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Comments
ESI Working Paper 18-21
The Tug-of-War in the Laboratory

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January 5, 2018

Abstract
The tug-of-war is a multi-battle contest often used to describe extended interactions in economics, operations management, political science, and other disciplines. While there has been some theoretical work, to the best of our knowledge, this paper provides the first experimental study of the tug-of-war. The results show notable deviations of behavior from theory derived under standard assumptions. In the first battle of the tug-of-war, subjects often bid less, while in the follow-up battles, they bid more than predicted. Also, contrary to the prediction, bids tend to increase in the duration of the tug-of-war. Finally, extending the margin necessary to win the tug-of-war causes a greater reduction in bidding than either a decrease in the prize or greater impatience despite all three having the same predicted effect. These findings have implications both for theorists and practitioners.

JEL Classifications: C91, D72, D74
Keywords: tug-of-war, all-pay auction, multi-stage contest, laboratory experiment

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We thank the special issue guest editor Kai Konrad and three anonymous reviewers for their helpful suggestions. We also thank Tim Cason, Alan Gelder, Dan Kovenock, participants at the North American Economic Science Association Meetings in Dallas and the Economics and Biology of Contests Conference in Brisbane, Australia for their helpful comments. We thank Domenic Donato for writing the computer program to run the experiment. Finally, we thank the Economic Science Institute and Chapman University for financial support. Any remaining errors are ours.
1. Introduction

The term “tug-of-war” derives from a rope pulling contest in which two contestants (or groups) pull a rope in different directions until one of the contestants pulls the other across a middle ground. But more generally, tug-of-war describes a contest consisting of a series of battles, where a battle victory of one contestant moves the game closer to the winner’s preferred terminal state, and where one contestant wins the war if the difference in the number of battle victories exceeds some threshold (Konrad and Kovenock, 2005; Agastya and McAfee, 2006). As a modeling device, the tug-of-war has a large number of applications like R&D races in economics (Harris and Vickers, 1987) and the interaction of viruses and cells in biology (Zhou et al., 2004). In political science, the back and forth between the legislature and the president (Whitford, 2005) and the status of Jerusalem (Organski and Lust-Okar, 1997) have been described as tug-of-wars as has the tradeoff between efficient production and improving customer satisfaction with early delivery in operations management (Schutten et al., 1996).

Harris and Vickers (1987) were the first to formally examine the tug-of-war game.1 They analyzed an R&D race as a tug-of-war in which two players engage in a series of multiple battles and the winner of each battle is determined probabilistically. The assumptions of their model, prevented Harris and Vickers from completely solving the model, and instead they were only able to obtain qualitative predictions. More recently, Konrad and Kovenock (2005) have explicitly solved the tug-of-war game and provided conditions for a unique equilibrium. They showed that the contest effort crucially depends on the number of needed victories, the value of the prize, and

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1 The structure of the tug-of-war game is reminiscent of a dollar auction, and so one could cite the seminal paper by Shubik (1971) as a starting point for this literature.
the discount rate. In a related paper, Agastya and McAfee (2006) also derive equilibrium conditions for an alternative formulation of the tug-of-war game.2

Despite an established theoretical literature, no effort has been devoted to empirically investigate individual behavior in the tug-of-war and compare such behavior with theoretical predictions. This is understandable, because it is not trivial to measure individual effort with naturally-occurring data, as the researcher can only observe the performance of contestants, which is a function of effort, ability and luck (see the discussion in Ericsson and Charness, 1994). In addition to measurement error, self-selection and endogeneity are unavoidable in dynamic settings (see the discussion in Kimbrough et al., 2018). Given the difficulties of testing the tug-of-war with naturally-occurring data, we chose to conduct a controlled laboratory experiment, which allows us to examine behavior in the tug-of-war without confounding effects and endogeneity issues.

To the best of our knowledge, this is the first study examining the tug-of-war experimentally (see Dechenaux et al., 2015 for a review of the experimental contest literature). Our experiment examines the theoretical predictions of Konrad and Kovenock (2005), using a three-by-one between-subjects design. In the Low Value treatment, the value of the prize $v$, is lower than in the other treatments. The Extended treatment involves more possible states $m$ and thus a greater necessary margin for victory, than the other two treatments. The Impatient treatment, reduces the discount rate $\delta$ as compared to the other treatments. We follow the standard procedure for inducing a discount rate by making continuation to the next round probabilistic (see Dal Bo, 2005; Duffy, 2008). The key aspect of the design is that for all three treatments $\delta^{m/2}v$ is fixed, which makes all three treatments theoretically equivalent. The prediction is that contestants should

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2 Agastya and McAfee (2006) assume that if both players bid the same amount in a given state, then neither player wins, while Konrad and Kovenock (2005) assume that if bids are the same, then the “advantaged” player wins. Another notable difference is that, Agastya and McAfee (2006) assume the final loser in the tug-of-war receives a strictly negative prize, while Konrad and Kovenock (2005) assume a loser prize of zero.
exert costly efforts (bids) in the first battle (round) using a mixed strategy as in the standard all-pay auction. In the follow-up rounds, when the state is not $m/2$, there should be no bidding. Moreover, bidding should not depend on the number of times a particular state has been reached. Finally, the aggregate behavior in each treatment should be the same as long as $\delta^{m/2}v$ is fixed.

We find notable deviations of behavior from theoretical predictions. First, we find that in the first round of the tug-of-war, bids are not drawn from the uniform distribution and there is significant underbidding. Second, we find that the bids are systematically greater than the predicted value of zero in the follow-up rounds. Third, contrary to the theoretical prediction, conditional on the state bids tend to increase in the duration of the tug-of-war. Finally, we find that bidding behavior is generally similar in the Low Value and Impatient treatments, but bidding is significantly lower in the Extended treatment, suggesting that extending the necessary margin of victory for the tug-of-war discourages subjects more from bidding initially than does lowering the prize or increased discounting. These findings have implications both for theorists and practitioners, which we discuss in the concluding section of our paper.

2. Related Experimental Literature

The most closely related experimental studies examine behavior in sequential multi-battle contests, also known as best-of-$n$ races. Mago et al. (2013), for example, examine behavior in a best-of-three race between two contestants and find that the leader exerts more effort than the follower. Zizzo (2002) implements a best-of-$n$ race and finds a positive correlation between investment and progress in the race. Ryvkin (2011) investigates a best-of-$n$ contest in which players who choose the high effort early in the competition decrease their probability of winning.

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3 There are also some studies on multi-battle elimination contests (Parco et al., 2005; Amegashie et al., 2007; Sheremeta, 2010a, 2010b; Altmann et al., 2012; Höchtl et al., 2015).
in later battles, imitating fatigue. Consistent with the theory, subjects abstain from high effort in early battles in the presence of fatigue. Deck and Sheremeta (2012) examine behavior in a multi-battle contest in which the defender must win each battle to secure the prize and the attacker needs only to win one battle to capture the prize. In the experiments, subjects’ behavior is consistent with the main qualitative prediction of the theory, except for one key pattern: when fighting, rather than lowering expected effort in each new battle, subjects increase effort. Finally, Gelder and Kovenock (2017) examine behavior in a multi-battle contest with a losing penalty, and also find escalation of conflict effort contrary to the theoretical predictions.

Our study differs substantially from the previous experimental studies. Specifically, the tug-of-war differs from the best-of-$n$ race because in the race the number of battles $n$ is fixed and the winner is determined by the absolute number of battles each player has won.\footnote{Similarly to the best-of-$n$ race, there is an active experimental literature examining behavior in simultaneous multi-battle contests, also known as Colonel Blotto games (Avrahami and Kareev, 2009; Hortala-Vallve and Llorente-Saguer, 2010; Kovenock et al., 2010; Arad, 2012; Arad and Rubinstein, 2012; Chowdhury et al., 2013; Holt et al., 2016; Mago and Sheremeta, 2015, 2018).} In contrast, in the tug-of-war victory is defined by the differential success of the players. The first player to win $m/2$ (where $m$ is even) more contests than her rival is the winner. In our experiment, the tug-of-war may continue for a very long time (infinity in the limit), potentially making it a very exhausting competition.

3. Theory and Hypotheses

3.1. The Tug-of-War Game

The experiment closely aligns with the theoretical model of Konrad and Kovenock (2005).\footnote{The model of Konrad and Kovenock (2005) is more general than what is presented here. We are only providing the detail needed for analyzing the specific situations we study in the laboratory.} There are two players: $A$ and $B$. There are $m+1 > 2$ ordered possible states (where $m$ is even)
located on the grid line \{0, 1, 2, \ldots, m\} along which the war can take place. Let \( x_t \in \{0, 1, 2, \ldots, m\} \) denote the state of the game at the start of round \( t \in [1, 2, 3, \ldots] \). The tug-of-war begins in round \( t = 1 \) in the initial state \( x_1 = m/2 \), halfway between the two terminal states of 0 and \( m \). At each round in which the game has not yet reached a terminal state, there is a contest resolved as an all-pay auction (Baye et al., 1996) where \( A \)'s bid is denoted by \( a_t \geq 0 \) and \( B \)'s bid is denoted by \( b_t \geq 0 \). If \( a_t > (\text{or} <) b_t \), then \( A \) (or \( B \)) wins the contest and the state becomes \( x_t - 1 \) (or \( x_t + 1 \)). If \( a_t = b_t \) then \( A \) wins if \( x_t < m/2 \), \( B \) wins if \( x_t > m/2 \), and the winner is determined randomly if \( x_t = m/2 \). If the game reaches state 0 (or \( m \)) then the game ends and \( A \) (or \( B \)) claims a prize of \( v \). Otherwise the game continues to the next round with the state in round \( t+1 \) determined by \( x_t \) and the outcome of the contest in round \( t \). The two players are assumed to have a common discount rate of \( \delta \). Figure 1 shows an example of the game with \( m = 4 \) in which \( A \) wins after the fourth round. In this example, \( A \) earns \( v-55 \) and \( B \) earns -40.

The unique Markov perfect equilibrium is \( a_t = b_t = 0 \) if \( x_t \in \{1, 2, \ldots, m-1\}\setminus\{m/2\} \) and \( a_t, b_t \) are drawn from the uniform distribution from 0 to \( \delta^{m/2} v \) if \( x_t = m/2 \).\(^6\) Intuitively, when the players are even (at state \( m/2 \)), they are in an all-pay auction and the expected payoff to each player is 0. If the game is at state \( m/2-1 \), then a winning bid by \( B \) will move the game to a point in which \( B \) expects to earn 0, so \( B \)'s optimal bid is 0 and given the tie breaking rule \( A \) should bid 0 as well.\(^7\)

Iterating this logic, \( B \) should never bid when the state is less than \( m/2 \) and similarly \( A \) should never bid if the state exceeds \( m/2 \). Because of the behavior that should occur when the state is not \( m/2 \),

\(^6\) For the details see Proposition 3 in Konrad and Kovenock (2005).
\(^7\) Konrad and Kovenock (2005) use a tie-breaking rule that awards the victory to a stronger player (with a higher continuation value). This is common in games with discontinuous payoffs, such as all-pay auctions (Konrad and Kovenock, 2009; Kovenock and Roberson, 2012). The reason for using such a rule is to avoid having to use \( \varepsilon \)-equilibrium concepts.
winning in the first round should result in winning the game in \( m/2 \) rounds making the prize for winning the first round \( \delta^{m/2}v \).

3.2. Hypotheses

The equilibrium solution provides the basis for the hypotheses to be tested in the laboratory. Specifically, we test the following hypotheses regarding the expected behavior in a tug-of-war.

**Hypothesis 1:** When the game begins (at state \( m/2 \)), a player’s bid is drawn from the uniform distribution over the interval \([0, \delta^{m/2}v]\).

**Hypothesis 2:** When the state is not \( m/2 \), subjects bid zero.

The model also provides predictions regarding different tug-of-war games. In particular, if two tug-of-war games have the same value for \( \delta^{m/2}v \) then behavior should be identical at state \( m/2 \). This leads to the following prediction.

**Hypothesis 3:** Behavior does not differ between games with differing values of \( m, \delta, \) and \( v \) if \( \delta^{m/2}v \) is held constant.

4. Experimental Design and Procedures

To test the hypotheses we conduct a three-by-one between-subjects experimental design. The three treatments (Low Value, Impatient, and Extended) differ in terms of the values of \( v, \delta, \) and \( m \) as shown in Table 1. In the Low Value treatment, the value of the prize, \( v \), is lower than in the other treatments. The Extended treatment involves more possible states, \( m \), than the other two treatments. The Impatient treatment, reduces the discount rate \( \delta \) as compared to the other
treatments. We follow the standard procedure for inducing a discount rate by making continuation to the next round probabilistic (see Dal Bo, 2005; Duffy, 2008).8

Our primary goal is not to identify how changes in a specific variable impact behavior, but rather to determine if strategic behavior is contingent upon $\delta m^2 v$, as predicted by the theory. Hence, the key aspect of our design is that for all three treatments $\delta m^2 v$ is held constant (at $\approx 66$). Although our design does not compare two treatments differing along a single dimension, one can identify the relative effects of specific parameters by comparing one treatment to the composition of the other two. The design of changing several treatment variables to keep the equilibrium prediction the same is commonly used in experimental economics to facilitate comparison between treatments and to test the most fundamental behavioral responses (Bull et al., 1987; Orrison et al., 2004; Chowdhury et al., 2014).

A total of 96 subjects participated in the experiment, which was conducted in the Behavioral Business Research Laboratory at the University of Arkansas. Subjects for each one hour long session were recruited through the lab’s database of volunteers and no subject participated in more than one session. For each of the three treatments, four sessions were completed. Each session involved 8 subjects who read written instructions (available in Appendix) and completed a comprehension worksheet. After the worksheets were checked for correctness and any remaining questions were answered, subjects completed two unpaid practice tug-of-war games, and then ten salient tug-of-war games.

8 The duration was not randomly generated for each pair. The randomization was done in advance so that the maximum number of rounds that could be played in a period was the same for every tug-of-war given $\delta$. For example, in every session of Impatient the first paid tug-of-war could last for up to three rounds. Drawing the duration separately for each pair could result in subjects not completing their own tug-of-war but still having to wait a long time until another pair resolved its game due to the rematching of subjects between games. The practice periods were not randomized to ensure that subjects experienced both a long game (5 rounds) and a short game (1 round).
Each game, referred to as a period in the experiment, subjects were randomly and anonymously paired with someone else in the session.\footnote{The instructions used the term tug-of-war in an effort to help subjects understand the nature of the game being played. Copies of the instructions are available upon request.} Figure 2 provides a screen shot for the \textit{Impatient} treatment. Subjects always saw themselves as the player who wins the game at state 0 on the far left of the screen as shown in Figure 2. The colored ball moved around based on the state of the game. The probability that the game would continue for one, two, five and ten more rounds if a terminal state was not reached is shown at the top left of the screen. The right hand portion of the screen records what has occurred in each round of the current period (game) and the outcome from previous periods.

After all 10 periods were completed, one was randomly selected and subjects were paid their earnings based on the outcome of the game in that period.\footnote{Other payoff procedures such as paying for every period could change the incentives of the subjects over the course of the experiment through wealth effects for example.} Experimental earnings were denoted in francs and converted into dollars at the rate 25 francs = $1. The average subject payment was $19.59.\footnote{Subjects were given a $20 endowment from which losses could be deducted.}

5. Results

The findings are presented as a series of results corresponding to the four hypotheses provided in the previous section. Figure 3 shows the distribution of bids in the first round of every tug-of-war by treatment.

Theory predicts (see Hypothesis 1) that all bids should be uniformly distributed between 0 and 66 ($= \frac{\delta m}{2\nu}$). Although, we find that by and large the bids are drawn from the same support as
the equilibrium distribution, none of these distributions appear to be uniform over the interval 0 to 66. Instead, bids are skewed to the left.

Statistically, we test that the mean and variance of each distribution is equal to the values that would be generated if subjects were behaving according to the theoretical predictions. Notice, that under the null hypotheses subjects are independently drawing their bids from the interval [0, 66] at the start of every tug-of-war. For each treatment, either the observed mean, the observed variance, or both differ from the theoretical predictions as shown in Table 2. For example, the average bid is significantly lower than the theoretical prediction in the Impatient treatment (27.8 versus 33, p-value < 0.001) and in the Extended treatment (16.8 versus 33, p-value < 0.001). Table 2 also reports Kolmogorov-Smirnov tests comparing the observed distributions with the predicted uniform distributions. Overall, these findings provide evidence against Hypothesis 1 and are the basis for our Result 1.

**Result 1:** Subjects do not bid according to the theoretical prediction in the first round of a tug-of-war. That is bids are not drawn from the uniform distribution over the interval [0, 66] and are skewed to the left.

Another prediction of the theory is that when the state is not \( m/2 \), subjects should bid zero. However, we find that in the second round bids are systematically greater than the predicted value of zero. Only 3% of all bids at state 1 in the Low Value treatment are 0 and only 20% of the bids at state 3 are 0 in that treatment. For the Impatience treatment the respective percentages are 1% and 29% for states 1 and 3. The percentage of bids equal to zero for states 1, 2, 4, and 5 in the Extended treatment are 4%, 3%, 32%, and 41% respectively. Figure 4 shows the average bid by

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12 The conclusions of each test are similar if attention is restricted to the last 5 periods, to control for possible learning. The one difference is that the variance in the Low Value treatment is no longer statistically different from the predicted value.
round and state for each treatment. Notice that when one player is in the state \( x \) the other player is in the state \( m-x \). Also, because the continuation to the next round is probabilistic, some pairs do not reach the terminal state nor do they reach the next round. The data in Figure 4 are taken to be sufficient evidence against Hypothesis 2.

**Result 2:** *When the game is not in a symmetric state, subjects do not bid zero contrary to the theoretical prediction.*

Turning to the question of whether or not subjects behave the same in each treatment (see Hypothesis 3), we note that Figure 3 and Figure 4 suggest that bids are lower in the *Extended* treatment than the other two treatments. This conclusion is supported statistically in Table 3. The first column of Table 3 estimates how bids are impacted by treatments when the state is \( m/2 \). The omitted treatment is *Low Value* so the lack of significance for *Impatient* suggests these two treatments yield similar behavior on average. The negative and significant coefficient for *Extended* indicates that on average bids are lower in this treatment than in the *Low Value* treatment. Average bids are lower in the *Extended* treatment than in the *Impatient* treatment (p-value = 0.052). The second and third columns of Table 3 compare *Impatient* to *Low Value* at states 1 and 3, respectively, omitting *Extended* because 1) it has already been shown to differ and 2) the states are not directly comparable. For both specifications, the coefficient on *Impatient* is not significant.\(^{13}\) Further, Kolmogorov-Smirnov tests that the distributions of bids in round 1 are the same in the *Extended* treatment and either of the other two treatments yield p-values < 0.001. For the comparison between the *Low Value* and the *Impatient* treatments, the p-value = 0.099.\(^{14}\) Together,

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\(^{13}\) If attention is restricted to the last half of the paid periods, the conclusions remain unchanged except that the significance of *Extended* in the first column is increased while the significance of *Impatient* disappears. That is, the evidence for Result 3 is even stronger when controlling for possible learning.

\(^{14}\) If attention is restricted to the last 5 periods, the *Extended* treatment remains highly significantly different than either of the other two treatments (i.e. p-value < 0.001), but for the comparison of *Low Value* and *Impatient* the p-value = 0.209. That is, the evidence for Result 3 is even stronger when controlling for possible learning.
the results indicate that behavior is similar in the Low Value and Impatient treatments, and different in the Extended treatment. This provides sufficient evidence against Hypothesis 3.

**Result 3:** Bidding behavior is similar in the Low Value and Impatient treatments, but bidding behavior in the Extended treatment differs from the other two. In particular, subjects bid less in the middle state when there are more states in the tug-of-war.

Figure 4 also reveals another stark pattern in the data. Regardless of treatment, conditional on the state, bids tend to increase the longer the tug-of-war has been going. For all 11 treatment-state combinations, the average bid is lower in the earliest round in which the state was reached than in the latest round in which the state was reached. Table 4 provides statistical evidence of the period trends in each situation. For both the Low Value and Extended treatments, the trends (given by State $i \times$ Round coefficients) are positive and significant. The trends are not significant in the Impatient treatment; however, one should be cautious given the relatively small number of observations occurring after round 3 in this treatment. Thus, rather than following a Markovian strategy as predicted by the model, behavior exhibits path dependence. These patterns lead to Result 4.

**Result 4:** Contrary to the theoretical prediction that bids are Markovian, bids are history dependent and generally increasing in the duration of the tug-of-war.

6. Behavioral Explanations

Taken together, our findings provide substantial evidence of behavioral deviations from theoretical predictions. While this result may not be surprising to those who have conducted laboratory experiments involving contests with different structures, documenting the relationship between standard theory and observed behavior is an important first step when examining a new
setting. As a second step, in this subsection we provide a discussion of the observed deviations and suggest two potential behavioral explanations in light of previous experimental results. The first explanation is that subjects perceive the actual tug-of-war in the laboratory as a game of incomplete information about players’ types. This is a reasonable assumption, given that subjects have a nonpecuniary utility of winning, which is unobservable and different across subjects (Sheremeta, 2013, 2015, 2016; Price and Sheremeta, 2011, 2015; Mago et al., 2016). The second explanation is that subjects may use a strategy of multi-dimensional reasoning (Arad and Rubinstein, 2012). Again, this is a reasonable assumption, given that subjects are boundedly rational and have been found to use dimension-reducing simplified strategies in complex situations (Harstad and Selten, 2013).

We begin with Result 1. Contrary to the prediction that the first round of the tug-of-war should resemble a one stage all-pay auction, we find that bids are not drawn from the uniform distribution but are left-skewed.15 This result is consistent with the behavioral explanation that subjects perceive the laboratory tug-of-war as a game of incomplete information. Indeed, there should be no full dissipation in the single-stage all-pay auction of incomplete information (Barut et al., 2002; Noussair and Silver, 2006). Second, underbidding in the first round could be explained using a framework of a multi-dimensional reasoning. Specifically, underbidding in the first round can be a “proper response” to the observation that most of the competition occurs in later rounds, as opposed to the predicted “frontloaded” competition. Since successful participation in later rounds requires substantial bidding, it seems prudent to bid less in the first round.

15 Some all-pay auction experiments find overbidding (Davis and Reilly, 1998; Gneezy and Smorodinsky, 2006; Lugovskyy et al., 2010; Llorente-Saguer et al., 2016), while others find underbidding (Potters et al., 1998; Gelder et al., 2015).
Results 2 documents that bids in the follow-up rounds are higher than the predicted bid of zero.\textsuperscript{16} Again, this type of non-equilibrium behavior could be explained by the fact that subjects have incomplete information about the opponent’s nonpecuniary utility of winning. Such a utility inherently transforms the game into a multi-battle contest with intermediate prizes; and one of the fundamental theoretical results with intermediate prizes is that “the player who is lagging behind may catch up, and does catch up with a considerable probability in the equilibrium” (Konrad and Kovenock, 2009, page 267). Alternatively, it could be the case that subjects first decide how much they are willing to spend on the tug-of-war and then they choose how to allocate this amount across the rounds (Mago and Sheremeta, 2015; Deck et al., 2017). This type of a multi-dimensional reasoning (Arad and Rubinstein, 2012) also could explain positive bids in asymmetric states.\textsuperscript{17}

Result 3 documents that bidding behavior in the \textit{Extended} treatment is significantly different from the \textit{Low Value} and \textit{Impatient} treatments, despite all three treatments being theoretically equivalent. Following the line of two previously suggested behavioral explanations, one explanation for this result is that subjects play a tug-of-war of incomplete information. This entails that subjects should expect for the game to proceed more rounds than theoretically predicted, as they extract information about the opponent in each round. This also means that a tug-of-war with a large number of states (as in the \textit{Extended} treatment) may be particularly discouraging for subjects since such a tug-of-war could potentially last for a very long time. Another explanation is that subjects use a strategy of multi-dimensional reasoning. To recall, in

\textsuperscript{16} A natural explanation of overbidding in asymmetric states is that the prediction of zero is at the boundary, and any mistake would lead to overbidding. The problem of boundary equilibrium predictions has been well recognized in dictator games (List, 2007), public good games (Laury and Holt, 2008), and contests (Kimbrough et al., 2014), and it has been proposed as an explanation for excessive giving, over-contribution to public goods, and excessive conflict.

\textsuperscript{17} We are grateful to the Editor who suggested yet another explanation for the non-equilibrium behavior. Specifically, since subjects observe that other subjects depart from full-dissipation behavior in the middle state, then the optimal reply to this is to make non-zero bids at states other than the middle state. As a result, the full dissipation at the middle state is also not optimal. Therefore, the behavior of subjects can be explained as conditionally optimal or conditionally consistent behavior.
such a case, subjects first decide how much they are willing to spend on the tug-of-war and then they choose how to allocate this amount across the rounds. At the start of the tug-of-war, the best case scenario is winning the first \( m/2 \) rounds and claiming the prize – a path along the top left edge of flow charts in Figure 4. For all three treatments, the sum of the average bid along the best case scenario path is similar (68.6 in \textit{Low Value}, 66.3 in \textit{Impatient}, and 60.8 in \textit{Extended}). Further, along each best case scenario path the average bids are fairly uniform. This suggests that subjects may begin by thinking about how much they want to spend along the path they hope to take and then bidding more or less equally along that path. As a result, bids at specific states along the best path in the \textit{Extended} treatment are smaller than in the other treatments because the same total amount is being divided over more states. As subjects win and remain on the best case scenario path, they continue to implement their plan. Once they are knocked off the best scenario path, they adjust as evidenced by Result 4.

Finally, Result 4 documents that bids increase in the duration of the tug-of-war. A natural explanation, based on incomplete information, is that the dynamic path is a function of the combination of the contestants’ types. Specifically, subjects who are more competitive (have a higher utility of winning) are more likely to reach later rounds. Also, as the game progresses, subjects should learn more about their opponents, turning the tug-of-war into a game of complete information, thus escalating the conflict. To examine this hypothesis, consider the \textit{Low Value} treatment. When a player bids in round 3 it is on average 3.7 higher than the same player’s round 1 bid. In round 5, the average increase from round 1 is 6.3. By rounds 7 and 9 the increases from round 1 are 21.5 and 34.3, respectively. Of course, this does not imply that the average change from round 7 to round 9 was 34.3-21.5 = 12.8, since not everyone who reached round 7 also reached round 9. Table 5 provides similar comparisons for the other two treatments as well as the
percentage of times a bidder in the center state at a particular round increased her bid relative to round 1. It is important to keep in mind that, while anyone returning to the center state in the Low Value or Impatient treatments must have returned to that state at every previous odd round, this is not true in the Extended treatment. For this reason, the Extended treatment is excluded from Figure 5 which shows the average bid in each odd numbered round conditional on the final round in which the tug-of-war returned to the center state. In Figure 5 solid lines are used for data from the Low Value treatment while dashed lines indicate the Impatient treatment. For those pairs that reach the center state for the last time in round 3 or round 5 there is not much difference in their round 1 or round 3 behavior in either the Low Value or Impatient treatment. Those pairs who return to the center state for the last time in round 7 or round 9 do appear to bid more in round 5 than those who do not return to the center after round 5 in the Low Value treatment, but there are only 3 pairs that reach the center state for the last time in round 7 and three pairs that do so in round 9 and the average bid is increasing as the tug-of-war progresses for these players. Thus, Result 4 provides evidence of conflict escalation, i.e., conditional on the state, bids tend to increase in the duration of the tug-of-war. Although this finding is contrary to the theoretical prediction, it is consistent with a well-documented escalation of commitment (Staw, 1976).

7. Conclusion

The tug-of-war is a multi-battle contest used to model extended interactions in economics, operations management, political science, and other disciplines. It has attracted the attention of prominent theorists (Harris and Vickers, 1987; Konrad and Kovenock, 2005; Agastya and McAfee, 2006), but there are no experimental tests of a tug-of-war (Dechenaux et al., 2015).
Our results show notable deviations of behavior from theory. In the first battle of the tug-of-war, subjects bid less, while in the follow-up battles, they bid more than predicted. Also, contrary to the theoretical prediction, bids tend to increase the longer the tug-of-war has been going. Finally, we find that the required margin of victory of the tug-of-war (how extensive the tug-of-war is) discourages subjects more from bidding in the first round than a theoretically equivalent reduction in the value of the prize or an increase in discounting.

Although our findings provide substantial evidence of behavioral deviations from theoretical predictions, such deviations have two potential behavioral explanations. The first explanation is that subjects perceive the actual tug-of-war in the laboratory as a game of incomplete information about players’ types. The second explanation is that subjects employ a multi-dimensional reasoning strategy. Both explanations are consistent with our main findings.

Our results contribute to several strands of the existing literature. First, our findings contribute to the experimental literature on races (Zizzo, 2002; Deck and Sheremeta, 2012; Mago et al., 2013; Mago and Sheremeta, 2015, 2018). Specifically, similarly to the findings in race experiments, we find that subjects bid more in asymmetric battles than predicted. Therefore, together our findings suggest that the discouragement effect, which is predicted to reduce conflict intensity in both the tug-of-war (Konrad and Kovenock, 2005) and the race (Klumpp and Polborn, 2006; Konrad and Kovenock, 2009), may not be as prevalent in laboratory settings.

Second, our results complement the existing literature in operations research on strategic attack and defense of reliability systems. Most of this literature is theoretical (Levitin, 2003; Zhuang et al., 2010; Hausken and Bier, 2011; Rinott et al., 2012) and direct empirical validation of the existing models is difficult with naturally occurring data.¹⁸ The fact that in all treatments of

¹⁸ Researchers recently have begun to exploit experimental methods in evaluating these models (e.g., Deck and Sheremeta, 2012; Deck et al., 2015).
our experiment subjects shift their effort from the first battle to the later battles suggests that there are important factors which are not captured by the theory. This also suggest that in practice, conflicts resembling the tug-of-war may be more extensive, lasting for longer time, than predicted by the standard theory. At the same time, potentially long conflicts may deter competing sides from exerting costly resources in early rounds of conflict.

Finally, our results could be of particular interest for economists studying patents and innovation. Beginning with the seminal papers by Loury (1979) and Lee and Wilde (1980), patent races have been often modeled as a contest between firms investing in R&D efforts with the aim of securing the ultimate prize, a patent. However, the literature is mostly theoretical (e.g., Harris and Vickers, 1987; Baye and Hoppe, 2003) and the predictions rely heavily on “unrealistic” assumptions making it difficult to establish its relevance to the real world patent races. For instance, our tug-of-war game is predicted to be behaviorally similar to a single-battle all-pay contest since winning the first battle (or $\varepsilon$-preemption) is all that is needed to win the entire contest. Our experimental results, on the other hand, show that the loss in the first battle does not deter the competitor from engaging in subsequent battles, and therefore mimic the real world patent races more closely.

Of course, analogies between our laboratory environment and the naturally-occurring problems in economics, operations management, and political science are imperfect. While our framework captures some of the most salient features of the tug-of-war, we have set aside empirically relevant issues, such as contestant strength differences, heterogeneous prizes, and resource constraints. Given the observed deviations of behavior from theoretical predictions, exploring these extensions is an important avenue for future research, and we hope that our study will encourage other researchers to evaluate more complex dimensions of the tug-of-war.
References


Figure 1: An Example Tug-Of-War

Bids in Period 1  
A \(a_1 = 20\) 
B \(b_1 = 15\)

Bids in Period 2  
A \(a_2 = 5\) 
B \(b_2 = 10\)

Bids in Period 3  
A \(a_3 = 25\) 
B \(b_3 = 10\)

Bids in Period 4  
A \(a_4 = 5\) 
B \(b_4 = 5\)

Figure 2: Sample Screen Shot in Impatient Treatment

<table>
<thead>
<tr>
<th>Time Remaining: 10</th>
<th>Round: 1</th>
<th>Period: 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rounds: 1 2 5 10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Likelihood: 73.00% 55.23% 20.73% 4.30%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Round</th>
<th>My Bid</th>
<th>Other Bid</th>
<th>Start</th>
<th>End</th>
<th>Winner</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Period Rounds My Bids Other Bids Result Payoff

Please enter your bid in the box below

Bid Amount:

Submit
Figure 3: Distribution of Bids in First Round by Treatment

Panel A. *Low Value* Treatment
average = 31.23, standard deviation = 21.47

Panel B. *Impatient* Treatment
average = 27.84, standard deviation = 19.97

Panel C. *Extended* Treatment
average = 16.81, standard deviation = 12.57
Figure 4: Average Bids by State and Round for Each Treatment

Panel A. Low Value Treatment

<table>
<thead>
<tr>
<th>Round t=</th>
<th>State m=</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

- $\mu = 31.2$  
  n = 320
- $\mu = 37.4$  
  n = 160
- $\mu = 26.5$  
  n = 160
- $\mu = 33.9$  
  n = 108
- $\mu = 36.4$  
  n = 43
- $\mu = 32.8$  
  n = 43
- $\mu = 40.1$  
  n = 34
- $\mu = 40.0$  
  n = 17
- $\mu = 41.1$  
  n = 17
- $\mu = 63.2$  
  n = 12
- $\mu = 73.2$  
  n = 6
- $\mu = 81.8$  
  n = 6
- $\mu = 104$  
  n = 2
- $\mu = 60.0$  
  n = 2
- $\mu = 65.0$  
  n = 2
- $\mu = 69.5$  
  n = 6
- $\mu = 73.2$  
  n = 6

Panel B. Impatient Treatment
Panel C. *Extended* Treatment

$\mu$ denotes the average and $n$ denotes the number of observations.
Figure 5: Average Bid by Round in Center State
Conditional on the Final Round in which Center State was Reached
Table 1: Experimental Treatments

<table>
<thead>
<tr>
<th>Treatment</th>
<th>$m$</th>
<th>$\delta$</th>
<th>$v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low Value</td>
<td>4</td>
<td>0.81</td>
<td>100</td>
</tr>
<tr>
<td>Impatient</td>
<td>4</td>
<td>0.73</td>
<td>125</td>
</tr>
<tr>
<td>Extended</td>
<td>6</td>
<td>0.81</td>
<td>125</td>
</tr>
</tbody>
</table>

Table 2: Statistical Comparison of Observed and Predicted Behavior in the First Round

<table>
<thead>
<tr>
<th>Test of</th>
<th>Treatment</th>
<th>Low Value</th>
<th>Impatient</th>
<th>Extended</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean ($t$)</td>
<td></td>
<td>p-value = 0.147</td>
<td>p-value &lt; 0.001</td>
<td>p-value &lt; 0.001</td>
</tr>
<tr>
<td>Variance ($\chi^2$)</td>
<td></td>
<td>p-value = 0.001</td>
<td>p-value = 0.190</td>
<td>p-value &lt; 0.001</td>
</tr>
<tr>
<td>Distribution (KS)</td>
<td></td>
<td>p-value &lt; 0.001</td>
<td>p-value &lt; 0.001</td>
<td>p-value &lt; 0.001</td>
</tr>
</tbody>
</table>

Table 3: Comparison of Treatments Conditional on State

<table>
<thead>
<tr>
<th>State = $m/2$</th>
<th>State = 1</th>
<th>State = 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>33.89***</td>
<td>38.91***</td>
</tr>
<tr>
<td>Impatient</td>
<td>-4.06*</td>
<td>-0.47</td>
</tr>
<tr>
<td>Extended</td>
<td>-13.00**</td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>1350</td>
<td>409</td>
</tr>
</tbody>
</table>

*, ** and *** indicate significance at the 10%, 5%, and 1% level respectively based on a two sided test. Standard errors are clustered at the session level.
Table 4: Estimation of Trend in Bids over Rounds Given Treatment and State

<table>
<thead>
<tr>
<th>State 1</th>
<th>State 2</th>
<th>State 3</th>
<th>State 4</th>
<th>State 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>× Round</td>
<td>× Round</td>
<td>× Round</td>
<td>× Round</td>
<td>× Round</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Low Value</th>
<th>Impatient</th>
<th>Extended</th>
</tr>
</thead>
<tbody>
<tr>
<td>State 1</td>
<td>34.44***</td>
<td>42.91***</td>
<td>19.10***</td>
</tr>
<tr>
<td>State 1 × Round</td>
<td>3.54***</td>
<td>0.58</td>
<td>3.41***</td>
</tr>
<tr>
<td>State 2</td>
<td>31.86***</td>
<td>29.24***</td>
<td>23.24***</td>
</tr>
<tr>
<td>State 2 × Round</td>
<td>3.92***</td>
<td>4.41</td>
<td>2.55***</td>
</tr>
<tr>
<td>State 3</td>
<td>20.17**</td>
<td>4.86</td>
<td>22.34***</td>
</tr>
<tr>
<td>State 3 × Round</td>
<td>5.52***</td>
<td>11.26</td>
<td>1.94**</td>
</tr>
<tr>
<td>State 4</td>
<td></td>
<td></td>
<td>19.55***</td>
</tr>
<tr>
<td>State 4 × Round</td>
<td></td>
<td></td>
<td>1.62***</td>
</tr>
<tr>
<td>State 5</td>
<td></td>
<td></td>
<td>6.44</td>
</tr>
<tr>
<td>State 5 × Round</td>
<td></td>
<td></td>
<td>4.54***</td>
</tr>
</tbody>
</table>

*, **, and *** indicate significance at the 10%, 5%, and 1% level respectively based on a two sided test. Standard errors are clustered at the session level. The regression included a Period variable to control learning; the coefficient is -0.77 and is significant at the 5% level. Based on 3054 observations.
### Table 5: Change in Bidding Behavior from Round 1 when Returning to Middle State

<table>
<thead>
<tr>
<th></th>
<th>Treatment</th>
<th>Low Value</th>
<th>Impatient</th>
<th>Extended</th>
</tr>
</thead>
<tbody>
<tr>
<td>Round 3</td>
<td></td>
<td>3.7</td>
<td>8.4</td>
<td>9.7</td>
</tr>
<tr>
<td></td>
<td>(63%)</td>
<td>(64%)</td>
<td>(74%)</td>
<td></td>
</tr>
<tr>
<td>Round 5</td>
<td></td>
<td>6.3</td>
<td>16.0</td>
<td>12.3</td>
</tr>
<tr>
<td></td>
<td>(68%)</td>
<td>(80%)</td>
<td>(69%)</td>
<td></td>
</tr>
<tr>
<td>Round 7</td>
<td></td>
<td>21.5</td>
<td></td>
<td>3.6</td>
</tr>
<tr>
<td></td>
<td>(83%)</td>
<td></td>
<td>(58%)</td>
<td></td>
</tr>
<tr>
<td>Round 9</td>
<td></td>
<td>34.3</td>
<td></td>
<td>15.3</td>
</tr>
<tr>
<td></td>
<td>(100%)</td>
<td></td>
<td>(83%)</td>
<td></td>
</tr>
<tr>
<td>Round 11</td>
<td></td>
<td></td>
<td></td>
<td>7.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(67%)</td>
</tr>
<tr>
<td>Round 13</td>
<td></td>
<td></td>
<td></td>
<td>13.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(100%)</td>
</tr>
</tbody>
</table>

Table entries give the average difference between the amount bid in Round 1 and the amount bid in the specified Round for those subjects who were at the middle state in the specified round. The percentage of bids that increases relative to Round 1 are given in parentheses. The number of observations for each cell is given along the central column of the appropriate panels in Figure 4.
Appendix (For Online Publication) – Instructions for the Low Value Treatment

General Instructions

This is an experiment in the economics of decision making. Various research agencies have provided the funds for this research. The instructions are simple and if you follow them closely and make careful decisions, you can make an appreciable amount of money.

The experiment consists of 10 decision periods. The currency used in the experiments is called Francs. At the end of the experiment your earnings in Francs from 1 randomly selected period will be converted to US Dollars at the rate 25 Francs = US $1. You are also being given a US $20 participation payment. Any gains you make will be added to this amount, while any losses will be deducted from it. You will be paid privately in cash at the end of the experiment. The period that will be used to determine your payoff will be randomly selected at the end of the experiment using a 10-sided die.

It is very important that you do not communicate with others or look at their computer screens. If you have questions, or need assistance of any kind, please raise your hand and an experimenter will approach you. If you talk or make other noises during the experiment you will be asked to leave and you will not be paid.

Instructions for the Experiment

Each period you will be randomly and anonymously paired with one of the other participants, but no participant will be able to identify if or when he or she has been paired with a specific person.

Every period you and the person that you are paired with for the period will have an opportunity to win a prize of 100 Francs. The person who wins the prize is determined by a game of tug-of-war that occurs over the course of multiple rounds, so at most of one of you will win the prize in a period.

Each period lasts for a randomly determined number of rounds. The way the number of rounds is determined is as follows: after each round there is an 81% chance that another round will occur. This means that there is a 19% chance that a period will end after a given round. Notice that the chance of the period continuing does not depend on how many rounds the period has already lasted. At any point in time, the probability of a period lasting at least N more rounds is 0.81^N. So the chance that a period will last for at least two more rounds is 0.81^2 = 65.61%. As you can see on the sample screen shot on the next page, by the heading "Rounds" your screen will show you the likelihood that the period will last at least 1, 2, 5, and 10 more rounds.

At the start of each period a green ball will be placed 2 spaces from you and 2 spaces from the participant with whom you have been randomly paired. Each round, you and the person you are paired with will make a bid. Any amount that you bid is instantly deducted from your payoff for the period. Bids cannot exceed the prize so bids can be anything from [0, 0.1, 0.2, … , 99.9, 100]. The ball will move 1 space closer to the person who bids the most that round. In the event of a tie, the ball will move towards the closer person. If the ball is equidistant from both of you then a tie will be broken randomly. This bidding process will continue until either 1) someone has moved the ball all the way to his side and thus claimed the prize of 100 or 2) the period ends due to the random process described above.
To place a bid you simply type it in the box on the lower left portion of your screen and press Submit. After both participants have placed their bids, each person will be informed of the bids and the outcome for that round in the upper right portion of their screen. The green ball will also be moved accordingly. The number above the green ball tells you how many more rounds you must win to claim the prize this period. This number is referred to as the location. The lower right portion of the screen will keep a record of what happened each period. Recall that at the end of the experiment, you will be paid based upon what happened in one randomly selected period.

Let’s look at a couple of examples:

1) Suppose that in Round 1 you bid 13 and the person you are paired with bid 45. Since the other person bid more, the ball would move one position to the right, away from you. If in Round 2 you bid 30 and the other person bid 10, the ball would move back to the left to its original position. At this point, it is as if the period just began except that you would have already spent $43 = 13+30$ and the other person would have already spent $55 = 45+10$.

2) Suppose that in Round 1 you bid 13 and the person you are paired with bid 45. Since the other person bid more, the ball would move one position to the right, away from you. If in Round 2 you both bid 18, the ball would move one more space to the right since there was a tie and the ball was closer to the other person. This would be the end of the period. The other person would claim the prize of 100 and earn a profit of $37 = 100–45–18$. You would earn a profit of $–31 = –13–18$.

3) Suppose that in Round 1 you bid 65 and the person you are paired with bid 30. Since you bid more, the ball would move one position to the left, towards you. If by random chance the period ended after that round, your profit would be $–65$ and the other persons’ profit would be $–30$.

If you are finished reading these instructions, please raise your hand and an experimenter will bring you a review sheet to complete. The review sheet will not impact your payoff in any way; rather it is intended to ensure that you and everyone else understand the experiment.
Review Sheet

Please answer each of the following. If you have a question at any point, you should raise your hand and an experimenter will assist you.

1) Complete the following table by determining where the green ball would start and end each round given the bids listed.

<table>
<thead>
<tr>
<th>Round</th>
<th>Starting Location</th>
<th>Your Bid</th>
<th>Other Person’s Bid</th>
<th>Ending Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>15</td>
<td>26</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>18</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>20</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>38</td>
<td>39</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>3</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

2) Using the table above, what would your profit be if the period was randomly ended after the third round? ___________ What would the other person’s profit be? ___________

3) Suppose instead that in round 1 you bid 10 and the other person bid 15. If both of you bid 0 in all subsequent rounds and the period was not randomly stopped before someone won the tug-of-war, what would your profit be? ____ What would the other person’s profit be? ____

4) True or False, at the end of the experiment you will be paid the sum of your earnings each period.