassical sense and utility as given by the demand price. Moreover, Marshall suggests that this postulate of constant marginal utility of wealth is inconsequential on the aggregate of many consumers, poor and rich combined (Marshall, [1890] 1920, pp. 15-16, 83).

6 Summary

[30] "We tacitly assumed that the sum which purchasers were willing to pay, and which sellers were willing to take, for the seven hundredth quarter would not be affected by the question whether the earlier bargains had been made at a high or a low rate. We allowed for the diminution in the buyers' need of corn [its marginal utility to them] as the amount bought increased. But we did not allow for any appreciable change in their unwillingness to part with money." (Marshall, 1890, pp 324-5) Thus indeed was Marshall vague about whether "money" meant wealth or "income," but the common interpretation as it became widely propagated was that the optimization problem was to maximize the utility of multiple goods subject to an income constraint on the sum of all expenditure, $I = p_1x_1 + p_2x_2 + \ldots p_Nx_N$.

We identify a consistent thread of development in classical economic thought that is directed toward a supply and demand theory of market price formation or discovery. The theory is articulated by Adam Smith and further developed and refined by his French, English, and Italian followers. The foundation is in classical descriptions of demand, expressed in markets as maximum willingness to pay reservation values, for given discrete quantities of goods desired for consumption by buyers. Sellers, likewise, harbor minimum willingness to accept reservation values for these quantities based on their unit costs.

Although goods clearly have hidden utility value, individual reservation values and costs were respected classical measures grounded in observation, with buyers trying to buy cheaper than their maximum willingness to pay, and sellers trying to sell dearer than their minimum willingness to accept. Mathematically, demand is a distribution function of individual values reordered from highest to lowest that Cournot acknowledged. Supply is a distribution function of individual unit costs. Consequently, buyers and sellers arrive in the market with aggregate distributions of values, $v = d(Q)$ and costs, $c = s(Q)$. However, price, p, in this narrative is yet to emerge.

Based on these reservation value data, Adam Smith's description of price formation in market "higgling" involves two coordinate features: (1) a dynamic price-change version of the "law of supply and demand," and (2) the concept that we call "short side rationing." At a quoted offer too low, purchases cannot exceed the supply offered, and buyer competition for the marginal unit offered raises the price. At a quoted price too high, sales cannot exceed the amount demanded and sellers cut the marginal unit's price. Hence, the dynamic implication is that price increases (decreases) if there is excess demand (supply). The

integral of this signed derivative, constrained by short side rationing, defines the short side rationed surplus profit of the traders, which is what directly motivates realized market gains from trade. Neither the so-called Walrasian excess demand, nor Marshall's excess of demand price over supply price, are fundamental drivers of price adjustment but are merely correlates of the more fundamental classical adjustment process. Convergence can be toward states that include short side rationing as in a constant (unit cost) industry or in an English auction of a unique item.

Contrastingly, in the neoclassical marginal revolution, demand is derived from individual utility functions defined over a continuous commodity space, subject to given prices and income. Demand is conceptualized as a price-conditional, pre-market maximization task, intended to be part of the equation structure of general equilibrium. Similarly, for individual producer-sellers, supply is a pre-market price conditional cost minimizing exercise. This equation structure end-objective, however, fatally undermines the task of articulating a theory of price formation emanating from interacting buyers and sellers. Price is "given" rather than a variable to be determined. Hence, knowledge of price seemed either to require complete information or an "as if" adjustment process whereby prices were determined by the law of one price in a market. This impasse ended with the Sonenshein-Mantel-Debreu theorems proving that general equilibrium was silent in yielding results and in failing even to imply the law of demand.

We claim that this vacuous result is a consequence of the axiom of price-taking behavior and the law of one price in a market, thus justifying a reexamination of the more

observationally grounded, consistent, and rigorous classical conceptions of individual behavior in markets.

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