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A Time-Symmetric Formulation Of Quantum Mechanics

feature article

A time-symmetric formulation of quantum mechanics

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Quantum mechanics allows one to independently select both the initial and final states of a single system. Such pre- and postselection reveals novel effects that challenge our ideas about what time is and how it flows.

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That quantum mechanics is a probabilistic theory was, by 1964, an old but still troubling story. The fact that identical measurements of identically prepared systems can yield different outcomes seems to challenge a basic tenet of science and philosophy. Frustration with the indeterminacy intrinsic to quantum mechanics was famously expressed in Albert Einstein's assertion that "God doesn't play dice."

By 1964 most physicists had abandoned the struggle and taken a more pragmatic view. The theory seemed to answer all questions in the workaday world of calculating ground states, energy levels, and scattering cross sections. Asking what actually happens at a measurement played no role in understanding, say, the properties of condensed matter or nuclei. The wavefunction and its evolution seemed to be all that was needed. The puzzle of indeterminism hadn't gone away, but it was safely marginalized.

But 1964 brought a reversal of fortune. Indeterminacy, until then an unpleasant feature of an indispensible theory, suddenly became an open door to new freedoms implicit in the theory. One such freedom, the possibility of nonlocal correlations, was discovered by John Bell.[1] Another is the freedom to impose independent initial *and* final conditions on the evolution of a quantum system. The inquiry into that latter freedom, started by one of us (Aharonov), Peter Bergmann, and Joel Lebowitz[2] (ABL), is the subject of this article.

Our inquiry evolved slowly at first, but by now it has led to a new approach to quantum mechanics, to the discovery of a number of new quantum effects, to a powerful amplification method, and to an admittedly controversial new view of the nature of time.

Pre- and postselection

Time evolution in classical mechanics is a relatively simple affair. If one knows the initial conditions and the dynamics (say, the Hamiltonian) of an isolated system, then one knows everything about the system for all time. No future measurement can reveal anything really new. One can, in principle, predict the result of any measurement. Nor does one need to be told about the system's past. That too can be calculated.

In quantum mechanics, of course, the situation is dramatically different. Even if one knows the complete wavefunction $|\Psi\rangle$ at time t_0, and the Hamiltonian at all times, one cannot, in general, predict with certainty the result of a measurement performed at a later time t_1. What one can do is calculate the probabilities for the different outcomes, but not which of them will actually occur in a particular experiment. So the measurement at t_1 yields genuinely new information.

All that was, of course, well known in 1964. What was novel in the ABL approach was the new point of view—the realization that the result of the measurement at t_1 has implications not only for what happens after t_1 but also for what happened in the past. Suppose we start with an ensemble of particles, each prepared at t_0 in the same initial state $|\Psi\rangle$. At some intermediate time t between t_0 and t_1, we subject each particle to some measurement and then we perform a final measurement at t_1.

We can then split the original ensemble into subensembles according to the result of the final measurement. The statistical distribution of results from the intermediate measurement at t is, in general, different for each such pre- and postselected ensemble, and different from the statistical distribution over the entire initial ensemble, which had only been preselected. So the results at t depend not only on what happened earlier at t_0, but also on what happens later at t_1.

Creating impossible ensembles

By pre- and postselection, one can prepare strange subensembles. We could, for example, start at t_0 with an ensemble of spin-½ particles, each one polarized "up" in the z-direction. Then at t_1 we measure each spin in the x-direction and select only the particles for which the spin turned out to be up again, but in the new direction. Thus, at any intermediate time t, the spin components in both the z and the x directions—two noncommuting observables—would seem to be completely determined.

Isn't that a quantum mechanical impossibility? How do we know that both spin components are completely determined? Well, suppose the Hamiltonian is zero (no magnetic field) so that the spin doesn't precess. Then, if at t we measure the spin in the z-direction we must find it up, because that's how the particle was prepared at t_0. On the other hand, if at t we measure the spin along x, we must also find it up, because otherwise the measurement at t_1 wouldn't find it up (see figure 1a).

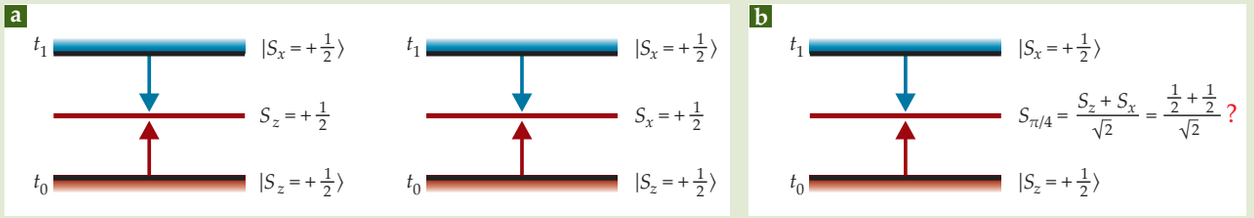

Figure 1. A spin-½ particle in a region free of external magnetic fields is preselected at time t_0 to be in the quantum state with spin up in the z direction, and postselected at t_1 to be in the state with spin up in the x direction. **(a)** At any intermediate time, such a particle would have well-defined values of the two noncommuting spin components S_z and S_x. Both would have to be +½. **(b)** It would seem to follow that the spin component $S_{\pi/4}$ along the 45° direction in the z–x plane would have to be $+\sqrt{2}/2$, an impermissible value.

One might well think that all this is trivial. Indeed, such pre- and postselected ensembles are quite common in classical physics. We can select them in classical probabilistic systems involving dice or coin flips. The fact that the statistics of a pre- and postselected subensemble differ from those of an ensemble that was only preselected follows trivially from conditional probability calculations. Suppose, for example, that a beam of classical particles is scattered by a target. We can postselect the particles that emerge in a particular direction. In general, what happened to a particle—for example, how some internal degree of freedom was affected—depends on the direction in which it emerged. So the statistical distribution of the internal variable in the pre- and postselected ensemble differs from that of the beam as a whole.

But in quantum mechanics, the situation is conceptually different. In the classical context, postselection is simply an issue of practical convenience. We could, in principle, have prepared the incoming beam with great care so that every particle had the same speed and hit the target in the same place. Then all particles would have emerged in the same direction, with the same internal state. There would be nothing left to postselect. In quantum mechanics, however, there is no way—even in principle—to know the result of the final measurement. Postselection is thus not just a surrogate for a more careful preselection.

We argue therefore that pre- and postselected ensembles should be considered *the* fundamental quantum ensembles. They contain more information than ensembles that are only preselected. A conventional preselected-only ensemble can be thought of as a collection of pre- and postselected subensembles one throws together by simply choosing to forget the results of the postselection measurements and thus losing information. So it shouldn't be surprising that the conventional understanding of quantum phenomena, which considers only preselected ensembles, is unnecessarily limited.

Nontrivial consequences?

Nonetheless, for more than two decades after the ABL paper, its new perspective yielded nothing more interesting than the observation about the ensemble of particles with well-defined spin components in both the x and z directions. The breakthrough to more significant consequences came in 1988 when Aharonov, David Albert, and Lev Vaidman (AAV) took another look at such an ensemble.[3] They asked what happens if in that pre- and postselected ensemble one measures $S_{\pi/4}$, the spin along a direction in the x–z plane making an angle of $\pi/4$ with the x axis.

By definition, the spin operator in that direction is $(S_x + S_z)/\sqrt{2}$. Taking seriously the idea that both S_x and S_z are well defined and equal to +½ in the ensemble, AAV predicted that

$$S_{\pi/4} = (1/2 + 1/2)/\sqrt{2} = \sqrt{2}/2$$

(see figure 1b). That answer is simple, natural, and obviously wrong! Like the spin component in any direction, $S_{\pi/4}$ can only take the values ±½. The value predicted above is simply not one of those two. Moreover, its magnitude exceeds that of either possible eigenvalue.

The correct answer is easy to get by following the standard method of conditional probabilities. The mistake, of course, was to be misled by the notion that during the interval between the pre- and postselection, S_x and S_z are both well defined. But it's a nice idea! Rather than give it up, AAV tried to better understand why it fails.

The measurement of $S_{\pi/4}$ is effectively a simultaneous measurement of S_x and S_z. The idea that they are both well defined stems from the fact that measuring *either one* yields +½ with certainty. But if we try to measure *both* of them, the situation becomes complicated. If we first measure S_z and then S_x, as in figure 2a, then, given the pre- and postselection, both measurements yield +½ with certainty. But if they are measured in the reverse order, as in figure 2b, each measurement can yield +½ or −½ with equal probability. In that order, S_z is no longer determined by the initial state, because the S_x measurement disturbed it. Similarly, because of the intervening S_z measurement, the outcome of the S_x measurement is no longer fixed by the postselection.

Thus, when the z and x spin components are measured simultaneously—as they are, in effect, when $S_{\pi/4}$ is measured—one shouldn't expect to find both of them well defined by their pre- and postselected values. The whole idea behind that expectation would seem to have been an illusion.

Weak measurements

But all is not lost. As every child who plays with magnets knows, bringing a small piece of iron close to a magnet, and thus implicitly measuring the strength of the magnetic field, doesn't destroy the magnet. In other words, when dealing with a large number N of spin-½ particles, all polarized in the same direction, one can measure the system's spin component along any axis without significantly disturbing the state. That such a measurement is possible despite the noncommutation of the different spin components is the result of a tradeoff. The child with its piece of iron is performing not a precise measurement but rather an approximate measurement with a typical error of order $\sqrt{N}$. So at the cost of precision, one can limit disturbance.

Such a tradeoff is at the heart of most of the effects to be discussed here. To emphasize the nondisturbing character of measurements that exploit the tradeoff, we call them "weak

measurements." But there is nothing weak about their precision; fractional errors of order $1/\sqrt{N}$ are negligible for large N.

So AAV suggested an expansion of the one-particle gedanken experiment. Suppose, they said, one starts at t_0 with a system of N spin-½ particles all polarized up along z. Then at t_1, every particle is subjected to a measurement of its x spin component. Suppose every single one of those measurements yields +½. Of course that's exponentially improbable; but play along for the sake of argument.

Thus we get a system preselected in the state $|S_z = +N/2\rangle$ and postselected in $|S_x = +N/2\rangle$, where S denotes the system's total spin. If we now measure S_x and S_z using a non-disturbing, approximate measurement like the child's piece of iron, both measurements will yield $+\sqrt{2}N/2 \pm \sqrt{N}$, no matter the order in which they are performed. Since neither measurement disturbs the other, they can be done simultaneously, and there is no longer any reason to dismiss the original intuition that a measurement of $S_{\pi/4}$ should now indeed yield the "impossible" result

$$S_{\pi/4} = (S_x + S_z)/\sqrt{2} = \sqrt{2}N/2,$$

$\sqrt{2}$ times the largest eigenvalue, with a negligible error of order $\sqrt{N}$.

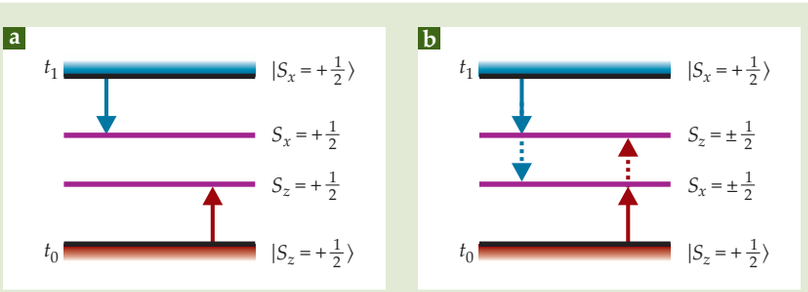

Figure 2. Measuring both the x and z spin components of a spin-½ particle in the interval between the pre- and postselections of figure 1 yields different results depending on the time order of the two measurements. **(a)** If the first interim measurement is along the direction of the preselection and the second interim measurement is along the postselection direction, both measurements are determined by the selections. **(b)** Reversing the order of the interim measurements destroys that certainty.

Physically significant errors

How is it possible that the measuring device indicates such a value for the spin? It's all a game of errors. A perfect measurement will yield, by definition, only one of the eigenvalues of the spin operator: values of $k/2$, where k is an integer such that $-N \le k \le N$. But we're now talking about an imperfect measurement with finite precision. So we don't require the measuring device's pointer to indicate precisely an eigenvalue. Sometimes it can even point, in error, to a value far outside the range of possible eigenvalues.

Although the typical error is of order $\sqrt{N}$, one can show that if nondisturbance is to be achieved, larger errors *must* be possible. In fact, there must be an exponentially decreasing tail of large errors. The remarkable point, though, is that in those ultrarare cases for which the postselection succeeded in finding $S_x = N/2$, the interim measurement of $S_{\pi/4}$ must have yielded, by mistake, the value $\sqrt{2}N/2 \pm \sqrt{N}$, way out in the tail (see figure 3).

Though the result is, technically, a measurement error, its significance goes well beyond that. First of all, it's by no means a random error. It is precisely the value predicted by intuition for the specified pre- and postselections. Second, it always occurs when the postselection succeeds. Third and most important, it is not limited to a particular kind of measurement. For that pre- and postselection, *anything* that interacts with the system weakly enough will be affected as if $S_{\pi/4}$ really is $\sqrt{2}N/2$.

Superoscillation

The process that yields such surprising results is complicated. Remember that we're no longer talking about ideal quantum mechanical measurements. So the whole eigenvalue–eigenstate paradigm no longer holds. Instead we must look into the actual physical measuring processes and model an approximate measurement that is less disruptive than the ideal one. That's not difficult: Take any physical model that, given a strong interaction with the system to be measured, approximates an ideal measurement well. Then let the interaction become weaker.

What happens next is what's complicated. A measurement shifts the measuring device's pointer. During the measurement, the pointer and the system being measured become quantum mechanically entangled. Postselection of the system's state destroys that entanglement. The pointer is left in a peculiar superposition of "legitimate" shifts, each corresponding to an eigenvalue of the measurement operator. But the superposition is such that an interference effect shifts it all the way out to the illegitimate value $\sqrt{2}N/2$.

How can a superposition of small legitimate shifts yield such a large illegitimate shift? At the mathematical core of the interference effect, and therefore of the weak-measurement idea, is a phenomenon we have called superoscillation.[4] One might think that by superposing waves with different wavelengths one cannot construct features with details smaller than the shortest wavelength in the superposition. But that's not true. Consider the function

$$f(x) = [(1 + \alpha)e^{2\pi i x/N}/2 + (1 - \alpha)e^{-2\pi i x/N}/2]^N,$$

where α is a real number larger than 1. By expanding the binomial, one can see that f is a superposition of waves with the shortest wavelength being 1. However, in the region of small x, one gets $f(x) \approx e^{2\pi i \alpha x}$. That is, the function behaves like a wave of wavelength $1/\alpha$, which is less than 1. Just such a superoscillation in the Fourier transform of the pointer's wavefunction is responsible for the pointer's large shift.

Weak values

The weak-measurement effect is clear and its existence is uncontroversial. It follows inevitably from standard quantum mechanics. But are we to think that $S_{\pi/4}$ really has the impossible large value? The measurement result, though indisputable, can nonetheless give rise to heated debate over its interpretation. One could argue that $S_{\pi/4}$ can, by definition, only have values up to $N/2$; so talk of a larger value simply makes no sense. Nonetheless, in every interaction in the weak regime the system does behave as if $S_{\pi/4} = \sqrt{2}N/2$.

One may wish, then, to revisit the whole quantum mechanical notion that physical variables are described by Hermitian operators and that their only possible values are the eigenvalues of the corresponding operator. In the early days of quantum mechanics, that seemed to be the most appropriate way to describe experimental results. But now that we have experiments that say otherwise, we may want to reconsider.

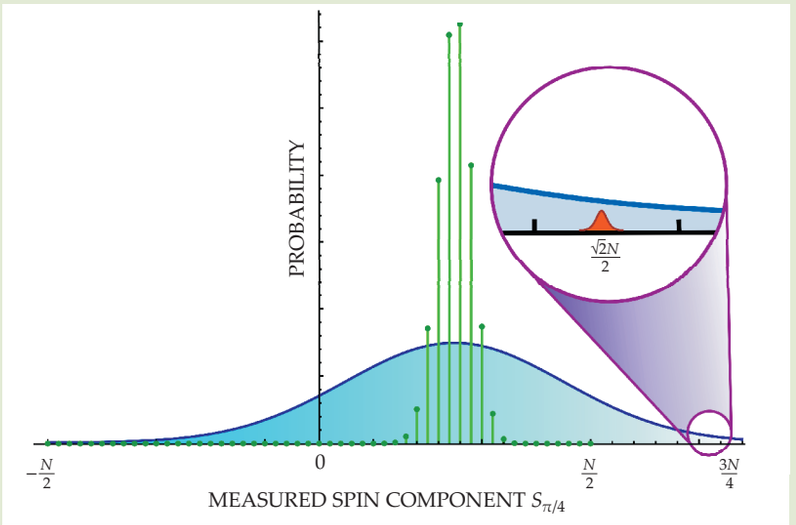

Figure 3. Probability distributions for different outcomes of the measurement of the spin component $S_{\pi/4}$ of a system of N spin-½ particles preselected in the state $|S_z = +N/2\rangle$. The green histogram represents measurement outcomes for an ideal measurement without postselection. The blue curve represents the probabilities in an approximate measurement. After postselection for the very improbable state $|S_x = +N/2\rangle$, only the red distribution way out in the tail survives.

At first sight, this seems to be one of those interpretational–philosophical issues that are best avoided. No such discussion is needed to prove the existence of the effect. But the discussion is important and more than just semantic; it has to do with guiding intuition. Taking seriously the idea that the spin component is indeed so large, one can now think of new situations in which the effect will play a role, and perhaps discover new quantum effects.

Let us discuss more generally a pre- and postselected ensemble defined by the preselected state $|\Psi\rangle$ and the postselected state $|\Phi\rangle$. In the weak-measurement regime, an arbitrary observable A behaves as if its value is

$$A_w = \langle\Phi|A|\Psi\rangle/\langle\Phi|\Psi\rangle.$$

We call A_w the "weak value" of the observable A.

The weak value can be arbitrarily large, far outside the range of the eigenvalues of A; all that's needed is a very small overlap $\langle\Phi|\Psi\rangle$ between the pre- and postselected states. Surprisingly, A_w can even be a complex number. Surprising but not wrong: It simply means that the effect the system has on other systems with which it interacts weakly is as if A were complex. If, for example, A_w is purely imaginary and the pointer's wavefunction is a Gaussian, the measurement would not shift the pointer's position. Instead, it would change the pointer's momentum.

The three-box paradox

Consider a single particle prepared at t_0 in a quantum state

$$|\Psi\rangle = (|1\rangle + |2\rangle + |3\rangle)/\sqrt{3}$$

that is a superposition of location eigenstates $|i\rangle$ in which the particle is definitely in the ith of three boxes. Suppose that at a later time t_1 the particle is measured and found in the state

$$|\Phi\rangle = (|1\rangle + |2\rangle - |3\rangle)/\sqrt{3}.$$

(A way of preparing such states with a three-armed interferometer is presented in reference 5.)

Imagine now that at some intermediate time we look in box 1. Given the pre- and postselections, the intermediate look finds the particle there with certainty. But if we look instead in box 2, we are certain to find the particle there![6] How is that possible, and what would we find if we look in both boxes?

To understand the confident predictions, consider the opposite: Suppose the experiment at time t didn't find the particle in box 1. Then the initial state $|\Psi\rangle$ collapses into its projection on the Hilbert subspace spanned by states $|2\rangle$ and $|3\rangle$, corresponding to the particle not being in state $|1\rangle$. But the projection of $|\Psi\rangle$ onto that subspace is (ignoring normalization) $|2\rangle + |3\rangle$, which has no overlap with $|\Phi\rangle$. Thus, such a postselection will never happen. The same argument holds for box 2.

So far, it may seem trivial. But our earlier spin example suggests that something really interesting is going on here. In the interval between the pre- and postselection there really is one particle in box 1 and one particle in box 2! Of course, if we open both boxes and look into them we will not find a particle in each. There is, after all, only one particle. But the exact measurement of opening the boxes strongly disturbs the state. Gentler measurements are required.

So suppose we start at t_0 with the same three boxes, but this time with a large number N of (nonidentical) particles, each one prepared in the state $|\Psi\rangle$; and suppose that at t_1 we happen to find every single one of them in the state $|\Phi\rangle$. Imagine furthermore that instead of actually looking in the boxes and carefully counting their occupants at the interim time t, we measure only the gravitational field near each box. (Assume the boxes themselves are capacious but almost weightless.) Because the field couples weakly to the particles, the measurement does not significantly disturb their quantum states. Neither is it an absolutely precise measurement of the number of particles in any one box. So one can arrange a measurement that is sufficiently nondisturbing while having an error of order $\sqrt{N}$ for the occupancy of each box.

In the absence of the postselection, we expect the measurements at t to indicate about $N/3$ particles in each box. But given the postselection, the measurement actually gives $N \pm \sqrt{N}$ particles in box 1 and also $N \pm \sqrt{N}$ particles in box 2. In other words, up to the negligible error of order $\sqrt{N}$, we find that box 1 has a gravitational field corresponding to all particles being there while, simultaneously, the gravitational field of box 2 says that all the particles are there!

What about box 3? There are, altogether, only N particles. But we already know from the pre- and postselections that there are N particles in box 1 *and* also N particles in box 2. So we are forced to predict that the third box contains $-N$ particles, a negative occupancy!

There's a joke about a mathematician who's asked how many people are in a building after only one person is seen entering and then two are seen leaving. "If one more person enters," he answers, "it will be empty!" That's meant to poke

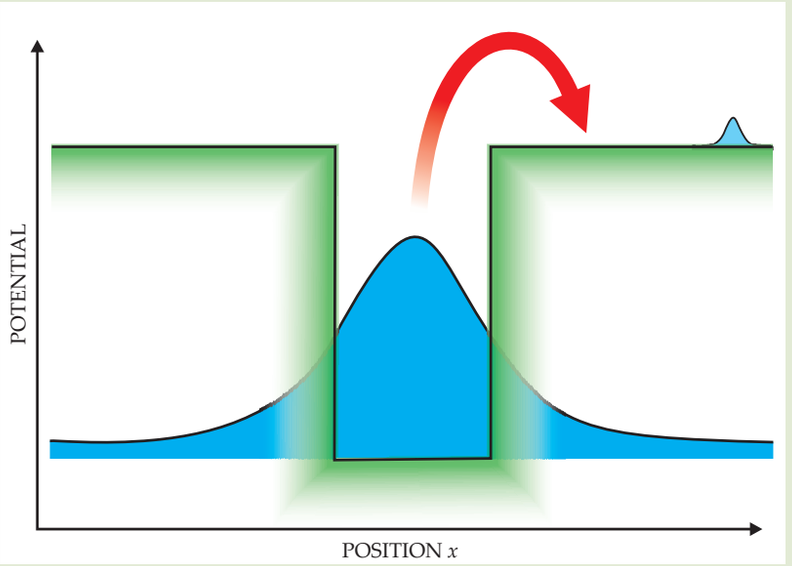

Figure 4. Preparing "genuine" tunneling particles. A quantum particle is preselected to have a particular total energy less than the depth of the potential well (green). The large bell-shaped curve shows its initial distribution in position x. But the particle is later postselected to have x far outside the well (small curve). In the interim between selections, a weak, imperfect measurement of the particle's kinetic energy K will always yield an "impossible" negative K—if the measurement's possible perturbing effect on position (red arrow) falls well short of the postselected x.

fun at pure mathematicians. But here it makes sense. The probe that measures the gravitational field of box 3, instead of being attracted to the box, is in fact repelled by it.

The paradoxical result is, of course, just a quantum fluctuation, a measurement error, but an error that happens with virtual certainty. And the effect is not restricted to the gravitational field. Any interaction (for example, electric or magnetic) sensitive to the number of particles will be as if there are $-N$ in box 3, so long as the coupling is small enough to be nondisturbing.

Negative kinetic energy

Tunneling is a quantum effect that allows a particle to be found in regions where its potential energy exceeds its total energy. One might assume there is nothing left to be learned about so paradigmatic a quantum effect. Yet pre- and postselection yield a completely new perspective on tunneling.[7]

How do quantum particles avoid the need for negative kinetic energy that classically forbids tunneling? The generally accepted explanation is that because the total energy E_T and position x operators do not commute, the very idea that a particle has both a well-defined x outside a potential well and a well-defined E_T smaller than the potential out there makes no sense and there is no need for negative kinetic energy K.

But by pre- and postselection we can prepare tunneling particles that are indeed outside the well and at the same time have E_T less than the potential. At t_0, we fix E_T to some value less than the well depth, and at t_1 we postselect only particles found outside the well. So between selections those particles have both E_T and x well defined—unless we disturb their state. But if we try to measure a particle's kinetic energy at some intermediate time, we do exactly that. An ideal K measurement completely disturbs x. So one can't know whether a particle found outside the well at t_1 was always there or was thrown out of the well by the measurement.

If, however, we accept a finite but arbitrarily small measurement error δ_K, we can limit the position disturbance to some large but finite δ_x. Then postselecting only particles further away from the well than δ_x guarantees that they were outside all along (see figure 4). These are "genuine" tunneling particles, for which K measurements yield "impossible" negative values.

Verification and amplification

Soon after weak measurement was proposed, some of our predictions were verified experimentally. In 1991 the "impossibly large spin" effect was demonstrated in a polarized-laser-beam setup by Nicholas Ritchie and coworkers.[8] Two years later Dietrich Suter and coworkers devised a nuclear-magnetic-resonance realization[9] of the predicted "time-translation machine,"[10] which returns the state of a spin back to what it was earlier without knowing its initial state or evolution. Last year, our prediction regarding the so-called Hardy paradox,[11] in which a measurement indicates a negative number of photon pairs, was confirmed experimentally by two groups.[12,13]

A recent development involves the exploitation of weak measurements as a new method of amplification. Consider again the gedanken experiment yields impossibly large spin. From one perspective, it raises a question about the spin's true value. But one can also regard it as an amplification effect. For all the particles in the pre- and postselected ensemble, the pointer of the measuring device moves much further than in a conventional measurement.

Even if you dismiss all the philosophy, you still have the amplification. And it doesn't even need to be quantum mechanical. The quantum interference responsible for the pointer's movement can be mimicked by classical waves. In 2008, Onur Hosten and Paul Kwiat used that technique to amplify the displacement of a laser beam by a factor of ten thousand.[14] That amplification allowed them to measure displacements of 0.1 nm and thus confirm the existence of the quantum Hall effect for light (see figure 5). And last year, Ben Dixon and coworkers[15], using a Sagnac ring interferometer, were able to detect mirror displacements of order 10 fm. The main point of the amplification method is that it offers a new kind of compromise between amplification and noise that may prove useful in many applications.

The flow of time

The most controversial—yet the most important—aspect of this research concerns the issue of time in quantum mechanics.

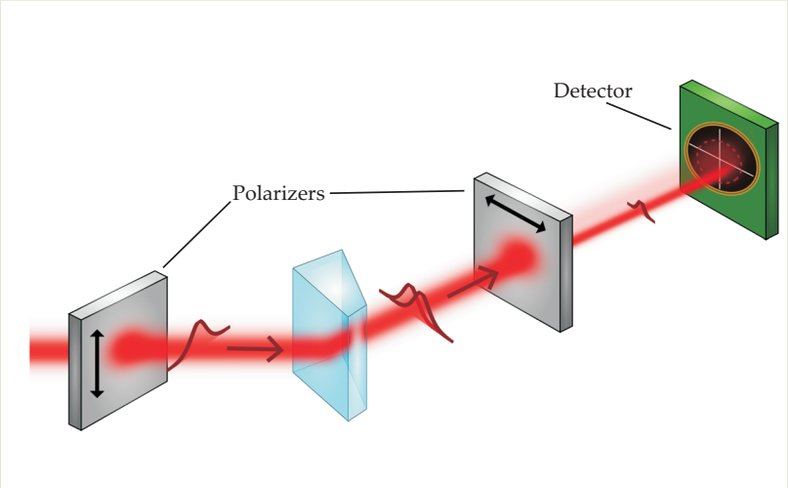

Figure 5. Amplifying the quantum Hall effect for light by weak measurement with pre- and postselection. The QHE produces a tiny lateral displacement of the light beam whose direction depends on whether the light is left- or right-circularly polarized. The beam undergoes the displacement while traversing the prism sandwiched between two polarizers. The polarizers, almost but not completely crossed, perform the pre- and postselection. As a result of those selections, the detector sees a beam that's much fainter but has a much greater QHE displacement. (Adapted from ref. 14.)

To be clear: The phenomena under discussion and the questions they raise are new, but they can be fully addressed in standard quantum mechanics. We have not modified quantum mechanics by an iota, nor have we tinkered with the notion of time. But it does seem to us that all these newly discovered phenomena indicate that the physicists' notion of time should be revisited.

Though all the effects discussed here can certainly be computed using the standard view of time flowing from past to future, such a description is extremely complicated. The weak values appear as a result of a game of errors in the measuring device. That game, in turn, relies on a complicated quantum interference in the state of the measuring device. That interference is the result of an unexpected mathematical phenomenon, namely superoscillation.

But all those complications disappear and the measurement result can be easily and intuitively understood when one takes seriously the idea that both time boundary conditions, the pre- and postselection, enter on equal footing and that during the interim the quantum system is influenced by both. Time thus propagates forward from the past boundary condition and backward from the future boundary condition.

So we propose an alternative formalism for quantum mechanics. A pre- and postselection experiment can better be described by two wavefunctions, one propagating forward in time and defined by preselection and the other, defined by postselection, propagating backward. Such "two-time" states are the basic objects of our formalism.[6,16,17] We can take that formalism a step further by using those objects even when there's no pre- or postselection. We discretize time and associate with each moment two Hilbert spaces, one a space of states propagating forward and the other of states propagating backward. In place of a single state changing over time by unitary interactions or measurement collapses, time evolution is viewed as correlations between forward and backward states at adjacent moments. Ordinary unitary evolution is obtained through maximal entanglement between them. A measurement breaks that entanglement.

These alternative formulations are completely equivalent to standard quantum mechanics in so far as their predictions are concerned. But the underlying concepts are dramatically different. So if in the future quantum mechanics has to be substantively changed to accommodate really new physics, the time-symmetric formulation might provide a starting point.

The reformulation may have one of its most exciting applications in cosmology. Our article thus far has regarded postselection only in experiments. Only after seeing the result of the final measurement does one decide whether a particle has passed the postselection. So when we spoke of the future affecting the past, we meant a very recent past affecting a slightly earlier past. But quantum mechanics lets one impose a true future boundary condition—a putative final state of the universe.

Philosophically or ideologically, one may or may not like the idea of a cosmic final state. The point is, however, that quantum mechanics offers a place to specify both an initial state *and* an independent final state.

What the final state would be, if there is one, we don't know. But if quantum mechanics says it can be done, it should be taken seriously.

References

1. J. S. Bell, *Physics* **1**, 195 (1964).
2. Y. Aharonov, P. G. Bergmann, J. L. Lebowitz, *Phys. Rev.* **134**, B1410 (1964).
3. Y. Aharonov, D. Albert, L. Vaidman, *Phys. Rev. Lett.* **60**, 1351 (1988).
4. Y. Aharonov, S. Popescu, D. Rohrlich, Tel Aviv University preprint TAUP 184790 (1990); M. V. Berry, S. Popescu, *J. Phys. A* **39**, 6965 (2006).
5. J. K. Resch, J. S. Lundeen, A. M. Steinberg, *Phys. Lett. A* **325**, 125 (2004).
6. Y. Aharonov, L. Vaidman, *J. Phys. A* **24**, 2315 (1991).
7. Y. Aharonov, S. Popescu, D. Rohrlich, L. Vaidman, *Phys. Rev. A* **48**, 4084 (1993).
8. N. W. M. Ritchie, J. G. Story, R. G. Hulet, *Phys. Rev. Lett.* **66**, 1107 (1991).
9. D. Suter, M. Ernst, R. R. Ernst, *Mol. Phys.* **78**, 95 (1993).
10. Y. Aharonov, J. Anandan, S. Popescu, L. Vaidman, *Phys. Rev. Lett.* **64**, 2965 (1990).
11. L. Hardy, *Phys. Rev. Lett.* **68**, 2981 (1992); Y. Aharonov et al., *Phys. Lett. A* **301**, 130 (2002).
12. J. S. Lundeen, A. M. Steinberg, *Phys. Rev. Lett.* **102**, 020404 (2009).
13. K. Yokota, T. Yamamoto, M. Koashi, N. Imoto, *New J. Phys.* **11**, 033011 (2009).
14. O. Hosten, P. Kwiat, *Science* **319**, 787 (2008).
15. P. B. Dixon, D. J. Starling, A. N. Jordan, J. C. Howell, *Phys. Rev. Lett.* **102**, 173601 (2009).
16. Y. Aharonov, L. Vaidman, *Phys. Rev. A* **41**, 11 (1990).
17. Y. Aharonov, S. Popescu, J. Tollaksen, L. Vaidman, *Phys. Rev. A* **79**, 052110 (2009). ■