10-2018

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Recommended Citation

Swanson, H. L., Alides, A., & Kong, J. E. (2017). Latent class analysis of children with math difficulties and/or disabilities: Are there cognitive differences? *Journal of Educational Psychology, 110*(7), 931-951. https://doi.org/10.1037/edu0000252

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This paper is based on a study funded by the U.S. Department of Education, Cognition and Student Learning in Special Education (USDE R324A090002), Institute of Education Sciences, and National Science Foundation (DRL 1660828) awarded to H. Lee Swanson. The author is indebted to Cathy Lussier, Amber Moran, Danielle Guzman-Orth, Olga Jerman, Loren Albeg, Catherine Tung, Dennis Sisco-Taylor, Kenisha Williams, Garett Briney, Kristi Bryant, Debbie Bonacio, Beth-Brussel Horton, Sandra Fenelon, Jacqueline Fonville, Alisha Hasty, Celeste Merino, Michelle Souter, Yiwen Zhu, and Orheta Rice in the data collection and/or task development. The report does not necessarily reflect the views of NSF, the U.S. Department of Education, or the school districts who participated in this project.
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Date Revision Submitted: 10/12/2017

Abstract

This study investigated whether a latent class of children with math difficulties (MD) or math learning disabilities (MLD) emerged within a heterogeneous sample of learners. A latent class analysis was computed on children (N = 447) in grade 3 who were administered a battery of math, reading, and cognitive measures. The analysis yielded four important findings. First, a discrete latent class of children with MD (15% of the sample) or MLD (10% of the sample) emerged when setting cut-off scores at or below the 25th and 11th percentile, respectively. Second, model testing yielded a high probability of finding children with MD or MLD with reading problems as well as a latent class of low problem solvers with average reading and calculation scores. Third, knowledge of problem solving component processes, estimation and the executive component of WM were significant and unique correlates of latent classes at both cut-off points. Finally, children defined as MD at 25th percentile cut-off but not 11th percentile cut-off yielded high effect sizes on measures of reading, but not on cognitive measures, when compared to children identified at risk at both cut-off points. The results suggest that a statistically distinct latent class of children at risk for MD or MLD can be separated from a heterogeneous sample of children who vary in math, reading and fluid intelligence.

Keywords: math disabilities, math difficulties, cognitive processes, latent class analysis

Abstract

Educational Impact and Implication

Using two traditional cut-off scores for defining serious math problems, a latent class of children with math difficulties (MD) and math learning disabilities (MLD) emerged within a heterogeneous sample of third grade learners. Regardless of cut-off score criterion, children with MD or MLD were found to have serious deficits related to both domain specific (i.e., estimation, knowledge of problem solving components), and domain general processes (i.e., executive component of WM). The results also showed a distinct latent class of poor problem solvers emerged, even though such children were average in calculation and reading abilities. There are at least three implications to the findings: (1) there is a low probability of finding children with MD or MLD independent of reading problems, (2) poor problem solvers shared similar deficits to children with MD and MLD on several cognitive measures (e.g., STM, identifying word problem solving components, numeracy, executive component of WM) and (3) an undue focus on reading and calculation for determining children at risk, may overlook the unique group of children at risk primarily in the area of problem solving.

Latent Class Analysis of Children with Math Difficulties and/or Disabilities:

Are There Cognitive Differences?

Math skills have been shown to have a significant impact on employability, apart from ethnic status, reading competence, and intelligence (Geary, 2011). Unfortunately, a significant number of children demonstrate serious math difficulties (e.g., Mazzocco, Devlin, & McKenney; 2008). School age children displaying characteristics of math difficulties (MD) have been estimated to be about 6.4% of the public school population (Badian, 1999; Geary, 1993). The estimates of prevalence, however, vary (Barbaresi, Katusic, Colligan, Weaver, & Jacobsen, 2005; Desoete, Roeyers, & DeClercq, 2004; Geary, Hoard, Nugent, & Bailey, 2012; Shalev, 2007; Shalev, Manor, & Gross-Tsur, 2005; Martin et al., 2013) and in some estimates comprise up to 10% of the public school population (Geary, 2013). A challenge in this field of research, though, is determining whether math learning disabilities can be considered as a distinct category from math difficulties (e.g., Geary, Hoard, Nugent, & Bailey, 2012, Murphy, Mazzocco, Hanich, & Early, 2007).

Researchers have used normative math test scores at various cutoffs points to determine math difficulties and/or disabilities. These cut-off points have ranged anywhere from the 5th through the 46th percentile (e.g., Murphy et al., 2007; Swanson & Jerman, 2006). Recent criterion among researchers includes using norm- referenced math scores below the 11th percentile to identify children with Math Learning Disabilities (MLD; e.g., Geary, 2013; Mazzocco, Myers, Lewis, Hanich, & Murphy, 2013). In contrast, math performance between 11th to the 25th percentile is a criterion used to indicate children with math difficulties (MD, or Low Achievers) and scores above the 25th percentile to establish the highest group (Typically

Achieving) (e.g., Cirino & Berch, 2010; Geary et al., 2012; Mazzocco, Feigenson, & Halberda, 2011). However, several studies have highlighted subgroup heterogeneity even when using strict cut-off score criteria to define children at risk for MD or MLD (Mazzocco et al., 2008). Thus, a challenge in this field of research is determining if the performance differences in children identified with MD can be differentiated from children with MLD on various achievement and cognitive measures (e.g., Geary et al., 2012; Murphy, Mazzocco, Hanich, & Early, 2007).

This study has two purposes. The first purpose was to determine if children at risk for math difficulties (MD) reflect a discrete latent class from those with math learning disabilities (MLD). Traditionally, as indicated above, children at risk for MD have been defined as at risk by performing below the 25th percentile on norm referenced standardized math measures (e.g., Cirino, Fuchs, Elias, Powell, & Schumacher, 2015; Fuchs et al., 2006; Geary, 2013; Jordon & Hanich, 2003; Mazzocco, 2007; Siegel & Ryan, 1989; Swanson, 2006; Vukovic & Lesaux, 2013). A further refinement in the sample selection of children at risk for math difficulties includes making sure that such children perform above the cut-off scores (> 25th percentile) on achievement measures other than math (i.e., reading). This refinement is necessary to establish that risk status resides in math and not another academic area. Likewise, further refinement in sample selection includes establishing that such children's math difficulties are not due to general intellectual difficulties (e.g., Geary, 2013). Thus, in school practice, children at risk for MD and MLD have been defined on normative measures as having intelligence scores in the average range (e.g., > 85 standard score) and reading scores above the 25th percentile. What differentiates the two groups is the severity of math performance on normative math measures: children with MD perform between the 11th and 25th percentile while children with MLD perform below the 11th percentile on normative math measures.

However, this selection process of determining children at risk for MD or MLD has been criticized because of reliance on an artificial cut-off score (e.g., Branum-Martin, Fletcher, & Stuebing, 2013; Tolar, Fuchs, Fletcher, Fuchs, & Hamlett, 2016). In addition, such procedures ignore the frequency of comorbidity in children with math problems. That is, reading disabilities and math disabilities co-occur more frequently than expected by chance (e.g., Landerl & Moll, 2010). Although a number of explanations emerge related to this comorbidity (e.g., sampling artifact, manifestations of a secondary disorder, alternative manifestations of the same etiology, see Willcutt, Petrill, Wu, Boada, DeFries, Olson, & Pennington, 2013 for a review), there is a high probability that children labelled as MD or MLD do not reflect a diagnostic category that is completely independent of reading difficulties (e.g., Cirino et al., 2015). For example, Cirino et al. administered a battery of measures to a large sample (N=660) of second graders and found that children designated with math difficulties (cut-off score for determining MD was < 25^{th} percentile on arithmetic subtest on the Wide-Range Achievement Test) shared a similar profile to children with both reading and math difficulties on several measures. Further, a number of comprehensive meta-analyses comparing children with MD and reading disabilities (RD) have also found minor differences between children with RD and MD groups on cognitive measures (e.g., Swanson & Jerman, 2006, Swanson, Jerman & Zheng, 2009), suggesting a common construct between the two disabilities. Thus, it is unclear as to whether a distinct latent class of children with MD or MLD can be clearly separated from children with comorbid disabilities. That is, the probability of identify children with serious math problems that are completely independent of reading problems is in question.

To address some of the above issues, it is important to note there have been notable methodological advances that have contributed to our understanding of children's math proficiency as it relates to children at risk for MD or MLD. Recently, there have been advances in modeling the development of discrete processes based on the latent class analysis (e.g., Collins & Lanza, 2010; Muthén, 2006). Latent class analysis (LCA) is a statistical method used to identify subgroups of individuals characterized by similar multidimensional patterns of responses (e.g., Collins, Hyatt, & Graham, 2000). In one sense, LCA is a categorical analog to factor analysis. Instead of defining attributes to a complex covariance structure, LCA posits unobserved classes to explain complex associations in a multidimensional contingency table. Studies that involve the analysis of unobserved classes from a heterogeneous sample are sometimes referred to as mixture models (e.g., Lubke & Muthén, 2005; Muthén, 2006). A rationale for using latent class or mixture modeling is that although math skills can be represented as a continuous outcome variable, the sample may be composed of different groups (or classes) of individuals. This group membership is not directly observed in latent growth models even though it is possible that the distribution of children's math proficiency reflects at least two different latent classes (e.g., children at risk and not at risk for math difficulties). The advantage of LCA when compared to other procedures, such as cluster analysis, is that it offers a probalistic model of the distribution latent classes in the data. Further, the selection process allows for goodness of fit indices, contrary to most clustering techniques that focus on algorithms related to distance measures. In this study, we test the notion that discrete latent classes or mixtures representing different states of math proficiency exist in children who may be identified as at risk or not at risk for MD or children with MLD.

The second purpose of this study determines the cognitive processes that correlate with performance of children at risk for math difficulties (MD) or math learning disabilities (MLD). On the assumption that a discrete subgroup of children at risk for MD or MLD emerges, it is important to know the cognitive processes associated with these risk groups. One of the most often referred cognitive process underlying serious math difficulties is working memory (e.g., Andersson, 2007; Bull, Johnston, & Roy, 1999; Cowan & Powell, 2014; David, 2012; Geary, 2011; Kolkman, Kroesbergen, & Leseman, 2014; Mammarella, Lucangeli & Cornoldi, 2010; Meyer, Salimpoor, Wu, Geary, & Menon, 2010; Simmons, Willis, & Adams, 2012; Swanson, Jerman, & Zheng, 2008). This domain general construct is viewed as a limited capacity system that is involved in the preservation of information while simultaneously processing the same or other information (e.g., Baddeley, 2012; Engle, Tuholski, Laughlin, & Conway, 1999).

Although the association between WM and mathematical performance has been established in the literature, the components of WM that underlie predictions of math performance are unclear (see Simmons et al.; for review). Some studies have suggested that the storage component of WM (referred to as the phonological loop or verbal STM) plays the major role in math performance, especially in the younger ages (e.g., Meyer, et al., 2010). Other studies have noted that difficulties with math problems are tied to visual-spatial component of WM (referred to as the visual-spatial sketchpad, e.g., Ashkenazi, Rosenberg-Lee, Metcalfe, Swigward, & Menon, 2013; Bull, Johnston, & Roy, 1999). In contrast, Lee, Ng, Ng, and Lim, (2004) found that neither the phonological loop nor the visual-spatial sketchpad, but rather the executive component of WM contributed significant variance to math problem solving solution accuracy.

Given that the literature is unclear as to those WM components that uniquely predict math performance, further study is necessary. We consider three competing models as an explanation of the role of WM in individual math performance in children: one focuses on processes related to phonological STM (phonological loop), another focuses information activated from long-term memory specifically related to math, and the third focuses on processes related to an executive system. These models are covered in depth (Swanson, 2004, 2006, Swanson et al., 2008; Swanson & Fung, 2016), but are briefly reviewed here.

One model tested in this study is that the influence of WM on children's mathematical performance is primarily influenced by the phonological storage component of WM because of its strong association with reading (e.g., Baddeley, Gathercole, & Papagno, 1998, for a review). The model follows logically from the literature that links phonological skills to new word learning, comprehension, and mental calculation. The model assumes that children with MD have deficits in the storage of phonological information that constrains higher levels of processing (e.g., Crain, Shankweiler, Macaruso, & Bar-Shalom, 1990; Lauro, Reis, Cohen, Cecchetto, & Papagno, 2010; Majerus & Lorent, 2009). Because of the prevalence of comorbid math and reading difficulties, several studies has suggested children with serious math or reading problems share difficulties related to phonological processing (e.g., Hecht, Torgesen, Wagner, & Rashotte, 2001; Landerl, Bevan, & Butterworth, 2004; Mazzocco & Grimm, 2013; Vukovic & Lesaux, 2013). A mechanism assumed to play a role in the storage of phonological information is naming speed. That is, subvocal rehearsal processes that reduce the decay of memory items in the phonological store prior to output are assumed to be related to naming speed (e.g. Henry & Millar, 1993; McDougall, Hulme, Ellis, & Monk, 1994). Poor performance on measures of naming speed that include numbers and letters have been attributed to both children with MD and children with MLD relative to typical achieving children (e.g., Mazzocco & Grim, 2013, see Table 1), suggesting difficulties in phonological processing (also see Geary, 2011).

A second model assumes that WM plays a key role in predictions of math, but does not identify specific components of WM in predicting math performance. Rather, WM is viewed as an activated portion of declarative long-term memory (Ericsson & Kintsch, 1995). Baddeley and Logie (1999) stated that a major role of WM "is retrieval of stored long term knowledge relevant to the tasks at hand, the manipulation and recombination of material allowing the interpretation of novel stimuli, and the discovery of novel information or the solution to problems" (p. 31). Information retrieved from long-term memory includes the accessing of information needed to making decisions related to number line estimation and magnitude judgments (e.g., Fuchs et al. 2012; Geary, 2011; Simmons et al., 2012) as well as the selection of appropriate operations and algorithms for math solutions (e.g., Mayer & Hegarty, 1996).

For example, children's estimation abilities, judging measurements and assigning numbers without counting, have been found to uniquely predict math skills (e.g., Fuchs et al., 2012; Geary, 2011; Rousselle & Noël, 2007). Number-line estimation tasks require children to estimate the position of target numbers on a line within numerals at end points (e.g., 0 and 100). The accuracy of number-line estimation has been found to correlate with general math achievement (e.g., Siegler & Opfer, 2003). It has been argued that accurate estimation of numerical magnitudes is important for children's mental representation of quantities (e.g., Booth & Siegler, 2008). That is, when children are solving arithmetic problems they activate both the answer to the problem as well as approximation of the answers magnitude to the accuracy of the problem. Thus, an adequate magnitude representation allows for a rejection of implausible answers. Children with MD have been found to have difficulties representing magnitudes accurately (e.g., Fuchs et al., 2012; Geary. 2011). In general, several studies have found the children with MD and MLD are less accurate than children with higher mathematical achievement (e.g., Geary, Hoard, Bryd-Craven, Nugent, & Numtree. 2007).

Another approach to assessing children's mental representation of quantities is assessing their performance related to making judgments of number magnitude. Children's basic understanding of mathematical thinking involves making judgments about numbers of larger or smaller magnitude. Judgments of number magnitude (e.g., which is larger: 8 or 5) have been found to underlie math performance (e.g., Rousselle & Noël, 2007). Fast binary judgments of smaller versus larger numbers have a strong relationship between numerical and spatial relations (e.g., Dehanane, Bossini & Giraux, et al. 1993). For example, Mazzocco and Thompson (2005) found that magnitude judgments of one digit numbers along with mental addition and reading were predictive of MLD in the later elementary grades.

Other information activated from LTM includes recognizing the components of word problems. Several studies have investigated whether the retrieval of contents in long-term memory, specifically the propositions within word problems outlined by Mayer and Hegarty (1996), mediate working memory and math problem-solving. These propositions within word problems are related to accessing numerical, relational, question and extraneous information as well as accessing the appropriate operations algorithms for solution. Thus, for children to effectively solve math problems they need to be able to translate each statement of the problem, integrate information to a coherent problem representation, devise and monitor the solution plan accurately and efficiently carry out the solution. Hegarty and colleagues (Hegarty, Mayer, & Green, 1992; Hegarty, Mayer & Monk, 1995) suggested that the identification of problem solving components plays a major role in translating key information within a word problem (e.g., converting text to a computation problem; e.g., Wong & Ho, 2017). Children who can problem-solve can directly translate key terms (e.g., less than, more than) whereas others (e.g., children with MD) pay attention to the numbers rather than to the relevant information within the problem to solve the problem (e.g., Swanson, Cooney & Brock, 1993). Previous studies have found the children with MD perform poorly on accurately identifying the components of word problems (e.g., Swanson& Beebe-Frankenberger, 2004).

In line with the other two models, a third model views executive processes as (1) accessing information from LTM (e.g., accessing the correct algorithm) and (b) providing resources to lower-order (i.e., phonological system) skills. That is, although math proficiency is related to the retrievability of contents in LTM and activities related to the phonological loop, activities of related to the executive system of WM may also underlie math proficiency. The executive component of WM (also termed “controlled attention”) is the residual variance captured in regression modelling when STM has been partialed-out out in the analysis (Engle et al., 1999). This residual variance (i.e., controlled attention) is assumed to reflect the inhibition of competing information from the targeted information (e.g., Unsworth, 2010). Several studies have shown a relationship between inhibition and poor performance in math (e.g., Blair & Razzo, 2007; Bull & Scerif, 2001; D’Amico & Passolunghi, 2009; Passolunghi & Pazzaglia, 2005). A random generation task was used to assess inhibition in this study. The use of Random Generation tasks has been well articulated in the literature as a measure of inhibition (e.g., Baddeley, 1996; Towse & Cheshire, 2007). The task is considered to tap inhibition because participants are required to actively monitor candidate responses and suppress responses that would lead to well learned sequences, such as 1-2-3-4 or a-b-c-d (Baddeley, 1996).

In summary, the purpose of this study was to identify whether children at risk for MD reflect a latent class. The study determined if this potential latent class could be differentiated in terms of severity of math deficiencies and whether this differentiation reflected qualitatively different cognitive processes. To extend the literature in these areas, the study sought to answer two questions:

1: Can a latent classification of children at risk for MD and MLD be identified within a heterogeneous sample of learners when performance in math, reading and intelligence measures are included in the analysis?

The present study determines the probability of identifying a latent class of participants with MD or MLD using the 25th percentile or 11th percentile as a cut-off point within a sample that includes a large range of math, reading, and cognitive abilities. As mentioned, LCA is a "model-based clustering" approach that derives clusters using a probabilistic model that describes distribution of data. So instead of finding clusters of children with math problems, LCA describes the distribution of the data based on a model that assesses probabilities that certain cases are members of certain latent classes. Thus, with goodness of fit indices, it is possible to test whether a "latent structure" underlies the data.

As previously mentioned, performance at or below the 25th percentile on normed referenced math measures is commonly used to designate risk for MD (e.g., Fletcher et al., 1989; Fuchs et al., 2012; Siegel & Ryan, 1989; Swanson & Beebe-Frankenberger, 2004; Vukovic & Siegel, 2010). However, as indicated earlier, we make a distinction in our data analysis between math difficulties (MD, math performance between the 11th to 25th percentile) and math learning disabilities (MLD, math performance < 11th percentile). Of interest is whether the profile (magnitude of differences on performance measures) differs among those children who retain risk status under both cut-off points (referred to as MLD) and those who only retain risk status at the 25th percentile cut-off point (referred to as MD). Such a comparison on cognitive measures would address the issue as to whether the two groups reflect qualitatively different profiles.

Given the issues related to comorbidity mentioned earlier, of particular interest is whether our modeling testing of latent classes within the current data set would yield a subgroup with low math performance but reading scores above the 25th percentile.

2. Do specific cognitive measures predict latent class membership?

Previous studies have emphasized WM as playing a major role in predictions of MD (e.g., Cowan & Powell, 2014; Swanson & Beebe-Frankenberger, 2004). However, there are a number of other processes that may underlie the relationship between WM and math skills. The processes considered in the current study are: knowledge of problem-solving processes, naming speed, estimation, number judgment, and inhibition. The importance of these processes was discussed earlier. For example, STM storage (phonological loop) and related phonological processes (rapid naming speed), domain specific measures (measures of word problem solving components, estimation) as well as measures of executive processing (e.g., inhibition), have been implicated along with WM as predictors math performance (e.g., Cowan & Powell, 2014; Fuchs et al., 2006; 2012; Lee et al., 2004; Swanson & Beebe-Frankenberger, 2004). What is of interest, however, is whether children identified as MLD (below the 11th percentile) have more general cognitive and academic delays than children with more moderate delays (children with MD). Some studies (Hanich et al., 2001; Jordon & Hanich, 2003; Jordon, Hanich et al., 2002) have suggested that children with MLD may have more circumscribed difficulties (e.g., problems related to estimation, magnitude judgment, naming speed). Therefore, when compared to average achievers, it was of interest children to determine if performance differences of children with MD are more generalized than children with MLD.

Method

Participants

The grade 3 children in this study were drawn from a longitudinal study that included at risk children in grade 2 to grade 5 (Fung, Orosco, Swanson, 2014; Swanson & Fung, 2016).After receiving signed parent permission forms, children in the third grade were administer a large battery of measures. In addition, children at risk and not at risk were identified by teacher nomination and previous test scores. Grade 3 was selected since it included the largest sample and the focus of classroom instruction included both calculation and math word problems. Third graders were also selected from this study since this is the grade where serious math difficulties are first identified (e.g., Swanson & Beebe-Frankenberger, 2004). In addition, the stability of math difficulties can be determined by considering math performance on high stake tests in the earlier grades. The total sample consisted of 447 children in the third grade (chronological age *M* = 8.39, *SD* = 0.50; 222 males, 225 females) selected from six Southwest public schools. The sample consisted of 199 Caucasians (49%), 133 Hispanics (33%), 20 African Americans (6%), 23 Asians (6%), and 24 children (6%) who were identified as Native American or Vietnamese. Forty children showed mixed ethnicity (e.g., Hispanic + African American, Hispanic + Caucasian). Based on school records, the sample was primarily low to middle SES based on free and reduced lunch eligibility, parent education levels, or parent occupation.

Measures Used for Identifying Latent Classes

Fluid Intelligence

Fluid Intelligence. Fluid intelligence was assessed by administering the Colored Progressive Matrices test (Raven, 1976). The dependent measure was the number of problems solved correctly, which yielded a standardized score (*M* = 100, *SD* = 15).

Calculation and Problem Solving Skills

Arithmetic calculation. The arithmetic computation subtest for the Wide Range Achievement Test-Third Edition (WRAT; Wilkinson, 1993) and the numerical operations subtest for the Wechsler Individual Achievement Test (WIAT; Psychological Corporation, 1992) were administered to measure calculation ability. The dependent measure was the number of problems correct, which yielded a standard score (*M* = 100, *SD* = 15).

Word problem-solving accuracy (WPS-accuracy). Word problem-solving accuracy was assessed using four measures. The Story Problem subtest of the Test of Math Ability (TOMA-2, Brown, Cronin, & McEntire, 1994) required children to silently read a short story problem and solve the computational problem. The Story Problem-Solving subtest from the Comprehensive Mathematical Abilities Test (CMAT; Hresko, Schlieve, Herron, Swain, & Sherbenou, 2003) required the examiner to read each of the problems to the children, asking children to read along on their own paper. Children were then asked to solve the word problem by writing out the answer. The KeyMath Revised Diagnostic Assessment (KeyMath; Connolly, 1998) word problem-solving subtest involved the tester reading a series of word problems to the children while showing a picture illustrating the problem, and then asking them to verbalize the answer to problem. Mental computation related to word problems was assessed from the arithmetic subtest of the Wechsler Intelligence Scale for Children, Third Edition (Psychological Corporation, 1992). Each word problem was orally presented and solved without paper or pencil. Questions ranged from simple addition to more complex calculations.

Reading Skills

Reading. Reading comprehension was assessed by the Passage Comprehension subtest from the Test of Reading Comprehension-Third Edition (TORC; Brown, Hammill, &

Weiderholt, 1995) and word recognition was assessed by the reading measure decoding subtest of the Wide Range Achievement Test-Third Edition (WRAT-Reading; Wilkinson, 1993).

Cognitive Measures Used for Determining Correlates of Latent Class Membership

Working Memory

Phonological loop. This component of WM was measured using three tasks. The Forward Digit Span subtest of the Wechsler Intelligence Scale for Children-Third Edition (WISC-III; Wechsler, 1991) assessed short term memory (STM) since it was assumed that forward digit spans presumably involved a subsidiary memory system (the phonological loop). The task involves a series of orally presented numbers which children repeat back verbatim. The Word Span task was previously used by Swanson and Beebe-Frankenberger (2004), and assessed the children's ability to recall increasingly large word lists (a minimum of two words to a maximum of eight words). Testers read lists of common but unrelated nouns to the children, and were asked to recall the words. Word lists gradually increased in set size from a minimum of two words to a maximum of eight. The Phonetic Memory Span task assessed the children's ability to recall increasingly large lists of nonsense words (e.g., des, seeg, seg, geez, deez, dez) ranging from two to seven words per list (Swanson et al., 2008).

Central executive. This component of WM was measured using three tasks. The Listening Sentence Span task assessed children's ability to remember numerical information embedded in a short sentence (Daneman & Carpenter, 1980). Testers read a series of sentences to each child and then asked a question about a topic in one of the sentences, and then children were asked to remember and repeat the last word of each sentence in order. The Conceptual Span task assessed children's ability to organize sequences of words into abstract categories (Swanson, 2013). The experimenter presented set of words (e.g., "shirt, saw, pants, hammer, shoes, nails”), asked a process question ("Which word, 'level' or 'saw', was said in the list of words?"), and then asked the participant to recall the words that went together.

Because WM tasks were assumed to tap a measure of controlled attention referred to as updating (e.g., Miyake, Friedman, Emerson, Witzki, & Howerter, 2000), an experimental Updating task, adapted from Swanson and Beebe-Frankenberger (2004) was also administered. A series of one-digit numbers was presented that varied in set length from 3, 5, 7, and 9. No digit appeared twice in the same set. The examiner told the child that the length of each list of numbers might be 3, 5, 7, or 9 digits. Children were then told that they should only recall the last three numbers presented. Each digit was presented at approximately one-second intervals. After the last digit was presented the child was asked to name the last three digits, in order. The dependent measure was the total number of sets correctly repeated (range 0 to 16).

Visual-spatial sketchpad. This component of WM was measured using two tasks (see Swanson, 1992, for review of these tasks). The Mapping and Directions Span task assessed whether the children could recall a visual-spatial sequence of directions on a map with no labels. Children were presented with a map for 10 seconds that contains lines connected to dots and square (buildings were squares, dots were stoplights, lines and arrows were directions to travel). After the removal of the map, children were asked a process question and then asked to draw the lines and dots on a blank map. The Visual Matrix task assessed the children’s ability to remember visual sequences within a matrix. Children were presented with a series of dots in a matrix and were allowed 5 seconds to study the pattern. After removal of the matrix, children were asked a process question and asked to draw the dots they remembered seeing in the corresponding boxes of a blank matrix.

Measures Assumed to Underlie the Relationship between WM and Math

The measures assumed to mediate the relationship between WM and math performance were related to accessing specific information from LTM (word problem solving components, estimation, and magnitude judgement), enhancing the storage of phonological information (naming speed) and inhibiting the accessing of irrelevant information (random generation). A description of each task follows.

Word problem-solving components. This experimental task assessed the child's ability to identify processing components of word problems (Swanson & Beebe-Frankenberger, 2004). Each booklet contained three problems that included pages assessing the recall of text from the word problems. To control for reading problems, the examiner orally read each problem and all multiple-choice response options as the students followed along. After the problem was read, students were instructed to turn to the next page on which they were asked a series of multiple-choice questions requiring them to identify the correct propositions related to (1) question (2) number, (3) goal, (4) operation and (5) algorithm of each story problem. Children were also to identify the extraneous propositions for each story problem.

Estimation. Two number line estimation tasks adapted from Siegler and Opfer's (2003) and Siegler and Booth’s (2004) study, were administered. For set 1 of the Estimation task, children were asked to examine five straight lines that were 25-cm long. Each line was identical in length and was marked with a zero at one end and one hundred on the other end, creating a blank number line. A single number (e.g., 50. 75, 45, 32, 6, 22) was placed above the center of each line. Children were asked to estimate where they thought the number presented should be placed on the line and indicated this by marking an X on the line. For set 2, children were asked to examine another set of five straight lines. For this set, however, each line was of a different length (25cm, 20cm, 12cm, 30cm, and 20cm) with end points of 0 and 100. The reason to manipulate the length of the line was related to issues raised as to whether spatial information or magnitude judgment underlines problem in estimation (Chew, Forte et al., 2016). Several studies suggest that children who are poor on mathematical tasks have a reduced visual-spatial span (Bull & Scerif, 2001) and therefore we varied to length to have a better sense if difficulties were related to magnitude or spatial judgment.

For each of the 10 lines (set 1 and 2), the point of accuracy was calibrated for each line. Accuracy was calculated by using a transparency template and counting how many units of measure the X was from the correct answer. For the five lines in Set 1, the distance from the accuracy point was computed for each ¼ inch. For set 2, arithmetically equivalent distances were used to count off the distance between the participant's X and the where actual placement the correct answer should be on the line. We converted difference scores (number of units from the exact point) to positive values by subtracting the difference score from 20 in each set. Thus, our estimate of the number line estimation varied from that of Siegler and Opfer (2003), in that they used group level median placements fitted to linear analog models to make inferences about the children's placements.

Magnitude comparisons. Two sets of digits were presented in 25 rows with three columns. Each row had the same number of digits (1 digit, 2 digits, and 3 digits) in each column. In the first set, children were asked to circle the largest number in each row as fast as they could in 30 seconds. The second set also had an additional 25 rows of numbers with three numbers in each row. Children were asked to circle the smallest number in each group as fast as they could in 30 seconds. The numerical distance between a symbolic magnitude comparison was alternated across rows so that each row had one comparison close in numerical distance (e.g., 2 and 3) and one far in numerical distance (2 and 9). Children were presented with 25 rows of numbers with three numbers (either in pairs or in three digits) in each row. The scores for set 1 were the number of correctly identified largest numbers (set 1) within 30 seconds, and the scores for set 2 were the smallest numbers correctly identified within 30 seconds.

Naming speed. The Comprehensive Test of Phonological Processing's (CTOPP; Wagner, Torgesen, & Rashotte, 2000) Rapid Digit and Rapid Letter Naming subtests were administered to assess speed in recalling numbers and letters. Children received a page that contained four rows and nine columns of randomly arranged numbers (i.e., 4, 7, 8, 5, and 2). Children were required to name the numbers as quickly as possible for each of the two stimulus arrays containing 36 numbers, for a total of 72 numbers. A stopwatch was used to time participants on naming speed. The dependent measure was the total time to name both arrays of numbers. The Rapid Letter Naming subtest is identical in format and in scoring to the Rapid Digit Naming subtest, except that it measures the speed children can name randomly arranged letters (i.e., s, t, n, a, k) rather than numbers.

Inhibition. The Random Number and Random Letter Generation Tasks were administered to assess inhibition (Swanson & Beebe-Frankenberger, 2004). Children were first asked to write, as quickly as possible, numbers (or letters) in a non-random sequential order to establish a baseline. They were then asked to write numbers as quickly as possible, out of order, in a 30-second period. Scoring included an index for randomness, information redundancy, and percentage of paired responses to assess the tendency of participants to suppress response repetitions. The measure of inhibition was calculated as the number of sequential letters or numbers, minus the number of correctly unordered numbers or letters, divided by the number of sequential letters or numbers, plus the number of unordered letters or numbers.

Nestedness

According to Maas and Hox (2005), at least 50 level-2 observations (classrooms in this case) are needed to assure that estimated parameters are unbiased. Because data were collected for children in 19 different classrooms, we considered utilizing a multi-level approach. However, the models would not converge due to the small number of clusters. Thus, multi-level modeling was not used in the final analyses.

Cut-off points

The aim of this study was to determine whether discrete subgroups in math ability emerged among a heterogeneous group of third graders. The manifest variables (calculation, problem solving, reading, fluid intelligence) in the first analysis to determine discrete groups were dummy coded as reflecting normative score as at or below the 25th percentile (1 = at or below 25th percentile, 2 = above the 25th percentile). The 25th percentile or a 90 standard score was based on the normative scores from the standardized math, reading and fluid intelligence measures. However, it is important to note that several researchers have suggested this cut-off point is more likely to capture children with general achievement difficulties and not necessarily children with MLD (i.e., math learning disabilities). A follow-up analysis we recomputed the latent classes and used scores below the 11th percentile as a cut-off point for determining children at risk for MLD. Obviously some of the children identified as at risk at the 25th percentile cut-off point would not necessarily be represented in a latent class at the more severe cut-off. However, on the assumption our sample is representative of children who experience serious difficulties in math; we will be able to compare the profiles (via probabilities of occurrence and effect sizes) of the latent classes that emerge at both cut-off points from those that emerge only at the 25th percentile cut-off. It was of interest to determine if the cognitive profile of children identified as at risk for moderate MD (children yielding normative scores between the 11th and 25th percentile) could be separated from children with MLD (severe MD). That is, do children identified as at risk only at the 25th percentile yield a distinct latent class when separated from children identified at risk below the 11th percentile cut-off?

Procedures

Ten graduate students trained in test administration tested all participants in their schools. One session of approximately 45–60 minutes was required for small group test administration, and one session of 45–60 minutes was required for individual test administration. During the group testing session, data were obtained from problem-solving process (components) booklets, Test of Reading Comprehension, Test of Mathematical Ability, and the Visual Matrix task. The remaining tasks were administered individually. Test administration was counterbalanced to control for order effects.

Results

Distribution of Measures

Table 1 shows the means, standard deviations, skewness, kurtosis and sample reliability (Cronbach alpha) for each measure**.** A preliminary analysis showed the classification measures met standard criteria for the univariate analysis (Kline, 2011). Skewness less than 3 and kurtosis less than 4 did not occur on the classification measures, but did occur for some of the cognitive processing measures (e.g., Estimation, Numeracy). However, a transformation of these measures did not change the pattern of the results and therefore the original scores were used in the analysis. Performance for the TOMA was of concern because mean scores were out of the normal range, even though standard score performance varied from 40 to 140 in the sample. It is important to note that this task required children to silently read and solve a story problem. We initially removed the outliers (SD > 3.5) for this task, but this did not change the pattern of results. Thus, on the assumption this was a heterogeneous sample and performance reflected a continuum of skills, no outliers were removed from the analysis.

Latent Class Analysis

Model fit. In order to evaluate the model fit, and because LCA is an exploratory analysis, a series of models were fit, varying the number of latent classes between one and six (Nylund, Asparouhov, & Muthén, 2007; see Masyn, 2013, for a comprehensive review). A combination of statistical indicators and substantive theory were used to decide on the best fitting model. Models with different numbers were compared using information criteria (i.e., Bayesian Information Criteria-BIC, Akaike Information Criteria-AIC, and Adjusted BIC). Lower values on these fit statistics indicated a better model fit. Statistical model comparisons included likelihood ratio tests: the Lo-Mendell-Rubin Test (LMR) and the Bootstrap Likelihood Ratio Test (BLRT). Both statistical procedures compared the improvement between neighboring class models (i.e., comparing models with two vs. three classes, and three vs. four, etc.) and provided *p*-values. *P*-values were used to determine if there was a statistically significant improvement in fit for the inclusion of one more latent class. A nonsignificant P-value indicated for a K-class that the previous K-class with a significant P-value fit the data better. Among the information criterion measures, the BIC is generally preferred, as is the BLRT for statistical model comparisons (Nylund et al., 2007). An additional consideration was the interpretability of the classes, as well as the size of the smallest class.

Given the indices reported in Table 2, the three and four class models were studied for interpretability. Both the LMR and BLRT yielded non-significant *p*-values for the four-class solution and significant *p*-values for the three class model, indicating that the three-class model provided an excellent fit to the data. The BIC was lower for the three than the four class model.

Thus, the four-class and five- class models did not represent an improvement over the three-class model. In addition, adequate sample proportionality and item probabilities for the three-class model were more easily interpreted than the four class model. Masyn (2013) suggested that class proportion values can be considered (i.e., “assign meaning to the classes” (p. 559) when determining the number of latent classes. The item probabilities for the three class model are reported in Table 3. The entropy for the three class model was .75, an acceptable value (Nylund et al., 2007).

Sample and item probabilities. Table 3 shows the proportion of the sample in each latent class (gamma estimates), as well as the probabilities (rho estimates) for each measure (manifest variable) for each response category as a function of each latent class for the total sample. Shown are the item probabilities for performance at or *under* the cut-off threshold of the 25th percentile and below 11th percentile cut-off score. These rho estimates reflected the latent class abilities of the given item-response, conditional on the given latent-class membership. To facilitate discussion, and because there is no set standard for determining meaningful probabilities, item latent class probabilities above 70% were selected as reflecting MD status and these values are shown in bold. That is, probabilities above .70 indicated risk of low performance for that particular manifest variable.

At Risk for Math Difficulties (MD)

Three latent classes emerged using the 25th percentile as a cut-off score. As shown in the left section of Table 3, item response probabilities at or greater than .70 indicated high probabilities for risk status (children who performed at or under the 25th percentile). The first latent class (LC=1) was labeled as average achievers across all manifest variables. Latent class group 2 (LC=2) was characterized by low achievement in calculation, problem solving, and reading comprehension, but average performance in areas of word identification and fluid intelligence. This group was labeled as children at risk for MD. Latent Class 3 (LC=3) was characterized by low achievement on selective measures of problem solving accuracy, but average achievement on the remaining manifest variables. This group was labeled as poor problem solvers.

To interpret these profiles further, the means and *SD*s for each of the normed manifest variables are reported in the top section of Table 4. Mean scores were in the normal range for the average achievers (LC=1), whereas mean scores for children with MD (LC=2) were below the 25th percentile (90 standard) for all classification measures, except fluid intelligence. In contrast, children with problem solving difficulties (LC=3) yielded average mean scores on measures of calculation and reading, but below the 25th percentile on three of the four problem solving measures.

Sample Distribution. The total sample proportional distributions for the three latent classes are shown in the top row of Table 3. These estimates (gamma estimates) represented the proportion of the sample expected to be members of a particular latent class. The largest proportional distribution of the sample occurs for LC= 3 (.54) followed by LC=1 (.32). The proportional distribution of gender across the three latent class groups for males was .50, .52 and .49, respectively. No significant effects were found for gender representation among the three latent classes, χ^2 (2, N = 447) = .29, p =.86.

In summary, given the research question and the statistical findings (BLRT) as well as the substantive meaning of each solution (item probabilities and proportional assignment discussed below); a three class model was selected. The model included average achievers, children with MD, and poor math problem solvers. The mean norm referenced scores for each latent class on the manifest variables are shown at the top of Table 4.

Math Learning Disabilities (MLD)

A follow-up to the above latent class analysis that used the 25th percentile as a cut-off score, the next analysis utilized the 11th percentile (based on the norms within the test manual) as a cut-off point to determine the latent classes. As shown in Table 2, three latent classes emerged as the best fit.[1] The right hand section of Table 3 shows that the largest proportional distribution of the sample occurred for LC=3 (.55) followed by LC=1 (.35). An interpretation of the item probabilities shows a profile similar to the more liberal cut-off score (at or < 25th percentile). Latent class 1 reflected average achievers, latent class 2 reflected children with MLD and latent class 3 reflected children with low math problem solving skills. The proportional distribution of gender for LC=1, LC=2, and LC=3 for males was .50, .51, and .49, respectively. No significant effects were found for gender representation, χ^2 (2, N = 447) = .07, p =.96. The mean norm referenced scores for each latent class on the manifest variables are shown at the bottom of Table 4.

Comparison of Cut-off points

As shown previously in Table 3, LC=2 was 15% of the total sample at the 25th percentile and 10% of the total sample at the < 11th percentile cut-off. A cross-classification of the two cut-off points was computed. As expected, there was a significant difference in sample representation for LC=1 thru LC=3 at the 25th percentile cut-off when compared to LC1 thru LC3 at the 11th percentile cut-off point, χ^2 (4, N = 447) = 621.33, p < .0001. The percentage of children identified as a MD latent class under the moderate cut-off (< 25th percentile) that retained their status under the severe cut-off (< 11^{th} percentile) was 57%. The other 43% of the MD group transitioned into the poor math problem solving group. The percentage of children identified as a latent class of poor math problem solvers (LC=3) under the moderate cut-off (< 25^{th} percentile) that retained status (LC=3) at the 11^{th} percentile cut-off was 89%. The remaining 11 percent transitioned into the average achieving latent group. Regardless of cut-off points, none of the average achieving children transitioned into the "at risk for MD" or poor problem solving group.

Table 5 shows the profile of the group that retained latent class status at both cut-off points and those who transitioned out of risk group status by lowering the cut-off point to the 11^{th} percentile. Consistent with our criteria, the retained group was defined as children with MLD and those that transition out of the latent class group at risk were defined as children with MD. As shown, except for fluid intelligence, all manifest (classification) variables for the retained MLD group yielded mean scores below the normal range (standard scores < 85). No significant differences were found in gender representation between the two (retained vs. transition) groups $\chi^2(1, N = 65) = .46, p=.49$. The transition group yielded normative scores in the average range on measures of fluid intelligence, calculation, and word identification (mean standard scores > 85).

As shown in Table 5, effect sizes (Cohen's *d*) were computed between those children who retained risk status (regardless of cut-off score) and those who transitioned out of the latent class related to risk by lowering the cut-off point to the 11^{th} percentile. To make *d*'s interpretable, statisticians have adopted Cohen's (1988) system for classifying *d*'s in terms of their size (i.e., .00 - .19 is described as trivial; .20 - .49, small; .50 - .79, moderate; .80 or higher, large). As shown in Table 5, the two groups, referred to as MD and MLD, respectively, where compared on manifest variables used to determine latent status. As expected, the shifting from one latent class to another was related to scores on the math measures. However, utilizing Cohen's criterion of a high effect size (ES's > .80), the magnitude of the ESs between the two groups was large for performance on reading measures. Children with relatively higher reading scores were more likely to transition out the latent class group at risk by lowering the cut-off point (< 11th percentile).

Correlates of Latent Classes

The next analysis determined those cognitive variables external to the classification measures that played a significant role in predicting latent class membership.

Confirmatory factor analysis. The cognitive measures were reduced to latent constructs for the subsequent analysis. Further, converting the measures to latent constructs eliminated measurement error and allowed for a focus on shared variance rather than isolated task variance (e.g., Kline, 2011). Therefore, in our next analysis we specified tasks as indicators of the problem solving process (question, number, goal, operations, algorithm, and irrelevant information), numeracy (numbers of high and low magnitude), estimation, speed (naming speed for numbers and letters), inhibition (random generation of numbers and letters),phonological loop or STM (Digit Forward Span, Word Span, and Phonetic Span), executive processing (Conceptual Span, Listening Sentence Span, updating), and visual-spatial sketch pad (matrix, mapping& directions).

Several indices were selected because of their widespread use and relative ease of interpretation with regards to the assessment of model fit. These indices included the χ^2 goodness-of-fit test, comparative fit index (CFI), Tucker-Lewis index (TLI), and the root-mean-square error of approximation (RMSEA), along with its associated confidence intervals. It is generally recognized that to support model fit a consensus among the following is needed: a non-significant χ^2 goodness-of-fit value; a CFI > .90; a TLI > .90; an RMSEA below .05 with the left endpoint of its 90% confidence interval markedly smaller than .05 (e.g., Hu & Bentler, 1999; Raykov & Marcoulides, 2008). The model fit indices indicated a good model fit: χ^2 (181) = 297.21, $p < .0001$; *CFI* = .95; *TLI* = .94; *RMSEA* = .04 (.032, .048); *SRMR* = .048. Thus, factor or latent scores were used as continuous variables in predicting latent class status.

A comparison of the latent class groups on the mean factor scores (latent variables) in z-score units are reported in Appendix A. Also reported are effect sizes comparing each latent class on these variables. Several large effect sizes according to Cohen's (1988) criteria (ESs > .80) emerged. As shown in Appendix A, average achievers (LC=1) superseded (magnitude of ESs were large) children at risk for MD (< LC=2 at 25th percentile cut-off) and children with MLD (LC= 2 at the 11th percentile cut-off) on all factor scores except on the latent measurement of naming speed.

Logistic regression. A logistic regression analysis was computed that included latent class membership as the criterion measure and cognitive processes (factor scores) as the predictor variables**.** Table 6 shows the unique contribution of each cognitive process in predicting the latent class in a full model. Regardless of the two cut-off points, as shown in Table 6, measures of domain specific component knowledge, estimation, and the executive component of WM were significant predictors of latent classes. In contrast to the 25th percentile cut-off, however, additional significant unique predictors at the 11th percentile cut-off occurred for STM (phonological loop), naming speed and visual-spatial WM. This finding suggests that children at the lower cut-off may have more generalized cognitive difficulties when compared to selecting children at risk at the 25th percentile. These findings will be qualified later.

Based on the three models discussed in the introduction, separate model predictions related to STM storage (phonological loop, naming speed), domain specific knowledge (components of word problems, magnitude judgments, and line estimation) and executive processing (WM, inhibition) were computed. Reported in Table 6 are the odds/ratio, estimates, and standard errors and significance of the odds ratios as indicated by the *p* value of the Wald statistic and model fit indices. The Akaike's Information Criterion (AIC) allowed for a comparison of models that were not nested, and the Bayesian Criterion (BIC) allowed for a comparison of nested models (Hox, 2010, pp. 47-50). In general, models with lower AIC, BIC and deviance values fit better than models with higher values.

Three important findings are shown in Table 6. First, lower AIC, BIC and deviance values emerged for the domain specific model when compared to the other two partial models. However, when comparing AIC, BIC and deviance values to the full model, all three comparisons were significant (all ps < .01) suggesting none of the three specific models in isolation provided a parsimonious fit to the data when compared to the full model. Second, regardless of the cut-off point, component knowledge, estimation, and the executive component of WM were significant predictors of latent class membership in the full regression model. Additional significant predictors (STM, Naming speed, visual-spatial sketch pad) of latent class membership occurred at the lower cut-off point (< 11^{th} percentile) when compared to the cut-off point at the 25^{th} percentile. Finally, regardless of the cut-off point, no significant unique variance was found for numeracy (small and large number magnitude judgements) and inhibition (random generation) in predictions of latent class membership.

Between latent class comparisons. Of interest in the next analysis was determining those cognitive processes that uniquely discriminated between each latent class. Table 7 shows the results of a multinomial regression that used the poor problem solving latent class as a reference group (LC=3). This group was selected because it showed the greatest overlap when transitioning from one latent class to another. Of interest was determining the cognitive variables that separated poor math problem solvers (LC=3) from average achieving children (LC=1) and those cognitive variables that separated poor math problem solvers from children with MD (LC=2 at the 25th percentile cut-off) or MLD (LC=2 at the 11th percentile cut-off). The odds ratios reported in Table 7 represented the ratio of change in the odds of an event (i.e., in this case not belonging to the "at risk for problem solving deficits" latent class) and varied from 0 to infinity. An odds ratio greater than 1 indicated a higher chance of not being in the reference group (poor problem solver). In contrast, an odds ratio less than 1 indicated a greater chance of belonging to the reference group. When the odds ratio is 1 or close to it, no effect was found. A nonsignificant odds ratio would suggest that the independent variable failed to provide reliable predictions to differentiate one latent class group from another.

To interpret the outcomes in Table 7, consider the multinomial logit for a one-unit increase in average achieving group (LC=1) on the STM measure. If a student in the average achieving group (LC=1) improved in STM by 1 point, the multinomial log odds of being classified as "not at risk for poor problem solving," when compared to the poor problem solving group (LC=3), would be expected to increase by 1.33 units, while holding all other variables in the model constant.

Table 7 shows a comparison between the poor problem solving latent class and average achieving children and between children with MD (cut-off < 25th percentile) and MLD (cut-off < 11th percentile). Because there were multiple comparisons, alpha was set to .01. As shown, regardless of the cut-off point, average achievers out performed poor problem solvers on measures of STM, knowledge of problem solving components, numeracy, and the executive component of WM. In contrast, children with MD or MLD could not be differentiated from poor problem solvers on measures of STM, numeracy, and measures of executive processing (WM and inhibition), suggesting that both latent classes suffer common difficulties on these measures. That is, given that poor math problem solvers were deficient on these measures relative to average achievers, we can assume that children with MD or MLD were deficient on these measures relative to average achievers. A significant effect was found contrasting poor math problem solvers (LC=3) and children with MD (LC=2 at 25th percentile) or MLD (LC=2 at 11th percentile) on two measures. The poor problem-solving latent class out-performed children with MD or MLD on measures of naming speed and estimation.

Stable group comparisons. Obviously, latent class comparisons may yield equivocal findings because the results are susceptible to transitions between classes as a function of variations in cut-off points. Thus, we compared children who retained (stable) their same latent classification as children at risk (LC=2) at both cut-off scores to those not retained. Children that retained their risk status at both cut-off we considered as suffering from MLD whereas children who transitioned out of the risk status group at the lower cut-off score were considered children with MD. Table 8 shows the effect sizes comparing the children with MLD to the two other latent classes. Using Cohen's (1988) criterion, large effect sizes (> .80) emerged in favor of average achievers (LC=1) when compared to children at risk for MLD (LC=2) on all cognitive measures, except naming speed and visual-WM. Large effect sizes also occurred between children with MLD (LC=2) and poor problem solvers (LC=3), suggesting that children with MLD yielded poor performance on measures related to knowledge of problem solving components and estimation.. However, it is important to note that several moderate effect sizes

(.50 to .80) occurred between children with MLD and poor problems solvers, suggesting children with MLD yielded poor performance on measures of STM, numeracy, and inhibition relative to poor problem solvers.

However, the results do not address the question as to whether children with MLD vary from children with MD on cognitive measures. The previous analysis (see Table 5) showed large effect sizes between the two groups (transition group vs. stable group) on manifest variables related to reading and calculation. However, according to Cohen's (1988) criteria, effect sizes between the two groups were small on measures of fluid intelligence and problem solving. Likewise, the magnitude of effect sizes between the two groups on the cognitive variables was miniscule. As shown in Table 9, means scores were higher for the transition group (children with MD) when compared to the stable group (children with MLD). However, according to Cohen's criteria, the only variable to approach a moderate effect size (ES=.50) was performance on the magnitude judgment factor score. An advantage emerged for children with MD when compared to children with MLD on this measure.

Discussion

The purpose of this study was to identify whether a discrete class of children at risk for MD or MLD emerged within a heterogeneous sample that varied in math, reading and fluid intelligence. The results yielded three important findings. First, the results showed that three latent classes emerged (average achievers, children with math difficulties or disabilities, and poor problem solvers) when setting cut-off scores at or below the 25th percentile and below the 11th percentile on manifest variables. As expected, the latent class referred to as average achievers outperformed the other two latent classes on a host of measures besides math (see Appendix A). When the influence of the various predictors was held constant in a logistic regression analysis, the cognitive variables that uniquely predicted these latent classes under both cut-off points were accuracy in identifying word problem solving components, estimating number values on a line, and span measures related to the executive component of WM. Finally, the results showed that poor problem solvers shared similar deficits to children with MD and MLD on several cognitive measures (e.g., STM, identifying word problem solving components, numeracy, executive component of WM), but yield advantages on measures of naming speed and estimation. Given these general findings, the results related to two questions that directed this study are now addressed.

Question 1: Can a latent classification of children at risk for MD and MLD be identified within a heterogeneous sample of learners when performance in math, reading and intelligence measures are included in the analysis?

The results show that a latent class emerges related to math difficulties (MD) and math learning disabilities (MLD) within a heterogeneous sample of learners. As shown in our analysis (see Table 3), the latent status membership probabilities for students at risk for both calculation and problem solving difficulties were approximately 15% of the total sample when the cut-off score was set at or below the 25th percentile and approximately 10% when set below the 11th percentile. The results showed that the lower of incidence was due to the fact that approximately half of the children identified as at risk with combined low calculation and problem solving performance identified at the 25th percentile (*N* = 65) were considered less likely to be identified as at risk for MLD when the cut-off scores were set to the 11th percentile (*N* was reduced to 37). Thus, the question emerges as to what is gained comparing the two cut-off points beyond lowering the incidence of MD by using the stricter cut-off score?

We suggest three advantages. First, reading performance plays a major role in defining MD and MLD. The effect sizes for reading were clearly in favor of children who were in the 11th to 25th percentile group when compared to children with MLD. Regardless of the degree of math severity, reading scores, especially comprehension, were in the below average range for both children with MD and children with MLD. As shown in Table 3, standard scores in passage comprehension were substantially lower than word identification, clearly suggesting that the children at risk in our sample were not proficient in reading comprehension. However, their math and reading problems did not reflect general problems in aptitude. That is, the normed scores for fluid intelligence measure for children with MD or MLD were in the normal range.

Second, our results showed that a latent class of children with MD or MLD and average reading did not emerge. In our sampling, we did not find a separate latent class of poor readers independent of math problems. Our findings are consistent with comprehensive meta-analyses of the published literature on math disabilities (Swanson & Jerman, 2006; Swanson et al., 2009) showing no clear cut differences on cognitive measures between children with math disabilities and children with reading disabilities (also see Swanson, 2012; Willcut et al., 2013).

Although it has been argued that clear contrasts between children with MD and RD on cognitive measures do not emerge because reading performance is not controlled (e.g., see Jordan, 2007, also see Swanson et al., 2009 for review), our results suggest that children with MD or MLD experience some of the same processing difficulties as children with reading disabilities. As expected and consistent with other studies (e.g., Cirino et al. 2015), variations in cognitive performance emerged related to variations in the cut-off score used for classifying children. When latent classes related to a 25^{th} percentile or 11^{th} percentile cut-off are compared to a latent class of average achievers, the 11^{th} percentile yielded more significant deficits on cognitive measures than the 25th percentile cut-off (see Table 6). However, a key issue was whether a separable latent class of children with math problems would emerge without reading problems. No doubt, we could have created a cluster of children with only math problems (i.e., via a cluster analysis), and compared this group on measures external to the classification (e.g., reading). However, this procedure would be limited because we could not determine the probability that such a latent class would emerge when reading was considered as part of the sampling process (e.g., Hagenaars & McCutcheon, 2009). The main difference between LCA and other clustering algorithms is that LCA offers a "model-based clustering" approach that derives clusters using a probabilistic model that describes the distribution of data. So instead of finding clusters with a distance measure, a model describes distribution of data based on the probabilities that certain cases are members of certain latent classes. LCA assumes an underlying latent variable gives rise to the classes, whereas the cluster analysis is an empirical description of correlated attributes from a clustering algorithm. LCA is considered methodologically superior given that it has a formal chi-square significance test, which the cluster analysis does not.

Finally, the results show that math problem solving difficulties are independent of calculation difficulties. Our results indicated that a large segment of the sample shows difficulties in math problem solving when using the 25th percentile (53% of the total sample) and 11th percentile as cut-off points (55% of the sample). This finding is consistent with the National Mathematics Advisory Panel (2008), and to PISA (Programme for International Student Assessment; OCED 2012a, b), showing that U.S. children show substantial weaknesses when asked to solve math word problems relative to other achievement domains and in comparison to other industrialized countries. In addition, longitudinal studies (e.g., Swanson et al., 2008) have shown that even when calculation and reading skills are at grade level, difficulties in math word problem solving are persistent across the elementary school years. Also finding a latent class of children at risk for MLD as well as a latent class of poor problem solvers fits within current categories of learning disabilities. These categories include specific disabilities in calculation and mathematical problem solving [see IDEA reauthorization, 2004, Sec. 300.8(c)(10)]. Although the majority of classification research on MLD has focused on calculation deficits (Andersson, 2010; Geary 2011; Gersten et al., 2009; Swanson & Jerman, 2006), we found a latent class group showing math classification related to poor problem solving separately from problems in calculation.

Question 2: Do specific cognitive measures predict latent class membership?

In terms of cognitive models that predict latent class status, three were considered. As reviewed in the introduction, these models considered whether STM storage (phonological loop), domain specific knowledge (components of word problems, magnitude judgments, and line estimation) and executive processing played a major role in predictions of latent class status. The results suggested that none of the above models in isolation provided a parsimonious account of the findings. The largest beta-weight loadings from the full logistic regression model were measures of domain specific knowledge (word problem solving components and estimation) and the executive component of WM. These findings fit the literature attributing MD and MLD to deficits in magnitude representation (e.g., Fuch et al, 2012; Martin et al., 2013; Geary, 2011), accessing specific knowledge related to word problem structure (e.g., Swanson & Beebe-Frankenberg, 2004) and the executive component of working memory (e.g., Lee et al., 2004) The domain specific knowledge of problem solving components played a significant role in the predictions of latent class, which is consistent with other studies identifying children at risk for math problems (Swanson et al., 1993; Swanson & Beebe-Frankenberger, 2003). In this study, problem solving components tapped into children's recognition of question, number assignment, goals, and irrelevant information propositions within word problems. The findings related to estimation also fit within the current literature on identify children with serious math difficulties (e.g., Geary, 2013). It has been argued the core of math abilities may be the ability to develop a mental number line on the assumption that numbers are arranged spatially or on a continuum. An adequate magnitude representation allows for a rejection of implausible answers and therefore helps children compute correct answers. The high loadings related to the executive component of WM were an expected finding. This finding is consistent with previous studies suggesting that the executive component of WM plays a major role in predicting math proficiency (e.g., Swanson & Beebe-Frankenberger, 2004).

Implications

There are two implications related to our findings. First, within a heterogeneous sample of third grade learners, an identifiable group of children with MD was identified at the 25th percentile cut-off point. Although the 25th percentile has been used as a common "a priori cut-off point" to identify children at risk the issue as to whether the cut-off score yields a latent class of children not proficient in math and/or a discrete identifiable group has not been established. Thus, this study contributes to the emerging literature that children with MD as well as MLD represent an identifiable group. The results provide empirical support for the commonly used 25th percentile as an a priori cut-off score for determining risk. That is, latent class analyses showed a discrete group of children emerged as at risk for MD at this cut-off point.

Second, the probability of finding a latent class of children with MD or MLD completely independent of reading problems may be quite low. Obviously, this implication is limited to our data set and may not generalize to other samples. However, our findings are consistent with a number of other studies finding the math difficulties are comorbid with reading difficulties and therefore share similar processing deficits (e.g., Hecht et al, 2001; Jordon et al., 2003). For example, Swanson and Jerman's (2006) meta-analysis of the published literature on MLD found no clear cut differences between children with arithmetic and reading difficulties on cognitive measures. Although the results of the meta-analysis suggested that effect sizes were in favor of children with reading disabilities (RD) when compared to children with MD across several measures, the substantive advantages for children with RD were isolated to measures of naming speed and visual-spatial WM. These overall findings were problematic because several studies have suggested that children with RD can be separated from children with MD (e.g., Jordan, Hanich, & Kaplan, 2003). No doubt, the poor differentiation between children with MD and those with reading difficulties may have occurred because the studies included samples with poor arithmetic skills accompanied by relatively low reading skills. Therefore, it was difficult to determine whether results attributed to MD were in fact due to arithmetic difficulties or whether they were outcomes related to generally poor academic skills that shared the same process that incorporated both reading and math skills.

Thus, there is a question as to whether children with MD or MLD suffer from the same processes associated with RD. For example, Jordan (2007) in her synthesis of the literature argued that authors have incorrectly assumed that MLD is related to language, which in turn suggests some commonality between math and reading. Other studies (e.g., Landerl, Bevan, & Butterworth, 2004) have suggested that all children with MD, with or without reading problems, showed general deficits in number processing. Other authors also find evidence (e.g., Fuchs et al., 2008) that problem solving rather than number and arithmetic skills differentiates children with MD from children with MD+RD. Thus, what may differentiate the two disabilities is the ability to solve complex word problems (e.g., Jordan et al., 2003).

As shown in this study, the ability to solve math word problems yields a discrete latent class g, in which the majority children in this discrete class yield calculation and reading scores in the average range (see Table 4). Thus, children with difficulties in math problem solving represent a distinct category (i.e., reading and calculation skills play a less important role) of risk that can be separated from children with a combination of reading and/or calculation deficits. Although the poor problem solving latent class shared similar cognitive deficits as children with MD or MLD relative to average achievers (e.g., STM, Knowledge of problem solving components, numeracy, executive component of WM), their skills related to naming speed and estimation supersede those of children with MD or MLD.

How is it that a poor math problem solver does not necessarily suffer deficiencies in reading or calculation? We raise that question since it is commonly assumed that poor mathematical word problem solving can be linked to reading proficiency (e.g., Swanson et al., 1993). This is because mathematical word problems are a form of text and the decoding and comprehension of text draws upon the phonological system. However, understanding mathematical word problems also involves a complex interaction of text comprehension and mathematical processes that are related to activities attributed to the WM system. There are some studies that clearly show that reading or reading related processes do not directly mediate the influence of WM on problem solving. For example, an earlier study by Swanson and Sachse-Lee (2001) found that for children with MLD and chronologically age-matched peers that phonological processing, verbal WM and visual WM contributed unique variance to word problem solution accuracy. Thus, they did not find support for the assumption that reading ability mediated the role of WM in solution accuracy. In a follow-up study, Swanson (2004) compared two age groups (7 and 11 years old) on WM and problem-solving measures. This study found that regardless of age, WM predicted solution accuracy in word problems independent of measures of problem representation, knowledge of operations and algorithms, phonological processing, fluid intelligence, reading, and math. Further, the results suggested that a general or executive system underlies age-related improvements in word problem-solving accuracy. Further, Swanson (2004) found that measures of LTM (such as calculation and knowledge of algorithms) entered into the regression analysis did not eliminate the contribution of WM to problem solving accuracy, a finding similar to ours. We did find that the retrievability of contents in LTM, propositions within word problems outlined by Mayer and Hegarty (1996) and estimation contributed unique variance to problem solving accuracy. In general, our findings are consistent with models of high-order processing that suggest that WM resources activate relevant knowledge from LTM (Baddeley & Logie, 1999; Ericsson & Kintsch, 1995), as well as include a subsystem that controls and regulates the cognitive system (Baddeley, 1986).

Limitations

There are at least four limitations to this study. First, although we used cut-off points identified in the literature as important in identifying children at risk for MD or MLD, we have not shown that the identification of latent classes validates a specific cut-off point. Rather the results suggest the measures were able to identify subgroups to the cut-off to which they were applied. That is, the same or other latent classes may have emerged with other cut-off scores. Second, we did not establish the stability of math performance across multiple grades. Although we used a variety of normed referenced measures to capture consistency in low math performance, performance across multiple grades was not assessed. Third, we have an absence of intervention information. Thus, our study is limited to discussing the risk classification within a heterogeneous sample and not whether a particular intervention program would later influence the classification of children at risk.

Finally, some reviewers raised questions about the power of our analyses on our classification measures. LCA has not at present provided procedures for conducting a power analysis. For example, the small sample size for the "MLD" group may have potentially yielded more identifiable items in the areas other than reading with a larger sample. In response to the sample size, Dziak Lanza, and Tan (2014) analyzed the predictive power of the Bootstrap likelihood ratio test (BLRT) in LCA analysis using a number of Monte-Carlo simulations. For example, they found that the simulated power for detecting a three-class over a two-class model at .90 (alpha = .05) would require an *N* of 150 (see Table 3, p. 537). Since our sample exceeded *N* = 150, we assume the BLRT procedure was adequately powered (see Table 2) to detect a three-class model from a four-class model. However, as useful as LCA is to determine meaningful patterns within the data, latent class assignment is not a definitive process. Our over-reliance on model fit compared to substantive theory can produce differing results. Thus, due to the exploratory nature of the current work and definite sample size limitations for a parameter intensive model, we take a cautionary approach to modeling math problems in third graders.

Summary

In summary, this study yielded three important findings. First, latent classifications of children at risk for MD and MLD could be identified among a sample of grade 3 children. The results provide support for the notion that children at risk for serious math difficulties within a heterogeneous sample reflect two discrete classes: those with combined calculation, problem solving, and reading comprehension difficulties and those with math problem solving difficulties. Second, approximately half the children who are initially identified as at risk for MD at the 25th percentile are less likely to be at risk when setting the cut-off score to the 11th percentile. Finally, regardless of the two cut-off score points, cognitive measures related to problem solving processes, estimation and the executive component of WM were the only cognitive measures that consistently predicted latent class status. Overall, the results support the notion that children at risk for MD or MLD reflect a latent class group that can be separated from a heterogeneous sample of children who vary in math, reading and fluid intelligence.

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Footnote

1. Although the fit indices supported a three latent class group, a four group LCA was also investigated on the assumption that a low math achieving but average reading group would emerge. No such group emerged. Gamma estimates (class membership probabilities) were .28 for average achievers, .09 for children with MD, .35 for poor problem solvers across all problem solving measures, and .32 for poor problems solvers on word problem measures that required reading. No latent class emerged in which risk for MD (low calculation and problem solving) reflected a group with average reading scores.

Table 1

Descriptive Information for Classification (standard scores) and Predictor Variables (raw scores) for Total Sample (N=447)

	Mean	SD	Min	Max	Kurtosis	Skewness	Reliability
Classification Measure							
Calculation							
WIAT	98.69	12.67	67	141	0.17	-0.24	0.84
WRAT-A	98.77	10.36	66	134	0.03	0.53	0.80
Math Prob. Solving							
TOMA	78.50	21.02	40	140	0.65	-0.17	0.94
WISC-A	97.61	31.46	10	180	-0.38	0.18	0.96
CMAT	66.73	30.02	0	150	0.05	-0.99	0.97
KeyMath	94.77	34.7	10	170	0.40	0.05	0.97
Reading							
WRAT-R	103.81	12.36	66	153	0.27	1.32	0.82
TORC	92.84	23.19	30	160	-0.16	0.09	0.94
Fluid Intelligence							
Raven	100.07	15.15	42	128	-0.69	0.64	0.93
Problem Solving Components							
Question	2.15	0.92	0	3	-0.79	-0.36	0.71
Number	2.32	0.86	0	3	-1.03	0.12	0.72
Goal	1.80	0.94	0	3	-0.29	-0.84	0.71
Operation	1.86	0.85	0	3	-0.35	-0.49	0.71
Algorithm	1.79	0.87	0	3	-0.31	-0.56	0.70
Irrelevant	2.35	0.86	0	3	-1.18	0.48	0.76
Estimation							
Estimation1	2.59	2.06	0	15	1.61	4.72	0.84
Estimation2	1.84	1.31	0	10	1.73	5.11	0.85
Numeracy							
Numeracy1	18.75	6.58	0	38	-0.15	0.34	0.83
Numeracy2	10.24	3.8	0	25	-0.01	1.67	0.65
Naming Speed							
Speed Digits	39.41	9.4	20	79	0.83	0.73	0.83
Speed Letters	42.82	10.64	0	107	0.63	4.37	0.82
Exec. WM							
Concept Span	3.07	3.18	0	21	2.15	7.00	0.84
Listspan	2.67	1.97	1	12	1.41	1.98	0.79
Update	5.56	4.21	0	16	1.03	0.11	0.88
Visual-Sketch Pad							
Map/Direction	1.75	0.55	1	5	1.91	4.41	0.65
Matrix	11.94	8.13	0	33	0.40	-0.41	0.92

Phonological Loop-STM							
Dforward	6.72	1.73	2	14	0.36	0.50	0.81
Word-Span	8.24	3.47	2	20	0.36	-0.54	0.68
Phon-Span	3.13	2.23	0	14	0.95	1.33	0.63

WIAT = Wechsler Individual Achievement Test. WRAT-A = Wide Range Achievement Test arithmetic subtest. TOMA = Test of Mathematical Abilities. WISC = Arithmetic subtest from Wechsler Intelligence Scale for Children. CMAT = Comprehensive Mathematical Abilities Test. KeyMath = KeyMath Revised Diagnostic Assessment. WRAT-Reading = Wide Range Achievement Test Reading Task. TORC = Test of Reading Comprehension. Raven = Colored Progressive Matrices Test. Estimation1=set 1-vary numbers, Estimation2=set 2-vary line length, numeracy1=circle large number, numeracy2=circle small number.Concept= conceptual span, Listspan=Listening Span, Dforward=digit forward, phon-span=pseudoword span task.

Table 2

Fix Indices for Six Latent Class Models

Cut off at or < 25						
	LC=1	LC=2	LC=3	LC=4	LC=5	LC=6
Log-likelihood:	-2232.86	-1994.98	-1950.58	-1946.89	-1933.29	-1921.38
G-squared:	835.95	360.2	271.4	264.01	236.82	212.99
AIC:	853.95	398.2	**329.4**	342.01	334.82	330.99
BIC:	890.87	476.15	**448.37**	502.01	535.84	573.04
CAIC:	899.87	495.15	**477.37**	541.01	584.84	632.04
Adjusted BIC	862.31	415.85	**356.34**	378.24	380.34	385.8
Entropy	1.0	0.75	0.75	0.80	0.82	0.85
Degrees	502	492	482	472	462	452
LMR (*p*-value)	-	0	0	.34	.27	.17
BLRT (*p*-value)	-	0	0	.66	.36	.42
Cut off < 11						
	LC1	LC=2	LC=3	LC=4	LC=5	LC=6
Log-likelihood:	-1882.47	-1687.33	-1643.29	-1633.22	-1624.42	-1618.48
G-squared:	633.81	243.53	155.45	135.3	117.71	105.83
AIC:	651.81	281.53	213.45	213.30	215.71	223.83
BIC:	688.73	359.48	**332.42**	373.30	416.73	465.88
CAIC:	697.73	378.48	**361.42**	412.30	465.73	524.88
Adjusted BIC	660.17	299.18	**240.39**	249.53	261.23	278.64
Entropy	1.0	0.72	0.75	0.74	0.71	0.72
Degrees	502	492	482	472	462	452
LMR (*p*-value)	-	0	.0180	.034	.19	.31
BLRT (*p*-value)	-	0	0	.065	.15	1.0

Note. *LC*=Latent Class, AIC = Akaike's Information Criterion; BIC = Bayesian Information Criterion; LMR = Lo-Mendell-Rubin Test; BLRT = Bootstrap Likelihood Ratio Test.

Table 3

Probabilities of Assignment to Latent Classes

		At or < 25th percentile				< 11th percentile		
GAMMA		Probabilities						
Latent Class		1	2	3		1	2	3
		0.32	0.15	0.53		0.35	0.10	0.55
Rho Estimates[a]		Math Difficulties (MD)				Math Learning Disabilities (MLD)		
Latent Class		1	2	3		1	2	3
Calculation								
WIAT		0.003	**0.82**	0.24		0.00	0.59	0.08
WRAT-A		0.006	0.46	0.13		0.01	0.40	0.01
Math Problem Solving								
TOMA		0.28	**0.92**	**0.81**		0.31	**0.96**	**0.83**
WISC-A		0.07	**0.80**	0.36		0.08	**0.76**	0.42
CMAT		0.23	**1.00**	**0.88**		0.26	**1.00**	**0.90**
KEYMATH		0.03	**1.00**	0.50		0.03	**1.00**	0.56
Reading								
TORC		0.07	**0.87**	0.33		0.09	**1.00**	0.36
WRAT-R		0.01	0.42	0.06		0.00	0.30	0.05
Fluid Intelligence								
Raven		0.11	0.42	0.19		0.06	0.23	0.10

[a] Note. = rho estimates >.70 (in bold) were considered high probability of Risk for MD (< 25th percentile) or MLD (< 11th percentile).

WIAT = Wechsler Individual Achievement Test. WRAT-A = Wide Range Achievement Test arithmetic subtest. TOMA = Test of Mathematical Abilities. WISC = Arithmetic subtest from Wechsler Intelligence Scale for Children. CMAT = Comprehensive Mathematical Abilities Test. KeyMath = KeyMath Revised Diagnostic Assessment. TORC = Test of Reading Comprehension. WRAT-Reading = Wide Range Achievement Test Reading Task. Raven = Colored Progressive Matrices Test.

Table 4

Normed Referenced Scores for Classification Measures as a Function Of Latent Class

	LC=1		LC=2		LC=3	
At or < 25th percentile						
Variable	Mean	*SD*	Mean	*SD*	Mean	*SD*
Calculation						
WIAT	108.01	9.27	83.68	8.12	97.29	10.8
WRAT-A	105.06	9.32	87.29	8.86	98.18	8.28
Math Problem Solving						
TOMA	96.45	19.57	64.15	14.13	71.87	15.98
WISC	119.15	23.25	65.69	23.78	93.61	28.44
CMAT	96.38	20.43	56.62	17.88	77.51	22.33
KEYMath	124.11	28.81	57.08	18.85	87.76	26.82
Reading						
TORC	107.16	18.72	64.77	15.92	92.03	19.82
WRAT-R	111.35	11.72	91.20	11.10	102.8	9.67
Fluid Intelligence						
Raven	105.69	15.49	92.26	14.22	98.88	14.01
	LC=1		LC=2		LC=3	
< 11th percentile						
Variable	Mean	*SD*	Mean	*SD*	Mean	*SD*
Calculation						
WIAT	106.82	10.32	81.26	7.57	96.64	10.94
WRAT-A	104.42	10.11	84.92	9.59	97.60	8.04
Math Problem Solving						
TOMA	95.84	19.52	60.51	10.50	71.24	15.92
WISC-A	119.33	23.56	64.87	28.27	90.04	27.99
CMAT	95.91	20.24	54.62	19.17	74.79	21.88
KEYMath	123.76	28.82	54.36	18.75	84.17	26.63
Reading						
TORC	106.51	19.24	59.49	14.86	90.00	20.02
WRAT-R	111.22	12.00	87.49	10.56	102.01	9.48
Fluid Intelligence						
Raven	105.39	15.75	93.20	14.18	98.04	14.05

WIAT = Wechsler Individual Achievement Test. WRAT-A = arithmetic subtest from the Wide Range Achievement Test. TOMA = Test of Mathematical Abilities. WISC-A = arithmetic subtest from the Wechsler Intelligence Scale for Children. CMAT = Comprehensive Mathematical Abilities Test. KeyMath = KeyMath Revised Diagnostic Assessment. TORC = Test of Reading Comprehension. WRAT-R = reading subtest from the Wide Range Achievement Test Reading Task. Raven = Colored Progressive Matrices Test.

Table 5
Comparison of Children on Normed References Score who Transitioned out of the Latent Class Risk Group (N=28) at the 11th Percentile and Those That Remained (Stable) at Both Cut-off points (N=37).

		Transition			Stable		
		M	*SD*		M	*SD*	ES
Classification							
Calculation							
WIAT		87.21	7.63		81.00	7.51	0.82
WRAT		90.61	6.44		84.78	9.67	0.72
Problem Solving							
TOMA		68.93	16.63		60.54	10.79	0.61
WISC-A		70.00	18.66		62.43	26.81	0.33
CMAT		58.93	15.48		54.86	19.53	0.23
KEYMath		60.71	18.04		54.32	19.23	0.34
Reading							
TORC		72.50	14.04		58.92	14.87	0.94
WRAT-R		96.82	9.40		86.95	10.46	0.99
Fluid Intelligence							
Raven		91.75	14.47		92.64	14.23	-0.06

Note. Transition= Children defined as math difficulties at 25th percentile cut-off but not 11th percentile cut-off
Stable=children who retained math risk status at both cut-off points (children with MLD).
ES=Cohen's effect size
Classification measures are normed referenced standard scores. WIAT = Wechsler Individual Achievement Test. WRAT = Wide Range Achievement Test. TOMA = Test of Mathematical Abilities. WISC-A = arithmetic subtest from theWechsler Intelligence Scale for Children. CMAT = Comprehensive Mathematical Abilities Test.KeyMath = KeyMath Revised Diagnostic Assessment. TORC = Test of Reading Comprehension. WRAT-R = reading subtest from the Wide Range Achievement Test Reading Task. Raven = Colored Progressive Matrices Test.

Table 6
Logistic Regression Model Predicting Latent Classes at Two Cut-off Points

	At or <25th Percentile				< 11th percentile			
Parameter	Odds	Estimate	SE	Wald χ2	Odds	Estimate	SE	Wald χ2
Full Model								
Phonological Storage								
STM	1.13	0.12	0.08	2.23	1.28	0.24	0.09	7.86**
Speed	0.87	-0.14	0.08	3.11	0.82	-0.19	0.09	5.04*
Domain Specific Knowledge								
Component	1.16	0.14	0.05	7.88**	1.25	0.22	0.06	15.58***
Estimation	0.77	-0.27	0.08	10.35***	0.83	-0.19	0.09	4.63*
Numeracy	1.12	0.11	0.06	3.47	1.1	0.1	0.06	2.48
Executive Processing								
WM-E	1.25	0.23	0.08	7.50**	1.29	0.25	0.09	8.26***
Inhibition	0.81	-0.21	0.13	2.74	0.87	-0.14	0.13	1.12
Vis-WM	1.14	0.13	0.09	2.30	1.19	0.18	0.09	3.73*
Model Fit Statistics								
Criterion								
AIC	734.928				642.713			
BIC	770.67				678.454			
Deviance	716.928				624.713			
Reduced Model 1								
Phonological Storage								
STM	1.29	0.26	0.07	12.73***	1.50	0.41	0.08	27.66***
Speed	0.73	-0.32	0.05	40.90***	0.74	-0.30	0.05	35.34***
Model Fit Statistics								
AIC	852.146				760.082			
BIC	864.345				772.280			
Deviance	846.146				754.082			
Reduced Model 2								
Domain Specific Knowledge								
Component	1.17	0.16	0.04	12.19***	1.28	0.25	0.05	25.80***
Estimation	0.70	-0.36	0.05	47.44***	0.72	-0.33	0.05	37.10***
Numeracy	1.16	0.15	0.05	7.28**	1.16	0.15	0.06	6.95*
Model Fit Statistics								
AIC	808.517				721.572			
BIC	824.697				737.752			
Deviance	800.517				713.572			
Reduced Model 3								
Executive Processing								
WM-E	1.37	0.32	0.08	16.3***	1.47	0.39	0.08	21.84***
Inhibition	1.00	0.01	0.11	0.01	1.09	0.09	0.12	0.59
Vis-WM	1.26	0.23	0.09	7.21**	1.31	0.27	0.09	9.25**

Model Fit Statistics		
AIC	869.037	776.048
BIC	885.264	792.275
Deviance	861.037	768.048

Note. WM-E=executive component of working memory, Component=identifying problem solving components. Vis-WM=visual-spatial sketch pad
AIC = Akaike’s Information Criterion; BIC = Bayesian Information Criterion

Table 7

Full Multinomial Logistic Regression Model Predicting Latent Classes at Both Cut-off Points

Full Model		**At or <25th**				**<11th**			
	Compare	Odds	Estimate	SE	Wald χ2	Odds	Estimate	SE	Wald χ2
Storage									
STM	LC1 vs. LC3	1.33	0.28	0.11	7.23**	1.44	0.36	0.11	11.76***
STM	LC2 vs. LC3	0.54	-0.62	0.17	13.61***	0.73	-0.32	0.17	3.37
Speed	LC1 vs. LC3	0.85	-0.17	0.11	2.18	0.79	-0.23	0.11	4.29
Speed	LC2 vs. LC3	0.68	-0.38	0.13	8.21**	0.51	-0.68	0.14	22.3***
Domain Specific Knowledge									
Component	LC1 vs. LC3	1.47	0.39	0.08	24.44***	1.52	0.42	0.08	29.06***
Component	LC2 vs. LC3	0.77	-0.26	0.08	10.03***	0.86	-0.15	0.09	2.90
Estimation	LC1 vs. LC3	0.72	-0.33	0.12	8.14**	0.79	-0.24	0.11	4.55
Estimation	LC2 vs. LC3	0.47	-0.75	0.14	29.64***	0.56	-0.57	0.14	17.85***
Numeracy	LC1 vs. LC3	1.30	0.26	0.08	10.61***	1.27	0.24	0.08	9.16**
Numeracy	LC2 vs. LC3	0.82	-0.20	0.10	3.64	0.81	-0.22	0.11	3.64
Executive Processing									
WM-E	LC1 vs. LC3	1.28	0.25	0.10	6.05**	1.34	0.30	0.10	8.19**
WM-E	LC2 vs. LC3	1.20	0.18	0.17	1.15	1.22	0.20	0.18	1.25
Inhibition	LC1 vs. LC3	0.85	-0.16	0.16	1.08	0.91	-0.10	0.16	0.36
Inhibition	LC2 vs. LC3	0.55	-0.61	0.26	5.44	0.75	-0.29	0.26	1.25
Vis-WM	LC1 vs. LC3	1.18	0.17	0.10	2.55	1.23	0.21	0.11	3.72
Vis-WM	LC2 vs. LC3	0.93	-0.07	0.17	0.19	1.12	0.11	0.18	0.37

LC1=Average Achiever, LC2=Math Disabled (at or < 25th percentile or MLD (< 11th percentile). LC3= poor problem solvers .Component= problem solving component, Inhibition = Random generation of numbers and letters; Speed = Naming speed; WM-E = Working memory executive component (Conceptual Span, Listening Sentence Span, and Updating task); Vis-WM = Visual–spatial sketchpad (Visual Matrix and Mapping & Directions); STM = Short-term memory (Forward Digit Span, Backward Digit Span, Word Span, and Pseudoword Span).

$^{**}p < .01$, $^{***}p < .001$

Table 8

Means, Standard Deviations and Effect Size Comparisons among Stable Latent Classes

	LC=1, *N*=141)		LC=2, *N*=37)		LC=3, *N*=231)		Effect Sizes		
	M	*SD*	M	*SD*	M	*SD*	LC1 vs. LC2	LC1 vs. LC3	LC2 vs.LC 3
Classification									
Calculation									
WIAT	108.01	9.27	81.00	7.51	97.79	10.74	3.22	1.02	-1.84
WRAT-A	105.06	9.32	84.78	9.67	98.45	7.80	2.14	0.77	-1.56
Math Problem Solving									
TOMA	96.45	19.57	60.54	10.79	71.52	15.85	2.37	1.41	-0.82
WISC-A	119.15	23.25	62.43	26.81	92.47	27.98	2.27	1.04	-1.10
CMAT	96.38	20.43	34.86	19.53	56.71	21.78	3.08	1.88	-1.06
KEYMath	124.11	28.81	54.32	19.23	87.01	26.11	2.91	1.35	-1.44
Reading									
TORC	107.16	18.72	58.92	14.87	92.12	19.61	2.87	0.78	-1.93
WRAT-R	111.35	11.72	86.95	10.46	102.64	9.32	2.20	0.83	-1.59
Fluid Intelligence									
Raven	105.69	15.49	92.64	14.23	98.8	13.83	0.88	0.47	-0.44
Cognitive Processes									
STM	0.63	1.09	-0.97	1.08	-0.09	1.33	**1.47**	**0.60**	**-0.73**
Speed	-0.41	1.39	0.6	2.08	-0.07	1.44	**-0.58**	-0.24	0.38
Component	1.38	1.67	-2.16	2.35	-0.27	2.26	**1.76**	**0.84**	**-0.82**
Estimation	-0.64	1.28	1.22	1.40	0.01	1.31	**-1.39**	-0.49	**0.90**
Numeracy	0.79	1.83	-1.51	1.69	-0.18	1.71	**1.31**	**0.55**	**-0.78**
WM-E	0.63	1.62	-0.59	1.06	-0.22	1.22	**0.91**	**0.60**	-0.32
Inhibition	0.18	0.88	-0.55	0.69	0.01	0.84	**0.93**	0.21	**-0.72**
Vis-WM	0.38	1.30	-0.27	0.84	-0.14	1.21	**0.61**	0.41	-0.13

Cognitive process measures are factor scores (z-scores) based on the total sample.
Stable=children who maintained same LC status for at risk at both cut-off points.
LC=1=Average Achiever, LC2=Math Disabled, LC3= poor problem solvers.
Components=component processes, Speed = Naming speed; STM = Short-term memory WM-E = Working memory executive component, VIS-WM = Visual–spatial sketchpad.
ES=Cohen's d, ES1=LC=1 vs. LC=2, ES2=LC=1 vs. LC=3, ES3=LC=2 vs. LC=3,
Bold=cognitive measures of moderate (> .50) and large (> .80) effect sizes.
Note. Scores were not partialed for the influence of other variables.

Table 9

Comparison of Children on Cognitive Measures who Transitioned out of MD Risk Group (N=28) and Those Who Remained Stable in the at Risk group (N=37) at Both Cut-Off Points

	Transition		Stable		
	M	SD	M	SD	ES
Cognitive Processes					
STM	-1.05	0.94	-0.97	1.08	-0.08
Speed	0.67	1.5	0.6	2.08	0.04
Component	-1.94	2.24	-2.16	2.35	0.1
Estimation	1.25	2.11	1.22	1.4	0.02
Numeracy	-0.78	1.9	-1.51	1.69	0.41
WM-E	-0.65	0.7	-0.59	1.06	-0.07
Inhibition	-0.46	0.71	-0.55	0.69	0.13
Vis-WM	-0.4	0.97	-0.27	0.84	-0.14

Note. Transition= Children defined as at risk at 25th percentile cut-off but not 11th percentile cut-off. Stable=children who retained risk status at both cut-off points. ES=Cohen's effect size. Cognitive process measures are factor scores (z-scores) based on the total sample. STM=short-term memory or phonological loop, Speed=naming speed, Component= accuracy identifying components of word problems, WM-E=executive component of WM, Inhibition=random generation ,Vis-WM= visual-spatial sketchpad.

Appendix A

Comparison of Latent Classes on Factor Scores (z-scores) for the Cognitive Measures

At or <25th percentile

	LC=1/N=141		LC=2/N=65		LC=3/N=240		ES1	ES2	ES3
	Mean	SD	Mean	SD	Mean	SD			
STM	0.63	1.09	-1.00	1.02	-0.06	1.33	1.60	0.52	-0.80
Speed	-0.41	1.39	0.63	1.84	-0.04	1.44	-0.57	-0.26	0.41
Components	1.38	1.67	-2.07	2.29	-0.23	2.26	1.51	0.71	-0.81
Estimation	-0.64	1.28	1.23	1.74	0.01	1.33	-1.07	-0.49	0.79
Numeracy	0.79	1.83	-1.19	1.81	-0.19	1.70	1.09	0.58	-0.57
WM-E	0.63	1.62	-0.61	0.92	-0.17	1.26	1.35	0.63	-0.40
Inhibition	0.18	0.88	-0.51	0.70	0.02	0.83	0.99	0.19	-0.69
Vis-WM	0.38	1.3	-0.33	0.89	-0.12	1.22	0.80	0.41	-0.20

< 11th percentile

	LC=1/N=149		LC=2/N=39		LC=3/N=258		ES1	ES2	ES3
	Mean	SD	Mean	SD	Mean	SD			
STM	0.65	1.09	-0.93	1.10	-0.2	1.33	1.44	0.64	-0.60
Speed	-0.36	1.41	0.58	2.04	0.02	1.46	-0.46	-0.26	0.32
Components	1.39	1.65	-2.14	2.29	-0.46	2.32	1.54	0.80	-0.73
Estimation	-0.64	1.25	1.29	1.49	0.14	1.47	-1.30	-0.53	0.78
Numeracy	0.75	1.8	-1.57	1.71	-0.24	1.74	1.36	0.57	-0.77
WM-E	0.65	1.62	-0.52	1.10	-0.26	1.19	1.06	0.76	-0.23
Inhibition	0.19	0.86	-0.54	0.68	-0.05	0.84	1.07	0.29	-0.64
Vis-WM	0.38	1.31	-0.28	0.83	-0.16	1.18	0.80	0.46	-0.12

LC=1=Average Achiever, LC2=Math difficulties(< 25th percentile) or math learning disabilities (< 11th percentile).
LC3= poor problem solvers. Cognitive process measures are factor scores (z-scores) based on the total sample.
STM = Short-term memory , Speed = Naming speed; Components=component processes, WM-E = Working memory
executive component, Vis-WM = Visual–spatial sketchpad. ES=Cohen’s d, ES1=LC1 vs. LC2, ES2=LC1 vs. LC3,
ES3=LC2 vs. LC3 . Note. ES values are not partialed for the influence of other variables.