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Minimal Frames and Transparent Frames for Risk, Time, and Uncertainty

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Behavior differs between transparent and nontransparent presentations of decisions, but 'transparent presentation' has not been precisely defined. We formally define 'transparent frames' for risk and time, establish their uniqueness, provide algorithms for constructing them, and compare them to 'standard' presentation formats. A logic emerges for predicting systematic shifts in choice under risk and over time, and how violations of rational choice theory will depend on frames. An experiment verifies most of those predictions in choice under risk. We extend results to choice under uncertainty and also predict frame dependence of ambiguity aversion, a result supported by recent experimental evidence.

1. Introduction

Among challenges to the descriptive validity of the expected utility hypothesis, none is more widely studied than Allais' (1953) common consequence effect, an example of which is shown below.

G1:	$500,000	with probability 1	G2:	$2,500,000	with probability 0.10
				$500,000	with probability 0.89
				0	with probability 0.01
G3:	$500,000	with probability 0.11	G4:	$2,500,000	with probability 0.10
	0	with probability 0.89		0	with probability 0.90

When asked to choose between G1 and G2 and between G3 and G4, many people, including prominent decision theorists like Leonard Savage, exhibit more risk aversion in the first choice than in the second, finding G1 more attractive than G2, and G4 more attractive than G3. This pattern violates expected utility theory. One suggested explanation is that people overweight certainty relative to possibility, as captured by the weighting function in Kahneman and Tversky's (1979) original version of prospect theory.

Savage (1954, p. 102) suggested that people choose G1 over G2 because "they do not find the chance of winning a very large fortune in place of receiving a large fortune outright adequate compensation for even a small risk of being left at the status quo" and select G4 over G3 because "the chance of winning is nearly the same in both gambles, so the larger prize appears preferable." Savage then reframes the choice problems as a state-based matrix where states across choice alternatives are correlated (shown below).

	Ticket Numbers		
	1	2 – 11	12 – 100
G1:	$500,000	$500,000	$500,000
G2:	0	$2,500,000	$500,000
G3:	$500,000	$500,000	0
G4:	0	$2,500,000	0

In these reframed options, the prize depends on the draw of a ticket from a bag containing 100 numbered tickets. Given this reframing, Savage chose in accordance with the independence axiom and in a

consistently risk averse manner, expressing a preference for G1 over G2 and for G3 over G4. Savage felt that his revision of the latter choice, brought about by this new frame, had "corrected an error" (p. 103).

Three decades later, Tversky and Kahneman (1986) presented additional and, perhaps, even more compelling evidence of decision errors that are sensitive to framing. They report that all 88 of their experimental subjects chose B over A given the choice shown below, in which probabilities of outcomes in each option are expressed as percentages of marbles of different colors in a box governing each option:

Option A	*90% white* *$0*	*6% red* *win $45*	*1% green* *win $30*	*1% blue* *lose $15*	*2% yellow* *lose $15*
Option B	*90% white* *$0*	*6% red* *win $45*	*1% green* *win $45*	*1% blue* *lose $10*	*2% yellow* *lose $15*

Option B stochastically dominates A since it offers a 1% chance of a larger gain ($45 versus $30) and a 1% chance of a smaller loss (-$10 versus -$15). However, they observed that a majority (58%) of subjects chose C over D with the alternatives presented differently, as below:

Option C	*90% white* *$0*	*6% red* *win $45*	*1% green* *win $30*	*3% yellow* *lose $15*
Option D	*90% white* *$0*	*7% red* *win $45*	*1% green* *lose $10*	*2% yellow* *lose $15*

The choice of C versus D reframes that between A and B. Option C is obtained from A by combining the 1% blue and 2% yellow chances of losing $15 while D is obtained from B by combining the 6% red and 1% green chances of winning $45.[1] This produces an economical or 'minimal' presentation in the sense that the matrix has fewer columns; yet this presentation masks the dominance relation and, moreover, juxtaposes the 1% chance of $30 against the 1% chance of -$10, which now drives the choice.

Intuitively the dominance relationship is 'more transparent' in the choice between options A and B. However, in addition to such intuitions concerning relative transparency—as held by Savage (1954) and Tversky and Kahneman (1986)—is there also a *logic of transparency* that can be formalized and generalized to a wider class of decision problems? Commenting on the Tversky and Kahneman (1986) paper, Hogarth and Reder (1986) note that "Tversky and Kahneman do not specify the conditions under which people perceive problems as transparent or opaque" (p. S192). This issue has apparently not been addressed in the subsequent literature: there is no general theory of 'transparency' of choice presentations.

We offer precise definitions of *presentations* or *frames* for choice under risk and over time, propose a property list for *transparent* frames, and show that these properties imply unique presentations of choice problems. We also define a *minimal* frame and identify it with many standard presentations of choice

[1] Birnbaum and Navarrette (1998) and Luce (1998) refer to this combining of the probabilities of identical outcomes as "coalescing." Earlier, Starmer and Sugden (1993) had called the opposite operation "event-splitting."

problems. We then apply a model of salience-based choice to derive behavioral predictions in minimal and transparent frames, and test for these predicted choice differences in a new experiment involving choice under risk. Finally, we extend the model to choice under ambiguity, where we derive the novel prediction that ambiguity aversion is frame-dependent.

2. Presentations or Frames

Figure 1. Presentations or Frames for Decisions under Risk and over Time

Choice Frame for Lotteries

	$(\mathbf{x_1},\mathbf{y_1})$	$(\mathbf{p_1},\mathbf{q_1})$	$(\mathbf{x_2},\mathbf{y_2})$	$(\mathbf{p_2},\mathbf{q_2})$		$(\mathbf{x_i},\mathbf{y_i})$	$(\mathbf{p_i},\mathbf{q_i})$		$(\mathbf{x_n},\mathbf{y_n})$	$(\mathbf{p_n},\mathbf{q_n})$
p	$\mathbf{x_1}$	$\mathbf{p_1}$	$\mathbf{x_2}$	$\mathbf{p_2}$	…	$\mathbf{x_i}$	$\mathbf{p_i}$	…	$\mathbf{x_n}$	$\mathbf{p_n}$
q	$\mathbf{y_1}$	$\mathbf{q_1}$	$\mathbf{y_2}$	$\mathbf{q_2}$	…	$\mathbf{y_i}$	$\mathbf{q_i}$	…	$\mathbf{y_n}$	$\mathbf{q_n}$

Choice Frame for Income Streams

	$(\mathbf{x_1},\mathbf{y_1})$	$(\mathbf{r_1},\mathbf{t_1})$	$(\mathbf{x_2},\mathbf{y_2})$	$(\mathbf{r_2},\mathbf{t_2})$		$(\mathbf{x_i},\mathbf{y_i})$	$(\mathbf{r_i},\mathbf{t_i})$		$(\mathbf{x_n},\mathbf{y_n})$	$(\mathbf{r_n},\mathbf{t_n})$
r	$\mathbf{x_1}$	$\mathbf{r_1}$	$\mathbf{x_2}$	$\mathbf{r_2}$	…	$\mathbf{x_i}$	$\mathbf{r_i}$	…	$\mathbf{x_n}$	$\mathbf{r_n}$
t	$\mathbf{y_1}$	$\mathbf{t_1}$	$\mathbf{y_2}$	$\mathbf{t_2}$	…	$\mathbf{y_i}$	$\mathbf{t_i}$	…	$\mathbf{y_n}$	$\mathbf{t_n}$

2.1 Lotteries, Income Streams and their Frames

Frames present pairs of options as shown above in Figure 1. Let X be a finite set of potential outcomes. A *lottery* is a mapping $p: X \rightarrow [0,1]$ such that $\sum_{x\in X} p(x) = 1$, and $\Delta(X)$ is the set of all such lotteries. Now consider a pair of one-dimensional finite arrays **p** and **q,** representing lotteries p and q and offering a finite and equal number of outcomes denoted $\mathbf{x_i}$ and $\mathbf{y_i}$, $i = 1,2, \dots, n,$ where each $\mathbf{x_i}$ occurs with probability $\mathbf{p_i}$ and each $\mathbf{y_i}$ occurs with probability $\mathbf{q_i}$: The top panel of Figure 1 illustrates the pair of arrays. We call $[\![\mathbf{p},\mathbf{q}]\!]$ a *frame* or *presentation* of lottery pair $\{p,q\}$, and say that $[\![\mathbf{p},\mathbf{q}]\!]$ *presents* $\{p,q\}$, if and only if $p(x) = \sum_{\{i\ |\mathbf{x_i}=x\}} \mathbf{p_i}$ and $q(y) = \sum_{\{i\ |\ \mathbf{y_i}=y\}} \mathbf{q_i}$. Let $supp(p)$ denote the support of p (the set of outcomes such that $p(x) > 0$). Note that n may exceed $|supp(p)|$ and/or $|supp(q)|$, where $|supp(p)|$ denotes the number of outcomes in a support: This is a key difference between frames $[\![\mathbf{p},\mathbf{q}]\!]$ and the pairs $\{p,q\}$ they present.

For decisions over discrete time periods $i \in \{0, 1, 2, \dots, T\}$, intertemporal *income streams r* and *t* are finite sequences of outcomes, each assigning one outcome to each period. Denote the set of income streams by C. We also study choices between income streams r and t, where $r := (x_0, x_1, \dots, x_T)$ and $t := (y_0, y_1, \dots, y_T)$. Here, a frame $[\![\mathbf{r},\mathbf{t}]\!]$ is again a pair of one-dimensional finite arrays **r** and **t** representing income streams r and t and offering a finite and equal number of outcomes, where each outcome $\mathbf{x_i}$ occurs in time period $\mathbf{r_i}$ and each $\mathbf{y_i}$ occurs in period $\mathbf{t_i}$: The bottom panel of Figure 1 illustrates such a pair of arrays. Bold fonts denote attributes as presented in a frame, while italicized fonts denote the underlying lotteries, income streams and the attributes in their supports.

2.2 Defining Minimal Frames and Transparent Frames

We provide here an intuitive treatment of minimal and transparent frames (our appendix gives formal definitions and uniqueness demonstrations). *Transparent frames* are reminiscent of Savage's (1954) state matrix representation of lotteries, but assume neither statistical independence nor correlation between payoffs. *Minimal frames* are compact presentations of choices and, for choice under risk, formalize the 'prospect' presentation of lotteries pioneered by Kahneman and Tversky (1979). Minimal frames for choice under risk contain no redundancy: the same outcome does not appear in the representation of a lottery more than once. Similarly, in minimal frames for choice over time, any time period appears just once in presentations of income streams, and the frame contains the fewest columns necessary to present (all of the non-zero payoffs in) the income streams.

Though both Savage (1954) and Tversky and Kahneman (1986) argued that people are more likely to make better decisions when choices are presented in a more transparent format, it appears that no prior work precisely articulated what properties would characterize a transparent presentation of decisions. To address this gap, we propose that a transparent frame for lotteries satisfies properties 1 – 5:

1. Common Consequence Separation: Identify the common consequences (the payoff-probability pairs that contain the same outcomes and corresponding probabilities) of the lotteries being compared and separate them from distinct consequences (the other payoff-probability pairs in the frame).

2. Monotonicity: Order the outcomes of distinct consequences in decreasing order such that the i^{th} best outcome in one lottery is in the same column as the i^{th} best outcome in the other lottery.

3. Alignment: Set the probabilities within each column equal to each other.

4. Completeness: Ensure that the probabilities for each row in the frame sum to 1.

5. Relevance: Ensure the probabilities in each column vector are positive.

Our appendix provides an algorithm for constructing a unique transparent frame satisfying these five properties for any lottery pair (including those with different numbers of outcomes in their support).

The five properties serve as heuristics for framing a decision to help articulate and focus on the relevant information: Together they simplify comparison of alternatives. The properties also have intuitive appeal. The common consequence separation property isolates shared payoff-probability pairs in lotteries, thus focusing attention on how the lotteries differ. Monotonicity ensures that the best payoffs in one lottery are compared to the best payoffs in another lottery. The alignment property sets the probabilities equal within each column vector in a frame so that when comparing any payoff-probability pair in the same column, one need only compare the two payoffs. The completeness property ensures that all outcomes in the support of a lottery are included so that all information about the choice alternatives is accounted for in the decision. The relevance property ensures that no irrelevant outcomes (those with probability zero) are considered. The properties may help a person focus on the tradeoffs needed to make a quality decision.

For decisions over time, we propose similar properties that a transparent frame should satisfy:

1. Common Consequence Separation: Identify the common consequences (the payoff-time pairs that contain the same outcomes and the same corresponding delays) of the income streams being compared and separate them from distinct consequences.

2. Monotonicity: Order the timing of distinct consequences in strictly increasing order such that the i^{th} soonest period in one income stream is in the same column as the i^{th} soonest period in the other.

3. Alignment: Set the time periods within each column equal to each other.

4. Completeness: Ensure that all time periods indexed by the income streams are included.

5. Relevance: Ensure that only time periods indexed by the income streams are included.

These five properties also have intuitive appeal and justification. Common consequence separation puts attention on where the two alternatives differ. Monotonicity reflects the intuition that time has a natural forward direction and it may help one to consider time periods sequentially. The alignment property ensures that the time periods within each column of a frame are the same, standardizing outcomes within each column to have the same 'time value of money'. Alignment also enables a decision maker who is comparing two payoff-time pairs in a given pair of columns to focus on the differences in payoffs, rather than trading off both risk and time within those columns. Completeness ensures that all relevant time periods and payoffs are considered. The relevance property ensures that only relevant time periods and payoffs are considered. In particular, it encourages the decision maker to be forward looking as it does not display sunk costs (e.g., from previous income outcomes) that occurred prior to the dates indexed by the income streams. We show in the appendix that for any pair of income streams there is a unique frame satisfying these five properties.

3. Salience Weighted Utility over Presentations

Leland and Schneider's (2017) Salience Weighted Utility over Presentations (SWUP) is a simple decision model that operates on frames as we define them here.[2] SWUP predicts differences between choice behavior for minimal and transparent frames, so we now briefly motivate and review the SWUP model.

3.1 Salience Weighted Utility of Presentations for Choice under Risk

In a standard expected utility model of choice under risk, p is chosen over q if and only if (1) holds:

$$(1) \qquad \sum_{x \in X} p(x)u(x) > \sum_{y \in X} q(y)u(y),$$

where $u(x)$ is a utility function denoting payoffs to the decision maker from outcomes x.[3] Leland and Schneider (2017) consider choices over frames as in the top panel of Figure 1, where (1) can be written equivalently as (2). (We use bold font to denote outcomes and probabilities in a frame):

[2] Leland and Schneider (2017) focus entirely on developing SWUP, the salience-based decision model for frames, but do not develop the predictions of this model for different frames (nor test these) as we do here.

[3] All of our theoretical results hold when $u(x) = x$. However we leave open the possibility that $u(x)$ is concave for gains (as in standard risk-averse EU), or additionally convex for losses and exhibiting loss aversion (as in Wakker and Tversky's nonstandard Sign-Dependent Expected Utility or SDEU).

(2) $\sum_{i=1}^{n} \mathbf{p_i} u(\mathbf{x_i}) > \sum_{i=1}^{n} \mathbf{q_i} u(\mathbf{y_i})$,

Inequality (1) pertains to choices over lotteries p and q (regardless of how they are framed), whereas (2) pertains to choices over a particular frame $[\![\mathbf{p}, \mathbf{q}]\!]$ presenting lotteries p and q. Note that (1) and (2) provide an *alternative-based evaluation* - one lottery is strictly preferred to another, if and only if it yields a greater expected payoff to the decision maker.

Building on Leland and Sileo (1998), the alternative-based evaluation in (2) may be rewritten equivalently as an *attribute-based evaluation* such that **p** is chose over **q** if and only if (3) holds:

(3) $\sum_{i=1}^{n}[(\mathbf{p_i} - \mathbf{q_i})(u(\mathbf{x_i}) + u(\mathbf{y_i}))/2 + (u(\mathbf{x_i}) - u(\mathbf{y_i}))(\mathbf{p_i} + \mathbf{q_i})/2] > 0$.

Note that (2) and (3) operate over frames rather than over lotteries directly. Leland and Schneider (2017) then allow for the possibility that the agent systematically overweights salient differences in probabilities and payoffs by introducing *salience weights* $\phi(\mathbf{p_i}, \mathbf{q_i})$ on probability differences and $\mu(\mathbf{x_i}, \mathbf{y_i})$ on payoff differences. This yields Leland and Schneider's Salience-Weighted Utility over Presentations (SWUP) model of choice under risk, in which **p** is chosen over **q** if and only if

(4) $\sum_{i=1}^{n}[\phi(\mathbf{p_i}, \mathbf{q_i})(\mathbf{p_i} - \mathbf{q_i})(u(\mathbf{x_i}) + u(\mathbf{y_i}))/2 + \mu(\mathbf{x_i}, \mathbf{y_i})(u(\mathbf{x_i}) - u(\mathbf{y_i}))(\mathbf{p_i} + \mathbf{q_i})/2] > 0$.

3.2 Salience Weighted Utility for Choice over Time

The SWUP model extends to choices over time as shown in the lower panel of Figure 1. In the discounted utility model, a person always chooses income stream a over b if and only if (5) holds:

(5) $\sum_{i=0}^{T} \delta^{i} u(x_i) > \sum_{i=0}^{T} \delta^{i} u(y_i)$,

where δ is a constant discount factor. Through an analogous development to (4), Leland and Schneider (2017) propose (6) as the corresponding generalization of discounted utility theory to allow for overweighting salient differences in payoffs and time delays. Placing salience weights $\theta(\mathbf{r_i}, \mathbf{t_i})$ on time differences and $\mu(\mathbf{x_i}, \mathbf{y_i})$ on payoff differences gives this salience-weighted evaluation in which **r** is always chosen over **t** if and only if

(6) $\sum_{i}^{m}[\theta(\mathbf{r_i}, \mathbf{t_i})(\delta^{\mathbf{r_i}} - \delta^{\mathbf{t_i}})(u(\mathbf{x_i}) + u(\mathbf{y_i}))/2 + \mu(\mathbf{x_i}, \mathbf{y_i})(u(\mathbf{x_i}) - u(\mathbf{y_i}))(\delta^{\mathbf{r_i}} + \delta^{\mathbf{t_i}})/2] > 0$,

whenever the frame $[\![\mathbf{r}, \mathbf{t}]\!]$ presents income streams r and t.

3.3 Salience Weighted Utility for Choice under Ambiguity

Leland and Schneider (2017) introduced SWUP for choices under risk and over time. Here we extend SWUP to the domain of uncertainty, where probabilities of some events are unknown. Suppose there is a finite set of possible states of nature $s \in \{1,2, \dots, S\}$, where a lottery is assigned to be played in each state. Denote uncertain prospects by f and g, where f assigns lottery $f(s)$ to each state s and g assigns lottery $g(s)$ to each state s. In the classic alternative-based evaluation, there is assumed to be a unique subjective probability distribution, π_s, over states (Anscombe and Aumann, 1963) such that f is preferred over g if and only if (7) holds (where $f_s(x)$ is the probability of outcome x in state s):

(7) $\sum_{s\in S}\sum_{x\in X}\pi_s[f_s(x)u(x)] > \sum_{s\in S}\sum_{y\in X}\pi_s[g_s(y)u(y)]$.

Let there be a frame for each state, where is indexes the i[th] attribute in the frame in state s. Given two multi-dimensional arrays $\mathbf{f} = \{\mathbf{f^1}, \dots, \mathbf{f^S}\}$ and $\mathbf{g} = \{\mathbf{g^1}, \dots, \mathbf{g^S}\}$, the analogous formula to (5) is **f** is always chosen over **g** if and only if (8) holds:

(8) $\sum_s^S \sum_i^{n(s)} \pi_s[\mathbf{f_{is}}u(\mathbf{x_{is}})] > \sum_s^S \sum_i^{n(s)} \pi_s[\mathbf{g_{is}}u(\mathbf{y_{is}})]$.

Next, we introduce the corresponding model of salience-weighted evaluation in which **f** is always chosen over **g** if and only if (9) holds:

(9) $\sum_s^S \sum_{i=1}^{n} \pi_s[\phi(\mathbf{f_{is}}, \mathbf{g_{is}})(\mathbf{f_{is}} - \mathbf{g_{is}})(u(\mathbf{x_{is}}) + u(\mathbf{y_{is}}))/2$

$+ \mu(\mathbf{x_{is}}, \mathbf{y_{is}})(u(\mathbf{x_{is}}) - u(\mathbf{y_{is}}))(\mathbf{f_{is}} + \mathbf{g_{is}})/2] > 0.$

We refer to agents who choose according to salience-based evaluation models (the representations 4, 6, and 9) as *focal thinkers* since such agents focus on salient differences in attributes. Such an agent chooses the alternative which 'looks better' with respect to that agent's perceptual system.

3.4 Properties of Salience Perceptions

The salience functions μ, ϕ and θ determine the only ways in which the behavior of a focal thinker differs from a rational agent who chooses according to formulas 1, 5 and 7. We assume a salience function exhibits the two properties of the perceptual system in Definition 1, introduced by Bordalo et al. (2012).

Definition 1 (Salience Function): A *salience function* $\sigma(\mathbf{a_i}, \mathbf{b_i})$ is any continuous, non-negative and symmetric ($\sigma(\mathbf{a_i}, \mathbf{b_i}) = \sigma(\mathbf{b_i}, \mathbf{a_i})$) function of $\mathbf{a_i}$ and $\mathbf{b_i} \in \mathbb{R}$ that satisfies the following two properties:

1. Ordering: If $[\mathbf{a_i}', \mathbf{b_i}'] \subset [\mathbf{a_i}, \mathbf{b_i}]$ then $\sigma(\mathbf{a_i}', \mathbf{b_i}') < \sigma(\mathbf{a_i}, \mathbf{b_i})$.

2. Diminishing Absolute Sensitivity (DAS): $\sigma(.)$ exhibits diminishing sensitivity if for any $\mathbf{a_i}, \mathbf{b_i}, \epsilon > 0$, $\sigma(\mathbf{a_i} + \epsilon, \mathbf{b_i} + \epsilon) < \sigma(\mathbf{a_i}, \mathbf{b_i})$.

In addition to properties 1 and 2 from Bordalo et al. (2012), we will allow for the possibility that a salience function satisfies a third property, increasing proportional sensitivity:

3. Increasing Proportional Sensitivity (IPS): $\sigma(.)$ exhibits increasing proportional sensitivity if for any $\mathbf{a_i}\mathbf{b_i} > 0$ and any $\alpha > 1$, $\sigma(\alpha\mathbf{a_i}, \alpha\mathbf{b_i}) > \sigma(\mathbf{a_i}, \mathbf{b_i})$.

There is a close relationship between DAS and IPS: DAS implies that for a fixed absolute difference, the perceptual system is more sensitive to larger ratios, while IPS implies that for a fixed ratio, the perceptual system is more sensitive to larger absolute differences. We will show that for focal thinkers, IPS for ϕ implies the general version of the Allais common ratio effect, and DAS for ϕ implies ambiguity aversion in Ellsberg's paradox in minimal frame presentations. Our approach thus provides a unified treatment of Allais-style and Ellsberg-style behavior and shows that they can be derived from basic properties of the probability salience function ϕ without requiring any parametric assumptions about the underlying salience functions or utility functions.

4. Minimal and Transparent Frames for Choice under Risk

We now consider what SWUP implies when alternatives are presented in a minimal versus a transparent frame, beginning with choices under risk. For frames with degenerate lotteries (those yielding a single outcome with probability 1), it seems almost unavoidable that one compares each outcome in the non-degenerate lottery **q** to the unique outcome in the degenerate lottery **p**. So when one lottery in a pair is degenerate, we adopt the convention that monotone minimal frames appear as shown in Figure 2.

Figure 2. Choice Frame with a Degenerate Lottery p

	(x_1, y_1)	(p_1, q_1)	(x_2, y_2)	(p_2, q_2)	…	(x_i, y_i)	(p_i, q_i)	…	(x_n, y_n)	(p_n, q_n)
p	**x**	q_1	**x**	q_2	…	**x**	q_i	…	**x**	q_n
q	y_1	q_1	y_2	q_2	…	y_i	q_i	…	y_n	q_n

4.1 The Stochastic Dominance Axiom in Minimal and Transparent Frames

One of the most basic axioms of rational choice is consistency with first-order stochastic dominance: If a lottery p offers at least as good an outcome at every probability increment as a lottery q and p offers a strictly better outcome at some probability increment, then p stochastically dominates q. Consider again the example due to Tversky and Kahneman (1986) from Section 2, shown in Figure 3.

Figure 3. Stochastic Dominance in Minimal and Transparent Frames

Stochastic Dominance in Minimal Frames

	x_1,y_1	p_1,q_1	x_2,y_2	p_2,q_2	x_3,y_3	p_3,q_3	x_4,y_4	p_4,q_4
p'	0	0.90	45	0.07	-10	0.01	-15	0.02
$\mathbf{q'}$	**0**	**0.90**	**45**	**0.06**	**30**	**0.01**	**-15**	**0.03**

Stochastic Dominance in Transparent Frames

	x_1,y_1	p_1,q_1	x_2,y_2	p_2,q_2	x_3,y_3	p_3,q_3	x_4,y_4	p_4,q_4	x_5,y_5	p_5,q_5
p	**45**	**0.01**	**-10**	**0.01**	**45**	**0.06**	**0**	**0.90**	**-15**	**0.02**
q	30	0.01	-15	0.01	45	0.06	0	0.90	-15	0.02

Given a transparent presentation of lotteries p and q, all subjects chose the stochastically dominant alternative, p (Tversky and Kahneman, 1986). However, when presented in a minimal frame as p' and q' many subjects violated dominance. Similar dominance violations have also been observed in several studies by Birnbaum and his colleagues (e.g., Birnbaum and Navarette, 1998; Birnbaum, 1999). For transparent frames, a focal thinker will satisfy stochastic dominance. Since payoffs are ordered monotonically, any differences in (3) will favor the stochastically dominant lottery.

4.2 The Independence Axiom in Minimal and Transparent Frames

The Allais common consequence paradox (Allais, 1953) involves choices like those in the left panel of Figure 4. A decision maker chooses between lottery q, offering \$2400 with certainty and a lottery p, offering a 33% chance of \$2500, a 66% chance of \$2400, and a 1% chance of \$0. The decision maker then

chooses between lottery $\tilde{q}$ offering a 34% chance of $2400 and lottery $\tilde{p}$ offering a 33% chance of $2500. In such choices, most subjects choose q over p and choose $\tilde{p}$ over $\tilde{q}$ (Kahneman and Tversky, 1979). This preference pattern is inconsistent with EU which predicts preferences of either p and $\tilde{p}$ or q and $\tilde{q}$.

Figure 4. The Allais Paradox in Minimal and Transparent Frames

Allais Paradox in Minimal Frames

	x_1,y_1	p_1,q_1	x_2,y_2	p_2,q_2	x_3,y_3	p_3,q_3
p	2500	0.33	2400	0.66	0	0.01
q	**2400**	**0.33**	**2400**	**0.66**	**2400**	**0.01**

	x_1,y_1	p_1,q_1	x_2,y_2	p_2,q_2
$\tilde{p}$	**2500**	**0.33**	**0**	**0.67**
$\tilde{q}$	2400	0.34	0	0.66

Allais Paradox in Transparent Frames

	x_1,y_1	p_1,q_1	x_2,y_2	p_2,q_2	x_3,y_3	p_3,q_3
p	2500	0.33	0	0.01	2400	0.66
q	**2400**	**0.33**	**2400**	**0.01**	**2400**	**0.66**

	x_1,y_1	p_1,q_1	x_2,y_2	p_2,q_2	x_3,y_3	p_3,q_3
$\tilde{p}$	2500	0.33	0	0.01	0	0.66
$\tilde{q}$	**2400**	**0.33**	**2400**	**0.01**	**0**	**0.66**

In the choice between p and q in minimal frames, the comparison of 2400 and 0 is more salient than that of 2500 and 2400, which favors q. However, in the choice between $\tilde{p}$ and $\tilde{q}$, the comparison between 2400 and 0 is not cued. Instead, the decision maker compares the upside of winning 2500 instead of 2400 with the downside of forfeiting a 1% chance in the probability of winning. To the extent this $100 difference is more salient than the 0.01 difference in probabilities, the decision maker chooses $\tilde{p}$ over $\tilde{q}$. Now consider the transparent frames in Figure 4. Here, the components common to each decision (i.e., ($2400, 0.66) in the choice between p and q and ($0, 0.66) in the choice between $\tilde{p}$ and $\tilde{q}$) are isolated and the decision in both cases depends on comparisons between 2500 and 2400 and between 2400 and 0.

Many scholars replicated Allais' (1953) demonstration of common consequence violations in minimal frames, but there is only mixed evidence regarding the prediction that common consequence effects will not occur in transparent frames. Moskowitz (1974) examined common consequence choice pairs presented either in a minimal written form as in the introduction, as tree diagrams, or as transparent matrices like the one proposed by Savage and shown in the introduction. Contrary to our predictions, there were no differences in the proportion of consistent responses among subjects given written or matrix presentations, although both were higher than for tree diagrams. However, Keller (1985), Incekara-Hafalir and Stecher (2012) and Harman and Gonzalez (2015) all find that more transparent presentations reduce the occurrence of common consequence violations. In their summary, Incekara-Hafalir and Stecher say, "We find that given a transparent presentation, expected utility theory performs surprisingly well, and that the alternative theories perform poorly except inasmuch as they make the same predictions as expected utility theory."

Allais' (1953) common ratio effect is a second well-known violation of EU. Figure 5 displays a classic version due to Kahneman and Tversky (1979). The minimal frames display a choice between lotteries p and q, offering an 80% chance of $4000 versus $3000 with certainty, and a choice between $\tilde{p}$ and $\tilde{q}$, offering a 20% chance of $4000 versus a 25% chance of $3000. In this example, a majority of subjects chose q over

p and chose $\tilde{p}$ over $\tilde{q}$ when the choices were presented in minimal frames. This response pattern violates EU which predicts choices of either p and $\tilde{p}$ or q and $\tilde{q}$. In the transparent frames in Figure 5, the salient comparisons in both choices are between 3000 and 0 and between 4000 and 3000. Since the same comparisons are focal in both choices, one might predict more consistent behavior in transparent frames.

Figure 5. The Common Ratio Effect Presented in Minimal and Transparent Frames

In Minimal Frames

	x_1,y_1	p_1,q_1	x_2,y_2	p_2,q_2
p	4000	0.80	0	0.20
q	**3000**	**0.80**	**3000**	**0.20**

	x_1,y_1	p_1,q_1	x_2,y_2	p_2,q_2
$\tilde{p}$	**4000**	**0.20**	**0**	**0.80**
$\tilde{q}$	3000	0.25	0	0.75

In Transparent Frames

	x_1,y_1	p_1,q_1	x_2,y_2	p_2,q_2
p	4000	0.80	0	0.20
q	**3000**	**0.80**	**3000**	**0.20**

	x_1,y_1	p_1,q_1	x_2,y_2	p_2,q_2	x_3,y_3	p_3,q_3
$\tilde{p}$	4000	0.20	0	0.05	0	0.75
$\tilde{q}$	**3000**	**0.20**	**3000**	**0.05**	**0**	**0.75**

Allais (1953) demonstrated the common ratio effect for choices presented in minimal frames. Harless (1992) showed that minimal presentations of common ratio choices violate independence while transparent frames do not. He presented different groups of subjects with choices such as a choice between S:($8,000, 0.10; $0, 0.90) and R:($20,000, 0.05; $0, 0.95) and one between S′:($8,000, 0.60; $0, 0.40) and R′:($20,000, 0.30; $0, 0.70). One group received both choices in a transparent matrix format in the top of Figure 6. Another group was presented with choices in a minimal 'tickets' frame in the bottom of Figure 6. As predicted by SWUP, Harless found that deviations from EU were unsystematic in the transparent frame and systematic and in the predicted Allais direction in the minimal frame. Harless also examined how different juxtapositions of payoffs across choices would influence the incidence and direction of common ratio violations in matrix frames. Those results are also consistent with the predictions of SWUP.[4] Keller (1985) also found that proportional matrix presentations reduce the occurrence of common ratio violations; yet

Figure 6. Transparent Matrix and Minimal 'Tickets' Presentations used by Harless (1992)

Transparent Tickets Matrix Presentation

	1-5	6-10	11-100
S	$8	$8	$0
R	$20	$0	$0

	1-30	31-60	61-100
S′	$8	$8	$0
R′	$20	$0	$0

Minimal Tickets Presentation

S	Ticket 1-10 drawn	Win $8
	Ticket 11-100 drawn	Win $0
R	Ticket 1-5 drawn	Win $20
	Ticket 6-100 drawn	Win $0

S′	Ticket 1-60 drawn	Win $8
	Ticket 61-100 drawn	Win $0
R′	Ticket 1-30 drawn	Win $20
	Ticket 31-100 drawn	Win $0

[4] Loomes and Sudgen's (1982) Regret theory and Bordalo et alia's (2012) salience based model of risky choice also predict many of these effects but only for statistically independent lotteries. Leland (1998) shows the behavior obtains for statistically dependent lotteries as well.

both Loomes and Sugden (1987) and Keller still report many common ratio violations of independence (with state matrix and proportional matrix presentations, respectively).

Starmer and Sugden (1993) and Humphrey (1995) studied another presentation effect known as event splitting: Presenting the same event split into many separate ones (for gains) makes that event more attractive. SWUP explains event splitting effects *when* splitting an event transforms a presentation from a minimal to a transparent frame, but *not* when the probabilities within each column vector of a frame are aligned in both presentations (so that salient payoff comparisons are the same in original and split frames).

5. An Experiment on Framing and Decisions under Risk

Our experiment compares the predictions of three models – the leading normative model of decision making (EU), cumulative prospect theory (CPT) due to Tversky and Kahneman (1992), and the model of salience weighted utility over presentations (SWUP). Since the outcomes in our experiment involve only gains, the predictions of CPT coincide with those of rank-dependent utility (RDU) due to Quiggin (1982). We use SWUP instead of the salience-based model in Bordalo et al. (2012) since our focus is on framing effects. The model in Bordalo et al. (2012) does not predict framing effects between minimal and transparent frames, but instead predicts choices are sensitive to correlations between lotteries.

We test general properties of the models. Choices satisfy stochastic dominance, independence and the property called frame invariance (different presentations of the same two lotteries will produce the same observed choices) under EU. RDU violates the EU independence axiom (allowing Allais paradoxes), but satisfies both stochastic dominance and frame-invariance. SWUP violates frame invariance, and makes the strong prediction that the independence axiom and stochastic dominance will be violated in minimal frames, but will be satisfied in transparent frames. However, we designed our experiment with a secondary purpose in mind: to support estimation of parametric probabilistic choice versions of EU, RDU and SWUP.

5.1 Experimental Procedure

Our 137 experimental subjects[5] were undergraduate students at a university in the western United States in April and May of 2016. We seated subjects at visually isolated computer terminals in lab cubicles. Each subject chose one lottery from a lottery pair (no indifference permitted) for 100 distinct pairs presented sequentially one at a time on a computer screen. After completing their 100 choices, each subject rolled a pair of ten-sided dice, randomly selecting one of their 100 chosen lotteries to count for payment: The subject then played out that lottery by selecting a numbered raffle ticket from an opaque bag, receiving a cash outcome to keep along with a promised flat $7.00 for timely arrival and participation.

Figure 7 shows how a pair appeared on subjects' computer screens (minimal and transparent versions are shown, though only one would appear on any screen). If a subject chose the 'red' lottery (and this choice

[5] We recruited and scheduled 144 students for six sessions of twenty-four subjects each, but seven failed to appear bringing the actual total to 137 subjects.

was randomly selected for payment), that subject would draw a ticket from an opaque bag containing 100 red raffle tickets. If the number on the ticket was between 1 and 25, that subject received \$30. If the number was between 26 and 100, that subject received \$0[6]. Computerized instructions (screen prints appear in the supplementary materials or SM) explained all this generally and with specific examples, following up with tests of understanding (which a subject had to correctly answer before proceeding).

Figure 7. A Choice Pair in a Minimal Frame (top) and a Transparent Frame (bottom)

Select One	$	Tickets	$	Tickets
Red	$30	25	$0	75
Blue	$40	20	$0	80

Select One	$	Tickets	$	Tickets	$	Tickets
Red	$40	20	$0	5	$0	75
Blue	$30	20	$30	5	$0	75

5.2 Experimental Design

Of the 100 lottery pairs, 40 are *test* pairs (shown in our SM) for hypothesis tests within and between transparent and minimal framed pairs, while 60 are *estimation* pairs to aid efficient estimation of structural model parameters. We present lottery pairs in blocked order (randomizing order within each block of ten pairs) to space related test pairs (also utilizing estimation pairs for this spacing purpose). Of the 40 test pairs, 18 test for common ratio effects (in two groups CR.A and CR.B of nine pairs each), 16 test for Allais' Paradox (common consequence effects) in four groups AP.A, AP.B, AP.C, and AP.D of four pairs each, and 6 pairs test for dominance violation effects (in three groups DV.A, DV.B and DV.C of two pairs each).

Table 1 shows predictions made by EU, CPT/RDU, and SWUP in our design. Under EU we should observe none of the usual regularities and frame invariance of choices. While CPT/RDU permit common ratio effects and Allais' Paradox, they rule out dominance violations and also imply frame invariance. SWUP predicts that none of the regularities are observed in transparent frames, but that all of them can be

Table 1. Predictions of Different Models for Minimal and Transparent Frames

	Common Ratio Effect		Allais Paradox		Dominance Violation	
Model	Minimal	Transparent	Minimal	Transparent	Minimal	Transparent
EU	*	*	*	*	*	*
CPT/RDU	✓	✓	✓	✓	*	*
SWUP	✓	*	✓	*	✓	*

Notes: ✓ indicates behavior is predicted; * indicates behavior is not predicted.

[6] Another bag of blue tickets was used to resolve "blue" lotteries. Using two separate devices for each lottery in a pair induces statistically independent lotteries as opposed to statistically dependent ones. While SWUP is insensitive to this distinction, other theories are not: We wished to preserve interpretability relative to these other theories.

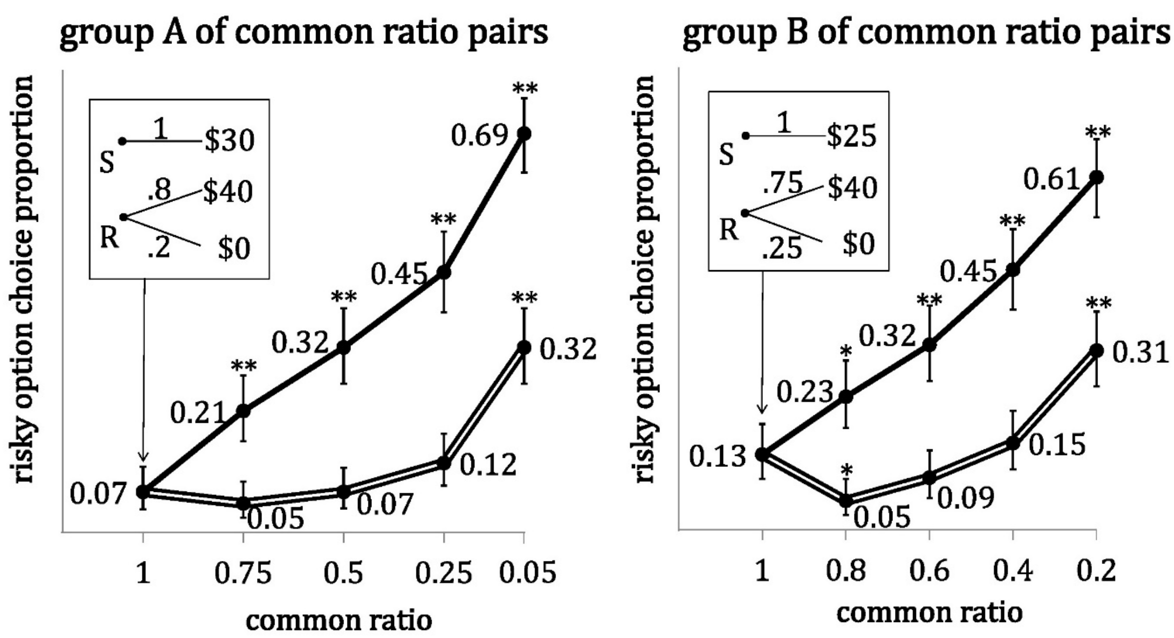
observed in minimal frames (and hence does not predict frame invariance). For instance, even with linear utility, SWUP explain the classical versions of the common consequence effect and common ratio effect in Kahneman and Tversky (1979) and the violation of stochastic dominance in Tversky and Kahneman (1986) in minimal frames, but SWUP satisfies independence and stochastic dominance in transparent frames.

5.3 Results: Hypothesis Tests

Experimental results in the two common ratio groups of pairs appear in the two panels of Figure 8. The inset in each panel is a prospect presentation of the "root pair" {S,R} generating each group of common ratio pairs. Because the safe lottery S is degenerate in each root pair, minimal and transparent presentations of the root pairs are identical: Hence in each panel, only one choice proportion is graphed above the common ratio of 1 (the root pair). However, minimal and transparent presentations differ for any pairs {S',R'} with common ratio less than 1 (the other pairs in each group), so two choice proportions are graphed above those common ratios. The solid (double) line connects minimal (transparent) frame observed risky lottery choice proportions within each group. We also show a Bayesian 90% confidence interval (based on the Jeffreys Prior as recommended by Brown, Cai and DasGupta 2001) for each proportion, and these illustrate two findings: Frame invariance dramatically fails in these common ratio groups; and relative to minimal framing, transparent framing strongly promotes choice of safe lotteries.

Under Conlisk's (1989) constant error model and EU, proportions of non-EU choice patterns $S \cup R'$ and $R \cup S'$ (involving the root pair and any other pair in a group) should be equivalent: Asterisks above each confidence interval in Figure 8 indicate rejection of this hypothesis (* at 5%, and ** at 1%) by a likelihood ratio test based on Conlisk's constant error model. Such rejections are universal in minimal

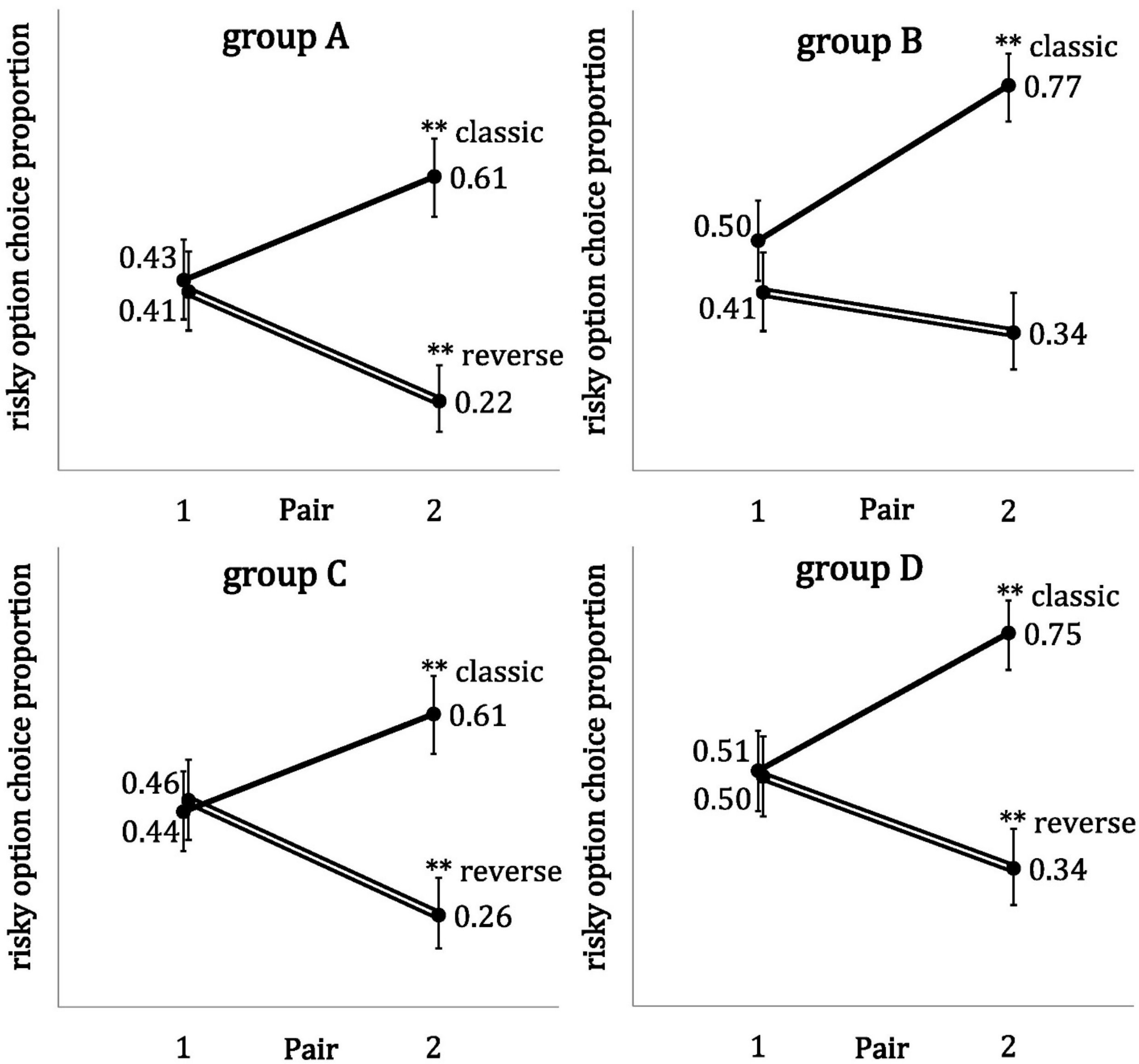
Figure 8: Common Ratio Effect in Minimal and Transparent Frames



frames and, moreover, observed minimal frame choice proportions display a characteristic crossing of the 0.5 Rubicon as the common ratio gets low enough. Neither result obtains in transparent frames: Risky choice proportions rise significantly at the lowest common ratios only, and never exceed 0.5—in keeping with simple strength-of-preference explanations (and so not clear evidence of an independence violation).

Experimental results in the four Allais Paradox (common consequence) groups of pairs appear in the four panels of Figure 9. Each group has a pair 1 $\{S_1, R_1\}$ and a pair 2 $\{S_2, R_2\}$ formed from pair 1 by changing a common consequence to zero. We presented these pairs in both minimal and transparent frames, so two choice proportions are graphed above each pair. As in Figure 8, the solid (double) line connects minimal (transparent) frame observed risky lottery choice proportions within each group, and we show the

Figure 9: Allais Paradox (Common Consequence Effect) in Minimal and Transparent Frames

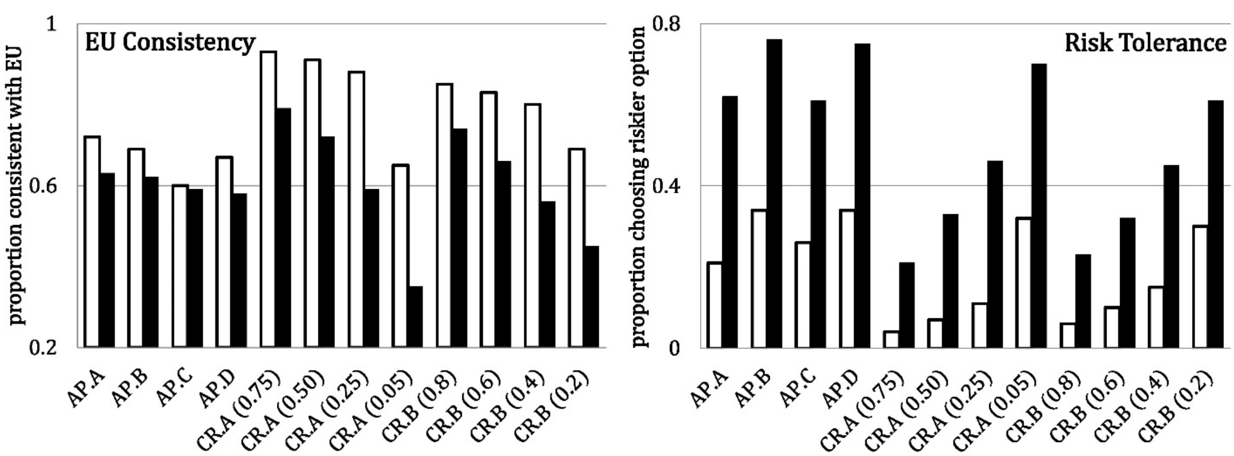


same type of Bayesian 90% confidence interval for each proportion. Note that in pair 1 of each common ratio group, the difference between the minimal and transparent frames of {S,R} is slight: SWUP predicts no difference between these choices,[7] and the data bear this out. However the pair 2 choice proportions always differ strongly across the two types of frames: as in the common ratio groups, transparent framing strongly promotes the choice of safe lotteries relative to minimal framing.

In these groups, under Conlisk's (1989) constant error model and EU, proportions of non-EU choice patterns $\mathrm{S}_1 \cup \mathrm{R}_2$ and $\mathrm{R}_1 \cup \mathrm{S}_2$ should be equivalent. Asterisks above each confidence interval in Figure 9 indicate rejection of this hypothesis (* at 5%, and ** at 1%) by a likelihood ratio test. Figure 9 shows that this hypothesis is almost always rejected in both minimal and transparent frames; but there is a strong difference in the direction of the rejection across these frame types. As predicted by both CPT/RDU and SWUP, we observe the pattern $\mathrm{S}_1 \cup \mathrm{R}_2$ significantly more than the pattern $\mathrm{R}_1 \cup \mathrm{S}_2$ in minimal frames: This is the 'classic' Allais result (and so labeled in Figure 9). However, significant 'reverse' Allais results appear in three of the four groups in transparent frames ($\mathrm{S}_1 \cup \mathrm{R}_2$ occurs significantly *less* often than $\mathrm{R}_1 \cup \mathrm{S}_2$) and no theory we know of predicts this. SWUP predicts that no significant Allais pattern (classic or reverse) should appear in transparent frames—as observed in group B, but not in groups A, C, and D.

Choice patterns $\mathrm{S}_1 \cup \mathrm{S}_2$ and $\mathrm{R}_1 \cup \mathrm{R}_2$ (in each Allais Paradox group) and choice patterns $\mathrm{S} \cup \mathrm{S}'$ and $\mathrm{R} \cup \mathrm{R}'$ (in each common ratio group) are consistent with EU. The left panel of Figure 10 shows how rates of EU consistency vary across transparent frames (the white bars) and minimal frames (the black bars). In all twelve pairs of pairs, EU consistency in transparent frames exceeds that in minimal frames, with nine of these twelve comparisons significant (at 10% or better) by likelihood ratio tests. The right panel of Figure 10 compares risk tolerance in minimal and transparent frames. In this case we use only data from the

Figure 10. EU Consistency and Risk Tolerance in Minimal and Transparent Frames



[7] Figure 4 illustrated this: There one can see that the top two framings $[\![\mathbf{p},\mathbf{q}]\!]$ of $\{p,q\}$ are identical *except* that the transparent frame isolates the common consequence column block at the right, while the monotone minimal frame places that column block at the center. Under SWUP such column block switching should have no effect on choice.

common ratio pairs $\{S', R'\}$ with common ratios less than 1 (where the minimal and transparent frames actually differ from one another) and pair 2 $\{S_2, R_2\}$ from each Allais Paradox group (recall that SWUP predicts no difference between the minimal and transparent framings of pair 1 in each common consequence group; see Figure 4 and fn. 5). Frame invariance (implied by EU and CPT/RDU) predicts that risk tolerance (R' or R_2 choice proportions) should not significantly differ between frames, while SWUP predicts greater risk tolerance in minimal frames. In all twelve comparisons (and significantly at 1% or better), risky choice proportions are greater in minimal than transparent frames: On average across these twelve pairs, risky choice proportions are 30.9 percentage points higher in minimal (than transparent) frames.

We expect that, as predicted by SWUP, dominance violations will be common in minimal frames (which is not predicted by EU or CPT/RDU) but very rare in transparent frames and our data bears this out. In transparent framings of the dominance violation pairs DV.A, DV.B and DV.C, dominance violations are just 3%, 2% and 3% of all choices, while in minimal framings dominance violations are 67%, 81% and 57% of all choices. This is a strong failure of frame invariance.

5.4 Results: Structural Parameter Estimates and Mixture Models

We also have subjects' choice data from the 60 estimation pairs, designed to better identify preference parameters than we could using only our 40 test pairs. We use 94 of the 100 total pairs[8] to estimate parametric probabilistic choice specifications of EU, RDU, and SWUP. We regard our sampled population as (possibly) consisting of subpopulations, each composed of either EU, or RDU, or SWUP *types*, and estimate seven specifications of such a population: three specifications that assume our sampled population is composed solely of one type (either EU, or RDU, or SWUP); three specifications that assume our sampled population is a mixture of just two of the three types; and a specification that assumes all three types exist in our sampled population. The specifications composed of just EU and SWUP types, or of just EU and RDU types, are mixtures of rational agents (EU) and behavioral agents (either SWUP or RDU). This interpretation of comparing EU and SWUP is further motivated by the observation that EU is grounded in basic normative axioms and SWUP is grounded in basic psychological principles of perception. The mixture model of RDU and SWUP can be interpreted as comparing agents who value alternatives in isolation with context-independent preferences (RDU) and agents who choose by comparing alternatives jointly and who are sensitive to context and framing (SWUP).

The specification containing all three types (EU, RDU, and SWUP) has a novel interpretation. Kahneman (2003) distinguishes three cognitive systems that he refers to as 'perception', 'System 1', and 'System 2'. We might conjecture that a different decision model is best suited for each system, given that

[8] We omit the six dominance violation pairs from the estimations for technical reasons we discuss in our SM. Including them (by any reasonable method skirting the technical difficulty) would simply improve the performance of SWUP relative to EU and/or RDU.

they engage in different processes. Since RDU allows for optimism and pessimism to influence decisions under risk, RDU seems a plausible candidate for a model of affective decision making. In addition, Barberis et al. (2013) have argued that prospect theory (which coincides with RDU in our experiment) is a natural model of System 1 decision making. To the extent that System 2 decision making resembles an unbiased rational agent, it seems plausible that EU is an appropriate model for System 2. Since SWUP explicitly models salience perception and visual changes in the framing of decisions, it seems plausible that SWUP captures some aspects of perception-based choice. The final mixture model combining all three theories thus allows for "System 2" agents (represented by EU), "System 1" agents (represented by RDU), and "Perceptual" agents (represented by SWUP). A similar interpretation holds even without a multi-system framework: People may decide based on different heuristics, where some people typically choose the lottery that 'looks better' (represented in our analysis by SWUP), others typically choose the lottery that 'feels better' (represented by their degree of optimism or pessimism as modeled by RDU), and that others choose the lottery that they can 'justify as better' through logical reasoning (represented by their conformance to EU). Given that choices may be arrived at by such diverse decision processes, it may be that a different decision model is best suited for each of these modes of decision making.

For each agent of each type, ρ is the degree of *utility* or *value* function curvature, very similar to a coefficient of relative risk aversion.[9] We allow subjects of all three types (EU, RDU and SWUP) to have their own personal ρ but we require that ρ has a normal distribution with mean and variance μ_ρ and σ_ρ (allowed to be specific to each type found in any mixture). The parameter λ is an inverse standard deviation of decision noise, frequently called *precision* or *sensitivity* in literature on probabilistic choice models. All three types (EU, RDU and SWUP) have the three parameters λ, μ_ρ and σ_ρ in their choice models. Agents of the RDU type additionally have a *weighting function* and we use a two-parameter (α and β) form due to Prelec (1998) for this weighting function.[10] The *mixture* parameters θ_{type} (such as θ_{eu}) are the proportion of the sampled population behaving according to each $type \in \{eu, rdu, swup\}$.[11]

Agents of the SWUP type additionally have two salience functions, one for payoffs and one for probabilities. To show the performance of SWUP in its parametrically leanest form, we use a parameter-free salience function. One alternative is a *BGS* (based on Bordalo, Gennaioli, and Shleifer 2012) salience function; whenever $\mathbf{a_i} > 0$ or $\mathbf{b_i} > 0$, this is

[9] The utility function is $u(z) = (1-\rho)^{-1}\left[-1 + (1+z)^{(1-\rho)}\right]$ for $z \geq 0$. It has two useful properties: (i) It is defined at $z = 0\ \forall \rho \in \mathbb{R}$, so that we can estimate specifications where ρ has a distribution on $\mathbb{R}$ (such as the normal distribution we employ); and (ii) $u(0) \equiv 0\ \forall \rho \in \mathbb{R}$ which is necessary for SWUP as well as RDU and CPT.
[10] This is $w(G) = \exp(-\beta(-\ln(G))^{\alpha})$, where G is the decumulative distribution function of a lottery.
[11] Our SM details the probabilistic choice model of each type.

(10) $\sigma(\mathbf{a_i}, \mathbf{b_i}) = \frac{|\mathbf{a_i} - \mathbf{b_i}|}{|\mathbf{a_i}| + |\mathbf{b_i}|}$, and $\sigma(0,0) = 0$.

Although simple and parameter-free, (10) does not satisfy IPS. Given behaviors such as the fourfold pattern of risk attitudes, and the simultaneous purchase of lottery tickets and insurance policies that follow from IPS (Leland and Schneider, 2017), along with the general form of the Allais common ratio effect, it seems desirable for a salience function to also satisfy that property. Therefore we introduce a new salience function satisfying IPS. To motivate this, consider the two frames in Figure 11: the salience function in (10) assigns the same salience value to the comparison between 300 and 100 in both frames ($\sigma(300,100) = 0.5$). But perhaps that comparison is more salient in the top frame in Figure 11 (in which other payoffs are in the hundreds of dollars, in which case \$200 is a big difference) than in the bottom frame in Figure 11 (where other payoffs are in the thousands of dollars, and the difference between 300 and 100 seems smaller).

Figure 11. Context-dependence and Scale-dependence of Salience Perception

	x_1,y_1	p_1,q_1	x_2,y_2	p_2,q_2	x_3,y_3	p_3,q_3	x_4,y_4	p_4,q_4
$\mathbf{p}$	300	0.25	200	0.25	100	0.25	0	0.25
$\mathbf{q}$	100	0.25	100	0.25	100	0.25	100	0.25

	x_1,y_1	p_1,q_1	x_2,y_2	p_2,q_2	x_3,y_3	p_3,q_3	x_4,y_4	p_4,q_4
$\tilde{\mathbf{p}}$	300	0.25	2000	0.25	1000	0.25	0	0.25
$\tilde{\mathbf{q}}$	100	0.25	1000	0.25	1000	0.25	1000	0.25

Such context-dependence and scale-dependence of salience perception can be accommodated by including a function that depends on all outcomes (or all probabilities) in a frame. One plausible candidate is the Euclidean norm which can be viewed as taking the second moment or the deviation of values from 0 for all outcomes (or all probabilities) in a frame. Let $(\mathbf{a}, \mathbf{b})$ denote a vector of length $2n$, formed by horizontally concatenating a pair of like dimension vectors in a frame (i.e., all outcomes in a frame, or all probabilities in a frame); and let $\|\mathbf{a}, \mathbf{b}\|$ denote the Euclidean norm of the vector $(\mathbf{a}, \mathbf{b})$. A *context-dependent parameter-free IPS* salience function can be defined as in (11):

(11) $\sigma(\mathbf{a_i}, \mathbf{b_i} | \mathbf{a}, \mathbf{b}) = \frac{|\mathbf{a_i} - \mathbf{b_i}|}{|\mathbf{a_i}| + |\mathbf{b_i}| + \|\mathbf{a}, \mathbf{b}\|}$.

This satisfies IPS for any frame, and under (11) the salience of (300, 100) is greater in the top frame in Figure 11 than in the bottom frame. We use this salience function for the SWUP type in our estimations.

Table 2 shows the results of our estimations of seven specifications. The fit (the negative log likelihood in the last row of Table 2) of the mixture of EU and SWUP is only exceeded by specifications that nest it (the mixture of RDU and SWUP, and the mixture of all three types). RDU has the best fit among single-type specifications, but keep several points in mind. First, RDU has two more preference parameters (its weighting function parameters) than do EU or SWUP; second, of the 94 pairs used in our estimations, 76

Table 2: Parameter Estimates, Single Theories and Mixture Models of Two or Three Theories

		Theory (or theories in mixture specifications) estimated						
Theory	Parameter	EU	RDU	SWUP	EU & RDU	EU & SWUP	RDU & SWUP	all three
EU	μ_ρ	0.99 (0.032)			0.89 (0.030)	0.97 (0.037)		1.14 (0.11)
	σ_ρ	0.54 (0.031)			0.36 (0.038)	0.43 (0.033)		0.34 (0.071)
	λ	13.71 (0.42)			21.38 (1.3)	22.03 (0.95)		20.33 (2.0)
	θ_{eu}	1 —			0.31 (0.051)	0.42 (0.047)		0.20 (0.043)
RDU	μ_ρ		1.20 (0.025)		1.25 (0.032)		1.05 (0.027)	1.06 (0.027)
	σ_ρ		0.30 (0.020)		0.28 (0.021)		0.30 (0.017)	0.28 (0.020)
	α		0.86 (0.0082)		0.79 (0.012)		0.99 (0.015)	0.89 (0.016)
	β		0.55 (0.015)		0.49 (0.018)		0.56 (0.019)	0.51 (0.020)
	λ		27.72 (0.78)		31.03 (1.2)		38.22 (1.5)	46.05 (2.5)
	θ_{rdu}		1 —		0.69 (0.051)		0.51 (0.048)	0.37 (0.050)
SWUP	μ_ρ			0.95 (0.066)		0.67 (0.090)	0.70 (0.11)	0.65 (0.10)
	σ_ρ			1.04 (0.073)		0.83 (0.069)	0.85 (0.076)	0.83 (0.083)
	λ			6.48 (0.17)		7.51 (0.27)	7.23 (0.28)	7.59 (0.34)
	θ_{swup}			1 —		0.58 (0.047)	0.49 (0.048)	0.43 (0.048)
	$-LL$	7771.69	7366.68	7478.05	7271.47	7219.4	7094.04	7050.4

pairs are in minimal frames—what we regard as the standard presentation that RDU was essentially designed for; and third, the six stochastic dominance pairs excluded from our estimation are also the pairs in which EU and RDU performs worst (among all 100 pairs) and SWUP performs best. In the three-type

specification, the largest sub-population is represented by SWUP (comprising 43% of the overall population), with RDU and EU representing, respectively, 37% and 20% of the population.

6. Minimal and Transparent Frames for Choice over Time: Present Bias and the Hidden Zero Effect

Consider the minimal frames in Figure 12. The stationarity axiom of Discounted Utility theory implies that people should choose either **r** and **r′** or **t** and **t′**. However, in choices such as these, experiments show that people frequently choose **r** and **t′**, a finding termed *present bias* (Laibson, 1997). Present bias occurs in the minimal frames in Figure 12 since the comparison between receiving money today versus in one year is more salient than the comparison of $75 versus $100, but this monetary comparison is more salient than receiving payment in 10 years or 11 years. One might make a different prediction in transparent frames, however, based on the intuition that the focal comparisons in both choices are between $75 and $0 and between $100 and $0. Indeed, switching from minimal to transparent frames provides a formal explanation of the hidden zero effect (Magen et al., 2008; Radu et al., 2011; Read et al., 2017) in which behavior becomes more patient when the opportunity costs of income (such as receiving $0 instead of $100 in 1 year) are made salient. Transparent frames retain the second choice of **t′** over **r′** from minimal frames, but shift the first choice toward preferring **t** over **r** via the hidden zero effect. The prediction of more patient behavior in transparent frames is also consistent with the finding by Fisher and Rangel (2014) that shifting attention from focusing on time to focusing on money reduces impatience since transparent frames increase the salience of the money dimension relative to minimal frames. Transparent frames may thus serve to induce more patient behavior and more time consistent behavior.

Figure 12. Present Bias in Minimal and Transparent Frames

In Minimal Frames

	x_1,y_1	Years
r	**75**	**0**
t	100	1

	x_1,y_1	Years
r′	75	10
t′	**100**	**11**

In Transparent Frames

	x_1,y_1	Years	x_2,y_2	Years
r	75	0	0	1
t	**0**	**0**	**100**	**1**

	x_1,y_1	Years	x_2,y_2	Years
r′	75	10	0	11
t′	**0**	**10**	**100**	**11**

7. Minimal and Transparent Frames for Choice under Ambiguity: Ellsberg's Paradox

SWUP predicts that ambiguity aversion will be observed in minimal frames but mitigated by transparent frames. Figure 13 depicts two choice pairs $\{A, B\}$ and $\{A', B'\}$ in minimal frames as Schneider, Leland, and Wilcox (2016) presented them to subjects. After making 60 such choices, one of the 60 choices is randomly selected for payoff: Suppose this was pair $\{A, B\}$ and the subject chose B. The subject then draws a ticket from a bag, and if the ticket is red she plays a lottery in which there is a 75% chance of winning $25 and a 25% chance of winning nothing. If the ticket is blue, she instead plays a lottery in which

there is a 25% chance of winning \$25 and a 75% chance of winning nothing. But notice that if she had chosen option A instead, she would play a lottery offering a 50% chance of winning \$25 and a 50% chance of winning nothing irrespective of the ticket color. The pair $\{A', B'\}$ is similar except that in option B' the "good" state is reversed. For these minimal frames, the experiment replicates Ellsberg's Paradox, finding that people do not assign well-defined subjective probabilities to states, but rather prefer alternatives with known probabilities over unknown probabilities—a preference pattern called *ambiguity aversion.*

Figure 13. The Ellsberg Paradox in Minimal Frames

	Red Ticket				Blue Ticket			
A	**\$25**	**0.50**	**\$0**	**0.50**	**\$25**	**0.50**	**\$0**	**0.50**
B	\$25	0.75	\$0	0.25	\$25	0.25	\$0	0.75
	Red Ticket				Blue Ticket			
A′	**\$25**	**0.50**	**\$0**	**0.50**	**\$25**	**0.50**	**\$0**	**0.50**
B′	\$25	0.25	\$0	0.75	\$25	0.75	\$0	0.25

Under SWUP, with a uniform prior over states[12] and normalizing $u(25) = 1$ and $u(0) = 0$, A is chosen over B in the minimal frame shown in the top panel of Figure 13 if inequality (12) holds:

(12) $$0.5\phi(0.5, 0.75)(-0.25) + 0.5\phi(0.5, 0.25)(0.25) > 0.$$

By symmetry and DAS, $\phi(0.5, 0.25) > \phi(0.5, 0.75)$, and (12) holds for any salience function and any utility function. Intuitively, the salient comparisons between A and B are between a 0.50 probability of winning and a 0.75 probability (in the 'red' state) and between probabilities of 0.50 and 0.25 (in the 'blue' state). Diminishing absolute sensitivity implies a focal thinker will be more sensitive to the latter comparison. Similarly, A' is chosen over B' in the bottom panel of Figure 13, yielding ambiguity aversion.

Figure 14 shows Ellsberg-style choices in transparent frames. If the decision maker has a uniform prior over red and blue ticket states, then SWUP predicts ambiguity aversion in minimal frames, and ambiguity-neutrality (indifference between A and B) in transparent frames. Ambiguity neutrality in transparent frames follows from symmetry of μ, in which case (13) holds:

(13) $$0.5\mu(0, 25)(-6.25) + 0.5\mu(25, 0)(6.25) = 0.$$

The transparent frame focuses attention on comparing \$0 and \$25 rather than probabilities. If the decision maker has a uniform prior over red and blue ticket states, then a focal thinker exhibits ambiguity aversion in minimal frames, and ambiguity-neutrality (indifference between A and B) in transparent frames.

Schneider, Leland, and Wilcox (2016) considered the simple setting of a world with two types of agents – those who are ambiguity-averse (agents who always choose A), and those who are ambiguity-neutral (agents who randomize between A and B with equal probability). They computed the unique

[12] In estimating a mean-dispersion model of ambiguity preference to explain their data, Schneider, Leland, and Wilcox (2016) estimate the subjective prior assigned to red and blue ticket states to be uniform.

proportion of ambiguity neutral agents which exactly fits the distribution of ambiguity-averse and ambiguity-seeking choices observed in their experimental data, for both minimal and transparent frames. This approach estimated there to be about 43% ambiguity-neutral agents in minimal frames but 63% ambiguity neutral agents in transparent frames. While this framing effect is predicted by SWUP, we are not aware of an alternative model that predicts this frame-dependence of ambiguity aversion.

Figure 14. The Ellsberg Paradox in Transparent Frames

	Red Ticket						Blue Ticket					
A	\$0	0.25	\$25	0.50	\$0	0.25	\$25	0.25	\$25	0.25	\$0	0.50
B	\$25	0.25	\$25	0.50	\$0	0.25	\$0	0.25	\$25	0.25	\$0	0.50

	Red Ticket						Blue Ticket					
A′	\$25	0.25	\$25	0.25	\$0	0.50	\$0	0.25	\$25	0.50	\$0	0.25
B′	\$0	0.25	\$25	0.25	\$0	0.50	\$25	0.25	\$25	0.50	\$0	0.25

8. Summary

We now gather our results on framing effects for risk, time, and ambiguity in three propositions. Earlier in sections 4, 6 and 7 we illustrated the claims in Proposition 1: formal demonstrations appear in our SM.

Proposition 1 (Minimal Frames and Behavioral Biases):

A focal thinker with linear utility $u(x) = x$ and either the BGS salience function (10) or our IPS salience function (11):

(i) *violates stochastic dominance in the minimal frame in Figure 3*;

(ii) *exhibits the Allais paradox in the minimal frames in Figure 4*;

(iii) *exhibits the common ratio effect in the minimal frames in Figure 5*;

(iv) *exhibits present bias in the minimal frames in Figure 12 (for annual discount factor $\delta \in [0.41, 0.95]$)*;

(v) *and exhibits Ellsberg's paradox in the minimal frames in Figure 13.*

For minimal frames, we prove more general results regarding two of the most robust and most well-known violations of expected utility theory: the Allais common ratio effect and the Ellsberg paradox. Although these paradoxes are two of the oldest violations of rational choice theory, there has been relatively little work investigating the precise relationship between them. Proposition 2 is proved in our Appendix, for general versions of the Allais common ratio effect and Ellsberg's paradox also defined in the Appendix.

Proposition 2 (Allais, Ellsberg, and Salience Perception): *For monotone minimal frames:*

(i) *A focal thinker exhibits the general Allais common ratio effect if and only if ϕ satisfies increasing proportional sensitivity.*

(ii) *Under a uniform prior, a focal thinker exhibits ambiguity aversion in Ellsberg's paradox if ϕ satisfies diminishing absolute sensitivity.*

Proposition 2 is general and establishes that two basic properties of the perceptual system (greater sensitivity to larger absolute differences for a fixed ratio (IPS), and greater sensitivity to larger ratios for a fixed absolute difference (DAS)) directly imply the most robust violations of expected utility theory without any parametric assumptions regarding the form of the agent's salience functions or utility functions.

Without defining them precisely, Savage (1954) and Tversky and Kahneman (1986) argued that transparent presentations would reduce violations of rational choice theory. We formalized 'transparent presentation' of choice alternatives for risk, ambiguity, and time using a set of properties which imply unique transparent frames. With this done we can now state a theorem concerning transparent frames—converting a long-standing suggestion into a set of falsifiable statements. Consider four types of systematic violations of rational choice theory: violations of stochastic dominance, Allais paradox violations of expected utility theory, present biased violations of discounted utility theory, and Ellsberg paradox violations of subjective expected utility theory. We show they should all vanish under our definition of transparent framing: This *Transparent Frame Theorem* is proved in our Appendix.

Proposition 3 (Transparent Frame Theorem): *For transparent frames, a focal thinker will not exhibit the following violations of rational choice theory, even if the focal thinker exhibits them in minimal frames:*

(i) *Violations of Stochastic Dominance*

(ii) *Allais Paradox violations of Expected Utility theory*

(iii) *Common Ratio violations of Expected Utility theory*

(iv) *Present Bias violations of Discounted Utility theory*

(v) *Ellsberg Paradox violations of Subjective Expected Utility theory*

The transparent frame theorem is general in that it does not depend on the form of the decision maker's salience functions or on the form of the decision maker's utility function or discount factor or subjective beliefs, or on the particular parameter values used for the paradoxes, and it applies to choices across the domains of risk, time, and uncertainty.

For risk, transparent frames are similar to the state matrices employed by Savage (1954) (but implying neither correlation nor independence between payoffs) and to the 'canonical split form' of Birnbaum and Schmidt's (2015) tree presentation of lotteries, but are distinct in that the canonical split form does not separate common consequences from distinct consequences. For decisions under risk, minimal frames are related to Birnbaum's (1999) tree presentation of lotteries in 'coalesced form:' In particular, a choice set in which all lotteries are in coalesced form generates a minimal frame. We are not aware of any previous attempt to formalize different presentation formats for income streams.

We suggest two avenues for subsequent research. First, one might apply SWUP to choices between complex multiple outcome gambles and investigate whether it can explain novel findings such as data supporting the probability of winning heuristic (Venkatraman, Payne, and Huettel, 2014). Second, one

might normalize salience weights to sum to one (see our probabilistic choice model for SWUP in our SM). One might interpret such normalized salience weights as the distribution of attention across the column vectors in the frame. Eye-tracking studies might then test whether subjects focus on particular comparisons in proportion to these normalized weights.

Our definitions of frames and our formal distinctions between minimal and transparent frames provide a unified foundation for analyzing choice presentations across three major domains of individual choice. Combine that foundation with a decision model that operates on frames (such as SWUP), and a formal logic of framing effects emerges. Our foundation may also be a useful tool to help control for non-random variation in experiments or decision analysis. We found, for instance, that changes from minimal to transparent frames for the same choice alternatives can generate increases in the proportion of safe choices by 20 to 30 percentage points. This is a large and systematic shift in risk preferences arising from small changes in framing. Earlier work by Harless (1992), Starmer and Sugden (1993), Humphrey (1995), and by Birnbaum and colleagues (e.g., Birnbaum and Chavez 1997; Birnbaum and Navarrete 1998) also observed significant framing effects for risk due to changes in presentation formats.

The same mathematical structure—and the same psychological intuition—explains a variety of the most robust and well-known behavioral biases across the domains of risk, uncertainty, and time, violating four of the most well-known axioms in rational choice theory (stochastic dominance, independence, stationarity, and the sure-thing principle). Focal thinkers will violate these axioms in minimal frames but satisfy them in transparent frames. Evidence from previous literature and from our own experiment suggests that biases are reduced, but not eliminated, when the presentation of choice alternatives is made transparent.

Appendix

Here, $\mathbf{v} \gtrdot \mathbf{w}$ $(\mathbf{v} \eqsim \mathbf{w})$ means that for focal thinkers, option v 'looks strictly better than' ('looks indifferent to') option w when a frame $[\![\mathbf{v}, \mathbf{w}]\!]$ (of type specified in each proposition) presents the choice pair $\{v, w\}$.

A.1 Proof of Proposition 2 (Allais and Ellsberg in Minimal Frames)

We define the general form of the Allais common ratio effect below:

Definition 2 (General Allais Common Ratio Effect): Consider monotone minimal frames (i) and (ii) in Figure A.1. The *general Allais common ratio effect* holds for focal thinkers if for all $\mathbf{y} > \mathbf{x} > \mathbf{0}, 1 \geq \mathbf{p_1} > \mathbf{q_1} > 0$ and $\boldsymbol{\alpha} \in (0,1)$, $\mathbf{p} \eqsim \mathbf{q}$, implies $\mathbf{q'} \gtrdot \mathbf{p'}$.

Figure A.1 Minimal Frame for the General Allais Common Ratio Effect

(i)	(x_1,y_1)	(p_1,q_1)	(x_2,y_2)	(p_2,q_2)	(ii)	(x_1,y_1)	(p_1,q_1)	(x_2,y_2)	(p_2,q_2)
$\mathbf{p}$	$\mathbf{x}$	$\mathbf{p_1}$	$\mathbf{0}$	$\mathbf{1-p_1}$	$\mathbf{p'}$	$\mathbf{x}$	$\boldsymbol{\alpha}\mathbf{p_1}$	$\mathbf{0}$	$\mathbf{1-\boldsymbol{\alpha} p_1}$
$\mathbf{q}$	$\mathbf{y}$	$\mathbf{q_1}$	$\mathbf{0}$	$\mathbf{1-q_1}$	$\mathbf{q'}$	$\mathbf{y}$	$\boldsymbol{\alpha}\mathbf{q_1}$	$\mathbf{0}$	$\mathbf{1-\boldsymbol{\alpha} q_1}$

Proof of Proposition 2 (i): Note that $\mathbf{p} \gtrsim \mathbf{q}$ in Figure A.1 choice (i) if and only if

$$\mu(\mathbf{x},\mathbf{y})\big(u(\mathbf{y})-u(\mathbf{x})\big)\left[\frac{\mathbf{p_1}+\mathbf{q_1}}{2}\right] = \phi(\mathbf{p_1},\mathbf{q_1})(\mathbf{p_1}-\mathbf{q_1})\left[\frac{u(\mathbf{y})+u(\mathbf{x})}{2}\right].$$

Also note that $\mathbf{q}' \gtrsim \mathbf{p}'$ if and only if $\mu(\mathbf{x},\mathbf{y})\big(u(\mathbf{y})-u(\mathbf{x})\big)\left[\frac{\mathbf{p_1}+\mathbf{q_1}}{2}\right] > \phi\big(\boldsymbol{\alpha}\mathbf{p_1},\boldsymbol{\alpha}\mathbf{q_1}\big)\big(\mathbf{p_1}-\mathbf{q_1}\big)\left[\frac{u(\mathbf{y})+u(\mathbf{x})}{2}\right]$.

By IPS, scaling $\boldsymbol{\alpha}\mathbf{p_1}$ and $\boldsymbol{\alpha}\mathbf{q_1}$ each by $\frac{1}{\boldsymbol{\alpha}}$ leads to $\phi\big(\boldsymbol{\alpha}\mathbf{p_1},\boldsymbol{\alpha}\mathbf{q_1}\big) < \phi\big(\mathbf{p_1},\mathbf{q_1}\big)$ for all $\boldsymbol{\alpha} \in (0,1)$. Letting $\mathbf{k} \equiv 1/\boldsymbol{\alpha}$, the common ratio effect holds if and only if $\big(\boldsymbol{\alpha}\mathbf{p_1},\boldsymbol{\alpha}\mathbf{q_1}\big) < \phi\big(\mathbf{k}\boldsymbol{\alpha}\mathbf{p_1},\mathbf{k}\boldsymbol{\alpha}\mathbf{q_1}\big)$ for all $\mathbf{k} > 1$. ■

Next, consider Ellsberg's (1961) two-color paradox. There are two urns. Urn 1 contains 50 red and 50 black balls. Urn 2 contains an unknown mixture of 100 red and black balls. A person is given two choices:

Choice 1: Choose between A and B	**Choice 2: Choose between C and D**
A. Win \$100 if red is drawn from Urn 1	C. Win \$100 if black is drawn from Urn 1
B. Win \$100 if red is drawn from Urn 2	D. Win \$100 if black is drawn from Urn 2

Subjective expected utility (SEU) requires choices of either A and D or B and C. However, Ellsberg found that most people choose A and C (options with objective probabilities) over B and D (options with ambiguous probabilities) thereby exhibiting ambiguity aversion. The minimal frame for these choices (for each state $s \in \{0,1,\dots,100\}$, the actual number of red balls in Urn 2) is displayed in Figure A.2, where $\mathrm{q}(s)$ is the probability of drawing a red ball from Urn 2 in state s.

Figure A.2. Minimal Frame for the Two-Color Ellsberg Paradox

A(s)	**\$100**	**0.5**	**\$0**	**0.5**
B(s)	\$100	q(s)	\$0	1 − q(s)
C(s)	**\$100**	**0.5**	**\$0**	**0.5**
D(s)	\$100	1 − q(s)	\$0	q(s)

Definition 3 (Ambiguity Aversion in Ellsberg's Paradox): For the frames in Figure A.2, a focal thinker exhibits *ambiguity aversion in Ellsberg's paradox* if $\mathbf{A} \gtrsim \mathbf{B}$ and $\mathbf{C} \gtrsim \mathbf{D}$.

Proof of Proposition 2 (ii): This proof is for Ellsberg's two-color paradox. An analogous argument resolves Ellsberg's three-color paradox. Let s denote the number of red balls in Urn 2. Since the number of black balls is $100 - s$, the state of the urn is fully characterized by s. For each state, the presentation for Choice 1 is given by Figure A.2, where $\mathbf{q}(\mathbf{s})$ is the probability of drawing a red ball from Urn 2 in state s. Without loss of generality, we normalize the payoffs such that $u(\mathbf{100}) = 1$, and $u(\mathbf{0}) = 0$. For a focal thinker, A is chosen over B if and only if inequality (14) holds. Under a uniform prior, (14) becomes (15):

(14) $\sum_{s=1}^{m} \pi_s \left[\phi\big(\mathbf{0.5},\mathbf{q(s)}\big)\big(\mathbf{0.5}-\mathbf{q(s)}\big)\right] > 0.$

(15) $\frac{1}{101}\left[\sum_{s=0}^{s=50} \phi\big(\mathbf{0.5},\mathbf{q(s)}\big)\big(\mathbf{0.5}-\mathbf{q(s)}\big) + \sum_{s=51}^{s=100} \phi\big(\mathbf{0.5},\mathbf{q(s)}\big)\big(\mathbf{0.5}-\mathbf{q(s)}\big)\right] > 0$, which implies

(16) $\sum_{s=0}^{s=50} \phi\big(\mathbf{0.5},\mathbf{q(s)}\big)\big(\mathbf{0.5}-\mathbf{q(s)}\big) + \sum_{s=0}^{s=50} \phi(\mathbf{0.5},\mathbf{1}-\mathbf{q(s)})(\mathbf{q(s)}-\mathbf{0.5}) > 0.$

To see that (14) holds, note that for each $\mathbf{q}(\boldsymbol{s}) \in [0,0.5)$ diminishing absolute sensitivity and symmetry of ϕ imply $\phi(\mathbf{0.5}, \mathbf{1} - \mathbf{q}(s)) < \phi(\mathbf{0.5}, \mathbf{q}(\boldsymbol{s}))$. In particular, by symmetry, $\phi(\mathbf{0.5}, \mathbf{1} - \mathbf{q}(\boldsymbol{s})) = \phi(\mathbf{0.5} + \mathbf{0.5} - \mathbf{q}(\boldsymbol{s}), \mathbf{q}(\boldsymbol{s}) + \mathbf{0.5} - \mathbf{q}(\boldsymbol{s})) = \phi(\mathbf{0.5} + \boldsymbol{\epsilon}, \mathbf{q}(\boldsymbol{s}) + \boldsymbol{\epsilon})$. Thus, by diminishing absolute sensitivity, (16) holds, yielding a choice for the risky over the ambiguous urn. The argument follows analogously for the choice between C and D, resulting in ambiguity aversion. ■

A.2 Proof of Proposition 3 (Transparent Frame Theorem)

Proof of Proposition 3 (i): Lottery p is defined to stochastically dominate q if $P(x) \leq Q(x)$ for all $x \in \mathrm{X}$, with at least one strict inequality, where $P(x)$ and $Q(x)$ are the cumulative distribution functions corresponding to p and q, respectively. Whenever p stochastically dominates q, $\mathbf{x_i} \geq \mathbf{y_i}$ and $\mathbf{p_i} - \mathbf{q_i} = 0$ for all i in a transparent frame. Thus, the salience weights in (4) favor **p** over **q** in each binary comparison where the differences are not zero. ■

Proof of Proposition 3 (ii), (iii): We show that this result applies to both Allais paradoxes – the common consequence effect and the common ratio effect. The Allais common consequence choices in transparent frames are displayed in Figure A.3, where $\mathbf{x} > \mathbf{y} > \mathbf{0}, \mathbf{q} > \mathbf{p}$, and $\mathbf{q}, \mathbf{p} \in (0,1)$.

Figure A.3. The Allais Common Consequence Effect in Transparent Frames

$\mathbf{p}$	$\mathbf{x}$	$\mathbf{p}$	$\mathbf{0}$	$\mathbf{q-p}$	$\mathbf{y}$	$\mathbf{1-q}$
$\mathbf{q}$	$\mathbf{y}$	$\mathbf{p}$	$\mathbf{y}$	$\mathbf{q-p}$	$\mathbf{y}$	$\mathbf{1-q}$
$\mathbf{p'}$	$\mathbf{x}$	$\mathbf{p}$	$\mathbf{0}$	$\mathbf{q-p}$	$\mathbf{0}$	$\mathbf{1-q}$
$\mathbf{q'}$	$\mathbf{y}$	$\mathbf{p}$	$\mathbf{y}$	$\mathbf{q-p}$	$\mathbf{0}$	$\mathbf{1-q}$

A focal thinker does not exhibit the common consequence effect if either $\mathbf{p} \gtrless \mathbf{q}$ and $\mathbf{p'} \gtrless \mathbf{q'}$ or $\mathbf{q} \gtrless \mathbf{p}$ and $\mathbf{q'} \gtrless \mathbf{p'}$. For focal thinkers, (4) implies that common consequences cancel, so $\mathbf{p} \gtrless \mathbf{q}$ iff $\mathbf{p'} \gtrless \mathbf{q'}$. ■

The choices for the general form of the Allais common ratio effect are shown in transparent frames in Figure A.4, where $\mathbf{y} > \mathbf{x} > \mathbf{0}, \mathbf{1} \geq \mathbf{p} > \mathbf{q} > \mathbf{0},$ and $\boldsymbol{\alpha} \in (0,1)$.

Figure A.4. The Allais Common Ratio Effect in Transparent Frames

$\mathbf{p}$	$\mathbf{x}$	$\mathbf{q}$	$\mathbf{0}$	$\mathbf{p-q}$	$\mathbf{0}$	$\mathbf{1-p}$
$\mathbf{q}$	$\mathbf{y}$	$\mathbf{q}$	$\mathbf{y}$	$\mathbf{p-q}$	$\mathbf{0}$	$\mathbf{1-p}$
$\mathbf{p'}$	$\mathbf{x}$	$\boldsymbol{\alpha}\mathbf{q}$	$\mathbf{0}$	$\boldsymbol{\alpha}(\mathbf{p-q})$	$\mathbf{0}$	$\mathbf{1-\boldsymbol{\alpha}p}$
$\mathbf{q'}$	$\mathbf{y}$	$\boldsymbol{\alpha}\mathbf{q}$	$\mathbf{y}$	$\boldsymbol{\alpha}(\mathbf{p-q})$	$\mathbf{0}$	$\mathbf{1-\boldsymbol{\alpha}p}$

A focal thinker does not exhibit the Allais common ratio paradox if either $\mathbf{p} \gtrless \mathbf{q}$ and $\mathbf{p'} \gtrless \mathbf{q'}$ or $\mathbf{q} \gtrless \mathbf{p}$ and $\mathbf{q'} \gtrless \mathbf{p'}$. For focal thinkers, (4) implies that the constant $\boldsymbol{\alpha}$ factors out, so that $\mathbf{p} \gtrless \mathbf{q}$ iff $\mathbf{p'} \gtrless \mathbf{q'}$. ■

Proof of Proposition 3 (iv): Choices for a test of present bias are shown in Figure A.5 in transparent frames.

Figure A.5. Present Bias in Transparent Frames

SS	$\mathbf{x}$	$\mathbf{0}$	$\mathbf{0}$	Δ	$\mathbf{SS'}$	$\mathbf{x}$	$\mathbf{t}$	$\mathbf{0}$	$\mathbf{t}+\Delta$
LL	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{y}$	Δ	$\mathbf{LL'}$	$\mathbf{0}$	$\mathbf{t}$	$\mathbf{y}$	$\mathbf{t}+\Delta$

Present bias is absent if either $\mathbf{SS} \hat{\gneq}_{\mathbf{t}} \mathbf{LL}$ and $\mathbf{SS'} \hat{\gneq}_{\mathbf{t}} \mathbf{LL'}$ or $\mathbf{LL} \hat{\gneq}_{\mathbf{t}} \mathbf{SS}$ and $\mathbf{LL'} \hat{\gneq}_{\mathbf{t}} \mathbf{SS'}$. For a focal thinker, (6) gives both $\mathbf{SS} \hat{\gneq}_t \mathbf{LL}$ iff $\mu(\mathbf{x},\mathbf{0})u(\mathbf{x}) > \mu(\mathbf{0},\mathbf{y})u(\mathbf{y})\delta^{\Delta}$ and $\mathbf{SS'} \hat{\gneq}_t \mathbf{LL'}$ iff $\mu(\mathbf{x},\mathbf{0})u(\mathbf{x})\delta^{\mathbf{t}} > \mu(\mathbf{0},\mathbf{y})u(\mathbf{y})\delta^{\mathbf{t}+\Delta}$. Since $\delta^{\boldsymbol{t}}$ can be factored out of the latter, $\mathbf{SS} \hat{\gneq}_t \mathbf{LL}$ iff $\mathbf{SS'} \hat{\gneq}_t \mathbf{LL'}$. ■

Proof of Proposition 3 (v): The transparent frames for Choices 1 and 2 in Ellsberg's two-color paradox are shown in Figure A.6, where state $s \in \{0,1,\dots,100\}$ indexes the number of red balls in Urn 2, and $p(s) = |50-s|/100$. Note that when $p(s) = 0.25$, Figure A.6 resembles the frame in Figure 14. Ellsberg's paradox absent if either $\mathbf{A} \hat{\gneq} \mathbf{B}$ and $\mathbf{D} \hat{\gneq} \mathbf{C}$ or $\mathbf{B} \hat{\gneq} \mathbf{A}$ and $\mathbf{C} \hat{\gneq} \mathbf{D}$ or if there is indifferent in both choices. Without loss of generality, set $u(100) = 1$ and $u(0) = 0$. Denote the set of states favoring A by $\overline{S}$ and the set of states favoring B by $\underline{S}$. The SWUP evaluation for the choice between A and B is:

$$\sum_{s\in\overline{S}} \pi_s\, \mu(100,0)\big(p(s)\big) + \sum_{s\in\underline{S}} \pi_s\, \mu(0,100)\big(-p(s)\big) \tag{17}$$

Figure A.6. The Ellsberg Paradox in Transparent Frames

	States favoring A: $s \in \{0,1,\dots,50\}$						States favoring B: $s \in \{51,52,\dots,100\}$					
A	\$100	$p(s)$	\$100	$0.5-p(s)$	\$0	0.5	\$0	$p(s)$	\$100	0.5	\$0	$0.5-p(s)$
B	\$0	$p(s)$	\$100	$0.5-p(s)$	\$0	0.5	\$100	$p(s)$	\$100	0.5	\$0	$0.5-p(s)$
	States favoring C: $s \in \{51,52,\dots,100\}$						States favoring D: $s \in \{0,1,\dots,50\}$					
C	\$100	$p(s)$	\$100	$0.5-p(s)$	\$0	0.5	\$0	$p(s)$	\$100	0.5	\$0	$0.5-p(s)$
D	\$0	$p(s)$	\$100	$0.5-p(s)$	\$0	0.5	\$100	$p(s)$	\$100	0.5	\$0	$0.5-p(s)$

where π_s is the subjective probability that the true state is s. All other differences within each column in the frame cancel. Under a uniform prior, the decision maker is indifferent between A and B (by symmetry of μ) in which case the evaluation in (17) equals zero. Moreover, even if the distribution is not uniform, (17) implies ambiguity neutrality since if (17) is positive, the decision maker would prefer A and D since the same set of states favor A and D. If (17) is negative, the decision maker prefers B and C. This argument extends analogously to Ellsberg's (1961) three-color paradox. ■

A.3 Minimal Frames

Definition 4 (Minimal Frame: Risk): For two non-degenerate lotteries $p, q \in \Delta(X)$, frame $[\![\mathbf{p},\mathbf{q}]\!]$ (such as in the top panel of Figure 1) is *minimal* if $\forall\ \mathbf{i} \neq \mathbf{j}$, $\mathbf{x_i} \neq \mathbf{x_j}$ and $\mathbf{y_i} \neq \mathbf{y_j}$.

Definition 5 (Monotone Frame: Risk): For two non-degenerate lotteries $p, q \in \Delta(X)$, frame $[\![\mathbf{p},\mathbf{q}]\!]$ (such as in the top panel of Figure 1) is *monotone* if $\mathbf{x_1} \geq \mathbf{x_2} \geq \cdots \geq \mathbf{x_n}$ and $\mathbf{y_1} \geq \mathbf{y_2} \geq \cdots \geq \mathbf{y_n}$.

The following result is immediate.

Proposition 4 (Uniqueness of Monotone Minimal Frames: Risk): *For any lotteries* $p, q \in \Delta(X)$ *with* $|supp(p)| = |supp(q)|$, *a monotone minimal frame* $[\![\mathbf{p}, \mathbf{q}]\!]$ *is unique up to the operation of row-switching.*

Proof: A frame that is minimal and monotonic is strictly monotonic. Hence, for two lotteries with the same support size, the i^th best outcome of p is in the same column vector as the i^th best outcome of q.■

Minimal frames can also be defined for choices over time.

Definition 6 (Minimal Frame: Time): For income streams $r, t \in C$, frame $[\![\mathbf{r}, \mathbf{t}]\!]$ (such as in the bottom panel of Figure 1) is *minimal* if it has the smallest number of columns necessary to present $\{r, t\}$.

Definition 7 (Monotone Frame: Time)[13]: For income streams $r, t \in C$, frame $[\![\mathbf{r}, \mathbf{t}]\!]$ (such as in the bottom panel of Figure 1) is *monotone* if $\mathbf{r_1} < \mathbf{r_2} < \cdots < \mathbf{r_n}$ and $\mathbf{t_1} < \mathbf{t_2} < \cdots < \mathbf{t_n}$.

Let the support of an income stream, r, be the set of non-zero outcomes in r, and denote it by $supp(r)$.

Proposition 5 (Uniqueness of Monotone Minimal Frames: Time): *For any streams* $r, t \in C$, *with* $|supp(r)| = |supp(t)|$, *a monotone minimal frame* $[\![\mathbf{r}, \mathbf{t}]\!]$ *is unique up to the operation of row-switching.*

The proof of Proposition 5 is analogous to that of Proposition 4. Propositions 4 and 5 guarantee uniqueness of monotone minimal frames when both lotteries or both income streams have the same support size.

A.4 Transparent Frames

We next define transparent frames and show they are uniquely defined under general conditions (even if the lotteries or income streams have different support sizes). Given two lotteries, p and q, a pair (x, r), consisting of an outcome x with probability r, is a *common consequence* if $x \in supp(p) \cap supp(q)$ and $p(x) \geq r$, $q(x) \geq r$. All other pairs of outcomes and corresponding probabilities are *distinct consequences*. That is, a common consequence between two lotteries is one with the same outcome occurring with the same probability in both lotteries. We say (x, r) is a *maximal common consequence* if (x, r) is a common consequence for which $p(x) = r$ or $q(x) = r$ (or both). Our definition of transparent frames is constructive: it uniquely specifies how to construct a transparent frame given any pair of lotteries.

Definition 8 (Transparent Frame: Risk): A *transparent frame* for lotteries $[\![\mathbf{p}, \mathbf{q}]\!]$ is the special case of the frame in the top panel of Figure 1 that has the following properties:

(1) Presentation of outcome-probability pairs (Common Consequence Separation): All (maximal) common consequences are separated from all distinct consequences such that all maximal common consequences are adjacent, and all distinct consequences are adjacent, as shown in Figure A.7.[14]

(2) Presentation of outcomes (monotonicity): Outcomes are ordered such that

$$\mathbf{x_n} \geq \cdots \geq \mathbf{x_1};\ \mathbf{y_n} \geq \cdots \geq \mathbf{y_1};\ \mathbf{z_k} > \cdots > \mathbf{z_1}.$$

[13] For our results, it does not matter whether monotone frames of income streams are monotonic in outcomes or monotonic in time periods, nor does it matter whether monotonic frames are increasing or decreasing.
[14] In Figure A.7, there are k common consequences (with corresponding outcomes $z_1, \ldots, z_k$) where $k \geq 0$. The remaining pairs of payoff column vectors and corresponding probability column vectors are distinct consequences.

(3) Presentation of probabilities (alignment): Probabilities are presented so that $\forall\ i \in [k+1, k+n]$:

(i) $\mathbf{p_i} = \mathbf{q_i}$; and

(ii) Given $\mathbf{x_i} = x$ and $\mathbf{y_i} = y$, $\mathbf{p_i} = min\left(p(x) - \sum_{j<i:\mathbf{x_j}=x} \mathbf{p_j},\ q(y) - \sum_{j<i:\mathbf{y_j}=y} \mathbf{q_j}\right)$.

(4) Relevance: $\mathbf{p_i} > \mathbf{0}$ for all $i \in [1, k+n]$.

Figure A.7. Transparent Frame of Lotteries p and q

$\mathbf{p}$	$\mathbf{x_n}$	$\mathbf{p_{k+n}}$	$\cdots$	$\mathbf{x_{k+i}}$	$\mathbf{p_{k+i}}$	$\cdots$	$\mathbf{x_1}$	$\mathbf{p_{k+1}}$	$\mathbf{z_k}$	$\mathbf{p_k}$	$\cdots$	$\mathbf{z_1}$	$\mathbf{p_1}$
$\mathbf{q}$	$\mathbf{y_n}$	$\mathbf{p_{k+n}}$	$\cdots$	$\mathbf{y_{k+i}}$	$\mathbf{p_{k+i}}$	$\cdots$	$\mathbf{y_1}$	$\mathbf{p_{k+1}}$	$\mathbf{z_k}$	$\mathbf{p_k}$	$\cdots$	$\mathbf{z_1}$	$\mathbf{p_1}$

In (3-ii), $p(x)$ is the overall probability of outcome x in lottery p, and $\mathbf{p_j}$ is the probability of outcome $\mathbf{x_j}$ in the jth payoff column vector in the frame. Note that $\sum_{j<i:\mathbf{x_j}=x} \mathbf{p_j}$ is the cumulative probability of outcome x that is summed over the preceding columns in the frame. The expression for $\mathbf{p_i}$ ensures that either the *entire* remaining probability mass for outcome x or for outcome y will be completely used in column **i** in the frame. The algorithm for computing these probabilities (property (3-ii)) thus ensures *compactness* (the frame has the fewest cells subject to satisfying properties (1), (2), (3-i), and (4)) and it ensures *completeness* – the probabilities in each row vector of the frame sum to 1. The algorithm in (3-ii) also ensures *uniqueness* (the frame is uniquely defined even for lotteries with different support sizes), and it is constructive, by specifying how to generate the frame given any pair of lotteries.

Proposition 6 (Uniqueness of Transparent Frames: Risk): *For any lotteries $p, q \in \Delta(X)$ with finite support, there is a unique transparent frame $[\![\mathbf{p}, \mathbf{q}]\!]$ up to the operation of row-switching.*

Proof of Proposition 6 (Uniqueness of Transparent Frames: Risk): By monotonicity, the display of the k maximal common consequences is strictly monotonic, and thereby unique. For distinct consequences, monotonicity, in conjunction with the algorithm for $\mathbf{p_i}$, (alignment properties (3-i) and (3-ii) uniquely determines each subsequent cell in the frame and so the frame is unique (There is only one possible specification implied by $\mathbf{p_i}$ for each probability and corresponding payoff column vector for each cell $i \in (k+1, k+n)$). ■[15]

Definition 9 (Transparent Frame: Time): A *transparent frame* $[\![\mathbf{r}, \mathbf{t}]\!]$ for income streams $r, t \in C$ is the special case of the frame in the bottom panel of Figure 1 that satisfies the following properties.

(1) Common Consequence Separation: Common consequences are separated from distinct consequences such that common consequences are adjacent and distinct consequences are adjacent as in Figure A.8[16].

(2) Alignment: $\mathbf{t_i} = \mathbf{r_i}$ for all i.

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[15] Note that in the cases of transparent risk frames, both alignment properties (3-i) and (3-ii) are used to confer uniqueness. For choices over time, we only require an alignment property similar to (3-i).

[16] In Figure A.8, there are m common consequences (with corresponding outcomes $z_1,\ldots,z_m$) where $m \geq 0$. The remaining pairs of payoff column vectors and corresponding time column vectors are distinct consequences.

(3) Monotonicity: Time periods are ordered monotonically such that $\mathbf{t_1} < \cdots < \mathbf{t_k};\ \mathbf{t_{k+1}} < \cdots < \mathbf{t_n}$.

(4) Completeness and Relevance: The first and last period in $[\![\mathbf{r}, \mathbf{t}]\!]$ are the same as in r and t and $\mathbf{n} = \mathrm{T}$, where $\mathbf{n}$ is the number of periods in the frame and T is the time horizon of the income streams.

Figure A.8. Transparent Frame of Income streams r and t

$\mathbf{r}$	$\mathbf{x_1}$	$\mathbf{r_1}$	…	$\mathbf{x_i}$	$\mathbf{r_i}$	…	$\mathbf{x_k}$	$\mathbf{r_k}$	$\mathbf{z_1}$	$\mathbf{r_{k+1}}$	…	$\mathbf{z_m}$	$\mathbf{r_n}$
$\mathbf{t}$	$\mathbf{y_1}$	$\mathbf{t_1}$	…	$\mathbf{y_i}$	$\mathbf{t_i}$	…	$\mathbf{y_k}$	$\mathbf{t_k}$	$\mathbf{z_1}$	$\mathbf{t_{k+1}}$	…	$\mathbf{z_m}$	$\mathbf{t_n}$

Proposition 7 (Uniqueness of Transparent Frames: Time): *For any income streams $r, t \in C$ over a horizon of T periods there is a unique transparent frame $[\![\mathbf{r}, \mathbf{t}]\!]$ up to the operation of row-switching.*

Proof of Proposition 7 (Uniqueness of Transparent Frames: Time): By completeness and relevance, all (and only) time periods that index the income streams are displayed in the frame. By alignment, the i^{th} period in stream *r* in the frame is in the same column vector as the i^{th} period in stream *t.* By monotonicity, the ordering of the periods in the frame is uniquely determined and the frame is unique. ■

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