2008

Color Transparency In Qcd and Post-Selection In Quantum Mechanics

Shmuel Nussinov
Tel Aviv University

Jeff Tollaksen
Chapman University, tollakse@chapman.edu

Recommended Citation

Nussinov, Shmuel, and Jeff Tollaksen. "Color transparency in QCD and post-selection in quantum mechanics." *Physical Review D* 78.3 (2008): 036007. doi: 10.1103/PhysRevD.78.036007

PHYSICAL REVIEW D **78,** 036007 (2008)

Color transparency in QCD and post-selection in quantum mechanics

Shmuel Nussinov[1] and Jeff Tollaksen[2,*]

[1]*Raymond and Beverly Sackler School of Physics and Astronomy, Tel Aviv University, Tel Aviv, Israel*
[2]*Chapman University, Department of Physics, Computational Science, and Engineering, Schmid College of Science, One University Drive, Orange, California 92866, USA*
(Received 26 March 2008; published 19 August 2008)

We discuss color transparency in the nuclear QCD context from the perspective of pre- and post-selected ensembles. We show that the small size of the hadronic states can be explained by the peculiar "force of post-selection," in contrast to the more standard explanation based on external forces.

DOI: 10.1103/PhysRevD.78.036007 PACS numbers: 12.38.Qk, 03.65.Ta

I. INTRODUCTION

Color transparency [1,2] and its electromagnetic precursors [3] follows from basic quantum mechanics, relativistic kinematics, and the vector nature of the theory (see Frankfurt *et al.*[4] and comments in [5]). It involves the fact that cross sections are proportional to the dipole size (and hence become small for heavy $\bar{Q}Q$ pairs). Various aspects of color transparency have been elaborated by many researchers [6–12]. The purpose of this article is to discuss color transparency from the perspective of pre- and post-selection.

Aharonov, Bergmann, and Lebowitz (ABL) [13] reformulated quantum mechanics in terms of *pre- and post-selected ensembles*. The traditional paradigm for ensembles is to simply prepare systems in a particular state and thereafter subject them to a variety of experiments. These are preselected only ensembles. For pre- *and post-selected*-ensembles, we add one more step: a subsequent measurement or post-selection. By collecting only a subset of the outcomes for this later measurement, we see that the preselected only ensemble can be divided into subensembles according to the results of this subsequent "post-selection measurement". Because pre- and post-selected ensembles are the most refined quantum ensemble, they are of fundamental importance and subsequently led to the *time-symmetric reformulation of quantum mechanics* (TSQM) [14,15] (for a review, see [16]). While TSQM is a new conceptual point of view that has predicted novel, verified effects, it is in fact a *reformulation* of quantum mechanics. Therefore, experiments cannot prove TSQM over quantum mechanics (or vice versa). The motivation to pursue such reformulations, then, depends on their usefulness. Indeed, we believe that to be useful and interesting, any reformulation of quantum mechanics should meet several criteria such as those met by TSQM:

(i) TSQM is consistent with all the predictions made by standard quantum mechanics.
(ii) TSQM brings out features in quantum mechanics that were missed before: e.g. the "weak value" of an observable which was probed by a new type of quantum measurement called the "weak measurement" [14,15].
(iii) TSQM leads to simplifications in calculations (as occurred with the Feynman reformulation) and stimulated discoveries in other fields: e.g. ABL influenced work in field theory [17], superluminal tunneling [18,19], quantum information such as the quantum random walk [20], new techniques for amplifying signals [14,21], foundational questions [22], the discovery of new aspects of mathematics, such as super-Fourier or superoscillations [23], etc.
(iv) TSQM leads to generalizations of quantum mechanics that were missed before [24].

We start by reviewing the subject of color transparency. We then illustrate this new approach in the context of filtering small atoms. Finally, we apply this novel approach to color transparency in the nuclear QCD context and show that the small size of the hadronic states can be explained by the peculiar "force of post-selection", in contrast to the more standard explanation based on external forces.

II. COLOR TRANSPARENCY: THE BASIC ISSUES

Rather than viewing hadrons as lumps of pionic fields or "hadronic matter", QCD (and its quark model predecessor) describes them as $\bar{q}q$ (or qqq) bound states.

When probed at large P_T, the individual pointlike quarks clearly manifest, and, due to asymptotic freedom, can be treated perturbatively. At low energy hadron collisions, the nonperturbative quark wave function are important. Color transparency involves an intermediate region of high energy, and smaller, yet appreciable, momentum transfer such that $E_{\mathrm{Lab}} m \gg P_T^2 > \Lambda_{\mathrm{QCD}}^2$.

For large targets, say a large nucleus, nuclear absorption is a key aspect. The incident, free particle states, can be decomposed in a basis of the nuclear propagation eigenstates. There is selective stronger attenuation of some of these propagating states. This will lead, after the nucleus has been traversed and we reexpress the final state in the original free basis, to "diffractive regeneration" of other free particle states [25]. In order to maintain coherence and

*tollakse@chapman.edu

not break the nucleus, we need that the longitudinal, momentum transfer, i.e.

$$\Delta_L \equiv \frac{(m^{*2} - m^2)}{2E_L}, \tag{1}$$

be small enough:

$$\Delta_L R(A) \leq 1. \tag{2}$$

In the above, m^* and m are the masses of the final and initial states, E_L is the (Lab) energy of the incident nucleon, and

$$R(A) \approx r_0 A^{1/3} \approx 1.2A^{1/3} \text{ Fermi} \tag{3}$$

is the nuclear radius.

It is intuitively suggestive that states which are small (in configuration space) have smaller cross sections and will be "filtered" in nuclear propagation. This indeed holds for $q\bar{q}$ (or e^+e^-) bound states in QCD (or QED) and manifests the vectorial nature of both theories. For color singlet hadrons, the two gluon exchange, "van der Waals like" force, is the lowest order hadron-hadron interaction. The strength of the latter grows with the induced dipoles, i.e. with the size of the hadrons [26].

It was suggested that color transparency be tested in the reaction $p + (A, Z) \rightarrow pp + (A - 1, Z - 1)$ [7]; events with maximal momentum transfer $|t_{\max}| = 2m(E_L - m)$ corresponding to 90° scattering in the pp center mass frame, and for which there was no appreciable particle production or nuclear excitation, were picked up. For $E_L \sim 10$ GeV, $\sqrt{t_{\max}} = 3$ GeV is higher than $\Lambda_{\rm QCD}$ or typical hadronic mass scale. This suggests that the hard pp elastic collision be treated perturbatively. If we adopt for this purpose the QCD diagrams consistent with the usual quark counting rules [27], then the dominant contribution comes from those rare components of the hadron's wave function where the incoming and outgoing hadrons are all small, pointlike configurations.

Once dominance of pointlike states is assumed, color transparency naturally follows: since we have small objects propagating in the nucleus, this quasielastic process should not appreciably decrease with nuclear number A, and hence, the total path length traveled by the three protons (the incident projectile and two outgoing protons) increases. More precisely, it was suggested that effective cross sections for absorption [i.e. further inelastic scattering leading to excited many particle final states $\sigma(t)$] should monotonically decrease with $|t|$. (Asymptotically, we expect on dimensional grounds $\sigma \approx \frac{1}{|t|}$.)

Because of the uncertainty principle, the small pointlike configurations have large relative momenta: $P_\perp \approx (\epsilon r_0)^{-1}$ with ϵr_0 the small size. The "small protons" emerging from the collision expand and reform the normal full size $r = r_0$ protons. Color transparency optimally manifests when the angle (in the lab frame) between the projectile's quarks is small enough, i.e. $\theta \equiv \frac{P_\perp}{E_L} < \frac{\epsilon r_0}{R_A}$. The condition that this will indeed not happen is

$$E_L \geq \frac{P_T}{r_0 \epsilon} R(A). \tag{4}$$

In this case, the size of the propagating hadronic system does not appreciably increase while the latter is still inside the nucleus.

Collisions with high momentum transfer do not transform the colliding protons into pointlike configurations. Indeed, imparting a large momentum transfer P_T to a quark in a bound state modifies the relative wave function as follows:

$$\psi^{(0)}(\vec{r}) \rightarrow \psi'_p(\vec{r}) = e^{i\vec{P}_T\cdot\vec{r}}\psi^{(0)}(\vec{r}). \tag{5}$$

While $\psi'(\vec{r})$ quickly oscillates as a function of $\vec{r}$, the probability of finding a specific $q\bar{q}$ separation in ψ' is the same as in the original $q\bar{q}$ wave function ψ^0 [28].

The converse however is correct. If, for example, due to some "external forces", the hadrons attained small size $r \approx \epsilon r_0$, then these "small hadrons" would not readily break in high momentum collisions. The typical momentum transfer in small hadron collision is not $P_\tau \approx \frac{1}{r_0}$ but $P_\tau \approx \frac{1}{\epsilon r_0}$.

Viewing the color transparency phenomenon in the time-symmetric formulation of quantum mechanics with pre- and post-selected ensembles can explain this squeezing effect. The hadronic states are kept small not by any "real" external force but rather by the peculiar force of post-selection. Before discussing this, we will first discuss this issue in the simpler setting of an idealized atomic physics experiment.

III. ATOMS SQUEEZED BY POST-SELECTION

Quantum systems which were preselected to be in a state $|\psi_{\rm in}\rangle$ at time $t_{\rm in}$ and subsequently post-selected to be in a different, almost orthogonal state $|\psi_{\rm fin}\rangle$ at time $t_{\rm fin}$ may exhibit surprising behavior at intermediate times $t_{\rm in} \leq t \leq t_{\rm fin}$. Specifically "weak measurements" (i.e. measurements that weakly change the time evolution of the system) of an operator A, yield average values given by

$$A_w \equiv \frac{\langle\psi_{\rm fin}|\hat{A}|\psi_{\rm in}\rangle}{\langle\psi_{\rm fin}|\psi_{\rm in}\rangle}. \tag{6}$$

For the case when

$$|\langle\psi_{\rm fin}|\psi_{\rm in}\rangle| \equiv \delta \ll 1, \tag{7}$$

these A_w values may be large and lie far outside the domain of (real) eigenvalues of the operator A.

By selecting this rare ensemble i.e. the $0(\delta^2)$ fraction of systems which satisfy both the pre- and post-selection criteria, we may "distill" and enhance certain effects [14,21]. Here, we consider the rare atoms in a beam which managed to go through a thin foil, without exciting it. These selected atoms have, during the traversal of the

foil, unusually small sizes so as to ensure the required weak interactions with the foil material. The post-selection of these can thus achieve a remarkable feat of squeezing the atom into a small dimension for a time $t = \frac{d_f}{V_A}$ with d_f the thickness of the foil and V_A the velocity of the atom traversing it.

To illustrate the basic notion, we focus first on a ‘‘gedanken experiment’’. Thus, we have our idealized ‘‘foil’’ consisting of a thin straight cylindrical tube of length d_f and radius ϵr_0, smaller than the atomic radius ($r_0 \approx A°$) in the ground state. The almost classical motion of the massive atom is a straight line along the tube's axis traversed with a fixed velocity V_A.

There are two general ways by which we could force the electron to be confined to the tube, i.e. to be at a small transverse separation $|\tilde{b}| \leq \epsilon r_o$ from the nucleus: 1) with a strong repulsive potential, and 2) via post-selection.

A. Confinement with strong repulsive potential

We postulate a strong repulsive potential acting between the electron and the wall material, say,

$$V(\vec{r}) = V_0 > 0 \tag{8}$$

for $\vec{r}$ inside the ‘‘walls’’ around the tube. In the original, unperturbed atom, the velocity of the electron relative to the nucleus is, according to the uncertainty principle,

$$v_e = \frac{p}{m} = \frac{\hbar}{r_o m}. \tag{9}$$

If the atom is further squeezed to a small transverse size $|\tilde{r}| = \epsilon r_o$, then the (transverse) velocity of the confined electron is higher (the squiggle sign over a momentum variable indicates the transverse part thereof):

$$\tilde{v}\,' = \frac{\tilde{p}}{m} = \frac{\hbar}{\epsilon r_o m}. \tag{10}$$

(The transverse squeezing does not dissociate the atom, but rather provides a further confining potential for the electron.) Indeed, due to squeezing, we have effectively a one-dimensional system and any attraction (and Coulombic force in particular [29]) will support bound states. If the velocity of the electron is higher than that of the center mass motion

$$v' \geq v > v_A \tag{11}$$

then we can treat the latter motion in an adiabatic (Born-Oppenheimer) approximation. The kinetic energy due to the transverse confinement of the electron

$$\Delta E = \frac{1}{2} m v^2 = \frac{\hbar^2}{2m\epsilon^2 r_o^2} \tag{12}$$

then becomes an effective potential barrier for the center-of-mass motion. Assuming that the initial kinetic energy of the latter is higher than this barrier

$$\tfrac{1}{2} M_A V_A^2 > \Delta E, \tag{13}$$

then this would minimally modify the center-of-mass translational motion.

B. Confinement by post-selection

Introducing the repulsive potential in Eq. (8) amounts to exerting a strong force on the electron. We next consider an alternative transverse confinement of the electron without a direct application of force. Let us postulate a short range interaction between the electron and the wall material, an interaction that can be neglected when $|\tilde{r}| \leq \epsilon r_o$ i.e. when the electron stays inside the tube.

This interaction is weak and, by itself, does not confine the electron to be within the tube. Let us assume however, that the wall is almost macroscopic with many closely spaced energy levels. The electron-wall interaction will readily excite the wall to one of these levels if, during the traversal of the foil, the electron meanders into the wall. Such an excitation could then be detected by careful, long time measurements made on the foil after the atom has passed through it. The small subset of post-selected atoms in which no such excitation occurred defines then the small subset of (transversally) ‘‘squeezed atoms’’. To avoid excitations, the latter had to have a small transverse size $|\tilde{r}| \leq \epsilon r_o$ (with $\tilde{r}$ the transverse part of r_o) when it impinged on the tube's opening *and* it also had to remain small throughout the entire time of transit t. The probability that in the initial ground state the electron is inside the tube ($|\tilde{r}| \leq \epsilon r_o$) is

$$|\hat{\mathsf{P}}^{\mathrm{tr}}_{\epsilon r_o}|\psi_0\rangle|^2 = \int_{|\tilde{r}| \leq \epsilon r_o} d^3\vec{r} |\psi_0(\vec{r}|^2 \tag{14}$$

with $\hat{\mathsf{P}}^{\mathrm{tr}}_{\epsilon r_o}$ the projection onto $|\tilde{r}| \leq \epsilon r_o$. The state $P^{\mathrm{tr}}_{\epsilon r_o}|\psi_0\rangle$ is not stationary, but a superposition of many (or a continuum of) excited eigenstates of the original Hamiltonian. The average energy of this state as dictated by the uncertainty principle is

$$\Delta E = \frac{\hbar^2}{2m\epsilon^2 r_o^2}. \tag{15}$$

The minimal time required for this state to change appreciably is [30]

$$\Delta t \approx \frac{\hbar}{2\Delta E} \approx \frac{m r_o^2 \epsilon^2}{\hbar} \tag{16}$$

[31]. The more precise definition of the ‘‘appreciable change’’ is that the overlap with the original wave function is $\frac{1}{e}$:

$$\langle \hat{\mathsf{P}}^{\mathrm{tr}}_{\epsilon r_o} \psi(t) | \hat{\mathsf{P}}^{\mathrm{tr}}_{\epsilon r_o} \psi(t + \Delta t)\rangle \approx \frac{1}{e}. \tag{17}$$

To ensure that the electron is at all times within the tube, i.e. that $|\tilde{r}| \leq \epsilon r_o$, we need to keep projecting with P^{tr} at time intervals Δt. The probability that the system will

survive all $N = \frac{t}{\Delta t}$ projections is therefore

$$P = \prod_{i=1}^{N} P_i = e^{-2N} = e^{-2t/\Delta t} = e^{-2\Delta E t} \quad \text{where } P_i \equiv |\langle \hat{\mathsf{P}}^{\rm tr}_{\epsilon r_o}\psi(t)|\hat{\mathsf{P}}^{\rm tr}_{\epsilon r_o}\psi(t+\Delta t)\rangle|^2. \tag{18}$$

Indeed $e^{-2\Delta E t}$ is the probability that the initial electron in the ground state ψ_0 will spontaneously jump to the squeezed state of average energy ΔE and remain in such a level for a time duration t.

The above is analogous to nuclear color transparency. The nucleus with a dense spectrum of excitation plays the role of the foil. The absence of nuclear excitation or breakup "post-selects" the rare events in which the proton was small and managed to traverse the nucleus with only one scattering (at 90° in the pp rest frame) [32].

In principle, if we had ultrarelativistic atom beams with $\gamma_A \equiv E_A/m_A \gg 1$, the relativistic time dilation would reduce the exponential damping of the post-selection event rate from $e^{-2\Delta E t}$ [Eq. (18)] to $e^{2\Delta E t/\gamma}$.

For the nuclear case, the hard scattering collision of the proton while traversing the nucleon was effectively the "weak" measurement at intermediate times. The surprising result of this measurement is the anomalously large cross section for such a collision reflecting the anomalous small size of the post-selected proton while transversing the nucleus.

In the case of squeezed atoms, we could witness the peculiar pattern of cascade of photons as the emergent quasi-one-dimensional atom relaxes into the ordinary ground state. However, it is not obvious what measurements are appropriate for verifying the small transverse dimensions of the squeezed atom while in transit [33].

IV. NUCLEAR COLOR TRANSPARENCY: THE POST-SELECTION POINT OF VIEW

In this section, we use the time-symmetric reformulation of quantum mechanics to discuss color transparency in the nuclear QCD context.

The experimental setup for measuring the reaction $p + (A, Z)_o \to pp + (A-1, Z-1)_o$ (where the $_o$ subscript indicates an unexcited ground state) is asymmetric in the colliding protons—the high energy incident projectile and the target proton almost at rest in the nucleus. It is convenient to consider the case described in Fig. 1, which is different from the real experimental setup because of the presence of the Lorentz noninvariant physical background of nuclear matter.

The two incident nucleons, say, a proton coming from the left and a neutron coming from below, have momenta of equal magnitude $|\vec{P}_1| = |\vec{P}_2| = P$, and trajectories "aimed" towards the center of the nucleus at the origin. Since $PR(A, Z) \gg \hbar$ we can specify the $\vec{P}_1$, $\vec{P}_2$ momenta and have the wave packets reasonably well localized in the

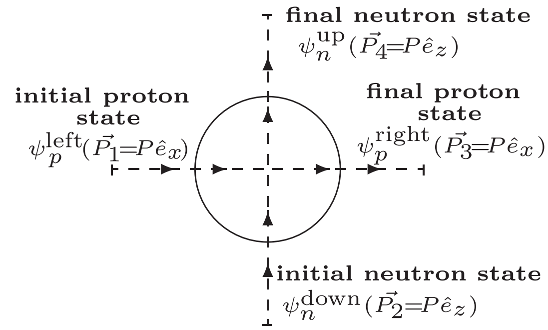

FIG. 1. Case (a)

respective transverse and longitudinal directions. We take $\vec{P}_1 = P\hat{e}_x$ (initial proton moving from the left along the x axis) and $\vec{P}_2 = P\hat{e}_z$ (initial neutron moving from below along the z axis). The initial wave function at some negative time $-t$ is a product

$$|\Psi_i(-t)\rangle = \psi_p^{\rm left}(\vec{P}_1 = P\hat{e}_x)\psi_n^{\rm down}(\vec{P}_2 = P\hat{e}_z)\psi^o(A, Z) \tag{19}$$

with $\psi^0(A, Z)$ the nuclear ground state wave function.

If neither projectile scatters from nucleons inside the nucleus the latter will stay in its ground state. Conversely, if we post-select these events in which the nucleus stayed in the ground state, then the incident proton and neutron suffered no collision with the nucleons inside the nucleus. We will consider two cases:

(a) There was no mutual scattering of the incident proton and neutron, and each continued along its initial path after reemerging from the nucleus at the points diametrically opposite the (respective) entry points (see Fig. 1), or

(b) the proton and neutron collided at time $t = 0$ (when their trajectories intersected at the origin) and scattered by 90° (see Fig. 2). Kinematics and symmetry then implies that $P_{\rm final}({\rm proton}) = \vec{P}_3$ and $P_{\rm final}({\rm neutron}) = \vec{P}_4$ have again the same common magnitude as the initial nucleons $|\vec{P}_3| = |\vec{P}_4| = P$.

The final state at positive time t for the no scattering case (a) is written in analogy with the initial state Eq. (19):

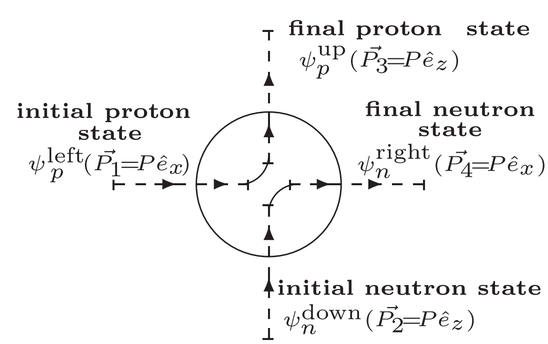

FIG. 2. Case (b)

$$|\Psi_{\text{final}}(t)\rangle_a = \psi_p^{\text{right}}(\vec{P}_3 = P\hat{e}_x)\psi_n^{\text{up}}(\vec{P}_4 = P\hat{e}_z)\psi^o(A,Z). \tag{20}$$

Where the up superscript on ψ_n^{up} indicates that at time t, the neutron wave packet is located above the nucleus (after having traveled along a diameter across the nucleus without exciting it). Likewise $\psi_p^{\text{right}}(\vec{P}_4 = P\hat{e}_x)$ is the proton wave packet emerging to the right of the nucleus. Finally, $\psi_0(A,Z)$ is the unexcited nuclear ground state.

The corresponding final state in case (b) is

$$|\Psi_{\text{final}}(t)\rangle = \psi_p^{\text{up}}(\vec{P}_3 = P\hat{e}_z)\psi_n^{\text{right}}(\vec{P}_4 = P\hat{e}_x)\psi^o(A,Z). \tag{21}$$

We next analyze the overlaps between the (post-selected) final state and the initial state, evolved from $-t$ to t for cases (a) and (b):

$$\langle\Psi_{\text{final}}(t)|U(t,-t)|\Psi_{\text{initial}}(-t)\rangle \equiv U_{if}. \tag{22}$$

Case (a): The overlap, Eq. (22), factorizes into two parts pertaining to the proton and to the neutron separately:

$$U_{if}^{(a)} = \langle\psi_p^{\text{up}}(\vec{P}_3 = P\hat{e}_z)|U(t,-t)|\psi_p^{\text{down}}(\vec{P}_1 = P\hat{e}_z)\rangle_{\psi^o(A,Z)} \otimes \langle\psi_n^{\text{right}}(\vec{P}_4 = P\hat{e}_x)|U(t,-t)|\psi_n^{\text{left}}(\vec{P}_2 = P\hat{e}_x)\rangle_{\psi^o(A,Z)}. \tag{23}$$

The $\psi^0(A,Z)$ subscript notation indicates that each overlap—including the time evolution and traversal of the whole nucleus along the z (or x) axis by the proton (neutron), respectively—is evaluated subject to the (post-selection) condition, namely, that the nucleus remains in the ground state $\psi^0(A,Z)$.

Since the motions of the proton along the x axis and of the neutron along the z axis are independent in the no collision case (a), the amplitude has the above product form. Any collision of the energetic incoming proton (or neutron), inside the nucleus leads to extra particle production and/or nuclear excitation. Thus the naive (and incorrect!) expectation is that the corresponding transmission probability (e.g. for the protons) is suppressed exponentially as a function of radius R:

$$P_{\text{trans}} \equiv |\langle\psi_p^{\text{up}}|U(t,-t)|\psi_p^{\text{down}}\rangle_{\psi^o(A,Z)}|^2 \approx e^{-2R/\lambda_N}. \tag{24}$$

With λ_N the mean free path of the incident nucleon inside the nucleus:

$$\lambda_N = (n\sigma_{NN}(E_L))^{-1} \tag{25}$$

with $n(\approx 0.16(\text{Fermi})^{-3})$ the nucleon number density inside the nucleus which can be taken as a constant for $R \gg$ Fermi and $\sigma_{NN}(E_L)$ is the nucleon-nucleon cross section at the relevant energy E_L.

Color transparency asserts that Eqs. (24) and (25) are wrong. In particular, P_{tran} falls only algebraically with nuclear size

$$P_{\text{trans}} \approx (R/r_o)^{-a} \tag{26}$$

providing that the initial projectile's energy $E_{\text{Lab}} \approx |P|$ is high enough:

$$E_L \geq \frac{\hbar R^2}{r_o^2\lambda_N} \tag{27}$$

with $r_0 \approx$ Fermi a typical hadronic scale. Just as with the atomic gedanken setup that we discussed earlier, we begin by estimating the probability that the projectile had, at the time it enters the nucleus, a small transverse size:

$$|\vec{r}| = \epsilon r_o. \tag{28}$$

If only the lowest Fock-space component [namely, the three quark (3q) component] was present in the initial proton's wave function, then, in analogy with Eq. (14) above, we find (with ψ^0 denoting the 3q component)

$$P(|\vec{r}| \leq \epsilon r_o, \quad \text{for 3q comp}) = \iint d^3\vec{r}_1 d^3\vec{r}_2 |\psi_p^0(\vec{r}_1,\vec{r}_2,\vec{r}_3)|^2 \times \Theta(\epsilon r_o - |\vec{r}_1|) \times \Theta(\epsilon r_o - |\vec{r}_2|) \tag{29}$$

with $\vec{r}_1 + \vec{r}_2 + \vec{r}_3 = 0$. The triangular inequality implies also that $|\vec{r}_3| \leq \epsilon r_o$. The symmetric quark wave function does not vanish when $|\vec{r}_i - \vec{r}_j| \rightarrow 0$. Normalizing $\iint d^3\vec{r}_1 d^3\vec{r}_2 |\psi_p(\vec{r}_1,\vec{r}_2,\vec{r}_3)|^2 = 1$, we find that

$$P_p(|r| \leq \epsilon r_o, \quad \text{for 3q comp}) \approx \epsilon^4. \tag{30}$$

The analog probability for a mesonic $q\bar{q}$ projectile is higher:

$$P_\pi(|r| \leq \epsilon r_o, \quad \text{for 2q comp}) \approx \epsilon^2. \tag{31}$$

Appealing now to the vector nature of QCD, the cross section of small ($|r_{ii}| \leq \epsilon r_0$) hadrons or dipoles scales with the (size)^2; and hence

$$\sigma_{P(|\vec{r}_{ij}| \leq \epsilon r_o)_N} = \sigma_{(\text{smallp})_N} \approx \epsilon^2 r_o^2 \tag{32}$$

or ϵ^2 times smaller than ordinary nucleon-nucleon scattering cross section:

$$\sigma_{NN} \approx r_o^2. \tag{33}$$

The probability that a "small nucleon" with $|r_i - r_j| \leq \epsilon r_0$ will traverse the nucleus without scattering is therefore

$$P_{\text{trans}}(\text{smallN}; |r_i - r_j| \leq \epsilon r_0) \approx e^{-(2R\epsilon^2/\lambda_N)}. \tag{34}$$

The above discussion neglected the higher Fock states in the high ("infinite") momentum wave function of the incident proton. We next argue that

$$P(3q) \equiv \text{prob of 3q Fock-space component} = \frac{E_L^{-\tilde{b}}}{E_0}. \tag{35}$$

Indeed, hadronic multiplicities at high energy collisions increase at least as

$$\bar{n}_g \approx \tilde{b} ln(E_L/E_o). \tag{36}$$

This is likely to reflect the average number of gluons $\bar{n}_g$ (of $q\bar{q}$ pairs) in the proton's wave function. Assuming a Poisson distribution, the probability of having no extra gluons or $q\bar{q}$ pairs is $\exp(-\bar{n})$ reproducing $P(3q)$ of Eq. (35). The above is modified by nonscaling behavior, e.g. rising rapidity plateaus and/or cross sections. However, in the energy range of the proposed color transparency experiments, $E_L \leq 20$ GeV, such effects are small. Combining then Eq. (30) and (35), we have

$$P_p(|\tilde{r}| \leq \epsilon r_o) \approx (E_L/E_o)^{-\tilde{b}} \epsilon^4. \tag{37}$$

Finally, the probability that the initial proton will traverse the nucleus without scattering by virtue of having components of size $|\tilde{r}| = \epsilon r_o$ i:

$$P_{\text{trans}} = \epsilon^4 (E_L/E_o)^{-\tilde{b}} e^{-(2R\epsilon^2/\lambda_N)}. \tag{38}$$

The dominant contribution comes from the saddle point in ϵ^2 space where $\frac{\partial P_{\text{trans}}}{\partial \epsilon^2} = 0$ and

$$\epsilon^2 = \lambda_N/R, \tag{39}$$

so that

$$P_{\text{trans}} = \left\{\frac{\lambda_N}{R}\right\}^4 \left\{\frac{E_L}{E_o}\right\}^{-\tilde{b}}. \tag{40}$$

This assumes that the small size "bare" system entering the nucleus maintains this small size during nuclear traversal. The transverse momenta of the quarks implied by the transverse confinement is

$$P_T \approx \hbar/|\tilde{r}| \approx \hbar/\epsilon r_o. \tag{41}$$

Substituting this in Eq. (4) yield:

$$E_L \geq \frac{\hbar R}{\epsilon^2 r_o^2}. \tag{42}$$

Upon substituting the optimal ϵ^2 of Eq. (39), we obtain our stated lower bound on E_L [i.e. Eq. (27)]. To maximize P_{trans} of Eq. (40), we choose the minimal energy $E_L = R^2/r_o^2 \lambda_N$ leading to the promised power dependence on R:

$$P_{\text{trans}}(R) \approx \left\{\frac{\lambda_N}{R}\right\}^{4+\tilde{b}} \left(\frac{r_o}{R}\right)^{\tilde{b}} \tag{43}$$

with $r_0 E_0 \approx O(1)$ factors omitted.

The above discussion leads to dramatic conclusions:

(1) The transverse size of the nucleons, if measured at intermediate times inside the nucleus, should be very small $|\tilde{r}|^2 = r_o^2(\lambda_N/R)$.

(2) The probability that a nucleus will be (diametrically) traversed by a high energy nucleon with no collisions falls off only as an inverse power $(R/r_o)^{-a}$ of the nuclear radius (rather than exponentially as e^{-R/λ_N}) so long as the projectile's energy is high enough: $E/m \equiv \gamma > R/r_o$. This dramatic, yet not directly measurable feature, has been already noted by Kopelovich, Lapidus, and Zamolodchikov [34].

If we could measure the transmission probability and verify that for $R_A = 2R_B$, $P_{\text{trans}}(R_A) \gg P^2_{\text{trans}}(R_B)$, then the above dramatic prediction could be tested. However, we need to make sure that the nucleon indeed traversed the nucleus rather than simply passing next to it. This could be done if we had a gedanken "nuclear" foil consisting of closely packed nuclei. Such foils are not available and the typical distances between nuclei are $a > A°$. For the post-selected small component to reach even the next atomic layer an angstrom away, requires $E \geq (mA°/r_o) \approx 10^5$ GeV. Thus, we need some extra, hard nuclear collisions to verify that the unperturbed nucleus was crossed by the proton or by the neutron.

In the symmetrized version, we look for collisions between the two nucleons at 90° with momentum transfers $(\Delta P_t)^2 \approx P^2$. These putative collisions occur for the initial state described above at time $t = 0$ and at the center of the nucleus. These scattered nucleons travel the same distance $2R$ through the nucleus as occurred and have the same energy. Hence, the "filtering" of small hadrons is the same in the case when proton-neutron scattering at 90° happened as in the case when it did not happen [35].

The weak average of the quantity of interest, namely, the large angle, 90°, scattering amplitude in the time-symmetric formalism is

$$\frac{\langle \psi_p^{\text{right}}(\vec{P}_4 = P\hat{e}_x)\psi_n^{\text{up}}(\vec{P}_3 = P\hat{e}_z)|U(t,-t)|\psi_p^{\text{left}}(\vec{P}_1 = P\hat{e}_x)\psi_n^{\text{down}}(\vec{P}_2 = P\hat{e}_z)\rangle_{\psi^o(A,Z)}}{\langle \psi_p^{\text{right}}(\vec{P}_3 = P\hat{e}_x)|U(t,-t)|\psi_p^{\text{left}}(\vec{P}_1 = P\hat{e}_x)\rangle_{\psi^o(A,Z)} \langle \psi_n^{\text{up}}(\vec{P}_4 = P\hat{e}_z)|U(t,-t)|\psi_n^{\text{down}}(\vec{P}_2 = P\hat{e}_z)\rangle_{\psi^o(A,Z)}}. \tag{44}$$

The hard 90° scattering of $pn \to np$ at $t = 0$ seems not to be a weak measurement. However, the discussion preceding Eq. (5) indicates that, insofar as the aspect of interest is concerned, namely, the filtering via nuclear absorption, the 90° scattering does not change anything at all! This justifies using in Eq. (44) the same weak value denominator [see Eq. (6)] as for the case of no scattering.

If the propagation of the proton and neutron through the nucleus—prior to and after the collision—were independent of the hard collision, then the numerator in Eq. (36) would be a product of two amplitudes for the nuclear propagation and the hard scattering amplitude in vacuum. The nuclear propagation amplitude would cancel in Eq. (44) leaving

$$U_{if} \approx A^{(0)}(pn \to np) = A^{(0)}_{NN}(\Theta = 90°), \tag{45}$$

and the hard scattering amplitude of the two energetic colliding particles—while traversing the nucleus—would then be the same as the corresponding amplitude in free space.

However, this is not the case. We can evolve the initial system of incident p, n from time $-t$ to time $-\delta t$ just prior to the collision and evolve backward in time the final emergent np from time $+t$ to time δt just after the hard collision.

The condition of leaving the nucleus unexcited filters small components of transverse size

$$\tilde{r}^2 = \epsilon^2 r_0^2 = \frac{\lambda_{NN}}{R} r_o^2 \tag{46}$$

in the propagating neutrons and protons. The hard scattering, then, involves no ordinary size nucleons of size r_0, but in the limit when $\frac{\lambda_N}{R} \ll 1$, it involves much smaller objects. Color neutrality implies that soft gluon exchanges between these pointlike configurations are strongly suppressed.

However, there is no such suppression of exchange of hard gluons with momentum transfer in the scattering of the nucleons:

$$(\Delta P)^2 = \frac{\hbar^2}{\epsilon^2 r_o^2} \approx \frac{\hbar^2}{r_o^2} \frac{R}{\lambda_N}. \tag{47}$$

Such gluons can resolve the quarks within the small pointlike objects. Further, since ΔP is comparable with the intrinsic transverse momentum spread in the wave function of the squeezed nucleon, there is $O(1)$ amplitude for 90° elastic scattering $p_s + n_s \to n_s + p_s$ of the latter [36].

V. SUMMARY AND CONCLUSIONS

We have seen above that color transparency in the nuclear QCD context and its relation to pre- and post-selected small nucleons naturally fits in the framework of the time-symmetric formalism of quantum mechanics. We also preceded this with a, hopefully, not completely academic discussion of a similar atomic filtering.

In closing, we note that while the use of post-selection is extremely common it is not always closely related to the pre- and post-selection (time-symmetric) formulation of quantum mechanics. An outstanding example is provided by the upcoming CERN LHC accelerator. The large $\sim 10^9/\mathrm{sec}$ collision rate and complex final states (~ 200 particles produced per collision) preclude analyses (and even storage) of all experimental data. Thus, one heavily relies on "triggers" which post-select interesting, relatively rare events where particles with large transverse momenta were produced. Only such events are stored and analyzed. These events are also preselected: all of the collisions have initial proton pairs moving towards each other along the z axis, say, with specific (large) energies of 7.10^3 GeV.

ACKNOWLEDGMENTS

We would like to thank Prof. Yakir Aharonov for many discussions. We acknowledge support by Grant No. 70NANB7H6138 of the National Institutes of Standards and Technology. J. T. acknowledges support by Grant No. N00173-07-1-G008 of the Naval Research Laboratory.

[1] L. L. Frankfurt, G. A. Miller, and M. Strikman, in *Annual Review of Nuclear and Particle Science* (Annual Reviews, Palo Alto, 1994), Vol. 44, p. 501.
[2] M. Strikman, arXiv:0711.1625v2 [Proceedings of Workshop on Exclusive Reactions at High Momentum Transfer, Newport News, Virgina, 2007 (to be published)].
[3] E. Chudakov, Izv. Akad. Nauk SSSR, Ser. Fiz. **19**, 651 (1955); G. Yekutieli, Nuovo Cimento **5**, 1381 (1957); D. Perkins, Philos. Mag. **46**, 1146 (1955); L. L. Nemenov, Yad. Fiz. **34**, 1306 (1981).
[4] L. L. Frankfurt and V. A. Khoze, in *Proceedings of 10th LNPI Winter School, Leningrad, USSR, 1975*, Vol. 2, p. 196 (unpublished); Yad. Fiz. **23**, 926 (1976).
[5] F. E. Low, Phys. Rev. D **12**, 163 (1975); S. Nussinov, Phys. Rev. Lett. **34**, 1286 (1975).
[6] A. H. Mueller, Columbia University Report No. CU-TP-232, 1982; S. J. Brodsky, SLAC Report No. SLAC-PUB-2970, 1982; in *Proceedings of the 13th International Symposium on Multiparticle Dynamics, Volendam, Netherlands, 1982*, edited by W. Kittel, W. Metzger, and A. Stergiou (World Scientific, Singapore, 1983), p. 0963.
[7] G. R. Farrar, H. Liu, L. L. Frankfurt, and M. I. Strikman, Phys. Rev. Lett. **61**, 686 (1988).
[8] B. Z. Kopeliovich, L. I. Lapidus, and A. B. Zamolodchikov, Pis'ma Zh. Eksp. Teor. Fiz. **33**, 612 (1981) [JETP Lett. **33**, 595 (1981)].
[9] B. Z. Kopeliovich and B. G. Zakharov, Phys. Rev. D **44**, 3466 (1991). In this reference, the enhanced production of radially excited states is discussed.
[10] J. P. Ralston and B. Pire, Phys. Rev. Lett. **61**, 1823 (1988); **65**, 2343 (1990); B. K. Jennings and G. A. Miller, Phys. Lett. B **318**, 7 (1993).
[11] N. N. Nikoleav, Z. Phys. C **32**, 537 (1986).
[12] B K. Jennings, G. A. Miller, Phys. Rev. C **49**, No. 5, 2637 (1994).
[13] Y. Aharonov, P. G. Bergmann, and J. L. Lebowitz, Phys. Rev. **134**, B1410 (1964); reprinted in *Quantum Theory and Measurement*, edited by J. A. Wheeler and W. H. Zurek

(Princeton University, Princeton, NJ, 1983), p. 680.

[14] Y. Aharonov, D. Albert, and L. Vaidman, Phys. Rev. Lett. **60**, 1351 (1988).

[15] Y. Aharonov and L. Vaidman, Phys. Rev. A **41**, 11 (1990); Y. Aharonov and A. Botero, Phys. Rev. A **72**, 052111 (2005); Y. Aharonov, A. Casher, D. Albert, and L. Vaidman, Phys. Lett. A **124**, 199 (1987); Y. Aharonov and L. Vaidman, J. Phys. A **24**, 2315 (1991); B. Reznik and Y. Aharonov, Phys. Rev. A **52**, 2538 (1995).

[16] Y. Aharonov and L. Vaidman, Lect. Notes Phys. **734**, 395 (2007); Y. Aharonov and D. Rohrlich, *Quantum Paradoxes* (Wiley-VCH, Weinheim, Cambridge, 2005); Y. Aharonov and J. Tollaksen, arXiv:0706.1232 [''Visions of Discovery,'' (Cambridge University Press, Cambridge, England, to be published)].

[17] R. Brout, S. Massar, R. Parentani, S. Popescu, and Ph. Spindel, Phys. Rev. D **52**, 1119 (1995).

[18] R. Y. Chiao and A. M. Steinberg, Prog. Opt. **37**, 345 (1997); D. R. Solli, C. F. McCormick, R. Y. Chiao *et al.*, Phys. Rev. Lett. **92**, 043601 (2004).

[19] A. M. Steinberg, Phys. Rev. Lett. **74**, 2405 (1995).

[20] Y. Aharonov, L. Davidovich, and N. Zagury, Phys. Rev. A **48**, 1687 (1993).

[21] J. Tollaksen, J. Phys. Conf. Ser. **70**, 012015 (2007); O. Hosten and P. Kwiat, Science **319**, 787 (2008); K. J. Resch, Science **319**, 733 (2008).

[22] Y. Aharonov, S. Massar, S. Popescu, J. Tollaksen, and L. Vaidman, Phys. Rev. Lett. **77**, 983 (1996); Y. Aharonov, A. Botero, S. Popescu, B. Reznik, and J. Tollaksen, Phys. Lett. A **301**, 130 (2002); J. Tollaksen, J. Phys. A **40**, 9033 (2007); Y. Aharonov, S. Massar, S. Popescu, J. Tollaksen, and L. Vaidman, Phys. Rev. Lett. **77**, 983 (1996).

[23] M. V. Berry, in *Fundamental Problems in Quantum Theory*, edited by J. A. Anandan and J. Safko (World Scientific, Singapore, 1994); M. V. Berry and S. Popescu, J. Phys. A **39**, 1 (2006); J. Tollaksen, J. Phys. Conf. Ser. **70**, 012016 (2007).

[24] Y. Aharonov, S. Popescu, J. Tollaksen, and L. Vaidman, arXiv:0712.0320.

[25] M. L. Good and W. D. Walker, Phys. Rev. **120**, 1857 (1960).

[26] Color confinement and the existence of a mass gap, i.e., a nonzero lowest mass glueball state, excludes a complete QCD- QED analogy. This is not important for the basic argument. The interaction can also be mediated via quark exchange and/or $q\bar{q}$ annihilation. However, at high energy, the gluon exchange (''pomeron exchange'') dominates [5].

[27] S. J. Brodsky and G. R. Farrar, Phys. Rev. Lett. **31**, 1153 (1973); Phys. Rev. D **11**, 1309 (1975).

[28] Note also that elastic scattering at large p_T can be achieved in nucleon-nucleon collisions via the ''Landshoff process'', namely, the exchange of three gluons between each pair of quarks with each carrying a momentum close to $p_T/3$. The relative momenta between quarks in the final nucleon remains small just like in the original nucleon, and no small configuration space states are expected to arise.

[29] L. D. Landau and E. M. Lifshiftz, *Quantum Mechanics Nonrelativistic Theory* (Pergamon, New York, 1977), p. 100.

[30] L. Vaidman, Am. J. Phys. **60**, 182 (1992).

[31] Δt is also the time required for an electron localized at $|\tilde{r}| \leq \epsilon r_o$ and with transverse momentum $\frac{\hbar}{\epsilon r_o}$ and transverse velocity $\frac{\hbar}{m\epsilon r_o}$ to travel a distance of order ϵr_o.

[32] In realistic high energy experiments optimized to deal with O (GeV) energies, it is difficult to verify that there has not been any O (few MeV) excitations of the hit nucleus. Rather one simply post selects the cases where an almost free two body quasielastic kinematics works. We still refer in the following to the ideal filter of no nuclear excitations.

[33] If anything like cold fusion could have been possible, then it would have occurred more readily with squeezed deuterium.

[34] B. Z. Kopelovich, L. I. Lapidus, and A. B. Zamolodchikov, JETP Lett. **33**, 595 (1981).

[35] In the actual experimental setup the struck nucleon is at rest before the collision and travels only half the nuclear diameter. However, since, as emphasized above, the hard collision does not squeeze it, it should be small prior to the collision as well.

[36] In the real experiment, the struck nucleon at the center of the nucleus was at rest and not subjected to any filtering. Thus the scattering $p_s + p \rightarrow p + p$ involved three squeezed nucleons and one normal full size target nucleon—somewhat diluting the effect.