News-driven Expectations and Volatility Clustering

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Abstract

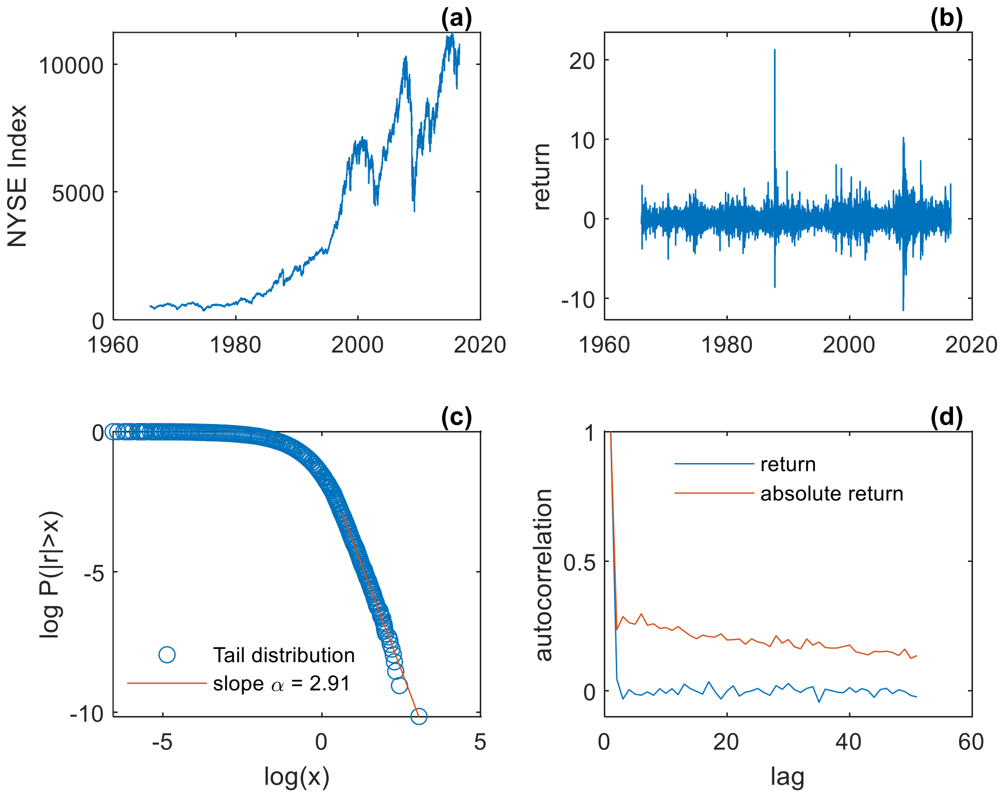
Financial volatility obeys two well-established empirical properties: it is fat-tailed (power-law distributed) and it tends to be clustered in time. Many interesting models have been proposed to account for these regularities, notably agent-based computational models, which typically invoke complicated mechanisms, however. It can be shown that trend-following speculation generates the power law in an intrinsic way. But this model cannot exaplain clustered volatility. This paper extends the model and offers a simple explanation for clustered volatility: the impact of exogenous news on traders' expectations. Owing to the famous no-trade results, rational expectations, the dominant model of news-driven expectations, is hard to reconcile with the incessant high-frequency trading behind the volatility clustering. The simplest alternative model of news-driven

expectations is to assume that traders have prior views about the market (an asset's future price change or its present value) and then modify their views with the advent of a news. This simple news-driven random walk of traders' expectations explains volatility clustering in a generic way. Liquidity plays a crucial role in this dynamics of volatility, which is emphasized in a dicussions section.

1. The two empirical regularities

Financial volatility obeys two well-established regularities: it is fat tailed, more precisely power-law distributed (with an exponent often close to 3), and it tends to be clustred in time (Fama, 1963; Mandelbrot, 1963; Gopikrishnan, Meyer, Amaral, & Stanley, 1998; Plerou, Gabaix, Stanley, & Gopikrishnan, 2006; Cont, 2007; Bouchaud & Challet, 2016). These are fascinating regularities that hold for various types of finanical products, on various markets, and on various time scales. The first regularity implies that extreme price changes are much more likely than would suggest the standard assumption of normal distribution. The second property, *volatility clustering*, holds that high-amplitude price changes tend to be followed by high-amplitude price changes, and low-amplitude price changes, by low-amplitude price changes. This corresponds to a nontrivial predictability of price changes: while their sign is uncorrelated, its amplitude (or volatility) is long-range correlated.

FIG. 1. NYSE composite daily index: (a) Price; (b) Return (in percent); (c) cumulative distribution of volatility in log-log scale, showing a linear fit of the tail, with a slope close to 3; (d) Autocorrelation function of return, which is almost zero at all lags, while that of volatility is nonzero over a long range of lags (a phenomenon known as volatility clustering).



Formally, let P_t be the price of a financial asset at the closing of period t, let the relative price change (or return) be $r_t = (P_t - P_{t-1}) / P_{t-1}$, and let volatility be measured by means of $| r_t |$, the amplitude (absolute value) or return. Then the two regularities hold that: (a) $P(| r_t |> x) \sim Cx^{-\alpha}$, for big values x, where $\alpha \approx 3$ and $C > 0$; and (b) $\mathrm{cor}(r_t, r_{t+h}) \approx 0$ for $h > 0$ (except perhaps for $h = 1$), but $\mathrm{cor}(|r_t|, |r_{t+h}|) > 0$ over a long range of lags h. FIG. 1 illustrates these two regularities for the NYSE daily index[1].

The universality of these laws suggests that there must be some basic, general, and stable mechanisms behind them. Standard financial economics, despite its important theoretical insights, is nonetheless silent on these empirical regularities. In fact, there seems to be an intrinsic difficulty in reconciling the high-frequency volatility of financial markets, caused by incessant trading at almost all time scales, with the dominant assumption of rational expectations, which often leads to a no-trade equilibrium, as is well-known (Milgrom & Stokey, 1982; Tirole, 1982). Agent-based models of financial markets, on the other hand, offers various realistic models of price fluctuations, but these models often involve relatively complicated mechanisms, which are handled computationally.[2] This paper, while it is closer in spirit to this alternative, complex-systems view, is nonetheless an attempt to pin down the empirical regularities to

[1] The linear fit is based on a maximum-likelihood algorithm developed by Clauset, Shalizi, and Newman (2009), which is also a good reference for the statistical test of empirical power laws. For an introduction to power laws, see, for example, Newman (2005) and Gabaix (2008, 2016).

[2] For a review of agent-based models of financial markets, see Samanidou, Zschischang, Stauffer, and Lux (2007). Agent-based models are, however, only one of the trends in the complex-systems approach to financial markets, which insists on the endogenous, emergent, dynamics of markets. For an introduction to this view on financial markets, see, e.g., Bouchaud (2011).

simple mechanisms. It can be shown that that the first regularity, the fat tail of volatility, derives naturally from trend-following speculative trading, which implies that the return process follows a random-coefficient autoregressive (RCAR) process (Inoua, 2016). Trend-following expectations, which are a popular practice on financial markets, are a realistic alternative to rational expectations, which are hard to reconcile with speculation (Tirole, 1982). The power-law tail follows by an important theorem due to Kesten (Kesten, 1973; Klüppelberg & Pergamenchtchikov, 2004; Buraczewski, Damek, & Mikosch, 2016). While the mathematics of this mechanism is rather involved, the underlying economics is elementary: the fat tail emerges because of the endogenous amplifying feedback intrinsic to speculative trend-following supply and demand. This model is not wholly satisfactory, however, because no such process could explain volatility clustering, as implies another theorem (Mikosch & Starica, 2000; Basrak, Davis, & Mikosch, 2002; Mikosch & Starica, 2003; Buraczewski et al., 2016). The basic reason for clustered volatility, this paper suggests, is the impact of exogenous news on expectations. The RCAR model is thus extended to include, as usual, a second class of agents, fundamental-value investors, who attach a value to the asset and buy it when they think it is underpriced, or sell it, otherwise; crucially, their valuations of the asset is entirely based on exogenous news. Owing once again to the no-trade results, the dominant model of news-driven expectations, rational expectations, is not assumed in this paper. Rather, it is simply assumed that a trader holds a prior belief about the market (on the future price change or the present value of the asset) and then revises this prior belief additively with the advent of news. This news-driven random walk of traders' expectations explains volatility clustering in a generic way. The fat-tail of volatility is preserved in the extended model; but for simplicity of exposition, and to avoid some technicalities inherent to a detailed study of the RCAR process, this paper enphasizes the power law passignly, and focuses on the second regularity. Finally, liquidity plays a crucial role in the volatility dynamics, which is emphasized in the discussions (section 3).

2. The model

Following a traditional dichotomy of market participants, consider a financial market populated by two types of traders: (short-term) *trend-following speculators*, who buy an asset when they anticipate a price rise (or sell, otherwise) by using standard moving averages of past price changes to detect trends; and (long-run) *fundamental-value investors* (or 'investors' for short), who buy the asset based on its anticipated real cash flows, buying the asset when they think it is worth more than its current price, or selling it, otherwise: the fundamental value is revised additively with the advent of an exogenous[3] (or real) news.

Let the (excess) demands of a speculator and an investor be respectively[4]

$$Z_{st} = \beta \frac{P_{st}^e - P_t}{P_t} = \beta r_{st}^e, \tag{1}$$

$$Z_{it} = \gamma \frac{V_{it}^e - P_t}{P_t}, \tag{2}$$

where r_{st}^e is the return the speculator expects to occur in period t, V_{it}^e is the value that the investor thinks the security is worth at the closing of period t, and $\beta, \gamma > 0$. Let M_t and N_t be respectively the numbers of investors and speculators active in period *t*. The market excess demand is

$$Z_t = \beta N_t r_t^e + \gamma M_t \frac{V_t^e - P_t}{P_t}, \tag{3}$$

where $r_t^e = N_t^{-1} \sum_s r_{st}^e$ and $V_t^e = M_t^{-1} \sum_i V_{it}^e$, namely, the average investor valuation (hereafter referred to simply as 'the value' of the security) and the average speculator anticipated future price.

[3] Throughout, exogenous is with respect to the asset price dynamics.

[4] Since demand and supply can treated symmetrically (supply being formally identified as formally identified as a negative demand) one can think formally in terms of excess demand of a trader, which is either a demand or a supply, depending on its sign.

Trend-following implies that speculators' overall anticipated return is of the form $r_t^e = \sum_{h=1}^{H} \omega_{ht} r_{t-k}$, to which we add an additive component to capture the impact of exogenous news on speculators' expectations. The weights ω_{ht} can be computed explicitly from standard trend-following techniques used by financial practitioners (Beekhuizen & Hallerbach, 2017). Let the arrival of exogenous news relevant to investors and speculators, respectively, be modeled as random events I_t and J_t, occurring with probabilities $\mathbb{P}(\mathrm{I}_\mathrm{t})$ and $\mathbb{P}(\mathrm{J}_t)$, and making them revise additively their prior views by the amounts ε_t and ν_t, respectively. That is, assume $V_t^e = V_{t-1}^e + \varepsilon_t \mathbf{1}(\mathrm{I}_t)$ and $r_t^e = \sum_{h=1}^{H} \omega_{ht} r_{t-h} + \nu_t \mathbf{1}(\mathrm{J}_t)$, where $\mathbf{1}(\mathrm{I}_t)$ and $\mathbf{1}(\mathrm{J}_t)$ are indicator functions. The pure news-driven random walk of investors' valuations conveys the notion that the asset's value incorporates all the exogenous news relevant to fundamental-value investors, in the sense that $V_t^e = V_0^e + \sum_{k=1}^{t} \nu_k \mathbf{1}(I_k)$. This makes V_t^e the natural definition of the asset's fundamental value in this model.

Finally, assume the following price adjustment, in accordance with the market-microstructure literature:

$$r_t = \frac{Z_t}{L_t}, \tag{4}$$

where L_t is the overall market liquidity (or market depth).

All in all, the the asset's price dynamics reads:

$$P_t = (1 + r_t)P_{t-1}, \tag{5}$$

$$r_t = n_t r_t^e + m_t \frac{V_t^e - P_t}{P_t}, \tag{6}$$

$$r_t^e = \sum_{h=1}^{H} \omega_{ht} r_{t-h} + \varepsilon_t \mathbf{1}(\mathrm{J}_t), \tag{7}$$

$$V_t^e = V_{t-1}^e + \nu_t \mathbf{1}(\mathrm{I}_t). \tag{8}$$

where the following notations are adopted:

$$n_t = \beta N_t \ / \ L_t, \tag{9}$$

$$m_t = \gamma M_t \ / \ L_t. \tag{10}$$

No general study of the model is attempted here, since the focus of this paper is clustered volatility. To this end, FIG. 2 illustrates a simplified version of the model using the following specification (where the parameters are chosen arbitrarily, except to reflect realistic orders of magnitude): $T = 10000$ periods; $P_0 = V_0^e = 100,\ r_1 = r_1^e = 0;\ n_t,\ m_t$ iid exponentially distributed processes with $\mathrm{E}(n_t) = 0.11, \mathrm{E}(m_t) = 0.1;$ ε_t, ν_t iid Gaussian processes with $\mathrm{E}(\varepsilon_t) = \mathrm{E}(\nu_t) = 0, \mathrm{std}(\nu_t) = 5, \mathrm{std}(\varepsilon_t) = 0.1; \mathbb{P}(\mathrm{I}_t) = 0.1,$ $\mathbb{P}(\mathrm{J}_t) = 0.01; H = 1,\ \omega_{1t} = \omega_1 = 0.99.$

FIG. 2 The model.

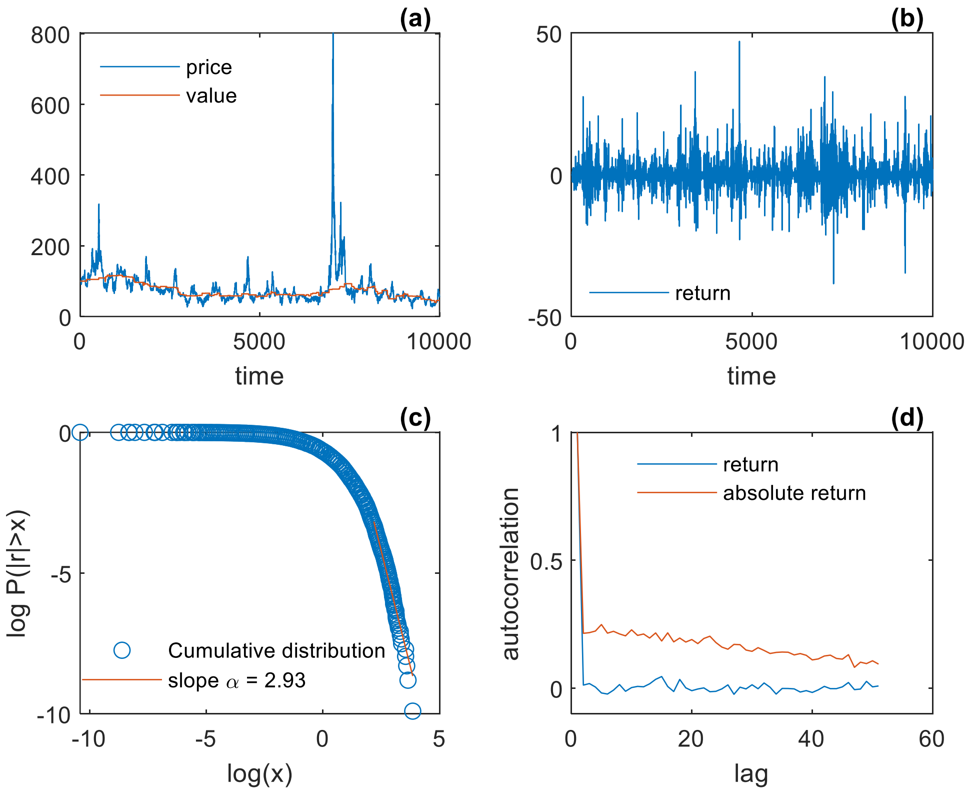
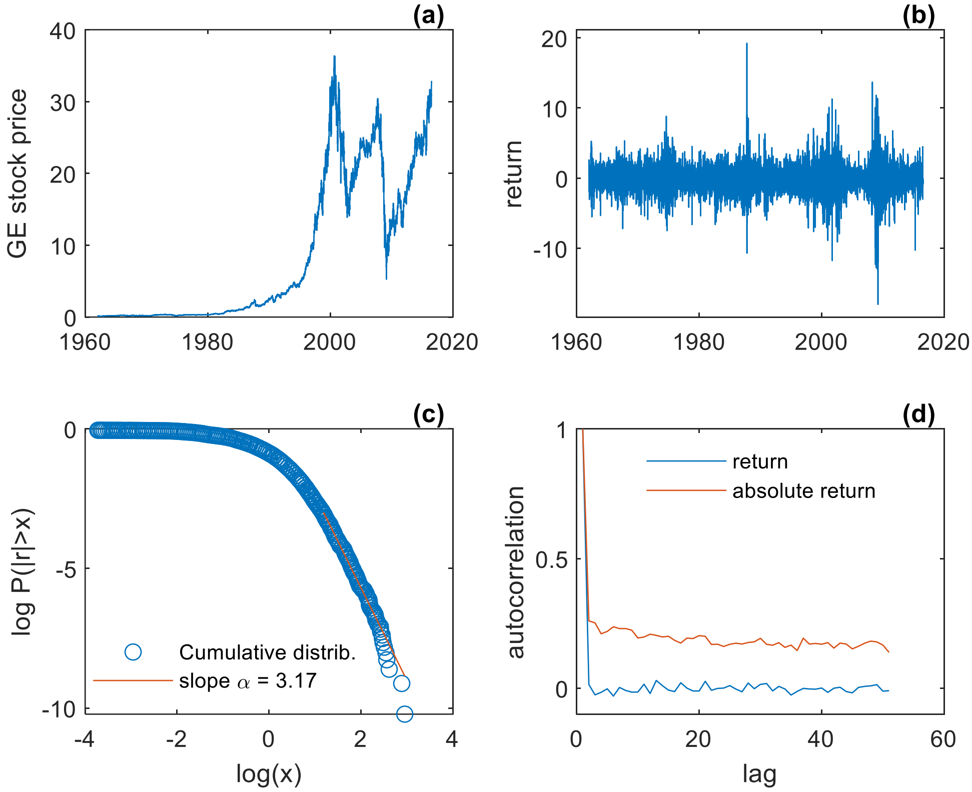


FIG. 3. For model-data comparison: General Electric's stock price.



3. Discussions

In the simulation, trend-following has been reduced to its simplest formulation. More generally, the key to the fat-tailed volatility is trend-following speculation (the systematic analysis of which being the subject of another paper): the purely speculative return process, namely the RCAR $r_t = n_t \sum_{h=1}^{H} \omega_{ht} r_{t-h} + \varepsilon_t \mathbf{1}(\mathrm{J}_t)$, genetrates a power law $\mathbb{P}(\mid r_t \mid > x) \sim Cx^{-\alpha}$ with the exponent α that depends solely on the joint distribution of $\{\omega_{ht}\}$, the trend-following coefficients, and the ratio of the number of speculators to liquidity. It can be shown that the less liquid the market is on average, the lower is the tail exponent $\alpha,$ hence the more extreme is volatility[5], as should be expected. The advent of exogenous news, on the other hand, is essential for clustered volatility, which the purely speculative model cannot explain, as noted earlier. That is, any such autorregresive model, and for any arbitary function $f, \operatorname{cov}[f(r_t), f(r_{t+h})],$ if well-defined, decays exponentially with the lag h (Mikosch & Starica, 2000; Basrak et al., 2002). So volatility, whether measured as $| r_t |,\ r^2,$ or any other function f, cannot be clustered in this version of the model.

In sum, this paper suggests a simple explanation for excess and clustered volatility in financial markets. Excess volatility means that that price fluctuations are too high given the underlying fundamentals; clustered volatility simply reflects, in this model, the flow of exogenous news affecting the traders' expectations.

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[5] There is an inverse relationship between the extremeness of a power-law and its tail exponent alpha: a very large alpha corresponds in fact to mild exponential tail; an alpha not greater than 2 has infinite variance, and below 1, an even infinite mean.

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