Abstract. We investigate how a player’s strategic behavior is affected by the set of notions she uses in thinking about the game, i.e., the “frame”. To do so, we consider matching games where two players are presented with a set of objects, from which each player must privately choose one (with the goal of matching the counterpart’s choice). We propose a novel theory positing that different player types are aware of different attributes of the strategy options, hence different frames; we then rationalize why differences in players’ frames may lead to differences in choice behavior. Unlike previous theories of framing, our model features an epistemic structure allowing for the case in which an individual learns new frames, given some initial unawareness (of the fact that her perception of attributes may be incomplete). To test our model, we introduce an experimental design in which we bring about different frames by manipulating subjects’ awareness of various attributes. The experimental results provide strong support for our theory.

KEYWORDS: frames, unawareness, focal points, coordination games.

JEL Classification Numbers: C72, C91.

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I. Introduction

Cognitive scientists define a “frame” as a bundle of information about the typical characteristics of a situation or problem: frames are stored in individuals’ minds and provide them with default information with which to interpret and respond to events (Schank and Abelson, 1977). Relatedly, artificial-intelligence pioneer Marvin Minsky (1975) utilized frames to codify knowledge structures in the context of visual-reasoning and communication-processing problems; for instance, the appropriate interpretation of (and response to) a hand sign or gesture depends on the agents’ mutual understanding of the situation in which the sign occurs. In these and other types of everyday problems, we face an implicit coordination puzzle whose solution depends on the frames we use. For example, how do you decide how to greet someone? (Do you shake hands? Hug? Cheek kiss? If so, how many times?) Such problems might seem trivial to many of us because we often interact with people from our own culture, or with a similar background and experience: in other words, we often interact with people with whom we share a frame. We therefore solve many of these problems effortlessly, to the point that we do not notice that there was a coordination problem in the first place. Yet, even within a tightly-knit community, miscoordination (such as a social faux pas) may arise whenever we interact with individuals who view a given problem through a different lens.¹

The cognitive process underlying everyday coordination problems may be described as follows: one categorizes (i.e., mentally frames) the problem on the basis of some recognizable characteristics, and then one follows the solution that is naturally associated with that category of problems. Thus, one’s perception (i.e., awareness) of the problem’s characteristics is key to one’s behavior. Accordingly, in this paper we aim to investigate – both theoretically and experimentally – how one’s perception may affect one’s behavior in abstract coordination games. We note that the issue of equilibrium selection is a long-standing consideration in the game-theoretic study of coordination problems (i.e., the class of symmetric, simultaneous-move games with multiple pure-strategy Nash equilibria). In this regard, Thomas Schelling (1960) was the first to draw attention to the importance of contextual cues in coordination problems. In particular, Schelling informally

¹ The theory of the firm considers coordination problems one of two main organizational hurdles (the other being the much more studied “agency problem”: Milgrom and Roberts, 1992; Camerer and Knez, 1997). It has been suggested that the coordination problem of organizations is inherently due to people’s cognitive limitations, in the sense that individuals often lack a common understanding of the tasks they need to integrate and coordinate upon (Heath and Staudenmayer, 2000). This line of research implies that agents come to develop a different understanding of their tasks as a result of a different focus or experience; different viewpoints may in turn imply different solutions to an identical or similar task (von Hippel, 1990; Weber and Camerer, 2003; Okhuysen and Bechky, 2009).
observed that the use of mutually-recognizable cues (i.e., characteristics or “attributes” of the problem) could help individuals solve a coordination game. Here we delve into the cognitive process underlying an individual’s strategic use of natural frames. To that end, we shall investigate the following perception-action chain: awareness of attributes \( K \rightarrow K\text{-induced frame} \rightarrow \text{behavior}. \) An example is provided below.

Consider a “matching game” where two players are presented with a set of objects from which each player must privately choose one (with the goal of matching her counterpart’s choice). In this case a frame may be thought of as a player’s categorization of objects (i.e., actions) on the basis of the attributes of which she is aware. For example suppose that, for any player \( i \), the action set in a matching game is formally given by \( \{s_{i,1}, s_{i,2}, s_{i,3}\} \), with options 1, 2, and 3 respectively representing a cyan triangle, a cyan diamond, and a lavender triangle. Note that the theorist’s conventional way of defining a game does not permit any characterization of the objects to enter the formal description of the game. On the other hand, as was first suggested by philosopher David Gauthier (1975), here a color-perceiving player \( i \) would naturally distinguish between the colors (cyan and lavender). In fact, the act of distinguishing between colors formally implies that she would be able to partition the action set so that each cell corresponds to an instance of the color attribute, that is, \( \{\{s_{i,1}, s_{i,2}\}, \{s_{i,3}\}\} \). By contrast, a shape-perceiving player \( i \) would partition the action set so that each cell corresponds to an instance of the shape attribute, that is, \( \{\{s_{i,1}, s_{i,3}\}, \{s_{i,2}\}\} \). If a player distinguishes between shapes but not colors, we say that that player is unaware of the color attribute (and so has “partial perception” of the qualitative characteristics of her action set). Generalizing, one could think of player types, where a type is endowed with the ability to distinguish among instances of some (or all) of the relevant attributes characterizing the action set.

Below we propose a novel theory positing that different player types are aware of different attributes of the strategy options, and hence use different frames. Given this, our model provides a rationalization of why differences in players’ frames may lead to differences in choice behavior (in particular, cells that contain fewer elements are more attractive). Thus, in the example above our model predicts that a “color type” would likely pick the odd-one-out with respect to the color attribute (i.e., the color oddity), whereas a “shape type” would likely pick the odd-one-out with
respect to the shape attribute (i.e., the shape oddity). As will be clear, our theory builds on two alternative approaches to the analysis of framings: (i) the models of Bacharach (1993), Casajus (2000), and Janssen (2001), in which strategy labels represent non-stochastic instances of attributes, as in the example above; and (ii) the Sugden (1995) model, in which labeled strategies are generated stochastically. All such theories explain the impact of frames on choice behavior by analyzing the structure of the strategy set, as experienced by a player. (A similar approach was followed by Crawford and Haller, 1990, and Blume, 2000, for the case of repeated coordination games, whereby previous play would implicitly label actions in such a way to generate a distinct option; e.g., “do what you did last time”, “do something else”.)

In what follows we focus on one-shot games and, unlike previous contributions, we incorporate an epistemic structure into the analysis of frames. In doing so, for the first time we formalize the relationship between a player’s partial perception (of the qualitative characteristics of the action set) and frame-dependent rationalizable actions. This exercise has both theoretical and practical implications. Whereas previous models could identify the impact of fixed frames on choice behavior, they would not account for changes in a player’s frames. That is, by providing static representations of individuals’ perceptual limitations, previous models could not explain the eventuality that a player might react to novel frames that initially eluded her (e.g., think of a player who suddenly focuses on some qualitative feature of an action that makes it uniquely distinctive). In fact, previous solutions such as Bacharach’s (1993) “variable universe equilibrium” and related specifications cannot convincingly be taken as equilibria proper, in the sense of being stable solutions. (The instability amounts to any behavioral changes disrupting an equilibrium outcome owing to the discovery of new frames; e.g., via an experimental manipulation.) To provide a more coherent solution concept that inherently allows for the discovery of new frames, we propose “frame-dependent rationalizability”. Such a solution rests on an epistemic structure allowing for the case in which an individual learns new frames, given some initial unawareness of one or more

2 The idea of partitioning the strategy space in order to model a player’s conception of the game was suggested by Gauthier (1975) and later used in Bacharach’s (1993) work on “variable universe games”. Following Bacharach, we refer to an item that differs from others (in some particular feature) as an “oddity”.

3 Sugden (1995) assumes that players distinguish between alternative strategies on the basis of a private description, which rests on independent realizations of a commonly-known random process. Here, we are going to focus on private descriptions that depend on publicly-observed realizations of a random process. For example, think of a matching game where objects are randomly drawn from an urn – in front of all players – and then privately labeled by players as “the first object”, “the second object”, “the third object”, etc.

4 We derive this concept from the interim correlated rationalizability notion of Dekel, Fudenberg, and Morris (2007), although we introduce frame-dependent restrictions on beliefs so as to account for (type-specific) partial perception.
attributes. Our solution implies that – given the attributes of which one is currently aware – one will best-respond solely to the types of which one can conceive; when one’s awareness expands, the process is coherently updated so as to account for newly-discovered frames (i.e., types).

To test whether subjects maximize a (possibly updated) frame-dependent expected utility, our between-subjects experimental design brings about different frames by manipulating subjects’ awareness of several attributes. We designed two main treatments, along with three supplemental ones (discussed later); all treatments involve matching games. In the “Baseline” treatment, we first allowed subjects to label the strategy space (i.e., the objects) as desired; we next asked subjects to estimate how others would play, and we finally had them pick one of their labeled strategies. The “All-Aware” treatment is the same as the Baseline except that, before having subjects estimate how others would play, we hinted at several attributes at once (so that each subject was privately made aware of multiple attributes, without inducing common awareness of any particular frame).

Unlike previous coordination experiments (e.g., Mehta, Starmer, and Sugden, 1994; Bacharach and Bernasconi, 1997; Crawford, Gneezy, and Rottenstreich, 2008; Bardsley, Mehta, Starmer, and Sugden, 2010; Blume and Gneezy, 2010), our design is instrumental in testing for our notion of frame-dependent rationalizability. In fact, contrasting the Baseline and the All-Aware treatments allows us to verify whether partial perception (i.e., awareness) of attributes, hence frames, affects choice behavior. In brief, we consider that All-Aware participants – prior to the treatment manipulation – are similar to their Baseline counterparts (on average), but eventually become aware of additional frames as a result of the All-Aware treatment manipulation. The implication is that an increase in frame awareness may affect the distribution of choices in the All-Aware treatment, as compared with the Baseline. Relatedly, we note that our proposed solution concept allows for the set of possible events (frames) to expand, and therefore for a decision-maker

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5 Notably, unawareness cannot be accounted for under the assumptions of the standard model of knowledge (Modica and Rustichini, 1994, 1999; Dekel, Lipman, and Rustichini, 1998). So, our framework’s epistemic structure does not feature the “negative introspection” property, according to which ¬[know E] entails know[¬[know E]], where E denotes a generic event (Dekel and Gul, 1997). As will be discussed, our model of knowledge draws on Heifetz, Meier, and Schipper’s (2006) specification of unawareness by adopting a system of multiple state spaces and surjective projections (onto weakly “less expressive” state spaces).

6 We do so by asking subjects the following questions: “How likely do you think it is that the other participants have noticed the order in which the objects have been drawn by the experimenter? [...] the different colors of the objects? [...] the different shapes of the objects?” We note that these questions can be identified with tautologies stating that “x is the case or ¬x is the case”. In fact, a question involving a tautological clause does not convey any specific information about the state of x, and yet it may raise awareness.
to react to novel information (in the form of a newly-discovered frame) by maximizing an updated frame-dependent expected utility.

More explicitly, we stress that the All-Aware treatment is identical to the Baseline except that, in the former, our experimental manipulation makes subjects privately aware of multiple attributes. It follows that the behavior of a subject who (prior to the All-Aware manipulation) had been unaware of some of those attributes might be affected by the fact that we mention them in the All-Aware treatment. Indeed, any differences in choice behavior between All-Aware and Baseline can be attributed to the fact that, in the former, one comes to consider additional frames; i.e., a fact that may impact one’s rationalizable actions.

Our empirical analysis indicates that subjects do best-respond to their (updated) beliefs about the opponents’ frames. Specifically, our data show that the distribution of choices varies significantly across Baseline and All-Aware treatments. In particular, we find that participants in the Baseline treatment tend to behave predominantly like “color types”, and to a lesser extent like “shape types” (by picking the oddity with respect to color labels and shape labels, respectively). More importantly, we observe that All-Aware participants choose color oddities significantly less often than Baseline participants: our interpretation is that whenever subjects are made aware of attributes (such as the shape or the order in which the blocks are displayed) that they had previously ignored, they end up realizing that their reasons for picking a color oddity may not be so compelling after all.

Notably, the effect of the All-Aware manipulation is to deterministically change the calculus of coordination, as is confirmed by the analysis conducted on the All-Aware sample. There, using the All-Aware participants’ beliefs (about others noticing each attribute) as predictors, we are able to verify the implications of frame-dependent rationalizability more directly. For example, if an All-Aware participant believes that (Baseline) subjects are more likely to notice, say, shapes than

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7 More generally, the behavior of an agent who had been unaware of an event might be affected by a sentence that merely hints that that event may or may not happen. (Note that if players’ frames are incomplete, then the space of events on which a player’s beliefs are defined is incomplete as well.) For example imagine that, as you prepare to go out for a stroll, your partner asks you whether it might rain. If you had already assigned probability zero to the event “rain”, then you will be unmoved by such a question. If, instead, you had ignored that event – and you now notice it is cloudy outside – then you may react by taking an umbrella along.

8 Note that our design controls both for a subject’s awareness level – which is exogenously varied – and for a subject’s probabilistic beliefs about others noticing each attribute (such beliefs are elicited in the All-Aware treatment): this allows us to directly test for frame-dependent rationalizability. As a side note, we stress that our design minimizes demand effects by mentioning multiple cues at once.
colors or order, then that participant should be more likely to pick a shape oddity (if there is one). We shall see that the regression analysis provides support for our notion of rationalizability.

To corroborate our framework, we contrasted the Baseline against three supplemental treatments, namely “Color-Aware”, “Shape-Aware”, and “Order-Aware”. Each of these treatments is the same as the Baseline except that the objects are now exogenously labeled in terms of colors, shapes, or the order in which the blocks are displayed, respectively. The purpose of these treatments is to bring about three pools of subjects, where each pool is characterized by common awareness of one frame. There, the experimenter should expect an increase (or, at worst, no decrease relative to the Baseline) in the frequency of the strategy associated with the relevant frame.

Data from the supplemental treatments show that Color-Aware subjects pick the color oddity slightly more frequently than our Baseline participants; moreover, Color-Aware subjects pick the shape oddity significantly less often than our Baseline participants. Similarly, the data show that Shape-Aware subjects pick the shape oddity significantly more often than our Baseline participants. Our interpretation is that subjects, who would have otherwise ignored the shape (color) attribute, end up noticing shape (color) oddities once they are assigned to Shape-Aware (Color-Aware). When contrasting the Baseline with the Order-Aware treatment, our data show no significant difference in the choices of either color or shape oddities, as predicted by the model; instead, we observe that subjects tend to pick the first block significantly more often than the third one. (This appears to be consistent with Schelling’s, 1960, p. 94 argument that people have a tendency to single out the first item in a numerical sequence.) As a final note, our data provide evidence of an increase in coordination rates when moving from the main treatments to the supplemental treatments (i.e., treatments inducing common awareness of one attribute).

In summary, we propose and test a model positing heterogeneity in individuals’ partial awareness of attributes (of the strategy options), hence heterogeneity in frames. We then rationalize why differences in players’ frames may lead to differences in choice behavior. Our experimental results provide strong support for our model, presenting a coherent account of the effect of manipulating awareness (of alternative framings) on strategic behavior. The remainder of the article is organized in this manner: section II presents the game, along with the experimental design and procedures; section III lays out the model, and IV discusses its predictions; section V presents

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9 In doing so, we substantially depart from the recent decision-theoretic literature on focusing (e.g., Köszegi and Szeidl, 2012; Bordalo, Gennaioli, and Shleifer, 2013; etc.), which posits that agents overweight some attributes of the strategy options under the assumption that they are always aware of all attributes.
II. Game, experimental design, and procedures

Our experimental design involves a coordination game we call “Choose Something”. In this game each subject is assigned to a computer terminal, and each is told that everyone is shown the same six objects on her own screen. The objects are blocks (i.e., colored shapes), which the computer program identifies by the numbers – invisible to the subjects – 1 to 6, as in Table 1 below (the experiment was conducted using the zTree software; Fischbacher, 2007).

<table>
<thead>
<tr>
<th>Object no.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Color</td>
<td>Cyan</td>
<td>Cyan</td>
<td>Lavender</td>
<td>Lavender</td>
<td>Turquoise</td>
<td>Turquoise</td>
</tr>
<tr>
<td>Shape</td>
<td>Triangle</td>
<td>Diamond</td>
<td>Triangle</td>
<td>Pentagon</td>
<td>Diamond</td>
<td>Pentagon</td>
</tr>
</tbody>
</table>

Table 1 - Characteristics of the objects in Choose Something.

Initially, the blocks are loosely arranged in a hexagonal fashion (one object per vertex) and collectively occupy the left-hand side of the screen. Each subject is assigned to an unknown partner. After each has viewed the six objects on her own screen, the computer program selects three of those objects – one by one – by sliding them and putting them in a column on the right-hand-side of the screen (note: the same three objects for each member of a pair). The rest of the objects subsequently disappear from the screen. Participants are then prompted to complete a few tasks. In particular, each subject is asked to indicate her choice of one of the three objects, with the goal of coordinating with her partner in the pair. Each member of a pair receives a payoff of $1.25 if each chooses the same object; each receives nothing otherwise. A copy of the experimental instructions and screen shots are in the Appendix.\(^{10}\) In what follows we extensively describe each of the experimental treatments in turn.

The list below specifies the entire sequence of events in the “Baseline” treatment.

i. Each subject is assigned to a computer terminal and everyone is shown the paper instructions.

\(^{10}\) A summary of the instructions, including the following message, was read aloud by the experimenter: “The computer program will select three of the aforementioned objects, and will display those three objects to every participant in the same fashion and order.”
ii. Pairs of subjects are formed at random. Subjects are presented with six objects on their screen, three of which are subsequently selected by the program and put in a column.

iii. An on-screen message prompts subjects to label those three objects [“PART A”].

iv. An on-screen message prompts subjects to estimate the probabilities of the three objects being chosen by others [“PART B”]; they are informed that good guesses will be rewarded with an additional payment.\(^{11}\)

v. An on-screen message prompts subjects to choose an object (by ticking the box located next to the label); they are reminded that their payoff will be $1.25 if both members of the pair chose the same object, $0 otherwise [“PART C”].

vi. Steps ii.-v. are repeated for 9 more rounds, whereby in each round a new three-object selection is implemented by the program and shown to each member of a pair.\(^{12}\) (In each round, subjects are randomly assigned to another pair and are so informed.) No feedback is given between rounds.

vii. Payment.

We stress that the design of our Baseline treatment involves \textit{no exogenous labels} associated with the strategy options (i.e., the blocks). In fact, in order to identify the blocks – for the purposes of performing tasks \textit{iv.} and \textit{v.} – in PART A of each round subjects are asked to type a short text in each of three boxes beside the objects. (Note that our hypothesis-testing does not rely on such qualitative data; the labeling task simply serves the purpose of pushing subjects to think of the game in their own terms.) After that, in PART B of each round, subjects are asked to estimate the likelihood of each block being chosen by others. Finally, in PART C of each round, subjects are asked to choose a block. The entire set-up is common information amongst all participants.

The design of the “\textit{All-Aware}” treatment is the same as the Baseline except for PART B, which presents 3 extra questions. Specifically, in PART B subjects see the following message.

«Recall that – in Part C of the experiment – you will be prompted to pick one object in order to coordinate with your partner. Now, prior to that we would like to know what you think about the other participants in this room. Please answer the following questions by moving the sliders to the desired percentages. Note that your partner will not be asked to answer these questions.

\(^{11}\) Subjects are presented with a pie chart with three spokes. The spokes are initially arranged in such a way that each sector corresponds to one-third of the area. Subjects are prompted to move the spokes to the desired percentages (note that within each sector there is a text box containing the label the subject has entered at step \textit{iii.}). We incentivized this task by informing participants that if at least one of their three estimates differed by no more than 5 percentage points from the realized value, they would receive an additional payment of $0.25 at the end of the experimental session. No feedback was provided before the end of the session.

\(^{12}\) At the beginning of every round, subjects are shown the same six blocks as in round 1, but \textit{three different blocks are subsequently selected in each round}. Note that the initial position of the six blocks is re-shuffled in every successive round, but is identical for all subjects taking part in the same round.
1) How likely do you think it is that the other participants have noticed the order in which the objects have been drawn by the computer program? Please move the below slider.
2) How likely do you think it is that the other participants have noticed the different colors of the objects? Please move the below slider.
3) How likely do you think it is that the other participants have noticed the different shapes of the objects? Please move the below slider.

A few comments are due. The order in which questions 1) - 3) are presented was randomized in each round. Subjects entered their guesses by moving a slider (i.e., one slider per question) to the desired percentage, with the slider ranging from 0% to 100%. Note that the questions above were not incentivized; the purpose of these questions is to make each subject privately aware of multiple attributes. Also note that, after presenting questions 1) - 3) above, the All-Aware treatment proceeds to the incentivized belief-elicitation task described at step iv. of the Baseline; the rest of the treatment is identical to the Baseline.

Finally, we designed three supplemental treatments to be the same as the Baseline, except for the labeling task of step iii. above (i.e., PART A): in such treatments the labeling task is omitted as the blocks are already labeled. Specifically, in PART A of the “Color-Aware”, “Shape-Aware”, and “Order-Aware” treatments, each of the available objects is displayed beside a color, shape, or order label, respectively. The purpose of such labels is to bring about common awareness of the relevant frame.13

III. The model

1. Frames and perceptual limitations

The model applies to any 2-player coordination game where each player’s payoff is a positive number \( \pi \) if both choose the same action, zero otherwise (“matching games”). We now distinguish between the theorist’s and the players’ way of identifying actions. To that end, we let an index denote the theorist’s (frame-free) identifier of a player’s action, as follows: a game has a set \( \{s_{i,1}, s_{i,2}, \ldots, s_{i,m}\} \) of pure actions, with generic element \( s_{i,b} \), where the first and second index subscripts respectively indicate some player \( i \in N = \{1,2\} \) and some option \( b \in \{1, \ldots, m\} \). For example, in Choose Something, the theorist might denote by \( s_{i,b} \) “player \( i \)’s act of picking block \( b \)”, where \( b \) is a number between 1 and 6, as in Table 1 above. In this regard, we note that in Choose Something only three of the actions above become available once Nature has drawn the

13 In the Color-Aware and Shape-Aware treatments we used the labels in Table 1 above. In the Order-Aware treatment we used the labels “first”, “second”, “third”. 
corresponding blocks; so, for the theorist Choose Something consists of \(\binom{6}{3} = \frac{6!}{3!3!} = 20\) symmetric games in strategic form, where Nature determines the one game to be played by the pair.\(^{14}\) We stress that, per the theorist’s way of identifying actions, each of those 3-action games is isomorphic to the others, in that each differs from the others solely in the indices; e.g., \([s_{i,1}, s_{i,2}, s_{i,3}],\) \([s_{i,1}, s_{i,2}, s_{i,4}],\) \([s_{i,1}, s_{i,2}, s_{i,5}],\) …

By contrast, the players’ way of identifying actions is based on a perceptual description, as follows. Let \(K\) denote a set of attributes, with generic element \(k\). With specific reference to Choose Something, we shall assume \(K = \{C, S, O\}\), with \(C, S, O\) respectively denoting the “color”, “shape”, and “order” attributes. Each \(k \in K\) is associated with a set of instances of that attribute, which are referred to as “\(k\)-instances”; e.g., in the case of the color attribute, \(C\)-instances include “cyan”, “turquoise”, etc. (see Table 1 above for color and shape instances in Choose Something).

The rest of the sub-section proceeds as follows: a) First, for any action set, we define alternative variously-expressive (i.e., detailed) descriptions of that action set. b) We then define projections from any description of an action set to a less expressive description of that action set. c) Next, we explicitly relate the variously-expressive descriptions above to different players: in doing so, we assume that players may be endowed with the ability to distinguish among instances of some or all the attributes in \(K\) (more explicitly, we characterize such an ability as a “type-specific awareness” by using an appropriate information correspondence). d) Given this, we impose a “labeling-invariance” constraint on the strategies one defines on the basis of the attributes of which one is aware. e) We finally specify equilibria under the assumption that there is no uncertainty about other players’ perceptual descriptions (“frame-induced equilibria”).\(^{15}\)

a. For any subset of attributes \(K’\) with \(K’ \subseteq K\), let \(\Omega_{K’}\) denote a set of tuples of labels, with generic member \(\omega\), so that each state \(\omega \in \Omega_{K’}\) represents a description of the available action set (i.e., each action is described by an instance of each attribute \(k \in K’\)).\(^{16}\) For example, suppose Nature has selected a game with action set \([s_{i,1}, s_{i,2}, s_{i,3}]\): here, if \(K’ = \{C, S\}\), then a state \(\omega \in \Omega_{K’}\) may be represented by \(\omega = ((\text{cyan, triangle}), (\text{cyan, diamond}), (\text{lavender, triangle}))\), so that

\(^{14}\) More precisely, each of those 20 games is a proper subgame of the larger game that begins with a chance move. However, since there is perfect information about the chance move, to simplify the exposition we shall treat each of the proper subgames as a distinct “game” and hence use the words actions and strategies interchangeably.

\(^{15}\) The case in which there is any such uncertainty is presented in section III.2 below.

\(^{16}\) The reader may anticipate that if \(K’ = \emptyset\), then every action is described by an instance of a “nondescript attribute” conveying no particular information; e.g., \((\text{object, object, object})\).
(cyan, triangle) describes the (attributes color and shape of the) first action, (cyan, diamond) describes the second action, and (lavender, triangle) describes the third action.

b. Let \( \{\Omega_{K'}\}_{K' \in K} \) denote a collection of nonempty sets \( \Omega_{K'} \) (i.e., there is one set for each \( K' \subseteq K \)). By construction, such sets are disjoint; moreover, for any \( K', K'' \subseteq K \), the construction naturally lends itself to the interpretation that \( \Omega_{K''} \) “is more expressive than” \( \Omega_{K'} \) if \( K'' \supseteq K' \). In other words, if \( K'' \supseteq K' \), then \( \Omega_{K''} \) is more expressive in that it provides a description of the action set in terms of a larger number of attributes than \( \Omega_{K'} \). Further, for any \( K', K'' \subseteq K \) with \( K'' \supseteq K' \), a surjective projection from \( \Omega_{K''} \) to \( \Omega_{K'} \) is naturally defined by “erasing” (in \( \omega \in \Omega_{K''} \)) any \( k \)-instance in \( K'' \setminus K' \). For example, if \( K' = \{C\} \) and \( K'' = \{C, S\} \), then \( \omega = ((\text{cyan, triangle}), (\text{cyan, diamond}), (\text{lavender, triangle})) \in \Omega_{K''} \) projects to \(( (\text{cyan}), (\text{cyan}), (\text{lavender})) \in \Omega_{K'} \). For a graphic representation, see Figure 1 below.

c. We move on to explicitly relate the variously-expressive descriptions above to different player types. In order to define players’ information correspondences, we begin by letting \( \Omega^* := \bigcup_{K' \in K} \Omega_{K'} \). Given this, we assume that for each player \( i \in N \) there is a correspondence \( l_i: \Omega^* \rightarrow \Omega^* \) such that, for each \( K' \subseteq K \) and for each \( \omega \in \Omega_{K'} \), \( l_i(\omega) \) is the projection to some \( \Omega_{K''} \) with \( K'' \subseteq K' \). Note that \( l_i(\omega) \) is interpreted as “player \( i \)'s possibly partial perception (hence, labeling) of the action set described in \( \omega \)”. For example, if \( \omega = ((\text{cyan, triangle}), (\text{cyan, diamond}), (\text{lavender, triangle})) \) and \( l_i(\omega) \in \Omega_{\{C\}} \), then player \( i \) ignores the shape attribute and focuses solely on the colors of the actions; that is, \( l_i(\omega) = ((\text{cyan}), (\text{cyan}), (\text{lavender})) \). It is easy to verify that \( l_i \) satisfies all the properties of the possibility correspondence introduced to model unawareness structures in Heifetz, Meier, and Schipper (2006). Note that our model is indeed a special case of an unawareness structure, whereby a player is aware of an attribute \( k \) if she distinguishes among different instances of that attribute. Also note that, so far, we have not modeled any uncertainty about the others’ perception; therefore, the information correspondence \( l_i \) simply amounts to an indicator of how each action set is labeled by player \( i \) (again, note that we model uncertainty in III.2 below).

d. Here we posit that, for each player \( i \in N \), there is a strategy \( \sigma_i \) assigning to each tuple \( \omega \in \Omega^* \) a probability distribution over the elements of \( \omega \), with the property that \( \sigma_i(\omega) \equiv \sigma_i(l_i(\omega)) \). This means that \( i \)'s strategy at \( \omega \) depends on \( i \)'s own perception of \( \omega \), that is, \( l_i(\omega) \). In this regard, while it is common in game theory to define a strategy as a mapping from information sets to actions, we

\[17\] The symbol \( \setminus \) denotes set difference.
stress that our definition rests on an unawareness structure: as such, it appropriately ties one’s behavior to one’s (however partial) perception of the game.

Now, denote the (ordered) elements in a tuple \( \omega \) by \( (a_1, a_2, \ldots) \) so that we refer to the first cell of \( \omega \) as \( a_1 \), to the second cell of \( \omega \) as \( a_2 \), etc. Given this, we say that:

**DEFINITION 1**—a strategy of player \( i \) is “labeling-invariant” if \( \sigma_i(\omega)(a) = \sigma_i(\omega)(a') \) whenever \( a = a' \) in \( I_i(\omega) \), for any two cells \( a, a' \in \omega \), with \( \omega \in \Omega^* \).

For example, if \( \omega = ((\text{first, cyan, triangle}), (\text{second, cyan, diamond}), (\text{third, lavender, triangle})) \) and \( I_i(\omega) = ((\text{cyan}), (\text{cyan}), (\text{lavender})) \), then a strategy is labeling-invariant if \( \sigma_i(\omega) \) assigns the same probability to cyan objects \( a_1 \) and \( a_2 \); i.e., if \( \sigma_i(\omega)(a_1) = \sigma_i(\omega)(a_2) \). We note that such a notion of labeling-invariance uses Bernoulli’s principle of insufficient reason in the following sense: if some actions are indistinguishable (given the player’s labeling), then those actions should be assigned the same probability in a mixed strategy.\(^{18}\)

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\(^{18}\)Interestingly, in Choose Something, the labeling-invariance principle says that players who describe each action merely as an “object” (i.e., players who are solely aware of the nondescript attribute \( \emptyset \)) should behave as was postulated by Harsanyi and Selten (1988); that is, they should randomize uniformly over all actions.
For any \(i, j \in N\), we define player \(i\)'s expected payoff from strategy profile \((\sigma_i)_{i \in N}\) at \(\omega \in \Omega^*\) by 
\[
U_i\left(\sigma_i(l_i(\omega)), \sigma_j(l_j(\omega))\right).
\]

Given this, we provide our first solution concept.

**DEFINITION 2**—a profile of strategies \((\sigma_i)_{i \in N}\) is said to be a "frame-induced" equilibrium if, for each \(i \in N\), \(\sigma_i\) is labeling-invariant and \((\sigma_i)_{i \in N}\) is a Nash equilibrium of the auxiliary game in strategic form defined as follows: The set of players is specified by the player-perception pairs 
\[
\{(i, l_i(\omega)): i = 1, 2; \omega \in \Omega^*\}.
\]

The set of actions of player \((i, l_i(\omega))\) corresponds to the labeling of the action set, as in \(l_i(\omega)\). The payoff to player \((i, l_i(\omega))\) is given by 
\[
U_i\left(\sigma_i(l_i(\omega)), \sigma_j(l_j(\omega))\right).
\]

The auxiliary game defined above is a modeling construct to incorporate the players’ way of identifying actions (as determined by their partial perception of attributes) into the analysis of play. Note that each player-perception pair \((i, l_i(\omega))\) defines a distinct (labeling) type, though such a modeling construct does not specify a type’s uncertainty about the labels used by other types. In a nutshell, it is assumed that each type chooses a strategy, with a strategy being defined as a probability distribution over the elements of \(\omega\), as perceived by the type herself. It is clear that in a frame-induced equilibrium each type chooses a best-reply conditional on her (possibly partial) perception. So, when player \(i\) with perception \(l_i(\omega)\) — i.e., “type \(i, l_i(\omega)\)” — considers the strategies of opponent \(j\), she actually imagines \(j\)’s action set at \(l_i(\omega)\) and not necessarily at \(\omega\) (this is because that type may not be aware of all the attributes used to describe actions at \(\omega\))!^{20}

The discussion above implies that at \(\omega\) player 1 may expect player 2 to play \(\sigma_2(l_2(l_1(\omega)))\), while in fact player 2 plays \(\sigma_2(l_2(\omega))\), with \(\sigma_2(l_2(l_1(\omega))) \neq \sigma_2(l_2(\omega))\). To ensure that no such surprise may arise in equilibrium, as a first remedy one could make the assumption that players have **common labeling**. In that case, for \(i, j \in N\), one would require that \(l_i\) and \(l_j\) are equal in that \(l_i(\omega) = l_j(\omega)\) at each \(\omega \in \Omega^*\). It is clear that a tuple \(\omega\) now unambiguously identifies an action set (hence, a game) in the players’ language. For example, suppose \(\omega = ((first, cyan, triangle), (second, cyan, diamond), (third, lavender, triangle))\) and \(l_i(\omega) = ((cyan), (cyan), (lavender)), \) with \(l_i(\omega) = l_j(\omega)\). Recalling that a strategy is labeling-invariant if it

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^{19} Payoffs are a function of perceived (i.e., labeled) strategies. Specifically, \(i\)'s strategy is defined by \(\sigma_i(l_i(\omega))\) as per point d) above. Then, from \(i\)'s viewpoint, \(j\)'s strategy is defined by \(\sigma_j(l_j(l_i(\omega)))\); that is, it depends on \(j\)'s perception of the state that \(i\) perceives at \(\omega\).

^{20} A frame-induced equilibrium may be viewed as a special case of a Bayes-Nash equilibrium for games with unawareness (Meier and Schipper, 2014).
assigns the same probability to actions that have the same label, it is easy to verify that this game has three frame-induced equilibria, whereby both players choose one of the following probability distributions over \( \omega \) (i.e., mixed strategies): \( \left( \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \right), \left( \frac{1}{2}, \frac{1}{2}, 0 \right), (0,0,1) \). Since – in a matching game – each player’s payoff is a positive number \( \pi \) if both choose the same action, the equilibria above have expected payoffs of \( \frac{1}{3} \pi, \frac{1}{2} \pi, \) and \( \pi \), respectively: this implies that playing the color oddity (i.e., lavender) with probability one is the payoff-dominant solution. The analysis above motivates the following concept.

**DEFINITION 3**—a strategy \( \sigma_i \) is “frame-appropriate” for player \( i \) with perception \( I_i(\omega) \) if it is labeling-invariant, and it does not induce a payoff-dominated outcome (of the auxiliary game) in the event that player \( j \) chooses a strategy \( \sigma_j \) such that \( \sigma_j = \sigma_i \).

The definition offers a rather weak (but intuitive) characterization of appropriateness in coordination games. In brief, frame-appropriateness discards a labeling-invariant strategy if it leads to inefficient outcomes whenever an opponent with a certain type-specific perception chooses the same strategy. This will later serve as a sensible constraint on beliefs about those players with a given type-specific perception.

2. **Frame-dependent rationalizability**

The frame-induced equilibrium is unsatisfactory as an equilibrium concept, in that it cannot convincingly be interpreted as a stable solution. The instability is due to the surprises that may arise if players do not happen to have common labeling, or simply due to the fact that players might learn new attributes (e.g., as a result of an experimental manipulation). To provide a more compelling solution concept for games with heterogeneous perception – and uncertainty about others’ labelings – we introduce “frame-dependent rationalizability”. This concept is derived from the interim correlated rationalizability notion of Dekel, Fudenberg, and Morris (2007), although in our case we shall account for (type-specific) partial perception.

To that end, we first define an extended state in such a way to specify not only instances of attributes, but also a labeling type for each player. Formally, we shall expand \( \Omega_K \) into the state \( \Omega_K' \). For example, suppose \( \omega = \{(first, cyan, triangle), (second, cyan, diamond), (third, lavender, triangle)\} \) like before, but now players ignore any attributes: in this case, every action is described by an instance of a “nondescript attribute”; e.g., \( (object, object, object) \). It is clear that this game has only one frame-induced equilibrium, where both players choose \( \left( \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \right) \).
space $\tilde{\Omega}_{K'}$, where $\tilde{\Omega}_{K'} := \Omega_{K'} \times 2^{K'} \times 2^{K'}$. For example, the extended state $\tilde{\omega} = ((\text{cyan, triangle}), (\text{cyan, diamond}), (\text{lavender, triangle})), \{\text{C}, \{\text{C}, \text{S}\}\}$ contains a description of the action set, as well as player 1’s labeling type $\{\text{C}\}$ (i.e., color) and player 2’s labeling type $\{\text{C}, \text{S}\}$ (i.e., color&shape). Next, we denote by $\tilde{\Omega}^*$ the union of the extended state spaces, that is, $\tilde{\Omega}^* \subseteq \bigcup_{K' \in K} \tilde{\Omega}_{K'}$. Given this, we introduce a type mapping $\mu_i$ such that $\mu_i : \tilde{\Omega}^* \rightarrow \bigcup_{K' \in K} \Delta(\tilde{\Omega}_{K'})$. We interpret $\mu_i(\tilde{\omega})$ as “player i’s belief about j’s perception, given i’s possibly partial perception at $\tilde{\omega}$”. Simply put, such a mapping associates with each state $\tilde{\omega}$ a probability over the $j$ types of which $i$ can conceive. (As such, unlike the information correspondence $l_i$ we used above, $\mu_i$ defines both i’s type and i’s beliefs about others.) Note that the type mapping is defined in such a way that $i$ knows i’s own “awareness level” at state $\tilde{\omega}$ is some $K'$. For example, if player $i$ is aware of color and order – but not shape – then $i$ can conceive of the opponent labeling the action set in terms of a (weak) subset of the attributes of which she herself is aware; that is, $i$ can conceive of the opponent labeling the action set in terms of color & order, only color, only order, and neither color nor order. More explicitly, this means that the type mapping $\mu_i$ for this particular player will assign a probabilistic belief only to $j$ types that describe the action set in terms of color & order, only color, only order, and neither color nor order. (One can verify that $\mu_i$ satisfies the common properties of type mappings in unawareness structures; for a formal presentation of such properties, see Heifetz, Meier, and Schipper, 2013.)

We now present our notion of rationalizability. Dekel et al. (2007) introduce interim correlated rationalizability to capture interactions where there is a correlation between the state of the world and players’ conjectures about the actions of others. Like the standard notion of rationalizability in complete-information games (Bernheim, 1984; Pearce, 1984), their concept is defined via an iterated-deletion procedure. At each round, an action survives for a type only if: (i) it is a best response to a belief attaching positive probability to type-actions pairs of the opponents that have not yet been deleted; (ii) it is consistent with that type’s beliefs about others and chance. In order to allow for unawareness, we modify such a notion so that $i$ does not best-respond to all $j$ types, but only to those of which $i$ can conceive (i.e., consistent with $\mu_i$). Another key difference is that we exploit frames by imposing a constraint on i’s beliefs about the actions of each $j$ type (of

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22 $2^{K'}$ denotes the power set of $K'$.

23 Obviously, if $i$ discovers new attributes (e.g., as a result of an experimental manipulation), then a different state obtains. In that case, the construction of $\mu_i$ implies that $i$ will know i’s new awareness level is $K''$, with $K'' \supseteq K'$. 

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which \( i \) can think): we do so by assuming that such beliefs are derived from strategies that are \textit{frame-appropriate} for each \( j \) type (in turn) to which \( \mu_i \) assigns a probability.

Formally, we define frame-dependent rationalizable actions at round \( q = 0 \) as \( R_{i,q=0}(\bar{\omega}) \)

\[ := S, \text{ where } S \text{ denotes a list of actions -- as identified by the theorist -- that is symmetric with respect to the players. The list } S \text{ has generic members } s_i \text{ and } s_j \text{ for players } i, j \in N, \text{ respectively } (s_i \text{ and } s_j \text{ are interpreted as the theorist’s representation of the choice each player type makes in her own language). We then denote by } u_i \left( (s_i, s_j), \bar{\omega} \right) \text{ player } i \text{'s utility at } \bar{\omega}. \text{ Frame-dependent rationalizability is defined inductively, as follows.}

\text{DEFINITION 4}——the set of frame-dependent rationalizable actions at \( q + 1 \) is

\[ R_{i,q+1}(\bar{\omega}) := \begin{cases} \text{there exists } \nu \in \Delta(S \times \bar{\Omega}_i,\bar{\omega}) \text{ such that} \\
(1) \nu \left( (s_j, \bar{\omega}') \right) > 0 \Rightarrow s_j \in R_{j,q}(\bar{\omega}')}} \\
(2) s_i \in \arg\max_{s_i \in S} \sum_{s_j,\bar{\omega}' \in S} u_i \left( (s_i, s_j), \bar{\omega}' \right)\cdot \nu \left( (s_j, \bar{\omega}') \right) \\
(3) \sum_{s_j \in S} \nu \left( (s_j, \bar{\omega}') \right) = \mu_i(\bar{\omega}')(\bar{\omega}')}} \\
(4) \nu \left( (s_j, \bar{\omega}') \right) \text{ is derived from strategies frame-appropriate at } \mu_i(\bar{\omega}')(\bar{\omega}')}} \end{cases} \]

and \( R_i(\bar{\omega}) = \bigcap_{q=1}^{\infty} R_{i,q}(\bar{\omega}). \)

In plain words, there exists a probability measure \( \nu \) defined on the Cartesian product of \( j \)'s actions \textit{and the states of which} \( i \) can conceive (note: \( \bar{\Omega}_i,\bar{\omega} \) denotes the collection of state spaces to which \( \mu_i(\bar{\omega}) \) assigns a probabilistic belief). Given this, \( i \) chooses a best response to \( \nu \) that puts positive probability only on type-actions pairs of the opponents that have not yet been eliminated (per conditions 1 and 2). Further, \( \nu \) must be consistent with \( i \)'s type belief \( \mu_i \) (condition 3). Finally, when possible (i.e., when the labeling-invariance principle has bite), \( \nu \) must be consistent with strategies that are frame-appropriate at \( \mu_i(\bar{\omega}')(\bar{\omega}')) \); that is, \( j \)'s actions are believed to be frame-appropriate according to \( j \)'s perception at state \( \bar{\omega}' \); given \( i \)'s own perception at \( \bar{\omega} \) (condition 4).

\text{EXAMPLE.} Suppose that, in Choose Something, Nature has selected object no. 5 (turquoise, diamond), no. 2 (cyan, diamond), and no. 6 (turquoise, pentagon); more specifically, let \( \omega = ((\text{first}, \text{turquoise}, \text{diamond}), (\text{second}, \text{cyan}, \text{diamond}), (\text{third}, \text{turquoise}, \text{pentagon})) \). As an example, we analyze a type of player \( i \) who is aware of \textit{color} and \textit{order}, which means that the type mapping \( \mu_i \) for this particular player assigns a probabilistic belief only to \( \bar{\Omega}_{i,\bar{\omega}} = \{ \bar{\Omega}_{i,(\text{cyan})}, \bar{\Omega}_{i,(\text{cyan})}, \bar{\Omega}_{(\text{cyan})}, \bar{\Omega}_{i,(\text{empty})} \} \). We begin by considering some \( i \) who attaches positive probability to a type
of $j$ who is aware of $\Omega_{\{C, O\}}$. In this case, the labeling-invariance principle imposes no constraint on $j$’s actions, as a description in terms of color & order implies three distinct labels, $(first, turquoise), (second, cyan), (third, turquoise)$; so, condition 4 above has no bite. We then consider some $i$ who attaches positive probability to a type of $j$ who is aware of $\Omega_{\{C\}}$: with some abuse of notation, we compactly refer to this belief as $\mu^C_i$. Here the labeling-invariance principle imposes a constraint, as a description in terms of color implies just two labels (i.e., $(turquoise), (cyan), (turquoise)$); it follows that the only frame-appropriate strategy here involves $j$ playing each of the turquoise objects with probability zero, and the cyan object with probability one.\footnote{Specifically, the labeling-invariance principle imposes a constraint whereby strategies that are indistinguishable with respect to the color labeling should be assigned the same probability. Then, definition 3 implies that a strategy is frame-appropriate for a player with color perception, if the strategy is labeling-invariant with respect to the color frame and it does not induce a payoff-dominated outcome in the event that a co-player chooses the same strategy. It follows that the only frame-appropriate strategy here involves playing the cyan object with probability one.}

We next turn to some $i$ who attaches positive probability to a type of $j$ who is aware of $\Omega_{\{O\}}$. Now the labeling-invariance principle imposes no constraint on $i$’s beliefs about $j$, as a description in terms of order here involves three labels (i.e., $(first), (second), (third)$). We then move on to some $i$ who attaches positive probability to a type of $j$ who is aware of $\Omega_{\{O\}}$, in which case $j$ describes each action as an “object”. Here the labeling-invariance principle imposes a constraint (playing each object with prob. $\frac{1}{3}$), which determines the frame-appropriate strategy.

Given the above, it follows that – for any $\mu_i$ assigning positive probability to the event that “others are aware of color differences” (i.e., compactly, for any positive $\mu^C_i$) – the unique frame-dependent rationalizable action is the color oddity (i.e., cyan). Finally, the reader might wish to consider alternative type mappings. Regardless of the specific mapping considered, the general insight is intuitive: given the attributes of which one is aware, one will best-respond solely to the actions of co-player types of which one can conceive. When different co-player types have different frame-appropriate strategies, then beliefs about the likelihood of those types (as defined by one’s type mapping $\mu_i$) become key to frame-dependent rationalizability.\footnote{As another example, suppose $i$ is aware of color and order like before, but now $\omega = (top, turquoise, diamond), (other, cyan, diamond), (other, turquoise, pentagon))$. Here the analysis is the same as above, except for the case of some $i$ who attaches positive probability to a type of $j$ who is aware of $\Omega_{\{O\}}$: we denote this belief by $\mu^O_i$. The labeling-invariance principle here imposes a constraint, as a description in terms of order now implies just two labels (i.e., $(top), (other), (other)$); so, strategies that are indistinguishable with respect to the order labeling should be assigned the same probability. It follows that the only frame-appropriate strategy here involves $j$ playing top with probability one. Finally, for any type mapping $\mu_i$ such that $\mu^C_i \leq \mu^O_i$, the unique frame-dependent rationalizable action is the top object. Instead, for any $\mu_i$ whereby order types are considered less likely than color types (formally, $\mu^C_i > \mu^O_i$), then the unique frame-dependent rationalizable action is the color oddity (i.e., cyan).}
IV. Predictions

Our experimental design brings about different frames by manipulating subjects’ awareness of several attributes. This allows us to investigate the perception-action chain:

\[ \text{awareness of attributes } K \rightarrow K\text{-induced frame } \rightarrow \text{behavior}. \]

A frame may be thought of as a player’s description of the available actions in terms of the attributes of which she is aware. Our theory provides a formal account of the effect of manipulating awareness (of alternative framings) on strategic behavior, via two “channels”, as follows. First, the player’s awareness level affects the frames, hence the types, one can consider. Second – for any fixed awareness level – behavior varies with one’s beliefs about the attributes of which (one thinks) others are aware.\(^{26}\) Given this, our experimental design controls both for a subject’s awareness level (which is exogenously varied) and for a subject’s probabilistic beliefs about the others’ awareness (with such beliefs being elicited in the All-Aware treatment). This allows for a direct test of frame-dependent rationalizability.

Our first hypothesis addresses the first channel above.\(^{27}\) In short, we assume that each Baseline participant is endowed with a particular frame (or set of frames); we further assume that All-Aware participants – prior to the treatment manipulation – are similar to their Baseline counterparts (on average), but eventually become aware of additional frames as a result of their exposure to the “All-Aware questions” (see pp. 8-9). A natural consequence is that an increase in frame awareness may reduce an individual’s reliance on the prior (unobservable) frames, thereby affecting the frequency with which participants choose oddities in the All-Aware treatment, as compared with the Baseline.

**H1.** An increase in frame awareness weakly reduces an individual’s reliance on the prior (unobservable) frames. As a result, the frequency with which participants choose (color or shape) oddities varies across the Baseline and the All-Aware treatments.

Our next hypotheses will delve into the directional effects of our experimental manipulation, addressing the second channel above. Before doing so, we shall first elaborate on the intuition behind H1. To that end, we stress that the All-Aware treatment is identical to the Baseline except that, in the former, our treatment manipulation makes subjects privately aware of multiple

\(^{26}\)In our model, such beliefs are specified by \(\mu_i(\bar{\omega})\).

\(^{27}\)The experimental design and hypotheses were deposited into the AEA’s registry for randomized controlled trials, prior to running the present study. The registration form is available at [https://doi.org/10.1257/rct.4219-1.0](https://doi.org/10.1257/rct.4219-1.0) (Charness, Gary and Alessandro Sontuoso. 2019. “The Doors of Perception.” AEA RCT Registry. May 20).
attributes. It follows that the behavior of a subject who (prior to the All-Aware manipulation) had been unaware of some of those attributes might be affected by the fact that we mention them in the All-Aware treatment. Indeed, any differences in choice behavior between All-Aware and Baseline can be attributed to the fact that, in the former, one comes to consider additional frames (i.e., a fact that may impact one’s rationalizable actions). For example, some All-Aware participants – who prior to the treatment manipulation had only been aware of color differences – might come to believe that a newly-discovered frame is most likely considered by others: this would imply a fall in the frequency of All-Aware participants choosing color oddities.\textsuperscript{28} Regardless of the specific example, we stress that the effect of the All-Aware manipulation is to deterministically change the calculus of coordination, as is clarified by the discussion of our next hypotheses.

Our second hypothesis – with its two components H2.a and H2.b – puts to test the notion of frame-dependent rationalizability more directly by focusing on the sample of All-Aware participants. We begin by introducing H2.a, which addresses the case in which a subject believes that others are more likely to consider (non-stochastic) attributes such as color and shape.\textsuperscript{29}

\textbf{H2.a.} The frequency with which All-Aware participants choose color (shape) oddities is positively related to their belief about others noticing the different colors (shapes).

What does H2.a specifically entail? To answer this question, recall that our model implies that when different co-player types have different frame-appropriate strategies, then beliefs about the likelihood of those types (as defined by one’s type mapping $\mu_i$) are key for determining

\begin{itemize}
\item [\textsuperscript{28}] Whether the All-Aware manipulation results in an increase or a decrease in the frequency of choice of (color or shape) oddities depends on two concurrent elements. In short, All-Aware participants are less likely to choose a color oddity than Baseline participants, if: (i) one typically considers the color frame and no other frames, unless exposed to the All-Aware questions; and (ii) after learning about all frames, one believes that others (who were not exposed to those questions) are more likely to consider another frame than the color frame. Conversely, All-Aware participants are more likely to choose a color oddity than Baseline participants, if: (i) one typically ignores the color frame, unless exposed to the All-Aware questions; and (ii) after learning about all frames, one believes that others (not exposed to those questions) are more likely to consider the color frame than another frame. Since the experimenter cannot possibly observe individuals’ prior frames, our next hypotheses will corroborate whether our interpretation (of any Baseline vs. All-Aware differences) is correct and consistent with our theory.
\item [\textsuperscript{29}] A “stochastic attribute” is an attribute that implies a stochastic labeling procedure, in the sense that the assignment of a label to an action is randomly determined. For example, consider a subject who refers to the blocks using the list of labels $\{(\text{first}), (\text{second}), (\text{third})\}$. Given this, a round of Choose Something entails 120 possible stochastic labelings (i.e., given a set of six blocks, there are 20 triplets and 6 permutations for each triplet). In each of those 120 labelings the blocks are named in the same way – i.e., $\{(\text{first}), (\text{second}), (\text{third})\}$ – but which object is labeled, say, “first” eventually depends on the random draw. Instead, consider a subject who refers to the blocks as a list of colors: in this case the random draw only determines which colors are “available”, but the assignment of a label to an action is not randomly determined, since color is an intrinsic property of objects (from the perspective of the human eye). In other words, whether a block is named blue rather than red here depends on a non-stochastic element, such as the stimulation of cone cells in the human eye.
\end{itemize}
rationalizable actions. Hence, if an All-Aware participant believes that her counterpart is more likely to notice colors than shapes or order, then she should choose a color oddity. Instead, if an All-Aware participant believes that her counterpart is more likely to notice shapes than colors or order, then she should choose a shape oddity. [As a side note, recall that All-Aware participants are told that they are assigned a (Baseline) partner who is not exposed to the extra questions in PART B of the experiment (see pp. 8-9 above). In order to keep Baseline participants’ awareness level unchanged, Baseline participants do not know about their partners’ extra questions.]

It is worth stressing that our hypotheses are incompatible with the standard Bayesian paradigm. According to that paradigm, behavioral responses to the acquisition of new information can only happen as a result of the shrinking (not the expansion!) of the set of possible events: a fact that precludes unawareness.$^{30}$ By contrast, our model allows for the set of possible events to expand, and therefore for a decision-maker to react to novel information (in the form of a newly-discovered frame) by maximizing an updated frame-dependent expected utility. More explicitly: if H2 is confirmed, then we will have evidence that subjects do attempt to maximize a frame-dependent expected utility function; further, if H1 is confirmed as well (i.e., if there are differences between Baseline and All-Aware), then we will have evidence that the expected utility function All-Aware participants are attempting to maximize has been updated.

We turn to the analysis of the order frame. Before formulating our next hypothesis, we note that while the color and shape attributes imply a fairly straightforward (and non-stochastic) description for each block, the same is not true in the case of the order attribute. Depending on the emphasis one puts on each of the objects’ positions, the order attribute might well entail any of the following lists of labels: (i) $((\text{top}), (\text{other}), (\text{other}))$; (ii) $((\text{other}), (\text{middle}), (\text{other}))$; (iii) $((\text{other}), (\text{other}), (\text{bottom}))$; (iv) $((\text{first}), (\text{second}), (\text{third}))$. $^{31}$ Whereas frame (iv) generates no order oddities, frames (i), (ii), (iii) would respectively pull a subject toward the top, middle, and bottom blocks, and hence possibly away from color or shape oddities. Now, the experimenter could use any of the order lists above as the basis for a null hypothesis, to formulate a prediction about the relationship between one’s order awareness and one’s tendency to (consistently) choose the distinct

$^{30}$ More precisely: in the standard Bayesian paradigm, the set of possible events shrinks whenever one acquires novel information; this is because conditioning on new data necessarily implies that one assigns probability zero to events that had previously been assigned positive probabilities.

$^{31}$ It is clear that each order frame has a number of equivalent translations that do not affect the application of the labeling-invariance principle. For example, list (iv) is equivalent to $((\text{primo}), (\text{sheni}), (\text{shlishi}))$, and to $((\text{primo}), (\text{secondo}), (\text{terzo}))$, and to $((\text{top}), (\text{middle}), (\text{bottom}))$, etc.
(nth) object in a list. That said, we know from research in memory and cognition that individuals’ order encoding is often characterized by a “primacy effect” where the first few items in a list are recalled more frequently than later items (Murdock, 1962; Rundus, 1971). Since recognizability and memory are key determinants of perceived salience, we shall make the default assumption that our mention of the order attribute (in the All-Aware treatment) likely induces frame (i) above. This in turn implies a relationship between one’s belief about the “others’ order awareness” and one’s tendency to consistently choose the top block (see example in footnote 25). This leads to the specification of H2.b below.

**H2.b.** The frequency with which All-Aware participants choose top or middle, or bottom objects – regardless of whether there are any color or shape oddities – varies with their belief about others noticing the order of the objects. In particular, if one believes that others are more likely to notice the objects’ order (rather than colors or shapes), then one is more likely to choose the top block.

We proceed to formulate hypotheses relating to our supplemental treatments. Before stating such hypotheses, we stress that each of the Color-Aware, Shape-Aware, and Order-Aware treatments is the same as the Baseline except that the objects are exogenously labeled in terms of colors, shapes, or the order in which the blocks are displayed, respectively. The purpose of the treatments is to bring about different subject pools, whereby each pool is characterized by common awareness of the relevant attribute. We begin by considering the Color- and Shape-Aware treatments.

**H3.** Color- and Shape-Aware participants choose (respectively color and shape) oddities weakly more frequently than Baseline participants.

Notably, H3 predicts that Color- and Shape-Aware participants will choose (respectively color and shape) oddities weakly more frequently than Baseline participants. This is because, by inducing common awareness of either attribute, such treatments make a subject believe that her assigned partner has considered the relevant frame, and hence oddity (if there is one).

---

32 More specifically, there is well established evidence of a strong – possibly exponential – primacy effect extending over the first 3 or 4 items in a list, as well as evidence of an S-shaped recency effect extending over the last 8 items in a list. This implies that the primacy effect is stronger in short lists (Murdock, 1962; Rundus, 1971).

33 We expect (at worst) a weak increase in the frequency of choice of the relevant (color or shape) oddity, since the treatment manipulation can impact only those subjects who – absent the manipulation – would not believe their partner to consider the relevant frame with certainty. Note that the experimenter has no a priori knowledge of such subjective probabilities.
Finally, we discuss a null effect of the Order-Aware treatment manipulation, predicted below.

**H4.** Order-Aware participants choose (color or shape) oddities *with the same frequency* as Baseline participants.

In the case of the Order-Aware treatment, we solely implemented frame (iv) above – i.e., “first”, “second”, “third” – thereby highlighting the sequential nature of the available actions without generating oddities. It follows from our notion of rationalizability that Order-Aware participants in this case will choose color or shape oddities just as frequently as Baseline participants. Again, this is because the order frame we implemented does not generate oddities, and therefore the labeling-invariance principle has no bite here. (This means that even if one were previously unaware of such a frame, its discovery would never entail a different best-reply with respect to one’s prior frames; see the main example in section III.2.)

**V. Experimental results**

1. *Summary statistics and regression models*

Sessions were conducted at UCSB, where subjects were recruited from a broad spectrum of academic departments with the help of ORSEE (Greiner, 2015). On average, a session had about 18 subjects and lasted about 40 minutes, with a mean payoff of about $12 per subject (including a $5 show-up fee), and with minimum and maximum earnings of $6.50 and $17.50, respectively. No subject was allowed to participate in more than one session.

A total of 212 subjects took part in our sessions, with each subject playing 10 different instances of Choose Something. The number of participants (per treatment) was 54 in each of the main treatments – i.e., Baseline and All-Aware – over a total of six sessions. The number of participants in our supplemental treatments was 34 in Color-Aware, 32 in Shape-Aware, and 38 in Order-Aware (with each of the supplemental treatments comprising two sessions).

For each of the five treatments, Table 2 presents mean choices (in the upper panel), along with mean guesses about the likelihood of the others’ choosing the top, middle, and bottom objects (lower panel). With specific reference to these guesses, we note that such data should not be confused with the beliefs about others noticing the various attributes, which were only elicited from All-Aware participants and shall be discussed later.
Here we report some preliminary tests addressing the above-discussed hypotheses. Later on we also perform a more formal regression analysis controlling for any round fixed-effects. Before doing so, to sketch a rough outline of the main patterns emerging from our dataset, we initially treat each choice as an independent observation. Note that this is justified by a few elements: (i) no feedback was given between rounds; (ii) in each round a different three-object selection (“triplet”) was implemented by the program; (iii) in each round subjects were randomly assigned to another pair, and were so informed (i.e., a “random matching” protocol).

<table>
<thead>
<tr>
<th>Choice (by location)</th>
<th>Base</th>
<th>All-A</th>
<th>Color-A</th>
<th>Shape-A</th>
<th>Order-A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top of column is chosen, %</td>
<td>40.18</td>
<td>50.00</td>
<td>43.23</td>
<td>33.13</td>
<td>45.53</td>
</tr>
<tr>
<td>Center of column is chosen, %</td>
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<td>26.30</td>
<td>33.53</td>
<td>41.56</td>
<td>33.42</td>
</tr>
<tr>
<td>Bottom of column is chosen, %</td>
<td>26.67</td>
<td>23.70</td>
<td>23.24</td>
<td>25.31</td>
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</tr>
<tr>
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<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Guess (by location)</th>
<th>Top of column is chosen, %</th>
<th>35.02</th>
<th>38.11</th>
<th>34.55</th>
<th>32.31</th>
<th>35.99</th>
</tr>
</thead>
<tbody>
<tr>
<td>Center of column is chosen, %</td>
<td>33.44</td>
<td>32.21</td>
<td>34.03</td>
<td>35.65</td>
<td>33.61</td>
<td></td>
</tr>
<tr>
<td>Bottom of column is chosen, %</td>
<td>31.54</td>
<td>29.68</td>
<td>31.42</td>
<td>32.04</td>
<td>30.40</td>
<td></td>
</tr>
<tr>
<td>Total, %</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td></td>
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<td># observations</td>
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<td>540</td>
<td>340</td>
<td>320</td>
<td>380</td>
<td></td>
</tr>
</tbody>
</table>

Table 2 - The upper panel shows the frequency of play of each object in a triplet, given a classification of the blocks based on their position (i.e., top, middle, or bottom of the column). The lower panel shows mean guesses about the others’ choices, again by position: see task iv. on p. 8 above. The number of observations equals the number of subjects in the relevant treatment times the number of triplets (i.e., 10 rounds).

A glance at the upper panel of Table 2 reveals that the distribution of choices varies across treatments. This provides some first evidence in support of the thesis that treatment-induced awareness affects choice behavior. In fact, we stress that all treatments featured exactly the same ordered triplets, so that the position of the oddities did not differ across treatments, whereas subjects’ awareness of any such oddities varied as a result of the treatments to which they were assigned. A chi-square test conducted on the entire choice dataset confirms that there is a
statistically-significant relationship between a subject’s choice and the treatment a subject was assigned to: \( N = 2,120 \) obs., \( \chi^2 = 32.797, p = 0.000, \) two-tailed.\(^{34,35}\) Similarly, the lower panel of Table 2 indicates that guesses about the others’ choices vary across treatments. In particular, our belief dataset shows statistically-significant differences across the five treatments (\( N = 2,120 \) obs.), with respect to a subject’s beliefs about others choosing the top block (\( \chi^2 = 54.965, p = 0.000, \) two-tailed Kruskal-Wallis test), or the middle block (\( \chi^2 = 58.113, p = 0.000, \) two-tailed), or the bottom block (\( \chi^2 = 51.896, p = 0.000, \) two-tailed).\(^{36}\) Our analysis below will further illuminate the relationship between beliefs, awareness-types, and choice behavior.

We move on to test for H1 by restricting our attention to the choice dataset from the Baseline and the All-Aware treatments only—see the relevant columns in the upper panel of Table 2. A chi-square test indicates that there is a statistically-significant relationship between one’s choice (by location) and the (Baseline vs. All-Aware) treatment to which one was assigned: \( N = 1,080 \) obs., \( \chi^2 = 10.973, p = 0.004, \) two-tailed. Such treatment-induced differences in choice distributions provide indirect evidence in support of H1, that is, the hypothesis that the frequency with which participants choose oddities varies across the Baseline and the All-Aware treatments (again, this is because the position of the oddities did not differ across treatments).

In order to delve deeper into the treatment effects, probe their robustness, and control for any clustering in the data, Table 3 reports a simple multinomial-logit model that consists of a subject’s choice as the dependent variable and of the following predictors: (i) a dummy for each treatment (indicating whether a subject was assigned to that treatment or to the Baseline); (ii) a set of dummies indicating whether there was a color or shape oddity in each position.

\(^{34}\) Similarly, a Hotelling’s T-squared generalized means test confirms that a subject’s virtual mixed strategy (i.e., a randomized strategy obtained by averaging the subject’s top, middle, and bottom choices over 10 rounds, i.e., triplets) significantly differed across treatments: \( N = 212 \) obs., \( \chi^2 = 17.62, p = 0.024. \) (The Hotelling’s test is simply a multivariate generalization of the t test.)

\(^{35}\) Additionally, Hotelling’s T-squared generalized means tests – conducted on the sample of per-subject mean choices – reveal that behavior in each of the treatments significantly differed from the fully-mixed equilibrium assigning equal probability to all actions (for Baseline: \( N = 54 \) obs., \( T^2 = 8.48, p = 0.021; \) for All-Aware: \( N = 54 \) obs., \( T^2 = 16.78, p = 0.000; \) for Color-Aware: \( N = 34 \) obs., \( T^2 = 21.80, p = 0.000; \) for Shape-Aware: \( N = 32 \) obs., \( T^2 = 14.94, p = 0.002; \) for Order-Aware: \( N = 38 \) obs., \( T^2 = 22.97, p = 0.000).\)

\(^{36}\) Furthermore, Hotelling’s T-squared generalized means tests – conducted on the sample of per-subject mean guesses – show that beliefs in each of the treatments significantly differed from the fully-mixed equilibrium assigning equal probability to all actions (for Baseline: \( N = 54 \) obs., \( T^2 = 8.69, p = 0.019; \) for All-Aware: \( N = 54 \) obs., \( T^2 = 15.40, p = 0.001; \) for Color-Aware: \( N = 34 \) obs., \( T^2 = 8.72, p = 0.023; \) for Shape-Aware: \( N = 32 \) obs., \( T^2 = 9.84, p = 0.016; \) for Order-Aware: \( N = 38 \) obs., \( T^2 = 7.94, p = 0.030).\)
<table>
<thead>
<tr>
<th></th>
<th>[A] Choice of the middle block</th>
<th>[B] Choice of the bottom block</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment All-Aware</td>
<td>-0.517 (.134) ***</td>
<td>-0.366 (.135) ***</td>
</tr>
<tr>
<td>Treatment Color-Aware</td>
<td>0.393 (.369) *</td>
<td>0.598 (.360) *</td>
</tr>
<tr>
<td>Treatment Shape-Aware</td>
<td>0.831 (.352) **</td>
<td>0.241 (.360) **</td>
</tr>
<tr>
<td>Treatment Order-Aware</td>
<td>0.335 (.309)</td>
<td>0.433 (.290) **</td>
</tr>
<tr>
<td>Top block is a color oddity</td>
<td>-1.738 (.279) ***</td>
<td>-1.083 (.616) *</td>
</tr>
<tr>
<td>Middle block is a color oddity</td>
<td>1.602 (.194) ***</td>
<td>-0.484 (.344) ***</td>
</tr>
<tr>
<td>Bottom block is a color oddity</td>
<td>-0.346 (.224) ***</td>
<td>0.053 (.173) ***</td>
</tr>
<tr>
<td>Top block is a shape oddity</td>
<td>-1.010 (.392) ***</td>
<td>-1.222 (.311) ***</td>
</tr>
<tr>
<td>Middle block is a shape oddity</td>
<td>1.269 (.333) ***</td>
<td>0.581 (.460) ***</td>
</tr>
<tr>
<td>Bottom block is a shape oddity</td>
<td>0.563 (.227) ***</td>
<td>0.352 (.170) ***</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.690 (.228) ***</td>
<td>-0.175 (.355) ***</td>
</tr>
<tr>
<td>Pseudo R²</td>
<td>0.064</td>
<td></td>
</tr>
<tr>
<td>Obs.</td>
<td>2,120</td>
<td></td>
</tr>
</tbody>
</table>

Table 3 - Logistic regression coefficients, with robust standard errors adjusted for (triplets’) [individuals’] clustering. *, **, and *** respectively indicate \( p<0.10 \), \( p<0.05 \), and \( p<0.01 \) for the relevant Z Statistic, two-tailed tests. The output of the regression has two parts – columns [A] and [B] – because our “choice by location” outcome variable has three values: top (the comparison group), middle, and bottom objects. Note: the reference category for each of the treatment dummies is the Baseline; the reference category for each of the oddity dummies is their negation. The regression uses the entire choice dataset.
Note that the output of the regression has two parts (which we respectively report in columns [A] and [B] of Table 3), since – if one classifies choices by location – the outcome variable has three values: top (the comparison group), middle, and bottom objects. Also note that below each coefficient are robust standard errors adjusted for triplets’ and individuals’ clustering, respectively in round and square brackets. A quick glance at Table 3 shows that the choice of an object located in the middle or bottom (vs. top) spots is significantly and negatively related to the presence of a color or shape oddity in the top spot. On the other hand, a middle block is more often chosen, if the middle block happens to be an oddity. Table 3 further confirms that our treatments – each to a different extent – significantly affected subjects’ choice distributions. So, the fact that our treatments have a different impact on each of the binary comparisons (i.e., middle or bottom vs. top) confirms our previous interpretation of the changes in the overall choice distributions as being driven by variations in subjects’ awareness of the oddities. (Again, recall that the position of the oddities did not differ across treatments.) The analysis below will further corroborate such a result.

To that end, we shall focus in turn on the sub-samples of the data containing color or shape oddities, which collectively account for 9/10 of our observations. We begin by examining all the triplets containing color oddities (triplet-class ‘CO’). Figure 2 below presents frequency distributions of individual-level choices in such triplets (note that we later analyze a perfectly symmetric sub-sample that features all the triplets containing shape oddities). Specifically, Figure 2 reports the frequency of play of each and every object in triplet-class ‘CO’, given a classification of the objects based on their position (see table in the upper panel), as well as a classification of the same objects based on their color and shape characteristics (see bar graph in the lower panel).

One can see from the bar graph in Figure 2 that there are differences in the frequency with which All-Aware participants chose (color and shape) oddities relative to Baseline participants, with All-Aware participants collectively picking oddities slightly less frequently than Baseline.

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37 By viewing the blocks in Table 1 above it is clear that all but two of the 20 possible three-object selections contain at least one oddity; that is, all but triplets {1,4,5} and {2,3,6}, implying a “theoretical probability of randomly selecting either one” equal to 1/10. Moreover, a quick glance at Table 1 reveals that the theoretical probability of randomly selecting a triplet containing only a color oddity (e.g., {1,2,6}) is 1/3; similarly, the theoretical probability of randomly selecting a triplet containing only a shape oddity (e.g., {2,3,5}) is 1/3; finally, the theoretical probability of randomly selecting a triplet containing both color and shape oddities (e.g., {4,5,6}) is 1/3 as well. That being said, we note that our computerized experiment involved a “constrained” pre-randomization of the blocks in such a way to ensure that participants in each treatment would be presented with a distribution of triplets representative of the theoretical distribution; this was also necessary in order to ensure that participants in each treatment would be presented with exactly the same triplets across rounds.
participants. Figure 2 further shows that Color-Aware participants have the highest rate of choosing color oddities, and All-Aware participants have the lowest such rate. By contrast, Shape-Aware participants have the highest rate of choosing shape oddities, and Color-Aware participants have the lowest such rate.

<table>
<thead>
<tr>
<th>Choice (by location)</th>
<th>Base</th>
<th>All-A</th>
<th>Color-A</th>
<th>Shape-A</th>
<th>Order-A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top of column is chosen, %</td>
<td>36.73</td>
<td>44.75</td>
<td>37.25</td>
<td>29.17</td>
<td>37.28</td>
</tr>
<tr>
<td>Center of column is chosen, %</td>
<td>34.26</td>
<td>28.70</td>
<td>34.80</td>
<td>42.71</td>
<td>35.53</td>
</tr>
<tr>
<td>Bottom of column is chosen, %</td>
<td>29.01</td>
<td>26.55</td>
<td>27.95</td>
<td>28.12</td>
<td>27.19</td>
</tr>
<tr>
<td>Total, %</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td># observations</td>
<td>324</td>
<td>324</td>
<td>204</td>
<td>192</td>
<td>228</td>
</tr>
</tbody>
</table>

**Figure 2** - The upper panel shows the frequency of play of each object, given a classification of the blocks based on their position (i.e., top, middle, or bottom), in the sub-sample of the data containing all the color oddities. The lower panel provides a breakdown of the same data, given a classification of the blocks based on color/shape oddities. Note that all treatments feature the exact same triplets. For each treatment, the first (i.e., blue) bar indicates the frequency with which the color oddity was chosen. The second (maroon) and third (gray) bars respectively indicate the frequency with which the shape oddity and a non-oddity were chosen in each treatment, with such values being reported solely for the sake of between-treatment comparisons—note that color oddities and shape oddities are respectively present in 100% and 50% of the sub-sample (i.e., all the triplets contain a color oddity, with half of such triplets containing a shape oddity as well). Note that Figure 3 below presents data from a sub-sample that is perfectly symmetric to this one.

A chi-square test conducted on the oddities’ choice data for the entire triplet-class ‘CO’ (as broken down in the Figure 2 bar graphs) confirms that there is a statistically-significant relationship between a subject’s choice of oddities and the treatment to which a subject was assigned: \( N = 1,272 \)
obs., $\chi^2 = 22.920$, $p = 0.003$, two-tailed. This test provides some suggestive insights into the relationship between awareness of an attribute and behavior, although it does not measure the effect ascribable to a specific treatment nor does it control for clustering or for differences between triplets within the same class.

So, to formally test our hypotheses, we ran a conditional (i.e., triplets’ fixed-effects) logit model consisting of a subject’s choice of the color oddity as the binary dependent variable and of treatment dummies as predictors (indicating whether a subject was assigned to that treatment or to the Baseline). The results – reported in column [I] of Table 4 below – confirm a significant negative change in the frequency with which color oddities were chosen, when comparing the All-Aware treatment with the Baseline: this provides corroborating evidence in support of H1. That is, an increase in frame awareness reduces All-Aware participants’ reliance on the prior frames (which in the case of our subjects predominantly consisted of the color frame). So, as a result of considering extra frames (i.e., a fact that may impact a subject’s rationalizable actions), the rate of choosing color oddities has fallen in the All-Aware treatment, relative to the Baseline.

Further, note that model [I] of Table 4 shows a positive but non-significant change in the frequency with which color oddities were chosen, when moving from the Baseline to the Color-Aware treatment, a fact that by itself does not warrant rejection of H3 (which states that “Color- and Shape-Aware participants choose – respectively color and shape – oddities weakly more frequently than Baseline participants”). Our below analysis of a subject’s choice of shape oddities will in fact provide evidence in support of H3. Prior to that, we conclude our commentary on model [I] of Table 4 by noting that there is no significant change between Baseline and Order-Aware treatments, which provides some first evidence in support of H4 (i.e., “Order-Aware participants choose color [or shape] oddities with the same frequency as Baseline participants”).

We proceed to examine a different slice of the dataset by presenting an overview of all the triplets containing shape oddities (triplet-class ‘SO’). Figure 3 shows frequency distributions of individual-level choices in such triplets. More specifically, Figure 3 reports the frequency of play of each and every object in triplet-class ‘SO’, given a classification of the objects based on their position (see the upper panel), as well as a classification of the same objects based on their color and shape characteristics (see the lower panel).

In particular, the bar graph in Figure 3 confirms changes in the frequency with which All-Aware participants chose oddities relative to Baseline participants, with All-Aware participants
collectively picking oddities slightly less frequently than Baseline participants. Also, Figure 3 indicates that Shape-Aware participants have the highest rate of choosing shape oddities, and Color-Aware participants have the lowest such rate. We also see that Color-Aware participants have the highest rate of choosing color oddities, with Shape-Aware participants having the lowest such rate.

<table>
<thead>
<tr>
<th>Choice (by location)</th>
<th>Base</th>
<th>All-A</th>
<th>Color-A</th>
<th>Shape-A</th>
<th>Order-A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top of column chosen, %</td>
<td>44.14</td>
<td>53.40</td>
<td>47.06</td>
<td>39.06</td>
<td>50.44</td>
</tr>
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<td>Center of column chosen, %</td>
<td>29.01</td>
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<td>35.42</td>
<td>28.95</td>
</tr>
<tr>
<td>Bottom of column chosen, %</td>
<td>26.85</td>
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<td>22.06</td>
<td>25.52</td>
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<td>Total, %</td>
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<tr>
<td># observations</td>
<td>324</td>
<td>324</td>
<td>204</td>
<td>192</td>
<td>228</td>
</tr>
</tbody>
</table>

**Figure 3** - The upper panel shows the frequency of play of each object, given a classification of the blocks based on their position (i.e., top, middle, or bottom), in the sub-sample of the data containing all the shape oddities. The lower panel provides a breakdown of the same data, given a classification of the blocks based on color/shape oddities. Note that all treatments feature the exact same triplets. For each treatment, the second (i.e., maroon) bar indicates the frequency with which the shape oddity was chosen. The first (blue) and third (gray) bars respectively indicate the frequency with which the color oddity and a non-oddity were chosen in each treatment, with such values being reported solely for the sake of between-treatment comparisons—note that shape oddities and color oddities are respectively present in 100% and 50% of the sub-sample (i.e., all the triplets contain a shape oddity, with half of such triplets containing a color oddity as well). Note that Figure 2 above presents data from a sub-sample that is perfectly symmetric to this one.

A chi-square test conducted on the oddities’ choice data for the entire triplet-class ‘SO’ (as broken down in the Figure 3 bar graphs) confirms a statistically-significant relationship between a
subject’s choice of oddities and the treatment to which a subject was assigned: \( N = 1,272 \) obs., \( \chi^2_8 = 16.719, p = 0.033 \), two-tailed. Yet, since this test does not measure the effect ascribable to each treatment nor does it control for differences between triplets within the same class, we next turn to the regression analysis.

<table>
<thead>
<tr>
<th>[I] Choice of the color oddity</th>
<th>[II] Choice of the shape oddity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment All-Aware</td>
<td>-.265 *** (.039)</td>
</tr>
<tr>
<td>Treatment Color-Aware</td>
<td>.090 (.123)</td>
</tr>
<tr>
<td>Treatment Shape-Aware</td>
<td>.052 (.334)</td>
</tr>
<tr>
<td>Treatment Order-Aware</td>
<td>.023 (.113)</td>
</tr>
<tr>
<td>Pseudo R²</td>
<td>0.003</td>
</tr>
<tr>
<td>Obs.</td>
<td>1,272</td>
</tr>
</tbody>
</table>

**Table 4** - Conditional logistic regression coefficients, with robust standard errors adjusted for (triplets’) clustering.38 *, **, and *** indicate \( p<0.10, p<0.05 \) and \( p<0.01 \), respectively, for the relevant Z Statistic, two-tailed tests. Note: the reference category for all the treatment dummies is the Baseline. Model [I] uses the entire triplet-class ‘CO’, and [II] uses the entire triplet-class ‘SO’.

Specifically, column [II] of Table 4 above presents a conditional (i.e., triplets’ fixed-effects) logit model consisting of a subject’s *choice of the shape oddity* as the binary dependent variable and of treatment dummies as predictors. The model shows a significant negative change in the frequency with which shape oddities were chosen, when contrasting the Color-Aware treatment with the Baseline; moreover, the model indicates a significant positive change when comparing Shape-Aware and Baseline, which provides support for H3 (which states that “Color- and Shape-Aware participants choose – respectively color and shape – oddities weakly more frequently than Baseline participants”). We further note that model [II] shows no significant change between Order-Aware and Baseline, thereby corroborating the previous evidence in support of H4 (i.e., “Order-Aware participants choose color or shape oddities with the same frequency as Baseline participants”).

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38 Unlike Table 3 above, here we do not report standard errors adjusted for individuals’ clustering, since our triplets’ fixed-effects models do not allow for clustering on individuals.
2. Tests involving one’s beliefs about others’ awareness

Here we test the hypothesis (“H2”) that there is a relationship between one’s behavior and one’s beliefs about the others’ awareness, as implied by our notion of frame-dependent rationalizability. We do so by presenting a few logistic regressions on the All-Aware dataset.

<table>
<thead>
<tr>
<th>Belief about others noticing different colors $\mu_i^C$</th>
<th>[I] Choice of the color oddity</th>
<th>[II] Choice of the shape oddity</th>
<th>[III] Choice of the top object</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>.026 *</td>
<td>- .045 ***</td>
<td>- .003</td>
</tr>
<tr>
<td></td>
<td>(.015)</td>
<td>(.012)</td>
<td>(.023)</td>
</tr>
<tr>
<td>Belief about others noticing different shapes $\mu_i^S$</td>
<td>- .009</td>
<td>.062 ***</td>
<td>-.014</td>
</tr>
<tr>
<td></td>
<td>(.007)</td>
<td>(.015)</td>
<td>(.018)</td>
</tr>
<tr>
<td>Belief about others noticing order $\mu_i^O$</td>
<td>- .015 **</td>
<td>.007</td>
<td>.031 **</td>
</tr>
<tr>
<td></td>
<td>(.007)</td>
<td>(.006)</td>
<td>(.013)</td>
</tr>
<tr>
<td>Constant</td>
<td>- .332</td>
<td>- 2.545 **</td>
<td>- .945</td>
</tr>
<tr>
<td></td>
<td>(.814)</td>
<td>(1.062)</td>
<td>(1.205)</td>
</tr>
<tr>
<td>Pseudo R²</td>
<td>0.030</td>
<td>0.036</td>
<td>0.019</td>
</tr>
<tr>
<td>Obs.</td>
<td>246</td>
<td>187</td>
<td>177</td>
</tr>
</tbody>
</table>

Table 5 - Logistic regression coefficients, with robust standard errors adjusted for (triplets’) clustering. *, **, and *** indicate $p<0.10$, $p<0.05$ and $p<0.01$, respectively, for the relevant Z Statistic, two-tailed tests. Note: [I] uses triplet-class ‘CO’ in All-Aware; [II] uses triplet-class ‘SO’ in All-Aware; [III] uses all the triplets in All-Aware.

We first examine whether a subject’s choice of the color oddity is frame-dependent rationalizable, given the subject’s beliefs. To that end, we ran a logit model consisting of a subject’s choice of the color oddity as the binary dependent variable and of belief variables as predictors. That is, we used continuous variables measuring the All-Aware participants’ beliefs about others noticing the different colors, shapes, and order of the blocks, respectively denoted $\mu_i^C$, $\mu_i^S$, and $\mu_i^O$ for ease of reference. More precisely, to test for (frame-dependent) rationalizability of a color oddity, we must restrict attention to the sub-sample of participants whose beliefs are such that $\mu_i^C \geq \mu_i^S$ and $\mu_i^C \geq \mu_i^O$. Results from model [I] of Table 5 reveal a positive and mildly significant effect of the belief about others noticing the different colors ($\mu_i^C$) on a subject’s choice of the color oddity. The model also indicates no significant effect of $\mu_i^S$, and a significant negative effect of $\mu_i^O$. In plain words, this means that All-Aware participants are more likely to pick a color oddity, the more they believe that others are likely to consider a color frame rather than an order frame. This provides some evidence in support of (the color-relevant part of) H2.a, which states that “the
frequency with which All-Aware participants choose color oddities is positively related to their belief about others noticing the different colors”.

Next, we examine if a subject’s choice of the shape oddity is frame-dependent rationalizable. To do so, we ran a logit model consisting of a subject’s choice of the shape oddity as the binary dependent variable and of the above belief variables as predictors. In this case, to test for (frame-dependent) rationalizability of a shape oddity, we must restrict attention to the sub-sample of participants whose beliefs are such that \( \mu_i^S \geq \mu_i^C \) and \( \mu_i^S \geq \mu_i^O \). Model [II] of Table 5 reveals a significant positive effect of the belief about others noticing the different shapes (\( \mu_i^S \)) on a subject’s choice of the shape oddity. The model also indicates no significant effect of \( \mu_i^O \), and a significant negative effect of \( \mu_i^C \). This means that All-Aware participants are more likely to pick a shape oddity, the more they believe that others are likely to consider a shape frame rather than a color frame. This provides evidence in support of (the shape-relevant part of) H2.a, which states that “the frequency with which All-Aware participants choose shape oddities is positively related to their belief about others noticing the different shapes”.

We finally verify if a subject’s choice of the top block is frame-dependent rationalizable. So, we consider a logit model consisting of a subject’s choice of the top object as the binary dependent variable and of the above belief variables as predictors. To test for (frame-dependent) rationalizability of the top block, we must restrict attention to the sub-sample of participants whose beliefs are such that \( \mu_i^O \geq \mu_i^C \) and \( \mu_i^O \geq \mu_i^S \). Results from model [III] of Table 5 reveal a significant positive effect of the belief about others noticing the order of the blocks (\( \mu_i^O \)) on a subject’s choice of the top object. The model also indicates no significant effects of \( \mu_i^C \) or \( \mu_i^S \). This provides evidence in support of H2.b., which states that “if one believes that others are more likely to notice the objects’ order (rather than colors or shapes), then one is more likely to choose the top block”.

To conclude, the data provide strong evidence in support of both components of H2, validating our notion of rationalizability: the results seem to indicate that subjects did attempt to maximize a frame-dependent expected utility. Given this, the curious reader might wonder how variations in individuals’ awareness ultimately affect coordination. To address this point, we report below the coordination rates resulting from the observed distribution of individual choices; specifically, in keeping with previous studies of coordination, we report expected coordination rates
(as opposed to actual frequencies of coordination; see for example Mehta et al., 1994, and Crawford et al., 2008), computed at the session level.\(^{40}\)

As a reference group, we shall consider the case in which an All-Aware participant is assigned to a Baseline participant – *as per our experimental design* – which yields a 44.6\% coordination rate (averaging across all sessions). We then turn to the hypothetical case in which a Baseline participant is assigned to another Baseline participant, yielding a 50.2\% coordination rate (averaging across all sessions). Contrasting the full distribution of per-session mean rates of <All-Aware, Baseline> pairs with that of the <Baseline, Baseline> pairs, we conclude that such rates significantly differ from each other: \(N = 12\) obs., \(z = -2.201, p = 0.027\), two-tailed Wilcoxon signed rank sum test. Additionally, one can use a binomial test to verify whether (same-direction) differences between the coordination rates are due to chance, as opposed to treatment-induced variations in awareness. To that end, we note that <All-Aware, Baseline> pairs exhibit lower coordination rates than hypothetical <Baseline, Baseline> pairs in six out of six sessions, thereby confirming that variations in individuals’ awareness have an impact on coordination rates (\(N = 6, p = 0.031\), two-tailed). In brief, these tests say that (perhaps counter to common intuition) awareness of additional solutions can hurt participants in coordination problems.\(^{41}\)

We further note that our supplemental treatments enhance coordination relative to the main treatments (i.e., <All-Aware, Baseline> pairs): that is, <Color-Aware, Color-Aware> pairs exhibit a 47.9\% coordination rate; moreover, <Shape-Aware, Shape-Aware> pairs have a 50.4\% coordination rate, and <Order-Aware, Order-Aware> pairs have a 50.3\% coordination rate. Since each of the supplemental treatments comprised only two sessions, for the purpose of hypothesis-testing we shall contrast the distribution of per-session mean rates of the <All-Aware, Baseline> pairs with that of the supplemental treatments as a whole. Despite the small sample size, a Wilcoxon-Mann-Whitney test shows mild evidence of a significant increase in coordination rates when moving from our main treatments to the supplemental treatments: \(N = 12\) obs., \(Z = -1.922, p = 0.054\), two-tailed. Finally,

\(^{39}\) Note that coordination rates depend on individual choices and on a purely random element, that is, the random assignment of partners; it is clear that – given a relatively small sample size – such a random element is likely to bias coordination rates. “Thus, the actual frequency of coordination has no special significance; it is more appropriate to consider the expected frequency of coordination”, Mehta et al. (1994, p. 663), italics in original.

\(^{40}\) Such rates are obtained by calculating the probability that two participants pick the same object in a certain round (i.e., triplet) and session, given the observed distribution of individual choices; *per-session mean rates* are then obtained by averaging across all the rounds in a given session.

\(^{41}\) It is clear from our analysis of individual behavior that such a negative impact does not mean that All-Aware participants behaved more randomly; instead, it indicates that participants best-responded to incorrect beliefs.
note that all the coordination rates are higher than chance even in the absence of exogenous labels, as in the case of the All-Aware and Baseline treatments. So, subjects exploited natural frames.\textsuperscript{42}

VI. Concluding remarks

Our model suggests that different players are aware of different attributes of the strategy options, and hence use different frames. Given this, our model provides a rationalization of why differences in players’ frames may lead to differences in choice behavior. Our data find support for our notion of frame-depend rationalizability. In particular, participants’ choices vary across the Baseline and the All-Aware treatments, as a result of considering new frames. To the best of our knowledge, we are the first to present a coherent – theoretical and experimental – account of the effect of manipulating awareness (of alternative framings) on strategic behavior. In this regard, we note that previous lab experiments have made the case for heterogeneity in subjects’ natural frames. For example, Bacharach and Bernasconi (1997) confirmed the existence of individual differences in attribute perception and behavior. However, neither their design nor others have manipulated subjects’ awareness of alternative framings, thereby investigating if subjects can learn new frames (and thus rationally maximize an updated frame-dependent expected utility).

In this connection, we note that Bacharach and Stahl (2000) propose a boundedly rational version of a framing model that builds on level-k theory (Nagel, 1995; Stahl and Wilson, 1995; Camerer, Ho, and Chong, 2004; Crawford and Iriberri, 2007; Crawford, Costa-Gomes, and Iriberri, 2013). Bacharach and Stahl introduce some strong assumptions about both the structure of labels – which implicitly vary with players’ perceptual limitations – and the players’ depth of reasoning about others’ limitations. Specifically, Bacharach and Stahl assume that non-strategic level-0 types disregard any other players’ behavior, and pick each of their labeled strategies with equal probability. (E.g., suppose that the set of possible labels is \{x, y, z\}; given this, the authors posit that level-0 types play x, y, z with equal probability, regardless of how many objects are assigned each of those labels; Bacharach and Stahl, 2000, pp. 230-231.) Higher types then best-respond to the strategies of lower-level players, as is typical of level-k models. Here, if one were to use the Bacharach-Stahl model to formulate hypotheses in the context of our treatments, some ad hoc assumptions (about the labels associated with level-0 players) would be needed. Whereas the

\textsuperscript{42} In fact, one should view the above coordination rates in light of the improvement over 1/3 (i.e., the expected rate of random coordination), so that for example the comparison between 44.6% and 47.9% effectively reflects an improvement of 11.3 percentage points versus one of 14.6 percentage points.
Bacharach-Stahl theory remains indeterminate insofar as our All-Aware manipulation is concerned, we note that their model is easily applied to treatments in which labels are exogenously assigned. In particular, a direct implication of their model is that if each object is assigned a distinct label (e.g., “first”, “second”, “third”), then choices should not significantly differ from a discrete uniform distribution. This prediction is clearly falsified by the data from our Order-Aware treatment (see the right-most column in the upper panel of Table 2, p. 23).

It is worth noting that a different line of research on framing has compared behavior between games with identical payoff structures but distinct game labels (e.g., the “Community game” versus the “Wall Street game”; Kay and Ross, 2003; Liberman, Samuels, and Ross, 2004; Ellingsen, Johannesson, Mollerstrom, and Munkhammar, 2012). Experiments have also considered games with identical payoff structures but different strategy labels (e.g., “cooperate”, “defect” versus “out”, “in”; Andreoni, 1995; Larrick and Blount, 1997). What those studies have in common is the fact that the experimenter induces a distinct “normative frame” by evoking individualistic versus cooperative norms. There, the experimenter affects players’ beliefs by directing their attention toward the actions one ought (or ought not) to take in the context that is evoked by the labels employed: as such, the theoretical explanation of a normative frame rests on the players’ guilt aversion (Dufwenberg, Gächter, and Hennig-Schmidt, 2011). In this respect, we depart from the studies above because – unlike the case of normative frames – we focus on pure coordination games, thereby ruling out confounds due to social preferences. Relatedly, we argue that a guilt-aversion explanation of the impact of manipulating frames is inadequate for our case.

That said, our framework may have useful application to (“impure” coordination) problems incorporating an element of conflict as well.44 Understanding the perception-action link has implications for virtually any class of strategic problems: indeed, successful coordination or anti-coordination depends on the fact that players share some action representations. While game theory traditionally assumes that this is always the case (in that players’ strategy labels are exogenously given), everyday life teaches us that the strategies available to an individual are not always clear cut; in fact, individuals who first face a problem may be unaware of some action representations.

43 Another line of research studies how labels can influence bargaining games with communication (Isoni, Poulsen, Sugden, and Tsutsui, 2014). For early experiments on the impact of labels in zero-sum games see O’Neill (1987), Rapoport and Boebel (1992), and Rubenstein, Tversky, and Heller (1997). See also Crawford and Iriberri (2007).
44 Among the possible applications, we note that a design implementing alternative games may allow testing for lower order frame-dependent rationality. In fact, an advantage of our solution concept is that – since it is defined inductively – it also generates predictions for every finite level-k mutual belief in (frame-dependent) rationality, without anchoring beliefs on arbitrary level-0 players as per the previous literature.
References


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APPENDIX (for online publication)

Experimental Instructions and Screen Shots

**Baseline and All-Aware treatments**

NOTE. The screen shots below refer to both treatments, unless otherwise noted.

→ Screen shot 1:
Consent form.

→ Screen shot 2:
*Hello and welcome to a decision making experiment. You will receive a show up fee, and can also earn additional money. The additional payment will be determined by your own choices and those made by some other participant, according to the rules described below.*

→ Screen shot 3:
NOTE. Here three objects just slid toward the right; i.e., in the screen shot above the program has selected three blocks – one by one – and put them in a column, on the right-hand side of the screen.
→ Screen shot 6:

NOTE. The (three) available strategy options are now shown on their own.

→ Screen shot 7:

NOTE. The subject is prompted to enter one label per object.
→ Screen shot 8.i (All-Aware ONLY):

PART B. Please answer the following questions.
Recall that— in Part C of the experiment—you will be prompted to pick one object in order to coordinate with your partner. Now, prior to that we would like to know what you think of the other participants in this room. Please answer the following questions by moving the sliders to the desired percentages. Note that your partner will not be asked to answer these questions.

1) How likely do you think it is that the other participants have noticed the objects drawn by the computer program? Please move the below slider.

2) How likely do you think it is that the other participants have noticed the different colors of the objects? Please move the below slider.

3) How likely do you think it is that the other participants have noticed the different shapes of the objects? Please move the below slider.

→ Screen shot 8.ii:

Note:
At this point, we ask you to estimate how likely you think it is that the participants in this room (excluding yourself) will choose each of the objects. To express your estimates, please move the spokes to the desired percentages, and then press CONTINUE to proceed to the next screen.

Note: if at least one of your estimates differs by no more than 5 percentage points from the realized value, at the end of the study you will receive an additional payment of $0.25.

Click and drag the small circles to move the spokes.

NOTE. The spokes are initially arranged in such a way that each sector corresponds to one-third of the area. Within each sector there is a text box containing the label the subject entered in Screen shot 7.
NOTE. The labels next to each of the options are those entered by the subject in Screen shot 7.

In what follows you will go through 9 other rounds, where each round will involve the same type of tasks as the ones you have performed so far.

NOTE. Screen shots 4 to 9 are repeated.
**Color-Aware treatment**

This treatment is the same as the *Baseline*, with the exception of Screen shots 7-9, where exogenous – color – labels are shown. See example below.

**Shape-Aware treatment**

This treatment is the same as the *Baseline*, with the exception of Screen shots 7-9, where exogenous – shape – labels are shown. See example below.
**Order-Aware treatment**

This treatment is the same as the Baseline, with the exception of Screen shots 7-9, where exogenous – order – labels are shown. See example below.