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Comments

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A Mathematical Statement of his Market Price Discovery Process

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Abstract: The relevance of Adam Smith for understanding human morality and sociality is generally accepted; witness the growing interest that his work is stimulating among scholars of various academic backgrounds (philosophers, political theorists, sociologists, economists). But, paradoxically, Adam Smith's theory of economic value enjoys a less prominent stature today among economists, who, while they view him as the 'father of modern economics', considered him more as having had the right intuitions about a market economy than as having developed the right concepts and the technical tools for studying it. Yet the neoclassical tradition failed to provide a satisfactory theory of price formation owing to the dominant axiom of price taking behavior; for if everyone takes prices as given, how do these prices emerge in the first place? Who is giving the prices? One early escape from this crucial price-discovery problem consisted of assuming that all traders should have complete information of supply and demand and the consequent equilibrium prices (Jevons, [1871] 1888) ; the other, that formed the

basis of general equilibrium theory, imagines a fictional auctioneer who finds the equilibrium prices by trial-and-error adjustments or tatonnement (Walras, [1874] 1954). Adam Smith's theory of the market mechanism (Ch. 7, Book 1, *Wealth of Nations*, 1776), as we shall argue in this paper, offers the right conceptual framework for understanding competitive price discovery, for which we offer a mathematical formulation. Mathematically, the driving force behind competitive price dynamics is not excess demand per se, but its integral; we make this concept, explored at the beginning in experimental economics (Smith, 1962), part of our formalization of classical competitive price dynamics. Finally, we explain key propositions of Smith's theory of value in the light of this mathematical formulation.

1 Introduction

The relevance of Adam Smith for understanding human morality and sociality is generally accepted; witness the growing interest that his work is stimulating among scholars of various academic backgrounds (philosophers, political theorists, sociologists, economists). But, paradoxically, Adam Smith's theory of economic value enjoys a less prominent stature today among economists, who, while they may view him as the 'father of modern economics', consider him as having had the right intuitions about a market economy but not as having developed the right concepts nor the

technical tools for studying it. The dominant historiography of economics draws indeed a picture of classical economics in which Adam Smith is overshadowed by the English classical followers (notably Ricardo), and it portrays the classical school itself as eclipsed in the 1870s by the neoclassical school. Yet the new school faces important difficulties, whose solution paradoxically leads back to the old school. First, on theoretical grounds: an important theorem established in the 1970s (by Sonnenschein, Mantel, and Debreu) uncovers an intrinsic lacuna in the core principle which consists of deriving economic regularities from individual rationality (for example the theorem shows that the demand of utility-maximizing agents is essentially arbitrary in the aggregate). So, we are left open to the possibility that economic regularities are better viewed as emergent properties in the classical school. Moreover, experimental economics established the stability, efficiency, and robustness of the market mechanism under conditions in which we should expect 'market failures' according to the standard neoclassical theory: markets with a few traders, who only know their private valuations of the good, and who generate the prices through their bids and asks, that converge to equilibrium and maximum efficiency. Adam Smith's theory of the market mechanism (*Wealth of Nations*, Book I, Ch. VII), as we shall argue in this paper, offers the right conceptual framework for understanding

competitive price discovery, for which we offer a mathematical formulation.

Our formulation derives from three classical economics principles:

Principle 1: It is a realistic portrayal of a market economy based on astute observation of individual behaviors and interaction in the marketplace.

Principle 2: From Principle 1, it derives deep emergent properties that are unintended consequences of these latter, the results of human actions and not of human design.

Principle 3: Supply and demand are classically given by an observable, operational, monetary value: the reservation price—the buyer's maximum willingness to pay (WTP) and the seller's minimum willingness to accept (WTA).

From these simple methodological principles, one can derive a rigorous picture of a market economy that has an integrity distinct from the neoclassical theory, as is shown elsewhere[1]. But this paper's goal is more limited to the fundamental problem of market price formation, as explained in Chapter 7 of Book 1 of *Wealth of Nations* (A. Smith, [1776] 1904). We explain key propositions of Smith's theory of value in the light of this mathematical formulation.

[1] This is proven in the authors' more general, unfinished manuscript on a rehabilitation of classical economics, to appear piecemeal in the forms of papers.

To understand Adam Smith's articulation of value theory today is uneasy because of at least three or four obstacles, which come down to an overall illusion of progress in economics whereby many think of history of economic ideas as if the present is regarded as subsuming the past, hence being legitimized to sit in judgment over its archaic version. The first difficulty in reading Adam Smith, and the classical school more generally, consists of treating Adam Smith's technical contributions to price theory as being superseded by those of his disciples (especially Ricardo, whose reduced formulation is largely responsible for the dismissal of classical economics as a mere labor theory of value that ignored the demand side of price formation); a fortiori, the classical school should not be viewed as subsumed and rendered obsolete by the neoclassical school. The second obstacle relates to the classical technical jargon (natural price, monopoly price, effectual demand, etc.), which to a large extent is outmoded today, and not always for good reasons. The third difficulty in dealing with Adam Smith's treatment of value relates to an equally outmoded tripartite articulation of value theory, which is often confused in modern commentaries. Until Marshall, value theory used to be organized around three problems: (1) the measure of value (consisting determining a universal and invariable stand-

ard of value); (2) the cause of value (consisting of deciding which cause, between utility and cost, is the ultimate, most primitive cause of value); (3) price theory proper, namely the problem of market price formation, to which Marshall reduce value theory. The first two problems are essentially metaphysical issues; yet they have been perhaps the greatest source of controversies among the classical economists, and perhaps the greatest source of misunderstanding about classical economics: it can be shown that the classical economists' obsession with labor is largely due to this scientifically peripheral issues, since labor was regarded to be the closest to being the invariable and universal value standard and ultimate cause of value (the first price paid for all commodities); yet none of the famous classical economists (except perhaps Ricardo) ever considered that this special status of labor implies that competitive market price formation amounts to a labor theory of value. For Adam Smith the relevance of the labor theory of value was confined to a hypothetical early and rude state of society, namely a primitive barter economy, in which all labor skills are identical (hunting skills, for example), land is not appropriated, and capital is non-existent as a separate factor of production; thus, starting from Ch. VII, Adam Smith expounded the theory of price formation from buyer-seller competition expressed in their aggregated supply and demand, which is relevant for a

modern economy. In fact, Ricardo is an exception in the classical school in his attempt to generalize the labor theory beyond the primitive case. Both Say and Malthus restated the classical value theory in the spirit of Adam Smith's formulation (Say, 1803 [2006]; Malthus, [1820] 1836; Say, [1828] 2010). But Ricardo had more disciples than the other classical economists!

Lastly, in terms of obstacles, classical economics is largely rooted in acute observational of real economic phenomena, but organized in informal, unsystematic way; but this unsystematic, nonmathematical nature does not make it less rigorous. Below is developed a general mathematical framework implicit in classical price theory; from this framework are derived key propositions in classical price theory.

2 Marshall's View on Adam Smith

Alfred Marshall is, of all the commentators of the history of economics, perhaps the author who most clearly understood the pivotal contributions of Adam Smith to modern economics. He regarded Adam Smith as having launched an epoch in economics when this latter built, from a core methodological principle overlooked in modern commentaries, a value theory that unifies all of economics ([1890] 1920, Appendix B, p. 627). This principle consists of dealing, as regards individual economic decisions, not directly with the unobservable ultimate psychological forces driving them

(need, desire, pleasure) but with the *monetary sacrifices* that people make in order to satisfy them: formally, their reservation prices.[2] Thus, the relevant concepts for demand and supply theory are the maximum money prices consumers are willing to pay and suppliers are willing to accept in the marketplace. This is a most fundamental classical principle the Marshall readopted in his reformulation of neoclassical value theory, in reaction to the hedonistic marginal utilitarianism of Jevons and Walras, who make pleasure the fundamental motivating category of economics. The WTP-WTA approach to supply and demand frames value theory throughout the classical literature; it is also adopted, not only by Marshall, but also by the Austrian marginalists in their explanation of competitive market price formation (Inoua & Smith, 2020). In the 1950s, moreover, experimental economists, inspired by Marshall's and Böhm-Bawerk's treatments of this classical principle, adopted it in their implementation of supply and demand functions (Chamberlin, 1948; V. L. Smith, 1962).

3 Adam Smith Belittled as an Economist

[2] The ongoing paragraph on classical methodology is a summary of a longer development on classical methodology as it applies to classical supply and demand theory.

Unlike Marshall, however, most commentators of the history of economics have belittled Adam Smith's technical contributions to value theory. A brief review is enough to show the extent to which even influential historians of economics have denigrated the classical school more generally as they interpret it in neoclassical terms.

J. Schumpeter, for example, considered that: '*There is no theory of monopoly [in the Wealth of Nations]. The proposition […] that "the price of monopoly is upon every occasion the highest which can be got" might be the product of a not very intelligent layman—taken literally, it is not even true. But neither is the mechanism of competition made the subject of more searching analysis. In consequence, A. Smith fails to prove satisfactorily his proposition that the competitive price is "the lowest which the sellers can commonly afford to take"—to the modern reader it is a source of wonder what kind of argument he took for proof. Still less did he attempt to prove that competition tends to minimize costs, though it is evident that he must have believed it.'* (Schumpeter, 1954 [2006], p. 294) Likewise, G. Stigler, in his historical essays, portrayed the classical concepts of utility and competition as archaic versions of their modern formulations (Stigler, 1957, 1982). He viewed Smith, not as the author of a unified theory of value, but as '*a manufacturer of traditions*', one of which is simply '*to pay no attention to the*

formal theory of monopoly' (Stigler, 1982). M. Blaug went further and concluded that '*Adam Smith had no consistent theory of wages and rents and no theory of profit or pure interest at all. To say that the normal price of an article is the price that just covers money costs is to explain prices by prices. In this sense, Adam Smith had no theory of value whatever*.' (Blaug, 1985, p. 39) Thus, it has become a common critique of Adam Smith that he held at best a confused view on value and income distribution. This misreading, we believe, is not solely induced by a neoclassical interpretation: it is reinforced by a Ricardian reading of Adam Smith, which is a lighter but similar bias. That is, the premise (or prejudice) that Adam Smith had a theory of income distribution à la Ricardo, one that is separate from value theory and primary with respect to it. Hence many commentators failed to see that Smith's views on wages, profits, and rents are merely consequences of his value theory, as explained in Ch. 7.

4 The Articulation of Smith's Value Theory

Today classical economics mostly refers to Ricardo's system, which is the central reference in much contemporary discussion on this school; thus, as noted above, Piero Sraffa's influential revival of this school is in fact a revival of Ricardo's tradition (Sraffa, 1960). Adam Smith is more generally over-

shadowed by his disciples. He is of course viewed as 'the father of economics'; but this is in fact an ambiguous title, which in any case does not follow, as we just saw, from the overall appraisal of his work even by the influential historians of economics (see section 3). It is as if he owed his stature in economics to an incidental reference to some 'invisible hand'. Yet his technical contribution to economics, be it insisted, is often belittled today because it is often misunderstood. Here is indeed how he dealt with value in his magnum opus, Book I (whose first part, Ch. I-III on the 'division of labor', pertains to economic development).

Chapter VII is theoretically the most important portion of this book, for it presents the general theory of market price formation, which Smith then applies, in Ch. VIII-XI, to explain the wages of labor (Ch. VIII and X), the profits of capital (Ch. IX and X), and the rents of land (Ch. XI). The previous chapters on value, Ch. IV-VI, are merely preparatory discussions on, first of all, *the nature of value*, which starts from the fundamental distinction between 'value in use', or the value a person attaches to a good in view of the good's utility, and 'value in exchange', or the ratio at which a good exchanges for another (p. 30). (A convention throughout the classical school consists of using the term 'value' (without qualification) in reference to exchange-value.) Smith then tackled the tricky problem of *the standard of value* (Ch.

V). When value is given in terms of this common measure, it is classically known as ‘price’, which is a much simpler notion (and so familiar one that the technical nature of its logical origin is easily forgotten). The problem then is to identify a medium that can serve as value standard. Smith first framed this problem in the most abstract way. He wanted a medium for comparing *‘the values of different commodities at all times and at all places’* (p. 38). Of course, this standard should be itself stable in value if it is to indicate the ‘real price’ of commodities. Adam Smith has already investigated the ‘origin of money’ (Ch. IV), for money being the universal medium of exchange and unit of account in modern economies, it is the natural value standard; but money being variable in value, ‘money price’ or ‘nominal price’ is a poor indicator of ‘real price’ over long periods. Moreover, money cannot serve as the standard for ‘all times’, for it was not used in the primitive state of society: barter, according to Smith, was the primordial type of exchange (Ch. IV).

There is in such speculation a clear temptation towards metaphysics, which Smith did not always resist, and which would obscure much of the later development on value theory, as emphasized below. For by ‘all times’ he literally meant all the times in which value is concerned, including the ‘early and rude state of society’; under this absolute requirement, it is easy to see

that no medium except labor can be used as standard, for labor is the only reality that is common to all exchangeable goods at all places and all times, including the hypothetical moneyless era or even beyond: '*Labour was the first price, the original purchase-money that was paid for all things. It was not by gold or by silver, but by labour, that all the wealth of the world was originally purchased.'* (p. 32) Thus the problem of the standard of value led Smith to consider *the origin of value*, which would be a second important topic in value theory. But we can even go further beyond and consider labor as the absolute origin of exchange-value; for whatever the primordial exchange was in human history, it must have indeed involved labor; assume, for example, that the first economic act in human history was the picking of a fruit: this then was the first time when emerged the phenomenon of exchange-value—someone has endured some labor in exchange for a fruit. The problem of the value standard, in other words, pushed Adam Smith on the verge of metaphysics: the origin of money, value, society, or even humanity. This is not to say that such discussion is uninteresting, irrelevant, or false; but to emphasize that it is an order of enquiry beyond the *science of value*, for its truth cannot be decided positively; hence we count this problem as part of the *metaphysics of value*. The most difficult problems on value, which triggered major controversies that involved most economists

from Smith to Marshall, pertain precisely to this metaphysics of value. For the modern reader, the difficulty in reading these controversies is compounded, because the different aspects of value are not often clearly distinguished. This confusion of science and metaphysics is therefore a serious obstacle against a modern reading of the history of value theory.

Fortunately, there is a clear enough demarcation between these two orders of investigation in Adam Smith's book. For, as already suggested, the metaphysics of value is mostly concentrated in Chapters 4-6, whereas the science of value truly begins in Ch. VII (and any note thereafter on the original conditions of humankind is passing and merely said by way of progression from the simple to the complex). Smith's pragmatism, moreover, eventually outweighed his metaphysics even in his preliminary speculations. For quantity of labor, he noted, is an abstract reality, which is not operational in ordinary transactions and is not easy to measure by its heterogeneity. In practice, money is enough: money price (or monetary valuation more generally) regulates almost the totality of ordinary economic life. Yet Smith needed also, as we would say today, a way of controlling for inflation when the very long run is concerned; hence his long digression (in Ch. XI) on the variations of the currency—primarily silver—whose real value he assessed in terms of corn, whose price, which was then available for four centuries back, Smith

believed to be stable from century to century, though it fluctuates from year to year. Thus, he measured the real price of silver by the amount of corn that this currency can buy. This important practical problem is of course handled today by the technique of index numbers.

Smith ended his preliminary discussions with a simple accounting of value, or 'the component parts of price' (Ch. VI), which is more a simplifying device than a necessary building block of his price theory. By construction, the total price of a good is the sum of the wages, profits, and rents that reward the three agents that produce this good—land, labor, and capital. So once price is explained in general, and wage and rent by implication, profit follows residually; thus if Smith explained the profit rate in general from the competition of capital (or 'stock'), he at times derived the overall pattern of profits directly from that of wages, which tend to evolve inversely (Ch. IX). Throughout the classical literature, the idea that price corresponds to cost has two senses, depending on the context, and this ambivalence is misleading if not kept in mind. The first one is the obvious accounting identity just noted: rent is the cost associated to land; wage is that associated to labor, and profit (which, as emphasized below, is classically viewed as a cost) is that associated to capital. Thus 'price equals cost' here is a mere truism.

The second meaning corresponds to a theoretical proposition of utmost importance in this school: it says that *price converges to minimum cost under free competition*. In the absolute, that is, as generalized to the whole economy, this proposition holds when the competition of landowners, of workers, and of capitalists is so intense that the rent, wage, and profit rates are the minimum they can be, and hence prices correspond to minimum costs. But more realistically, this proposition holds locally, to a given market, when the competition on the side of supply is so intense that only the most efficient suppliers succeed to sell the good, and at the lowest possible price. We will come back to this proposition below.

Sadly, this articulation of Smith's investigation has been so often misunderstood. It is in his metaphysics of value (and other preliminary discussions) that many readers sought his science of value; and hence they were misled to find a confusing web of views (including a so-called 'adding-up theory of value', namely the mere accounting of value, which has no greater status in his book than any accounting identity has in contemporary economics). In particular, some attribute to him a 'labor theory of value', owing to a mere speculation on the primordial era of humankind: '*In that early and rude state of society which precedes both, the accumulation of stock and the ap-*

propriation of land, the proportion between the quantities of labour necessary for acquiring different objects seems to be the only circumstance which can afford any rule for exchanging them for one another.' (p. 49) Clearly Smith is not thus explaining price formation in this primitive economy but was merely emphasizing the only objective exchange-value system that can be implemented therein: this is, in other words, a normative view, not a theory. Anyhow, as he turns to the relevant case of advanced market society, he emphasizes so many qualifications to this simple rule that it amounts to an opening or passing note, after which he turned to the theory of value proper (Ch. VII), which is in essence as follows.

The 'market price' of a good is regulated by competition of supply and demand. When, moreover, the competition on the side of suppliers is free, in the sense of being limitless and hence the most intense, the market price converges to the 'natural price', the lowest price at which the good can sell, namely the lowest cost at which it can be produced and brought to market. Classically, 'cost' includes indeed all that it takes to produce and bring a commodity to market, including notably a 'normal' or 'ordinary' profit expectation, that is, a minimum profit anticipation that makes worthwhile the toil of production (for none will produce who does not expect at least a minimum profit from sale): so technically, cost is classically a synonym for

supplier reservation price, and by price convergence to cost or natural price, the classical economists meant, not convergence to zero profit, of course, but to zero *surplus* above the overall minimum acceptable profit, namely the natural profit rate. Smith, and all of his disciples, insisted on *free competition* because they viewed it as a norm, and in two senses: it is classically the ideal case, the socially optimal state under which price are so low that consumers of all orders of society can afford it (in short, a state of cheapness and plenty); but they also considered it to prevail reasonably in practice, more frequently than one might expect, as long as market supply is not artificially restricted, and even if this is so, the natural course of a market *in the long run*, is towards a removal of these barriers. All limitations to competition, whether natural or artificial, are collective referred to as 'monopoly', a term whose classical meaning is so easily misunderstood that it might be better to avoid it altogether. The early neoclassical economists who, following an unfortunate innovation by Cournot, reduce monopoly to its etymology: a market supplied by a lone seller. But what if this lone seller happens to be the most efficient supplier of the good, who managed to undersell all rivals and thus brings the price to its natural level; the etymological use of monopoly, in other words, is a simplistic and self-defeating; and the classical economists, who were thinking at a higher level of understanding

than their commentators and critics, have not made such mistake. Under monopoly, as classically understood, namely under a limitation of competition among suppliers, the market price trends to the 'monopoly price', or the maximum price that the set of all demanders are willing to pay, depending on the extremeness of the limitation of competition. The classical notion of 'monopoly' may seem peculiar to the modern reader; but it is the modern reader's terminology that is questionable.

As simple as it seems, this theory is fascinatingly profound by its implications. First, it is universal, in that it applies to any market, whatever its size. Moreover, it leads directly to the characterization of the market mechanism announced earlier and formally proven below: *the market price of a good converges to a best summary of all the individual valuations of the good, namely the demanders' use-values and the suppliers' costs.* This proposition is proven below (Section 6

Smith's exposition is rigorous all along, except for one conceptual lapse. At times, he treated the natural price as if it were a synonym of equilibrium price, or even the price attractor in all markets; hence his referring to the natural price at times as the 'normal price' or 'ordinary price', as if free competition were a norm in the even stronger sense of being the normal state of affairs in all cases; hence also his oft-quoted yet misleading gravitation

metaphor, namely that market price gravitates around the natural value; hence finally his restricting demand to 'those who are willing to pay the natural price', in his explanation of how the price returns to equilibrium in response to an excess demand near this latter. All this would be benign if he were throughout assuming free competition, which is obviously not the case. Still, Smith's formulation of the theory of value has overall hardly been surpassed, and he left it at a position that calls for no more than some clarification, few conceptual additions, and a mathematical formulation.

5 The Technical Jargon of Chapter VII

Chapter VII introduces a number of technical terms, most of which we encountered and explained in the previous section, but which have also been used in different senses that should be discussed here, since this polysemy has created much confusion and controversy among the classical economists themselves. We mean the concepts of cost, natural price, effectual demand. This polysemy, which is largely due to Adam Smith, is one of the major limitations of classical economics, and an important source of confusion to the modern reader; it was a major source of logomachy among the classical economists themselves. A mathematical theory has no place for ambiguity of language; hence the mathematical formulation (Section 6 below) adopts a unique definition, chosen among the possible meanings

based on consistency. In fact, for the sake of formal theory, it is perhaps better, while we should keep the classical concepts, which as we shall see are much needed, to avoid the outmoded classical terminology in favor of formal equivalents, as follows (not that old vocabulary is less useful, but because of the polysemy and the different meaning that the modern reader prejudges they have; that is, we are suggesting a less ambiguous, formal version of classical terms, not to replace it with the neoclassical jargon, which it would be a mistake to think is more precise than the classical one).

We suggest the following correspondence:

Classical term	Unambiguous version
Cost	*Seller's reservation price*
Natural price	*Minimum price*
Monopoly price	*Maximum price*
Absolute demand (decided by need or desire)	*Quantity needed*
Effectual demand (decided by need, constrained by wealth)	*Quantity demanded*
Free competition	*Maximum competition on the supply side*

Here is a sketch of the different meanings attached to these terms, in alphabetical order.

Cost. Classical cost includes profit: *Cost = Prime Cost + Profit = Wages + Rent + Profit*. Thus, the proposition *Price = Cost* is strictly speaking an accounting identity: *Price* $\equiv$ *Wage + Rent + Profit*. It is a mistake therefore to read this

mere accounting decomposition of price as a price theory (a so-called 'adding-up value theory' and treating Adam Smith as confusing switching to various value theories from paragraph to paragraph. For example, J.-B. Say's recurrent statement that 'price cannot exceed cost of production' obviously assumes cost in the broader classical sense: he is thus merely enouncing an identity [see, e.g., the clear explanation by Say ([1815, 1848] 1966, Ch. VII, p. 31)]. But *Price = Cost* has a second theoretical meaning, adopted in this paper, which is a more important reason why cost plays such a central role in classical value theory, and which is the equilibrium of free competition, in which profit is minimum, which is by definition the natural profit rate: competition of firms, incumbent as well as potential entrants, drives profit to its minimum: *price converges to minimum cost under free competition*. Thus, in all rigor, *Price = Cost* in this theoretical sense means more precisely *Price* $\rightarrow$ *Min Price = Min Cost = Natural Price* (see definition below). As generalized to the whole economy, this proposition holds when the competition of landowners, of workers, and of capitalists is so intense that the rent, wage, and profit rates are the minimum they can be, and hence prices correspond to minimum costs. But, more realistically, this proposition when it holds locally, to a given market, when the competition on the side of supply

is so intense that only the most efficient suppliers succeed to sell the good, and at the lowest possible price.

Effectual demand. 'It is different from the absolute demand. A very poor man may be said in some sense to have a demand for a coach and six; he might like to have it; but his demand is not an effectual demand, as the commodity can never be brought to market in order to satisfy it.' (*WN* [1776] 1904, Book I, Ch. VII, p. 58) Absolute demand corresponds to the quantity needed or desired, independently of affordability; *effectual demand*, in contrast, corresponds to quantity (effectually) demanded, which is constrained by wealth. Thus, *effectual demand* is none other than the classical equivalent of the modern notion of *demand*, without the classical qualification, which is no longer needed since absolute demand falls in disuse (unfortunately). So far, so clear. But, unfortunately, Adam Smith first uses effectual demand, not in the general sense he intended, but in a specific context explained below (see natural price): thus '*effective demand*', in the first occurrence, is restricted to *'the demand of those who are willing to pay the natural price* (see definition)*'*; then Adam Smith goes on to give the general idea he intended. J.S. Mill clarifies the matter as follows:

> But what is meant by the demand? Not the mere desire for the commodity. A beggar may desire a diamond; but his desire, however great,

> will have no influence on the price. Writers have therefore given a more limited sense to demand, and have defined it, the wish to possess, combined with the power of purchasing. To distinguish demand in this technical sense, from the demand which is synonymous with desire, they call the former *effectual* demand. (Mill, [1848] 1965, Book III, Ch. II, §3, p. 465)

Monopoly price. See natural price.

Natural price. As used in this paper, natural price means minimum price, in contrast to monopoly price, the maximum price:

> The price of monopoly is upon every occasion the highest which can be got. The natural price, or the price of free competition, on the contrary, is the lowest which can be taken, not upon every occasion indeed, but for any considerable time together. The one is upon every occasion the highest which can be squeezed out of the buyers, or which, it is supposed, they will consent to give: The other is the lowest which the sellers can commonly afford to take, and at the same time continue their business. (A. Smith, [1776] 1904, Book I, Ch. VII, p. 63)

That Adam Smith allows for price to be temporary below the natural price is not in itself a contraction to the meaning of natural price as the minimum price: in his characteristic realism, pushed at times to the details, he allows

for the producer to temporary sell units at a loss (perishable good), rather than incur the bigger loss of not selling at all. This paper simplifies the discussion by avoiding casual cases of sales at a loss, without loss in generality mathematically, moreover, since this exceptional case is easily included by a mere redefinition of the seller's reservation price. But Adam Smith also used natural price as a synonym of: (2) cost (see definition); (3) equilibrium price or even attractor price (hence his oft-quoted gravitation metaphor); (4) 'normal price' or 'ordinary price'. The justification for these other meanings is a general attitude among the classical economists, which Adam Smith inaugurated, and which consists of treating *free composition* as a norm, as if the normal state of affairs in all markets. Thus, Adam Smith explains price adjustment in disequilibrium assuming supply-demand imbalances around the natural price; hence also his restricting *effective demand* (see definition) to *'those who are willing to pay the natural price'*. This is not a definition of effectual demand, but merely a specification of it in the given context.

Scarcity. Classically, the scarcity of a good is given by the ratio between the overall number of units consumer need of a good and the total number of units of the good that can be supplied; that is, classical jargon, the ratio of absolute demand to absolute supply. Scarcity thus understood is an indicator of competition between suppliers and demanders in a market, and is

therefore an indicator of the competitive price of a commodity: the scarcer a good (e.g. a diamond) the more competition will there among the demanders of the commodity, hence the greater will be its equilibrium market price; on the other hand, the more abundant is a good, the more competition there is among the suppliers of the good, who, by underselling each other to supply the relatively few customers in need of the good (e.g. water), will bring the market price to closer and closer to its lowest possible value, the natural price (see definition).

This scarcity ratio is ubiquitous in the classical school, so much so that Ricardo noticed that *'The opinion that the price of commodities depends solely on the proportion of supply to demand, or demand to supply, has become almost an axiom in political economy'* (Ricardo, [1817] 2004, Ch. XXX, p. 382). But for this global, aggregate, objective measure of scarcity, the marginal school substituted an local, individual, subjective one: marginal utility, or its property of being diminishing function of the quantity an individual possesses of a good: the more units of a good an individual already has, the less valuable is an additional unit to the consumer. Later, after the ordinal turn in the neoclassical school, this subjective relation between value and scarcity (diminishing marginal utility), thus rendered is irrelevant

to the ordinal value theory, is replaced by a more elaborate one: diminishing marginal rates of substitution, the idea that the more units of a good an individual already has, the more units of this good he or she would be willing to exchange for another more desire commodity (Hicks, [1939] 1946, pp. 20-22).

Free competition. The market price of a good varies between two limits, $\min(c) \leq p \leq \max(v),$ depending on the degree and type of competition in it. When competition is the most intense on the supply side, then the lowest-cost firms, the most efficient producers, undersell all rival firms, and the good sells at the lowest price, which is called the 'natural price': this case is classically known as free competition, because it happens when no constraint (natural or artificial) limits the capacity to supply to supply the good, which formally amounts to assuming that the good is infinitely abundant, namely $\alpha = \bar{S}/\bar{D} \to \infty.$ On the other extreme is maximum competition on the demand side, in situation of extreme scarcity, $\alpha = \bar{S}/\bar{D} \to 0,$ whereby the highest-value buyers outbid all the rival buyers to gain the few units available for sell: then the price converges to the maximum willingness of pay, or 'monopoly price'. The classical concept of *monopoly* is a very profound one, which Walras and other marginalists (following Cournot's unfor-

tunate innovation) misread, dismissed, and replaced with the literal meaning of the term—the situation of a market supplied by a lone seller, insulated from competition—which fails to capture the concept of a single seller as the most efficient firm that defeated all rivals by underselling them. Hence it is the result of maximum competition on the supply side. Classically, monopoly simply means the state of a market whose supply is so scarce that the price of the good is the maximum it can be. Where the extreme scarcity is natural (such as a diamond or a picture by an old master), we have a natural monopoly. The classical economists saw the greatest evil and were critical of artificial monopoly, wherein the scarcity is artificially created through some restriction of supply or entry, whereby a seller or a group of colluding sellers (often protected by state-granted mercantilist privileges) restrict the supply of a good and entry into the market, and hence raise the price to its maximum to the detriment of consumers. (Natural monopoly affects only those few consumers who have both the 'wealth and the fancy' of engaging into competition over the rarity at stake.)

Water and diamonds. Let be it said in passing the classical solution to the paradox of value, which was long known before Adam Smith to be solved by the concept of scarcity, and thus also did he explain it passingly to his students in his *Lectures on Jurisprudence* ([1763] 1869, p. 177). Water, by

its abundance, usually involves little or no competition to acquire; so, though it is a vital good, its market price is close to its natural value, which is relatively low. A diamond, in contrast, has a much higher price by its rarity: its possession involves intense competition at the top of the distribution of WTP values, so its price is near its monopoly value, as would be achieved in an auction for example.

6 The Mathematics of Chapter VII

The essence of Adam Smith's Chapter VII on competitive price formation in a market for a non-retradable good or service can be reduced to three assumptions. The justifications and rationale of this assumptions, based on textual evidence, will be thoroughly expounded elsewhere in the authors' follow-up papers. These three assumptions are:

1. (Motivation) An individual is willing to trade is willing to trade, if there is any gain from trading (namely if there is surplus to be gained).
2. (The law of supply and demand) Price change and excess demand have the same sign.
3. (Short-side principle) Quantity traded is the minimum between quantity supplied and quantity demanded.

For mathematical simplicity (and merely to go directly to the main points), assume further a large market, namely one in which the distribution of values and costs are continuous, smooth functions, which we assume as given,

hence taken wealth distribution across consumers and the prices of related good as given.

We adopt the following notations:

Notation

p : the market price, the standing market price in a given period.

c : sellers' costs or reservation prices.

v : buyers' values or reservation price.

F : cumulative distribution function of costs: $F(x) = P(c \leq x)$.

G : complementary distribution function of values: $G(x) = P(v \geq x)$.

S : market supply.

D : market demand.

Q : total quantity traded.

Z : market excess supply: $Z = D - S$.

α : abundance of the good (the inverse of scarcity): $\alpha = \bar{S}/\bar{D}$, where $\bar{D} = D(0)$ is total (maximum) number of units demanders need of the good, and (by a simplifying abuse of notation) $\bar{S} = S(\infty)$, the total number of units producers can supply.

Given that the basic concept in classical price theory is the reservation price, the three assumptions formally amounts to saying:

By the first assumption, market demand and market supply are respectively the total number of units of the good that the buyers and sellers can afford at the prevailing (the number of values and costs that are respectively

above and below the prevailing market price). In other words, market demand and market supply are given by the cumulative distribution functions of the values and costs:

$$D(p) = \bar{D}G(p), \tag{1}$$

$$S(p) = \bar{S}F(p), \tag{2}$$

where $\bar{D} = D(0)$ is total (maximum) number of units demanders need of the good, and (by a simplifying abuse of notation) $\bar{S} = S(\infty)$, the total number of units producers can supply.

The second assumption, the law of supply and demand, formally reads

$$Z(p)\frac{dp}{dt} \geq 0. \tag{3}$$

Finally, the third assumption, the short-side principle, simply says that

$$Q = \min(D, S). \tag{4}$$

Consider the following distance between the market price and the individual valuations of the good, where individual values, costs, supplies and demands are indexed:

$$V(p) = \sum\nolimits_i | v_i - p | D_i(p) + \sum\nolimits_j | c_j - p | S_j(p), \tag{5}$$

It can be shown that $V(p)$ is an integral of excess supply[3]:

[3] See the authors' follow-up paper on the theory of competitive price formation rooted in classical and experimental economics.

$$V(p) = V(0) - \int_0^p Z(x)dx. \tag{6}$$

In a large market, where this function is not only continuous but also smooth, we have, by the chain rule:

$$\frac{dV}{dt} = \frac{dV}{dp}\frac{dp}{dt} = -Z(p)\frac{dp}{dt}. \tag{7}$$

Thus, by the law of supply and demand (3), this distance between price and the valuations is nonincreasing (technically, it is Lyapunov function of competitive price dynamics):

$$\frac{dV}{dt} \leq 0. \tag{8}$$

In fact, it can be shown that this property is more fundamental than the law of supply and demand, though the two laws happen to be equivalent in this large-market model assumed merely for simplicity. The property (8) has a fascinating interpretation that echoes Hayek's intuition about the informational function of a competitive market price, which reveals the sum of information about consumers' needs, means, tastes, and producers' production capacities: a sum of dispersed information not in the reach of any single mind. Property (8) means that the market price of a good evolves so as to reflect the traders' valuations and costs better and better, until the distance between the market price and the distribution of values and costs is mini-

mized. Mathematicians refer to such minimum abstractly as a Fréchet median, after the mathematician Fréchet, who generalized the concept of summary statistic (mean, median, mode) to abstract mathematical spaces. Let therefore this emergent informational optimization of the market be referred to as the principle of maximum information. Fascinatingly, it was first discovered in early experimental data as explaining well the dynamics of lab markets and was referred to as the 'minimum rent hypothesis' (V. L. Smith, 1962).

Coming back to Ch. VII more specifically, the following result, illustrated in Figure 1, summarizes formally the key propositions of classical value theory heuristically derived so far. The core ingredient of the proof, not detailed in this paper, is none other than the PMI derived above, which guarantees directly the stability of price dynamics by a classic theorem by Lyapunov. Since Sraffa (1960), the labor theory of value is known to have a natural formulation in terms of input-output analysis. The derivation below is slightly different from Sraffa's, however, in that profit (and the profit rate) is classically counted as a cost and does not therefore appear explicitly.

Theorem: Consider a good traded in a large market. Then its competitive equilibrium price is an increasing function of its scarcity: that is, over some range, $p^* = f(\alpha)$ with $f'(\alpha) < 0$. The competitive price, moreover, tends to

the natural value, if the good is extremely abundant, and to the monopoly value, if the good is extremely scarce: $p^* \to \min(c)$ as $\alpha \to \infty$ and $p^* \to \max(v)$ as $\alpha \to 0$. If all goods in the economy can be produced in abundant amounts, at proportional costs, using homogenous labor, then the natural general equilibrium of the economy (an equilibrium under which all goods are traded at their minimum prices or minimum WTA) is a Leontief price system; hence all the goods would be priced according to the total labor involved in their production: $\mathbf{p^* = \ell(I - A)^{-1}}$, where ℓ is the vector of direct labor requirement per unit of output and $\mathbf{A}$ is the matrix of direct material input requirements per unit of output (and $\mathbf{I}$ being the identity matrix).

Proof: A large-market competitive equilibrium reduces to the traditional market-clearing concept, defined here by the equation $\bar{D}\mathbb{P}(v \geq p^*) = \bar{S}\mathbb{P}(c \leq p^*)$, or $\mathbb{P}(v \geq p^*) = \alpha\mathbb{P}(c \leq p^*)$. Since each probability is between 0 and 1, it follows $0 \leq \mathbb{P}(c \leq p^*) \leq 1/\alpha$ and $0 \leq \mathbb{P}(v \geq p^*) \leq \alpha$. Thus $\mathbb{P}(c \leq p^*) \to 0$ as $\alpha \to \infty$ and $\mathbb{P}(c \leq p^*) \to 0$ as $\alpha \to 0$, implying, respectively, $p^* \to c_{\min}$ and $p^* \to v_{\max}$. By the implicit function theorem, the equation $G(p^*) = \alpha F(p^*)$ implies that, over some interval in $[c_{\min}, v_{\max}]$, $p^* = f(\alpha)$ with $f'(\alpha) = [G'(p) - \alpha F'(p)]/F'(p) < 0$. (Labor theory of value) The unit cost of commodity i can be written as $c_i = w_i \ell_i + \sum_j a_{ij} p_j$, where ℓ_i is the labor requirement,

w_i the average wage, and a_{ij} are direct nonlabor input requirements. Proportional costs means that both the matrix $\mathbf{A} = [a_{ij}]$ and the vector $\ell = [\ell_i]$ are constant; homogenous labor implies a uniform wage rate $w_i = w,$ which can be taken as the value standard, setting $w = 1.$ All commodities are sold at their minimum WTA means $p_i = c_i,$ for $i = 1,...,n,$ hence $p_i = \ell_i + \sum_k a_{ik} p_k,$ which is a Leontief price system, whose existence and uniqueness are a standard result [see, e.g., Meyer (2000, p. 681)]■

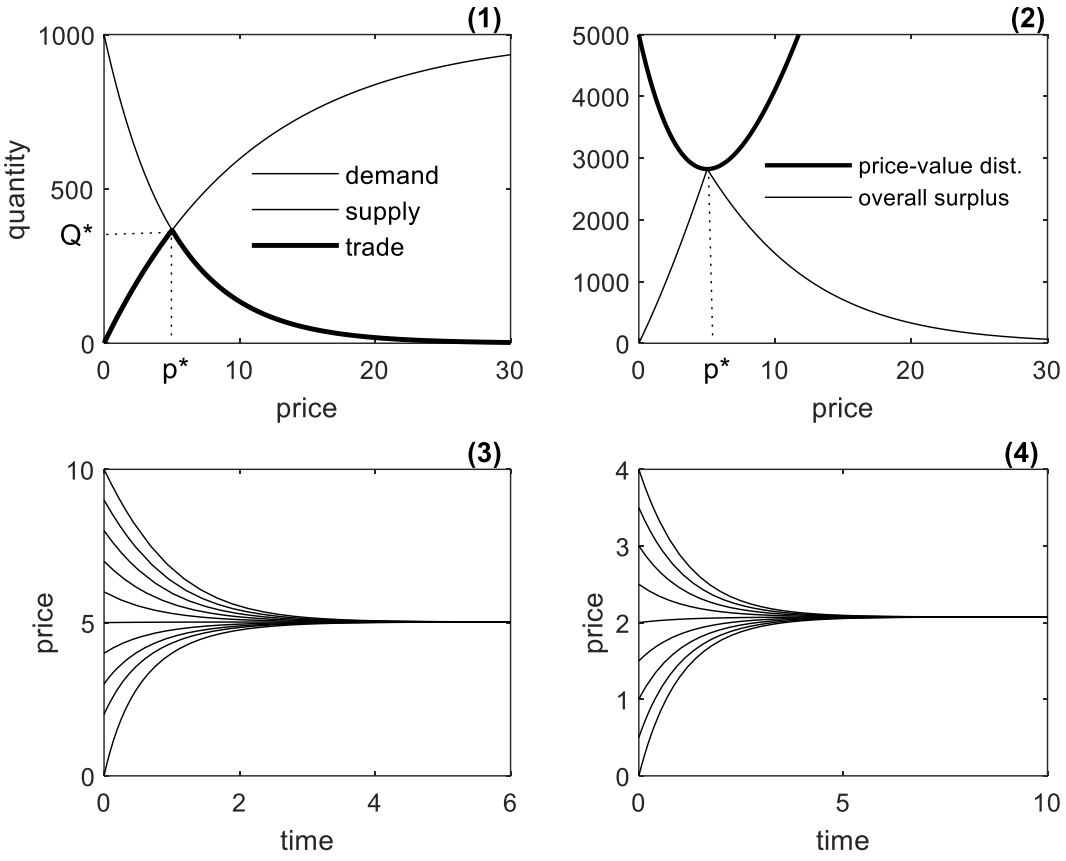

Figure 1: Price formation in a large market. The reservation prices are here exponentially distributed. In (1)-(3), mean(v)=5, mean(c)=11, and scarcity=1, making for p*≈ 5. Idem in (4), save for min(c)=2 and scarcity=0.01. In (3)-(4) are price trajectories for various initial conditions.

Proof: A large-market competitive equilibrium reduces to the traditional market-clearing concept, defined here by the equation $\bar{D}\mathbb{P}(v \geq p^*) = \bar{S}\mathbb{P}(c \leq p^*)$, or $\mathbb{P}(v \geq p^*) = \alpha\mathbb{P}(c \leq p^*)$. Since each probability is between 0 and 1, it follows $0 \leq \mathbb{P}(c \leq p^*) \leq 1/\alpha$ and $0 \leq \mathbb{P}(v \geq p^*) \leq \alpha$. Thus $\mathbb{P}(c \leq p^*) \to 0$ as $\alpha \to \infty$ and $\mathbb{P}(c \leq p^*) \to 0$ as $\alpha \to 0$, implying, respectively, $p^* \to c_{\min}$ and $p^* \to v_{\max}$. By the implicit function theorem, the equation $G(p^*) = \alpha F(p^*)$ implies that, over some interval in $[c_{\min}, v_{\max}]$, $p^* = f(\alpha)$ with $f'(\alpha) = [G'(p) - \alpha F'(p)]/F'(p) < 0$. (Labor theory of value) The unit cost of commodity i can be written as $c_i = w_i \ell_i + \sum_j a_{ij} p_j$, where ℓ_i is the labor requirement, w_i the average wage, and a_{ij} are direct nonlabor input requirements. Proportional costs means that both the matrix $\mathbf{A} = [a_{ij}]$ and the vector $\ell = [\ell_i]$ are constant; homogenous labor implies a uniform wage rate $w_i = w$, which can be taken as the value standard, setting $w = 1$. All commodities are sold at their minimum WTA means $p_i = c_i$, for $i = 1,...,n$, hence $p_i = \ell_i + \sum_k a_{ik} p_k$, which is a Leontief price system, whose existence and uniqueness are a standard result [see, e.g., Meyer (2000, p. 681)] ■

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