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Learning to Notice Mathematics Instruction: Using Video to Develop Pre-service Teachers' Professional Vision of Ambitious Pedagogy

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Abstract: This study examines the development of a cohort of pre-service mathematics teacher candidates' noticing after they participated in a video-based course designed to develop their professional vision of ambitious mathematics pedagogy. We trace their development from the beginning to the end of the course, as well as three months after the course to investigate whether they developed a vision of ambitious mathematics instruction to frame their noticing of teaching. We propose different trajectories in candidates' noticing to gain insight into the nature and development of their professional vision of ambitious teaching. This study contributes to research on the construct of noticing and the design of learning environments to help prospective teachers cultivate expert ways of noticing mathematics teaching and learning.

Key words: pre-service teacher education, noticing, video, mathematics teaching

Research in teacher education in the last decade has drawn attention to an important skill for teaching - the ability to attend to and reason about teaching and learning - what is referred to as teacher noticing (Sherin, Jacobs, & Philipp, 2011). One reason is that it captures teachers' in-the-moment decision making, which relies on teachers being able to attend to what students are thinking and doing, and to reason about student ideas to make informed choices about how to proceed with a lesson (Ball & Cohen, 1999; Mason, 2002, 2011; Rodgers, 2002; Schoenfeld, 2011). This ability to notice is central to the types of teaching advocated by mathematics education reform initiatives that promotes developing students' procedural fluency, conceptual understanding, and mathematical reasoning, as well as their dispositions and identities as mathematics learners (National Council of Teachers of Mathematics [NCTM], 2000; National Governors Association Center [NGAC], 2010; National Research Council [NRC], 2001). Within this vision, teachers are called on to create discourse-rich environments where students' ideas are the center of classroom activity and students share, discuss, and reason together about mathematical ideas. In this model, the work of teaching involves attending closely to student thinking – what their work represents about their understanding, similarities and differences in their thinking and the development of their mathematical understanding in a lesson as well as over time. This is similar to what Schoenfeld (2011) calls "diagnostic teaching" (p. 463) – a model of teaching whereby teachers have well formulated, content-rich goals, they recognize that students have different ideas and understandings of the mathematical focus of the lesson, and they use these ideas to inform teaching decisions. Research suggests that when teachers attend carefully to students' ideas throughout a lesson, both teacher and student learning result (Franke, Carpenter, Levi, & Fennema, 2001). Thus, classroom environments that achieve the vision of reform become generative learning spaces for both teachers and students.

In this study, we use the construct of teacher noticing – both what teachers attend to and how they reason about events (van Es & Sherin, 2008) – to investigate the nature and development of teacher candidates' *professional vision* of ambitious mathematics instruction. Professional vision refers to the ability to observe and make sense of events, objects, and interactions that are aligned with the interests of a particular social group (Goodwin, 1994). We seek to understand whether teacher candidates who participated in a video-based course, *Learning to Learn from Teaching*, develop ways of looking at and reasoning about instruction that aligns with the vision put forth by the mathematics education community that is focused on elevating the rigor and responsiveness of instruction in secondary mathematics classrooms.

We are interested in secondary mathematics candidates' professional vision of teaching for several reasons. First, students' mathematics achievement and motivation declines as they enter middle and high school (Fredricks & Eccles, 2002; Jacobs, Lanza, Osgood, Eccles, & Wigfield, 2002; Köller, Baumert, & Schnabel, 2001). Research focused on how to support secondary teachers in adopting a more relational, responsive perspective to instruction may play an important role in helping secondary students have more positive experiences in mathematics in upper grades (Boaler & Greeon, 2000; Greeno, 2006). Second, with greater numbers of students enrolling in Algebra, a gateway for advanced mathematics learning, but fewer of them experiencing success (Stein, Kaufman, Sherman, & Hillen, 2011), learning to see teaching as an interactional activity that is largely informed by student thinking may be a crucial piece for improving student learning and persistence at this critical juncture. Third, a variety of standards documents (NCTM, 2000; NGAC, 2010; NRC, 2001) emphasize students engaging in mathematical reasoning to promote conceptual understanding, while also developing productive dispositions in the discipline. To accomplish this vision, teachers need conceptual frameworks to guide their instructional practice to attend more closely to student ideas, to identify ideas that are noteworthy with respect to the learning goal, and to navigate conversations that are mathematically substantive and productive for student learning. We view the construct of professional vision as a critical component for addressing these issues. By equipping prospective teachers with frameworks to help them learn to notice important elements of instructional interactions, as well as with experiences in which they dissect the work of teaching to understand what constitutes a pedagogy of ambitious instruction, we propose that they can develop a more robust professional vision of mathematics teaching and learning that can guide their instructional decision making.

Although a good deal of research on teacher noticing has focused on teachers' ability to attend to and make sense of student mathematical thinking (Jacobs, Lamb, & Philipp, 2010; van Es & Sherin, 2008), we expand the study of teacher noticing to examine if secondary mathematics pre-service teachers who participated in a video-based teacher education course develop a professional vision of mathematics instruction that accounts for the relationship between particular teaching practices, student learning, and mathematical content. We then investigate whether and how they draw on the vision of teaching they developed in the course to analyze their own instruction in the context of a portfolio assessment for teacher credentialing. Recognizing that developing expertise takes time and that there is more than one trajectory to competence (Kazemi & Hubbard, 2008; Lajoie, 2003; Schoenfeld, 2011), we also explore different ways that teacher candidates develop a professional vision of teaching that shapes their observations and sense-making of teaching by examining the relationship between shifts in noticing over the duration of the course and their noticing in the portfolio assessment they complete several months later. Such an analysis provides a framework for understanding how teacher candidates' professional vision of teaching influences what they notice in classroom interactions, as well as how they use the language of the discipline to decompose the work of mathematics teaching. To be clear, this study does not examine candidates' noticing during instructional practice, a subject we take up elsewhere (see Sun & van Es, under review). The focus here is on how teacher candidates come to see the work of teaching – to attend to salient features of classroom interactions and how they make sense of these features – to develop a professional vision of ambitious mathematics instruction. Thus, the central questions for this study include: a) Do secondary mathematics teacher candidates develop a professional vision of ambitious pedagogy as they analyze video records of teaching in the context of the *Learning to Learn from Teaching* course?; b) Do they draw on a vision of ambitious mathematics pedagogy they develop in the course to guide their noticing of their own practice several months after the conclusion of the course?; and c) What are the variations in the ways candidates notice instructional interactions over time and what might these differences reveal about developing a professional vision of ambitious mathematics pedagogy?

Theoretical Framework

This study is framed by research on professional vision and teacher noticing, as well as research on the teaching and learning of professional practice. Goodwin (1994) defines professional vision as "the ability to see and understand events that are answerable to the distinctive interests of a particular social group" (p. 606). Goodwin explains that experts in a domain use tools, language, and gesture to foreground some elements of their practice and background others as they apprentice newcomers to the profession. By doing so, the expert helps the learner dissect their practice, construct a language system for talking about the elements of their practice, and develop ways of reasoning about what is observed. Goodwin describes this process as highlighting and coding. *Highlighting* involves selecting those parts of an object of focus to draw attention to and *coding* refers to categorical schemes that are created to organize what is observed. The coding scheme becomes a way for novices to transform the complexity of what is attended to into particular categories, which provides a shared language for talking about observed phenomena. Thus, as newcomers enter the profession, they are apprenticed by more experienced others to learn to identify what is relevant in their work and to develop a language system and discourse for making sense of and reasoning about phenomena that are relevant to their practice (Gee, 2008; Goodwin, 1994).

In the last decade, research in teacher education has sought to understand the nature of teachers' ability to observe and make sense of classroom interactions (Kersting, 2008; Seidel & Stürmer, 2014; Sherin et al., 2011). This line of inquiry has been spurred by research that reveals important differences between novice and expert teachers: expert teachers have greater sensitivities to particular aspects of their practice and strategies for analyzing, using, and inquiring into these aspects in their work; novice teachers tend to offer general descriptions or judgments about what they observe, whereas, expert teachers reason about what they observe to make sense of these events with respect to broader issues in teaching and learning; and experts see meaningful patterns in their area of expertise and use what they observe to navigate and enact meaningful instructional interactions (Berliner, 1994; Erickson, 2011; Mason, 2002; Russ, Sherin, & Sherin, 2011; Seidel & Stürmer, 2014). In addition, research shows that teachers who have more sophisticated ways of looking at and making sense of classroom interactions show greater gains in student achievement (Kersting, 2008), suggesting that this is an important skill to cultivate in prospective teachers. Moreover, reform efforts in mathematics education require that teachers develop ways of attending to and responding to student thinking during instruction

(NCTM, 2000; NGAC, 2010; NRC, 2001). Central to this image of teaching is that teachers pose cognitively complex tasks to students, that they establish classroom norms where students articulate, explain, and justify their individual and collective thinking, and where they compare and contrast their ideas and reasoning for purposes of developing conceptual understanding (Franke, Kazemi, & Battey, 2007; Hiebert & Grouws, 2006; Stein, Engle, Smith, & Hughes, 2008; Stein, Smith, Henningsen, & Silver, 2009; Yackel & Cobb, 1996). These types of instructional interactions require that teachers attend closely to the interactions that unfold to gain insight into student thinking and to use what they learn about student thinking to inform teaching decisions. Finally, research advocates for teacher education to provide prospective teachers with conceptual tools that will support them in learning in and from their practice to guide future instructional decisions (Feiman-Nemser, 2001; Hiebert, Morris, Berk, & Jansen, 2007; Lampert, 2001). We propose that to develop routine and systematic ways of learning from practice entails learning to observe and make sense of classroom interactions guided by a professional vision of mathematics reform pedagogy.

Developing pre-service teachers' professional vision of ambitious pedagogy is not without its difficulties. Having been students in classrooms for over a dozen years, prospective teachers bring their preconceptions of teaching and learning to the teacher education experience (Kagan & Tippins, 1991; Lortie, 1975; Pajares, 1992). It is not uncommon for pre-service teachers to observe a handful of students leaning forward with hands raised, voicing their ideas, and taking notes and infer that *all* students are learning. Likewise, they might see a student tapping a pencil or another walking around the room and infer that these students are not paying attention. From their many years as students, beginning teachers enter the profession as *insiders,* already deeply familiar with the profession of which they seek to become a part (Pajares, 1992).

Developing a vision of teaching from the perspective of the student can be problematic. It may be that the student tapping a pencil is paying attention and that the student who takes careful notes does not understand the content (Ball, 2011). Because teachers' prior experiences and pedagogical commitments inform their observations and reasoning about instruction, it can be difficult then to shift one's focus to elements of classroom interactions that promote a pedagogy of teaching mathematics for understanding (Erickson, 2011; Schoenfeld, 2011).

Research on teacher noticing has made progress in documenting the processes for seeing and making sense of instruction. In particular, there is broad agreement that noticing consists of an ability to attend to noteworthy features of instruction, to reason about what is observed in meaningful ways, and to decide how to respond (Jacobs et al., 2010; Sherin, 2007; Sherin et al., 2011; Stürmer, Konings, & Seidel, 2012). Yet, there is less agreement about the content of teacher noticing. For instance, Seidel and colleagues (Stürmer et al., 2012) study of pre-service teachers' professional vision examined the nature and development of their generic pedagogical knowledge, specifically their attention to goal clarity and learning climate, because they have proven to be relevant to student learning (Seidel & Shavelson, 2007). Others have focused on learning to attend to the particulars of student thinking with respect to specific content, such linear functions or algebraic reasoning (Goldsmith & Seago, 2011; Walkoe, 2013), because it is in the work of teaching particular content that teachers need to navigate student ideas. Star and Strickland (2008) recognized this tension and developed a framework that captures a range of features relevant to mathematics instruction, such as tasks and discourse, while also capturing more generic categories (e.g. classroom environment) that reflect the features of instruction teachers typically observe. Yet, central to Goodwin's (1994) notion of developing professional vision is the idea that novices are apprenticed to learn to look at features of a profession that are valued by a social group. Being explicit about the vision of teaching to which teachers are being apprenticed to notice is necessary for investigations into this construct for teaching, as well as examining how particular learning environments create opportunities for pre-service teachers to develop new ways of looking at their practice.

In using the term ambitious mathematics pedagogy, then, we draw on research that is focused on creating and sustaining learning environments in mathematics classrooms where student work is the center of activity and the goal is to develop procedural fluency, deep and enduring mathematics understanding, and positive dispositions and identities as mathematics learners (Carpenter et al., 1997; Carpenter & Lehrer, 1999; NRC, 2001). In this vision, the work of teaching entails providing students opportunities to grapple with complex tasks and generate solutions, as well as to communicate their mathematical reasoning in small group and whole class contexts (Hiebert & Grouws, 2007; NCTM, 2000; Stein et al., 2009). This requires shifting roles for teachers and students, with students explaining and questioning one another and taking on increased responsibility for one another's learning (Hufferd-Ackles et al., 2004). In addition, teachers create classroom norms that cultivate presentation, argumentation and justification of mathematical ideas (Carpenter & Lehrer, 1999; NCTM, 2000), while also honoring the unique and valuable contributions that students brings to the learning setting (Boaler & Staples, 2008). It is this vision of teaching, promoted by national reform documents and the mathematics education research community (Franke et al., 2007; Lampert, Beasley, Ghousseini, Kazemi, & Franke, 2010; NCTM, 2000; NRC, 2001), as well as reflected in assessment systems for improving teaching (Darling-Hammond, 2006) which we ascribe and use as the foundation for a vision of ambitious pedagogy to which we seek to apprentice future teachers.

Learning to Decompose Classroom Interactions

Given this backdrop, research in mathematics education has made important progress in documenting ways to prepare beginning teachers to enact practices that achieve the vision of ambitious instruction (Franke et al., 2009; Ghousseini, 2009; Grossman, et al., 2009; Lampert et al., 2010, 2013). Grossman and colleagues (2009) study of how people are prepared for various professions identified three key concepts that underscore the pedagogies for preparing practitioners: using representations of practice to gain insight into and study the work of teaching, decomposing practice into its constituent parts, and engaging in approximations of practice that provide access to the core practices of a profession. Much of the work in mathematics education has focused on the third facet, developing prospective teachers' beginning repertoire – the techniques, skills, and approaches that when routinely enacted, can achieve the vision of ambitious pedagogy. However, Grossman, Smagorinsky, and Valencia (1999) underscore the importance of conceptual tools for informing instructional decisions. Conceptual tools are the principles, frameworks, and ideas about teaching and learning in the content area that guide decision-making. One of the dilemmas inherent in providing prospective teachers with a practice-based pedagogy is that in the work of teacher preparation the goals and intentions of enacting particular practices may be lost, resulting in what Brown and Campione (1996) refer to as lethal mutations. Like Cohen's (1990) depiction of Mrs. Oublier, teacher candidates may believe that they are enacting reform oriented teaching practices, yet without deep understanding of the purpose and goals for their practice guided by a professional vision of ambitious teaching the intent of particular pedagogical approaches may be lost.

We propose that a core component of teacher preparation include providing teacher candidates opportunities to decompose the work of teaching as captured in representations of practice to learn to identify the salient features of the profession, as well as develop a shared, precise language for characterizing this model of instruction (McDonald, Kazemi, & Kavanagh, 2013). We contend that, in the context of current efforts to improve mathematics teaching and learning, supporting beginning teachers to learn new ways of looking at and making sense of classroom interactions can help prepare them to adopt a more student-centered approach to mathematics instruction, as well as develop a student-centered frame for instruction (Levin, Hammer, & Coffey, 2009; Russ & Sherin, 2012).

Using Video to Support Pre-Service Teacher Learning

Although research suggests that over time teachers can develop expertise in noticing (Jacobs et al., 2010), video has proven to be a particularly powerful tool for helping prospective teachers learn new ways of noticing teaching practice (Santagata & Guarino, 2011; van Es & Sherin, 2002). Representations of teaching captured in video are useful for helping prospective teachers learn to decompose and use a common language for describing ambitious mathematics instruction (Hatch & Grossman, 2009; Sherin, 2007). By viewing video records of teaching, prospective teachers can develop shared ways of seeing "classroom discourse," "developing conceptual understanding and mathematical reasoning," and "equitable instruction" as they arise in classroom interactions. Moreover, the affordances of video offer prospective teachers opportunities to adopt analytic practices for viewing and discussing teaching (Santagata & Guarino, 2011; Sherin, 2007). Because of the permanence of video, prospective teachers can take time to study a classroom interaction together and contemplate a variety of explanations they may not have previously considered. This is in contrast to typical fieldwork experiences, where candidates typically observe an experienced classroom teacher alone and have few if any opportunities to explore the details of their observation with others to make sense of the work of teaching (Gomez, Sherin, Griesdorn, & Finn, 2008). Finally, video libraries can organize video segments in a variety of different ways, by classroom routines, the chronological sequence of a particular lesson or a collection of lessons over time, or student ideas related to specific content domains (Hatch & Grossman, 2009). In this way, prospective teachers can study a particular routine, such as teachers orchestrating productive discussions, or they can examine how classroom norms are established over time to create a community focused on student ideas.

Several studies show that prospective teachers can develop their observation and analysis skills of teaching in the context of a video-based course focused on these goals (Santagata & Guarino, 2011; Star & Strickland, 2008; van Es & Sherin, 2002). These studies find that pre-service teachers attend more closely to student ideas and the interaction between particular teaching moves and student learning. They also become more precise in their analyses – shifting from oversimplified generalizations to careful examinations of the specific interactions that unfold in a lesson and how those interactions influence students making progress toward to the learning goal. Yet, less of this research investigates candidates noticing and analysis of teaching after the conclusion of a course focused on this goal, nor does it account for variations in the ways that candidates see the work of teaching both when they begin and as they progress through their teacher education program. Kazemi and Hubbard (2008) argue that because of teachers varied experiences and contexts prior to and during their participation in a learning event, they will not take up opportunities that are afforded to them in the same ways. We investigate then not just if teacher candidates develop a professional vision of ambitious pedagogy, but also how they come to notice and analyze teaching in different ways over time.

Research Context: Learning to Learn from Teaching Project

This study takes place in the context of the *Learning to Learn from Teaching Project* as part of the teacher preparation program at a large western university. The goal of this project is to develop elementary and secondary candidates' ability to notice ambitious teaching and to systematically analyze instructional interactions through video analysis of teaching (Santagata & Guarino, 2011; Santagata & van Es, 2010). In the 2008-09 academic year, the *Learning from Teaching* course was incorporated into the first quarter of a three-quarter credential program. There were a total of 12 class meetings over a three-month period for this class. The first author was the instructor for the course. The class met once a week with each meeting lasting three hours. Candidates were concurrently assigned a field placement in which they observed a secondary mathematics teacher for one to two hours each week. The subjects of this study include one cohort of secondary mathematics teaching candidates (n=33)[1] who enrolled in a section of the *Learning from Teaching* course. To understand if this group of pre-service teachers drew on the ways of noticing teaching they developed in the course to analyze their own practice, we also analyzed data from a cohort of secondary mathematics teacher credential students from a previous year who did not participate in the course (n=12). The program grew substantially in size the year we collected data for the study, which explains the difference in numbers of participants in each cohort. Other than the addition of the *Learning from Teaching* course, the program curriculum was the same for both cohorts.

The design of the *Learning to Learn from Teaching* course for secondary mathematics teacher candidates (referred throughout as *Learning from Teaching* course) was informed by research on mathematics teacher education, teacher noticing, and lesson analysis and reflection. First, we drew on frameworks from mathematics education research to draw attention to the centrality of student thinking and ideas, as well as how teachers enacted cognitively demanding

[1] Our analysis includes 25 of the 33 cases. Two students were dropped from the analysis because data was missing for the post video task, two other students were dropped because they left the program before completing the PACT Teaching Event, and four other candidates' data was not included because we could not access the files on their PACT Teaching Event CDs.

mathematical tasks and created discourse-rich classroom environments focused on student ideas (e.g. Hufferd-Ackles, et al., 2004; Stein et al., 2009). Second, research on noticing and reflection emphasize attending to the details of interactions, and particularly student thinking as it unfolds in a lesson, and using these details to make sense of and offer various explanations about the interactions that occurred (Rodgers, 2002; van Es, 2011). Finally, we used Hiebert and colleagues (2007) approach to systematic analysis of teaching to provide a structure for examining the influence of particular teaching moves on student learning. This process consists a four part cycle of defining clear learning goals, gathering evidence of student thinking and learning toward the learning goal, examining the teaching with respect to the evidence of student learning, and then offering suggestions. Our work focused primarily on the relationship between the second and third phase of this model. Taken together, we think of these as conceptual tools for analyzing teaching (Grossman et al., 1999). The collection of research-based frameworks related to mathematics education and teacher noticing inform the *substance* of what teachers attend to and how they reason about these events and interactions, and the framework for analyzing teaching provided a *structure* for engaging in evidence-based analysis of instruction.

The *Learning from Teaching* course consisted of three phases. The main goal of the first phase was to attune candidates to student thinking and to examine different ways students thought about and understood the mathematics (Lampert, 2001). In whole class and small groups, candidates viewed video segments of individual students solving mathematical tasks and sharing their reasoning behind their solutions (Walbert, 2001), as well as student thinking that arose during classroom interactions (Boaler & Humphreys, 2005). Class discussion centered on what the candidates found noteworthy and surprising about student thinking (e.g. a student using a strategy with which they were unfamiliar or a student making errors solving that candidates did not anticipate). During this phase, the instructor emphasized the importance of attending closely to what students said and did and using their descriptions to infer what students seemed to understand about the content. The culminating activity for this phase involved each candidate identifying a student in the field site, conducting and recording an interview using several pre-selected problems, and then transcribing and analyzing student thinking.

In the second phase, the focus shifted from a sole focus on student thinking to analysis of pedagogical practices that influenced learning. We drew on research-based frameworks to examine the nature of mathematical tasks, teacher questioning, classroom discourse and students' roles in constructing mathematical understanding, and classroom norms that position students as authors of mathematical ideas (Boaler & Humphreys, 2005; Herbel-Eisenman & Breyfogle, 2005; Hufferd-Ackles et al., 2004; Stein et al., 2009). Throughout this phase, the focus of activity was on viewing and discussing videos of practicing teachers (Boaler & Humphreys, 2005; Carnegie Foundation for the Advancement of Teaching, 2006) and using these frameworks to decompose the work of teaching and ascribe a language to the practices represented. As in the first phase of the course, in both whole class and small group contexts, the candidates viewed and analyzed the videos with a focus on developing a shared account of the events that unfolded and drawing inferences about student thinking and the influence that particular teaching moves had on student learning. Hiebert and colleagues' (2007) four-part approach to analyzing teaching and learning provided a structure for examining the influence of teaching on student learning.

Finally, in the third phase of the course, the candidates observed lessons of their host mentor teacher in the field site using frameworks from the first two phases to analyze a series of lessons together. In addition, in pairs, they co-designed a mini-lesson specifically focused on eliciting and gaining insight into student thinking and each enacted and videotaped this lesson with a small group of students from their field site. When possible, candidates observed each other teaching the lesson. They submitted a written analysis of their lessons as captured in video using the frameworks from the course to guide their observations.

Data Collection

Data for the study include a pre-post video analysis task administered in the *Learning from Teaching* course and candidates' analyses of their own teaching in the context of the Performance Assessment for California Teaching - Teaching Event (referred throughout as the PACT Teaching Event). The video analysis task consisted of viewing two video clips, each between 3-6 min long, during the second and final class meetings. The clips came from published materials (Seago, Mumme, & Branca, 2004; The Concord Consortium, 2005) and were selected because they represented a range of student ideas, students engaging in classroom discourse, the teacher and students interacting in different settings (e.g. a whole class and a small group setting), and students working on cognitively demanding tasks. In completing the video analysis task, the candidates viewed the first video clip and then responded to four prompts: a) What do you notice?; 2) Describe what's going on in the clip; 3) How did the teaching support student thinking and learning?; and 4) Was anything else noteworthy? They were then shown the clip a second time and were able to modify or add to their responses. The same format was followed for viewing the second clip. The candidates had one hour to complete this task.

The second data source comes from the PACT Teaching Event, one of several teaching portfolio assessments used in California that candidates are required to complete and pass to obtain licensure. It assesses candidate competencies in five areas: context for learning, planning, instruction, assessment, and reflection. For each area, candidates respond to 3-8 prompts. The portfolio consists of lesson plans for a 3-5 day teaching sequence with accompanying assessments and student work; one video segment of 15 continuous minutes or two combined segments of 10 minutes each, along with commentary and analysis of video segment(s); and reflections for the entire lesson segment. Because we are interested in understanding what candidates attend to and how they reason about what they see when they observe teaching, we focused our analysis on the PACT Teaching Event task that best captured candidate noticing. Task 3 is dedicated to candidates demonstrating their instruction with a self-selected video from their own classroom teaching and commenting on the instruction represented in this video. We identified two questions in Task 3 that we thought captured candidate noticing: 1.) *How did you further the students' knowledge and skills and engage them intellectually in understanding mathematical concepts, procedures, and reasoning*? and 2.) *Describe the strategies you used to monitor student learning during the learning task shown in the video clip. Cite examples of what students said or did in the clips that indicate progress toward accomplishing the learning objectives.* In the next section, we describe our analysis for each set of data.

Data Analysis

Data analysis focused on understanding if candidates developed in their noticing of ambitious pedagogy in the context of the course and if they used the frameworks that were promoted in the course to analyze their own teaching in the context of the PACT Teaching Event. Analysis occurred in several phases. In the first phase, we investigated whether the candidates shifted in their noticing from the beginning to the end of the course. In the next phase, we examined development of noticing of ambitious teaching from the end of the course to the PACT Teaching Event and also compared the responses of the candidates in the *Learning from Teaching* course to a cohort who did not enroll in the course. Finally, in the third phase, we conducted a qualitative analysis to examine differences in the nature of candidates' noticing and professional vision. We describe each phase of analysis below.

Phase 1: Development of Candidate Noticing in the *Learning from Teaching* Course.

The first phase of analysis involved investigating the development of teacher candidates' professional vision from the beginning to the end of the course. Our coding scheme was informed by the research on professional vision, teacher noticing, systematic reflection of teaching, and lesson analysis. We reviewed this literature to identify common dimensions for characterizing what teachers observe and how they analyze teaching and learning (Davis, 2006; Jansen & Spitzer, 2009; Morris, 2006; Santagata & Guarino, 2011; Star & Strickland, 2008; van Es & Sherin, 2008). At the same time, we were aware of the goals of the video-based course and took into account particular ways of observing and analyzing teaching that the course promoted. With both the literature and the course goals in mind, we identified several dimensions as central to our framework for analysis: what pre-service teachers observe and highlight as noteworthy, the extent to which they shift from describing to interpreting what they observe, the connections they make between events and interactions they highlight, and the level of specificity with which they observe and reason about teaching and learning. We were particularly interested in whether the candidates attended to elements in the video segments that were the subject of focus throughout the *Learning from Teaching* course, including student thinking, task quality, teacher questioning, classroom discourse, and norms for making student thinking visible. In other words, we wanted to understand if the candidates improved in their attention to and sense-making of these elements in the video segments they viewed.

We also wanted to capture the extent to which they became more descriptive and interpretive in their responses by attending to the details in the clip and by making connections between what they observed. The course sought to help teacher candidates shift from offering general, oversimplified observations, what is typical of novice teachers (Erickson, 2011), to providing more specific observations in their analyses and unpacking the details of the events in a lesson (Rodgers, 2002). Additionally, we wanted the candidates to begin to examine classroom episodes as interactions between the teacher, learners, and content in the particular learning context because research suggests that opportunities for worthwhile learning arise through the interaction among these elements and not in any one area alone (Ball & Cohen, 1999).

With this framework in mind, we randomly selected a sub-set of four cases from the pre and post data sets, for a total of eight pre analysis task responses and an additional eight post analysis responses, and reviewed their responses to each question. Through an iterative process, we constructed a three-level coding scheme related to six different dimensions (Miles, Huberman, & Saldaña, 2014). Four of the six dimensions capture the substance of their noticing, including *Mathematical Task, Student Thinking, Pedagogies for Making Thinking Visible,* and *Classroom Discourse*. The two other dimensions, *Specificity* and *Making Connections*, capture the level of detail and integration of their observations. To be clear, we did not enter the analysis with a pre-established coding scheme. Instead, our initial reading of the cases was informed by research and by the goals that were established for the course, which informed the broad categories of focus for our analysis.

Once we defined the categories, we returned to the subset of cases and examined similarities and differences in the candidates' responses with respect to the six categories. Using the constant comparative method (Glaser & Strauss, 1967), we then developed a three-level framework to capture variations we observed in the data. This approach is consistent with other research that examines the nature and development of teacher noticing and lesson analysis (Davis, 2006; Santagata & Guarino, 2011; van Es & Sherin, 2002; van Es, 2011). In particular, for the four dimensions that capture what the candidates observed, we noted differences in the level of detail and inferential nature of their responses. Thus, a Level 0 for each dimension reflects little or no attention to the elements of focus in the course (e.g. no mention of the mathematical task or student thinking) and an ambiguity with respect to that topic; Level 1 referred to responses that were descriptions or evaluations with respect to the category of focus, and Level 2 captured an interpretive, sense-making stance toward what was observed. Similar to the features of what was noticed, we developed a three-level coding scheme for the dimensions *Making Connections* and *Specificity*. For these two categories, Level 0 constituted no linking of events and vague commentary that was not particular to the clip; Level 1 responses began to make connections but also treated events as isolated from each other and began to include specific details with some vague language; and Level 2 consisted of clear links between observed events and interactions, as well as specific, detailed and elaborate responses, including specific quotes and points in time in the clip with references to events that were particular to the clips viewed. The complete framework, with definitions and examples can be found in Appendix A.

To ensure that the research team had a common understanding of the coding framework, we randomly selected an additional four candidates' responses to both clips and independently coded them in terms of the six categories. We also wrote analytic memos to justify coding decisions (Miles et al., 2014). We then met to review and discuss our individual coding and our memos. This process is consistent with qualitative research methods that recognize that working with participants' responses requires that research teams engage in cycles of analysis to develop shared lenses for data analysis (Weston et al., 2001). In our discussions, we identified two categories that were difficult to distinguish from each other, *Pedagogies for Making Thinking Visible* and *Classroom Discourse*. Thus, we returned to the literature and the cases we had reviewed until we gained confidence that we had a shared understanding of these categories, how they reflected the goals of ambitious mathematics pedagogy, and how particular events and interactions in classroom interactions represent each category.

Next, we divided the remaining 17 cases among the first three authors who then individually coded the candidates' responses to both clips for the pre and post video analysis tasks, assigning a score of 0, 1, or 2 for each of the six dimensions. To ensure inter-rater reliability, the second and third author randomly selected a subset of these cases and coded them individually and then met to discuss their coding. Inter-rater reliability across the six categories was 93%. Any disagreements were resolved through discussion until consensus was reached. To compare their noticing from the pre to post analysis task, we averaged the scores they received for each of the six dimensions for each clip, to arrive at an overall score for each candidate. We then conducted a one tailed t-test to examine significance of the differences we observed from the pre to post video analysis task. We performed a one tailed t-test because we hypothesized that teachers would positively change from one time point to the next, thus a one tailed test was used as there was a clear direction hypothesized.

Phase 2: Development of Candidate Noticing from the *Learning from Teaching* Course to the PACT Teaching Event.

In the second phase of analysis, we used the coding framework we developed in the first phase to code the candidates' responses to the two questions in Task 3 of the PACT Teaching Event. The first author coded the PACT Teaching Event responses for the cohort of candidates who enrolled in the *Learning from Teaching* course and the fourth author coded the PACT Teaching Event responses for the comparison cohort (n=12). The first author randomly selected a subset of these cases and coded them to ensure inter-rater reliability. The two coders had 85% agreement. Again, any disagreements were resolved through discussion until consensus was reached. To compare the *Learning from Teaching* participants' noticing from the post analysis task to the PACT Teaching Event, we conducted a t-test to examine significance. We also conducted a one tailed t-test to examine differences in noticing between the two cohorts.

Phase 3: Investigating Candidate Noticing and Professional Vision Over Time.

The third phase of analysis was more qualitative in nature. After we coded the three data sources with this framework, we observed that at the aggregate level, the candidates shifted in ways we hypothesized from the pre to post task, and that they sustained this shift three months later in the context of the PACT Teaching Event. However, having reviewed their written responses and our own memos, it was clear that the development of their noticing was more complicated than this analysis revealed. That is, the candidates appeared to enter the course with different noticing capacities and they did not all shift in the same ways from the beginning to the end of the course or draw on their ways of professional vision of ambitious teaching in the same ways in the context of the PACT Teaching Event. This raised questions for us about differences in the development of candidate noticing, as well as about what these differences reveal about developing a professional vision of ambitious pedagogy. We then returned to the data and further elaborated our analytic memos to capture the qualitative nature of their noticing in each context and how they appeared to shift in their noticing over time (Miles et al., 2014). These analytic memos were again informed by the vision of mathematics teaching and learning to which they were being apprenticed, as well as by research on systematic analysis of teaching (Carpenter & Lehrer, 1999; Hiebert et al., 2007; Hiebert & Grouws, 2007; Hufferd-Ackles et al., 2004; NCTM, 2000; NRC, 2001; Rodgers, 2002; Stein et al., 2009). More specifically, we reviewed their responses and looked to see if in their writing they used language to highlight and analyze mathematics interactions in ways that reflect the values of this community, if they were more precise in their depictions of the events that unfolded, if they were more analytic in the ways they approached their observations, and if they linked observations of teaching practices, student thinking and learning, and content. Through an iterative process of reviewing the data and memos, we investigated variations in the development of their noticing of ambitious pedagogy over time. We report our findings below.

Results

Data analysis revealed three important results. First, participants in the *Learning from Teaching* course shifted in their noticing of teaching and learning from the beginning to the end of the course. In addition, the participants in the course sustained this shift in the PACT Teaching Event context. Finally, qualitative analysis of participant responses revealed important variations in the ways they came to notice teaching over time. Each of these findings is elaborated below.

Development of Candidates' Noticing in the *Learning from Teaching* Course

We first sought to understand if candidates developed a professional vision of ambitious pedagogy in the context of the course. We used the pre and post video analysis task to investigate what the candidates attended to at the beginning and end of the course and how they made sense of what they observed. We used the construct of teacher noticing to infer whether the candidates developed a professional vision of mathematics instruction that had been advocated through the course. We were particularly interested in understanding if, over time, the course participants attended to elements of ambitious mathematics pedagogy - the nature of the mathematical tasks, student mathematical thinking, pedagogies for making thinking visible, and norms for classroom discourse – and adopted an interpretive stance toward the analysis of what they observed. In addition, we examined whether their analyses became more precise in characterizing what they noticed and considered the relationships between what they observed and how they interpreted these interactions.

Using a one tailed t-test[2] to examine differences in candidates' noticing on the six dimensions from the beginning to the end of the video-based course reveals that in all six areas, there was a significant, positive shift in candidates' noticing ($p < .001$) (see Table 1).

Table 1 *Average Scores by Cohort for Pre and Post Video Analysis Task and PACT Teaching Event Responses*

	Pre	Post	Change Pre to Post	LfT PACT-TE	Non-LfT PACT-TE	Cohort Differe
Overall	0.70	1.37	0.67***	1.43	0.8	.62**
Math content	0.70	1.58	0.88***	1.56	1.17	.40+
Student Thinking	0.46	1.2	0.74***	1.28	0.75	.53*
Ped Make Thinking Visible	1.04	1.44	0.40***	1.56	0.67	.89***
Discourse	0.72	1.18	0.46***	1.16	0.5	.66**
Specificity	0.72	1.68	0.96***	1.56	1.08	.48+
Make Connections	0.56	1.14	0.58***	1.44	0.67	.77**

Note. + $p < .10$, * $p < .05$, ** $p < .01$, *** $p < .001$.

When we compare the averages from the pre to the post task, combining the six dimensions into one overall score, the candidates' overall average in the pre-video analysis task is a .7 and in the post-analysis task was 1.37. This means that early on, as a group, the candidates attended to general ways that the teachers in the clips made student thinking visible, with minimal attention to the mathematical content, student thinking, and classroom discourse. They offered vague descriptions of what they observed, and made few connections between elements they highlighted. By the end of the course however, they highlighted elements of ambitious pedagogy and their accounts of what they observed became more specific and detailed. At the same time, as a group, they began to draw inferences about what they noticed (e.g. inferring the relevance of a math task for achieving a learning goal or inferring student understanding from

[2] To test for robustness of results, a two tailed t-test was also run and results were consistent with the one tailed test.

analysis of their errors, confusions, explanations). They not only observed these elements but they sought to understand how these features functioned to achieve the vision of ambitious pedagogy. Finally, they began to make connections between the features they observed and became increasingly more specific in their analyses.

Development of Candidates' Noticing in the PACT Teaching Event

We also investigated if the candidates from the *Learning from Teaching* cohort framed their noticing and analysis of their own teaching in the context of the PACT Teaching Event in terms of the professional vision of ambitious teaching promoted in the course. When we compared their noticing from the end of the course in the post video analysis task to their noticing of teaching in the PACT Teaching Event, we found that the candidates in the course sustained the same levels of noticing that they demonstrated at the end of the course (see Table 1). A comparison of the overall average scores and the average scores by dimension reveal little difference between the averages on the post task and the PACT Teaching Event. Like the post video analysis task, in the PACT Teaching Event context, the candidates sustained a descriptive stance with respect to the dimensions of ambitious pedagogy emphasized in the course, but they also began to interpret and make connections between events they observed. While the differences in the averages per dimension from the post video task to the PACT Teaching Event context were not statistically significant, we did find a marginally significant difference (one-tailed t-test, p value = 0.08) for the dimension, *Making Connections*. This further supports our conjecture that, in the PACT Teaching Event context, the candidates began linking student thinking to their teaching and examined those elements with respect to the mathematical goals of the lesson. This marginally significant shift suggests that they were now seeing these elements as integrated and connected in a lesson.

Finally, we compared this cohort's performance on the PACT Teaching Event to a cohort of candidates who enrolled in the same credential program two years prior, but were not offered the video-based course. Table 1 shows that for each category of analysis, there was a highly statistically significant difference between the two cohorts on all dimensions, except for one, Classroom Discourse.

Taken together, we see that as a cohort, the candidates who participated in the *Learning from Teaching* course began to develop a professional vision of ambitious pedagogy. They came to attend to the particulars of the mathematical content and task, student thinking, pedagogies for making thinking visible, and classroom discourse. Their observations became more specific and interpretive over time, suggesting that they were seeking to understand the meaning behind what they observed and making connections between features of instruction that stood out to them. The fact that they shifted in the context of the course is consistent with other research that shows that teacher candidates who participate in a course designed to help them learn to observe and analyze central features of classroom interactions develop their noticing and analysis skills in the context of a course (Santagata & Guarino, 2011; Star & Strickland, 2008). Less of this research examined, however, if they sustained these changes after the conclusion of the course, and the results of our analysis suggest that the cohort we studied drew on their professional vision of mathematics teaching and learning to notice and analyze their own teaching practice several months after the conclusion of the course.

Examining Variations in Noticing of Teaching

Though these results are encouraging, we propose that an analysis by cohort does not reveal the full complexity of developing new ways of observing and making sense of teaching practice. Because we had reviewed their responses at each point in time, we were aware that the candidates' ways of noticing varied in different ways. Moreover, with an overall score of 1.37 in the post analysis and 1.43 in the PACT analysis, it appeared that the group was making progress in developing a professional vision of ambitious pedagogy yet there was more room for development either across dimensions or in different groups of candidates. Thus, we returned to the data to gain a more robust picture of how these candidates shifted in their ways of noticing teaching and learning. Using the analytic memos, along with the overall average scores for each candidate for each task, we identified four different approaches candidates used to notice teaching and learning. We focus our discussion on the variations in the PACT Teaching Event because we conjectured that we would observe greater differences in their noticing in this context for two reasons. First, they were viewing videos of their own teaching; thus, the content of their videos likely shaped what they observed and how they analyzed their instruction, unlike the pre-post task where they all watched the same clips. Second, completing the portfolio assessment task several months after conclusion of the course allowed for a range of issues and experiences to come into play that likely influenced how they viewed teaching, such as their field placements and experiences in other courses. Understanding how these informed their professional vision of teaching and their noticing of instructional interactions is beyond the scope of this analysis. Here, we focus on characterizing the variations in their professional vision of mathematics instruction as reflected in their noticing and analysis of their own teaching.

Using Framework Terminology.

Data analysis revealed that in the context of the PACT Teaching Event, five candidates used the framework of ambitious teaching that was promoted in the course to highlight features of instruction. That is, they used the language of the framework to define events and interactions that stood out to them, such as the mathematics and the task, student thinking, pedagogies for making thinking visible, and classroom discourse. For example, one candidate, Candace, when reflecting on her teaching in the PACT Teaching Event, wrote: “Both video clips include examples of inquiry. When a student is called upon and does not know the answer, I use inquiry to facilitate the student's thinking through the problem and further their understanding of the topic." Candace highlighted in a general way how she provided opportunities for student thinking to become visible during the lesson. Similarly, another candidate, Nida, highlighted several practices she used to promote the development of a discourse community, such as having students explain their thinking, solve problems in multiple ways, and compare methods to convince one another. She attended to student thinking as it arose during instruction. For instance, she wrote, "It is easy for students to get confused between the sum of the interior angels and the exterior angles." However, like Candace and others in this group, her observations were more general in nature. Thus, these candidates appeared to adopt the language of ambitious teaching to highlight events, yet they did not provide the level of specificity to understand how these types of events arose in their teaching or how they interacted to support students in making progress toward the learning goals.

Exploring Details of Ambitious Instruction.

Like the candidates in the previous group, four other candidates used the framework of ambitious teaching to highlight features of instruction to inform their analyses of instruction. However, these candidates also focused on the details of the interactions, providing more specific and elaborate observations and analyses. The distinction between the two groups is that this group appears to be getting inside the features to understand how they played out in their instruction. For example, when writing about her own teaching, Hailey wrote that the purpose of the task was for students to analyze two diagrams – one was a circle with a number of inscribed angles and another was a circle with a number of non-inscribed angles – to generate a definition of an inscribed angle. She further explained that this task challenged students' abilities to compare and contrast geometric figures, analyze them and then define and understand the essential characteristics of inscribed angles. This more elaborate explanation suggests that she not only attended to the task but also how the task quality afforded students the opportunity to construct a collective definition through comparison and analysis in small and whole class discourse, addressing particular dimensions of high quality tasks that were discussed in the course.

Hailey's analysis also suggested that she had a more robust vision of the role that students' ideas play in creating a discourse community:

> I used the worksheet that was displayed on the board and filled in the table using students' measures. We used the data that students got as a result of their work. It was not given to them. Students made conclusions based on their own results. Instead of being told, they took an active part in a discovery of the mathematical relationship.

This more elaborate depiction of how she used student thinking during instruction suggested that she took up an important idea about developing a mathematics discourse community – that eliciting student talk is not solely for the teacher to assess learning but also concerns positioning students as sources of mathematical ideas. Hailey also described how she created a learning context to allow student thinking to be made public. She wrote, "Talking is making students' thinking and understanding visible. When students talk to each other, I had opportunities to listen to their conversations and monitor if they were discovering important relationships and to address their misconceptions." Statements like these that were focused on features of the task, as well as particular dimensions of the classroom discourse that supported student thinking and placed them in positions to be authors of their learning, suggest that the candidates in this group were not only drawing on frameworks of ambitious pedagogy to characterize what they observed, but also using the detailed observations to understand how they play out in the particulars of their practice.

Blending Visions of Teaching to Analyze Instruction.

Another group of five candidates appeared to use an alternative approach to observe and analyze their instruction. Like the second group, these candidates focused on the detailed interactions that arose to make sense of their teaching and student learning. In addition, this group's responses were more interpretive, in that they moved from detailed descriptions to seeking to make sense of the meaning of these features for their instruction. However, these candidates appeared to blend different frameworks of mathematics instruction when analyzing their own teaching in the PACT Teaching Event.

For example, attention to student thinking was a core component of the framework for ambitious instruction and in particular, the importance of attending carefully to student ideas to make sense of their thinking. This group demonstrated attention to student thinking with respect to concepts, procedures and reasoning, and they also drew inferences about what their observations revealed about their understanding. Jacey, for instance, explained that she designed the worksheet students used so that it could become a resource to help students explain mathematical concepts in a coherent way. She also pointed out that she had students explain "their how's and why's" because she wanted them to have a reason for using certain steps to enhance their conceptual knowledge and mathematical reasoning skills. However, when analyzing the details of student thinking, the focus of their analysis was typically related to student errors, mistakes and correct or incorrect answers. This excerpt from Jacey's response exemplifies this focus:

> When doing the whiteboard activity, students were able to practice and understand the procedures…they were able to see where in the procedure they misunderstood. For example, at 2:53 in the video clip, we went over what to do with the exponents; to add them and make sure the powers with negative exponents are switched to the other side. They were able to practice many problems and receive guidance from those sitting around them, me, their partner, or from the other whiteboards. If they

> did not understand a specific procedure, we would go over it and explain why it was wrong.

Similarly, when analyzing how their teaching supported learning, the framework of mathematics instruction that operated around their analysis was mixed. In some cases, they used language specific to pedagogies for making thinking visible and developing a discourse community to examine how their teaching influenced learning. Abigial, for example, explained that she created a lesson that allowed students to explore mathematical ideas and to explain and clarify the concepts in small group discussions that they had been exploring. Yet, as she provided detailed accounts to support this observation, she used language that was not consistent with this perspective. In describing her efforts to monitor learning, Abigail wrote:

> I used this time to walk around the classroom and monitor student work, and checked for their understanding based upon the steps shown on their paper… As part of helping students further their knowledge and skills during this process, I attended to the students' questions and addressed any mistakes in their work.

Though Abigail created a context for students to discuss their ideas with each other and explain their findings to each other, when she interacted with them, the focus was on fixing mistakes and ensuring students reached a correct answer. Thus, these candidates appear to be blending different visions of teaching – one vision focused on putting the work in students' hands, letting them work through the mathematics and share their thinking publicly and another in which the teacher is more directly in control of the lesson and guides students to an answer and maintains student engagement. We do not intend to suggest that understanding procedures or engaging students in learning are not important dimensions of mathematics learning or that teachers do not have a central role in monitoring and facilitating the instructional interaction. However, these candidates' analyses of their teaching suggest that they are taking up elements different frameworks to make sense of their instruction and that they do not see the consistency or inconsistency among these frameworks for achieving the vision of instruction advocated by the mathematics education community.

Using a Vision of Ambitious Teaching to Systematically Analyze Instruction.

Finally, data analysis revealed that the remaining 12 candidates used the substantive frameworks from the course to highlight and interpret particular features of the classroom episodes they observed and that they made connections between the features they observed in systematic ways. In particular, as they viewed their own teaching, these candidates demonstrated increased attention to the mathematics, student thinking, classroom discourse and pedagogies for making student thinking visible, such as having students explain and justify their reasoning or creating and sharing different strategies and mathematical representations. In addition, they used what they observed related to student thinking as evidence to draw inferences about the effectiveness of their teaching, making connections between elements they observed and students' substantive mathematics learning. Finally, their depictions of what they observed were specific to the events that arose in the video and their analyses of these events were more elaborate. These findings suggest that this group developed a professional vision of mathematics instruction that informed their noticing of noteworthy mathematical moments and interactions and provided a structure for making sense of what they observed. The following excerpts from one candidate, Angela's, analysis illustrates this vision of ambitious teaching.

In Angela's PACT Teaching Event response she drew on the frameworks from the course to frame her analysis of teaching. When explaining how she monitored student learning, she wrote:

> During this clip a major confusion was brought up, which was when to use an open or solid dot when graphing an inequality. It quickly became apparent that many students were struggling with the same misunderstanding when they were asked to share their thought about the dot by showing thumbs up or down. A quick poll of the class demonstrated there was a mix of ideas. At that point I decided to take some

> time to discuss everyone's confusion before moving on. In the beginning of the discussion, I asked if anyone could share when to use an open or solid dot, and one student volunteered to share his thinking. When referring to a solid dot, he explained that this is used "when you have that line under the inequality sign" (Clip #2, 00:55). I then asked for clarification of what "*that line*" represented, and many students knew and responded that it means "or equal to." From this occurrence I was able to determine that the students knew the academic vocabulary and meaning of the mathematical symbols, but some continued to struggle with how the symbols applied to graphing the inequality.

In this excerpt, Angela attended to the particular mathematics and the students' thinking of the mathematics. She also provided a detailed account of the interaction that unfolded to illustrate how she monitored student thinking during the lesson. This detailed account allowed her to then infer how students came to understand the mathematics. In addition, she highlighted how she learned something about student thinking through her monitoring efforts and was then able to use that to pursue student ideas. As she continued to comment on how she monitored student learning, she wrote:

> Later on when discussing the second problem on the board, the students were asked to clarify why the graph should include a solid dot at the 6. Luis explained that the dot on the 6 should be solid because, "*x* could be…. equal to 6" and furthered his explanation by saying, "6 could be a solution" (Clip #2, 06:08). When asked to clarify how he knew that 6 could be a solution, Ashley helped out by adding that you could tell because the inequality sign was "greater than *or equal to.*" I intentionally had the students explain their ideas and thinking because this helped me monitor the class' understanding, solidified the participants' learning, and gave the rest of the class a chance to hear an explanation that did not come from the teacher.

Angela continued to draw from the framework of ambitious instruction to highlight important features of instruction and use evidence-based reasoning to analyze her instruction. First, like the previous example, she honed in on a particular interaction and student idea to understand how the students were thinking about the mathematics. She also noted the importance of students explaining their thinking and how such a teaching move positions students as knowers. Finally, she provided an instance of students supporting each other's learning to illustrate her efforts to create an environment where students communicate their mathematical thinking.

These excerpts characterize the 12 candidates' ways of noticing. In the PACT Teaching Event, they attended to features of instruction that reflected the goals of ambitious mathematics instruction, and they used evidence of student thinking to make claims about how particular teaching moves that were characteristic of this vision of instruction supported student thinking and learning. We view this group as integrating the frameworks provided in the course to make sense of their practice by using a systematic approach to analysis – linking evidence of student learning as it arose in instruction to particular teaching moves – guided by a vision of mathematics teaching where student ideas, thinking and participation are central to the activity, where the students are part of a collective that is working together to promote each others' learning, and where the teacher enacts strategies to elicit student thinking and attends to, reasons about and responds to student ideas during instruction. We now turn to discuss these results.

Discussion

We sought to understand if participation in a video-based course supported teacher candidates in developing a professional vision of ambitious mathematics pedagogy. We used the construct of teacher noticing – both what teachers attend to and how they reason about these events - as a way to capture the substance of teacher candidates' professional vision. We conjectured that by providing candidates with discipline specific frameworks to guide their attention to particular features of instruction, as well as a structure for making sense of what they observe, they would develop a professional vision of mathematics instruction focused more squarely on students and their learning. Consistent with other research, we found that participation in the video-based course specifically designed to support secondary candidates in learning to notice ambitious instruction supported them in learning to observe and analyze classroom instruction in more substantive ways (Santagata & Guarino, 2011; Star & Strickland, 2008; Stockero, 2008; van Es & Sherin, 2002). In particular, they came to attend to the details of the mathematics, student thinking, and the ways that classroom discourse and pedagogies for making thinking visible supported student learning. We also found that several months later, in the context of the PACT Teaching Event, they continued to attend to these features of instruction and they used detailed observations to draw inferences about the relationship between teaching and student learning. This suggests then that a video-based course focused on learning from teaching can provide a context for teacher candidates to learn to frame the work of teaching in terms of student learning, what has been identified as a central goal for teacher education (Levin et al., 2009).

We also found variations in candidates' ways of noticing their own instruction in the context of the PACT Teaching Event. We observed that all of the candidates who enrolled in the video-based course came to use a common language to characterize their instruction, one that was informed by a vision of ambitious instruction. A major shortcoming of teacher education is the lack of a shared language to characterize instruction (Grossman & McDonald, 2008). Our findings suggest that engaging teacher candidates in cycles of analysis of teaching as represented in video, guided by frameworks that define the core work of mathematics instruction, can provide them with a way in to noticing the complexity of mathematics instruction. Importantly, however, we observed that one group of candidates blended visions of instruction, using alternative frameworks to characterize teaching and learning. We conjecture that other coursework in the teacher education experience, as well as the contexts of the student teaching experience likely informed the ways they came to see instruction (Grossman, Smagorinsky, &

Valenica, 1999). We also observed that attending to the details in the interactions that unfold is an important dimension for developing a vision of ambitious instruction. All but the four candidates in the first group attended to the *particular* ways that students interacted with the mathematics or the specific approaches they used to promote students understanding mathematical concepts, procedures, and reasoning. This finding supports prior research that proposes that learning to see the details in the interactions that unfold during instruction is an essential step in learning to systematically reflect on instruction (Davis, 2006; Rodgers, 2002).

An important issue for consideration relates to the video clips the candidates submitted and analyzed as part of the PACT Teaching Event. First, these clips came from the classroom contexts where the candidates were assigned to student teach, they videotaped and self selected the clips themselves, and they were captured early in the student teaching experience. This raises several issues about what the candidates could notice when they observed their own teaching. In particular, the kinds of practices being advocated in the course are not widely enacted in US classrooms (Raudenbush, 2009; Stigler & Hiebert, 1999). It is likely that the candidates were not teaching in classrooms where they were mentored to have students engage in cognitively complex tasks in discourse rich classroom environments focused on student thinking and reasoning. It is also the case that capturing quality video that provides insight into student thinking is not a simple matter and that candidates may have had difficulty capturing and selecting clips that represented the vision of teaching advocated by reform (Sherin, Linsenmeier, & van Es, 2008). Finally, because the PACT Teaching Event is a high-stakes assessment for teacher credentialing, the candidates may have been less inclined to discuss features of instructional interactions that they noticed were *not* present but could lead to more productive learning opportunities for students. Though the frameworks in the course facilitated a formative approach to improving teaching by learning from analysis of practice, the summative nature of the PACT Teaching Event may not encourage candidates to highlight ways they perceive they were less effective in their instruction.

It is also the case that the *Learning from Teaching* course focused on the development of a particular type of teacher noticing, one focused on the goals for mathematics teaching and learning defined by the broader research and education community (NCTM, 2000; NRC, 2001). Seidel and colleagues assess pre-service teachers' professional vision around three general pedagogical domains: goal clarity and orientation, teacher support and guidance, and learning climate (Seidel et al., in press). While these dimensions are worthwhile and have been found to be predictive for future teaching success (Seidel & Shavelson, 2007), we agree with Ball and Cohen (1999), Lampert (2001), and Hiebert and colleagues (2003; 2007) that a critical goal for teacher preparation is providing candidates with tools to learn in and from their practice and this involves learning to see and makes sense of the particulars in one's practice and how they relate to promote student learning. Recent calls for the improvement of teacher education advocate for a practice-based approach that focuses the curriculum of teacher education on the core work of the practice of teaching (Ball & Cohen, 1999; Grossman et al., 2009; Grossman & McDonald, 2008). This study contributes to this effort by documenting the different dimensions of learning to notice classroom interactions, arguably a central goal for teacher preparation (Feiman-Nemser, 2001), while also offering a design that achieves this goal (see Santagata & van Es, 2009). Grossman and colleagues (2009) report that teacher education programs often offer candidates opportunities to learn to reflect on teaching. However, we propose that the cycles of observation and analysis in which the candidates engaged in the *Learning from Teaching* course promoted a more systematic, deliberate way of looking at teaching, one that is guided by frameworks in the discipline and one that directly focuses teacher candidates' attention on students' disciplinary ideas. Moreover, if part of the core of teaching is attending and reasoning about student thinking during instruction, then the *Learning from Teaching* course provided a context to engage in approximations of this practice (see Grossman et al., 2009) by providing opportunities for candidates to notice and make sense of student ideas through structured video analysis, without the need to act on them in the moment of instruction.

We propose that several features of the course design may have influenced the shifts we observed. Schoenfeld (2011) discussed the challenges inherent in teachers developing a student-centered orientation to teaching and strategies for helping teachers shift to attend to students and their thinking. We propose that several features of the course provided opportunities for candidates in the *Learning from Teaching* course to get closer to the ways that students engage with the mathematics during instruction. In particular, they viewed and studied students being interviewed about mathematical concepts; they designed, conducted, and analyzed an interview with a student around mathematical tasks; and they also viewed and discussed several segments of video that highlighted student thinking during classroom lessons. These kinds of activities provided opportunities for candidates to develop a vision of mathematics teaching as a student-centered activity. Because the student interviews did not resemble classroom activity, the candidates could focus on students' ways of thinking about the mathematics. Moreover, when they viewed video segments from classroom lessons, the class discussions began by unpacking the student thinking and students' ways of participating in the lesson. Careful selection of the videos, along with structured guidance in viewing the clips, appeared to help orient the candidates' attention to students and their ideas. After working with the candidates to develop this orientation to attend to students, the focus of the course shifted to learning to examine how particular teaching moves influenced student thinking and learning. Thus, we propose three design features as central to supporting candidates in learning new ways of noticing and developing a professional vision of instruction: *foregrounding student learning of mathematics in the analysis*, *describing and naming pedagogical practices as they arise in representations of teaching*, and *engaging in joint design, enactment and shared noticing*. An important issue for future inquiry is to look inside the candidates' experiences in the video-based course, and others like it, to investigate how such features afforded candidates opportunities to delve into the details of the work of mathematics teaching and learning to develop a more robust vision of the work entailed to achieve the vision of reform. Findings of such research would contribute to recent calls to define a pedagogy for teacher education that will prepare teachers for the ambitious goals of mathematics education (Grossman & McDonald, 2008).

Though we are encouraged by our findings, we recognize several limitations to this study. First, one may wonder if we observed the same kind of variation in noticing from the pre to post task. Our preliminary analysis suggests this was not the case, but this is an important subject for future inquiry. Second, we defined a particular vision of mathematics instruction to which we sought to apprentice candidates in the course. It may be that this vision of teaching does not capture all the features of ambitious instruction or perhaps different features are necessary to consider for the local contexts in which teacher candidates teach. For example, issues of equity and participation are certainly important dimensions for providing access to students who are typically not served by the education system (Boaler & Staples, 2008; Hand, 2012). Further refining the vision of teaching to which candidates are apprenticed and studying their development of this vision of teaching is an important area for future inquiry. Finally, we recognize that the candidates' analyses of teaching are shaped by the instructional task and videos of teaching that were the subjects of their observation. In this study, we did not analyze the complexity of the tasks or lessons, nor did we examine their teaching practice. Future research will examine the relationship between tasks as designed and enacted and their observations of what occurred and their analyses of teaching. Such an analysis will provide greater insight into how teacher candidates' vision of teaching arises in their instruction.

Conclusion

Returning to Goodwin's (1994) construct of professional vision, essential to understanding how teachers develop ways of seeing and understanding their practice that are answerable to a particular group means that the profession has shared ways of seeing and talking about teaching. Further inquiry into how different participants in the teacher education system help candidates learn to look at and make sense of their practice, analyzing what tools they use, how different language systems are used to talk about teaching, and the consistency and inconsistency within and across contexts in the use of these tools and language to highlight and code classroom interactions. We conjecture that such a systemic approach to studying teacher noticing and the development of a vision of ambitious pedagogy will raise new questions that will inform the design of learning opportunities for teachers, while also pressing the field to adopt alternative conceptual frameworks for studying this critical construct for teaching.

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Appendix A Coding Framework for Assessing Candidate Noticing

Mathematical Content & Learning Goal The extent to which the mathematical focus of the lesson is identified. Level 0 to 2 captures a shift from inattention to the mathematics, to noticing the task, to inferring the mathematical learning goal from the task and problem posed. 0 – No mention of the math content/ learning goal; little or not attention to the mathematics of the lesson 1 – Identify the mathematics, the task or problem and/ or evaluate the appropriateness of the task for accomplishing the learning goal 2 – Infer from the task or the math problem to a broader learning goal and/ or upon analyzing student math thinking, draw inference(s) about the appropriateness of the task for accomplishing the learning goal
Student Thinking This category captures the extent to which the candidates attend to the students' thinking of the mathematics. Level 0 to 2 represents a shift from a focus on student answers and student participation or behavior to examining the whole class's learning in general to attention to and analysis of different ideas particular students raise, errors or confusions they have, and their mathematical thinking. 0 – Little or no attention to student mathematical thinking; focus on whole class behavior, participation, and/ or learning of group as a whole; minimal attention to student answers 1 – Some attention to individual student thinking - identifies thinking and understanding of the group/ class as a whole; some focus on student answers; minimal attention to individual student ideas & thinking 2 – Drawing inferences about student thinking – what student ideas, confusions, errors, understandings reveal about their thinking; differentiating between student mathematical thinking
Pedagogies for Making Thinking Visible This category captures the extent to which the candidates attend to the teacher, the choices the teacher makes throughout the lesson, and the teachers' role for making student thinking visible. Development in this category captures a shift from how the teacher organizes the class for learning and manages the students/ class, to the choices the teacher makes in general, to particular strategies the teacher uses to elicit ideas and explore mathematical ideas (type of task, ways of questioning and assessing, tools for learning) 0 – Class management; Student organization and arrangement of the class 1 – Describes teaching strategies (e.g the task; questions), choices the teacher makes in the lesson, and strategies for making thinking visible through the use of visual representations, materials; and/ or evaluates the effectiveness of particular strategies without reasoning about the teaching moves 2 – Draws inferences about the influence of teaching strategies on learning and hypothesizes about influence/ outcome of strategy on learning
Classroom Discourse Norms This category captures candidates' attention to the classroom discourse norms – ways of communicating about mathematical ideas, the roles of teachers and students in creating a discourse environment, and how individuals participate (or are expected to participate) in the

discourse community through explaining, questioning, taking on responsibility for each other's learning, and who is the source of mathematical ideas.
0 – Overall feeling of a classroom interaction/ environment
1 – Imply that norms exist for communicating mathematically and/ or focus primarily on the teachers' role in classroom discourse
2 – Identify norms for communicating about mathematics (explaining, questioning, source of ideas, responsible for learning) and infer how participating in discourse influences student thinking and learning; and/ or attend to and analyze both the teacher and the students' role in the classroom discourse

Specificity
This category refers to the specificity of the comments the PST makes and the extent to which it accurately depicts the events in the clip.
0 – Vague commentary; not particular to the clip
1 – Points to some specific events and interactions in the clip but not consistent and continues to use vague language
2 – Particular to the events in the clip; detailed and elaborate, referring to particular quotes, excerpts, and points in time.

Making Connections
This category concerns the extent to which the PST view teaching as an interactional space, where there is an interaction between members of the class – between students, and between teachers, as well as the interaction between teacher, students, and content.
0 – Little or no linking of noticed events
1 – Some connections are made between elements that are noticed but evidence remains that candidate is treating the observation in discrete elements; little elaboration of the connections that are made.
2 – Integrated analyses that examine the relationship between teaching, content, and student thinking and learning; More elaborate in the reasoning and sense-making of what is observed.