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Abstract

I study the effect of task difficulty on workers' effort and compare it to the effect of monetary rewards in an incentivized lab experiment. I find that task difficulty has an inverse-U effect on effort, and that this effect is quantitatively large when compared to the effect of conditional monetary rewards. Difficulty acts as a mediator of monetary rewards: conditional rewards are most effective at the intermediate or high levels of difficulty. I show that the inverse-U pattern of effort response to difficulty is not consistent with the Expected Utility model but is consistent with the Rank-Dependent Utility model that allows for non-linear probability weighting. I structurally estimate the model and find that it successfully captures the treatment effects observed in the data. I discuss the implications of my findings for the design of optimal incentive schemes for workers and modeling effort.

Keywords: incentives, task difficulty, monetary rewards, effort provision, risk preferences, probability weighting

JEL codes: C91, D91, D81, J20, J33

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1 Introduction

Labor economics has long recognized the role of monetary incentives in determining workers' effort (Lazear, 2018). The effectiveness of monetary incentives that are tied to performance is well established. Behavioral economics added new insights to the effect of these conditional rewards by showing that the effectiveness of rewards is subject to psychological factors, such as crowding out of intrinsic motivation (Gneezy and Rustichini, 2000) and choking-under-pressure (Ariely et al., 2009; Hickman and Metz, 2015).[1] Simultaneously, this literature discovered alternative incentive mechanisms, often inspired by studies in psychology, that can boost workers' effort at no extra monetary cost. Among such mechanisms are goal-setting (Goerg and Kube, 2012; Corgnet et al., 2015), performance ranking (Blanes i Vidal and Nossol, 2011), and framing (Hossain and List, 2012).[2] The role of such behavioral incentives has been recognized in a recent World Bank Report (World Bank Group, 2015).

Despite the convergence of economic and psychological insights on the determinants of effort, an important strand of psychological literature remains, somewhat surprisingly, completely untapped by economists. A prominent psychological theory of motivation called motivational intensity theory posits that the primary determinant of effort is task difficulty (Brehm and Self, 1989; Wright, 1996). Task difficulty is a characteristic of a task that, when increased, reduces the probability of success in the task for any given level of effort.[3] Task difficulty is conceptually different from the cost of effort, even though the two concepts are sometimes mixed in the economics literature (Bremzen et al., 2015).[4]

Motivational intensity theory argues that effort is determined by the minimum amount of work needed to complete the task, as long as success seems possible and beneficial. The implication of the theory is that effort increases with task difficulty, up to a point, after which effort drops leading to an inverse-U pattern. At the same time, motivational intensity theory views monetary rewards, not as a primary, but rather mediating factor. Task difficulty has been argued to play an

[1] See Gneezy et al. (2011) for an excellent overview of the cases when monetary rewards do and do not work.

[2] DellaVigna and Pope (2018) conduct an elaborate horse race among monetary and behavioral incentives in terms of their effectiveness in stimulating effort.

[3] Equivalently, achieving a given probability of success in a difficult task requires more effort than in an easy task. This definition is nothing more than a slightly adapted dictionary definition of difficulty.

[4] Cost of effort is disutility from exerting effort and depends both on the task and preferences of an individual, whereas difficulty is a characteristic of a task. Low difficulty does not imply low disutility: it might be easy to wash dirty plates, but few people would find it pleasant!

essential role in activating analytical reasoning (Alter et al., 2007) and achieving goals (Labroo and Kim, 2009). Empirical studies in psychology provide extensive support for the inverse-U relation between task difficulty and effort.[5] A natural question arises of whether these results hold up in an incentivized economic environment. Understanding the role of difficulty and its interaction with other incentive elements, such as extrinsic monetary rewards, is relevant for managers who design optimal incentive schemes for workers. Answering these questions would help to address the points raised by Holmström (2017) who advocates studying the whole array of incentive tools that are not limited to pure monetary rewards.

I seek to accomplish two goals: to empirically study the effect of difficulty on effort and its interaction with monetary rewards and to use this evidence to improve our understanding of the mechanisms behind effort exertion. In the empirical part, I set up a lab experiment that follows a chosen effort framework (Fehr et al., 1997). Subjects assume the roles of agents who choose how much effort to exert in a series of projects. The probability of a project's success depends on a subject's chosen effort level and a project's difficulty. Higher difficulty reduces the probability of success for any given level of effort. Monetary rewards consist of an unconditional (wage) and conditional (bonus) parts. The cost of effort is monetary and is subtracted from a project's monetary outcome. I vary difficulty, monetary rewards, and cost within-subjects and observe how this exogenous variation affects subjects' effort levels. The chosen effort framework allows me to precisely define and observe difficulty and effort and to derive sharp testable hypotheses.

To establish theoretical predictions, I use a very general model of effort choice under risk that allows for a utility function that is potentially non-separable in money and effort, as in Mirrlees (1971). Allowing for such a general utility function is necessary since the pattern of complementarity/substitutability between effort and money in the utility function determines the effect of difficulty on effort. I consider two alternative models of risk preferences. First, I use a benchmark Expected Utility (EU) model in which an agent chooses effort to maximize the weighted average of outcomes utilities with the weights being the probabilities of each outcome. I show that a risk-averse agent with a monetary cost of effort would monotonically decrease their effort in response to higher difficulty. A risk-neutral agent would not change their effort in response to difficulty at

[5] See Gendolla et al. (2012) for a comprehensive overview of the motivational intensity theory and behavioral evidence on its empirical validity.

all. Second, I consider a Rank-Dependent Utility (RDU) model (Quiggin, 1982) in which the agent also chooses effort to maximize a weighted average of outcomes utilities, but now the weights are outcomes probabilities that are transformed using a probability weighting function. This function, in addition to the utility function, captures the agent's risk attitudes. I show that allowing for the probability weighting makes it possible to generate a non-monotonic response of effort to difficulty. In particular, when the probability weighting function is inverse-S-shaped (respectively, S-shaped), the pattern of effort response to difficulty is U-shaped (respectively, inverse-U-shaped).

I find that monetary rewards affect effort in the predicted direction. Conditional rewards, on average, have a strong positive effect on effort, while the effect of unconditional rewards is positive but weak. These results are consistent with the previous findings in the literature (Gneezy and List, 2006; DellaVigna and Pope, 2018) and can be viewed as a validation of the experimental design. Despite strong statistical significance, the economic significance of conditional rewards is somewhat disappointing: doubling the conditional rewards increases effort only by 20%. Making the cost of effort steeper leads to a sharp decrease in effort, as predicted. This result is consistent with the results reported in the contest literature (Dechenaux et al., 2015), as well as the studies employing real-effort tasks (Goerg et al., 2019).

I find that the effect of difficulty on effort is inverse-U-shaped. This result is new to the economics literature. It does match, however, the findings in psychology on motivational intensity theory (Gendolla et al., 2012). Interestingly, the magnitude of the effect of difficulty is on par with the magnitude of the effects of conditional rewards or costs. Difficulty mediates the effect of rewards. In particular, conditional rewards are most effective at the intermediate and high levels of difficulty. The inverse-U effect of difficulty on effort suggests an S-shaped probability weighting function. I confirm this in a structural analysis by estimating the parameters of the probability weighting function. While a typical finding in the literature on risk preferences is an inverse-S-shaped probability weighting function (Wu and Gonzalez, 1996; Bruhin et al., 2010; l'Haridon and Vieider, 2019),[6] DellaVigna and Pope (2018) report that subjects underweight small probabilities (implied by an S-shaped weighting) in a real-effort experiment.

The present findings have several important implications for the design of incentive schemes and modeling effort. The strong incentive effect of task difficulty suggests that managers should

[6] There are, however, some exceptions (Wilcox, 2015b).

take this effect into account when assigning tasks to their subordinates. A manager can expect that it will take workers more effort to complete a moderately hard task than an easy task. However, a very hard task might not be devoted as much effort as desired. This behavioral response to high difficulty can substantially lower a worker's performance by amplifying the direct negative effect of difficulty on performance. A manager, however, can counter this detrimental behavioral effect of difficulty using conditional rewards, since they are most effective when difficulty is high. From a theoretical perspective, the present findings suggest that economic agents might perceive chances differently depending on whether they face a labor context or an abstract context. Given that the studies of risk preferences are typically conducted in an abstract context, the role of risk preferences in other contexts, such as a labor context, deserves a closer investigation. If different contexts do produce different patterns of risk preferences, one would need a theory to explain such an effect.

I proceed as follows. Section 2 discusses the related literature. Section 3 describes the experimental procedures, design, and treatments. Section 4 presents the theoretical framework and derives testable hypothesis. Section 5 discusses the results of the experiment and estimates a structural model motivated by the observed behavioral patterns. Section 6 concludes.

2 Related Literature

Most closely related to my design is the research by Vandegrift and Brown (2003) that studies the interactive effect of task difficulty and conditional monetary rewards on performance in a tournament environment. It finds that monetary rewards do not have any effect on performance for easy tasks, but do have a positive effect for difficult tasks.[7] A related strand of literature looks at the effect of goals on workers' effort. Corgnet et al. (2015) conduct a laboratory principal-agent experiment and find that principals tend to set challenging but attainable performance goals for agents, which increases agents' performance relative to a no-goals baseline. They also report that the effect of goal-setting on effort[8] is stronger under high monetary rewards. Smithers (2015) conducts a laboratory experiment and finds that setting exogenous goals in an addition task increases

[7] McDaniel and Rutström (2001) report that for a difficult task, increasing penalty for bad performance tends to induce more effort from subjects, though the study does not vary task difficulty.

[8] It should be noted that higher induced effort, or performance, due to higher incentives does not necessarily increase a worker's welfare. The focus on performance is motivated from the viewpoint of a principal, but not a worker.

subjects' performance, with the effect being most pronounced in the male participants. Goerg and Kube (2012) conduct a field experiment at a campus library in which subjects were paid to sort books. They report that both exogenous and endogenous goal-setting lead to higher performance. These results are similar to the present findings, however, goal difficulty and task difficulty are not equivalent (Campbell and Ilgen, 1976). Apart from having a difficulty component, goals also have a reference point component (Heath et al., 1999).

The present work ties in to the literature on the behavioral effects of monetary rewards. Hossain and List (2012) conduct a field experiment at a Chinese manufacturing firm. They randomly assign workers to one of two conditions: in one condition, workers are paid a bonus upon reaching a given performance goal; and in the other condition, workers are paid a bonus in advance and lose it if they do not reach a performance goal. Consistent with the loss aversion hypothesis, the workers' performance was higher in the second condition. Gneezy and List (2006) conduct a field experiment at a campus library in which they vary the unconditional rewards paid to their subjects for arranging books. They find that increasing the rewards has a positive but short-lived effect on the subjects' effort. A similar finding is reported by Jayaraman et al. (2016) who study the effort response of tea pluckers in India to an increase in their unconditional rewards caused by a change in their contracts. Hennig-Schmidt et al. (2010) show that the positive effect of unconditional rewards occurs only when workers understand the benefit of their work to the principal, which triggers positive reciprocity. The weakly positive effect of unconditional rewards reported in the present chapter is consistent with their findings.

Finally, the present work contributes to studies of the role of alternative behavioral tools in stimulating workers' effort. Charness et al. (2012a) conduct a laboratory gift-exchange experiment in which the principal can delegate the choice of wage and effort to the agent, as well as the choice of a desired effort goal. They find that delegation is profitable both for the principal and the agent, contrary to the standard economic prediction. Blanes i Vidal and Nossol (2011) present quasi-experimental evidence from a firm in which workers compensated by piece rates receive information about their relative performance. They find that providing this information has a long-term positive effect on worker's performance.

3 Experimental Design

3.1 Procedures

The experiment was conducted at the ExCEN lab at Georgia State University (GSU) in May–June 2015. A total of 98 subjects participated in the experiment over the course of six sessions. The subjects were recruited using an automated system that randomly invited participants from a pool of more than 2,000 students who signed up for the participation in economic experiments. The subjects in the study were undergraduate students at GSU invited to participate via email. Each session was run on computers and lasted for roughly 1.5 hours. The subjects received a show-up fee of \$5 and the payoffs from decisions tasks.[9] The average payment per subject was \$45.89 (the minimum payment was \$5, and the maximum was \$100), including the show-up fee.

Table 1 summarizes the demographic characteristics of the sample. The sample had equal shares of males and females.[10] The racial composition was dominated by African American students: they account for 62% of the sample, while Caucasian students are just 14% of the sample. An average student's age was slightly above 21 years. The majority of subjects were in the advanced stages of a college program: more than half of participants were either juniors or seniors. Only 10% of the subjects came from an Economics or Finance major, which alleviates the concern that the observed behavior could be driven by the subjects sophisticated in economics.

3.2 Effort Task

In each round of the effort task, a subject was given a project that had two possible outcomes: success or failure.[11] Each project was characterized by four treatment variables that varied across rounds. These variables are: bonus (z), wage (w), difficulty (θ), and cost (k). In the case of success a project yielded a high revenue (the sum of a wage and a bonus), and in the case of failure it yielded a low revenue (a wage only). Wage represents an unconditional (on performance) reward and bonus represents a conditional reward. The difficulty of a project affected the probability of success. Higher difficulty resulted in a lower probability of success for any given level of effort. The

[9] In each session subjects also participated in a supplementary incentivized risk elicitation task, the results of which are not reported here.

[10] The shares do not sum up to one, since subjects had an option not to answer or state their gender as "other." These options were used by 4% of the subjects.

[11] Appendix A provides sample screenshots of the software, which was programmed in z-Tree (Fischbacher, 2007).

Table 1: Socio-Demographic Characteristics of the Sample

Characteristic	Mean
Gender	
Male	0.48
Female	0.48
Race and Age	
White or Caucasian	0.14
Black or African American	0.62
Age	21.29
Year in Program and Major	
Freshman	0.04
Sophomore	0.18
Junior	0.24
Senior	0.45
Econ or Finance Major	0.10

cost variable affected the steepness of the cost-of-effort function. The experiment was presented to subjects in the meaningful labor context with terms such as "revenue," "effort," and "difficulty."[12] It is possible that the meaningful context had a framing effect (Alekseev et al., 2017), however, this context was chosen to enhance the understanding of an arguably complex decision-making task and because the labor context is precisely the setting, behavior in which is of interest in the present study.

Subjects could choose any effort level a between 0 and 100 per cent. The chosen level of effort had a twofold effect: on the probability of success, and on a project's profit (revenue minus cost). Higher effort increased the probability of success but led to lower profits. The subjects incurred the cost of effort regardless of the outcome of a project. When deciding what effort level to choose, the subjects therefore faced a trade-off between a higher chance of the project being successful and lower profits.

The use of the chosen, rather than real, effort framework is common in tournament (Bull et al., 1987) and principal-agent experiments (Fehr et al., 1997; Charness et al., 2012a). The primary motivation for this design is to allow for a clean test of theoretical predictions. Brüggen and Strobel (2007) demonstrate that the chosen effort framework yields qualitatively similar results to the real effort framework in their setting, while allowing for a greater control in experiments.

[12] The experimental instructions, however, did use terms "wage" and "bonus." Only terms "high" or "low revenue" were used, where a high revenue is a sum of a wage and a bonus, and a low revenue is a wage.

The probability of success p as a function of effort a and difficulty θ was computed as

$$p(a,\theta) = a/2 + (1-\theta)/2, \tag{1}$$

so that, effectively, the probability of success was a simple average between an effort level and a project's ease, $1-\theta$.[13] The cost of effort was computed as the square of effort multiplied by the cost variable k:

$$c(a) = ka^2 \tag{2}$$

to induce a convex cost schedule. The subjects were informed of the linear relationship between the probability of success and effort and of the convex relationship between the costs and effort.

Subjects received feedback on the outcome of a project in every round.[14] The feedback was provided to ensure that subjects had a good understanding of the task, since quick feedback is crucial for learning and improving performance (Balzer et al., 1989; Hoch and Loewenstein, 1989). This was justified by the complexity of the decision task, which had many alternatives to choose from (the subjects could choose from 101 levels of effort) and several decision-relevant variables to consider. After the outcome of a project was determined, the subjects could review the results of the current round on a summary screen.

Each subject played between 15 and 19 rounds of the effort task, which were preceded by five practice rounds. After completing all the rounds, the payoff for the effort task was determined by randomly selecting one round. While paying randomly for one round is theoretically not incentive compatible with the Rank Dependent Utility model (Harrison and Swarthout, 2014; Cox et al., 2015) that is used for subsequent analysis, the provision of feedback after each round (i.e., playing out choices sequentially but paying for one choice randomly in the end) might alleviate this concern in practice (though not in theory), as suggested by Cox et al. (2015). They show that estimated risk preferences are not significantly different between a treatment in which subjects made, and were paid for, only one choice (theoretically incentive compatible with RDU) and a treatment in which subjects made multiple choices with feedback and were paid for a random round.

[13] In terms of a cognitive production function, this is equivalent to having constant returns to scale. This form was chosen to simplify the analysis and because it is easy to convey to subjects.

[14] All random outcomes were realized by a computer. While using a computer may raise credibility concerns among some subjects, this design was implemented due to its procedural convenience.

The four project characteristics (difficulty, wage, bonus, and cost) changed randomly across rounds, and subjects were aware of this. Each distinct combination of the values of these treatment variables represents a treatment. The three monetary treatment variables (w, z, and k) assumed one of the two possible values. Wage w assumed the values of 1 and 2, bonus z assumed the values of 2 and 4, and cost k assumed the values of 1 and 2. These values were multiplied by \$10 and then presented to subjects. For instance, a treatment with $w = 1$ and $z = 2$ corresponded to a project with a low revenue of \$10 and a high revenue of \$30 = \$10 + \$20. Similarly, given an effort level of 40% and a treatment with $k = 2$, the cost of effort would be \$3.2 = \$20$\times 0.4^2$. Difficulty assumed five values: 0, 0.25, 0.5, 0.75, and 1, which was important for identifying its potentially non-monotonic effect on effort. The treatments were constructed from the permutations of the values of the treatment variables. The order of treatments was randomized on the subject level. Appendix A provides the summary of the treatments used in the experiment.

The within-subject design was chosen for three main reasons. First, the large number of treatments made it impractical to use a between-subject design. Second, the use of the within-subject design allows for a deeper analysis of the subjects' data and improves statistical power. Third, even though it is possible that subjects experience the experimenter demand effect by observing all the treatments (Charness et al., 2012b), it is unlikely in the present experiment. A relatively large number of treatment variables (four) changing randomly between rounds would make it extremely hard for subjects to infer patterns and the desired response, especially since the response to difficulty is theoretically ambiguous.

4 Theoretical Framework

4.1 Environment

Consider an agent who works on a project with a stochastic binary outcome: success or failure. The project yields a wage $w \geqslant 0$ regardless of the outcome and a bonus $z \geqslant 0$ in the case of success. The probability of success in the project is determined by two factors: the agent's effort $a \in [0, 1]$ and the project's difficulty $\theta \in [0, 1]$. The vector of the values of the project's characteristics is denoted as $\pi \equiv (w, z, \theta)$.

Let X be a Bernoulli random variable encoding the project's outcome. The agent's revenue from the project is $Y = w + zX$. The probability of success $p : [0,1] \times [0,1] \mapsto [0,1]$ is a function of effort a and difficulty θ. I assume that p is twice continuously differentiable, increasing and concave in effort, and decreasing in difficulty.

If the cdf of X conditional on effort and difficulty is $F(x \mid a, \theta)$ and pmf is $f(x \mid a, \theta) = xp(a, \theta) + (1 - x)(1 - p(a, \theta))$ (for $x \in \{0, 1\}$, and 0 otherwise), then for $0 \leqslant x < 1$ we have $F_a(x \mid a, \theta) = -p_a(a, \theta) < 0$. This implies that the agent has an incentive to exert more effort, since the project endowed with a high effort level first-order stochastically dominates the project with a low effort level. However, there is a trade–off in that more effort leads to higher disutility from exerting it.

4.2 Preferences

I assume that the agent has preferences over money y and effort a represented by a utility function $u : \mathbb{R}_+ \times [0,1] \mapsto \mathbb{R}$. The u function is assumed to have the standard properties: it is twice continuously differentiable, strictly increasing and concave in money, and is strictly decreasing and concave in effort, i.e., the marginal disutility of effort increases. I use a very general utility function that is potentially non-separable in money and effort, as in Mirrlees (1971), as opposed to a more commonly used additively separable function (Abeler et al., 2011; Hossain and List, 2012; Jayaraman et al., 2016; DellaVigna and Pope, 2018). The reason for this choice is that the additively separable specification turns out to be extremely restrictive in terms of the comparative statics effect of difficulty on effort.

I consider two alternative models of the agent's risk preferences and show that the nature of risk preferences plays a crucial role in determining the effect of difficulty on effort. The first natural assumption is an Expected Utility (EU) model.

Assumption 1.A (EU)**.** *The agent's risk preferences are characterized by the Expected Utility model:*

$$U(a \mid \pi) \equiv \mathbb{E}u(Y, a) = \sum_{x=0}^{1} u(w + zx, a) f(x \mid a, \theta).$$

I also explore the implications of the Rank-Dependent Utility (RDU) model, which extends the EU model by allowing for non-linearity in probabilities. This property turns out to be critical for producing the non-monotonic effect of difficulty on effort.[15]

Assumption 1.B (RDU)**.** *The agent's risk preferences are characterized by the Rank Dependent Utility model:*

$$\tilde{U}(a \mid \pi) \equiv \tilde{\mathbb{E}}u(Y, a) = \sum_{x=0}^{1} u(w + zx, a)\tilde{f}(x \mid a, \theta),$$

where $\tilde{f}(x \mid a, \theta) = x\tilde{p}(a, \theta) + (1-x)(1-\tilde{p}(a, \theta))$ *is the decision weight of an outcome* x*, and* $\tilde{p}(a, \theta) = \omega(p(a, \theta))$ *is the success probability weighted by the probability weighting function* $\omega : [0, 1] \mapsto [0, 1]$*, twice continuously differentiable and strictly increasing with* $\omega(0) = 0$ *and* $\omega(1) = 1$*.*

4.3 Optimal Effort

Under Assumption 1.A the agent chooses the optimal effort level a^* given the parameters of the problem π by maximizing $U(a \mid \pi)$. If $a^* = \arg\max U(a \mid \pi)$, then the first-order necessary condition must hold:[16]

$$\mathbb{E}\left[u(Y, a^*)\frac{f_a(X|a^*, \theta)}{f(X|a^*, \theta)}\right] = -\mathbb{E}u_a(Y, a^*). \tag{3}$$

Equation (3) means that in the optimum the marginal benefit from exerting more effort on the left-hand side must be balanced by the marginal cost of effort on the right-hand side. The marginal benefit represents the expectation of the utility weighted by f_a/f, since the gain comes from the increased probability of success. It can be rewritten as $p_a\Delta u$, where $\Delta u \equiv u(w + z, a) - u(w, a)$ is the utility gain between the success and failure given effort a. Written in this form, the marginal benefit is the marginal increase in the probability of success multiplied by the utility gain. The marginal cost is the expected marginal disutility of effort. It can be rewritten using the Mean Value Theorem as $u_a + pzu_{ya}$, where the cross-partial is evaluated at some point $(\bar{y}, a^*), \bar{y} \in [w, w+z]$. This

[15] Another popular preference assumption is the Cumulative Prospect Theory (CPT) model, which adds reference-dependence to the RDU model. Below I show that reference-dependence cannot produce a non-monotonic effect of difficulty on effort and that probability weighting is sufficient.

[16] See Appendix E for all the derivations and proofs.

form will be useful for understanding the comparative statics of the model.[17] All the above results also hold under Assumption 1.B after replacing U, $\mathbb{E}$, f and p with $\tilde{U}$, $\tilde{\mathbb{E}}$, $\tilde{f}$ and $\tilde{p}$, respectively.

4.4 Special Cases

Several special cases of the utility function u permit closed-form solutions. The first case is a separable utility function, $u(y, a) = v(y) - c(a)$, which is a popular specification in the literature. In this specification the utility of money v is linearly separable from the cost of effort c. Assume that $v : \mathbb{R}_+ \mapsto \mathbb{R}$ is twice continuously differentiable, increasing and concave. With a quadratic cost of effort as in (2) and a linear probability of success as in (1), the optimal effort is

$$a^* = \frac{v(w+z) - v(w)}{4k}. \tag{4}$$

A notable feature of this specification is that the optimal effort does not depend on the project's difficulty. As will be shown shortly, this is the consequence of the linearity of p and the additive separability of u, under which the cross-partial derivatives of both functions are zero. The optimal effort increases with the bonus and decreases with the cost k, which is intuitive. The increase in the wage causes the optimal effort to go down if v is strictly concave. This makes sense since if the agent can guarantee herself good result regardless of the outcome she will not have a strong incentive to exert effort. However, if v is linear, the optimal effort does not depend on wage.

The second case is a non-additively separable specification in which effort has a monetary cost to the agent, just like in the current experimental design, and the utility of money is exponential, $u(y, a) = v(y - c(a)) = -e^{-\gamma(y-c(a))}$, with $\gamma > 0$ being the constant absolute risk aversion (CARA) parameter. The benefit of using the exponential (CARA) utility is its analytical tractability, which has been exploited in various settings, e.g. von Gaudecker et al. (2011). Assume that the cost of effort is quadratic and the probability of success is linear. It can be shown that in this case the

[17] Given the generality of the assumed utility function, the first-order condition is not sufficient since the problem is not guaranteed to be globally concave. When U is not globally concave, it is possible to get a local maximum, in which case one also needs to check the function values at the boundaries of the choice set to find the global maximum The global maximum will always exist by the Extreme Value Theorem. Simulations show that for some parameter values, it is possible to run into corner solutions, which however does not invalidate the comparative statics results. They will still hold as weak inequalities. In what follows, I assume that a^* is the interior global maximum of the agent's problem.

optimal effort is given by

$$a^* = \frac{A + \theta - \sqrt{(A+\theta)^2 - 2/(k\gamma)}}{2}, \text{ where } A \equiv \frac{1+e^{-\gamma z}}{1-e^{-\gamma z}},$$

and the effect of difficulty on the optimal effort is negative because of the positive risk aversion parameter. Unlike in the additively separable case, u has a positive cross-partial derivative, which drives this result. It is worth noting that the optimal effort will not depend on the wage. As in the previous case, the optimal effort will increase with the bonus and decrease with the cost.

4.5 Comparative Statics, Difficulty

Under Assumption 1.A, the effect of the project's difficulty on the optimal effort is given by

$$\frac{da^*(\pi)}{d\theta} = -\frac{z[p_\theta(a^*,\theta)u_{ya}(\bar{y},a^*) + u_y(\bar{\bar{y}},a^*)p_{a\theta}(a^*,\theta)]}{U''(a^* \mid \pi)}, \tag{5}$$

where $\bar{y}, \bar{\bar{y}}$ are some numbers on $[w, w+z]$. The sign of the effect crucially depends on the signs of the cross-partial derivatives of both u and p. This effect can be concisely stated as follows.

Proposition 1.A. *If* $\operatorname{sgn}(u_{ya})\operatorname{sgn}(p_{a\theta}) < 1$*, then* $\operatorname{sgn}\left(\frac{da^*(\pi)}{d\theta}\right) = \operatorname{sgn}(p_{a\theta} - u_{ya})$*.*

Assuming that $p_{a\theta} \neq 0$, the effect of difficulty will have the same sign as this cross-partial derivative. Intuitively, this implies that if effort and difficulty are complements in the probability function, it is optimal to increase effort in response to a higher difficulty. Since higher difficulty reduces the probability of success, the optimal response is to compensate this reduction. On the other hand, if effort and difficulty are substitutes in p, it is optimal to reduce effort, which makes the reduction in the probability of success even higher.

Using Assumption 1.B instead, the formula in (5) remains valid after the appropriate change of non-tilde characters to tilde characters. Expanding, one obtains

$$\frac{da^*(\pi)}{d\theta} = -\frac{z[\omega' p_\theta(a^*,\theta)u_{ya}(\bar{y},a^*) + u_y(\bar{\bar{y}},a^*)(p_{a\theta}(a^*,\theta)\omega' + p_\theta(a^*,\theta)p_a(a^*,\theta)\omega'')]}{\tilde{U}''(a^* \mid \pi)}, \tag{6}$$

where ω, ω' and ω'' are evaluated at $p(a^*,\theta)$. Note that if $\omega'' = 0$ we are back to the EUT case, since there would be no probability weighting. The sign of the effect of difficulty on effort now depends, in addition to the signs of the cross-partial derivatives of u and p, on the sign of ω''. Assuming that $\omega'' \neq 0$, the effect of difficulty in this case can be concisely stated as follows.

Proposition 1.B. *If* $\operatorname{sgn}(u_{ya})\operatorname{sgn}(p_{a\theta}) < 1$ *and* $\operatorname{sgn}(u_{ya})\operatorname{sgn}(\omega'') > -1$*, then* $\operatorname{sgn}\left(da^*(\pi)/d\theta\right) = -\operatorname{sgn}(\omega'')$.

This result has an intuitive interpretation. If the agent exhibits probability pessimism, $\omega'' > 0$, she will reduce effort in response to a higher difficulty, as if she does not believe in her ability to affect the chances of success strong enough to accept the challenge. On the other hand, if the agent exhibits probability optimism, $\omega'' < 0$, she will raise effort in response to a higher difficulty.

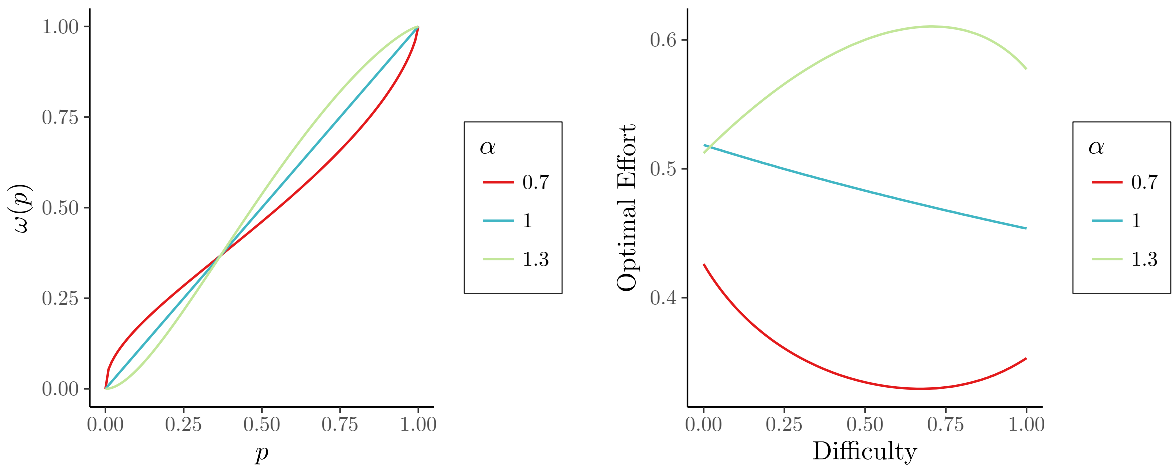
The results obtained so far predict only a monotonic effect of difficulty on effort. The findings in psychology (Gendolla et al., 2012, 2008), however, suggest an existence of a non-monotonic inverse-U pattern. The question is what features of the model can produce such a non-monotonic effect. An inspection of the comparative statics formulas (5) and (6) reveals that in order to achieve a non-monotonic effect, the terms containing θ need to change their sign as θ changes. The only term that has this feature is ω'' in the RDU model, since experiments sometimes find an inverse-S shape of the probability weighting function (Wu and Gonzalez, 1996; Bruhin et al., 2010), which is concave for low values of p and convex for high values of p.[18]

Assume for simplicity that both cross-partial derivatives or u and p are zero and that ω is inverse-S-shaped. The sign of the effect of difficulty then would change from positive, for small p, to negative, for high p. The difficulty is inversely related to p, hence one could expect that effort would increase (decrease) for high (low) values of difficulty, which implies a U-shaped pattern. Figure 1b illustrates this point by plotting the optimal effort as a function of difficulty for three different shapes of ω. In the simulation, I use the monetary cost of effort specification with the CRRA utility of money $u(x) = x^{1-\gamma}/(1-\gamma)$ with $\gamma = 0.2$ and the one-parameter Prelec (1998) weighting function $\omega(p) = \exp(-(-\ln(p))^{\alpha})$. The cost and probability of success functions are specified as before. The

[18] The term p_θ is always negative by assumption. The term $p_{a\theta}$ has an undetermined sign. While it is conceivable that this cross-partial derivative might change its sign depending on the value of θ, such an assumption would be hard to justify *a priori*. If one were to add reference dependence by allowing u to be concave above the reference point and convex below the reference point, this would not allow for a non-monotonic effect of difficulty on effort. Assume that the reference point, $\tilde{y}$, lies on $[w, w+z]$. Then in the comparative statics formulas (5) or (6), the term u_y would be negative for $y \geqslant \tilde{y}$ and positive for $y < \tilde{y}$. The change of the sign of u_y, however, would not depend on θ, hence the sign of the comparative statics effect of difficulty would not change with θ. Note that the Disappointment Aversion model (Gul, 1991), which also allows for a probability transformation, collapses to the RDU model in the present binary-outcome case.

optimal effort as a function of difficulty is U-shaped for an inverse-S-shaped probability weighting function and is inverse-U-shaped for an S-shaped probability weighting function. When there is no weighting, the optimal effort monotonically declines in difficulty as predicted by Proposition 1.A, since $u_{ya} > 0$ in this specification.

Figure 1: Comparative Statics Under RDU



(a) Probability Weighting Function (b) Effect of Difficulty on Effort

4.6 Comparative Statics, Wage and Bonus

The effect of bonus on optimal effort is given by

$$\frac{da^*(\pi)}{dz} = -\frac{p(a^*, \theta)u_{ya}(w + z, a^*) + p_a(a^*, \theta)u_y(w + z, a^*)}{U''(a^* \mid \pi)}, \tag{7}$$

which leads to the following result:

Proposition 2. *If* $u_{ya} \geqslant 0$, *optimal effort will increase with bonus.*

This prediction makes sense intuitively, as a higher bonus means a better outcome in case of success, which in turn justifies higher effort. Both additively separable and exponential utility cases are examples of this result. Proposition 2 clarifies, however, that this result holds unambiguously only when money and effort are complements in the utility function.

The effect of wage on optimal effort is given by

$$\frac{da^*(\pi)}{dw} = -\frac{\mathbb{E}u_{ya}(Y, a^*) + zp_a(a^*, \theta)u_{yy}(\bar{y}, a^*)}{U''(a^* \mid \pi)},$$

which leads to the following result:

Proposition 3. *If $u_{ya} \leqslant 0$, optimal effort will decrease with wage.*

This means that if the agent can guarantee herself higher revenue regardless of the outcome, she has an incentive to exert low effort, provided that u is submodular. This is true, for example, in the additively separable case.

Propositions 2 and 3 will still hold for the RDU model, since $\tilde{p}$ and $\tilde{p}_a = \omega' p_a$ have the same signs as p and p_a, respectively.

4.7 Testable Hypotheses

In the experiment, the probability of success function in (1) is linear and therefore $p_{a\theta} = 0$. Moreover, the use of the chosen effort framework allows me to determine the sign of the cross-partial u_{ya} unambiguously. Since effort has a monetary cost to the agent, the utility function becomes $u(y, a) = v(y - c(a))$, where $v : \mathbb{R}_+ \mapsto \mathbb{R}$ is the utility of money, assumed to be twice continuously differentiable, increasing and concave, and $c : [0, 1] \mapsto \mathbb{R}_+$ is the cost of effort function given by (2), twice continuously differentiable, increasing and convex. Then the cross-partial of the utility function is $u_{ya} = -c'v'' \geqslant 0$, which implies that $\operatorname{sgn}(da^*/d\theta) = -\operatorname{sgn}(u_{ya}) \leqslant 0$.

The analysis of the comparative statics shows that the effect of difficulty cannot be signed unambiguously, as different preference assumptions (EU, RDU with inverse-S-shaped weighting, or RDU with S-shaped weighting) yield competing predictions. The use of the chosen effort framework does allow me to sign the predicted effect of difficulty within a given preference assumption, hence the data will ultimately determine the winning assumption. Below I summarize the competing hypothesis derived so far.[19]

Hypothesis 1.A (Difficulty, decreasing)**.** *Subjects' average effort will monotonically decrease with the project's difficulty.*

[19] The comparative statics predictions are derived under the assumption of deterministic choice. Appendix D.3 shows that the model's predictions also hold under the assumption of stochastic choice. In the case of stochastic choice the predictions are in terms of expected, or average, effort, which is reflected in the hypotheses stated below.

Hypothesis 1.B (Difficulty, U-shape)**.** *Subjects' average effort will first decrease, reach a minimum, and then increase with the project's difficulty.*

Hypothesis 1.C (Difficulty, inverse-U-shape)**.** *Subjects' average effort will first increase, reach a maximum, and then decrease with the project's difficulty.*

Turning to the effect of incentives, in the experiment u is supermodular and Proposition 2 applies directly leading to the following hypothesis:

Hypothesis 2 (Bonus)**.** *Subjects' average effort will increase with the project's bonus.*

In the experiment, $u_{ya} \geqslant 0$ and therefore Proposition 3 cannot be applied to sign the effect of wage. We have seen that in the special case of exponential utility the optimal effort level did not depend on the wage. In the numerical simulations with CRRA utility, effort monotonically increases with wage. It is, therefore, reasonable to expect the following behavior.

Hypothesis 3 (Wage)**.** *Subjects' average effort will monotonically increase with the project's wage.*

The experimental design includes one more treatment variable, k, which is the scaling factor of the cost of effort function. The comparative statics for the cost variable cannot be derived in a general model, since k only arises in the parametrizations of the model. The effect of k on the optimal effort, however, can be evaluated in the special cases we considered earlier. These special cases suggest the following behavior.

Hypothesis 4 (Cost of effort)**.** *Subjects' average effort will decrease with the cost of effort.*

5 Results

5.1 Treatment Effects

Bonus

Figure 2a shows the empirical CDFs[20] of subjects' effort levels for the two values of bonus.[21] The increase in bonus causes a clear increase in subjects' efforts across the entire distribution. The mean

[20] Figures D.1 and D.2 in the Appendix D.1 also show the means and histograms of effort.

[21] Since the dataset has a panel structure (every subject experiences multiple treatments), I first compute the mean effort levels for each value of a treatment variable for each subject and then report statistics for these subject-level means.

effort under $z = 2$ is 0.53 and the mean effort under $z = 4$ is 0.64, which implies a positive average treatment effect (ATE) of 0.11. The treatment effect of bonus is highly statistically significant ($p-$value < 0.001, Wilcoxon signed rank test with continuity correction).

Wage

Figure 2b shows the empirical CDFs of subjects' effort levels for the two values of wage.[22] The increase in wage causes a weak increase in subjects' efforts. This effect is present for low effort levels, while being virtually absent at high effort levels. The mean effort under $w = 1$ is 0.58 and the mean effort under $w = 2$ is 0.61, which implies a positive ATE of 0.03. The treatment effect of wage is borderline statistically significant ($p-$value $= 0.04$, Wilcoxon signed rank test with continuity correction).

Cost

Figure 2c shows the empirical CDFs of subjects' effort levels for the two values of cost. The increase in cost causes a strong decrease in subjects' efforts across the entire effort distribution. The mean effort under $k = 1$ is 0.61 and the mean effort under $k = 2$ is 0.49, which implies a negative ATE of -0.12. The treatment effect of cost is highly statistically significant ($p-$value < 0.001, Wilcoxon signed rank test with continuity correction).

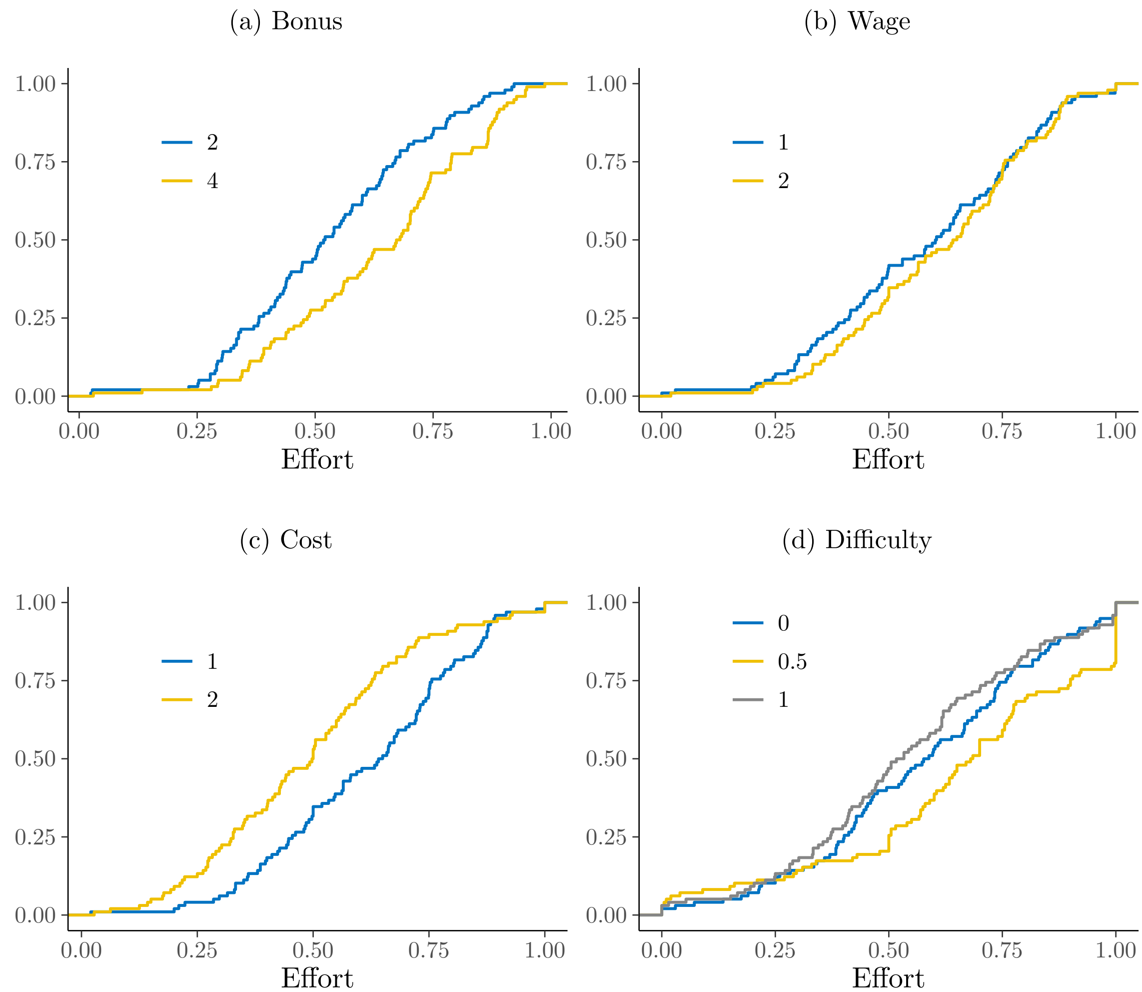
Difficulty

Figure 2d shows the empirical CDFs of subjects' effort levels for the three values of difficulty. The first increase in difficulty (from 0 to 0.5) causes a strong increase in subjects' efforts across the entire (with the exception of a lower tail) effort distribution. The mean effort under $\theta = 0$ is 0.57 and the mean effort under $\theta = 0.5$ is 0.65, which implies a positive ATE of 0.08. The treatment effect of the first increase in difficulty is highly statistically significant ($p-$value $= 0.001$, Wilcoxon signed rank test with continuity correction). The second increase in difficulty (from 0.5 to 1) causes a strong decrease in subjects' efforts across the entire (again with the exception of a lower tail)

[22] To analyze the effect of wage, I use the subset of observations with $k = 1$. For the full sample, wage and cost are highly correlated, which is due to the nature of the design. In the experiment, cost cannot exceed wage, otherwise a negative profit could occur and subjects could lose money. When analyzing the effect of cost, the sample is restricted to observations in which $w = 2$ for the same reason.

effort distribution. The mean effort under $\theta = 1$ is 0.65, which implies a negative ATE of the second increase in difficulty of -0.11. The treatment effect of the second increase in difficulty is, again, highly statistically significant ($p-$value $= 0.002$, Wilcoxon signed rank test with continuity correction). Interestingly, the dramatic increase in difficulty from the lowest to the highest value results only in a slight change in mean effort by -0.03, which is not even statistically significant ($p-$value $= 0.195$, Wilcoxon signed rank test with continuity correction).

Figure 2: Empirical CDFs of Effort by Treatment Variable



Note: The graph shows empirical CDFs of effort levels broken down by a treatment variable and a value of the variable. Effort levels used to plot CDFs are subject-level mean effort levels for each value of a treatment variable.

Robustness

The panel design of the experiment allows for a robustness check based on using *ceteris paribus* pairs. A pair of samples of effort is called a *ceteris paribus* pair (*CP*-pair) for a treatment variable x, if only x changes within a pair while the rest of the treatment variables are constant. Let δ denote the vector of values of the treatment variables. Each treatment $j \in J$ represents a particular combination of the values of treatment variables from the set of all treatments J, $\delta_j \equiv (z_j, w_j, k_j, \theta_j)$. Set J_x is the set of all *CP*-pairs for a treatment variable x and is defined as

$$J_x = \left\{ (j_1, j_2) \mid j_1, j_2 \in J, x_{j_1} = x^1, x_{j_2} = x^2, x^1 < x^2, (\delta_{-x})_{j_1} = (\delta_{-x})_{j_2} \right\},$$

where δ_{-x} denotes the vector of values of treatment variables except x, and for convenience the first index corresponds to a treatment with a lower value of the treatment variable. Let k index the elements of J_x and $K_x \equiv ||J_x||$ be the number of elements in this set. For every *CP*-pair $k = 1, \ldots, K_x$ the two samples used in the *ceteris paribus* tests are

$$\left\{ a_{i,(j_1)_k} \right\}_{i=1,\ldots,n}, \left\{ a_{i,(j_2)_k} \right\}_{i=1,\ldots,n}, (j_1, j_2)_k \in J_x,$$

where $a_{i,j}$ denotes effort chosen by a subject i in a treatment j, and n is the total number of subjects in the experiment. Since hypotheses are derived from comparative statics results, the *ceteris paribus* test is a more direct test of hypotheses. Table D.1 in Appendix D.2 confirms that treatment effects hold under a *ceteris paribus* test: the direction of the treatment effects, with a few exceptions, is consistent across the *CP*-pairs. The treatment effects are also confirmed in a regression analysis. See Table D.2 in Appendix D.2 for the estimates of a panel regression with fixed effects.

Discussion

The effects of conditional and unconditional rewards are consistent with the previous findings in the literature and can be viewed as a validation of the experimental design. The positive effect of higher bonus supports Hypothesis 2 and is in line with the results reported in (Lazear, 2000; Hossain and List, 2012) and more recently by (DellaVigna and Pope, 2018; de Quidt et al., 2018).

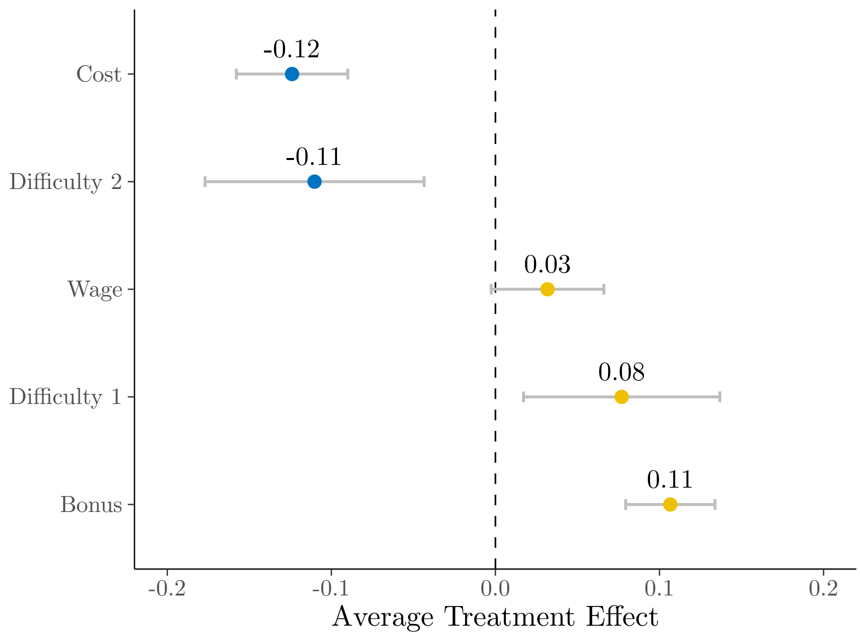
Some papers suggest that when conditional rewards are too small or too high, they can backfire and lead to lower effort either through crowding out of intrinsic motivation or chocking-under-pressure (Gneezy et al., 2011). I do not observe these negative effects in my data because the chosen effort framework leaves little space for these two channels and also because the level of conditional rewards in the experiment apparently did not hit either extreme. I do note, however, that while the effect of bonus is highly statistically significant, its economic significance is somewhat disappointing. Recall that in the experiment the monetary value of a bonus doubles from \$20 to \$40. This substantial change in stakes, however, does not lead to doubling effort, as a simple risk-neutral model (4) would suggest. Effort increases only by 20% or by 0.52 standard deviations.

The weakly positive effect of higher wage supports Hypothesis 3 and is also consistent with the previous studies on the effect of unconditional rewards. A common finding is that unconditional rewards are effective in the short-run (Gneezy and List, 2006; Jayaraman et al., 2016) and when the reciprocity channel exists (Cohn et al., 2014; Hennig-Schmidt et al., 2010). Since the subjects in the experiment were unlikely to have reciprocal motives—their performance did not benefit anyone else but them—the increase in wage did not result in a significant increase in effort.

The strong negative effect of higher cost supports Hypothesis 4. While the effect of cost of effort is relatively unexplored in the studies of individual effort, the contest literature, as reviewed by Dechenaux et al. (2015), finds that higher costs generate lower effort. The results in the present individual-effort experiment are therefore consistent with the results in contests. The present results are also consistent with a recent real-effort study by Goerg et al. (2019) that employs an individual-choice setting.

The inverse-U effect of difficulty support Hypothesis 1.C and is new to the economics literature. This result, however, is consistent with the previous findings in psychology on the motivational intensity theory, as reviewed by Gendolla et al. (2012). This literature finds that subjects increase their effort in response to higher difficulty up to a certain point after which effort is decreased. A remarkable feature of the present result is that changing difficulty produces a powerful incentive effect that is comparable to the effect of doubling the conditional reward or doubling the cost of effort, see Figure 3. The effect of difficulty on effort, however, is more noisy than the effects of monetary rewards or cost.

Figure 3: Comparison of ATEs

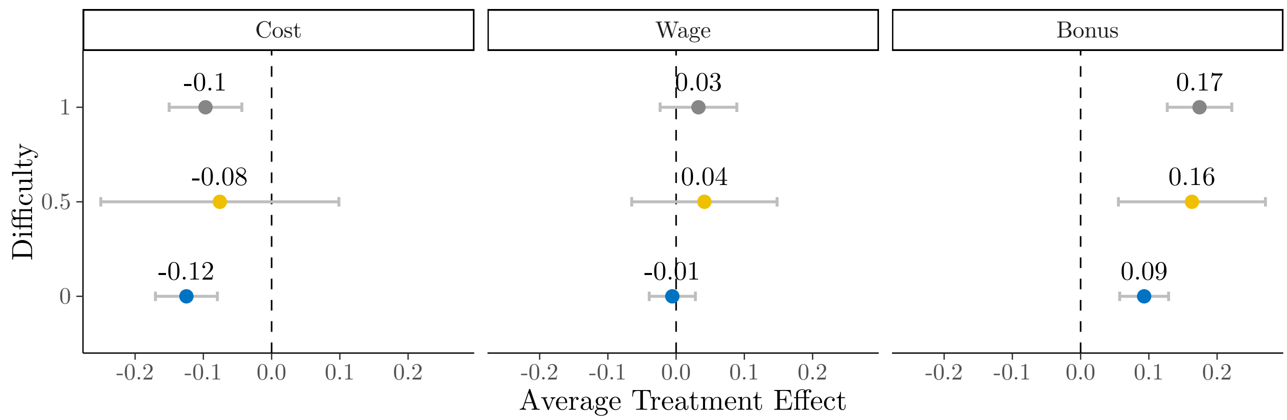


Note: The graph plots the ATEs of increasing the value of each treatment variable, ordered by the magnitude of the effect. The change in difficulty is broken down into the first increase from 0 to 0.5 (Difficulty 1) and a second increase from 0.5 to 1 (Difficulty 2). Horizontal bars represent 95% confidence intervals from a paired $t-$test.

5.2 Interaction Effects

Is it possible that difficulty mediates the effects of monetary rewards and cost? To explore this possibility, I compute the ATEs of bonus, wage, and cost at different values of difficulty and plot them on Figure 4. The figure shows that higher difficulty in general increases the treatment effects of the rest of variables but in many cases this interaction is too small to be statistically significant. In particular, increasing difficulty from 0 to 1 increases the ATE (reduces the negative ATE) of cost by 0.03 ($p-$value $= 0.565$, Anova), increases the ATE of wage by 0.04 ($p-$value $= 0.214$, Anova), and increases the ATE of bonus by 0.08 ($p-$value $= 0.041$, Anova). Hence the data suggest that conditional rewards are most effective in stimulating effort when difficulty is high. This result is consistent with Vandegrift and Brown (2003) who also find that conditional rewards are more effective on difficult tasks.

Figure 4: ATEs Conditional on Difficulty



Note: The figure plots the ATEs of cost, wage, and bonus computed at different values of difficulty. The vertical bars represent 95% confidence intervals from a $t-$test.

5.3 Individual Heterogeneity

Figure 5 shows the distributions of the average treatment effects for each treatment variable computed at a subject level. Colors highlight the probability masses of the negative (red) and positive (blue) ATE's. The subjects are typed based on the sign of their ATE as either Decreasing (negative ATE) or Increasing (positive ATE) for bonus, wage, and cost. The typing for difficulty is more complicated and explained below.

Bonus

The distribution of the ATE's for bonus is unlikely to be normal (Shapiro-Wilk test, $p = 0.02$). The majority of subjects, 81%, increase their effort in response to higher bonus. The proportion of Increasing types is significantly higher than the proportion of Decreasing types (test for equality of proportions, $p < 0.001$). The behavior of Decreasing types is hard to rationalize (it would imply that those subjects prefer less money to more money), and thus is probably due to confusion. While the proportion of Decreasing types is non-negligible, the mean ATE for them is only -0.059. Excluding the subjects who make errors would increase the mean effect size to 0.147 from the unconditional average of 0.107.

Wage

The ATE's for wage are well approximated by a normal distribution (Shapiro-Wilk test, $p = 0.621$). The increase in wage results in higher effort for 62% of the subjects. The remaining 38% reduce their effort on average by 0.14, which is comparable to the mean effect size for Increasing types, 0.136. The proportion of Increasing types is significantly higher than the proportion of Decreasing types (test for equality of proportions, $p = 0.02$), but the mean effect size across all the subjects, 0.032, is close to zero. The behavior of Decreasing types, given their strong presence in the distribution, is unlikely to be entirely driven by errors and might reflect actual preferences.

Cost

The ATE's for cost are also well approximated by a normal distribution (Shapiro-Wilk test, $p = 0.794$). The effect of increasing cost is negative for 74% of the subjects. The proportion of Decreasing types is significantly higher than the proportion of Increasing types (test for equality of proportions, $p < 0.001$). The proportion of Increasing types is non-negligible, but the mean effect size for them, 0.087, is relatively small. Their behavior is likely to be caused by errors. Excluding them would reduce (increase in absolute terms) the mean effect size from -0.124 to -0.196.

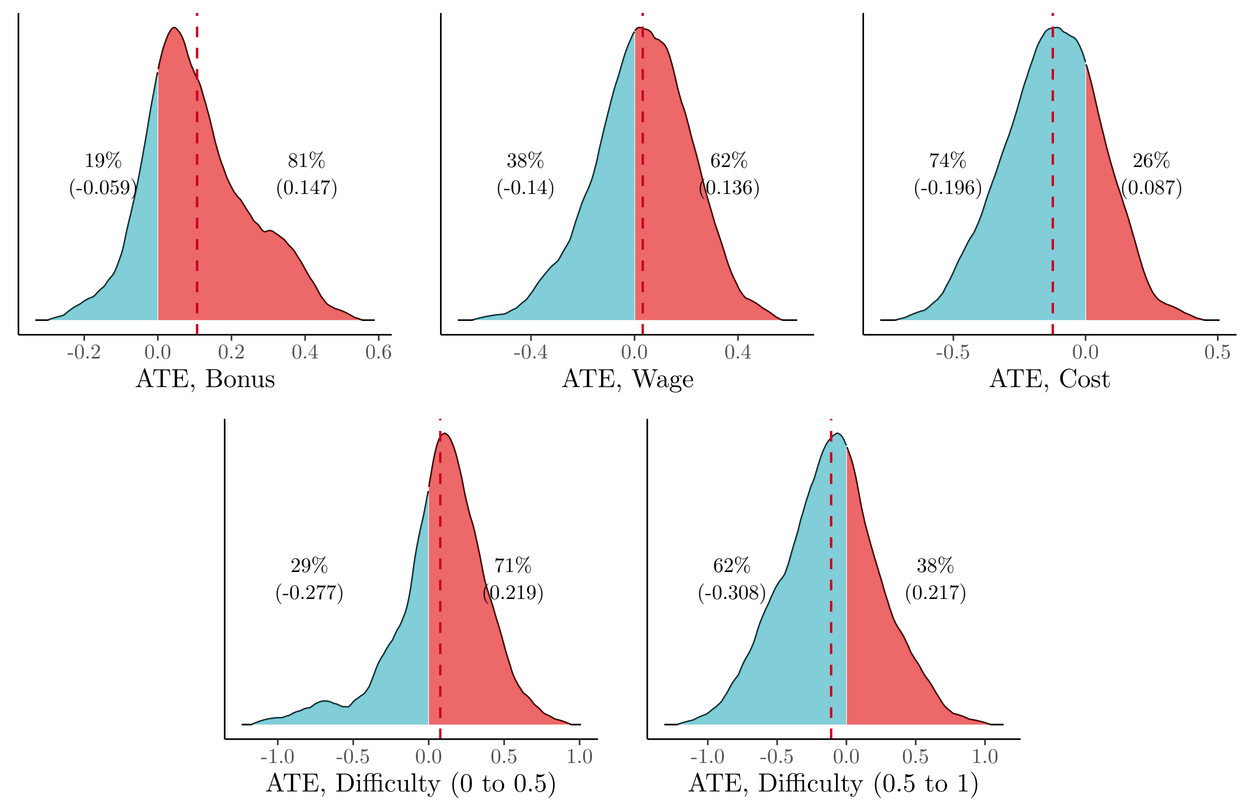
Difficulty

The first increase in difficulty results in higher effort for 71% of the subjects. The mean effect size for those subjects is 0.219. The distribution of the ATE's for the first increase in difficulty is unlikely to be normal (Shapiro-Wilk test, $p = 0.001$). The second increase in difficulty leads to lower effort for 62% of the subjects, with the conditional mean effect size of -0.308. The ATE's for the second increase are well approximated by a normal distribution (Shapiro-Wilk test, $p = 0.968$). The subjects who increase their effort initially are not necessarily the same subjects who then reduce their effort (e.g., some of them increase their effort even further). The response to difficulty is thus characterized by Increasing, Decreasing, U, or Inverse-U types,[23] which requires knowing the full response path and cannot be shown on a distributions picture like in Figure 5.

[23] Specifically, Increasing types increase their effort on both intervals of difficulty (from 0 to 0.5 and 0.5 to 1), Decreasing types reduce their effort on both intervals, U types reduce their effort on the first interval and increase their effort on the second interval, and Inverse-U types increase their effort on the first interval and reduce their effort on the second interval. I omit the intermediate values of difficulty 0.25 and 0.75 to simplify the classification of types.

The dominant type is Inverse-U, with 50% of subjects conforming to it (see Table 2d). The equality of proportions of each type is clearly rejected (test for equality of proportions, $p < 0.001$). The subjects are split roughly equally among the three other types, with Increasing type being the second largest group after Inverse-U type.

Figure 5: Individual Heterogeneity in ATEs



Note: The figure plots the ATEs computed at a subject level. The vertical bars represent the means of distributions. The numbers are the percentages of subjects with negative and positive ATE, with the conditional ATEs for each type in parenthesis.

Individual Heterogeneity and Gender

An important question is whether the differences between the types can be attributed to the observable subjects' characteristics, such as gender. In the cases of bonus and cost, this amounts to asking whether females or males are more likely to make errors. Table 2 shows the contingency tables for each treatment variable, with subjects' gender in rows and subjects' response type in columns. For bonus (Table 2a), the proportion of females belonging to a decreasing type, 17%, is lower than the corresponding proportion of males, 23%, however, this difference is not statistically significant (Fisher's exact test, $p = 0.608$). A similar result holds for wage (Table 2b), the proportion of females belonging to a decreasing type, 34%, is slightly lower than the corresponding proportion of males, 40%, but not significantly so (Fisher's exact test, $p = 0.67$). The proportion of females who decrease their effort in response to higher cost, 70%, (Table 2c) is slightly lower than the corresponding proportion of males, 79%, with no significant difference (Fisher's exact test, $p = 0.478$). The effect of difficulty (Table 2d), however, does reveal a marginally significant association between gender and response type (Fisher's exact test, $p = 0.048$). The proportion of males belonging to the Inverse-U type, 64%, is higher than the corresponding proportion of females, 36%. The females are more likely than males to exhibit the U and Increasing effort response to difficulty.

Table 2: Association Between Gender and Response Type by Treatment Variable

(a) Bonus

	Decreasing	Increasing	Total
Female	8	39	47
Male	11	36	47
Total	19	75	94

Fisher's exact test, $p = 0.608$

(b) Wage

	Decreasing	Increasing	Total
Female	16	31	47
Male	19	28	47
Total	35	59	94

Fisher's exact test, $p = 0.67$

(c) Cost

	Decreasing	Increasing	Total
Female	33	14	47
Male	37	10	47
Total	70	24	94

Fisher's exact test, $p = 0.478$

(d) Difficulty

	U	Inv-U	Decr	Incr	Total
Female	10	17	7	13	47
Male	5	30	6	6	47
Total	15	47	13	19	94

Fisher's exact test, $p = 0.048$

5.4 Structural Analysis

In this section I ask whether a simple structural model can explain the observed behavioral patterns. As highlighted by the theoretical analysis in Section 4, the inverse-U pattern of effort response to difficulty can be accommodated by the RDU model, but not by the EUT model. Therefore, I will be estimating the parameters of the RDU model.

Estimation Procedures

Consider an agent with preferences β who faces treatment δ. The rank-dependent utility of effort choice $a \in A = \{0, 0.01, \ldots, 1\}$ will be given by

$$U(a \mid \delta, \beta) = \omega(p(a,\theta) \mid \beta_p)u(w+z, a \mid \beta_u) + (1 - \omega(p(a,\theta) \mid \beta_p)u(w, a \mid \beta_u),$$

where $\beta = (\beta_p, \beta_u)$, $\omega : [0,1] \mapsto [0,1]$ is the probability weighting function parametrized by β_p, and $u : \mathbb{R}_+ \times [0,1] \mapsto \mathbb{R}$ is the utility function parametrized by β_u. In the experiment, the utility function u is $u(y, a \mid \beta_u) = v(y - c(a) \mid \beta_u)$. The cost function c and the probability of success function p are defined, as before, by equations (2) and (1), respectively.

I assume that ω takes the two-parameter Prelec form (Prelec, 1998), which is frequently used in applied work (Filiz-Ozbay et al., 2015; Wilcox, 2015a; l'Haridon and Vieider, 2019) and has been show to have good empirical properties (Stott, 2006), with $\beta_p = (\alpha, \psi)$

$$\omega(p \mid \beta_p) = \exp(-\psi(-\ln p)^{\alpha}),$$

where $\psi > 0, \alpha > 0$. I also assume that v takes the standard constant relative risk aversion (CRRA) form, with $\beta_u = \gamma$

$$v(x \mid \beta_u) = \frac{x^{1-\gamma} - 1}{1-\gamma}.$$

I estimate a representative agent model on the pooled data and use i to index individual observations. In each observation, effort is chosen to maximize $U(a \mid \delta_i, \beta)$, and the optimal effort as a function of δ and β is denoted $a^*(\delta, \beta)$. No closed-form expression for $a^*(\delta, \beta)$ exists for a given model specification, therefore, I rely on a numerical solution for optimal effort. I assume that the observed effort follows

$$a_i = a^*(\delta_i, \beta) + \epsilon_i,$$

where ϵ_i is a mean-zero error term. To estimate the model, I use a non-linear least squares estimator,[24] which minimizes the sum of squared deviations between the observed and predicted

[24] One could estimate the model using Maximum Likelihood estimator (MLE).

choices:

$$\hat{\beta} = \arg\min_{\beta} \sum_{i=1}^{N} (a_i - a^*(\delta_i, \beta)).$$

The risk-neutral parameter vector $\beta = (0, 1, 1)$ is used as a starting value.

Estimation Results

Figure 6 compares the predicted effort from the estimated model to the actual effort, split by treatment variable. The figure shows that the estimated model reproduces the comparative statics found in the data reasonably well. In particular, the model does capture the inverse-U pattern of effort response to difficulty. It over-predicts, however, the mean effort for $\theta = 0.75$ leading to a more prolonged increase in effort as difficulty increases and a sharper drop in effort when difficulty changes from 0.75 to 1. The estimated model generates somewhat larger changes in effort in response to changes in wage, bonus, and cost than observed in the data.

Table 3 presents the estimation results. The estimates show that subjects are moderately risk averse in terms of the contribution of the curvature of the utility function to risk aversion. This finding is consistent with the previous findings in the laboratory experiments (Holt and Laury, 2002; Andersen et al., 2008), however the estimate for the CRRA parameter is higher than is typically reported for binary lottery choices (Harrison and Rutström, 2008)[P. 121]. The 95% confidence interval for γ covers the value of one, which implies a special case of a logarithmic utility.

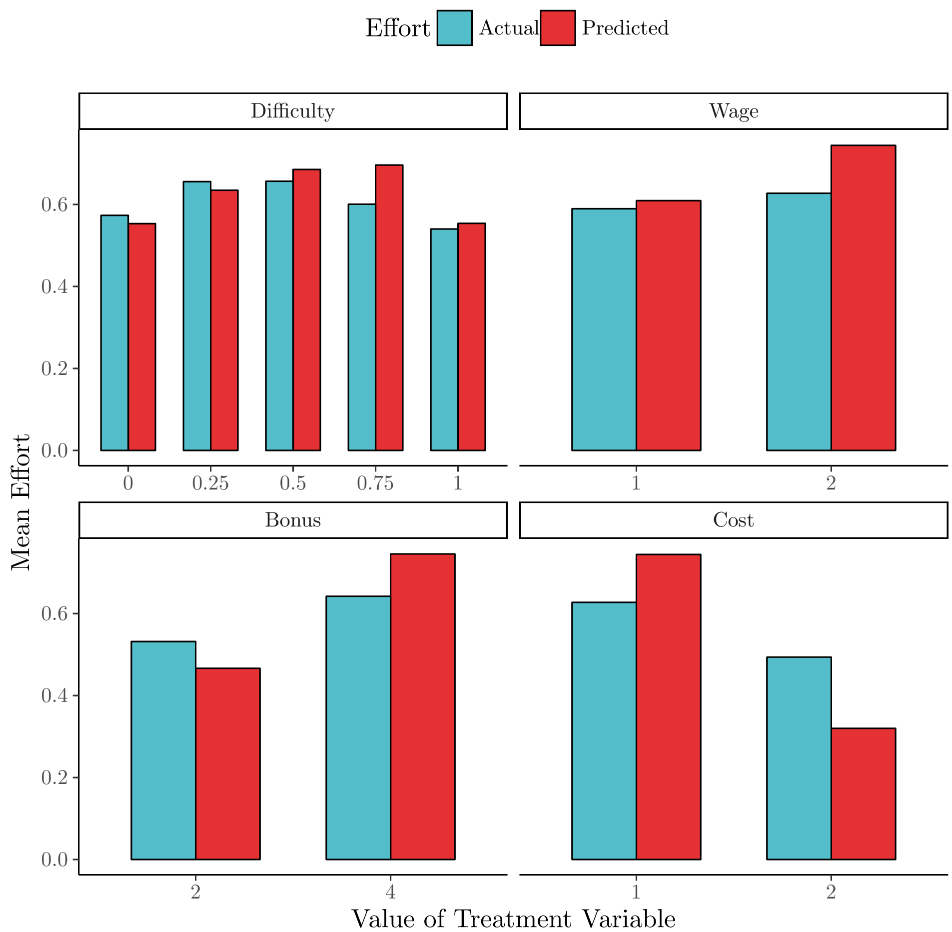
Table 3: Estimates of RDU Model

Parameter	Estimate	SE	2.5%	97.5%
γ	0.978	0.117	0.748	1.208
α	1.603	0.071	1.464	1.743
ψ	0.821	0.054	0.716	0.926

N obs = 1625

The estimate of α, which determines the shape of the probability weighting function, is significantly greater than one and leads to an S-shaped probability weighting. The S-shaped probability weighting implies likelihood sensitivity: subjects underweight the likelihoods of rare outcomes. As highlighted by the theoretical analysis in Section 4, such a shape arises precisely to fit the inverse-U pattern of effort response to difficulty that is observed in the data. Figure 7 (left panel) shows the

Figure 6: Actual and Predicted Mean Effort Levels

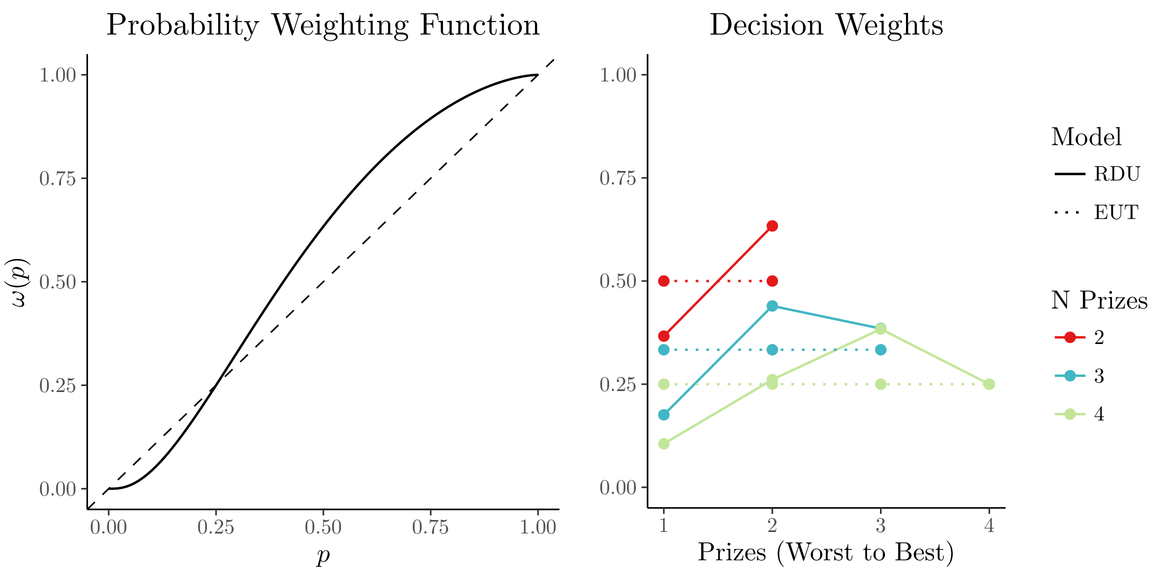


Note: The figure plots actual mean effort and mean effort predicted from the estimated model by treatment variable.

estimated probability weighting function. The underweighting occurs for probabilities roughly less than 0.25. Probabilities greater than 0.25 are overweighted. The right panel of Figure 7 shows the implied decision weights from the estimated probability weighting function. The decision weights are computed using *equiprobable* lotteries with different number of outcomes (two, three, or four). The picture shows that extreme outcomes are underweighted when there are more than two outcomes, and the worst (best) outcome is underweighted (overweighted) when there are exactly two outcomes.

The estimated shape of the weighting function is in stark contrast with some of the previous estimates (Wu and Gonzalez, 1996; Bruhin et al., 2010; l'Haridon and Vieider, 2019) that report an inverse-S shaped probability weighting function first proposed by Kahneman and Tversky (1979).[25] While an inverse-S-shaped weighting function leads to overweigthing of small probabilities, I find underweighting of small probabilities. Interestingly, underweighting of small probabilities in a labor context was reported recently by DellaVigna and Pope (2018) who find that subjects significantly underweight a 1% chance of winning the prize. Their estimates indicate that subjects perceive 1% as 0.2–0.38%, depending on the estimated model. My estimates, however, imply a more extreme underweighting: a 1% chance of receiving a prize is perceived as only 0.007%. The present results are also related to the results reported by Wilcox (2015b) who finds that the second most common type of probability weighting (after the concave shape) is also S-shaped.

Figure 7: Probability Weighting Function and Implied Decision Weights from Equiprobable Lotteries



Individual Heterogeneity

To explore individual heterogeneity in preference parameters, I re-estimate the model allowing the parameters to depend on demographic characteristics. I assume that each behavioral parameter β^j can be written as a linear combination of demographic indicators for gender and race, with Black

[25] Also see a meta-analysis of probability weighting estimates in DellaVigna and Pope (2018)[Online Appendix Table 3].

male being the base category:

$$\beta^j = \beta^j_{Constant} + \beta^j_{Female}\,\mathbb{I}(Gender_i = Female) + \\ + \beta^j_{White}\,\mathbb{I}(Race_i = White) + \beta^j_{Asian}\,\mathbb{I}(Race_i = Asian).$$

Table 4 presents the estimation results, which reveal preference differences between demographic groups. Females tend to have a higher estimate of the coefficient of CRRA than males, while Whites tend to have a lower estimates than Blacks or Asians. There are no significant differences in the coefficient of CRRA between Blacks and Asians. Turning to the estimates of the probability weighting function, females tend to have a slightly higher estimate of the shape parameter α than males, though the difference is not statistically significant. Similarly, there are no statistically significant differences in the estimates of α between racial groups. The estimates of the scale parameter ψ show that females tend to have lower estimates than males, while Whites tend to have higher estimates than Blacks. There are no statistically significant differences in the estimate of ψ between Asians and Blacks or Asians and Whites.

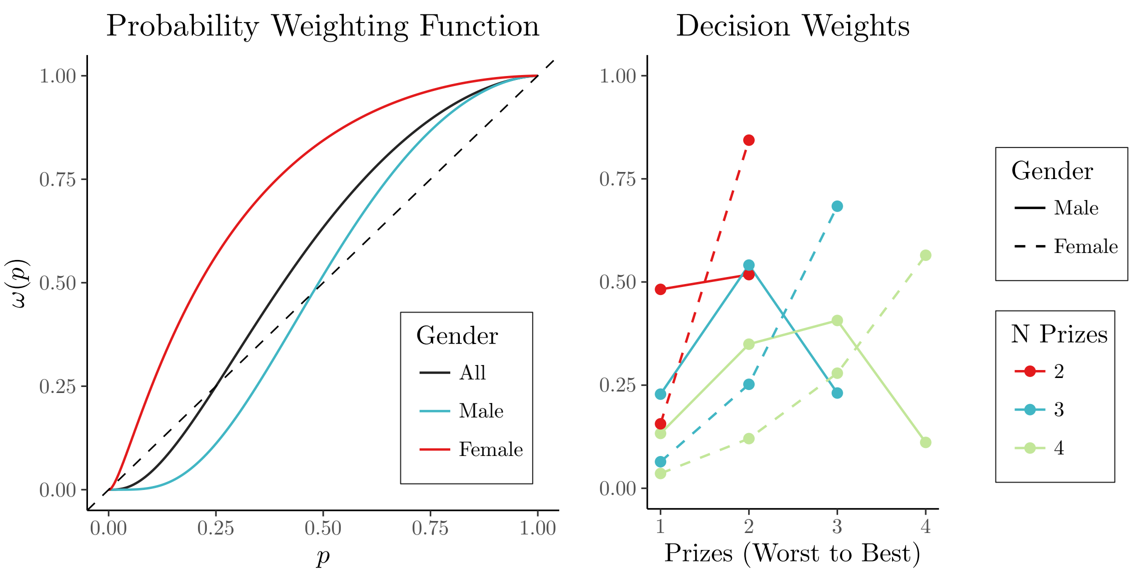
Table 4: Estimates of RDU Model from Main Task with Demographic Covariates

Parameter	Estimate	SE	2.5%	97.5%
γ				
Constant	0.528	0.150	0.234	0.821
Female	1.339	0.388	0.577	2.100
White	−1.375	0.486	−2.327	−0.422
Asian	0.250	0.193	−0.129	0.629
α				
Constant	1.479	0.084	1.314	1.644
Female	0.121	0.174	−0.219	0.461
White	−0.204	0.195	−0.586	0.178
Asian	0.034	0.148	−0.256	0.325
ψ				
Constant	1.075	0.079	0.921	1.230
Female	−0.796	0.095	−0.982	−0.611
White	0.615	0.161	0.300	0.930
Asian	0.222	0.114	−0.001	0.446

N obs = 1625.

Figure 8 interprets these estimates graphically by showing the estimated probability weighting functions and implied decision weights from equiprobable lotteries for males and females separately. For males, the probability weighting function is clearly S-shaped, while for females, it is closer to being simply concave. As a result, males tend to overweight outcomes between the extremes, while females tend to overweight best outcomes. Figure D.3 in Appendix D.1 presents similar results for races. The differences between races are less pronounced than the differences between genders, which results in similar S-shaped probability weighting functions and similar decision weights.

Figure 8: Estimated Probability Weighting Functions and Implied Decision Weights from Equiprobable Lotteries by Gender (Effort Task)



6 Conclusion

Research in labor economics has traditionally focused on monetary rewards as the primary incentive tool for principals to incentivize agents. Research in behavioral economics shows that monetary rewards are subject to psychological factors and that alternative behavioral incentives are also relevant for building incentive schemes for agents. I contribute to this strand of behavioral economics by testing a prominent psychological theory of motivation, motivational intensity theory, in an economic environment. Motivational intensity theory argues that the primary determinant of effort is task difficulty, while monetary rewards serve as a mediating factor.

I study the effect of task difficulty and monetary rewards on effort in an incentivized lab experiment. I find that difficulty has an inverse-U effect on effort: effort first increases as difficulty goes up, reaches a peak, and then drops as difficulty continues to increase. The incentive effect of difficulty is on par with the incentive effect of conditional monetary rewards. Difficulty mediates the effect of monetary rewards: conditional rewards are most effective at inducing effort when task difficulty is intermediate or high. I theoretically show that the inverse-U effect of difficulty is inconsistent with the EU model. It can be generated, however, by the RDU model that allows for non-linear probability weighting. I estimate a structural RDU model and find evidence of an S-shaped probability weighting function.

What causes the estimates of probability weighting in the present setting to differ so dramatically from previous estimates, which typically find an inverse-S-shaped probability weighting function? The first possible explanation is that the present experiment is framed in a labor context, while previous studies typically used an abstract lottery context. Contextual instructions can affect subjects' behavior, as is evident from a variety of studies (Harrison and List, 2004; Alekseev et al., 2017). This explanation is consistent with the recent evidence in DellaVigna and Pope (2018) who report underweighting of small probabilities in a field experiment on effort. However, this explanation raises the follow-up question that would deserve further investigation: what features of a labor context make it different from an abstract context.

Another difference between the present study and previous studies on lottery tasks is the frequency of feedback. In the effort task, the feedback is provided every round, while in the lottery tasks, the feedback is typically provided only after all the choices are made. Evidence from incentivized lab experiments suggests that providing frequent feedback can make subjects more sensitive to probabilities. Frequent feedback thus can result in linear probability weighting (Van de Kuilen, 2009), or even in S-shaped probability weighting (Hertwig et al., 2004; Hertwig and Erev, 2009) in a binary lottery choice setting.

The present findings are relevant for the design of optimal incentive schemes for workers. The strong incentive effect of difficulty calls for taking it into account when assigning tasks to workers. A manager can expect that it will take workers more effort to complete a moderately hard task than an easy task. However, a very hard task might not be devoted as much effort as desired.[26] This behavioral response amplifies the direct negative effect of difficulty and thus can substantially lower a worker's performance. A principal can counter this detrimental behavioral effect of difficulty using conditional rewards since they are found to be most effective when difficulty is high.

[26] While this is true for most subjects in this experiment, one has to be mindful about possible heterogeneity in responses, especially between males and females.

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Appendices

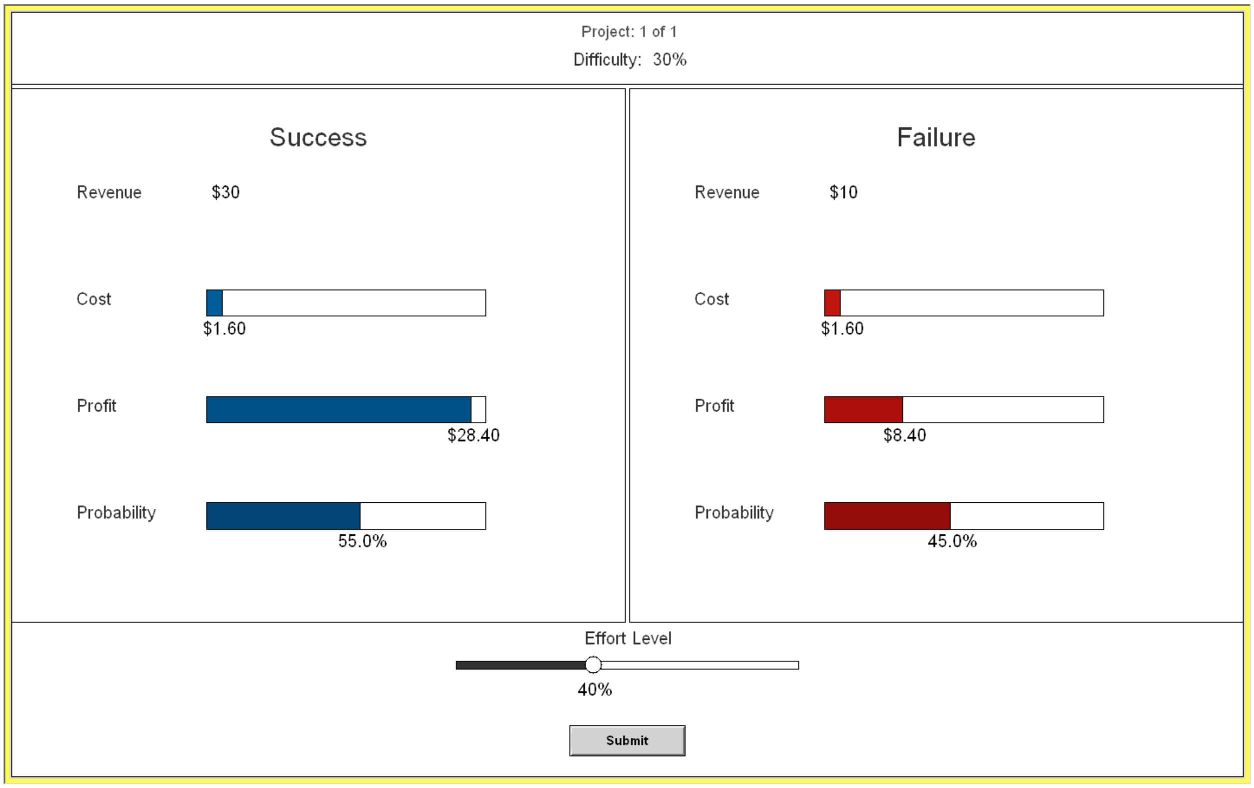
A Design of the Effort Task

Table A.1: Summary of Treatments

id	θ	w	z	k	5_21_15	5_28_15	6_2_15	6_3_15	6_4_15	6_5_15
1	0	1	2	1	1	1	1	1	1	1
2	0	1	4	1	1	1	1	1	1	1
3	0	2	2	1	1	1	1	1	1	1
4	0	2	2	2	1	1	1	1	1	1
5	0	2	4	1	1	1	1	1	1	1
6	0	2	4	2	1	1	1	1	1	1
7	0.25	1	2	1	1	1	1	0	0	0
8	0.25	2	2	1	0	0	0	1	1	1
9	0.25	2	2	2	0	0	1	0	0	0
10	0.25	2	4	1	0	0	0	1	1	1
11	0.50	1	2	1	1	1	1	0	0	0
12	0.50	2	2	1	0	0	1	1	1	1
13	0.50	2	2	2	0	0	1	0	0	0
14	0.50	2	4	1	0	0	0	1	1	1
15	0.75	1	2	1	1	1	0	0	0	0
16	0.75	2	2	1	0	0	1	1	1	1
17	0.75	2	2	2	0	0	1	0	0	0
18	0.75	2	4	1	0	0	0	1	1	1
19	1	1	2	1	1	1	1	1	1	1
20	1	1	4	1	1	1	1	1	1	1
21	1	2	2	1	1	1	1	1	1	1
22	1	2	2	2	1	1	1	1	1	1
23	1	2	4	1	1	1	1	1	1	1
24	1	2	4	2	1	1	1	1	1	1

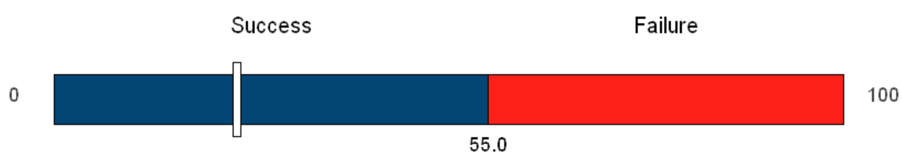
Notes: The first column is an id of a treatment, and the next four columns show the values of the treatment variables for that treatment. The last six columns correspond to the six sessions and indicate whether a treatment was (1) or was not (0) used in a session.

Figure A.1: Choice Screen for the Effort Task



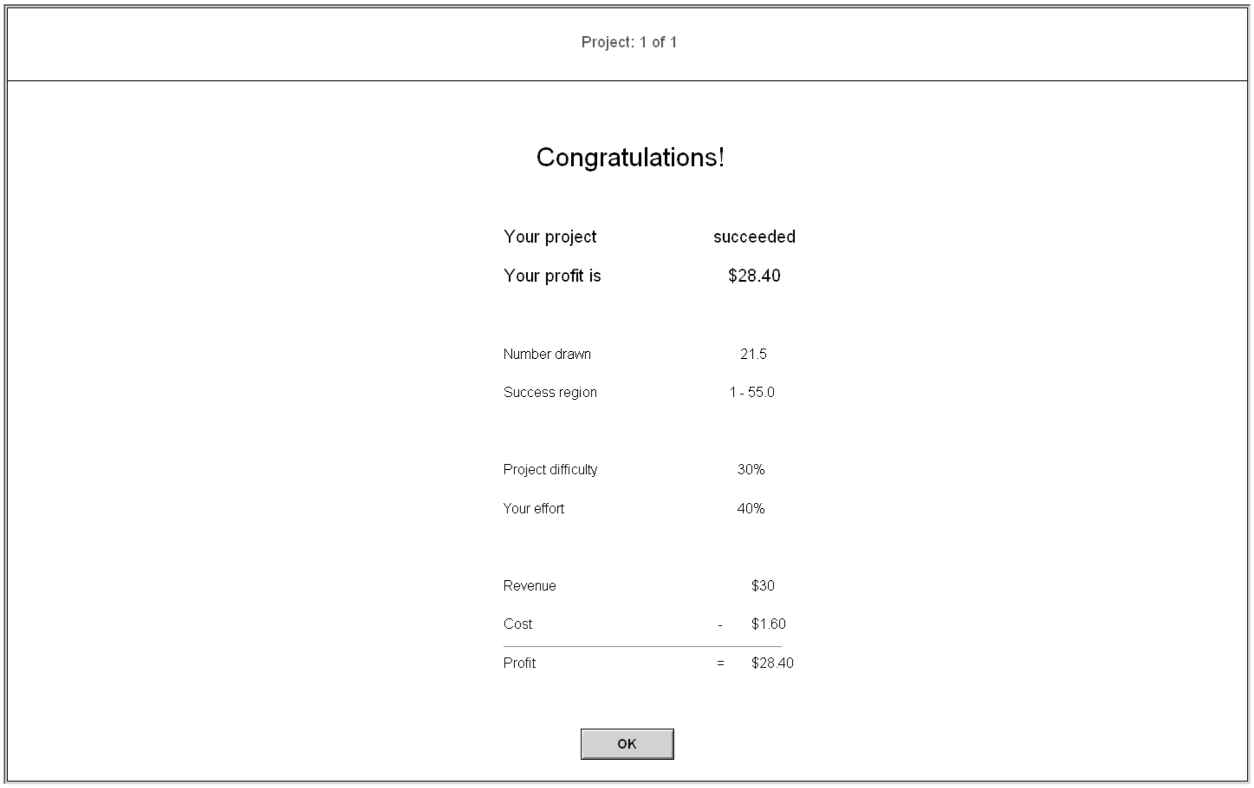
Note: The figure shows the graphical interface of the decision screen. The top of the screen showed the current round and a current project's difficulty. The middle of the screen was split into the Success and Failure regions that showed the corresponding values of revenue, cost, profit, and probability for each outcome. Subjects could choose their effort level by dragging a slider at the bottom of the screen. The slider could take one of 101 positions from a set $\{0, 1, 2, \ldots, 100\}$%. Subjects could observe how the values of probabilities, costs, and profits were changing as they experimented with the effort level. These variables were represented both numerically and graphically as colored bars.

Figure A.2: Project Outcome



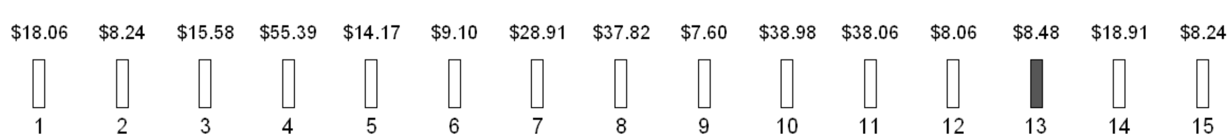
Note: The figure shows the graphical representation of a project's outcome. An outcome screen featured a bar with the success and failure regions, which corresponded to the success and failure probabilities determined by a subject's choice of effort. Along this horizontal bar, a white slider was moving quickly. The slider's position at each fraction of a second was determined by a draw from a uniform random distribution. After three seconds, the needle stopped either in the success or failure region, which determined whether the project succeeded or failed.

Figure A.3: Project Summary



Note: The figure shows a summary screen. The screen presented the outcome of a project and the realized profit, along with the details on the random draw, the characteristics of a project, realized revenue and cost.

Figure A.4: Payoff for the Effort Task



Note: The figure shows the final screen that determined the payoff for the effort task. The screen showed all the past rounds and profits made in those rounds. Every fraction of a second a random bar was highlighted, and after three seconds the highlighting stopped, which determined the payoff for the effort task.

B Subject Instructions

Introduction

Welcome and thank you for participating! This is an experiment in individual economic decision-making. Please, mute/turn off all of your electronic devices for the duration of the experiment.

Payment

Your total payment will consist of a participation payment of $5 and the sum of the payments from the two decision tasks. Your payment can be considerable and will depend on your decisions and chance. You will be paid in cash privately at the end of the session.

Time

Today's session will consist of a quiz on probabilities, two decision tasks and a demographic survey. The session will take up no more than 2 hours.

Payment Protocol

Each of the 2 decision tasks will have several rounds. At the end of each task we will randomly select one of the decision rounds to determine your payment. It is worthwhile to think carefully about each decision, since you don't know which decision round will be picked.

Privacy

You will not interact with other participants. Please, do not reveal your identity to anyone. You must not talk to other participants during the experiment.

Final Notes

Please read these instructions carefully. You are welcome to ask questions at any point. Just raise your hand and we will answer your question in private.

In this task you will choose an effort level for a project (Figure 1). A project has two possible outcomes: success or failure. In case of success the project will yield you a high revenue (i.e., payoff), in case of failure it will yield you a low revenue. The exact values of high and low revenues will be shown on the screen. The task is to select the level of effort you prefer the most. There are no right or wrong answers, just pick whatever suits you the most.

Effort

By choosing a higher effort level you increase the chances that the project will be successful. Equivalently it means that the chances of failure are reduced, because the probability of success and failure must add up to 100%. Effort is costly to you: the higher is the effort level, the higher is the cost. The cost will be subtracted from the revenue of the project. On the screen you can observe how chances of success/failure, cost of effort and the profit (=revenue minus cost) change as you change the effort level.

Example

On the Figure 1 you can see a project that gives you a revenue of \$30 if it's successful and \$10 if it fails. Suppose you chose an effort level of 40%, which leads to a 55% probability of success (45% probability of failure) and costs you \$1.60. In case of success your profit will be: \$30 (revenue) - \$1.60 (cost of effort) = \$28.40 (profit). If the project fails your profit will be: \$10 (revenue) - \$1.60 (cost of effort) = \$8.40 (profit). Note that you bear the cost of effort regardless of whether the project succeeds or fails.

Each additional unit of effort will increase the probability of success by the same amount but will cost you more than the previous one.

Example

Increasing effort from 0% to 1%, or from 1% to 2%, or from 2% to 3% (and so on) increases the probability of success by the same amount. Increasing effort from 99% to 100% costs more than an increase from 98% to 99%, which in turn costs more than an increase from 97% to 98% (and so on).

Difficulty

Another important characteristic of the project is its difficulty. Difficulty affects the chances of success, just like effort, but in the opposite way. A more difficult project is less likely to succeed than an easier one, for any given level of your effort. The difficulty of the project will appear on the top of the choice screen.

Example

In the previous example, suppose the difficulty was 30%. Now imagine that the project's difficulty increased to 70%. Given the same effort level as before, 40%, the probability of success might decrease to 25% (equivalently, the probability of failure might increase to 75%). Note that difficulty does not affect revenues or cost of effort, only the chances of success/failure.

Payoff

The outcome of the project will be determined right after you submit your choice. You will see a bar with a success region, a failure region and a randomly moving white needle (Figure 2). The regions are determined by your choice of effort. The needle is equally likely to appear at any position along the bar. It will stop after 3 seconds. If it ends up in the success region the project will succeed, if it ends up in the failure region the project will fail (Figure 3). The next screen will show you the summary of the current round (Figure 4).

Example

On the Figure 2 the probability of success is 55% and the probability of failure is 45%. If at the moment you stop the needle it is in the region 0–55, the project will succeed (Figure 3). If it is in the region 55.1–100, it will fail.

There will be 15 rounds, as well as 5 practice rounds. Rounds will differ in the project difficulty, costs and possible revenues. After you complete all the rounds, one of them will be randomly picked for payoff. You will see bars with rounds' numbers and their results (Figure 5). These bars will be randomly highlighted. Each bar is equally likely to be highlighted at any given moment. It will stop after 3 seconds and the payoff will be determined.

C Demographic Survey

1. What is your age?
2. What is your gender?
    - Male
    - Female
3. What is your racial or ethnic background?
    - White or caucasian
    - Black or African American
    - Hispanic
    - Asian
    - Native American
    - Multiracial
    - Other
    - Prefer not to answer
4. What is your marital status?
    - Married
    - Single
    - Divorced
    - Widowed
    - Other
    - Prefer not to answer
5. What is your major/field of study?
    - Accounting
    - Economics
    - Finance
    - Business Administration
    - Education
    - Engineering
    - Health and Medicine
    - Biological and Biomedical Sciences
    - Math, Computer Sciences, or Physical Sciences
    - Social Sciences or History
    - Law
    - Psychology
    - Modern Languages and Cultures

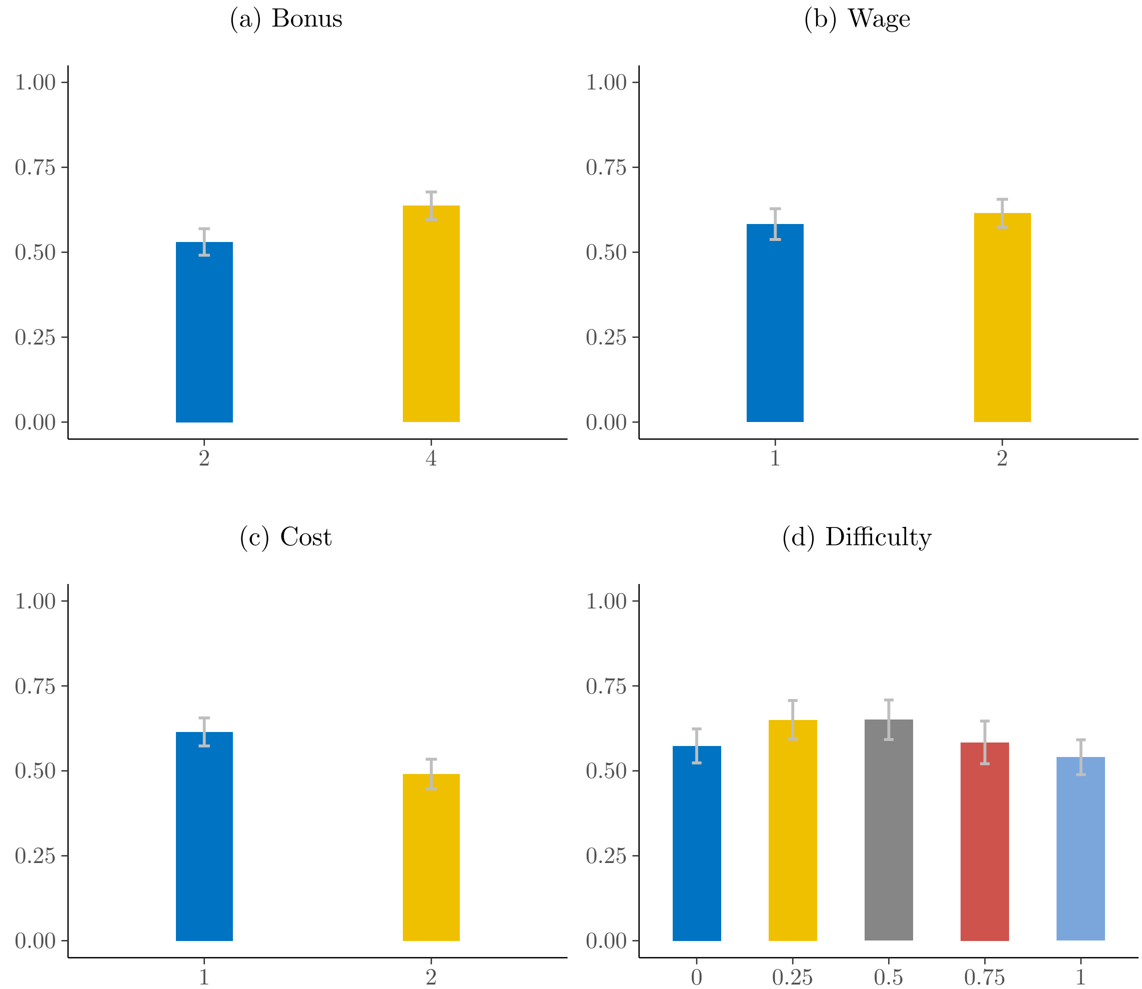
- Other

6. What is your GPA?
7. What is your year in school?
   - Freshman
   - Sophomore
   - Junior
   - Senior
   - Masters
   - Doctoral
8. What is the number of people in your household?
9. What is the total income of your household?
   - Under $5000
   - $5000—$15000
   - $15001—$30000
   - $30001—$45000
   - $45001—$60000
   - $60001—$75000
   - $75001—$90000
   - $90001—$100000
   - Over $100001
   - Prefer not to answer
10. What is the total income of your parents?
    - Under $5000
    - $5000—$15000
    - $15001—$30000
    - $30001—$45000
    - $45001—$60000
    - $60001—$75000
    - $75001—$90000
    - $90001—$100000
    - Over $100001
    - Don't know
    - Prefer not to answer

D Additional Analysis

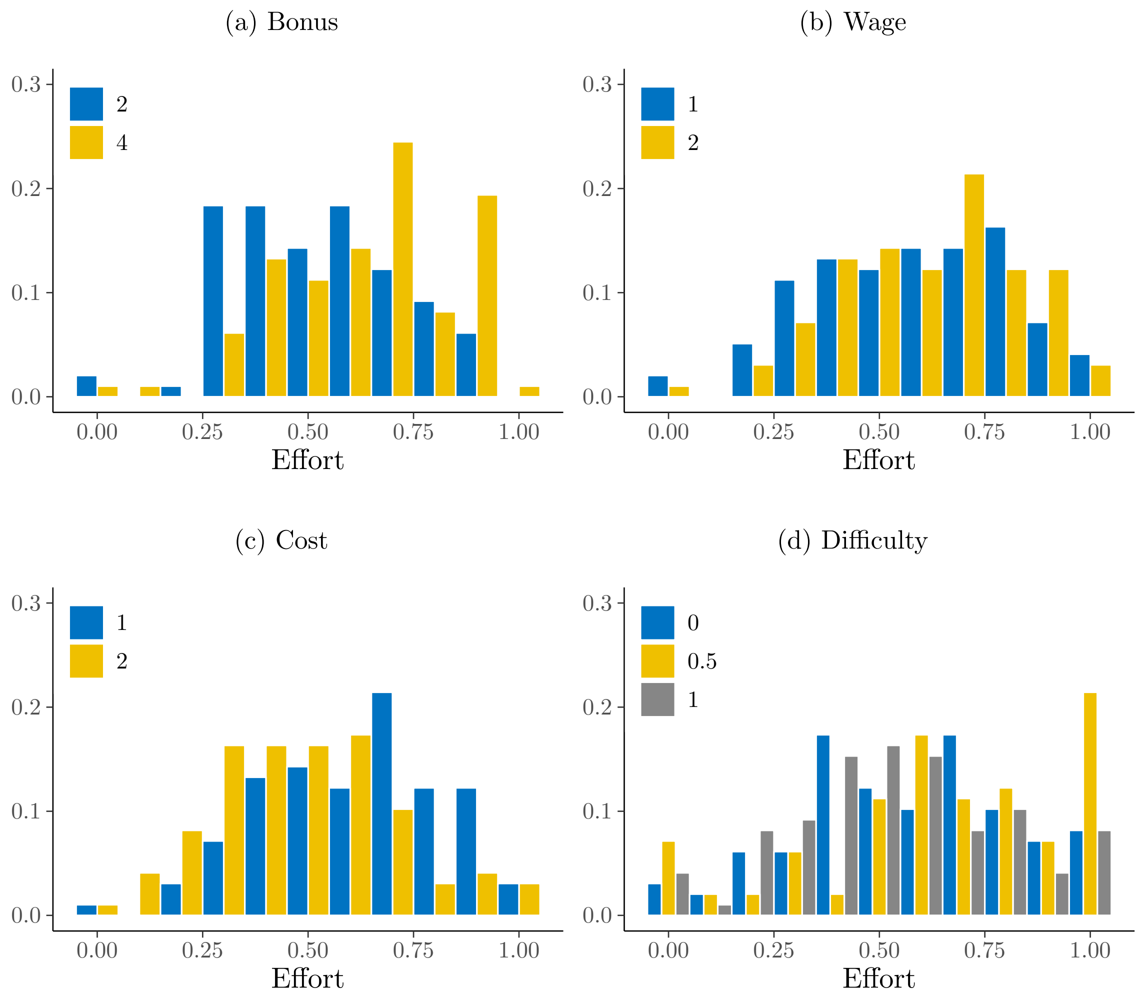
D.1 Additional Graphs

Figure D.1: Means of Effort by Treatment Variable



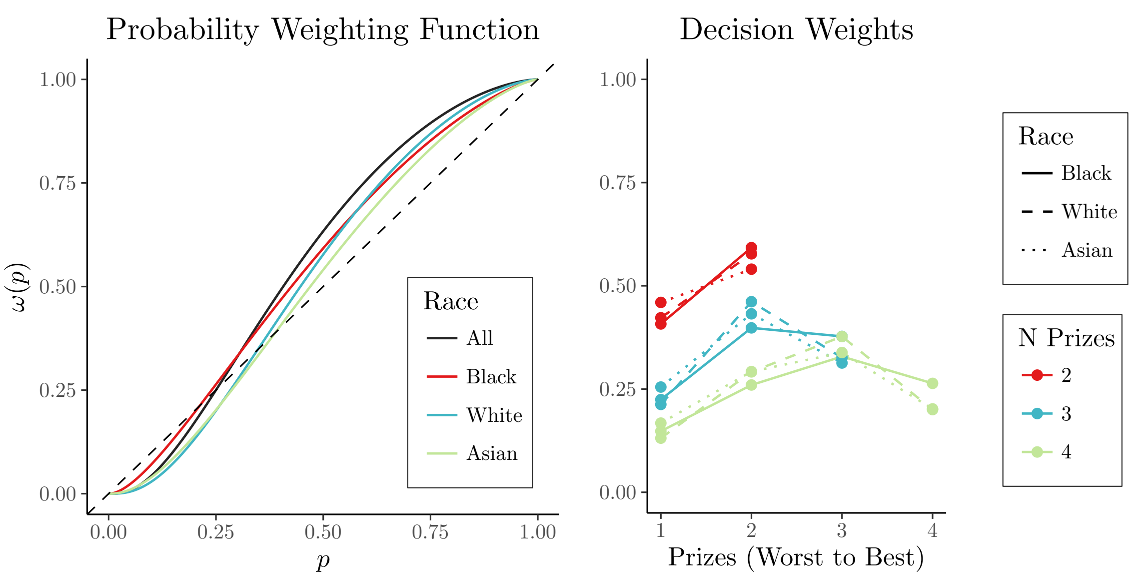
Note: The graph shows the means of effort levels broken down by a treatment variable and a value of the variable. Effort levels used to plot the means are subject-level mean effort levels for each value of a treatment variable. Vertical grey error bars show the 95% confidence intervals from a $t-$test.

Figure D.2: Histograms of Effort by Treatment Variable



Note: The graph shows the histogram of effort levels broken down by a treatment variable and a value of the variable. Effort levels used to plot the means are subject-level mean effort levels for each value of a treatment variable. The vertical axis is scaled to show the relative proportions of observations falling into the bins within each value of the variable.

Figure D.3: Estimated Probability Weighting Function and Implied Decision Weights from Equiprobable Lotteries by Race (Effort Task)



D.2 Additional Tables

Table D.1: Summary Results for the *Ceteris Paribus* Analysis

		p-values		
CP-pair	ATE	t	Wilcoxon	sign
		z		
1	0.083	0.003	<0.001	<0.001
2	0.05	0.153	0.042	0.007
3	0.146	<0.001	<0.001	<0.001
4	−0.002	0.971	0.765	0.856
5	0.174	0.004	0.001	
6	0.174	0.016	0.006	
7	0.167	<0.001	0.001	0.014
8	0.202	<0.001	<0.001	0.001
9	0.154	<0.001	<0.001	0.001
		w		
1	0.011	0.7	0.463	1
2	−0.022	0.407	0.598	0.902
3	0.077	0.187	0.245	
4	−0.027	0.634	0.558	
5	0.035	0.617	0.714	
6	0.015	0.688	0.835	0.815
7	0.05	0.304	0.25	0.228
		k		
1	−0.173	<0.001	<0.001	<0.001
2	−0.077	0.019	0.001	<0.001
3	−0.15	0.024	0.089	
4	−0.007	0.93	0.654	
5	−0.01	0.926	0.876	
6	−0.073	0.036	0.112	0.336
7	−0.121	0.002	0.002	<0.001
		θ (0 to 0.5)		
1	0.055	0.292	0.25	
2	0.018	0.729	0.816	
3	0.183	0.039	0.042	
4	0.142	0.015	0.038	
		θ (0.5 to 1)		
1	−0.165	0.004	0.004	
2	−0.123	0.023	0.028	
3	−0.189	0.036	0.045	
4	−0.096	0.105	0.394	

Notes: The first column is an index of a *CP*-pair, the second column shows the average treatment effect of a treatment variable in a given pair, the last three columns show the *p*-values from either a) the paired t test, the Wilcoxon signed rank test, and the sign test, or b) the unpaired t test and the Wilcoxon rank sum test. The *CP*-pairs with an empty value in the column for the sign test are the non-paired samples. All the tests are two-sided.

Table D.2: Panel Regression Results

	Dependent variable:
	effort
z	0.07*** (0.01)
w	0.01 (0.02)
k	−0.11*** (0.02)
theta	0.37*** (0.08)
I(theta^2)	−0.41*** (0.08)
Observations	1,625
R^2	0.09
Adjusted R^2	0.03
F Statistic	30.54*** (df = 5; 1522)
Note:	*p<0.1; **p<0.05; ***p<0.01

D.3 Stochastic Choice

The model considered so far assumes that the agent's choices are deterministic, even though she makes them in a stochastic setting. For a fixed set of parameters $\bar{\pi}$, the agent always chooses $a^* = \arg\max U(a \mid \bar{\pi})$. However, a large body experimental of experimental evidence shows that subject's choices are stochastic (Starmer and Sugden, 1989; Camerer, 1989; Ballinger and Wilcox, 1997) and a large and growing theoretical literature rationalizes this behavior (Matêjka and McKay, 2015; Gul et al., 2014). It is natural to ask whether and how the predictions change if one allows the agent's choices to be stochastic.

The main difference between a deterministic and a stochastic model of choice is that the stochastic model can only make predictions about *expected* effort, since choice is now a random variable for an outside observer. To illustrate the point, consider a simple case when the choice set consists of only two elements, full effort and no effort, $A = \{1, 0\}$. Assign probabilities q and $1-q$ to choices of full and no effort, respectively, according to the usual multinomial logit formula (Luce, 1959)

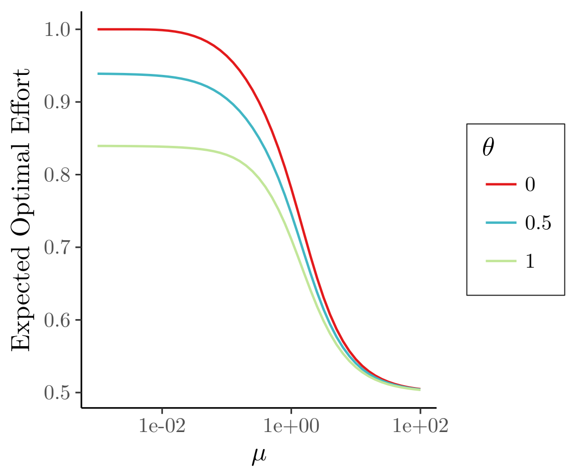
$$q = \frac{\exp(U(1 \mid \pi)/\mu)}{\exp(U(1 \mid \pi)/\mu) + \exp(U(0 \mid \pi)/\mu)} = \Lambda\left(\frac{U(1 \mid \pi) - U(0 \mid \pi)}{\mu}\right),$$

where Λ denotes the logistic cdf and μ is the noise parameter: higher values of μ make choices more "random" in the sense of not taking into account the respective utilities.

Suppose that under some set of parameters $\bar{\pi}$ the utility of full effort is higher than the utility of no effort, $\Delta U(\bar{\pi}) \equiv U(1 \mid \bar{\pi}) - U(0 \mid \bar{\pi}) > 0$. In this case, the deterministic model would predict that the choice will always be the full effort. In the stochastic model, we would have that the choices *on average* will be closer to the full effort, because $\mathbb{E}(a) = q$ and $\Delta U(\bar{\pi}) > 0$ implies that $p = \Lambda(\Delta U(\bar{\pi})/\mu) > 1/2$. In the limit, as noise goes to zero, one would have that $\lim_{\mu \to 0} \mathbb{E}(a) = 1$: small noise values would push the expected effort towards the prediction of the deterministic model, as the difference in utilities becomes more salient. As noise increases, the difference between utilities becomes less salient and the expected effort is sucked towards $1/2$, since $\lim_{\mu \to \infty} \Lambda(\Delta U(\bar{\pi})/\mu) = 1/2$: effort choice becomes uniformly distributed. The deterministic model is therefore a special case of a stochastic model with zero noise.

Figure D.4 demonstrates the result in a general case when the agent's choice set is $[0, 1]$ for three levels of difficulty. I use the monetary cost of effort specification with the CRRA utility of money function $u(x) = x^{1-\gamma}/(1-\gamma)$ with $\gamma = 0.2$ and no probability weighting with the cost of effort and probability of success functions defined as before. Proposition 1.A predicts declining effort levels in response to higher difficulty. The picture confirms that the ordering of optimal choices in the deterministic case is preserved for the expected choices in the stochastic case.

Figure D.4: Expected Effort as a Function of Noise



E Derivations and Proofs

Consider the derivative of $U(a \mid \pi)$ w.r.t. a:

$$U' = \sum_{x=0}^{1} \big(u(w+zx,a) f_a(x \mid a,\theta) + u_a(w+zx,a) f(x \mid a,\theta) \big) \tag{E.1}$$

$$= \sum_{x=0}^{1} u(w+zx,a) \frac{f_a(x \mid a,\theta)}{f(x \mid a,\theta)} f(x \mid a,\theta) + \sum_{x=0}^{1} u_a(w+zx,a) f(x \mid a,\theta) \tag{E.2}$$

$$= \mathbb{E}\, u(Y,a) \frac{f_a(X \mid a,\theta)}{f(X \mid a,\theta)} + \mathbb{E}\, u_a(Y,a). \tag{E.3}$$

At the maximum point a^* the FONC must hold:

$$\mathbb{E}\, u(Y,a) \frac{f_a(X \mid a,\theta)}{f(X \mid a,\theta)} = -\,\mathbb{E}\, u_a(Y,a). \tag{E.4}$$

The LHS of this equation is

$$\mathbb{E}\, u(Y,a) \frac{f_a(X \mid a,\theta)}{f(X \mid a,\theta)} = \sum_{x=0}^{1} u(y,a) f_a(x \mid a,\theta) \tag{E.5}$$

$$= u(w,a)(-p_a(a,\theta)) + u(w+z,a) p_a(a,\theta) \tag{E.6}$$

$$= p_a(a,\theta)[u(w+z,a) - u(w,a)] \tag{E.7}$$

$$= p_a(a,\theta) \Delta u(w,z,a) \tag{E.8}$$

$$= z p_a(a,\theta) u_y(\bar{y},a), \tag{E.9}$$

where the last equality follows from the Mean Value Theorem and $\bar{y} \in [w, w+z]$. The RHS of the FONC is

$$-\,\mathbb{E}\, u_a(Y,a) = -[u_a(w,a)(1-p(a,\theta)) + u_a(w+z,a) p(a,\theta)] \tag{E.10}$$

$$= -[u_a(w,a) + p(a,\theta)(u_a(w+z,a) - u_a(w,a))] \tag{E.11}$$

$$= -[u_a(w,a) + z p(a,\theta) u_{ya}(\bar{y},a)], \tag{E.12}$$

where the last equality follows from the Mean Value Theorem and $\bar{y} \in [w, w+z]$.

Consider the second derivative of U w.r.t. a:

$$U'' = \sum_{x=0}^{1} \big(u(w+zx,a) f_{aa}(x \mid a,\theta) + 2u_a(w+zx,a) f_a(x \mid a,\theta) + u_{aa}(w+zx,a) f(x \mid a,\theta) \big) \tag{E.13}$$

$$= \sum_{x=0}^{1} u_{aa}(w+zx,a) f(x \mid a,\theta) + \sum_{x=0}^{1} 2u_a(w+zx,a) \frac{f_a(x \mid a,\theta)}{f(x \mid a,\theta)} f(x \mid a,\theta) \tag{E.14}$$

$$+ \sum_{x=0}^{1} u(w+zx,a) \frac{f_{aa}(x \mid a,\theta)}{f(x \mid a,\theta)} f(x \mid a,\theta) \tag{E.15}$$

$$= \mathbb{E}\, u_{aa}(Y,a) + 2\,\mathbb{E}\, u_a(Y,a) \frac{f_a(X \mid a,\theta)}{f(X \mid a,\theta)} + \mathbb{E}\, u(Y,a) \frac{f_{aa}(X \mid a,\theta)}{f(X \mid a,\theta)}. \tag{E.16}$$

The first term is the expectation of u_{aa}, which is negative by assumption. The last term can be rewritten as

$$\sum_{x=0}^{1} u(w+zx,a)f_{aa}(x \mid a,\theta) = u(w,a)(-p_{aa}(a,\theta)) + u(w+z,a)p_{aa}(a,\theta) \tag{E.17}$$

$$= p_{aa}(a,\theta)\Delta u(w,z,a) \tag{E.18}$$

$$= zp_{aa}(a,\theta)u_y(\bar{y},a), \tag{E.19}$$

where the last equality follows from the Mean Value Theorem and $\bar{y} \in [w, w+z]$. This term is also negative, since the utility gain Δu is positive and p_{aa} is negative by assumption. The middle term is

$$\sum_{x=0}^{1} 2u_a(w+zx,a)f_a(x \mid a,\theta) = 2\left(u_a(w,a)(-p_a(a,\theta)) + u_a(w+z,a)p_a(a,\theta)\right) \tag{E.20}$$

$$= 2p_a(a,\theta)\left(u_a(w+z,a) - u_a(w,a)\right) \tag{E.21}$$

$$= 2zp_a(a,\theta)u_{ya}(\bar{y},a), \tag{E.22}$$

where the last equality follows from the Mean Value Theorem and $\bar{y} \in [w, w+z]$. While $p_a > 0$ by assumption, the term u_{ya} cannot be signed without an additional assumption about the cross-partial derivative of u. If u is submodular, $u_{ya} \leqslant 0$, the expected utility function U is strictly concave and the first-order condition is sufficient. If u is supermodular, $u_{ya} \geqslant 0$, however, one needs to check the second-order condition as well.

Consider the special case when $u(y,a) = v(y) - c(a)$, $p(a,\theta) = 1/2(a+1-\theta)$, $c(a) = ka^2$. Then $p_a(a,\theta) = 1/2$, $\Delta u(w,z,a) = v(w+z) - v(w)$, $u_a(y,a) = -c'(a) = -2ka$, and $u_{ya}(y,a) = 0$. Plugging these values into the FONC yields

$$1/2\left(v(w+z) - v(w)\right) = -(-2ka^*) \tag{E.23}$$

$$a^* = \frac{v(w+z) - v(w)}{4k}. \tag{E.24}$$

Assume now that $u(y,a) = -e^{-\gamma(y-c(a))}$, $p(a,\theta) = 1/2(a+1-\theta)$, $c(a) = ka^2$. Then at $a = a^*$

$$\Delta u(w,z,a) = -e^{-\gamma(w+z-c(a))} + e^{-\gamma(w-c(a))} = e^{-\gamma(w-c(a))}(-e^{-\gamma z} + 1), \tag{E.25}$$

$$u_a(y,a) = -\left(e^{-\gamma(y-c(a))}(-\gamma)(-2ka)\right) = -2ak\gamma e^{-\gamma(y-c(a))} \tag{E.26}$$

and

$$u_a(w+z,a) - u_a(w,a) = -2ak\gamma e^{-\gamma(w+z-c(a))} + -2ak\gamma e^{-\gamma(w-c(a))} \tag{E.27}$$

$$= -2ak\gamma e^{-\gamma(w-c(a))}(-e^{-\gamma z} + 1). \tag{E.28}$$

Plugging these values into the FONC yields

$$\frac{1}{2}e^{-\gamma(w-c(a))}(-e^{-\gamma z}+1) = -\left(-2ak\gamma e^{-\gamma(w-c(a))} + \frac{a+1-\theta}{2}2ak\gamma e^{-\gamma(w-c(a))}(-e^{-\gamma z}+1)\right) \tag{E.29}$$

$$\frac{1-e^{-\gamma z}}{2} = 2ak\gamma - a^2k\gamma(1-e^{-\gamma z}) - ak\gamma(1-\theta)(1-e^{-\gamma z}) \tag{E.30}$$

$$\frac{1}{2k\gamma} = \frac{2a}{1-e^{-\gamma z}} - a^2 - ak(1-\theta) \tag{E.31}$$

$$\frac{1}{2k\gamma} = -a^2 + a\left(\frac{2}{1-e^{-\gamma z}} - 1 + \theta\right) \tag{E.32}$$

$$\frac{1}{2k\gamma} = -a^2 + a\left(\underbrace{\frac{1+e^{-\gamma z}}{1-e^{-\gamma z}}}_{A} + \theta\right) \tag{E.33}$$

$$a^2 - (A+\theta)a + \frac{1}{2k\gamma} = 0. \tag{E.34}$$

The roots of this quadratic equation are

$$a_{1,2} = \frac{A+\theta \pm \sqrt{(A+\theta)^2 - 2/(k\gamma)}}{2}. \tag{E.35}$$

Note that the negative root, a_1, is less than $(A+\theta)/2$, while the positive root, a_2, is greater than $(A+\theta)/2$. The first derivative of U can be written as

$$U' = k\gamma(1-e^{-\gamma z})e^{-\gamma(w-c(a))}\left((a-a_1)(a-a_2)\right). \tag{E.36}$$

It is easy to see that the first derivative changes its sign from positive to negative at a_1 and from negative to positive at a_2, hence a_2 is a local minimum, while a_1 is a local maximum, $a^* = a_1$.

Consider the optimality condition at $a = a^*$

$$a^2 - (A+\theta)a + \frac{1}{2k\gamma} = 0. \tag{E.37}$$

Differentiate w.r.t. θ to get

$$2a\frac{\partial a}{\partial\theta} - (A+\theta)\frac{\partial a}{\partial\theta} - a = 0 \tag{E.38}$$

$$\frac{\partial a}{\partial\theta}(2a - (A+\theta)) = a \tag{E.39}$$

$$\frac{\partial a}{\partial\theta} = \frac{a}{2a-(A+\theta)}. \tag{E.40}$$

Since $a^* < (A+\theta)/2$, the sign of the partial derivative is negative.

To derived the general comparative statics, consider again the general FONC:

$$zp_a(a^*,\theta) + u_a(w,a^*) + zp(a^*,\theta)u_{ya}(\bar{y},a^*) = 0. \tag{E.41}$$

It defines an implicit function $F(a^*, \theta) = 0$. Using the Implicit Function Theorem, we get

$$\frac{da^*}{d\theta} = -\frac{F_\theta}{F_a} = -\frac{zp_{a\theta}(a^*, \theta)u_y(\bar{y}, a^*) + zp_\theta(a^*, \theta)u_{ya}(\bar{\bar{y}}, a^*)}{U''(a^*)} \tag{E.42}$$

$$= -\frac{z\left[p_\theta(a^*, \theta)u_{ya}(\bar{\bar{y}}, a^*) + p_{a\theta}(a^*, \theta)u_y(\bar{y}, a^*)\right]}{U''(a^*)}. \tag{E.43}$$

Proposition 1.A

Proof. Consider the terms in the square brackets. Since $p_\theta < 0$ and $u_y > 0$, the terms u_{ya} and $p_{a\theta}$ must have the opposite signs (with the possibility that one or both of them are zero) to unambiguously determine the effect of difficulty on optimal effort. Hence, the condition $\operatorname{sgn}(u_{ya}) \operatorname{sgn}(p_{a\theta}) < 1$ must hold. If the condition holds, the sign of the effect of difficulty on optimal effort will coincide with the sign of u_{ya}, if $u_{ya} \neq 0$, or be the opposite of the sign of $p_{a\theta}$, if $p_{a\theta} \neq 0$. If both $u_{ya} = 0$ and $p_{a\theta} = 0$, difficulty will have no effect on optimal effort.

□

Under the RDU model, the FONC is

$$z\tilde{p}_a(a^*, \theta)u_y(\bar{y}, a^*) + u_a(w, a^*) + z\tilde{p}(a^*, \theta)u_{ya}(\bar{\bar{y}}, a^*) = 0 \tag{E.44}$$

$$z\omega'(p(a^*, \theta))p_a(a^*, \theta)u_y(\bar{y}, a^*) + u_a(w, a^*) + z\omega(p(a^*, \theta))u_{ya}(\bar{\bar{y}}, a^*) = 0. \tag{E.45}$$

Using the Implicit Function Theorem yields (the points at which functions are evaluated are dropped to improve readability)

$$\frac{da^*}{d\theta} = -\frac{z\left[\omega' p_\theta u_{ya} + u_y\left(\omega'' p_\theta p_a + \omega' p_{a\theta}\right)\right]}{U''(a^*)} \tag{E.46}$$

Proposition 1.B

Proof. Consider the terms in parentheses. The sign of $p_\theta p_a$ is negative, since $p_\theta < 0$ and $p_a > 0$. The sign of ω' is positive, since the probability weighting function is a strictly monotonically increasing transformation. Hence, for the expression in parentheses to have an unambiguous sign, one must have that ω'' and $p_{a\theta}$ are of the opposite signs (with the possibility that $p_{a\theta} = 0$), or $\operatorname{sgn}(\omega'') \operatorname{sgn}(p_{a\theta}) < 1$. Consider now the remaining terms in square brackets. Since $u_y > 0$, to sign the expression in brackets unambiguously, one must have that the expression in parentheses has an unambiguous sign and that this sign is the opposite of the sign of u_{ya} (with the possibility that $u_{ya} = 0$). Hence the condition $\operatorname{sgn}(u_{ya}) \operatorname{sgn}(p_{a\theta}) < 1$. If both conditions hold, the effect of difficulty on optimal effort will have the sign opposite to the sign of ω''.

□

To derive the comparative statics w.r.t. bonus, consider the FONC written as

$$p_a(a^*, \theta)\left(u(w + z, a^*) - u(w, a^*)\right) + u_a(w, a^*) + p(a^*, \theta)\left(u_a(w + z, a^*) - u_a(w, a^*)\right). \tag{E.47}$$

Implicit differentiation w.r.t. z yields

$$\frac{da^*}{dz} = -\frac{p_a(a^*, \theta)u_y(w + z, a^*) + p(a^*, \theta)u_{ya}(w + z, a^*)}{U''(a^*)} \tag{E.48}$$

Proposition 2

Proof. Consider the numerator of da^*/dz. The terms p_a, u_y, and p are non-negative, but the sign of u_{ya} is undetermined. Hence, to unambiguously sign the effect of bonus, one must have $u_{ya} \geqslant 0$ in which case the effect of bonus will be non-negative. $\square$

To derive the comparative statics w.r.t. wage, consider the FONC written in the form

$$p_a(a^*, \theta)\left(u(w+z, a^*) - u(w, a^*)\right) + \mathbb{E}\, u_a(w + zX, a^*). \tag{E.49}$$

Implicit differentiation w.r.t. w yields

$$\frac{da^*}{dw} = -\frac{p_a(a^*, \theta)\left(u_y(w+z, a) - u_y(w, a)\right) + \mathbb{E}\, u_{ya}(w + zX, a^*)}{U''(a^*)} \tag{E.50}$$

$$= -\frac{zp_a(a^*, \theta)u_{yy}(\bar{y}, a^*) + \mathbb{E}\, u_{ya}(w + zX, a^*)}{U''(a^*)}, \tag{E.51}$$

where the last equality follows from the Mean Value Theorem and $\bar{y} \in [w, w+z]$.

Proposition 3

Proof. Consider the numerator of da^*/dw. The first term is non-positive, since $z \geqslant 0$, $p_a \geqslant 0$, and $u_{yy} \leqslant 0$. The sign of the second term is undetermined, since the sign of u_{ya} is undetermined. Hence, in order to unambiguously sign the effect of wage, one must have $u_{ya} \leqslant 0$ in which case the effect of wage will be non-positive. $\square$