An Experimental Investigation of Simultaneous Multi-battle Contests with Complementarities

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Abstract

This paper reports the results of laboratory experiments that are designed to test theoretical predictions in a multi-battle contest with complementarities. The specific setting is a game of Hex where control of each region is determined by a Tullock contest and the overall winner is determined by the combination of claimed regions. We find that in a game with only a few regions, aggregate behavior across regions is largely consistent with the theoretical predictions. However, examining individual level behavior suggests that bidders are not behaving in accordance with the model, but rather pursuing focused attacks. This intuitive behavioral approach is also found to occur in larger games where the theory is undeveloped.[1]

JEL CLASSIFICATION: C7, C9, D7

KEYWORDS: Contests, Multibattle Complementarities, Hex Game, Experiments

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1 Introduction

Contests have long been used to model competitive situations like lobbying (Krueger (1974), Tullock (1980), and Synder (1989)) and patent races (Fudenberg et al. (1983), Haris and Vickers (1985, 1987)). While much of the work on contests has focused on the outcome of a single event, many situations can be better viewed as a series of inter-related contests. Changing policies and regulations often requires gaining the support of multiple lawmakers. Bringing a new product to market may require a large number of innovations. For example, Apple holds over 1300 patents related to the iPhone (Thomson Reuters (2012)). While not all of those patents may be necessary, clearly a smartphone is of no use without a user interface, a power system and an antenna. Having three user interfaces and no power system or antenna would not result in a successful product. In both of these multi-battle contest examples, the value of winning a particular contest (patent or lawmaker) depends on the combination of other contests one wins. Similarly, after gaining the support of a majority of lawmakers, the marginal value of an additional vote may drop to zero.

Concern about multi-battle contests goes back to discussion of the Colonel Blotto game (Borel (1921), Borel and Ville (1938), Gross (1950) Gross and Wagner (1950), and Freidman (1958)) in which two militaries allocate soldiers to a series of n battles, and in the standard version of the game, the winning side is the one that wins the most battles, similar to the lobbying example above. In these games the outcome of a given battle depends on who has the larger army in that battle and the battlefields are linked by a budget constraint which captures the fact that the number of soldiers available is fixed.

More recent work allows for asymmetric budgets and a positive opportunity cost of the resource in games with both continuous and discrete strategy spaces (see Hart (2008), Kvasov (2007), Laslier (2002), Laslier and Picard (2002), Roberson (2006), and Weinstein (2005)). Protecting pipelines and computer networks, which are only as strong as the weakest link, are examples of multi-battle contests where the parties have different objectives. In these settings one party has to win all of the battles to come out ahead while the other party only needs to win a single battle (see Clark and Konrad (2007), and Golman and Page (2009) for simultaneous move games, and Deck and Sheremeta (2012) for a sequential game of siege). Szentes and Rosenthal (2003a, b) examine a more generalized game in which one needs to win m battles to claim the prize (the majority rule is a special case where $m = \frac{N+1}{2}$. More thorough surveys of the theoretical work in this area are available in Kovenock and Roberson (2010a) and Konrad (2009). Recent experimental work in this

area includes Chowdhury et al. (2013) who study both deterministic and probabilistic versions of the Colonel Blotto game, Kovenock et al. (2010) who study weakest-link games, and Hortala-Vallve and Llorentr-Saguer (2010) who allow for heterogeneity of player values (see Dechenaux et al. (2012) for a more general review of contest experiments).

Thus far, work on multi-battle contests has focused on situations where the overall outcome depends on simple counts of individual battle outcomes, but not the identity of the specific battles. The exception is Kovenock et al. (2012), who consider a multi-battle contest where each battle is for a given region in a 2×2 game of Hex. Hex is a simple game independently invented by Piet Hein and John Nash where each player attempts to create a path connect opposing sides (see Figure 1). In the traditional version of Hex, individual cells in the gird are claimed alternatingly by the two players, but Kovenock et al. (2012) consider the situation where each region is allocated based upon a standard Tullock contest function.

The game of Hex can be thought of as mimicking a communications network with redundancies like the internet as there are multiple ways to navigate through the network and no particular relay is critical by itself. Therefore, an attack on such a network has to successfully block all possible communication routes. Hex can also be thought of as a contest literally in geographic terms like when environmentalists want to establish a greenway and developers want to build a highway. In equilibrium, contestants should compete for every region, but should compete more fiercely for certain regions.

The design and structure of Hex is such that if every region is claimed there will be exactly one winner. Hence, Hex creates an environment where there are complementarities between certain battles. Of course, there are an unlimited number of other multi-battle contests with complementarities that one could construct. Even with only four battles, one could arbitrarily assign many different sets of combinatorial values. This is the typical approach that is used in the combinatorial auction experiment literature (e.g. Banks et al. (2003), Porter et al. (2003), and Kagel and Levin (2005)). The advantage of Hex is that it creates a natural and intuitive set of combinatorial values that is easy for contestants to understand and internalize, which is important for understanding how people view complementarities.

In this paper we report the results of controlled laboratory experiments designed specifically to test the theoretical predictions of Kovenock et al. (2012) and to explore behavior in contests with complementarities more generally. We also report the results of experiments involving a larger 4×4 game board. While the size of this larger game is still relatively small, the computations

involved make developing theoretical predictions for it overly cumbersome which is why Kovenock et al. (2012) restrict their attention to the 2×2 game. Such scalability is a clear advantage of a behavioral approach to studying contests with complicated complementarities. As a prelude to the results, we find that aggregate behavior in the 2×2 game is generally consistent with the model in the sense that relative bids among different regions are in line with the equilibrium predictions. However, we do observe substantial overbidding, a robust phenomenon in contest experiments (see Dechenaux et al. (2012)). Despite the aggregate success we find that players bid on winning combinations instead of bidding on every region as predicted. This same behavioral pattern is observed in the larger game as well, with the result being that on average a larger amount is bid on those cells that have a greater number of winning paths running through them.

The remainder of the paper is organized as follows. The next section presents the specific hypotheses to be tested based upon the theoretical model. The third section details the experimental design and the fourth section presents our results. Concluding remarks are given in the final section.

2 Model and Hypotheses

The model presented here is from Kovenock et al. (2012). Consider the 2×2 game of Hex shown in Figure 1. There are two players, X and Y. Player X controls the top left and bottom right corners of the board, while Y controls the other two corners. The objective for each player is to form a contiguous path connecting their pair of corner regions. Thus player X wins a prize valued at V if he captures any of the following sets of regions {North, South, East, West}, {North, South, East}, {North, South, West}, {North, East, West}, {South, East, West}, {North, South}, {North, East}, {South, West}. There are also 8 winning combinations for Player Y, some of which are the same as those for player X and some that are different. For example, either player wins by capturing the set {North, South}, while only player X wins with the set {North, East} and only player Y wins with the set {South, East}. Let X* and Y* denote the set of winning sets for players X and Y respectively.

The players are assumed to be risk neutral, have a common value for winning the game, and do not face a budget constraint. Further, the winner of each region is determined by a standard Tullock contest success function. Letting $X_r(Y_r)$ denote the investment by player X (Y) in region $r \in R = \{North, South, East, West\}$, the probability that player X wins region r is $\frac{X_r}{X_r+Y_r}$. Winning a region allows the player to use that region is completing a path between their assigned corners. Hence the
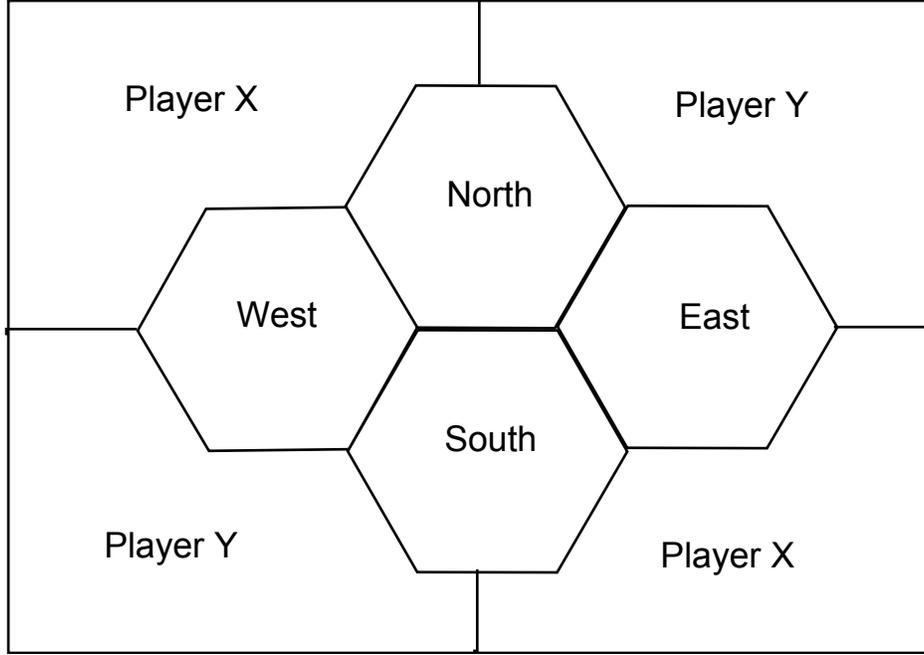
Figure 1: The 2 × 2 Hex Game of Kovenock et al. (2013)

probability that X wins the overall prize is given by $P = \sum_{\alpha \in X^*} \left(\prod_{r \in \alpha} \left(\frac{X_r}{X_r + Y_r} \right) \prod_{r \notin \alpha} \left(\frac{X_r}{X_r + Y_r} \right) \right)$ with a similar calculation for the probability that Y wins the prize. As any investment is forgone regardless of the outcome, player X's profit function is given by $\pi_X = VP - \sum_{r \in R} X_r$ and similarly for player Y.

The unique Nash equilibrium of this game is $X_{North} = X_{South} = Y_{North} = Y_{South} = \frac{V}{8}$ and $X_{East} = X_{West} = Y_{East} = Y_{West} = \frac{V}{16}$. Notice, that each payer should invest a positive amount in each region, but should invest more in North and South which are in more winning combinations and are thus of higher strategic value. The equilibrium calculation is straightforward, but tedious involving the simultaneous solution to four first order conditions of the profit maximization problem for each player. In equilibrium, each player has a 50% chance of winning and an expected payoff of $\frac{V}{8}$. Notice also that aggregate investment by the two players together is $4(\frac{V}{8}) + 4(\frac{V}{16}) = \frac{3V}{4}$. Details of how this equilibrium was obtained can be found in Kovenock et al. (2013).

For comparison, if there were no complementarities and each of the regions was valued at V_r then the standard result would hold for each region. Specifically, each bidder should bid $\frac{V_r}{4}$ for each region, resulting in a 50% chance of claiming any region and an expected profit of $\frac{V_r}{4}$ in each region. The total investment over R would be $\sum_r \frac{V_r}{2}$. If $\sum_r V_r = V$ so that the total prize was the same in the two games then without complementarities each player would invest a total of $\frac{V}{4}$,

the expected profit per player would be $\frac{V}{4}$, and the total investment would be $\frac{V}{2}$. Therefore, for the same total prize the complementarity increases total investment, and the expected profits are lower.

The equilibrium investments serve as the basic hypotheses to be tested in the lab. However, given the robust result from previous experiments that people tend to overbid in simple contests we also investigate the following relative hypotheses that focus on the role of complementarities.

1. Without complementarities, bids in a region are proportional to the value of that region and thus also proportional to the equilibrium bid for the region.

2. With complementarities, bids in a region are proportional to the equilibrium bid for the region.

Before continuing to the experimental design, we briefly point out a few additional facts. First, the theoretical problem described above can be extended to an $n \times n$ size game of Hex. The equilibrium condition is determined by the simultaneous solution of $2n^2$ first order conditions. Further, the profit function itself depends on the elements of X* and Y* which each contain 2^{n^2-1} entries. So for a 4×4 game there are 32768 winning combinations for player X and 32768 winning combinations for player Y and the equilibrium level of investment depends on 32 simultaneous equations based on those winning combinations. Interestingly, while this problem quickly becomes intractable, due to the large number of wining combinations, it is trivial for a person to look at a 4×4 game and see who has won, a fact that we make use of when determining the winner in the experiments.

Finally, notice that in the 2×2 game every winning path for X is either (North, South), (North, East), (South, West), or some superset of one of these. With this in mind we define the notion of a *minimal winning set*. Minimal winning sets are the sets of cells which are sufficient for victory, but no proper subset of which is a winning set. There are also three minimal winning paths for player Y and again each involves 2 regions. For both players, the regions North and South are in two of their minimal wining sets and East and West are only in one of the minimal winning sets. Therefore, if a player were to invest uniformly along a randomly selected minimal winning path and not bid off of that path (a pattern that emerges in the experimental results), then on average twice as much would be invested in North and South than in East and West, the same aggregate pattern predicted by the equilibrium despite the difference in individual behavior. As the game becomes larger, $n > 2$, it is no longer the case that every minimal winning path is of length n, although each player does always have $\sum_{i=1}^{n} 2^{i-1}$ minimal wining paths of length n.

3   Experimental Design

To evaluate the theoretical predictions of the model, we conducted a series of contest experiments using a between subjects design. In one treatment subjects participated in a series of contests that did not involve complementarities and in the other treatment the contests involved complementarities.

Throughout the experiment, monetary amounts are denoted in Experimental Currency (EC), which would be converted to $US at the rate of $EC 25 = $US 1. This exchange rate was explained to the subjects when the experiment began. Unless otherwise noted, all monetary amounts below are in EC. Subjects also received a $US 5 payment for showing up to the lab on time for the one hour session, as is standard policy in the Behavioral Business Research Laboratory at the University of Arkansas where the experiments were conducted. Participants were 72 undergraduate students from the lab's database of approximately 2000 volunteers. While some subjects had previously participated in other economics experiments, none had participated in any related studies.

Both treatments involved three phases as described below. Subjects read phase specific instructions just prior to the start of the phase. Subjects did not know how many periods were in any phase nor did they know of the existence of future phases.

Each phase consists of multiple repetitions of a single game. Each game consists of acquiring regions, with the difference between phases consisting in how the regions are acquired and the number of regions that are being contested. After all regions have been claimed, the set of regions claimed by each player is examined. Players then receive payouts based on the regions they have claimed in the game. At the start of each game all regions are unclaimed (claims do not carry over between games) and players are matched with a new, unknown, opponent.

Phase 1: Sequential Play in a 2 × 2 Game with No Investment

The first phase consisted of 10 games on a 2 × 2 board. For technical ease, the regions were actually squares instead of hexagons, but the arrangement was such that the winning combinations were the same as discussed in the previous section for the game with complementarities (see Figure 2). In each game one player moved first and was able to claim a region (at no cost) by simply clicking on their screen. The second player could observe the choice of the first player and then select a region to claim. This choice was revealed to the first mover who could then claim a second region leaving the final region for the second mover.

For the no complementarities treatment, the North and South regions were valued at 8 while the other two regions was valued at 4 each (see Figure 3). Thus, the first mover should take either North or South and the second mover should take the other of these relatively high valued regions. For the complementarities treatment, the value of completing a winning path was 48. In this case, the first mover should pick North or South initially. The second mover should then select whichever remains unclaimed of North or South in the hope that the first mover makes a subsequent mistake. The first mover should then make a wining move and hence the first player should always win. This first mover advantage holds in the sequential game of Hex regardless of the board size, In fact, the board game from Parker Brothers is played sequentially on an 11 × 11 board.

Each player alternated between playing the first and second mover in these games. Further, there were 6 people in each session and players were randomly and anonymously matched each game to eliminate any issues with repeated play or reputation.

The purpose of phase 1 is twofold. First, it allows players to discover the strategic value of each region to both players in the game of Hex without explicitly being instructed about the importance of the North and South regions, which might bias bidding behavior. Second, it provides an opportunity for the subjects to earn money which can be used be as an endowment during the later phases of the experiment where one risks losing money. In fact, since both players forfeit their investment and only one player can claim the prize, there must be a financial loser in each contest. Therefore, to maintain control over subject incentives, the standard procedure of providing an endowment from which losses can be deducted is used. Because the values in the two games are different, for reasons explained below, the subjects were also given a treatment specific initial endowment. Those in the complements treatment received an endowment of 160. Those in the no complements treatment received an endowment of 280. The end result is that after phase 1, each subject should have earned a cumulative profit of 400.[2]

Phase 2: 2 × 2 Game with Regions Decided by Contests

The second phase of the experiment consisted of 20 games on a 2 × 2 board. Again, players were randomly and anonymously matched each period. In this phase the players privately and simultaneously submitted bids for each region shown in Figure 2 with each region being independently awarded probabilistically according to the Tullock success function. For the games with

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[2]In the complements treatment, each player should win five of the ten rounds played in phase 1, winning 48 in each round for a total gain of 240. In the no complements treatment, each player should win 12 in each of the ten rounds of phase 1, for a total gain of 120.
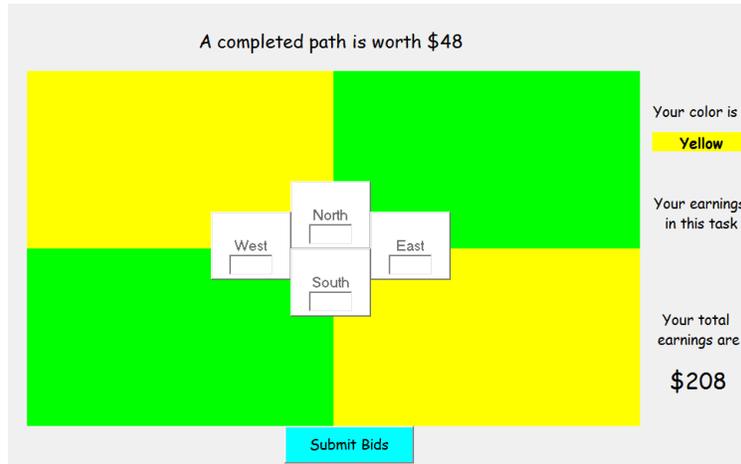
Figure 2: Screenshot of 2×2 game with complementarities

complementarities, the prize for winning was again 48. Without complementarities, North and South were valued at 8 and East and West were valued at 4. Table 1 summarizes the equilibrium bids and profits for both treatments. As described in the model section, for the same total value available, the expected profits differ between treatments. Therefore, the values in the no complements treatment were adjusted so that (1) the expected profit per bidder is held constant across treatments, and (2) the relative value of each region is held constant across treatments. The first point is important for ensuring that subjects have the same incentives in both treatments. The second point enables a clearer test to determine how the complementarity impacts investment as well as allowing for a test of the standard contest model as the value changes.
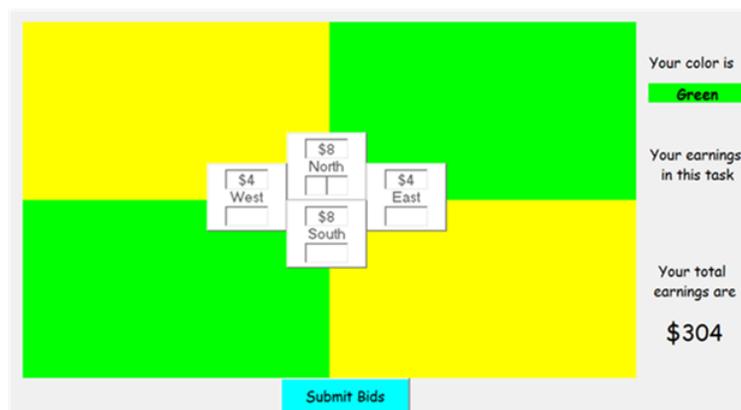

Figure 3: Screenshot of 2×2 game with no complementarities

Phase 3: 4×4 Game with Regions Decided by Contests

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	No Complementarities	With Complementarities
Value of Winning	North, South worth 8 East, West worth 4	Completed Path worth 48
Equilibrium Bid on North, South	2	6
Equilibrium Bid on East, West	1	3
Expected Profit per Player	6	6

Table 1: Summary of Experiment Parameters and Predictions for 2×2 Games

The procedures for this phase were identical to those in phase 2, except that the game board was increased to 4×4 and that subjects only played the game 5 times (and thus placed the same total number of bids as in phase 2 since there are four times as many regions in phase 3). Figure 4 shows a sample outcome of the 4×4 game with complementarities. The value of a winning path remained 48 for the treatment with complementarities. For the game without complements, each of the 16 regions was valued at 3 so that the total value was the same between the two treatments, allowing for an evaluation of the impact of incentives and additional variation in prize values for standard stand-alone contests. Given the exploratory nature of this game and the inherent increase in complexity when there are complementarities, this phase was always conducted last so as to 1) not influence behavior in the 2×2 game which is the main focus of the project and 2) provide subjects an opportunity to learn in the simpler game so that observed behavior in this game is meaningful. While these two concerns are not as important when there are no complementarities, the two treatments are kept parallel to make direct between treatment comparisons.

Figure 4: Screenshot of 4×4 game with complementarities

4   Behavioral Results

We begin our analysis with behavior in Phase 1 of the experiment. Overall, players made an optimal choice 98% of the time in the no complementarities case, where optimal is defined as claiming the higher valued region if it is available. For the more complicated sequential Hex game, optimal behavior, defined as following the subgame perfect equilibrium strategy conditional on the decision point, is observed 95% of the time. Further, no suboptimal behavior was observed in the last three rounds for either treatment. This finding clearly indicates that subjects understood the strategic value of the different regions before beginning phase 2.

We now turn to analyzing behavior in phase 2. Given the symmetry in the game, regions are standardized around a vertical line drawn through the center of the game and reported with respect to the position of a region from the perspective of the Yellow player who is trying to complete a path from the top left to the bottom right. For the 2×2 game this means that for reporting purposes bids Green placed nominally for the West region are combined with Yellows' bids for the East and vice versa. The average bid for each region is shown in Table 2.

We begin by considering the no complementarities treatment, the results of which are shown in the left column of Table 2. Several interesting features of the data are readily apparent. First, for all four regions players are bidding almost twice the equilibrium level on average. This is clear evidence that the subjects are not bidding according to the equilibrium predictions. Such overbidding is actually typical in simple contests experiments. The second main feature is that players are bidding basically the same amount for North and South, consistent with Hypothesis 1. The players are also bidding nearly identically on average for East and West, also consistent Hypothesis 1. Finally, players are bidding twice as much for North and South as for East and West, again consistent with Hypothesis 1.

| | No Complementarities | | With Complementarities | |
	Observed	Percent Overbid	Observed	Percent Overbid
North	3.81	91%	7.38	23%
South	3.75	88%	8.36	39%
East	1.89	89%	3.56	19%
West	1.85	85%	4.65	55%
North / South	1.02		0.89	
East / West	1.02		0.77	
$\frac{North+South}{East+West}$	2.03		1.92	

Table 2: Average Bids by Region in the 2×2 Games

Table 3 reports the estimation results from regression analysis where the dependent variable is the amount bid on a region relative to the equilibrium bid for that region and South and West are dummy variables for those respective regions and Side is a dummy variable that takes the value of 1 for the East and West regions and is 0 otherwise. Thus the omitted case is the North region. Side captures the effect of being in the East region while West captures any differential effect between the East and West regions. Standard errors are clustered at the session level while each individual bidder is treated as a random effect. The joint lack of significance for the dummy variables in Table 3 provides statistical support in favor of Hypothesis 1. To test whether or not players are bidding according to the equilibrium predictions, involves comparing the constant term in Table 3 to the predicted value of 1. This hypothesis can be rejected in favor of systematic overbidding at even the 1% significance level. We can see this overbidding in Figure 5, showing the cumulative distribution of the total amount spent by players in the no complementarities treatment. The dashed line indicates the equilibrium total spending $= 6(= 2 + 2 + 1 + 1)$, while the solid line marks the total value of prizes for all four areas. In fact, Figure 5 suggests that total expenditure is almost uniformly distributed and centered around $\sum_r \frac{V_r}{2}$.

	Coefficient	Robust Std. Error
Constant	1.9070***	0.0447
Side	-0.0535	0.0736
West	0.0238	0.0278
South	-0.031	0.0385
***indicates 1% Confidence Level		

Table 3: Region Game Ratio of actual bids to theoretical bids, regressed on region dummies

Figure 6 shows the cumulative distribution of the fraction of total spending on high value regions (North and South) and low value regions (East and West). The long (short) dashed line indicated the theoretical prediction for high (low) valued regions. This figure suggests that subjects are reacting proportionally to the value of the regions. The theoretical predictions capture the central tendency of the fraction of money spent on regions is a parallel shift to the right of the low value regions distribution.

Turning to the treatment with complementarities, the data presented in the right column of Table 2 reveal that subjects are again overbidding in each region, but not as dramatically as in the no complementarities case. Table 2 also provides strong evidence in support of Hypothesis 2. The average bids are very similar in North and South as predicted. Further, average bids are similar in
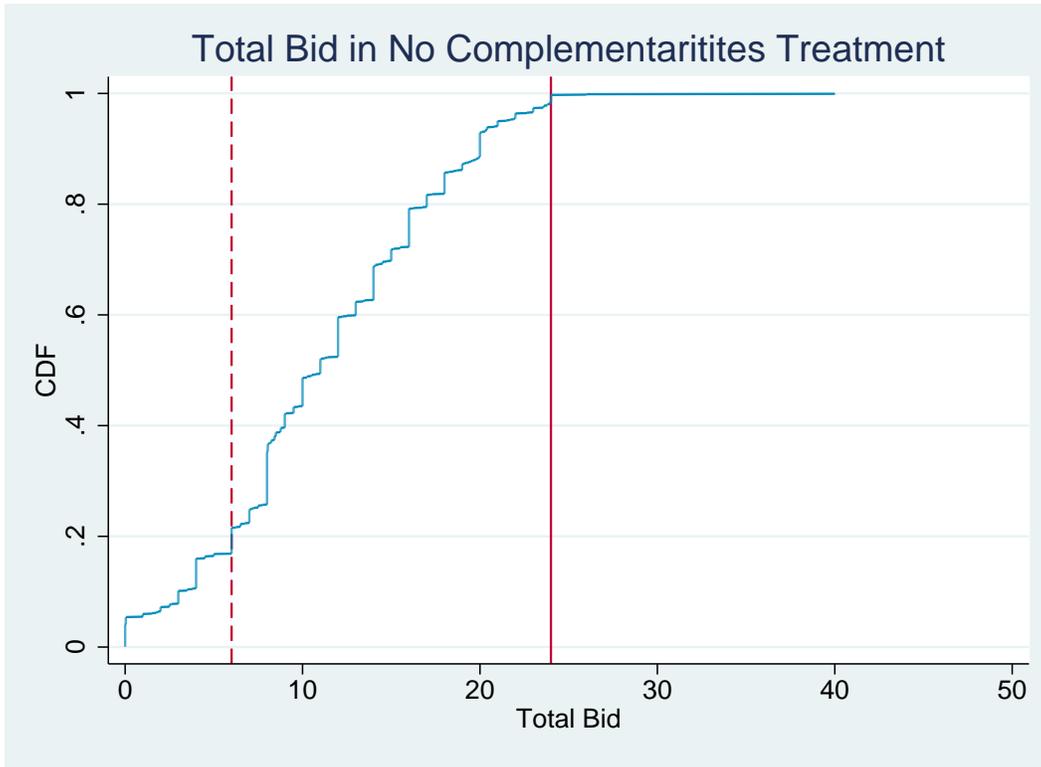
Figure 5: Total Spending in No Complementarity Treatment

the East and West, as predicted. Finally, bids are predicted to be twice as high in the high strategic value regions (North and South) as in the low strategic value regions (East and West) and this is what is observed. Statistical evidence is provided in Table 4, which reports a similar regression to that done for the no complementarities treatment. Hypothesis 2 is supported by the joint lack of significance on the three dummy variables. In this case however we cannot reject the hypothesis that the constant term equal 1 at the 5% significance level. This is due to the lower overbidding that we find when complementarities are present.

Figure 7 gives the cumulative distribution of total spending in the path formation game. The dashed line indicates the theoretical equilibrium total spending, while the solid line is where spending equals the prize. As in Figure 5, total bidding appears to be relatively uniform. The reduction in overbidding is driven by the fact that the predicted total bid is a greater percentage of value in this treatment.

The above analysis focuses on aggregate behavior and in general the results seem to support the relative theoretical predictions. However, these results are masking a distinct behavioral pattern. The theoretical prediction is that a player should bid on every region. This pattern is observed in
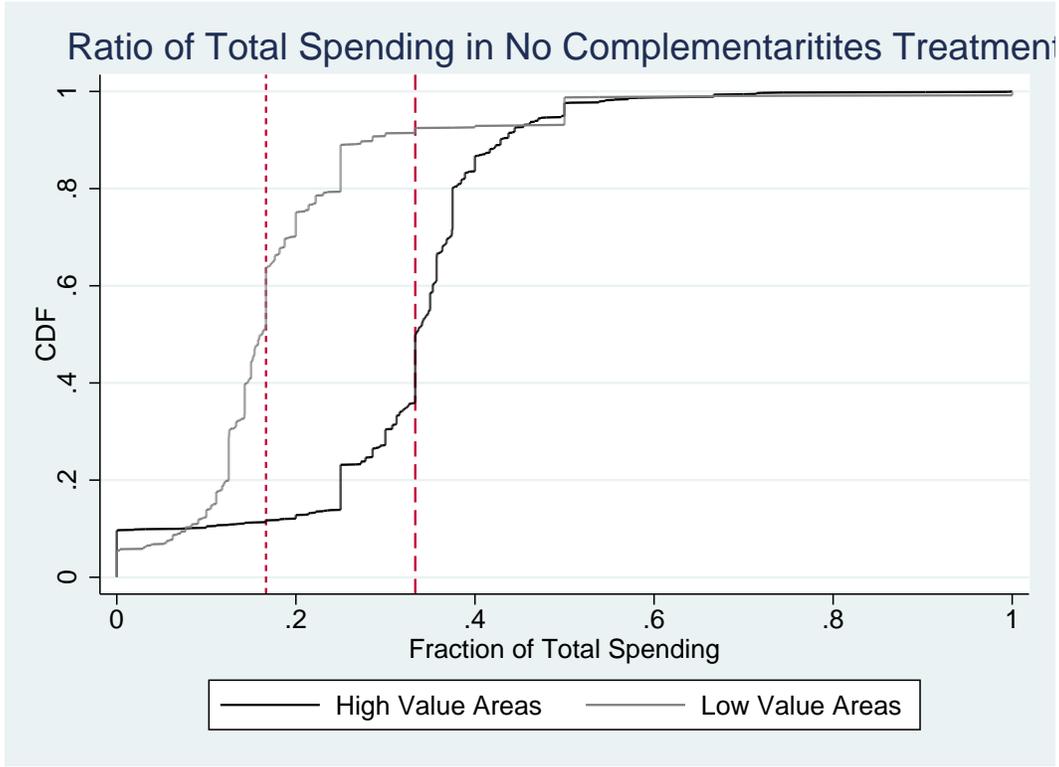
Figure 6: Fraction of Total Spending Bid by Region Type

	Coefficient	Robust Std. Error
Constant	1.2297***	0.1503
Side	0.2409	0.2457
West	-0.2066	0.3342
South	0.1626	0.1318
∗∗∗indicates 1% Confidence Level		

Table 4: Path Formation Game Ratio of actual bids to theoretical bids, regressed on region dummies

only 35% of the realizations, see Figure 8. Only 1% of the time does the set of regions on which a subject bid constitute a non-winning set. The missing majority of observations, 64%, are such that subjects bid on a path through the game such as {North, West}, or a set of three regions containing a winning set such as {North, South, West}. As discussed previously, if one randomly selects a minimum winning path and bids uniformly on it, the result would also be average bids that are twice as high for North and South as for East and West. Further, if the total bid equaled half of the value of winning, as occurred in the independent contests with the no complementarities treatment, then one would bid 12 on North and South with two-thirds probability and would bid 12 on East and West with one-third probability. The result would be an average bid of 8 for North
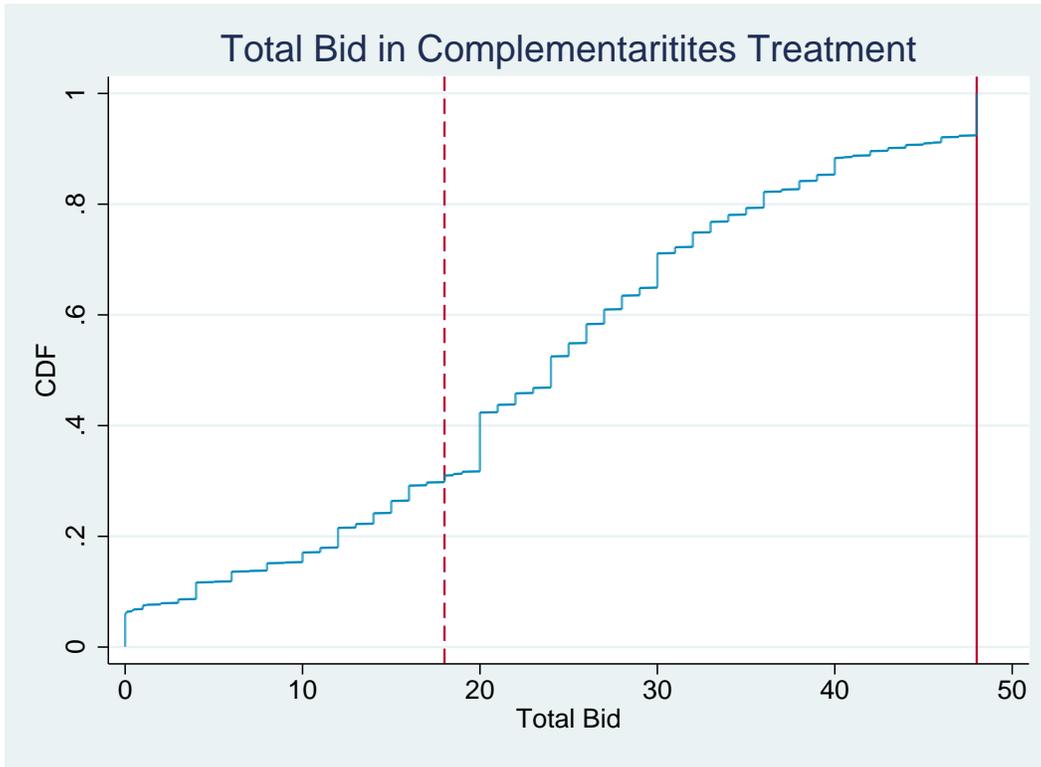
Figure 7: Total Spending in Complementarity Treatment

and South and an average bid of 4 for East and West, the pattern revealed in Table 2. Hence it appears that theoretical model of Kovenock et al. (2013) works in aggregate, but not for the right reason.

This targeted behavior can also be seen in Figure 9, which plots the ratio of bidding on the high strategic value areas to total spending, along with the ratio of bidding on the low strategic value areas. Figure 9 shows spending in low strategic value areas is a parallel shift up from spending in high strategic value areas. This suggests that the likelihood of bidding on low strategic value areas is lower, but that conditional on placing a bid the distribution of bids is similar. The figure also reveals the subjects frequently spend either half or a third of their total bid on either type of region, consistent with bidding equal amounts on the regions for which they place positive bids. While the theoretical prediction is that one third of spending should be on a high strategic value region, this is not the case for low strategic value areas, but there is no real behavioral difference between the two treatments in this respect.

Related to this point is that in the path formation game we see a lack of players who consistently place 4 positive bids in each round, with only 2 of the 36 subjects doing so. The lack of such
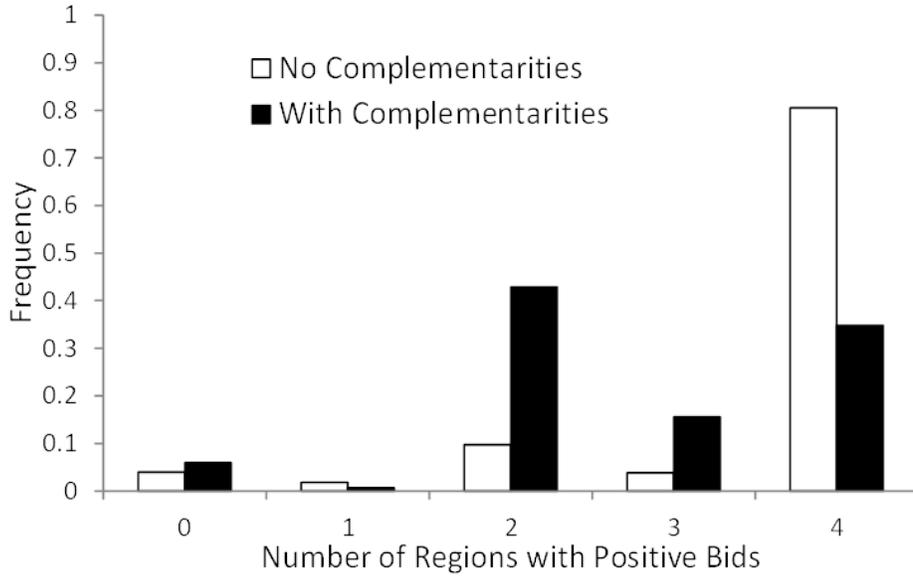
Figure 8: Number of Positive Bids in 2×2 Game

behavior further reinforces that players are averaging the correct choices for the wrong reasons. In the individual rounds, bidding on all four regions does increase the average profit, see Table 5. The mean profits of those bidding on all four regions is significantly different from those of players who bid on a path or on a non-winning set at the 95% level, while the mean profits of those bidding on paths and non-winning sets do not statistically differ.

	Mean	Standard Error
All Four Regions	2.683	1.520
Path	-1.227	1.212
Non-Winning Set	-2.200	1.670

Table 5: Mean Profits by Positive Bids in Complementarity Treatment

Further analysis of the data indicates that it is not simply that there are two types of bidders: those who bid on winning paths and those that bid on the entire region. Instead, over 80% of the subjects in the treatment with complementarities follow both strategies at some point during phase 2. Regression analysis indicates that subjects do not change the frequency with which they bid everywhere over the course of play, nor do they change their strategy based on the behavior of their opponent in the previous round; however, a switch in strategy is more likely to occur after incurring a loss, see Table 6.
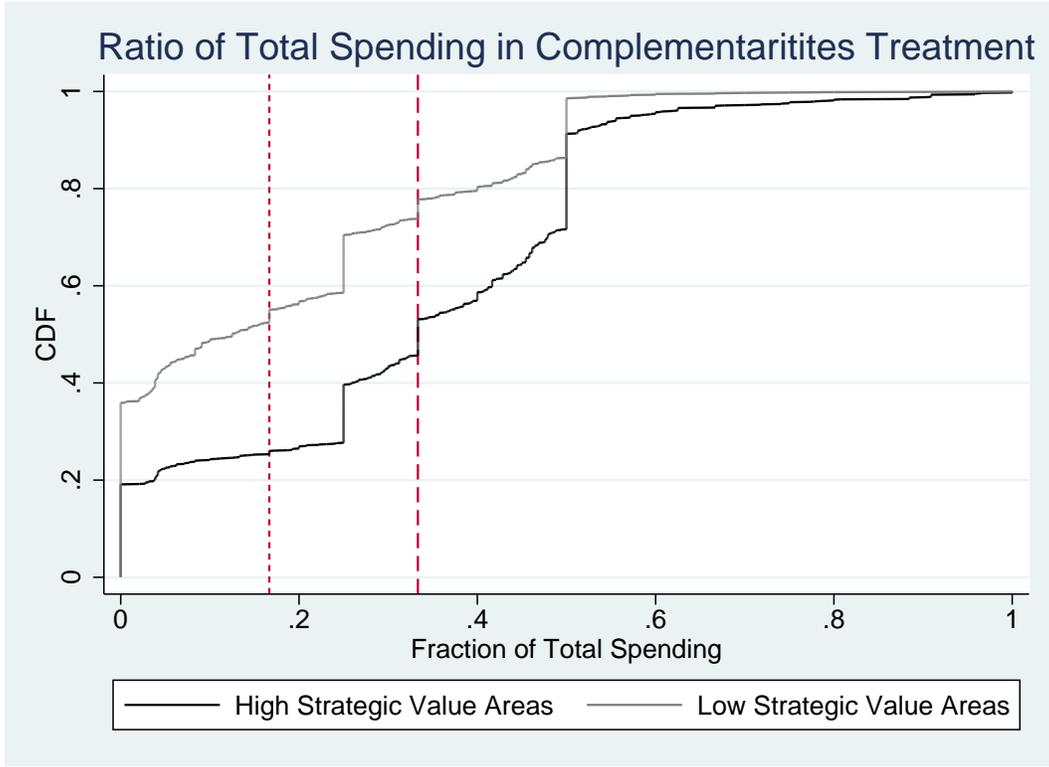
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Figure 9: Ratio of Spending on High and Low Strategic Value Areas with Complementarities

	Coefficient	Standard Error
Previous Round Profit	-0.00958***	0.00247
Constant	-0.98045***	0.05834
***indicates 1% Confidence Level		

Table 6: Switching Between Strategies in Complementarity Treatment

We now turn to behavior in the exploratory third phase of the experiment. Figure 10 shows the average bid in each region for the no complementarities treatment. In this case, each region had a value of 3 and thus the equilibrium bid is 0.75. The average observed bid across all 16 independent regions was 1.09 and there are no major differences between regions. The average rate of overbidding was 45%, which is smaller than in phase 2. One reason for this drop in overbidding is that subjects only bid on every region 64% of the time with the other observations typically involving a few ignored regions.[3] Once non-bids are accounted for the rate of overbidding is similar to that in phase 2, suggesting that the level of incentives are not driving overbidding directly. Of

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[3]87% of the bids in this treatment involved more than half of the regions. For comparison, in the 2×2 game with complementarities, 81% of bids were for all four regions.

course, failure to bid on every region could be due to fatigue or the large number of choices that had to be made, which may in turn be influenced by the level of the stakes.
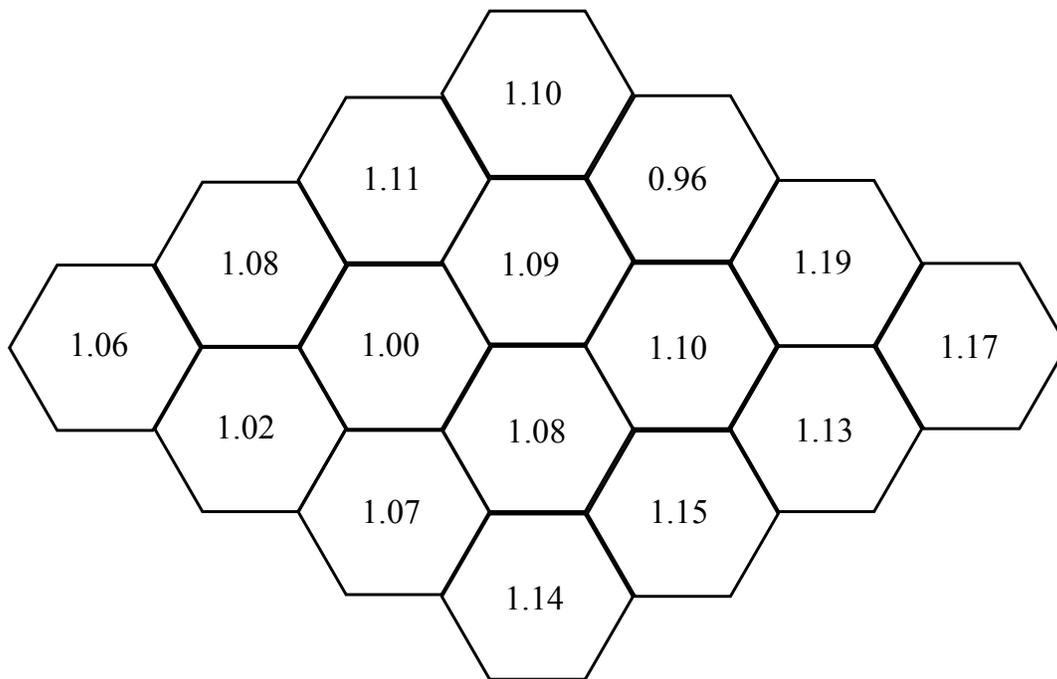

Figure 10: Average Bid By Area

In the 4×4 game with complementarities, the optimal bid for each region is related to the number of winning sets that contain the region. Figure 11 shows the number of winning sets and Figure 12 the average observed bid by region for this case. The correlation between the two measures is quite high at $\rho = 0.898$. As in the 2×2 game, it appears that on average relative bids between regions are in line with what the model would predict. However, this aggregate success again masks individual heterogeneity that is distinctly out of equilibrium. Rather that bidding on every region as predicted, 36% of bids are for a 4 region path and only 31% of bids involve more than half of the regions, see Figure 13.[4] This figure also reveals a non-trivial number of subjects bidding on 5 to 8 regions, giving themselves multiple possible winning paths. The same phenomenon is occurring in the 2×2 game when people in this treatment bid on 3 regions as shown in Figure 8.

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[4]Note that 3% of bids are for only one, two or three regions and thus could not result in a winning path. In this treatment 4% of the bids are zero for all 16 regions. For comparison, 3% of bids in the no complementarities treatment were zero in all 16 regions.
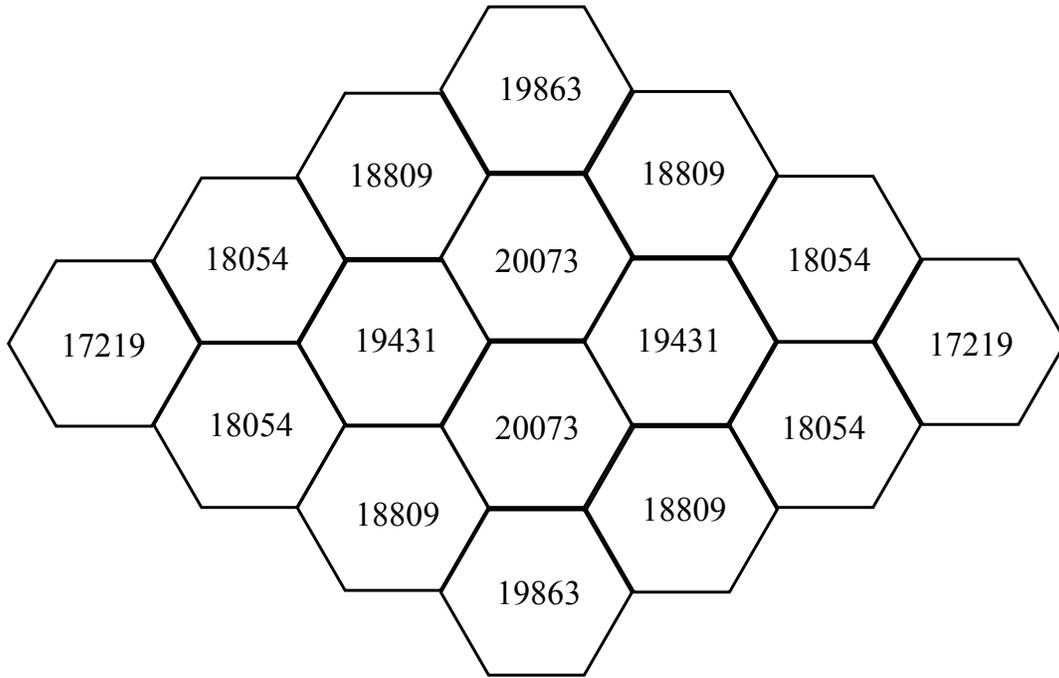
18

Figure 11: Number of Winning Sets Containing Region

5   Concluding Remarks

Contests have long been used to model a wide array of activity. Recently, researchers have begun to look at ever more complicated strategic situations where outcomes are based on a series of inter-connected battles. One such example is the recent model by Kovenock et al. (2013), which focuses on the game of Hex, but is representative of a wider class of games that involve complementarities in outcomes. This model is particularly relevant for the protection of networks that have built in redundancies, such as telecommunications or computer networks. Unfortunately, the tradeoff of additional complexity is often tractability. For example, Kovenock et al. (2013) restrict attention to a 2×2 game board.

In this paper we report the results of controlled laboratory experiment designed to test the predictions of the Kovenock et al. (2013) model specifically and also explore how players actually approach more complicated contests. After all, if people really solved the problem the way theorists do, there would not be much point in researchers considering such models as the solutions would already be apparent. What we find from the experiment is that in aggregate the model's predictions
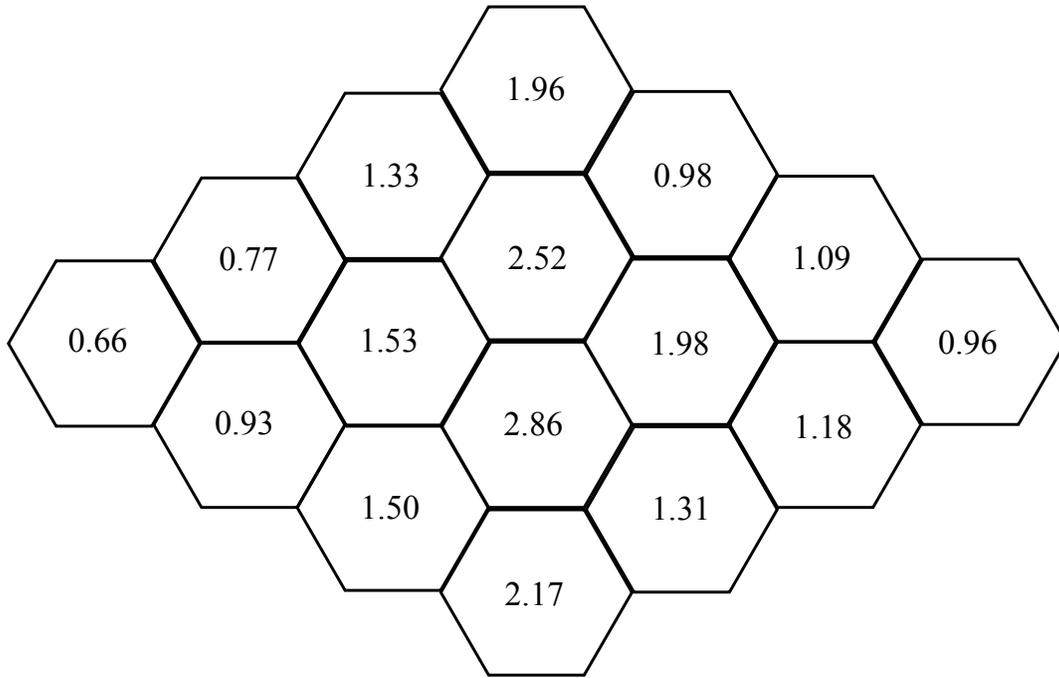
Figure 12: Average Bid By Region

accurately reflect the relative bids in the different regional battles. Overbidding is observed with and without complementarities, the later result being consistent with previous contest experiments. However, the models predictions are correct for the wrong reason. Rather than fighting for every region in accordance with the model, subjects actually concentrate on specific winning combinations and largely ignore the other battles. As in the case of independent values, people tend to bid just less than half of the prize value, but in the Hex game they often spread that amount along a minimal winning path.

An advantage of a behavioral approach to investigating contest behavior is that one can investigate situations beyond what is computationally attractive. Here we also considered a more complex 4×4 that involves 32768 wining combinations for each player. Again aggregate behavior was consistent with theoretical play in that players bid more for regions that are in more winning combinations. However, this aggregate pattern is again explained by individuals focusing on some winning combination. This finding indicates that overbidding (which is lower in the presence of complementarities because predicted spending is higher) and concentrated attacks are robust be-
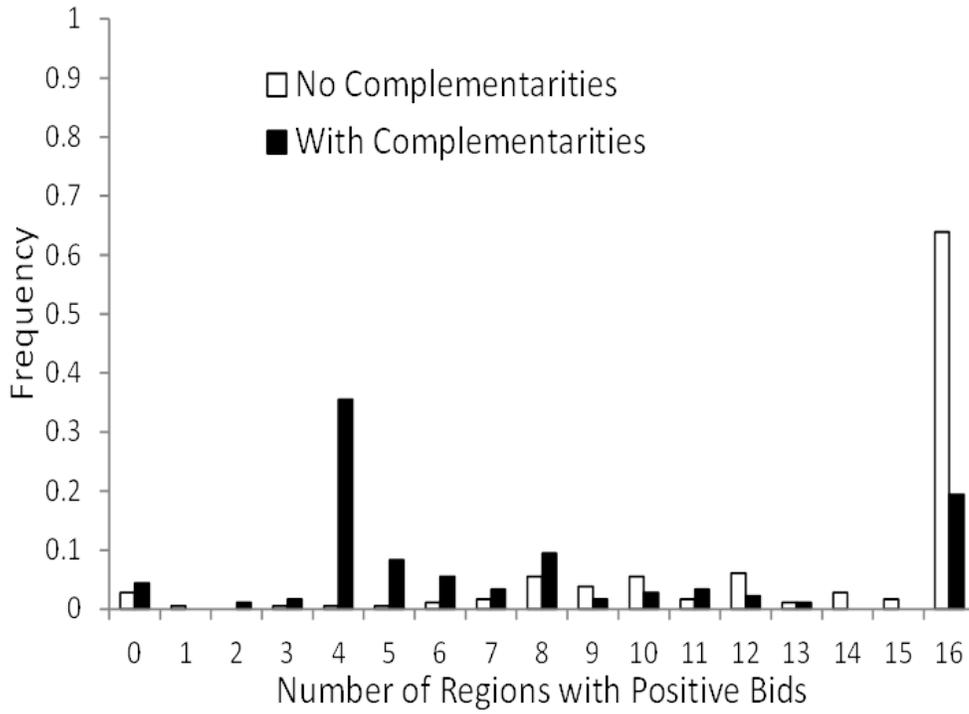
Figure 13: Number of Positive Bids in 4×4 Game

havior. The implications of these patterns are potentially quite broad as researchers attempt to identify strategies for attack and defense of networks in many naturally occurring applications such as cyber-security or supply chains.

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6   Appendix: Subject Instructions

On the following pages there are two sets of instructions. The first set is for the treatment with no complementarities and the second set is for the treatment with complementarities. Text in [brackets] was not observed by subjects.

[No Complementarities Phase 1]

This is an experiment in the economics of decision making. You will be paid in cash at the end of the experiment based upon your decisions, so it is important that you understand the directions completely. Therefore, if you have a question at any point, please raise your hand and someone will assist you. Otherwise we ask that you do not talk or communicate in any other way with anyone else. If you do, you may be asked to leave the experiment and will forfeit any payment.

The experiment will proceed in three parts. You will receive the directions for part 2 after part 1 is completed and for part 3 after part 2 is completed. What happens in part 2 does not depend on what happens in part 1, and what happens in part 3 does not depend on what happens in part 1 or 2. But whatever money you earn in part 1 will carry over to part 2 and whatever you earn in part 2 will carry over to part 3.

Each part contains a series of decision tasks that require you to make choices. For each decision task, you will be randomly matched with another participant in the lab. None of the participants will ever learn the identity of the person they are matched with on any particular round.

In each task you have the opportunity to earn Lab Dollars. At the end of the experiment your lab dollars will be converted into US dollars at the rate $25 Lab Dollars = $US 1.

You have been randomly assigned to be either a "Green" or a "Yellow" participant. You will keep this color throughout the experiment. For every task you will be randomly matched with a person who has been assigned the other color. Your color is indicated in a box on the right side of your screen. This portion of your screen also shows you your earnings on a task once it is completed and your cumulative earnings in the experiment. You will start off with an earning balance of $280.

In the center of your screen you can see a map with four regions: North, South, East, and West. The North is worth $8. The South is worth $8. The East is worth $4. The West is worth $4. This information is displayed at the top of your screen above the map. In part 1 of the experiment, when it is your turn you can claim any one unclaimed region on the map by clicking on it. When

you claim a region, that part of the map will be colored in with your color and your earnings will be increases by the value of the region you claimed. When the person you are matched with claims a region, it will be colored with that persons color and that persons earnings will be increased by the value of the claimed region.

On each task Green and Yellow alternate turns until all of the regions on a map are claimed. Once all of the regions are claimed the task is complete. At that point, you will be randomly rematched with another participant for the next task. Which color gets to go first alternates between tasks. You will go through this process several times.

[No Complementarities Phase 2]

Part 2 of this experiment is very similar to part 1. The map is the same, as are the values of each region. Your color will be the same and you will be randomly and anonymously rematched with someone in the opposite role for each task.

What is different is how the regions are claimed. Now you and the person you are matched up with have to bid for each of the four regions at the same time. However, any amount you bid on a region is deducted from your earnings regardless of whether or not you get to claim the region. Since you have to pay what you bid, the sum of your bids for the four regions cannot exceed the amount of earnings you have when placing your bids.

Bidding for a region works in the following way. The chance that you claim a region is proportional to how much you bid relative to the total amount bid for that region. For example, suppose that Yellow bid $6 for the North and Green bid $2 for the North then the chance that Yellow would claim North is $6/(6+2) = 6/8 = 75\%$. The chance that Green would claim North is $2/(6+2) = 2/8 = 25\%$.

As another example, suppose that Yellow bid $0 for the North and Green bid $0.25 for the North then the chance that Yellow would claim North is $0/(0+0.25) = 0\%$. The chance that Green would claim North is $0.25/(0+0.25) = 100\%$.

If both bidders bid $0 for a region then each would claim the region with a 50% chance.

You and the person you are matched with will both privately and simultaneously place your bids for all four regions at one time. The computer will then determine who claims each region based upon the probabilities associated with the bids. Each region will turn Yellow or Green to indicate who claimed that region.

As before, whoever claims a region will receive the value for that region and have that value added to their earnings. Suppose Yellow bid $5 for the North and Green bid $2 for the North. If Yellow claims the North then Yellow will earn $8 - $5 = $3 and Green will earn -$2. Thus $3 will be added to Yellows earnings and Green will have $2 subtracted from their earnings. However, if Green claims the North then Yellow will earn -$5 and Green will earn $8 - $2 = $6. Earnings for the other three regions will be determined in the same fashion.

[No Complementarities Phase 3]

Part 3 of this experiment is just like part 2, except that there are now 16 regions on the map and the value of each region is $3. Regions will be claimed in the same way, your color will be the same and you will be randomly and anonymously rematched with someone in the opposite role for each task.

[Complementarities Phase 1]

This is an experiment in the economics of decision making. You will be paid in cash at the end of the experiment based upon your decisions, so it is important that you understand the directions completely. Therefore, if you have a question at any point, please raise your hand and someone will assist you. Otherwise we ask that you do not talk or communicate in any other way with anyone else. If you do, you may be asked to leave the experiment and will forfeit any payment.

The experiment will proceed in three parts. You will receive the directions for part 2 after part 1 is completed and for part 3 after part 2 is completed. What happens in part 2 does not depend on what happens in part 1, and what happens in part 3 does not depend on what happens in part 1 or 2. But whatever money you earn in part 1 will carry over to part 2 and whatever you earn in part 2 will carry over to part 3.

Each part contains a series of decision tasks that require you to make choices. For each decision task, you will be randomly matched with another participant in the lab. None of the participants will ever learn the identity of the person they are matched with on any particular round.

In each task you have the opportunity to earn Lab Dollars. At the end of the experiment your lab dollars will be converted into US dollars at the rate $25 Lab Dollars = $US 1.

You have been randomly assigned to be either a "Green" or a "Yellow" participant. You will keep this color throughout the experiment. For every task you will be randomly matched with a person who has been assigned the other color. Your color is indicated in a box on the right side of your screen. This portion of your screen also shows you your earnings on a task once it is completed and your cumulative earnings in the experiment. You will start off with an earning balance of $160.

In the center of your screen you can see a map with four regions: North, South, East, and West. These regions are surrounded by large colored areas. The top left and bottom right areas are colored "Yellow". The top right and bottom left areas are colored "Green". In part 1 of the experiment, when it is your turn you can claim any one unclaimed region on the map by clicking on it. When you claim a region, that part of the map will be colored in with your color. If you can complete a continuous path of regions in your color connecting your two large colored areas you will earn $48. When the person you are matched with claims a region, that part of the map will be colored in with that persons color. If the person you are matched with completes a continuous path, that person will earn $48. Exactly one person can complete a path each time and the person that does not complete a path will earn $0.

On each task Green and Yellow alternate claiming regions until all four regions are claimed. At that point, you will be randomly rematched with another participant for the next task. Which color gets to go first alternates between tasks. You will go through this process several times.

[Complementarities Phase 2]

Part 2 of this experiment is very similar to part 1. The map is the same, as is the value of completing a path. Your color will be the same and you will be randomly and anonymously rematched with someone in the opposite role for each task.

What is different is how the regions are claimed. Now you and the person you are matched up with have to bid for each of the four regions at the same time. However, any amount you bid on a region is deducted from your earnings regardless of whether or not you get to claim the region. Since you have to pay what you bid, the sum of your bids for the four regions cannot exceed the amount of earnings you have when placing your bids.

Bidding for a region works in the following way. The chance that you claim a region is proportional to how much you bid relative to the total amount bid for that region. For example, suppose that Yellow bid $6 for the North and Green bid $2 for the North then the chance that Yellow would

claim North is $6/(6+2) = 6/8 = 75\%$. The chance that Green would claim North is $2/(6+2) = 2/8 = 25\%$.

As another example, suppose that Yellow bid $0 for the North and Green bid $0.25 for the North then the chance that Yellow would claim North is $0/(0+0.25) = 0\%$. The chance that Green would claim North is $0.25/(0+0.25) = 100\%$.

If both bidders bid $0 then each would claim the region with a 50% chance.

You and the person you are matched with will both privately and simultaneously place your bids for all four regions at one time. The computer will then determine who claims each region based upon the probabilities associated with the bids. Each region will turn Yellow or Green to indicate who claimed that region.

As before, whoever completes a path will receive the value for it and have that value added to their earnings. Suppose Yellow bid a total of $23 on the four regions and Green bid a total of $18 for the four regions. If Yellow completes a path then Yellow will earn $48 - $23 = $25 and Green will earn -$18. Thus $25 will be added to Yellows earnings and Green will have $18 subtracted from their earnings. However, if Green completes a path then Yellow will earn -$23 and Yellow will earn $48 - $18 = $30.

[Complementarities Phase 3]

Part 3 of this experiment is just like part 2, except that there are now 16 regions on the map. Regions will be claimed in the same way, a completed path is still worth $48, your color will be the same and you will be randomly and anonymously rematched with someone in the opposite role for each task.