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Research article

A dynamical systems approach to cryptocurrency stability

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Abstract: Recently, the notion of cryptocurrencies has come to the fore of public interest. These assets that exist only in electronic form, with no underlying value, offer the owners some protection from tracking or seizure by government or creditors. We model these assets from the perspective of asset flow equations developed by Caginalp and Balenovich, and investigate their stability under various parameters, as classical finance methodology is inapplicable. By utilizing the concept of liquidity price and analyzing stability of the resulting system of ordinary differential equations, we obtain conditions under which the system is linearly stable. We find that trend-based motivations and additional liquidity arising from an uptrend are destabilizing forces, while anchoring through value assumed to be fairly recent price history tends to be stabilizing.

Keywords: ordinary differential equations; mathematical finance; asset bubbles; quantitative finance
Mathematics Subject Classification: 65L07

1. Introduction

Blockchain technology enables large numbers of participants to make electronic transactions directly without intermediaries, and has led, in recent years to a new form of payment, and essentially to a new set of currencies called cryptocurrencies. During 2017 the spectacular nine-fold rise in the price of Bitcoin focused the spotlight of public attention on cryptocurrencies that evolved into a new asset class. Following the pattern of other nascent assets, speculators dominated trading and pushed prices toward a bubble.

As with some other asset bubbles of the past, notably the dot-com frenzy of the late 1990s, the emergence of a new technology clouded judgements about the basic value of the asset.

Cryptocurrencies offer both opportunities and risks to society. On the one hand, cryptocurrencies and technology underpinning them – if designed appropriately – could be used to make transactions faster, safer and cheaper, alongside other societal benefits [14, 22, 23]. A less apparent feature is that they can make it more difficult (though not theoretically impossible; see [3, 19]) for totalitarian governments to expropriate savings, either directly or indirectly through currency inflation, thereby depriving savers of a large fraction of their assets. In this way, a proper cryptocurrency could lead to greater economic freedom, and render more difficult the financing of a dictatorship. Indeed, this can be modelled by a choice of two alternatives: either their home currency or cryptocurrency that cannot easily be seized [21, 38].

The risks presented by existing cryptocurrencies are multi-faceted. The difficulty in tracing transactions facilitate illicit activity and its financing. The vulnerability of cryptocurrency to hijacking or even forgetfulness is another concern. One less obvious – and possibly most significant – risk arises from the instability of prices of major cryptocurrencies. As the market capitalization (number of units times the price of each unit) of the cryptocurrencies rises, there is growing risk that a sharp drop in the price of a cryptocurrency could have a cascading effect on other sectors of world economy, particularly if borrowing is involved. During the period October 2017 to April 2018, the price of Bitcoin unit rose from $6,000 to $20,000 and back to $6,000. The market capitalization of all cryptocurrencies during that time period increased from $170 billion to $330 billion, peaking together with Bitcoin in December 2017. While attention is often focused on the rise and fall of the trading prices of these assets, the magnitude of the problem of stability increased significantly during this six month period, as fears about its volatility have been borne out. As people become more accustomed to using these instruments, the market capitalization may increase to several trillion – i.e., a few percent of the $ 75 trillion Gross World Product – and many of the challenges of managing such a behemoth will be critical. For example, one would want to prevent large swings in its price from impacting other sectors of the economy.

Generally, the features of a financial instrument that might make it attractive to speculators are undesirable to those who seek to use it as a currency in daily transactions. Speculators see a greater opportunity in a volatile market, as they can use technical analysis and expertise to profit at the expense of the layperson. Conversely, large fluctuations on a day-to-day basis create obstacles for common purchases or the pricing of service contracts [37]. Without stability in the marketplace, the cryptocurrencies may simply become "a mechanism for a transfer of wealth from the late-comers to the early entrants and nimble traders" [6]. Thus, a set of questions of critical importance deals with the potential stability (or lack thereof) of Bitcoin or other cryptocurrencies, which is the main topic of our paper.

The turbulence arising from the collapse of the housing bubble was a major challenge for markets, but from a scientific perspective, it could be addressed largely with classical methods [20, 32, 34]. However, classical methods are not readily adaptable to studying cryptocurrencies, as discussed below. We use a modern approach whereby an equilibrium price can be determined and the stability properties established within a dynamical system setting [5, 8, 9, 15, 16, 18, 24, 25, 29, 31, 34–36, 41].

2. Modelling prices and stability

Most of classical finance such as the Black-Scholes option pricing model has its origin in the basic equation

$$\frac{1}{P} dP = \mu dt + \sigma dW \tag{2.1}$$

for the change in the relative price $P^{-1}dP$ in terms of the expected return, μ, the standard deviation of the return, σ, and independent increments of Brownian motion, dW. It is widely acknowledged that this equation does not arise from compelling microeconomic considerations, nor empirical data. But rather, it is mathematically convenient and elegant for expressing and proving theorems (see [10] for discussion). Much of risk assessment is based upon this model with an increasing array of adjustments.

The limitations of this basic model are apparent, for example, if one examines the standard deviation of daily relative changes in the S&P 500 index, which is typically around 0.75%. This leads to the conclusion that a 4.5% drop is a sixth standard deviation event, i.e., it occurs once every billion trading days, while empirical data shows it is on the order of a few times per thousand [2].

Thus identifying risk on a large time scale based on the variance of a small time scale can vastly underestimate risk.

Furthermore, the modeling of asset prices is generally based on the underlying assumption of infinite arbitrage. While there may be some investors who are prone to cognitive errors or bias in assessing value, the impact of their trades will be marginalized by more savvy investors who manage a large pool of money. Of course the inherent assumption is that there is some value to an asset, based for example on the projection of the dividend stream, replacement value, etc., and that the shareholder has a vote that allows him ultimately to extract this value. For assets such as US Treasury bills, the model works quite well, as the owner is assured of receiving a particular dollar amount from the US at a specified time.

Herein lies the central problem for the application of classical theory to cryptocurrencies: there is no underlying asset value, as noted above. Cryptocurrencies constitute the opposite end of the market spectrum to US Treasury Bills, in which an arbitrageur can confidently buy or sell short based on a clear contract that will deliver a fixed amount of cash at a predetermined time.

If fact, classical game theory would conclude that since everyone knows the structure of the cryptocurrency, and understands that everyone else is also aware, then the price should never deviate much from zero. Furthermore, classical finance (2.1) would suggest that there is some measure of risk based on the historical average of σ. This will be less helpful than it is for stock indexes as discussed above, as these factors are more difficult to measure for cryptocurrency.

Our analysis begins with the fact that despite the absence of underlying assets or backing, various groups have incentive to use it over traditional currencies. In particular there are large groups who need to make transactions outside of the usual banking system. Among these are (i) people with poor credit who cannot obtain a credit or even a debit card, (ii) citizens of totalitarian countries who fear expropriation of their savings, (iii) citizens of countries with high inflation and a much lower interest rate, (iv) people engaged in illicit activity, and (v) people who espouse utilizing a new idea or technology.

Collectively, these groups constitute a core ownership of cryptocurrencies, investing a sum that gradually grows with familiarity [13, 17, 26]. Meanwhile, the rising prices catch the attention of speculators who provide additional cash into the system, but also bring motivations inherent in speculation, namely momentum trading, or the tendency to buy as prices rise, and analogously sell as prices fall [40].

We assume a single cryptocurrency and that the price is established by supply and demand without infinite arbitrage, and apply a modern theory of asset flow [9]. This alternative approach relies on the notion of liquidity price. The experimental asset markets presented a puzzle to the economics community by demonstrating the endogenous price bubbles in which prices soared well above any possible expectation of outcome [29]. Caginalp and Balenovich [9] observed that in addition to the trading price and fundamental value (defined clearly by the experimental setup), there was an additional important quantity with the same units: the total cash in the system divided by the number of shares. Denoting this by liquidity value or price, L, they adapted earlier versions [7] of the asset flow model.

This approach leads to a system of ordinary differential equations, as summarized below, whereupon equilibrium points can be evaluated and their stability established as a function of the basic parameters.

3. Modeling cryptocurrency with asset flow equations

For brevity, we first present the full model which will be a nonlinear evolutionary system that is based on [9] but with some key differences for cryptocurrencies. We can then consider simpler models in which some features are marginalized by setting parameters to zero and obtaining 2×2 or 3×3 systems, enabling us to understand the key factors in stability.

We denote the trading price by $P(t)$, the number of units by $N(t)$, the amount of cash available by $M(t)$, and the liquidity price by $L(t) = M(t)/N(t)$. With B as the fraction of wealth in the cryptocurrency, i.e., $B = NP/(NP + M)$, the supply and demand are given by $S = (1 - k)B$, $D = k(1 - B)$ respectively, where k is the transition rate from cash to the asset. We use an adaptation of the basic "excess demand" price equation in [41]. In order to apply this nonlocally, we need to normalize excess demand and take the continuum limit, as discussed in [9].

$$\tau_0 \frac{1}{P}\frac{dP}{dt} = \frac{D}{S} - 1, \tag{3.1}$$

where τ_0 is a constant that determines the timescale between supply/demand imbalance and the corresponding change in price. It follows that $B(1 - B)^{-1} = NP/M = P/L$, so that the price equation is

$$\tau_0 \frac{1}{P}\frac{dP}{dt} = \frac{k}{1-k}\frac{L}{P} - 1 . \tag{3.2}$$

The variable k is assumed to be a linearization of a tanh type function and involves the motivations of the traders which are expressed through sentiment, $\zeta = \zeta_1 + \zeta_2$ where ζ_1 is the trend component and ζ_2 is the value component. This construct has been studied, for example, in closed-end funds [1, 11, 12, 27] which frequently trade either at a discount or premium to their net asset value. Writing the term $k/(1 - k)$ in terms of the ζ_1 and ζ_2 and linearizing we have then

$$\frac{k}{1-k} \cong 1 + 2\zeta_1 + 2\zeta_2 \tag{3.3}$$

and the price equation is then

$$\tau_0 \frac{1}{P}\frac{dP}{dt} = (1 + 2\zeta_1 + 2\zeta_2)\frac{L}{P} - 1 . \tag{3.4}$$

Next, the variable ζ_1 is defined as the trend-based component of the total sentiment. This quantity expresses the idea that in an uptrend, momentum traders will be encouraged to buy. This is paired with an aggregate timescale, c_1, through which they examine the history of price patterns. For traders with a short time horizon, the value of c_1 would be small. The parameter q_1 is the amplitude of this factor, i.e., the effective weighting of this momentum trading. Thus, we write

$$\zeta_1(t) = \frac{q_1}{c_1}\int_{-\infty}^{t} e^{-(t-\tau)/c_1}\frac{1}{P(\tau)}\tau_0\frac{dP(\tau)}{d\tau}d\tau \tag{3.5}$$

The valuation is more subtle for a cryptocurrency. The only concept of value relates to fairly recent trading prices. The first purchase with Bitcoin was for two slices of pizza for 10,000 Bitcoins [30]. The sense of value at that time was probably much less than 2018 when people became accustomed to prices in the thousands of dollars. We thus stipulate the definitions

$$P_a(t) = \frac{1}{c_3}\int_{-\infty}^{t} e^{-(t-\tau)/c_3}P(\tau)\,d\tau, \tag{3.6}$$

$$\zeta_2(t) = \frac{q_2}{c_2}\int_{-\infty}^{t} e^{-(t-\tau)/c_2}\frac{P_a(\tau)-P(\tau)}{P(\tau)}d\tau, \tag{3.7}$$

i.e., ζ_2 represents the motivation to buy based on the discount from the perceived value of the cryptocurrency, $P_a(t)$. This perceived value is based on an exponentially weighted average of previous prices, with more recent prices having a strong influence. Analogous to q_1, the parameter q_2 measures relative emphasis on the discount in valuation. In other words, if q_2 is large and $(P_a - P)/P$ is positive, i.e. the asset is undervalued, then ζ_2 will be positive and large. For a small q_2, the impact will be smaller given identical other parameters. Likewise, c_2 determines the timescale on which reaction to undervaluation or overvaluation will occur. Finally, the liquidity will not be constant but will be the sum of the core group's capital L_0 plus the additional amounts arriving from speculators that is correlated with the recent trend:

$$L(t) = L_0 + \frac{L_0}{c}q\int_{-\infty}^{t} e^{-(t-\tau)/c}\frac{\tau_0}{P(\tau)}\frac{dP(\tau)}{d\tau}d\tau \tag{3.8}$$

The parameters q and c play the same role as q_1, q_2 and c_1, c_2, representing amplitude and timescales for liquidity. Note trend's dual role in influencing both the decision-making process, and also an uptrend draws more money into the system. Both factors can contribute to price fluctuations.

Note that L and ζ_1 are linear functions of one another, but we retain L as a variable so the system is more easily generalized to incorporate a time-dependent L_0. We assume that L_0 is constant, but one can easily adapt the model to include temporal changes in L_0 due to, for example, greater public acceptance of cryptocurrencies. By differentiating (3.5–3.8) and combining the resulting equations with (3.4) we obtain the 5x5 system of ordinary differential equations:

$$\begin{aligned}
c_3P_a' &= P - P_a,\\
c_2\zeta_2' &= q_2\frac{P_a - P}{P_a} - \zeta_2,\\
\tau_0P' &= (1 + 2\zeta_1 + 2\zeta_2)\,L - P,\\
cL' &= 1 - L + q\,\{(1 + 2\zeta_1 + 2\zeta_2)\,L - P\},\\
c_1\zeta_1' &= q_1\left((1 + 2\zeta_1 + 2\zeta_2)\,\frac{L}{P} - 1\right) - \zeta_1.
\end{aligned} \tag{3.9}$$

We find a unique equilibrium at $(P, P_a, L, \zeta_1, \zeta_2) = (1, 1, 1, 0, 0)$. In other words, the only steady-state of the system occurs when the price, the anchoring notion of fundamental value, and liquidity price all coincide with the base liquidity value L_0 [33]. The time scale for price adjustment will be short as markets adjust rapidly to supply/demand changes. Much longer will be the time scale for observing the trend and reacting to under or over-valuation, as well as the time scale for liquidity. Moreover, one might expect that the valuation is on an even longer time scale. Thus one expects three time scales such that $\tau_0 \ll c, c_1, c_2 \ll c_3$, which we can scale as $c = c_1 = c_2 = 1$, and we allow arbitrary τ_0, c_3 in the analysis. We thus linearize the system about the aforementioned unique equilibrium point and obtain the following.

$$\begin{pmatrix} \tau_0 P \\ c_3 P_a \\ L \\ \zeta_1 \\ \zeta_2 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 1 & 2 & 2 \\ 1 & -1 & 0 & 0 & 0 \\ -q & 0 & q-1 & 2q & 2q \\ -q_1 & 0 & q_1 & 2q_1 - 1 & 2q_1 \\ -q_2 & q_2 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} P \\ P_a \\ L \\ \zeta_1 \\ \zeta_2 \end{pmatrix}. \tag{3.10}$$

Thus, the system is determined entirely by three parameters: q, the amplitude of liquidity; q_1, the effective weighting on the trend; and q_2, the influence of fundamental value, along with the timescale parameters τ_0 and c_3 as discussed above.

Stability of these systems has been studied extensively in previous works. In [28], a Hopf bifurcation analysis was carried out, and periodic solutions are found for critical values of the bifurcation parameter. Other work [16] has demonstrated that if traders focus on fundamentals, i.e. the value of the asset rather than trend sentiment or other factors, stable equilibria can be found. In an asset flow model [4], two disparate groups of participants are considered, with one focused on price trend and the other on valuation. Numerical computations establish the existence of regions of stability and instability separated by precise boundaries in the parameter space.

The question of stability here can be investigated by calculating the eigenvalues in the relevant parameter space, i.e. $(q, q_1, q_2) \in \mathbb{R}^3_+$ (the first octant), along with τ_0 and c_3. In particular, the main question is whether the maximal real part of the eigenvalues is positive, leading to instability, or if they are all negative, yielding stability. One sees that there is a double eigenvalue at $\lambda = -1$, and the other three eigenvalues remain negative if the Routh-Hurwitz conditions [39] below are satisfied

$$\begin{aligned} \frac{1}{\tau_0} + \frac{1}{c_3} + Q > 0, \\ \left(\frac{Q}{c_3} + \frac{1}{\tau_0} + 2\frac{q_2}{\tau_0} + \frac{1}{\tau_0 c_3}\right)\left(\frac{1}{\tau_0} + \frac{1}{c_3} + Q\right) > \frac{1}{c_3 \tau_0}. \end{aligned} \tag{3.11}$$

where we have set $Q = 1 - q - 2q_1$. A sufficient set of conditions for (3.11) to hold is the following:

$$\begin{aligned} &\frac{1}{c_3} + \frac{1}{\tau_0} > q + 2q_1 =: K, \\ &\frac{1}{c_3} + \frac{1}{\tau_0} > \frac{K}{c_3} - 2\frac{q_2}{\tau_0}. \end{aligned} \tag{3.12}$$

However, one can observe numerically that (3.12) are not necessary conditions to satisfy (3.11). Also, if we set $q_2 = 0$, we obtain the simpler condition

$$\frac{1}{c_3} + \frac{1}{\tau_0} > K \tag{3.13}$$

for stability, which we will see describes a simpler model that excludes valuation and the component of investor sentiment associated with it. We sketch various cross-sections holding one of these parameters constant and numerically compute eigenvalues across values of the other two. Note in Figures 1 and 2 below that increasing K induces a destabilizing effect. We choose various values of τ_0 and c_3 in Figures 1 and 2.

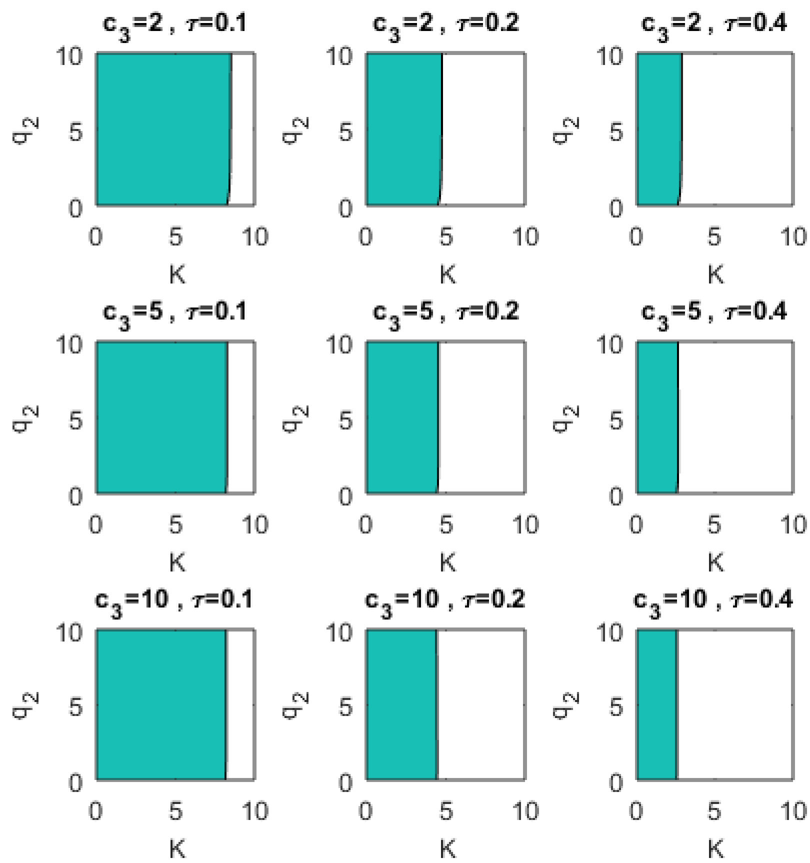

Figure 1. Stability of the 5×5 system in the $K-q_2$ plane for different values of the time scales c_3 and τ. The shaded region in the figures refer to stability for those values of parameters. Increasing c_3 and decreasing τ_0 increases the region of linear stability for the equations.

This yields a number of results. First, as market participants focus greater attention to the deviation of the asset from the acquired fundamental value driven from the liquidity price, there is less room for prices to stray from equilibrium. In addition, for a fixed q_2, the asset would experience stability given that K is not too large. Finally, for K large enough, one sees that we have instability for a large range of q_2, i.e., if investors place too much emphasis on the relative trend, the asset price becomes unstable. The shaded regions in Figures 1, 2, and 3 indicate the range of parameters for which the system (3.10) is stable.

When we set $q_2 = 0$, the model simplifies somewhat, leaving a linear interface between the regions of stability and instability. We then have the following theorem. We define $Q := 1 - q - 2q_1$.

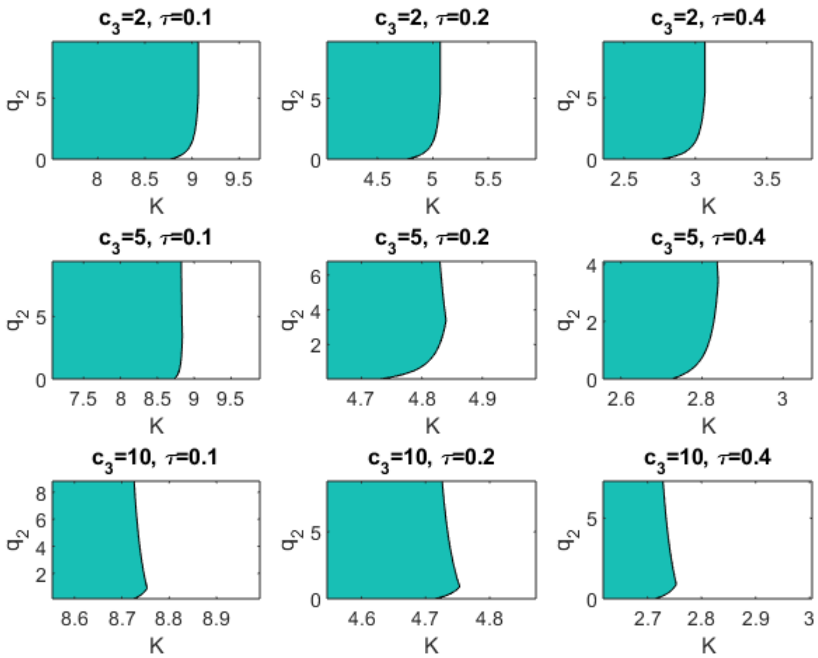

Figure 2. Zoom views of panels in Figure 1 to illustrate curvature of the interface between stability and instability. The shaded regions indicate stability.

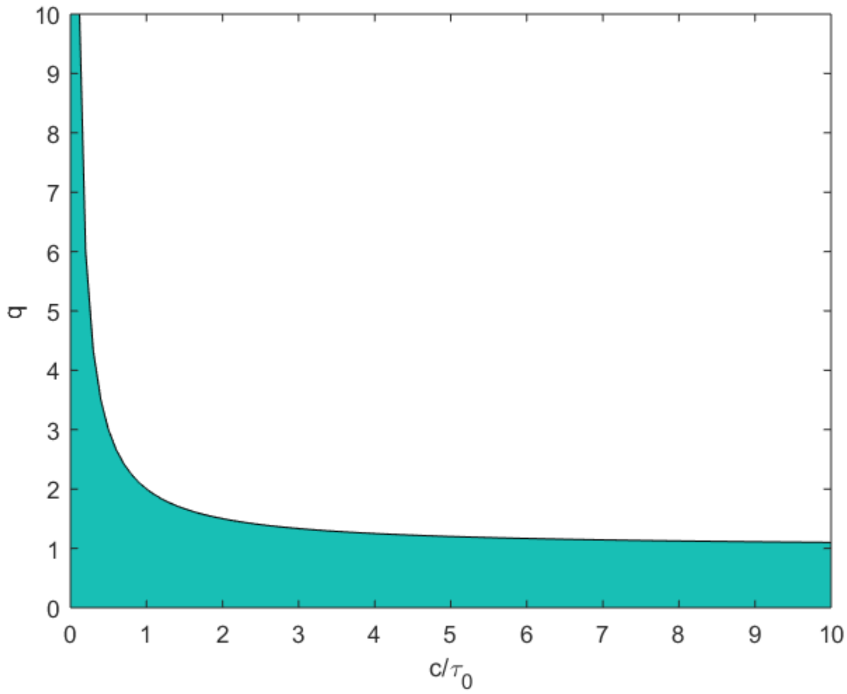

Figure 3. Stability for our simplified model without the presence of fundamental value or sentiment. The system is stable in the shaded region for the parameters q and $\frac{c}{\tau_0}$.

Theorem 1. *Consider the system (3.10). With $q_2 = 0$, one has stability of the system (3.10) if and only if*

$$Q + \frac{1}{\tau_0} > 0 \tag{3.14}$$

Proof. Setting $q_2 = 0$, the necessary conditions become

$$\left(Q + \frac{1}{\tau_0}\right)\left(\frac{1}{\tau_0} + \frac{Q}{c_3} + \frac{1}{c_3\tau_0} + \frac{1}{c_3^2}\right) > 0 \text{ and } Q + \frac{1}{\tau_0} + \frac{1}{c_3} > 0 \tag{3.15}$$

We prove this is equivalent to $Q + \frac{1}{\tau_0} > 0$.

(i) Assume $Q + \frac{1}{\tau_0} > 0$. Then clearly the second inequality in (3.15) is satisfied. Also, one has

$$\frac{1}{\tau_0} + \frac{Q}{c_3} + \frac{1}{c_3\tau_0} + \frac{1}{c_3^2} = \left(Q + \frac{1}{\tau_0}\right)\left(\frac{1}{c_3}\right) + \frac{1}{\tau_0} + \frac{1}{c_3^2} > 0, \tag{3.16}$$

satisfying the first inequality.

(ii) Suppose (3.15) holds. Then clearly

$$0 < \left(Q + \frac{1}{\tau_0} + \frac{1}{c_3}\right)\frac{1}{c_3} + \frac{1}{\tau_0} = \frac{1}{\tau_0} + \frac{Q}{c_3} + \frac{1}{c_3\tau_0} + \frac{1}{c_3^2}, \tag{3.17}$$

implying (3.14). □

4. The effect of liquidity with or without sentiment

In order to isolate the effect of liquidity, we eliminate the role of investor sentiment and value by setting the associated parameters to zero. To this end, we are left with the system

$$\begin{aligned} \tau_0 P' &= L - P, \\ cL' &= 1 + (q-1)L - qP. \end{aligned} \tag{4.1}$$

One readily calculates that there will be positive eigenvalues of the linearized system if and only if $q > 1 + \frac{c}{\tau_0}$ In other words, in a system where only price and liquidity are relevant, a large amplitude q of liquidity is destabilizing while a large time scale τ_0 for the price is stabilizing. The stability is illustrated in the Figure 3.

Another nontrivial subcase is obtained from examining the full model (3.9) in the case where we set the value component of the sentiment, ζ_2, and the fundamental value equal to zero. In other words, we are considering the case where market participants are aware that there is no real fundamental value, and thus no accounting for deviations from it either. We then have the system of equations

$$\begin{aligned} \tau_0 \frac{dP}{dt} &= (1 + 2\zeta_1)L - P \\ c\frac{dL}{dt} &= 1 - L + q(1 + 2\zeta_1)L - qP \\ c_1\frac{d\zeta_1}{dt} &= q_1(1 + 2\zeta)\frac{L}{P} - q_1 - \zeta_1 \end{aligned} \tag{4.2}$$

One then observes that the only equilibrium point is $L = P = L_0$ and $\zeta = 0$. Recalling that $Q := 1 - q - 2q_1$, one has the following result.

Theorem 2. *The system (4.2) incorporating liquidity and sentiment (with $c := c_1$) is stable if and only if*

$$Q + \frac{c}{\tau_0} > 0, \tag{4.3}$$

i.e. if the perturbations from trend and liquidity sentiment are sufficiently small as a relative comparison to the timescale of reaction with respect to price.

Proof. By scaling, assume without loss of generality that $c_1 = c = 1$; then we can linearize the system as follows:

$$\begin{pmatrix} P \\ L \\ \zeta \end{pmatrix}' = \begin{pmatrix} -1/\tau_0 & 1/\tau_0 & 2/\tau_0 \\ -q & q-1 & 2q \\ -q_1 & q_1 & 2q_1 - 1 \end{pmatrix} \begin{pmatrix} P \\ L \\ \zeta \end{pmatrix} =: A \begin{pmatrix} P \\ L \\ \zeta \end{pmatrix}. \tag{4.4}$$

Leaving aside the eigenvalue of -1 that is present for all values of the parameters, the matrix A has eigenvalues with positive real part if and only if

$$q + 2q_1 > 1 + \frac{1}{\tau_0}. \tag{4.5}$$

After rescaling, this is the statement of the theorem. □

Furthermore, we have either zero or two roots with positive real parts, so that we will have a stable spiral for $Q + \frac{c}{\tau_0} > 0$ and an unstable spiral for $Q - \frac{c}{\tau_0} < 0$ for the equilibrium point at $(1, 1, 0)$. This matches our intuition from an economics perspective since one has instability when $q + 2q_1 > 1 + \frac{c}{\tau_0}$, i.e., there will be stability if $q + 2q_1 < 1$ regardless of c and τ_0. For $q + 2q_1 > 1$, one sees that instability arises when $\frac{c}{\tau_0}$ is sufficiently small, i.e. traders are focused on short term trends.

The analysis above clearly shows that the potential stability of a crypto-asset may be contingent on several parameters that one may be able to influence. With this information, further research may be useful to examine the correlations and fit of these parameters with the effects of news and government policy. A problem of future interest would be whether, and if so how, governmental policy might be developed to diminish the volatility in cryptocurrencies. Another alternative would be a decentralized cryptocurrency with a concrete value. A good index to base this on would be either current or future gross world product (which could be estimated via futures markets). For a nominal fee, holders of this currency would be able to demand a basket of underlying currencies (such as dollar, euro, yen, etc.) representing, say, one trillionth of the gross world product in that currency. This would keep the value of such a currency relatively close to its true fundamental value.

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Conflict of interest

The author declares that there is no conflicts of interest in this paper.

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