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Selection in the Lab: A Network Approach

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Abstract

We study the dynamics of the selection problem in economic experiments. We show that adding dynamics significantly complicates the effect of the selection problem on external validity and can explain some contradictory results in the literature. We model the dynamics of the selection problem using a network model of diffusion in which agents' participation is driven by the two channels: the direct channel of recruitment and the indirect channel of agent interaction. Using rich recruitment data from a large public university, we find that the patterns of participation and biases are consistent with the model. We find evidence of both short- and long-run selection biases between student types. Our empirical findings suggest that network effects play an important role in shaping the dynamics of the selection problem. We discuss the implications of our results for experimental methodology, design of experiments, and recruitment procedures.

Keywords: selection problem, experiments, external validity, networks, diffusion, peer effects

JEL codes: C32, C90, D85

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1 Introduction

Experimental methods have become an indispensable tool in economic science (Smith, 1989; Falk and Heckman, 2009). Lab and field experiments provide a steady source of behavioral insights for new models and serve as a convenient test bed for existing models. With increasing reliance on experimental methods, however, comes the need to confirm the robustness of their procedures. An important issue that has gained considerable attention in recent years is external validity, or whether the results obtained with experimental subjects generalize to a relevant reference population.[1] A major threat to external validity is the selection problem, i.e., the fact that subjects represent a non-random sample of a reference population.

A large and growing literature has emerged to address the selection problem. This literature has produced contradictory results.[2] A major limitation of the existing literature, which could also serve as a potential source of contradictory results, is a view that the selection problem is static and individual in nature. In this paper we attempt to overcome this limitation by studying, both theoretically and empirically, how the dynamic and network effects shape the selection problem.

We begin our analysis by showing that the presence of dynamics significantly complicates the effect of the selection problem on external validity. Conditions under which selection problem does not pose a threat to external validity in the static case are no longer valid in the dynamic case. In particular, even if treatment effects are homogeneous across types (a sufficient condition for external validity in the static case), the estimated treatment effects may still deviate from the population treatment effect in the dynamic case. The presence of dynamics is likely to create a treatment effect bias even a subject pool does not have a selection problem initially.

Having established the importance of dynamic effects in the selection problem, we proceed by developing a dynamic network model of participation in a subject pool. Our model is based on a classic Bass (1969) network model of diffusion. The distinctive feature of our model, as

[1] For example, Levitt and List (2007) and List (2009) argue that social preferences, which remain one of the most actively studied topic in behavioral and experimental economics, are grossly over-estimated in laboratory experiments. Henrich *et al.* (2010) argue that the behavior and characteristics of student subjects in Western countries are somewhat atypical when compared with other societies across the world. Gächter (2010), on the other hand, argues that student subjects are a great sample for the research questions economists usually ask.

[2] Some papers (Cleave *et al.*, 2013; Exadaktylos *et al.*, 2013; Falk *et al.*, 2013) report that the behavior of students who sign up for economic experiments is not significantly different from the behavior of student or general population in social and risky settings. Other papers (Slonim *et al.*, 2013; Cappelen *et al.*, 2015) do find significant differences between student participants and reference populations.

opposed to the models typically used in the literature (e.g., Abeler and Nosenzo (2015), Slonim *et al.* (2013)), is that it recognizes the dynamic and social (via networking) nature of the decisions to participate in a subject pool. The central assumption of the model is that agents can sign up for participation via two channels. The first channel is a direct channel represented by an experimenter's recruitment efforts, such as e-mail invitations or class visits. The second channel is an indirect channel represented by the interactions between agents within their network.[3] This channel highlights the idea that an agent may be convinced to participate by her peers. In each period, the agents who are not in a subject pool may arrive into the pool via either direct or indirect channels at given type-specific rates.[4]

In our first specification of the model, an agent's type is a time-invariant characteristic. The model generates three key features of the participation dynamics: rapid growth immediately after recruitment, a slowdown in growth later on, and steady growth towards an upper limit in subsequent periods. A natural implication of this model is that over time the participation of each type asymptotically approaches a value that we refer to as a *potential proportion*. Differences in potential proportions, which may result from differences in preferences, such as risk preferences (Harrison *et al.*, 2009), is what drives the long-run selection biases between types in this version of the model. Additionally, if types have different initial participation levels or have different participation growth rates, a subject pool will also exhibit short-run selection biases. Simulations show that a typical dynamic pattern is a high initial bias that declines steadily in subsequent periods. Such a pattern is consistent with the results in two recent studies (Slonim *et al.*, 2013; Cleave *et al.*, 2013) that have almost identical designs but differ in the points in time at which biases were evaluated.

In our second specification of the model we allow types to be time-variant. We consider a case when types are cohorts of agents. An important insight from this version of the model is that long-run biases are possible even when potential proportions are the same across types that is not the case with time-invariant types. The model predicts oversampling of older cohorts in a subject pool relative to younger cohorts. The participation levels among types will be asymptotically lower than their respective potential proportions, unlike in the model with time-invariant types, due to

[3] Recruitment through an indirect channel is related to a recruitment technique in sociology known as *snowball sampling* (Biernacki and Waldorf, 1981).

[4] Our decision to introduce two separate channels is motivated by two considerations. First, it is a more accurate description of an actual recruiting process. Second, having two channels leads to different dynamics of the selection problem, and hence to different policy implications, as compared to having only one channel.

the constant outflow of agents. The dynamic path and magnitude of the selection problem between cohorts will depend on the network structure of a reference population.

We use recruitment data from the Interdisciplinary Center for Economic Science (ICES) at George Mason University (GMU) and GMU's registrar enrollment data to construct daily time-series of student participation in the subject pool. Our observation period is from the Fall semester of 2014 until the Spring semester of 2017. We have data on when each subject entered and left the subject pool, as well as subjects' characteristics. We compare the number of students of each type who are in the subject pool to the number of students of that type in the student population to identify selection biases.

While our theoretical model allows for any reference population, in the empirical analysis we use the student population as our reference population. Our choice of the reference population is motivated by convenience and the fact that the GMU registrar provides high-quality data on the student population. Using alternative reference populations would have changed the scales of our time-series but not the dynamic trends, which are the focus of the present study.

Similarly, even though our theoretical model allows for types to be unobservable, as well as observable, we choose to use observables in our empirical analysis. This choice is dictated by necessity: the analysis of the dynamics of selection requires high-frequency data on student types. We are aware of no recruitment system that currently features such high-frequency data on unobservable characteristics. The analysis of the dynamics of observable types, however, does provide insights into the dynamics of the unobservable types. If a decision to participate in a subject pool reflects a subject's preferences, then the trends in observable types should reflect these preferences. Put differently, if the dynamics of unobservable types are flat and there is a mapping between observable and unobservable characteristics, the dynamics of observable types must also be flat.

Our empirical analysis yields three main findings. First, we show that the dynamics of the subject pool is consistent with the dynamics predicted by the model. In particular, we confirm the existence of potential proportions for each type. These patterns are observed regardless of whether we define gender (time-invariant) or cohort (time-variant) as a type. Second, we find evidence of both short- and long-run biases between types. Males tend to be consistently oversampled relative to females both in the short- and long-run, though the size of the bias is small. Cohorts data reveal that younger cohorts are initially oversampled in the subject pool relative to older cohorts but over time a reversal of this ranking occurs. Finally, we conduct the structural estimation of the model and document the heterogeneity between types in terms of the strengths of the direct and indirect channels. The counterfactual analysis shows that the indirect channel accounts for one-third of the total participation gain and plays a substantial role is shaping the selection bias.

Three policy recommendations emerge from our analysis of the dynamics of the selection problem. First, future studies of the selection problem should recognize its dynamic nature. The measurements of the selection problem, for example, could be done at various points in time throughout a recruitment cycle. Ignoring the dynamic nature of the selection problem could lead to seemingly contradictory results about the presence of the selection problem in a subject pool. Second, studies using a between-subject design would benefit from randomizing treatments on a subject-level within a session. That would minimize the potential treatment effect bias caused by the dynamic component the selection problem. If randomization to treatment within a session is infeasible, keeping sessions temporally close (or even running them simultaneously at different places) would be the second-best option. Third, it would make sense to leave a short (one to two weeks) burn-in period after a main recruitment event. Selection biases tend to stabilize in the long-run. Therefore, leaving a burn-in period would minimize the selection biases caused by short-run fluctuations.

Our paper contributes to the growing literature on the selection problem in economic experiments by providing a theoretical framework that captures the dynamic and network nature of participation in a subject pool, as well as empirical evidence on the dynamics of student participation and the selection problem. Other papers have proposed theoretical models of participation in a subject pool. Abeler and Nosenzo (2015) assume that the aggregate utility of participation consists of three parts: monetary utility, pro-social utility, and a fixed cost. Monetary and pro-social parts have different weights, and these weights determine which motive is stronger. The model assumes that recruitment changes potential participants' priors about the expected monetary reward and the need for social approval. Slonim *et al.* (2013) introduce a more general utility-of-participation function that has four components: monetary reward, leisure time, intellectual curiosity, and social preferences. Their model predicts that students with lower income, more leisure time, higher curiosity, and higher pro-social preferences are more likely to participate in a subject pool.

Our theoretical analysis highlights the mechanisms that drive short- and long-run selection biases and helps to explain some contradictory results in the existing literature. Falk *et al.* (2013)

look at the entire student population of the University of Zurich from 1998 to 2004 and compare social preferences of students who signed up for participation in economic experiments with social preferences of the general student population. As a measure of social preferences, they use information from the school about students' donation choices. The main finding is that pro-social behavior does not differ between the two groups. Cleave *et al.* (2013) study the student population at the University of Melbourne by conducting experiments in classes and also inviting students to participate in economic experiments. Five months later, they compare the choices of students in the class experiments to the choices of those students who agreed to participate in experiments, registered in the database and showed up for a laboratory experiment. Their analysis reveals that the choices are very similar across the two groups, as are their demographic characteristics. Slonim *et al.* (2013), using a design similar to that of Cleave *et al.* (2013) but a shorter waiting period of several weeks, do find significant differences in some characteristics and preferences of participants and non-participants in lab experiments. Taken at face value, the results from the two former papers contradict each other, however, viewing these results from the perspective of our dynamic model alleviates the contradiction. It is likely that the results in Slonim *et al.* (2013) reflect the short-run biases in the data, while the results in Cleave *et al.* (2013) reflect the long-run biases, which are typically smaller. None of the existing models of participation can account for this pattern.

The remainder of the paper is organized as follows. Section 2 defines the selection problem and illustrates how the selection problem affects external validity. Section 3 introduces the model with time-invariant types and presents theoretical results for the dynamics of participation and selection bias. Section 4 extends the model by allowing for time-variant types. Section 5 describes the data and presents the reduced-form results for participation and selection. Section 6 contains the procedure and results of the structural analysis of the data. Section 7 concludes and discusses directions for further research.

2 Selection Problem

Let $\mathcal{N} = \{1, \ldots, N\}$ be the set of types of agents in a population based upon a certain characteristic. We call the characteristics that do not change over time *time-invariant* characteristics. Characteristics that change over time are called *time-variant* characteristics. Characteristics can be observable (such as demographic characteristics) or unobservable (such as preferences and personality traits).

Let $\tilde{\mathbf{x}}_t \equiv (\tilde{x}_t^1, \ldots, \tilde{x}_t^N)'$ be a vector of proportions of the number of agents of each type in a subject pool relative to the total number of agents in a reference population at time t. A reference population can be the population of people in a school that hosts a subject pool, the population of a local area in which the school is located, the general population of a country, or any other population deemed relevant by a researcher. For example, if we use risk aversion as a characteristic and assign Type I to risk averse/neutral people and Type II to risk loving people, then $\tilde{\mathbf{x}}_{t=10} = (0.1, 0.2)'$ means that at the 10th period of observation a subject pool contained 10% of people in the reference population who are risk averse/neutral and 20% of people in the reference population who are risk loving people. If we denote S_t to be the total number of people in a reference population at time t, then a subject pool can be defined by the number of agents of each type in it: $S_t\tilde{\mathbf{x}}_t$. For example, if there are $S_{10} = 10{,}000$ people in a reference population at time 10, a subject pool would be composed of 1,000 risk averse/neutral subjects and 2,000 risk loving subjects.

While there are more risk loving subjects than risk averse/neutral subjects in this subject pool, this does not necessarily cause a selection problem, since the shares of each type in a reference population may also be different. For instance, if there are twice as many risk loving people in a reference population than risk averse/neutral people, then the composition of the subject pool perfectly matches the composition of the reference population, and there is no selection problem. Let $\mathbf{m} \equiv (m^1, \ldots, m^N)'$, $\boldsymbol{\imath}'\mathbf{m} = 1$, ($\boldsymbol{\imath}$ is a sum vector of order N), be a vector of shares of each type in a reference population. In the previous example, if there are twice as many risk loving people as risk averse/neutral people, the reference population is 33% risk averse/neutral and 66% risk loving, and so $\mathbf{m} = (1/3, 2/3)'$. Let $\mathbf{M} \equiv \text{diag}(\mathbf{m})$ be a diagonal matrix with the population proportions of each type on the diagonal. Then $\mathbf{x}_t \equiv \mathbf{M}^{-1}\tilde{\mathbf{x}}_t = (\tilde{x}_t^1/m^1, \ldots, \tilde{x}_t^N/m^N)'$ is a vector of proportions of the number of agents of each type in a subject pool relative to the shares of agents of these types in a reference population. This quantity will be called a vector of *relative proportions* and its dynamics is the main object of study in this paper.

What does it mean, then, to have a selection problem? A selection problem occurs when the composition of a subject pool does not match the composition of a reference population. We can formally define the selection problem as follows.

Definition 1 (Selection Problem)**.** *A subject pool is said to have a selection problem at time t (asymptotically) if there is no constant $\bar{\alpha} \in [0,1]$, such that the vector of relative proportions can be written as $\mathbf{x}_t = \bar{\alpha}\boldsymbol{\imath}\,(\lim_{t\to\infty}\mathbf{x}_t = \bar{\alpha}\boldsymbol{\imath})$.*

Suppose that for some subject pool $\mathbf{x}_t \neq \bar{\alpha}\boldsymbol{\imath}$ for any $\bar{\alpha}$ so that there is a selection problem at time t. How bad is this problem? To answer this question, we can look, for instance, at *pairwise* biases between types and define $b_t^{ij} \equiv x_t^i/x_t^j,\; i,j \in \mathcal{N}$. This number determines the ratio of a relative proportion of agents of type i to a relative proportion of type j at time t. If $b_t^{ij} > 1$, there are more agents of type i in a subject pool than agents of type j relative to their population shares, and the bigger is b_t^{ij}, the stronger is the selection bias between these two types. On the flip side, b_t^{ji} in this example will be less than one. The strength of the bias is therefore determined by how far a pairwise bias is from 1. If all the pairwise biases are equal to one, there is no selection problem. On the other hand, if there is at least one pairwise bias that is not equal to one, there is a selection problem, and a subject pool is biased.

Proposition 1. *A subject pool does not have a selection problem at time t (asymptotically) iff for any two types $i,j \in \mathcal{N}$ the pairwise bias $b_t^{ij} = 1 \left(\lim_{t\to\infty} b_t^{ij} = 1\right)$.*

The existence of a selection problem makes it harder to extrapolate the results obtained from a subject pool to a reference population. Two inference problems arise because of the selection problem. First, the estimated shares of each type will not be representative of the reference population. Consider the example with two types of risk preferences. The share of Type I (risk averse/neutral people) in the subject pool is given by[5]

$$\tilde{m}^{\mathrm{I}} \equiv \frac{\tilde{x}^{\mathrm{I}}}{\tilde{x}^{\mathrm{I}} + \tilde{x}^{\mathrm{II}}} = \frac{m^{\mathrm{I}}}{m^{\mathrm{I}} + m^{\mathrm{II}} b^{\mathrm{II,I}}}.$$

[5] To simplify the exposition, below we focus on the case with only two types. However, the logic carries over to a more general case.

The existence of a selection bias between the two types ($b^{\mathrm{II,I}} \neq 1$) will drive the wedge between the shares of each type in a subject pool and the actual shares in a reference population. The following proposition follows.

Proposition 2. *The shares of each type in a subject pool, $\tilde{m}^i$, will be identical to a reference population shares, m^i, iff a subject pool does not have a selection problem.*

If a research question is to measure the prevalence of a certain type in the population (e.g., based on risk aversion or altruism), then the presence of a selection problem will bias the results. A selection problem will also lead to a second inference problem: biased estimates of treatment effects. To see this, consider again the case of two types. The average treatment effect in a reference population, ΔY, is given by

$$\Delta Y = m^{\mathrm{I}} \Delta Y^{\mathrm{I}} + m^{\mathrm{II}} \Delta Y^{\mathrm{II}} = \Delta Y^{\mathrm{II}} + m^{\mathrm{I}} \left(\Delta Y^{\mathrm{I}} - \Delta Y^{\mathrm{II}} \right),$$

where ΔY^{I} and ΔY^{II} are the treatment effects for the two types. In a subject pool, the average treatment effect, $\Delta \tilde{Y}$, is given by

$$\Delta \tilde{Y} = \tilde{m}^{\mathrm{I}} \Delta Y^{\mathrm{I}} + \tilde{m}^{\mathrm{II}} \Delta Y^{\mathrm{II}} = \Delta Y^{\mathrm{II}} + \tilde{m}^{\mathrm{I}} \left(\Delta Y^{\mathrm{I}} - \Delta Y^{\mathrm{II}} \right).$$

The following proposition follows.

Proposition 3. *A treatment effect in a subject pool will coincide with a treatment effect in a reference population iff either (or both) condition holds: a) treatments effects are homogeneous across all types; b) a subject pool does not have a selection problem.*

This implies that if neither condition holds, i.e., treatment effects are heterogeneous and there is a selection problem, a treatment effect in a subject pool will be different from a treatment effect in a reference population. The existence of heterogeneous treatment effects across types is a common scenario in many experimental settings (l'Haridon *et al.*, 2018; Castillo and Freer, 2018). Hence, selection problem will be sufficient to cause biases in estimated treatment effects.

If a selection problem is dynamic, then the inference problems become more complicated. Consider the case with two types when the relative proportions of types vary in time and treatments are assigned in different moments in time. It is a common practice for experimental economists to assign treatments on a session level in a between-subject design and then run different sessions in sequence, which creates a temporal separation of treatments (Charness *et al.*, 2012). Assume that at time t both types receive the baseline condition that produces response Y_0^i in Type i and that at time $t+1$ both types receive the treatment condition that produces response Y_1^i in Type i. Then an average treatment effect in a subject pool is given by

$$\Delta\tilde{Y} = \Delta Y^{\mathrm{II}} + \tilde{m}_t^{\mathrm{I}}\left(\Delta Y^{\mathrm{I}} - \Delta Y^{\mathrm{II}}\right) + \Delta\tilde{m}^{\mathrm{I}}\left(Y_1^{\mathrm{I}} - Y_1^{\mathrm{II}}\right),$$

where $\tilde{m}_t^{\mathrm{I}}$ is share of Type I in a subject pool at time t, $\Delta\tilde{m}^{\mathrm{I}} \equiv \tilde{m}_{t+1}^{\mathrm{I}} - \tilde{m}_t^{\mathrm{I}}$ is the change in a subject pool share of Type I from time t to $t+1$. The difference between the treatment effects in a subject pool and a reference population, a *treatment effect bias*, is then

$$\Delta Y - \Delta\tilde{Y} = \underbrace{\left(\Delta Y^{\mathrm{I}} - \Delta Y^{\mathrm{II}}\right)\left(m^{\mathrm{I}} - \tilde{m}_t^{\mathrm{I}}\right)}_{\text{Static component}} - \underbrace{\Delta\tilde{m}^{\mathrm{I}}\left(Y_1^{\mathrm{I}} - Y_1^{\mathrm{II}}\right)}_{\text{Dynamic component}},$$

where the first term represents the static component of the bias in an average treatment effect, and the second component represents the dynamic component of the bias. The following proposition then holds.

Proposition 4. *In the presence of a dynamic selection problem, an estimated treatment effect in a subject pool will coincide with a treatment effect in a reference population iff the static component equals the dynamic component.*

In contrast to the previous case with no time-varying selection problem, it is possible to have a bias in an estimated average treatment effect even if treatment effects are homogeneous across types. On the other hand, even if there is a selection problem at time t and treatment effects are heterogeneous across types, it does not necessarily imply that there will be a bias in an average treatment effect. It is possible that the change in the selection bias will be just right to compensate for the differences between treatment responses.[6] Such a fortunate scenario, however, is unlikely. If we further assume that types differ in their treatment effects and treatment responses, which is

[6] The precise condition is $\dfrac{Y_0^{\mathrm{I}} - Y_0^{\mathrm{II}}}{Y_1^{\mathrm{I}} - Y_1^{\mathrm{II}}} = \dfrac{m^{\mathrm{I}} - \tilde{m}_{t+1}^{\mathrm{I}}}{m^{\mathrm{I}} - \tilde{m}_t^{\mathrm{I}}}$, assuming that the denominators are not zero.

reasonable to expect, the presence of the dynamic component will create a treatment effect bias even a subject pool does not have a selection problem initially. If a subject pool has a selection problem initially, it is unclear *a priori* whether the dynamic component will make a treatment effect bias smaller or larger. It is clear, however, that the dynamic component complicates the interaction between a selection problem and related inference problems and thus deserves a closer consideration.

3 Model For Time-Invariant Types

In order to guide our empirical analysis in the next section and make testable predictions, we build a simple model of the participation in a subject pool and potential bias among types with time-invariant characteristics. We use Bass (1969) model of diffusion as a starting point and adapt it to the environment of a subject pool.[7] Suppose that at the start of the observation period, $t = 0$, there are $S\tilde{\mathbf{x}}_0$ agents in a subject pool. We allow the relative proportions $\mathbf{x}_t$ to have upper limits given by a vector of *potential* proportions $\boldsymbol{\alpha} = (\alpha^1, \ldots, \alpha^N)'$, $\alpha^j \in [0, 1], j \in \mathcal{N}$. These potential proportions will allow for the existence of asymptotic biases.

New subjects can arrive into a subject pool via two channels. The first channel is a direct channel of recruitment efforts, such as emails, posters, and class visits. The growth rate at which agents who are not a pool will join it immediately after a recruitment event is given by a vector of *spontaneous* rates $\mathbf{p} \equiv (p^1, \ldots, p^N)'$, $p^j \geqslant 0, j \in \mathcal{N}$. These rates can be different across types. We allow for a decay in the spontaneous rates, since recruitment events are typically infrequent and short-lived. Let $\boldsymbol{\delta} \equiv (\delta^1, \ldots, \delta^N)'$ be a vector of *decay* rates and let $\mathbf{d}_t \equiv \exp(-\boldsymbol{\delta} t)$ be a vector of decay factors.[8] Then at each time t, the vector of spontaneous rates is defined as $\mathbf{p}_t \equiv \mathrm{diag}(\mathbf{d}_t)\mathbf{p}$.

The second channel is an indirect channel of agent interactions: agents who are already in a subject pool can induce their peers to join the pool. For example, a person who earned a decent amount of money in an experiment or just enjoyed being a part of it may share this experience with her friends. This channel is captured by a matrix of *imitation* rates $\mathbf{Q} \equiv \{q^{ij}\}$, $i, j \in \mathcal{N}$. Each

[7] We are certainly aware of more advanced network models with types, e.g., Jackson and López-Pintado (2013). However, we chose in favor of a simple model of diffusion to make the exposition more intuitive. Despite its simplicity, the model is flexible enough to incorporate the relevant features of the student pool.

[8] Research in psychology suggests that exponential decay is a reasonably close approximation of the so-called "forgetting curve" (Murre and Dros, 2015; Averell and Heathcote, 2011; Ebbinghaus, 1885/1974).

element $q^{ij} \geqslant 0$ represents the growth rate in the participation of type i because of the interaction with agents of type j. The diagonal elements of $\mathbf{Q}$ are *own*-imitation rates (imitation of the own type), and the off-diagonal elements are *cross*-imitation rates (imitation of other types). Matrix $\mathbf{Q}$ can be thought of as a network structure of a reference population. The own-imitations rates are allowed to differ between types. It is natural to assume that the own-imitation rates are higher than the cross-imitation rates, but this is not required. The strength of the indirect channel will depend on the relative proportions of agents of each type in a pool: there has to be someone to imitate, and the more agents of a given type are in the pool, the more likely they will influence the decision of their peers. We assume that imitation rates are constant, but this can be relaxed by assuming that they depend on the activity of an experimental laboratory. In this case, running more experiments would result in a higher likelihood that participants will share their experience with their friends.

Our decision to introduce two separate channels, as opposed to the direct channel only, is motivated by two considerations. First, having two channels is a realistic description of an actual recruiting process. For example, in some schools the majority of the agent inflow into a subject pool occurs via word-of-mouth (indirect channel). Ignoring the indirect channel thus would considerably limit the model's descriptive ability. The second, and more pragmatic, consideration is that the model with two distinct channels produces very different dynamics of the selection problem as compared to the model with the direct channel alone. Since different dynamic patterns of the selection problem will have different policy implications, entertaining the possibility of the two distinct channels and comparing their relative strength is relevant for policy recommendations.

In the practice of laboratory experiments, agents arrive into a subject pool from a student population. However, a student population does not have to be a reference group. If a reference group is chosen to be the general population, the rates in the model will simply reflect the joint rate of transitioning from the general population into a student population and transitioning from a student population into a subject pool. The choice of a reference group does not affect the intra-semester or intra-year dynamics of a subject pool (measured at a daily frequency in our application), since the transition into a student population occurs at a much lower frequency (typically, yearly). The choice of a reference group will, however, affect the levels of time series.

Let the spontaneous and imitation rates be defined over a small time period $\Delta t \geqslant 0$. The dynamics of the subject pool participation can then be described by the following system of equations:

$$\tilde{x}^i_{t+\Delta t} = \tilde{x}^i_t + p^i_t \Delta t(\alpha^i m^i - \tilde{x}^i_t) + \left(q^{i1} \frac{\tilde{x}^1_t}{m^1} + \ldots + q^{iN} \frac{\tilde{x}^N_t}{m^N} \right) \Delta t(\alpha^i m^i - \tilde{x}^i_t),\ i \in \mathcal{N}. \tag{1}$$

This equation attributes the growth of a subject pool participation to the two channels described above. The second term on the right-hand side represents the inflow of subjects due to recruitment efforts. The spontaneous rate is applied to the proportion of agents who are not currently in the subject pool. Note that we multiply the share m by the potential proportion α to allow for non-full participation, as typically only a fraction of a reference population participates in a subject pool.[9] The third term reflects the inflow due to the imitation of the behavior of own types and other types. The own- and cross-imitation rates are scaled by the relative proportions of agents: the more agents participate in the subject pool, the stronger is the indirect channel.

Dividing both parts of equation (1) by the share m^i, rearranging the terms, and taking the limit $\Delta t \to 0$, we obtain the following system of differential equations:

$$\dot{x}^i = (\alpha^i - x^i)(p^i_t + q'_{i\cdot}\mathbf{x}_t),\ i \in \mathcal{N}, \tag{2}$$

where $q'_{i\cdot}$ is an i-th row of $\mathbf{Q}$, or in matrix form,

$$\dot{\mathbf{x}} = \mathrm{diag}(\boldsymbol{\alpha} - \mathbf{x}_t)(\mathbf{p}_t + \mathbf{Q}\mathbf{x}_t). \tag{3}$$

Since the second terms on the right-hand side are positive, the steady state of the system is simply the vector of potential proportions $\boldsymbol{\alpha}$. Starting from any initial value $\mathbf{x}_0 \leqslant \alpha$, the relative proportions will converge to the potential proportions: $\lim_{t\to\infty} \mathbf{x}_t = \boldsymbol{\alpha}$. The data on the relative proportions at the end of a semester can then be used to infer the potential proportions. If the potential proportions are different across types, long-run biases will occur. The differences in the potential proportions can be caused by differences in preferences. For example, since women are

[9] In the case when the reference population is different from a student population, the potential proportion reflects the joint long-term rate of transitioning from the reference population into a student population and from a student population into a subject pool.

typically found to be more risk-averse than men (Falk *et al.*, 2018), women may have lower potential proportions than men. We summarize this result in the following proposition.

Proposition 5. *A subject pool has a selection problem asymptotically iff there is no constant $\bar{\alpha}$, such that the vector of potential proportions can be written as $\boldsymbol{\alpha} = \bar{\alpha}\boldsymbol{\imath}$.*

The short-run dynamics of the relative proportions and biases are determined by the starting values $\mathbf{x}_0$, the potential proportions and the rate parameters of the model. The time path of relative proportions can be found analytically in some special cases. The first case arises when there are only own-imitation and no cross-imitation between types, and there are no subjects in the pool at the beginning of the observation period. The formula below also assumes that the direct channel is always present, i.e., there is no decay in the spontaneous rates. This case would apply in a situation when, for example, recruitment posters are posted on a school's announcement board and remain there until the end of a semester.

Example 1. *Assume that there is no cross-imitation between types, i.e., all the off-diagonal elements of the imitation matrix $\mathbf{Q}$ are zero, $\mathbf{x}_0 = \mathbf{0}$, and there is no decay, $\boldsymbol{\delta} = \mathbf{0}$. Then the relative proportions of each type evolve according to*

$$x_t^i = \alpha^i \frac{1 - e^{-(p^i + \alpha^i q^{ii})t}}{1 + \frac{\alpha^i q^{ii}}{p^i} e^{-(p^i + \alpha^i q^{ii})t}}, \; i \in \mathcal{N}.$$

The second case arises when there is no direct channel, and the indirect channel works only through own-imitation. For example, some time might have passed after recruitment and spontaneous rates decayed to almost zero. In order for the subject pool to grow, some initial amount of subjects has to be there. A subject pool will exhibit logistic growth in this case.

Example 2. *Assume that there is no cross-imitation between types, (all the off-diagonal elements of the imitation matrix $\mathbf{Q}$ are zero), that own-imitation rate is non-zero, that there is no direct channel ($\mathbf{p}_t = \mathbf{0}$), and that the subject pool starts with $S\mathbf{x}_0$ subjects in it. Then the relative proportions of each type evolve according to*

$$x_t^i = \frac{\alpha^i}{1 + \frac{\alpha^i - x_0^i}{x_0} e^{-\alpha^i q^{ii} t}}, \; i \in \mathcal{N}.$$

The model, in general, generates a time path of relative proportions that is characterized by a quick growth immediately after a recruitment event followed by a slow growth towards potential proportions, as illustrated on Figure 1 (panel A). Three features of the model produce this pattern. First, a quick growth immediately after a recruitment event is caused by spontaneous rates. In the model, spontaneous rates enter the growth equation directly, while imitation rates are scaled by the relative proportions of imitated types. Therefore, imitation rates cannot account for initial rapid growth. Figure 1 (panel B) illustrates this point. It simulates the dynamics of the relative proportions of two types when the direct channel is shut down (spontaneous rates are zero). The only source of growth in the simulation is the indirect channel, which works because of non-zero starting values. The relative proportions show steady but slow growth, reaching potential proportions only by the end of the simulation period.

A sharp decrease in the growth rate of relative proportions is produced by the presence of decay rates that dampen the effect of recruitment and reduce spontaneous rates over time. In the absence of decay, the relative proportions would very quickly converge to the potential proportions. Figure 1 (panel C) shows simulation results for the case of zero decay rates and when only the direct channel is present. Relative proportions show a steep growth immediately after the beginning of the simulation period and quickly reach potential proportions.

Slow growth of relative proportions in the subsequent periods is produced by imitation rates. In the absence of imitations rates, the growth rate would plummet to almost zero, and relative proportions would never reach potential proportions. Figure 1 (panel D) shows the simulation results featuring the direct channel with decaying rates and completely no indirect channel. After initial rapid growth, relative proportions flatten out and converge to steady states that are well below potential proportions.

The model can produce several distinct dynamic patterns of biases. On Figure 2 we focus on the case with two types, in which there is a high initial bias and no asymptotic bias. Panel A shows the simulation that features both direct and indirect channels, as well as decay, and all the rates are identical across types. The only difference between types is in the starting values of potential proportions. The initially high bias produced by different starting values quickly converges, following a recruitment event, to the bias implied by the ratio of potential proportions.

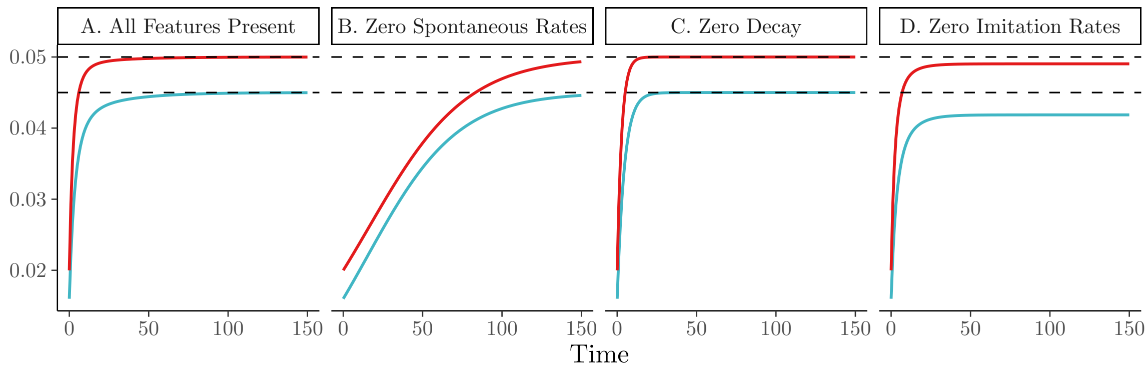

Figure 1: Simulated Relative Proportions Under Different Parameter Values

Differences in spontaneous rates between types can alter the picture in two distinct ways.[10] In particular, if the initial spontaneous rate of a type that starts low is higher than the initial spontaneous rate of a type that starts high, a dip in the bias is observed, as Panel B illustrates. A different pattern occurs if the type that starts high also has a higher initial spontaneous rate, as illustrated on Panel C. After an initially high level, the bias increases even further following a recruitment event, after which the bias drops and converges to the level implied by the ratio of potential proportions.

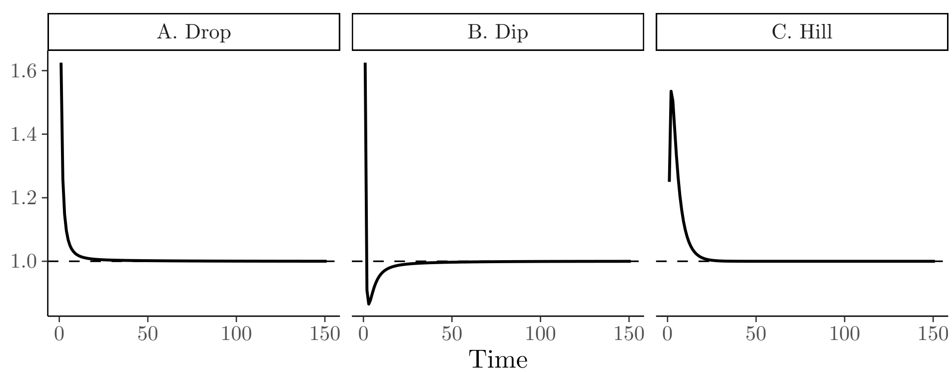

Figure 2: Simulated Biases Under Different Parameter Values

4 Model For Time-Variant Types

In the previous section we assumed that the population is static in that it consists of only time-invariant types. The natural result of this assumption is that all the agents who can potentially

[10] We focus on the effect of spontaneous rates on the dynamics of the bias since these rates have the most impact. Adding differences in imitation rates can produce even more complex patterns.

be in a subject pool eventually end up there. In order to model low overall participation and asymptotic biases between types, we had to set potential proportions exogenously and make them different across types. In this section we are relaxing the assumption about a static population by introducing *cohorts* of agents as time-variant types and look at how this changes our results. One important consequence of this change is that non-full participation and biases between types will arise endogenously, even when potential proportions are the same across types.

We assume that a refernce population consist of cohorts indexed by elements of a set $\mathcal{N} = \{0, \ldots, N-1\}$, $N \geqslant 2$. For simplicity, we assume that the shares of each cohort in a population are identical: $\mathbf{m} \equiv (m^0, \ldots, m^{N-1})' = (1/N, \ldots, 1/N)'$. Agents who belong to cohort i in one period move to cohort $i+1$ in the next period. Agents of cohort $N-1$ in the next period automatically drop out of a subject pool (if they were there). At the same time, a new cohort, $n = 0$, arrives every period. Each cohort represents a different type of agents. For example, in the case of college students, as students advance through their programs, their type (e.g., risk or time preferences) changes due to new experiences and acquired knowledge.[11] To make the point about endogenous biases and non-full participation stronger, we assume that a fixed proportion $\alpha \in [0, 1]$ of *each* cohort can potentially be recruited for a subject pool.[12]

As before, we assume that there are two channels through which agents may be induced to participate in a subject pool. The direct channel is given by a vector of spontaneous rates $\mathbf{p} \equiv (p^0, \ldots, p^{N-1})'$, $p^i \in [0, 1], i \in \mathcal{N}$. Importantly, the spontaneous rates are defined over the *same period* as cohorts, and thus we do not consider decay in these rates within a period. In other words, the rates in this version of the model are defined over academic years (or semesters) rather than days.

The indirect channel is given by a matrix of imitation rates $\mathbf{Q} \equiv \{q^{ij}\}$, $i, j \in \mathcal{N}$. It is natural to assume that the probability to imitate own type is at least as big as that of other types, $q^{ii} \geqslant q^{ij}, \forall i, j \in \mathcal{N}$. For instance, imitation rates can decline with the distance between cohorts, $q^{ij} = q^{ii}/(1 + d(i, j))$, where $d : \mathcal{N} \times \mathcal{N} \mapsto \mathbb{R}_+$ is some distance function, such as $|i - j|$. This specification

[11] For example, Dohmen *et al.* (2017) show, using representative panel data from Germany and Netherlands, that people become more risk averse as they age. In particular, the willingness to take risks declines almost linearly with age.

[12] As we will show shortly, the equality of the potential proportions does not lead to the asymptotic equality of relative proportions in the case of time-variant types. Differences in the potential proportions, therefore, will only make this result stronger.

captures the idea that, for example, sophomores are more likely to interact with freshmen and juniors than with seniors, and are even more likely to interact among themselves.

Let $\tilde{\mathbf{x}}_t \equiv (\tilde{x}_t^0, \dots, \tilde{x}_t^{N-1})'$ be a vector of proportions of the total number of agents in each cohort, who are in a subject pool at time t, to the total number of agents, S. Given some initial number of people in a subject pool, $S\tilde{\mathbf{x}}_0$, the dynamics of these proportions are described by the following system of equations:[13]

$$\tilde{x}_{t+1}^{i+1} = \tilde{x}_t^i + p^i \left(\alpha m^i - \tilde{x}_t^i\right) + \sum_{j\in\mathcal{N}} q^{ij} \frac{\tilde{x}_t^j}{m^j} \left(\alpha m^i - \tilde{x}_t^i\right), \; i \in \mathcal{N}/\{N-1\}. \tag{4}$$

These equations state that the number of agents of cohort $i+1$ who are in the subject pool at time $t+1$ consists of agents who belonged to a previous cohort one period ago plus those who arrived at the subject pool via direct and indirect channels. Equations (4) can be viewed as a natural extension of the system (1), with the difference being that types change over time. We assume that the newly arrived cohort does not provide any agents for a subject pool, since recruitment efforts and agents' interactions happen after cohorts are formed, so that $\tilde{x}_{t+1}^0 = \tilde{x}_t^0 = 0$, $\forall t \geqslant 0$.

The proportion of all agents who are in a subject pool at time t relative to the total number of agents is

$$\tilde{x}_t = \sum_{i\in\mathcal{N}} \tilde{x}_t^i.$$

Using this equation and equation (4), we find that the dynamics of the proportion of all agents relative to a total number of agents in a population are given by

$$\tilde{x}_{t+1} = \tilde{x}_t - \tilde{x}_t^{N-1} + \sum_{i=0}^{N-2} p^i \left(\frac{\alpha}{N} - \tilde{x}_t^i\right) + \sum_{i=0}^{N-2}\sum_{j=0}^{N-1} q^{ij} \frac{\tilde{x}_t^j}{m^j} \left(\frac{\alpha}{N} - \tilde{x}_t^i\right). \tag{5}$$

Note that we have to subtract the agents who were in cohort $N-1$ at time t since they leave the subject pool in the next period.

Denote $\mathbf{x}_t \equiv (\tilde{x}_t^0/(\alpha m^0), \dots, \tilde{x}_t^{N-1}/(\alpha m^{N-1}))$ to be a vector of the proportions of agents of each cohort who are in a subject pool at time t relative to their potential shares in a reference population. We will call $\mathbf{x}_t$ a vector of relative proportions. Note that it is defined differently than in the previous section. We scale $\tilde{\mathbf{x}}_t$ not only by the shares $\mathbf{m}$, but also by the potential proportions

[13] In this version of the model we are focusing on the discrete time, so $\Delta t = 1$.

α. In the context of time-invariant types, this definition would imply the absence of selection bias in the long-run. In the context of time-variant types, however, this will not be the case. Similarly, define $x_t \equiv \tilde{x}_t/\alpha$ to be the proportion of all agents in a subject pool relative to their potential share. We will call this quantity a total relative proportion.

We now show that the selection problem occurs in the model with time-variant types even if the types are *a priori* identical. We consider a symmetric case with no heterogeneity in spontaneous or imitation rates across cohorts, i.e., $p^i = p, q^{ij} = q, \forall i, j \in \mathcal{N}$, and (without loss of generality) the potential share $\alpha = 1$. We bound the rate parameters by the condition $p + qx_t < 1, \forall t$ to ensure that cohorts do not immediately reach their potential proportions. Under these assumptions, the dynamics of the relative proportions can be rewritten as follows.

$$x_{t+1}^{i+1} = x_t^i + p(1 - x_t^i) + qx_t(1 - x_t^i),\ i \in \mathcal{N}/\{N-1\}. \tag{6}$$

Similarly, the dynamics of the total relative proportion are given by

$$x_{t+1} = x_t - \frac{x_t^{N-1}}{N} + p\left(\frac{N-1}{N} - \left(x_t - \frac{x_t^{N-1}}{N}\right)\right) + qx_t\left(\frac{N-1}{N} - \left(x_t - \frac{x_t^{N-1}}{N}\right)\right).$$

First, consider the asymptotic behavior of the subject pool. We are interested in the steady state of the system when $x_{t+1}^i = x_t^i = x^i$ and $x_{t+1} = x_t = x$. Plugging these values into (6) yields

$$x^{i+1} = x^i \big(1 - (p + qx)\big) + p + qx.$$

This is a first-order linear difference equation, whose solution is $x^i = 1 - \big(1 - (p + qx)\big)^i$. Using this solution, we can find the stead-state equation for the total relative proportion:

$$x - 1 + \frac{1 - \big(1 - (p + qx)\big)^N}{N(p + qx)} = 0. \tag{7}$$

While a closed-form solution for x cannot be obtained, note that it will depend on the rate parameters of the model, as well as on the number of cohorts. Therefore, not everyone ends up in the subject pool among those who can potentially be there, even if we wait long enough. Moreover, this result has implications for the nature of the selection problem that would arise in a subject pool.

Proposition 6. *A subject pool has a selection problem asymptotically in the presence of cohorts. The agents from older cohorts are oversampled in a subject pool, while agents from younger cohorts are undersampled. The pairwise bias between two subsequent cohorts decreases in the cohort number. The further cohorts are apart from each other the larger is the bias between them.*

The intuition for this result is straightforward: agents in older cohorts have more chances to be recruited over time. In the example with college students, if recruiting occurs every year, juniors experience two recruitment periods (at the beginning of a school year), while sophomores experienced only one. The problem will also exist for any time t. Thus we have shown that even in the simplest symmetric case and potential proportion equal to 1, non-full participation and selection problem arises endogenously.

We now consider possible asymmetric cases and compare them with the baseline results. We consider four cases:

1. *Baseline.* Spontaneous, own- and cross-imitation rates are the same ($p^i = p, q^{ij} = q, \forall i, j \in \mathcal{N}$). This case is one from Proposition 6.

2. *Constant.* Spontaneous and own-imitation rates are constant for every cohort ($p^i = p, q^{ii} = q, \forall i \in \mathcal{N}$), but the cross-imitation rates decline with the distance between the cohorts according to
$$q^{ij} = \frac{q^{ii}}{1 + |i - j|} \tag{8}$$
This represents the case when the cohorts are identical in terms of their interaction with their own members and their sensitivity to recruitment, but their interaction with other cohorts declines with the distance between the cohorts.

3. *Increasing.* Spontaneous and own-imitation rates increase exponentially with the cohort number ($p^i = p^{1/(i+1)}, q^{ii} = q^{1/(i+1)}, \forall i \in \mathcal{N}$), and the cross-imitation rates decline with the distance between the cohorts according to (8). This pattern of rates would occur when not only the interaction with other cohorts is different from the interaction within a cohort, but also the interaction within a cohort becomes more pronounced, and the propensity to be recruited

increases. In the example with college students, students might get to know each other better and build stronger bonds among themselves over time. Also, students can become less risk-averse and more willing to engage in new activities, as they overcome the initial stress of first college years and feel more comfortable in school.

4. *Decreasing*. Spontaneous and own-imitation rates decrease exponentially with the cohort number ($p^i = p^{(i+1)}, q^{ii} = q^{(i+1)}, \forall i \in \mathcal{N}$), and the cross-imitation rates decline with the distance between the cohorts according to (8). In the example with college students, this case reflects the possibility that as students progress through their programs, they become more focused on their major, which leads to less interaction with their peers. Students also become less willing to engage in new activities such as participation in experiments, because a larger chunk of their time is now dedicated to completing thesis papers and finding jobs.

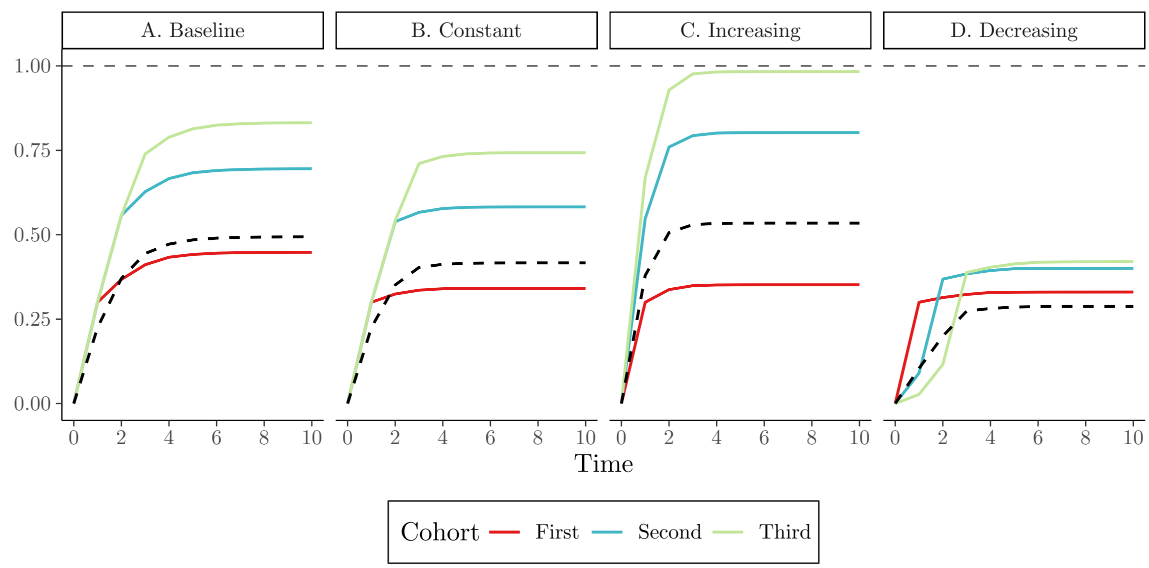

Figure 3: Simulated Relative Proportions for Cohorts

Figure 3 shows the simulated time-series of relative proportions x_t^i for each cohort, as well as the total relative proportion x_t (thick dashed line), for the four cases described above. The figures do not show the results for cohort $i = 0$ since it never participates in the subject pool by assumption. Comparing the Constant (panel B) and the Baseline (panel A) cases, we see that the reduced interaction between the cohorts leads to lower long-run participation in the subject pool. We can also see that the long-run participation increases with the cohort number since older cohorts have more chances to end up in a subject pool, in line with the theoretical prediction.

Panel C shows that the increasing interaction within cohorts can more than offset the reduction caused by lower interaction between cohorts. The oldest cohort almost reaches its potential maximum of participation in the long-run. In the Decreasing case (panel D), the participation is even lower than in the Constant case, and thus decreasing rates is another important factor that could drive low overall participation in a subject pool. Interestingly, the participation is no longer greater for older cohorts at each time t: initially, younger cohorts are more represented in a subject pool than older cohorts, but over time the older cohorts catch up and start to dominate the subject pool.

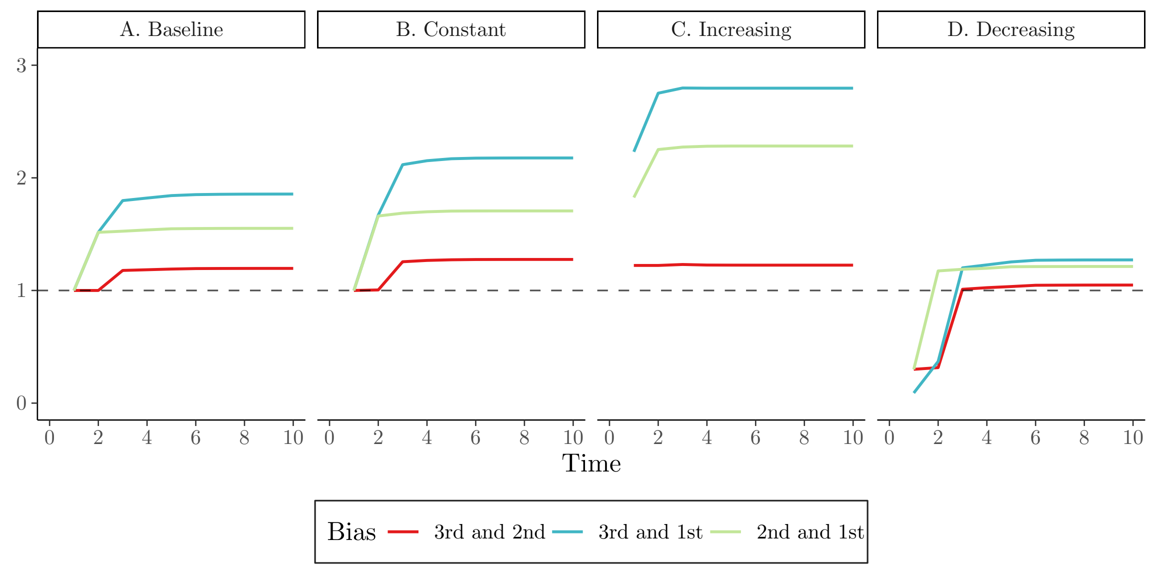

Figure 4: Simulated Biases Between Cohorts

This point is worth developing further by looking at the graphs of pairwise biases over time on Figure 4. In all the cases, except for the Decreasing one, we observe the growth of the bias over time. Older cohorts are oversampled in a subject pool relative to younger cohorts in the long-run. The bias increases in the distance between cohorts, e.g., the bias between cohorts 3 and 1 is higher than the bias between cohorts 3 and 2. As we depart from the Baseline, the selection problem becomes worse. It is particularly strong in the Increasing case. The Decreasing case is a notable exception since the pattern of the spontaneous and imitation rates is offsetting the oversampling of older cohorts. Even though the asymptotic bias is still present in the Decreasing case, its magnitude is much lower than in other cases.

5 Empirical Results

5.1 Data Description

The data we use come from an online recruitment system (the recruiter, henceforth) used at the ICES at GMU. Our observation period is from the beginning of the Fall semester of 2014 until the end of the Spring semester of 2017.[14] Two types of recruitment events are conducted at the ICES: email invitations sent to the entire student population at GMU and selected class visits (typically large undergraduate classes). Both types of events encourage students to sign up for participation in economic experiments. In order to sign up, a student has to create an account and fill out personal information, such as gender, ethnicity, age, starting year at college, and major. Moreover, the recruiter records when an account was created, as well as the first and the last dates of participation in a session for each subject.

We use an account creation date as the date when a subject enters the subject pool. Upon entering the pool, a subject's account becomes *active*. Only subjects with active accounts are allowed to participate in experiments. All accounts are deactivated at the beginning of each semester, and a corresponding email notification is sent to the entire subject pool. The email notifies subjects that they will not receive any invitations for experiments until they reactivate their accounts. In order to reactivate an account, a subject only needs to login to the recruiter. Subjects who do not reactivate their accounts become *non-active* and are assumed to have dropped out of the subject pool. We use the last day of a semester in which a non-active subject last participated in a session as the date when they exited the subject pool. By the end of Spring 2017 the ICES subject pool consisted of around 1,000 active accounts with complete demographic information.

We concentrate our analysis on two subject characteristics: gender (time-invariant type) and year in college, or cohort, (time-variant type). Gender is self-reported by a subject upon registering

[14] The ICES recruiter has been launched in Fall 2014. Unlike the previously used ORSEE system, the ICES recruiter allows us to collect the data necessary for our analysis. We exclude Fall 15/Spring 16 academic year from the analysis, however, due to a temporary shift in the recruitment procedures used. We also include summer months in the Spring semester, since there are some changes in the subject pool during summer, but these changes are not enough to have a separate Summer semester.

in the recruiter. Year in school is imputed from the self-reported starting year at college (the year in which a subject started their program at GMU). In our analysis we use only undergraduate student population since most economic experiments are conducted with undergraduate students. We use year in college instead of a "class year" (freshman, sophomore, junior, senior), because the information about the class year is not regularly updated by the subjects.

While our model allows the types to be observable or unobservable, we opt for using observables. The primary reason for our choice is data availability. The analysis of the dynamics of selection requires high-frequency data on students' types, which are simply not available for the unobservable characteristics.[15] And even if data on unobservables were available, it would have likely been much noisier than the data on observables (Gillen *et al.*, 2019), which in turn would have made uncovering the dynamic trends harder. We do believe, however, that the dynamics of the observable types are still informative of the dynamics of the unobservable types. If a decision to participate in a subject pool reflects a subjects' preferences, as the literature suggests (Abeler and Nosenzo, 2015; Slonim *et al.*, 2013; Krawczyk, 2011), then the trends in the observable types, which we uncover in our data, should reflect some of these unobservable preferences.

We use the data from the Office of Institutional Research and Effectiveness (OIRE) at GMU to construct the characteristics of the reference population, which in our case is the student population. Each semester OIRE publishes an overview of the entire student population, which includes gender and ethnic composition, the total number of students enrolled, the number of undergraduate students enrolled,[16] as well as how many new students were admitted and what are the retention rates for every class starting from 2007. Retention rate is a share of students who are still in college after a given number of years. Students are considered to be in college only if they are still pursuing a bachelor degree. For instance, for the cohort who started their program in 2007, 83.5% of the students remain in college after one year, while only 5% of them remain in college after six years.

[15] It is conceivable, and even advisable, that a recruitment system supplements basic demographic questions with preference elicitation instruments. It will be impractical, of course, to deploy fully incentivized risk elicitation tasks at the recruitment stage. However, survey questions, such as the ones proposed by Falk *et al.* (2018), can be easily incorporated into a recruitment system and will not add too much burden on potential student participants during registration. While arguably being crude, these instruments will shed at least some light on the unobservable characteristics of entire subject pools.

[16] The OIRE demographic composition data combines both undergraduate and graduate students. We use the gender ratio for the total student population to impute the undergraduate gender composition.

Using these data in combination with the initial number of students in every cohort allows us to impute the number of students who spend between 1 and 6+ years in college.[17]

While our model allows for any reference population, in our empirical analysis we opt for using the student population at GMU as a reference. This choice is motivated by convenience and the fact that the university's registrar provides high-quality data on the student population. There is little loss of generality from using the student population at GMU as a reference, as opposed to the general population of the city of Fairfax, the state of Virginia, or the United States. Using these alternative reference populations would change the scales of our time-series but not the dynamic trends, the focus of this study. At the same time, the distributions of types in these alternative reference populations are arguably measured with more noise than in the GMU registrar's data.

	Admitted Freshmen	Total Undergraduate	% Female
2014-2015	3,113	21,672	54%
Fall 2015	3,226	22,304	53%
Fall 2016	3,254	23,174	53%

Table 1: Description of the Population by Semester

Table 1 provides a brief description of the student population at GMU by semester. Overall we observe around 22,000 students every academic year of whom there are slightly more females than males. About 3,000 new undergraduate students are admitted every Fall.

5.2 Results

5.2.1 Gender Types

We begin the analysis by looking at gender as a time-invariant type and present the dynamics and summary statistics of the relative proportions of male and female participants in the subject pool in Figure 5 and in Table 2 (see table notes for variables definitions). The shaded vertical bars in Figure 5 indicate the start and end of recruitment. The long-run participation in the subject pool is very limited among both types, rarely exceeding 5%. Apart from the Fall 2014 semester, there is a constant gap between the relative proportions of males and females, with males participating at higher rates than females. The average potential proportion of males is slightly higher than the

[17] We lump together the students who spend 6 and more years in college to increase the sample size in this small group of students.

average potential proportion of females indicating the existence of a small but persistent long-run bias.

Table 2: Summary Statistics

	Spontaneous sign-up		Imitation sign-up		Drop-out		Potential proportion
	Absolute	Relative	Absolute	Relative	Absolute	Relative	
			Gender types				
Female	0.014	1.018	0.006	0.190	0.009	0.213	0.043
Male	0.015	0.839	0.006	0.201	0.009	0.191	0.046
			Cohort types				
First-years	0.002	0.851	0.009	0.897	0.011	0.328	0.022
Second-years	0.037	1.800	0.010	0.150	0.034	0.359	0.075
Third-years	0.020	0.800	0.006	0.108	0.026	0.355	0.066
Fourth-years	0.013	0.419	0.004	0.083	0.024	0.356	0.067
Fifth-years	0.018	0.517	0.004	0.053	0.028	0.382	0.077
Sixth-years	0.009	0.378	0.001	0.023	0.014	0.392	0.065

Notes: Spontaneous sign-up is defined as an average (across all semesters) 1-week increase in a relative proportion of a type since the start of a recruitment event. Imitation sign-up is defined as an average (across all semesters) change in a relative proportion of a type between the end of semester and a 7-day point after the start of recruitment. Drop-out is defined as an average (across all semesters) decrease in a relative proportion of a type at the end of semester. Absolute values are defined as a difference between the ending and starting values of a given measure. Relative values are defined as a difference between the ending and starting values of a given measure divided by the starting value. Potential proportion is defined as an average (across all semesters) relative proportion at the end of semester.

There are distinctive dynamic patterns of the relative proportions in Figure 5 that are consistent with the patterns predicted by the model. Immediately after recruitment, relative proportions of both types exhibit a rapid growth, which, however, decays within about a week. The relative proportions continue to grow in the months after recruitment and slowly converge to an upper limit. Table 2 shows that the spontaneous sign-up rates for males and females are almost identical.[18] If we look at the relative sign-up, females increase their participation in the subject pool by more than 100% immediately after recruitment, while males increase their participation only by 84%. Females and males are equally active in the imitation sign-up and increase their participation in the subsequent months by around 19% and 20%, respectively. Table 2 also shows that females are relatively more likely to drop out of the subject pool.

Figure 6 shows the dynamics of the pairwise bias between males and females, $b_t^{\text{male,female}}$. As noted earlier, there is virtually no bias throughout the Fall 2014 semester. After that, a small but persistent bias emerges. The immediate response of the bias to recruitment tends to fall within one of the three categories predicted by the model. The response is either monotonically declining

[18] The spontaneous and imitation sign-up measures used in Table 2 do not directly map into the structural parameters of the model. These parameters are estimated in Section 6.

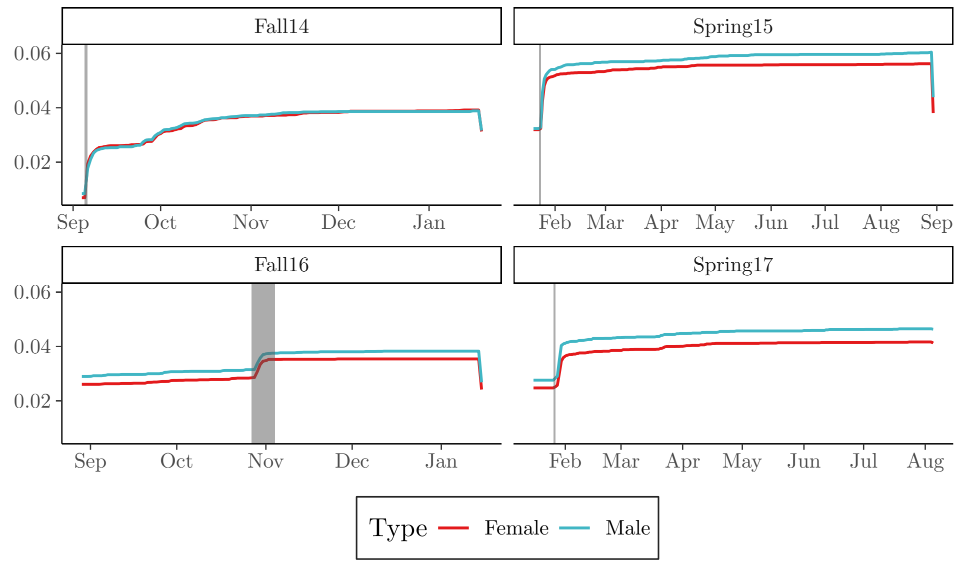

Figure 5: Relative Proportions of Males and Females

(as in Fall 2016), hill-shaped (as in Spring 2017), or dip-shaped (as in Fall 2014). The mid-term dynamics of the bias are rather complex and exhibit both high- and low-frequency oscillations. Over time the bias tends to converge to a fixed value. By the end of the observation period the bias is just above 1.1.

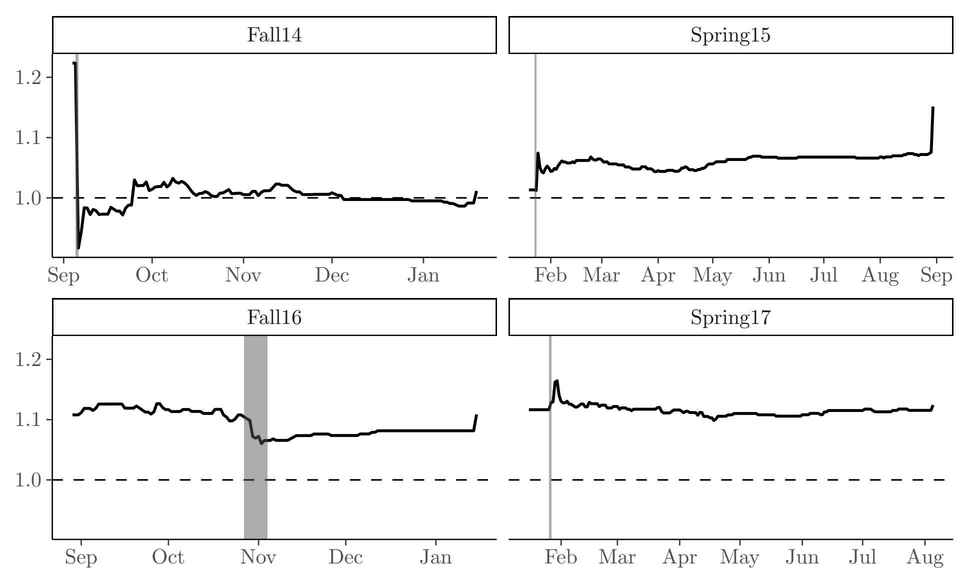

Figure 6: Bias Between Males and Females

5.2.2 Cohort Types

On Figure 7, we present the results for the time-variant types. The potential proportions among the cohorts range between 2.2% for the first-years and around 7.7% for students with 6+ years in school. The participation gaps among different cohorts are large and persistent, though the ranking of the cohorts by their participation varies across semesters. During the first two semesters younger cohorts tend to participate more in the subject pool than older cohorts. By the Fall 2016 semester, the ranking of the cohorts is completely reversed: each older cohort dominates the cohort a year earlier. This pattern continues, except for the abnormally high participation among second-years, throughout the Spring 2017 semester. Such a pattern is in line with the predictions Proposition 6 of the Decreasing case of the time-variant version of the model.

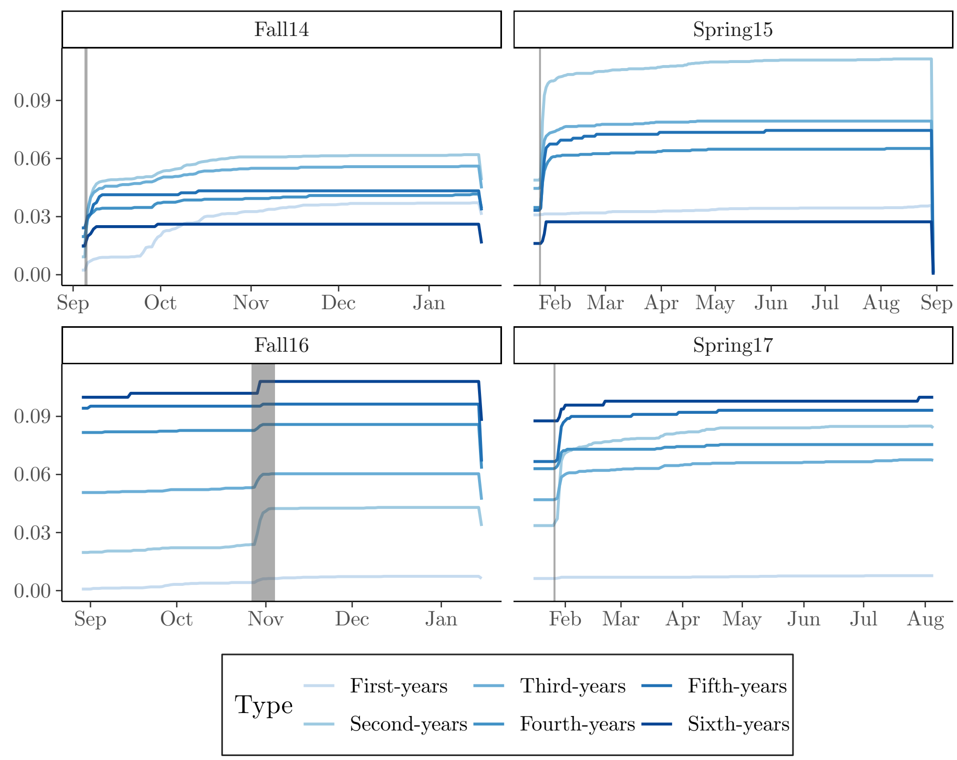

Figure 7: Relative Proportions of Cohort Types

The dynamic response of the relative proportions to recruitment in the case of cohort types is very similar to the case of gender types. There are large differences between cohorts, however, in terms of their immediate and long-run response to recruitment. The second-years beat all other cohorts in terms of the spontaneous sign-up, both absolute and relative, and increase their participation by 180%, on average, immediately after recruitment. The closest competitors are the first-years and the third-years with relative spontaneous sign-up rates of 85.1% and 80%, respectively. The sixth-years have the lowest relative sign-up of 38%, while the first-years have the lowest absolute sign-up of 0.002. On the other hand, the first-years have the strongest relative imitation sign-up rate of 89.7%, while the second highest rate, belonging to the second-year students, is only 15.2%. The relative drop-out rates are very similar across the cohorts and range between 32.8% for the first-years and 39.2% for the sixth-years.

Figure 8 shows the dynamics of the pairwise biases between subsequent cohorts.[19] Consistent with the Decreasing case, we observe the prevalence of younger cohorts at the beginning of the period of observations, which is reversed as time goes on. Note that by the end of the 2016-2017 academic year the biases between cohorts are limited, as compared to the previous periods. This is consistent with the predictions of the Decreasing case.

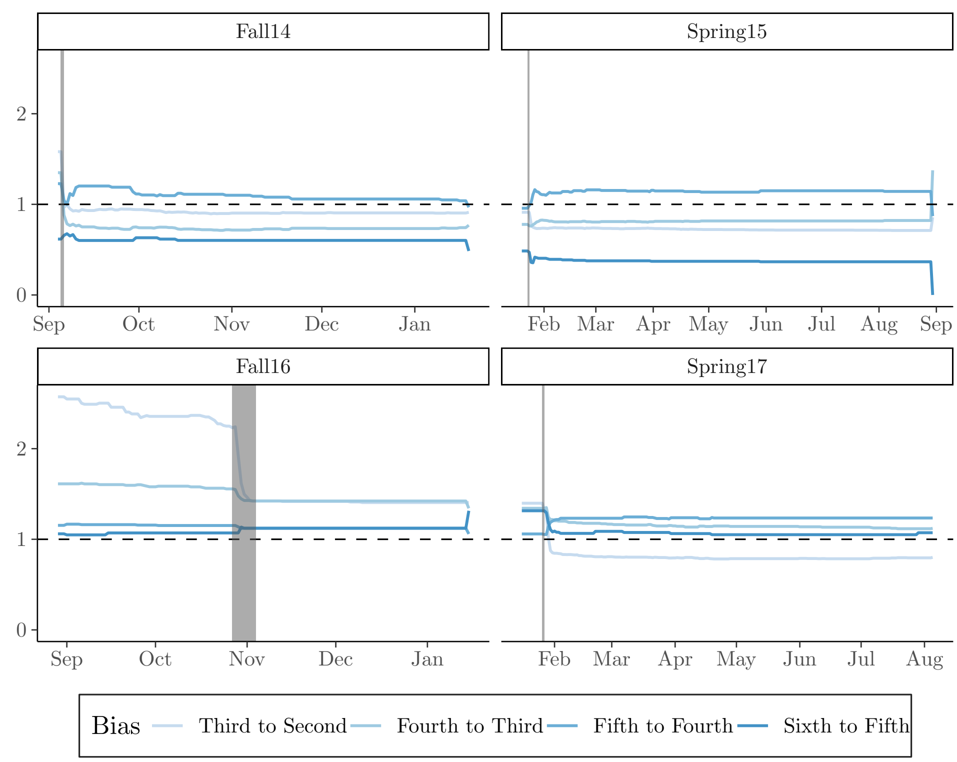

Figure 8: Biases Between Cohort Types

[19] We look only at the biases between an older cohort and a cohort a year younger to make the picture readable. We note, however, that the ordering of cohorts by their relative proportions and resulting pairwise biases during the last periods are consistent with the predictions of the last part of Proposition 6. The bias between the second-years and first-years is omitted because of its huge size that trumps the sizes of all other biases.

Figure 8 shows the possibility of decreasing, increasing, hill-shaped, and dip-shaped patterns that are broadly consistent with the patterns generated by the model. Medium-run dynamics of the biases often have fluctuations, but by the end of the semester, the biases tend to converge to fixed values. The sizes of the long-run biases range between 0.6 and 2.5 with little stability between semesters.

5.2.3 Summary

We summarize the patterns observed in the data in the following results.

Result 1 (Response of relative proportions to recruitment)**.** *The dynamic response of relative proportions to recruitment is consistent with the model's predictions and is characterized by three features. First, there is rapid growth in relative proportions immediately after recruitment. Second, this growth decays within about a week after recruitment. Third, relative proportions continue to grow in the subsequent months until converging to upper limits.*

Result 2 (Selection bias)**.** *The subject pool exhibits short-run and long-run selection biases between types. There is a small long-run selection bias between males and females, while the long-run biases between cohort types are much larger. For cohorts, by the end of the observation period each older cohort dominates a cohort a year earlier, which is consistent with the Decreasing case of the time-variant version of the model.*

Result 3 (Response of selection bias to recruitment)**.** *The immediate dynamic response of biases to recruitment is consistent with the model and tends to be either decreasing, increasing, dip-shaped, or hill-shaped. The most common type of response is a decreasing response to recruitment. Over time, biases converge to fixed values.*

6 Structural Estimation

6.1 Estimation Procedure

Consider a discrete-time version of equation (2) with Δt equal to one day (the data are daily time-series in our case). For each type $i \in \mathcal{N}$, the growth in relative proportion follows the following difference equation.

$$x_{t+1}^i - x_t^i = (\alpha^i - x_t^i)(p^i e^{-\delta^i t} + q^{i1} x_t^1 + \ldots + q^{iN} x_t^N). \tag{9}$$

Our goal is to estimate the structural parameters of the model, i.e., the immediate spontaneous rate p^i, the decay rate δ^i, the potential proportion α^i, as well as the imitation rates $q^{i1}, \ldots, q^{iN}$.

We first note that due to an almost perfect linear correlation between relative proportions among gender types and cohort types, the imitation rates cannot be separately identified. We can estimate, however, the aggregate imitation rate q^i for each type. If $x_t^j = k^{ij} x_t^i$, then

$$q^{i1} x_t^1 + \ldots + q^{iN} x_t^N = \underbrace{(k^{i1} q^{i1} + \ldots + k^{iN} q^{iN})}_{q^i} x_t^i.$$

We then assume a non-linear trend model of the form

$$x_t^i = f(t, x_0^i \mid p^i, q^i, \delta^i, \alpha^i) + \epsilon_t^i,$$

where the non-linear trend $f(\cdot)$ is a function of the starting value x_0^i and time t conditional on the parameters of the model $\theta^i \equiv (p^i, q^i, \delta^i, \alpha^i)$, and ϵ_t^i is mean-zero noise term. The non-linear trend function $f(\cdot)$ is defined implicitly as a solution to the difference equation (9). The identification of the parameters governing the two channels is ensured by our assumption on the functional form of decay and the fact that the recruitment at GMU occurs over a short period of time at the beginning of each semester.[20] The potential proportion parameter is identified by the data on the relative proportions at the end of a semester. We estimate the model by minimizing the sum of squared deviations of the observed time-series x_t^i from the predicted values $\hat{x}_t^i(\theta^i) = f(t, x_0^i \mid \theta^i)$:

$$\hat{\theta}^i = \arg\min_{\theta^i} \sum_{t=0}^{T} \left(x_t^i - \hat{x}_t^i(\theta^i) \right)^2.$$

[20] If recruitment procedures occurred throughout a semester that would complicate the separate identification of the two channels.

6.2 Estimation Results

6.2.1 Gender Types

Table 3 shows the estimation results for gender types. The parameter estimates vary across semesters for both males and females, potentially due to differences in recruitment procedures and students' response to them. For females, the estimates of p range from 0.17 to 0.462, with $p = 0.279$ for the average semester. The estimate for the average semester implies that in a case when there is no indirect channel, and the starting relative proportion is zero, the relative proportion would grow to the 28% of the potential proportion in the first day after recruitment. Males, on average, tend to be more responsive to recruitment than females. However, the results are mixed if one looks at the estimates for individual semesters. For males, the estimates of p range from 0.089 to 0.456, with $p = 0.303$ for the average semester. The estimate for the average semester implies that in a case when there is no indirect channel and the starting relative proportion is zero, the relative proportion would grow to the 30% of the potential proportion in the first day after recruitment.

Table 3: Estimation Results for Gender Types

Parameter	Fall 14	Spring 15	Fall 16	Spring 17	Average
Panel A. *Female*					
p	0.389 (0.009)	0.462 (0.005)	0.170 (0.040)	0.375 (0.034)	0.279 (0.009)
q	1.061 (0.036)	0.358 (0.009)	0.004 (0.001)	0.487 (0.026)	0.629 (0.023)
δ	1.449 (0.149)	0.429 (0.010)	0.727 (0.031)	0.876 (0.332)	0.520 (0.035)
α	0.039 (0.0001)	0.056 (0.00003)	0.046 (0.007)	0.042 (0.0001)	0.046 (0.0001)
Panel B. *Male*					
p	0.316 (0.022)	0.456 (0.037)	0.089 (0.062)	0.407 (0.060)	0.303 (0.005)
q	1.240 (0.031)	0.202 (0.043)	0.010 (0.184)	0.327 (0.111)	0.629 (0.014)
δ	1.239 (0.312)	0.443 (0.019)	0.520 (0.085)	0.518 (0.045)	0.522 (0.014)
α	0.039 (0.0001)	0.061 (0.003)	0.063 (0.020)	0.047 (0.003)	0.048 (0.00005)
Observations	136	218	78	188	135

Notes: The table reports the estimates of the structural parameters of the model for each semester, as well as for the average semester, broken down by gender. The standard errors are based on 500 bootstrap replication.

The estimates of the imitation rate q show that the indirect channel is important for recruitment. For females, the estimates of q range from 0.004 to 1.061, with $q = 0.629$ for the average semester. The strength of the indirect channel, on average, is virtually identical for males and females despite some variation across individual semesters. The estimates of q for males range from 0.01 to 1.24, with $q = 0.629$ for the average semester. Despite the high chances of being recruited through interaction with peers, the strength of the indirect channel is limited due to low overall participation.[21]

Another characteristic of recruitment effectiveness is the decay rate δ. For females, the estimates of δ range from 0.429 to 1.449, with $\delta = 0.52$ for the average semester. This estimate implies that the spontaneous rate drops by half, on average, in 1.332 days. After a week, the spontaneous rate drops by 97%. For males, the decay rate, on average, is not significantly different from the decay rate for females. This result becomes mixed, however, if one looks at the individual semesters. The estimates of δ for males range from 0.443 to 1.239, with $\delta = 0.522$ for the average semester. This estimate implies that the spontaneous rate drops by half, on average, in 1.328 days. After a week, the spontaneous rate drops by 97%.

The low overall participation in the subject pool is evidenced by low estimates of the potential proportions α for both types. For females, the estimates of α range from 0.039 to 0.056, with $\alpha = 0.046$ for the average semester, meaning that, on average, only 4.6% of the female student population is in the subject pool in our sample. For males, the estimates of α range from 0.039 to 0.063, with $\alpha = 0.048$ for the average semester, meaning that, on average, 4.8% of the male student population is in the subject pool, which is slightly higher than the corresponding value for females. This difference implies a long-run pairwise bias between males and females of 1.054.

Overall, the proposed model does a remarkably good job at quantitatively matching the patterns observed in the data. Figure 9 illustrates this point by plotting the actual relative proportions for the average semester against the values predicted by the model (left and middle panels), as well as the actual pairwise bias between males and females for the average semester against the predicted bias (right panel). The dashed horizontal lines on the graphs for relative proportions correspond to the estimated values of α. The time is measured in weeks starting from the date of recruitment.

The predicted values track the actual data very closely. In particular, the estimated model is capable of producing the three main features of the dynamics of relative proportions: a rapid growth

[21] In other words, conditional on interacting with a peer who is already in a subject pool, the chances of being recruited are high, but since few peers are in a subject pool, the overall effect is small.

immediately after recruitment, a quick decay in this growth within a week after recruitment, and a slow growth in subsequent months with a convergence to an upper limit by the end of a semester. The predicted bias makes the trend in the data sharper and can be described by a hill-shaped pattern. Immediately after recruitment, the bias spikes due to a higher spontaneous rate for males than for females. The bias then slowly converges to its long-run value. The graph of the bias also highlights the importance of the dynamic effects in evaluating the selection problem. Evaluating the selection problem at the beginning of the recruitment cycle would yield different results as compared to the end of the cycle.

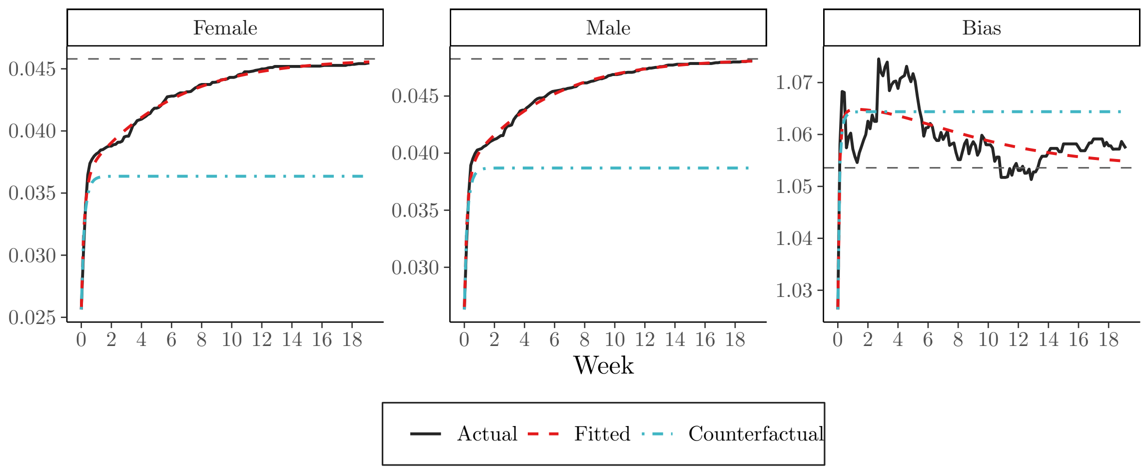

Figure 9: Time-series of Relative Proportions and Male-Female Bias for the Average Semester

To further illustrate the importance of the indirect channel, we conduct a counterfactual simulation in which this channel is completely shut down. Figure 9 presents the results for the simulated relative proportions of females and males (left and center panel) and the implied male-female bias (right panel). As is evident from the picture, shutting down the indirect channel would have a dramatic effect on the relative proportions and bias. The relative proportions of both types would never reach their potential proportions by the end of the semester. The relative proportion would be virtually stuck at 0.036 for females and 0.039 for males. These numbers represent 53% and 64% of the total gain in participation (the difference between a potential proportion and a starting value) for females and males, respectively. The indirect channel thus accounts for roughly one-half to one-third of the total participation gain. The bias would be stuck at 1.064, which is 1% larger than the ratio of the potential proportions.[22]

6.2.2 Cohort Types

Table 4 presents the estimation results for cohort types. We restrict our attention to the average semester and the first four cohorts. Individual semesters, for the most part, do not provide enough variation in the data to allow for a meaningful estimation, which is also the case for the cohorts of fifth- and sixth-years for the average semester.

Table 4: Estimation Results for Cohort Types

Parameter	First-years	Second-years	Third-years	Fourth-years
p	−0.009 (0.015)	0.338 (0.012)	0.342 (0.007)	0.317 (0.021)
q	1.865 (0.582)	0.284 (0.024)	0.427 (0.017)	0.166 (0.030)
δ	0.048 (4.895)	0.382 (0.016)	0.449 (0.016)	0.403 (0.019)
α	0.026 (0.0001)	0.086 (0.002)	0.068 (0.0002)	0.063 (0.001)
Observations	136	135	135	135

Notes: Reports the estimates of the structural parameters for each cohort for the average semester. The standard errors are based on 500 bootstrap replications.

The table reveals dramatic differences among cohorts regarding their responsiveness to direct and indirect recruitment, as well as potential proportions. The effectiveness of the direct channel is lowest for the cohort of first-years. The estimate of the spontaneous rate for this cohort is virtually zero,[23] suggesting that the first-years do not respond, or respond weakly, to direct recruitment events. The estimates of the spontaneous rates for the three older cohorts are significantly different from zero. The third-years have the highest point estimate of $p = 0.342$. This number implies that if there was no indirect channel and a starting relative proportion was zero, the relative proportion would grow to the 34% of the potential proportion in the first day after recruitment. The differences in spontaneous rates between the second-, third-, and fourth-years are small and not statistically significant.

The effectiveness of the indirect channel is highest for the first-years with an estimate of $q = 1.865$. This estimate is in stark contrast to the virtually non-existent direct channel for the first-

[22] The absence of the indirect channel does not necessarily imply a larger bias. The differences in the spontaneous and decay rates will determine the size of the bias.

[23] While the point estimate is, in fact, negative, which is not allowed by the model, we interpret it as being virtually zero based on its magnitude and standard error.

years, suggesting that the indirect channel plays a dominant role in the participation decisions of this cohort. There is, however, a substantial amount of uncertainty around the estimate of q for the first-years. Compared to the first-years, the estimates of the imitation rates among the remaining three cohorts are much smaller, with the fourth-years having the lowest estimate of $q = 0.166$. Overall, the effectiveness of the indirect channel tends to decline with a cohort's age.

There is substantial uncertainty in the estimate of δ for the first-years, which is not surprising given the low estimate of the spontaneous rate. The estimates of the decay rate for the remaining three cohorts range between 0.382 for the second-years and 0.449 for the third-years. These numbers imply that, for instance, the effectiveness of the direct channel for the third-years would drop by half in 1.544 days and that in a week following recruitment the spontaneous rate would drop by 96%, which is similar to what we found using gender types. Overall, the estimates of the decay rates tend to increase with a cohort's age.

The estimates of the potential proportions show low overall participation across the cohorts. The second-years have the highest potential proportion of 0.086, while the first-years have the lowest potential proportion of 0.026. These numbers imply that, on average, only 8.6% of the second-years and 2.6% of the first-years participate in the subject pool. The third- and fourth-years lie in between the other two cohorts in terms of their potential proportions.[24]

Just like in the case of gender types, the estimated model for cohorts does a remarkably good job at matching the data. This is evident from Figure 10 in which each panel shows the time-series of the actual relative proportion for a given cohort along with the model's fit. The horizontal dashed lines, as before, indicate the estimates of α, and the time is measured in weeks starting from the date of recruitment. The three key features of the data—a rapid growth immediately after recruitment, a quick decay in this growth within a week after recruitment, and a slow growth in subsequent months with convergence to an upper limit by the end of a semester—are well-captured by the model.

Figure 11 shows the time-series of pairwise biases between subsequent cohorts: the bias between the second- and first-years, the bias between the third- and second-years, and the bias between the fourth- and third-years. The fitted values of the bias track the actual values quite closely. The bias

[24]Note that since we are dealing with the average semester in cohort estimates, it is not possible to track how the patterns of potential proportions across cohorts change by semester.

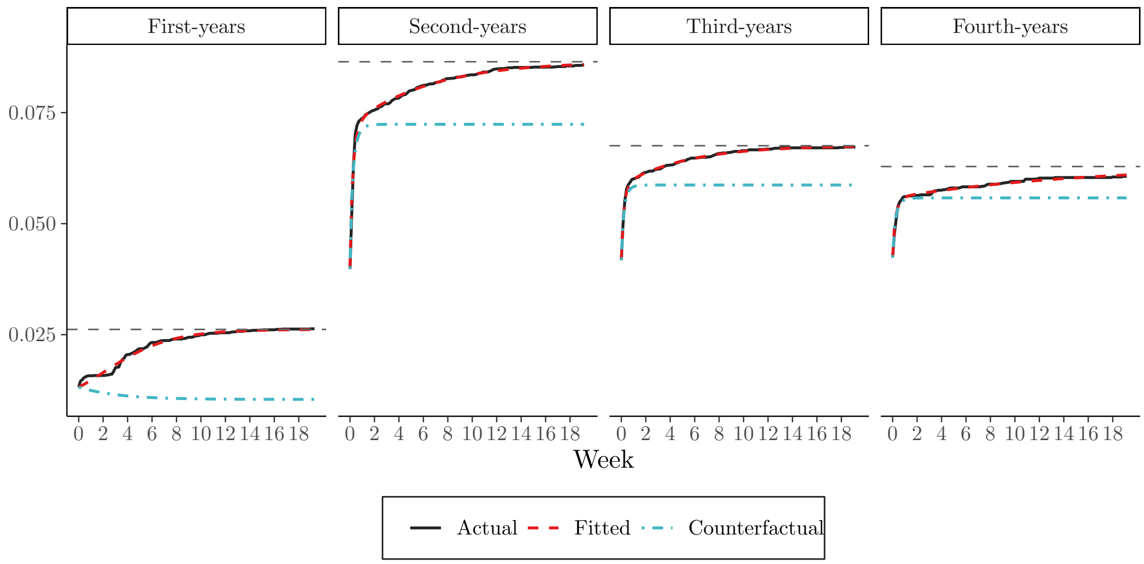

Figure 10: Time-series of the Relative Proportions of Cohorts for the Average Semester

between the second- and first-years has a hump-shaped pattern. This bias grows initially, reaches a peak, and then slowly converges to the long-run bias of 3.302 defined by the ratio of the potential proportions. The bias between the third- and second-years exhibits a decreasing pattern, however, it is worth noting that since the bias value goes well below 1, it implies a stronger bias between the two cohorts. The bias between these two cohorts converges over time to the long-run value of 0.781. The bias between the fourth- and third-years exhibits a similar decreasing pattern but is more complex. After the decline in the first half of the semester, the bias starts to increase slightly in the second half of the semester towards the long-run value of 0.931.

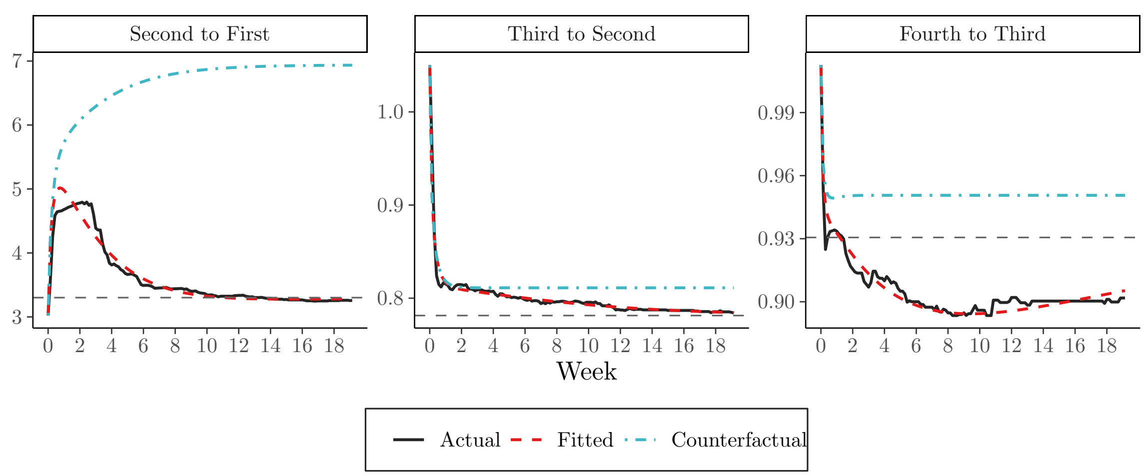

Figure 11: Time-series of Biases between Cohorts for the Average Semester

To highlight the importance of the indirect channel of recruitment, we conduct a counterfactual analysis in which we completely shut down the indirect channel. The resulting time-series of counterfactual relative proportions are shown in Figure 10. In the absence of the indirect channel, the relative proportions would virtually stop growing after two weeks and would never reach the potential proportions. The long-run values of the counterfactual relative proportions would only attain around two-thirds of the total gain in participation (the difference between a potential proportion and a starting value) for the three older cohorts. The relative proportion of the first-years would remain virtually constant in the absence of the indirect channel. We can conclude that the indirect channel accounts for roughly one-third of the total gain in participation.

Figure 11 shows the time-series of counterfactual biases between cohorts in the absence of the indirect channel. The bias between the second- and first-years would be increasing and exceed the ratio of the potential proportions by a large degree. The bias between the third- and second years would be slightly lower (a weaker bias) than the ratio of potential proportions. The same would also hold for the bias between the fourth- and third-years.

7 Conclusion

We study the dynamics of the selection problem in economic experiments. Our analysis shows that while a static selection problem leads to biases in the estimates of types shares and treatment effects, a time-varying selection problem further complicates potential treatment effect biases. The introduction of dynamics also helps to explain some of the existing contradictions in the literature (Slonim *et al.*, 2013; Cleave *et al.*, 2013). In order to understand the dynamic nature of the selection problem, we develop a model of participation in a subject pool. The model assumes that agents' participation evolves over time and is driven by the two channels: the direct channel of recruitment and the indirect channel of agents' interaction. Differences in potential proportions drive the long-run selection biases, while differences in initial participation levels and participation rates drive the short-run biases between agent types. The modification of the model in which types are time-variant results in the possibility of long-run biases even when potential proportions are identical across types.

In our empirical analysis of the recruitment data from ICES at GMU we find that the participation dynamics are consistent with the model's predictions. We find evidence of short- and long-run selection biases between males and females, as well as between cohorts. The counterfactual analysis of the data using an estimated model shows that the indirect channel accounts for roughly one-third of the total participation gain for both gender and cohort types. We use the model to show that the selection bias would be higher or lower, depending on the type, in the absence of the indirect channel.

Our findings imply that networks effects play a crucial role in shaping the dynamics of the selection problem. The presence of the dynamic effects in the selection problem, in turn, leads to several important policy implications for methodology, design of experiments, and recruitment. The methodological implication is that future studies of the selection problem should address its dynamic nature. For example, the measurements of selection biases could be done at various point in time throughout the semester. This would help to avoid the seemingly contradictory results about the presence of the selection problem in a subject pool. The implication for experimental design is that studies using a between-subject design would benefit from randomizing treatments within a session. This practice would minimize the potential treatment effect biases caused by the variation in selection biases over time. Alternatively, if randomization to treatment within a session is infeasible, sessions should be kept temporally close. The implication for recruitment is that it would help to leave a short burn-in period after a main recruitment event. Since selection biases tend to stabilize in the long-run, leaving a burn-in period would minimize the selection biases caused by short-run fluctuations.

We propose that future research on the selection problem should focus more on its dynamics. A major obstacle to studying the dynamics of the selection problem is the absence of high-frequency subject pool data on unobservable subject characteristics, such as preferences and personality. The lack of such data forced our analysis to focus on observable characteristics that are recorded at a time of registration for a subject pool. While the dynamics of observable characteristics may provide some insights into the dynamics of unobservable characteristics, a more direct measurement of unobservables is desirable. Supplementing basic demographic data with preference and personality data elicited at a time of registration would be highly beneficial.

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Appendices

A Experimental Instructions

In this paper we are using the entire recruitment data from an experimental laboratory at the Interdisciplinary Center for Economic Science (ICES) at George Mason University (GMU) rather than data from a particular experiment. Therefore, instead of describing experimental instructions for a particular experiment, we describe the recruitment procedures of the experimental laboratory.

The recruitment system at the ICES experimental laboratory in its current form (the recruiter, henceforth) was launched in Fall 2014. The recruiter is used to manage the subject pool and, in particular, to invite subjects to participate in experimental sessions. Two types of recruitment events are conducted on campus to populate the subject pool. First, at the beginning of each semester the recruiter sends a generic email to the entire student population at GMU. The email encourages students to sign up for participation in economic experiments by registering in the recruiter. A typical email is presented in Figure A.1. The email emphasizes the time commitment that a typical experiment requires and that students can expect monetary compensation for their time. A second type of recruitment events, which however did not occur within our observation period, is class visits. This type of recruitment involves representatives of the ICES who come to large undergraduate classes, deliver a short talk designed to encourage participation in economic experiments, and distribute flyers with information on how to sign up. The content of the talk, for the most part, mirrors the content of the recruitment email. Table A.1 shows the timeline of recruitment events at the ICES during our observation period.

Upon following the link to the recruiter, a potential subject fills out demographic information, such as gender, ethnicity, birth year, and major. He or she also gives consent to receiving invitations from the recruiter to participate in experimental sessions. After a subject's account is created, the recruiter records information on when an account was created, when an account was last updated, how many sessions a subject participated in, and when were the first and last times a subject participated in a session.

The recruiter also keeps track of whether an account is active or not. At the beginning of each semester, the recruiter de-activates all accounts and sends an email to all registered subjects

Figure A.1: An Example of a Recruitment Email

Table A.1: Timeline of Recruitment Events

Event ID	Type	Semester	Start Date	End Date
1	email	Fall14	2014-09-05	2014-09-06
2	email	Spring15	2015-01-23	2015-01-24
3	email	Fall15	2015-09-13	2015-09-14
4	email	Fall16	2016-10-27	2016-11-04
5	email	Spring17	2017-01-26	2017-01-27
6	class	Fall15	2015-09-28	2015-09-30
7	class	Fall15	2015-10-01	2015-10-02
8	class	Fall15	2015-10-06	2015-10-07
9	class	Fall15	2015-12-08	2015-12-13
10	class	Spring16	2016-02-01	2016-02-06
11	class	Spring16	2016-02-08	2016-02-13
12	class	Spring16	2016-04-11	2016-04-16
13	class	Spring16	2016-04-25	2016-04-30

asking them to re-activate their account thus confirming their willingness to further participate in economic experiments. Subjects who successfully re-activate their accounts become active, while subjects who do not re-activate their accounts remain non-active. Those latter subjects effectively drop out of the subject pool.

B Proofs

Proposition 1

Proof. If a subject pool does not have a selection problem, then there is a constant $\bar{\alpha}$, such that $\mathbf{x}_t = (\bar{\alpha}, \bar{\alpha}, \ldots, \bar{\alpha})$. Clearly, any pairwise bias will be equal to 1 in this case. Similarly, if all the pairwise biases are equal to 1, they can be written as $b_t^{ij} = \bar{\alpha}/\bar{\alpha}$, which implies that $\mathbf{x}_t = \bar{\alpha}\boldsymbol{\imath}$. $\square$

Proposition 2

Proof. Consider the ratio of the shares

$$\frac{m^{\mathrm{I}}}{\tilde{m}^{\mathrm{I}}} = m^{\mathrm{I}} + m^{\mathrm{II}} b^{\mathrm{II},\mathrm{I}} = 1 + m^{\mathrm{II}} \left(b^{\mathrm{II},\mathrm{I}} - 1 \right).$$

If the ratio of the shares is one, then $m^{\mathrm{II}} \left(b^{\mathrm{II},\mathrm{I}} - 1 \right) = 0$, which implies that $b^{\mathrm{II},\mathrm{I}} = 1$ (apart from the trivial case when there is just one type in the population). Similarly, if $b^{\mathrm{II},\mathrm{I}} = 1$ then $m^{\mathrm{I}}/\tilde{m}^{\mathrm{I}} = 1$. $\square$

Proposition 3

Proof. Consider the difference between an estimated treatment effect and a treatment effect in a reference population:

$$\Delta Y - \Delta\tilde{Y} = \left(\Delta Y^{\mathrm{I}} - \Delta Y^{\mathrm{II}}\right)\left(m^{\mathrm{I}} - \tilde{m}^{\mathrm{I}}\right).$$

If the two treatment effects are identical, $\Delta Y - \Delta\tilde{Y} = 0$, then either $\Delta Y^{\mathrm{I}} = \Delta Y^{\mathrm{II}}$ (homogeneous treatment effects) must be true or $m^{\mathrm{I}} = \tilde{m}^{\mathrm{I}}$ (no selection problem) must be true, or both. Similarly, if either condition is true (or both), then $\Delta Y = \Delta\tilde{Y}$. □

Proposition 6

Proof. Consider the asymptotic pairwise bias between two subsequent cohorts:

$$\lim_{t\to\infty} \frac{x_t^{i+1}}{x_t^i} = \frac{1-(1-p-qx)^{i+1}}{1-(1-p-qx)^i}.$$

Since $p + qx < 1$, the fraction is greater than one. Differentiating the fraction w.r.t i yields

$$\frac{d}{di}\left(\frac{1-(1-p-qx)^{i+1}}{1-(1-p-qx)^i}\right) = \frac{\ln(1-p-qx)(1-p-qx)^i(p+qx)}{(1-(1-p-qx)^i)^2}.$$

This expression is negative since $p + qx < 1$. Moreover, consider cohorts i and $i + j$. The pairwise bias is increasing in j, since

$$\frac{1-(1-p-qx)^{i+j}}{1-(1-p-qx)^i} > \frac{1-(1-p-qx)^{i+j'}}{1-(1-p-qx)^i} \text{ if } j > j'.$$

□