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Preparing Elementary School Teachers to Learn from Teaching: A Comparison of Two Approaches to Mathematics Methods Instruction

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Abstract

Teacher preparation programs face a significant challenge in determining how to design learning experiences that develop the combination of knowledge, practices, and dispositions needed for effective classroom teaching. Time constraints and the theory-practice divide are two well-documented concerns. We introduce the conceptual framework and design elements of a video-enhanced mathematics methods course that targets these concerns. The course centers on systematic reflection and analysis of practice intended to foster career-long learning. We then examine the impact of this course on several facets of learning-from-teaching competencies, including teacher knowledge, beliefs, and practices. Sixty-two pre-service teachers, enrolled in a one-year post-bachelor elementary teacher preparation program, were randomly assigned to attend this course or a more typical mathematics methods course. Findings suggest that teacher preparation experiences centered on systematic reflection and analysis create opportunities to develop certain aspects of learning-from-teaching competencies that remain otherwise underdeveloped. Implications for the design of teacher preparation include the integration in mathematics methods courses of cycles of analysis through video-enhanced discussions; collaborative planning, implementation, and reflection on teaching; and live observation and co-constructed interpretations and considerations of next steps.

Keywords: analysis of practice, mathematics teaching, teacher preparation, video

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Introduction

Conceptualizations of what makes teachers effective differ, but most agree on a combination of knowledge, practices, and dispositions that are central to quality teaching. Teachers need content and pedagogical content knowledge, and they need to master learning and development theories, so that they can make content accessible to all learners (Shulman, 1986). Knowledge must be coupled with a positive disposition towards the subject matter and towards students (Darling-Hammond & Bransford, 2005; Grossman, Hammerness, & McDonald, 2009; Wilson, Floden, & Ferrini-Mundi, 2003). Because of racial, ethnic, and socio-economic disparities in opportunities to learn, teachers also need to develop political knowledge (Gutiérrez, 2013). Finally, some have argued that teachers must have an inquiry orientation towards their profession; in other words, they must be motivated to continue to improve their practices throughout their careers (Bransford, Brown, & Cocking, 1999; Brouwer & Korthagen, 2005; Cochran-Smith & Lytle, 2001; Leinhardt, Young, & Merriman, 1995).

Teacher preparation programs face a significant challenge in determining how to design learning experiences that develop the combination of knowledge, practices, and dispositions outlined above. Teacher educators often have limited time to prepare teachers for their profession. Decisions are continuously made that privilege specific content and experiences over others. This is true for structural and organizational decisions that relate to programs as a whole and for individual courses as well.

In addition to time constraints, another common obstacle is the divide between theory and practice (Black & Halliwell, 2000; Brouwer & Korthagen, 2005; Smagorinsky, Cook, &

Johnson, 2003). Teacher preparation programs struggle to integrate theories of teaching and research-based knowledge of learning into skilled practice. Too often, pre-service teachers perceive university coursework as a required step to receive a degree and as secondary to their ability to function in the realities of classrooms; fieldwork experiences are perceived instead, as the only setting where learning to teach occurs (Feiman-Nemser & Buchmann, 1985; Rosaen & Florio-Ruane, 2008).

Educators have tried to close this divide. In the late 1960s through the mid-1970s, when behaviorism influenced teacher education research and practices, microteaching experiences focused on training future teachers to adopt specific teaching techniques through cycles of modeling, imitation, and feedback. Later in the 1980s, the research paradigm shifted to cognitivism and teacher educators' attention likewise shifted from teachers' behavior to teachers' thinking and knowledge. It became clear that professional judgment is essential in teaching and that knowledge must translate into action to respond to in-the-moment situations (Santagata, Gallimore, & Stigler, 2005). The field has come to the agreement that teachers need to learn practices that are much more complex than the discrete techniques rehearsed in microteaching sessions (McDonald, Kazemi, & Schneider Kavanagh, 2013). Also, learning to teach entails much more than applying theoretical knowledge, or research-based strategies that are designed and tested in psychology labs, to classroom teaching (Ermeling, Hiebert, & Gallimore, 2015; Putnam & Borko, 2000).

Far from being resolved, the theory-practice divide has been re-conceptualized. Some have argued that the nature of knowledge that is useful for classroom teaching is substantially different from the knowledge generated by researchers (Hiebert, Gallimore, & Stigler, 2002). Teacher knowledge is detailed, concrete and specific; it is linked to practice and is organized around problems of practice. For this knowledge to be useful to the profession and to newcomers, teachers must make teaching public and have mechanisms for sharing their knowledge and for improving practice over time (Santagata, et al., 2005).

This study addresses the two challenges outlined above: time constraints and the theory-practice divide. We designed and studied a mathematics methods course that structures teacher learning around artifacts of teaching and learning—particularly videos of instructional episodes and samples of student work, with the explicit goal of situating learning in the context of practice. In addition, to make effective use of limited teacher preparation time, we also prioritized the development of learning-from-teaching competencies that we conjecture will be generative of new learning as teachers enter the profession. To examine whether this approach to preparing elementary-school teachers to teach mathematics is promising, we compared learning outcomes for a group of teachers who attended this course and a control group who attended a course structured around the most frequently used mathematics methods textbook in the United States. This study design allowed us to ask the following broad question: Should time be set aside in teacher preparation to deliberately teach learning-from-teaching competencies or do these competencies develop through more typical experiences?

Before we describe the two courses and the study participants and methods, we contextualize the study within the broader approach of practice-based teacher preparation and discuss our conceptual framework.

Practice-Based Teacher Preparation and Learning-from-Teaching Competencies

In accordance with the reconceptualization of the theory-practice divide we discussed above, scholars advocate for an organization of teacher preparation around a core set of practices that are grounded in research-based theories of how students learn (Ball and Forzani, 2009; Ball, Sleep, Boerst, & Bass, 2009; Grossman, Hammerness, & McDonald, 2009; McDonald, et al., 2013). Core practices are not checklists of competencies nor techniques such as those promoted by the microteaching approach in the 1960s and 1970s. Some examples of these core practices for teaching mathematics include: eliciting and responding to students' ideas; leading a whole-class discussion; and making content explicit through explanations, modeling, representations, and examples (Grossman & McDonald, 2008).

While we situate our research within this practice-based approach to teacher education (and we join the effort of that community of teacher educators and scholars to develop conversations and a shared language about the details of our work (McDonald, et al, 2009; Grossman, et al., 2009)), we also propose and study something different and complementary.

In our study, pre-service teachers examined and enacted core practices. For example, they learned to elicit and build on student mathematical thinking, to orchestrate classroom discussions, and to use a variety of mathematical representations to foster children's understanding of mathematical concepts. This work was complemented by a deliberate focus on an additional practice that fosters *teacher learning*: systematic reflection and analysis. Other scholars have included reflection and analysis as a core practice of the work of teaching. For example, the Teaching Works team, led by Deborah Ball at the University of Michigan, includes "analyzing instruction for the purpose of improving it" among their list of core practices (see http://www.teachingworks.org/work-of-teaching/high-leverage-practices).

This study is framed by the working hypothesis that the engagement in systematic analysis of practice facilitates pre-service teachers' development of learning-from-teaching competencies that will become useful once they enter the profession.

We conceptualize learning-from-teaching competencies as comprised of particular kinds of knowledge, beliefs, and dispositions that interact with teaching and reflection practices in ways that facilitate teacher learning and improvement over time (see Fig. 1).

[INSERT FIG. 1 APPROX. HERE]

Specifically, we conjecture that learning from teaching in mathematics is facilitated by *usable knowledge for teaching mathematics* (Kersting, Givvin, Sotelo, & Stigler, 2010). This conception of knowledge is defined by its use in context, which is different from other more static notions of teacher knowledge (Shulman, 1986). Building on Shulman's notion of pedagogical content knowledge, Ball and colleagues (Ball, Thames, & Phelps, 2008) have examined mathematical knowledge for teaching by analyzing the work that teachers do in teaching mathematics and the mathematical demands of teaching. This research has led them to identify two aspects of mathematical knowledge for teaching: subject-matter knowledge and pedagogical content knowledge.

Similarly, Kersting and colleagues have examined the knowledge needed for teaching mathematics, and have identified four facets of usable knowledge that are described as practices in which teachers engage. These are: (a) consideration of mathematics content at the core of instruction; (b) attention to student mathematical thinking and learning during teaching; (c) reasoning about effects of teaching on learning and suggesting improvement; and (d) analysis of instruction that goes beyond description and includes elaboration and interpretations of the teaching/learning process (Kersting et al., 2010). Usable knowledge is situated knowledge, in that it is elicited in the process of making sense of teaching. These four facets integrate both aspects of knowledge included in Ball and colleagues' definition (such as specialized content knowledge and knowledge of content and students), and other aspects that relate to analysis of practice, such as elaboration and interpretation, and considerations of ways teaching can be improved to facilitate learning of mathematics. Because of this integration, we argue that this type of knowledge is a key aspect of learning-from-teaching competencies. In studies involving practicing teachers, those with superior usable knowledge for teaching mathematics showed higher quality of instruction and more learning by their students (Kersting, Givvin, Thompson, Santagata, & Stigler, 2012).

In addition to usable knowledge, learning from teaching requires teachers to hold student-centered beliefs. These manifest themselves as appreciation for their students' mathematical thinking as legitimate even when it deviates from formalized mathematical thinking. It also encompasses the belief that teachers need to give space to children's thinking in the classroom—both to foster their students' learning and to engage in teaching improvement.

Lastly, learning from teaching requires a disposition towards professional learning and continuous improvement that motivates teachers to engage in practices that facilitate on-the-job learning.

As illustrated in Fig.1, knowledge, beliefs, and dispositions interact with practices that we hypothesize are central to learning from teaching. First, teachers need to act on their student-centered beliefs and create opportunities in their lessons for students to think creatively about mathematical ideas and problem solutions. They also need to create opportunities for students to make their thinking visible.

Second, we conjecture that learning from teaching involves utilizing evidence of student thinking for making claims about the effectiveness of one's instructional decisions (vis-a-vis a specific learning goal for students). These evidence-based reflections also involve using the results of this analysis to plan next steps and to make modifications that might improve student learning and teaching over time.

As illustrated by the bi-directional arrows in the diagram, each component of learning-from-teaching competencies interacts with and reinforces one another. For instance, while knowledge, beliefs, and dispositions inform decisions that teachers make in practice, student-centered practices can reinforce student-centered beliefs. Similarly, evidence-based reflections on the effectiveness of teaching choices might contribute to the development of usable knowledge and foster dispositions towards continuous improvement. Before making evidence-based reflections, teachers must create opportunities to see student thinking in practice. Finally, evidence-based reflections can prompt teachers to exercise further efforts in their teaching to make student thinking visible, so that multiple forms of evidence are accessible and reflections can be more productive.

This conceptual model is supported, in part, by our prior studies. For example, we have shown that structured opportunities for developing teachers' evidence-based reflections improve aspects of usable knowledge (see Santagata, Zannoni, & Stigler, 2007; Santagata & Angelici, 2010; Santagata & Guarino, 2011). We have also shown that pre-service teachers who are given opportunities to learn to conduct systematic and student-centered analyses of teaching, use more student-centered instruction than a control group. Those pre-service teachers, who learned to attend to the details of student mathematical thinking when reflecting on the successes or missed opportunities in their teaching, also elicited, pursued, and built on student mathematical thinking during instruction (Santagata & Yeh, 2014). Finally, longitudinal case studies of teachers, who participated in the project summarized in the present manuscript, provide qualitative evidence for the interplay among different components of learning-from-teaching competencies (Santagata, 2014; Santagata & Yeh, 2016). These case studies followed teachers up to two years after graduation from the teacher preparation program and triangulated evidence from a measure of usable knowledge, videos of classroom lessons, and post-lesson interviews.

Study Goals and Design

As we discussed in the introduction, this study addresses two challenges the field has identified as characterizing teacher preparation: time constraints and the theory-practice divide. We take the position also maintained by other scholars (Bransford, Brown, & Cocking, 1999; Hiebert, Morris, Berk, & Jansen, 2007) that given the limited time, teacher preparation should prioritize preparing prospective teachers to learn from their teaching. We propose a framework for reflecting on and analyzing practice that is intended to facilitate the development of learning-from-teaching competencies. We hypothesize that because of the interplay among different components of learning-from-teaching competencies, engagement in the core practice of systematic reflection and analysis promotes evidence-based reflections as well as the development of usable knowledge for teaching mathematics, student-centered beliefs, and dispositions towards continuous improvement. In addition, it promotes a student-centered approach to teaching. Finally, this approach to teacher preparation, by foregrounding systematic analysis of practice, contributes to closing the theory-practice divide. Teachers develop new knowledge as this becomes relevant to teaching situations; knowledge thus is linked to practice from the start.

As mentioned above, the study's fundamental question is: Should time be set aside in teacher preparation to deliberately teach reflection and analysis of practice, or do pre-service teachers learn to reflect and analyze practice (and also develop other interrelated learning-from-teaching competencies) through more typical experiences, such as coursework and fieldwork?

To answer this question, we compared two groups of pre-service teachers. One attended a math methods course structured around a Learning to Learn from Teaching (LLMT) curriculum. The other attended a course structured around the most frequently adopted mathematics education textbook in the United States, “Elementary and Middle School Mathematics: Teaching developmentally” by Van de Walle, Karp, & Bay-Williams (2013). We called this course the Mathematics Methods Course (MMC). Below, we describe each curriculum and highlight their differences.

The Learning to Learn from Teaching Curriculum

The two-quarter elementary mathematics methods course was structured around two mathematics domains: whole numbers and rational numbers. The former was discussed during the first quarter of the course and the latter during the second quarter. The nature and sequence of activities and tasks in the course were carefully chosen to provide opportunities for decomposition and approximation of practice through various representations of teaching (Grossman, et al., 2009).

Below we draw on Grossman’s framework to describe the pedagogy of the course. Grossman and colleagues define decomposition of practice as involving “breaking down practice into its constituent parts for the purposes of teaching and learning.” (p.2058). They refer to approximations of practice as “opportunities for novices to engage in practices that are more or less proximal to the practices of a profession.” (p.2058). Finally, they discuss the affordances that different representations create for making key aspects of practice visible to novices.

As mentioned above, central to our approach is mathematics teaching that builds on student thinking. Several core practices are entailed in this approach: eliciting and responding to student ideas; designing and sequencing instructional episodes that build conceptual understanding as the basis for procedural fluency; using multiple mathematical representations to support students' development of conceptual understanding; and orchestrating classroom discussions.

The curriculum included videos and samples of student work as representations to decompose these core practices. It also included activities that purposely reduce complexity to approximate gradually the authenticity of the work of teaching. Similarly, decomposition and approximations were used to teach reflection and analysis practices. Pre-service teachers had opportunities to learn their essential elements, to practice them with the support of their instructor and peers, and to incorporate them in their routines, as they did for other teaching core practices.

Pre-service teachers began by learning to attend to the details of student thinking and to infer student understanding by drawing from research conducted in Cognitive Guided Instruction (Carpenter, Fennema, Franke, Levi, & Empson, 1999; Empson & Levi, 2011) and the New Zealand Numeracy Projects (http://nzmaths.co.nz/numeracy-projects).

Video clips of children solving mathematics problems were used to engage pre-service teachers in discussions about students' solutions and mathematical understanding. Then, they learned to elicit student thinking by conducting a videotaped interview with a student who was asked to solve a series of mathematics problems. These activities isolated types of teacher-student interactions that may occur in a classroom when a teacher approaches a student during seatwork to discuss the student's solution to a math problem. They were also relevant to classroom discussions and instances when teachers engage with a student's solution to unveil the student thinking for herself/himself and the class. These activities thus served as approximations of more complex practices. They allowed the instructor to highlight the work entailed in listening carefully to student explanations; asking questions that make student thinking visible; and, connecting student ideas and solution strategies to specific mathematical understandings and placement on a learning trajectory (i.e., the core practice of eliciting and building on student ideas).

Subsequently, pre-service teachers refined their questioning and analysis skills by working in triads and taking turns at leading the planning, teaching, and commentary of a videotaped mini-lesson with a small group of students. The instructor limited the number of students they taught to three or four. She also required them to work in triads and help each other throughout the process of planning, enacting, and reflecting. In short, she simplified the task for them—again approximating activities that typically engage the whole class and decomposing them into key instructional moves. Detailed lesson plans, student work from the mini-lessons, and videos of the mini lessons constituted the representations of practice intended to support pre-service teacher learning. Work on these lessons involved the core practices of using multiple mathematical representations to support students' development of conceptual understanding and orchestrating classroom discussions. The "5 Practices for Orchestrating Productive Mathematics Discussions" book (Stein and Smith, 2011) was used as an additional resource. The focus on systematic reflection and analysis was achieved by frequent discussions to compare and contrast pre-service teachers' images of mathematics teaching and learning (in most cases characterized by traditional, teacher-centered practices) with new standards-aligned conceptions (Santagata, Jovel, & Yeh, 2016).

During the second quarter, we extended the work on core practices of teaching and analysis by focusing on rational numbers. This quarter also included participation in collaborative planning and a focus on designing and sequencing instructional episodes that build conceptual understanding as the basis of procedural fluency. Activities were organized around a week-long unit on fractions. Through the use of digital video, copies of student work, and a blog, pre-service teachers followed the course instructor into a fourth grade classroom to analyze the nature of children's understanding represented in their work. They built on these analyses to provide suggestions on lesson plans, next steps, revisions, and solution of problems of practice that may arise. Through collaboration, the processes entailed in teaching a unit that is responsive to student thinking, was unveiled and made explicit. Every step in the process was highlighted, unpacked, and discussed. Various options for designing a lesson in the unit were considered based on students' experiences and work from the prior lesson. Options were assessed by hypothesizing ways different mathematical tasks and activities might support student development of mathematical ideas. This experience culminated with a final in-person visit to the classroom during which pre-service teachers observed the last lesson in the sequence and debriefed with the instructor.

The work of decomposing complex teaching practices and the sequence of approximations prepared pre-service teachers to design and teach mathematics lessons in their placement classrooms. Here they were confronted with the real work of teaching, but with the support of their mentor teacher.

The Lesson Analysis Framework

A central feature of the course curriculum was a framework that the first author developed collaboratively in the context of prior projects. This framework was designed to guide reflections and analyses of classroom lessons (Santagata, et al., 2007). The Lesson Analysis Framework structures analyses around four fundamental steps: (1) Specifying learning goals; (2) conducting empirical observations to collect evidence of student learning; (3) generating hypotheses about features of the teacher's instruction that promoted student learning; and (4) using hypotheses to propose improvements in teaching.

By grounding analyses of teaching in evidence of student learning, the framework pushes prospective teachers to become aware of tacit knowledge they frequently bring to teacher preparation from their own experiences as students. Sometimes, teachers take for granted teaching methods that have become ubiquitous in the United States. They do not realize that for each decision by a teacher, many other options have been rejected (Stigler & Hiebert, 1999; Gallimore, 1996). The Lesson Analysis Framework encourages evidence-based reasoning about the effectiveness of instructional decisions and consideration of alternative teaching strategies that might improve student learning.

Below, we briefly describe each step of the framework and explicate how, through engagement in this process, pre-service teachers learn important aspects of teaching and can use them to improve their practices over time. They can use this framework to analyze their own lessons and those by other teachers. In both cases, video serves as critical support as it allows them to view segments several times and helps ground discussions in observable evidence.

Specifying lesson learning goals. Most teachers begin a lesson with a learning goal for their students. Often this learning goal is broad and is described in terms of what students will be able to do, with little attention to the key ideas that students may develop as they engage in the lesson activities. Broadly-defined goals may lead to assessment of students' performance as either right or wrong because gradual progress towards the goal is not recognized. Take for example a lesson using base-ten blocks to model a take-from subtraction word problem requiring regrouping. A teacher could phrase the lesson learning goal as: students will be able to solve a variety of subtraction word problems. The teacher could specify the goal further: Students will model the mathematical context of the problem (modeling with mathematics). Using base-ten blocks, students will represent the value of each digit as determined by its position in the numeral; for example, 853 is 8 hundreds, 5 tens, and 3 ones (place value notation). Using base-ten blocks, students will demonstrate how the unit in a place-value position can be exchanged for 10 units in the position to the right (i.e., regrouping concept). When teachers specify the key ideas of a topic, they are more likely to attend to the nuances of student thinking and learning during instruction and when evaluating student work (Morris, Hiebert, & Spitzer, 2009).

Conducting empirical observations to collect evidence of student learning. After specifying the lesson learning goals, teachers can collect different forms of evidence to assess whether students made progress towards these goals. To be useful, evidence needs to be (1) focused on what students can demonstrate (not on what the teacher did effectively); and (2) relevant to the learning goals of that instructional episode. Frequently, teachers make the error of attributing conceptual understanding to students when the evidence they collected is only about the final answer ("Her answer is correct"), the operation ("She is subtracting"), or the tool ("She used base-ten blocks"). Instead, they should pay attention to student solution strategies (e.g., accurate representation of the value 853 using base ten blocks and accurate regrouping) and what those strategies tell us about the student's understanding (e.g., understanding of place value) (Bartell, Webel, Bowen, & Dyson, 2012). In addition, teachers need to know how to collect evidence and to plan for the collection of multiple sources of evidence, so that their claims of teaching effectiveness can be stronger.

Generating hypotheses about features of the teacher's instruction that promoted student learning. This practice involves the ability to make claims about teaching approaches, activities, and tasks that promoted student progress towards the learning goal. This practice is tied strongly to the previous two practices. Claims that lead to improvements in teaching are focused on students' progress towards the stated learning goal and are based on relevant evidence. Without a strong foundation in the previous two practices, teachers are likely to make claims that are unjustified; they focus on assumptions they have about the effectiveness of certain instructional strategies rather than on evidence of student learning.

Let us return to the subtraction situation above. What are some conjectures that could be generated? Imagine the following scenario captured in a lesson video. After reviewing the ways that three-digit numbers can be formed using different combinations of one-, ten-, and hundred-blocks, the teacher guides the students to the solution of a problem in which reconfiguring blocks helps to solve a subtraction problem that requires regrouping. The class discusses the solution process together, and the teacher answers students' questions. She then structures student work in pairs and asks them to use base-ten blocks to solve a new word problem that can be represented by the number sentence 853 – 65. The video then follows the teacher as she walks around students' desks to monitor their solution processes. Different situations arise. For example, the teacher walks by two students' desks, and the viewers can hear one student taking the lead at solving the problem correctly. The student is using the blocks to represent the two numbers and to model regrouping, while the other student passively watches his classmate. Another pair of students works collaboratively but fails to represent 853 correctly with the blocks. Yet, another pair represents the two numbers correctly, but fails to use the blocks to regroup correctly and it is unclear whether they understand the 1-for-10 relationship in the trade.

After viewing the video, pre-service teachers are asked to evaluate the existing evidence to discuss aspects of the lesson that worked, aspects that seem problematic, and situations in which further evidence would be necessary to make justified claims. Significant clips are reviewed, sometimes multiple times, and analyzed closely to attend to the details of student solution steps and to the language that students use to describe their thinking. Based on this close analysis of existing evidence, pre-service teachers make several hypotheses about why the teacher's initial modeling of a subtraction problem with regrouping was sufficient for some students to solve a new problem collaboratively with a peer, but ineffective for other students who failed to use the blocks to regroup a tens block into 10 ones. The pre-service teachers might review the segment of the video when the teacher introduced regrouping and discuss aspects of the teacher's modeling that were clear and aspects that might have caused the kind of confusion seen in the students' clip. They may discuss also how, in at least one case, it was unclear whether the quiet student had an understanding of his classmate's strategy and how the design of the pair activity did not require both students in the pair to take an active role. Based on these analyses, a series of hypotheses are generated, and these lead to the next step.

Using hypotheses to propose improvements in teaching. Once pre-service teachers have generated learning-goal oriented and evidence-based hypotheses, they are ready to propose next steps and improvements in teaching; they are ready to make decisions about aspects of teaching they want to maintain and aspects they want to change. This is an important step in the process because it links the systematic analysis of teaching and learning to subsequent action. A final visit to the subtraction situation above: one possible instructional revision would be to slow down the modeling phase of the lesson and involve more students in the solution of a few problems together as a class, before moving on to the activity in pairs. Another suggestion might involve using more precise language when explaining how base-ten blocks can be used to regroup. Yet, another suggestion might involve providing a greater window into students' individual thinking by asking students to first solve the problem individually and then share their solution strategy with a classmate.

Each step in the process involves teachers' knowledge of mathematics, mathematics teaching, and of children's learning; hence, each step has the potential to generate new usable knowledge that is directly relevant to practice. In this sense, engaging in this process provides a mechanism for future teachers to continue learning about teaching over time—a process for career-long learning. For example, when engaging in the analysis of students' thinking and learning, teachers use their knowledge of mathematics and of children's understanding of mathematics to attend to the details of student thinking and to interpret their writing, words, or gestures as evidence of specific mathematical understanding or misunderstanding. The practice of attending and interpreting at the same time provides opportunities for teachers to ask their students further questions about their mathematical thinking and to learn something new. As we discussed above, this systematic, student-centered analysis of teaching also has the potential to foster student-centered beliefs and practices.

In the next section, we describe the control group methods course and highlight differences between the two courses.

The Mathematics Methods Course Curriculum and Differences between LLMT and MMC Courses

Pre-service teachers enrolled in the MMC spent a considerable amount of time learning about children's development of mathematical understandings and the same core practices included in the LLMT course. This curriculum did not include opportunities for pre-service teachers to learn to reflect and analyze practice in systematic ways. Instead, the control group learned about mathematics topics in the K-6 curriculum other than whole numbers, operations, and rational numbers providing opportunities to develop content and pedagogical content knowledge for a broader range of content in elementary mathematics. Fig. 2 below summarizes the differences between the LLMT and MMC courses math topics and lists the core practices included in each course.

[INSERT FIG. 2 APPROX. HERE]

We chose to design the control group course this way because we wanted both courses to promote the same vision of mathematics classroom practices and learning, but to differ with respect to systematic opportunities to develop dispositions, knowledge, and practices for analyzing and learning from teaching. Consequently, how pre-service teachers engaged in the core practices differed in the two courses.

MMC participants focused mostly on enactment of core practices (whereas this enactment was always interspersed with opportunities to closely analyze practice and reflect on one's own teaching for LLMT participants). In both MMC and LLMT courses, approximations and decomposition characterized many of the activities. Instructors reduced complexity by designing tasks that diminished the demands of enactment in real classrooms and pointed out essential elements of core practices. Drawing on Grossman's conception of decomposition, LLMT participants had more extensive opportunities to learn about core practices' essential elements because they were often asked to identify them, reflect on them, and analyze them for their impact on student learning experiences. These opportunities simultaneously served as activities for the instructor to approximate and decompose the core practice of systematic reflection and analysis. Video for these participants served as an essential representation of practice; it allowed them to review the enactment of a certain practice multiple times and engage in collaborative analysis using the Lesson Analysis Framework described above.

For example, both the LLMT and MMC groups learned about the core practice of eliciting and responding to student mathematical ideas. Both groups were given the opportunity to focus on questioning as an essential component of this core practice that allows teachers to make student thinking visible. Using mathematical tasks developed by the New Zealand Numeracy Project, both groups of pre-service teachers interviewed three students at their fieldwork site and practiced asking questions to elicit their thinking. They then wrote a report that documented students' solution strategies and inferred students' placement on a trajectory of number sense development.

What distinguished the learning experience of the LLMT group, were two additional activities. Before enacting the questioning moves, LLMT participants reviewed and analyze several video clips portraying an adult interviewing a child solving a mathematical problem. They then collaboratively analyzed the interviewer's questioning moves and the child's solution strategy as indicative of her/his mathematical understanding. When they completed their own interviews, they videotaped themselves, and then reflected on their own questioning. They were asked to attend closely to how they framed the questions, to students' responses and challenges, and to how interviews could be improved. The Lesson Analysis Framework framed this self-reflection process. In addition, the LLMT instructor facilitated a group activity, in which pre-service teachers shared their students' solutions and responses with their colleagues, and discussed ways they revealed different understandings of number sense ideas as well as instances in which their questioning strategies fell short.

Both courses were taught by instructors with several years of experience both in classroom teaching and mathematics methods instruction. While the authors worked closely with the instructors of both courses and planned activities that reflected the goals of each course, they were not directly involved in teaching the courses and were invested in creating successful courses for both groups of pre-service teachers.

Now that we have described in detail the two different mathematics methods course experiences, we introduce the study specific research questions, its methods, and findings.

Study Research Questions

The idea of preparing teachers to learn from teaching implies examining learning that carries on after graduation—exploring whether mathematics method instruction was effective in preparing participants to learn from their practice over time. This requires a longitudinal study that extends into the first years of professional practice. In this study, we focus on examining whether mathematics methods instruction may place teachers on an upward trajectory with regard to learning-from-teaching competencies.

As we discussed above, time for teacher preparation is always limited. We were thus interested in examining whether a focus on deliberate and systematic reflection and analysis is necessary to develop knowledge that is readily usable in practice (thus also addressing the theory-practice divide) as well as to develop other facets of learning-from-teaching competencies included in our conceptual framework (see Fig. 1). While, as reviewed above, prior research points to the benefits of engaging pre-service teachers in reflection and analysis of practice, there is a dearth of research examining the relative advantage of a curriculum centered on systematic reflection and analysis compared to more typical experiences that are offered by mathematics methods instruction. In addition, most existing studies have focused only on one specific aspect of learning-from-teaching competencies (in most cases, attending to student thinking). In this study, we utilized several measures to capture the components of learning-from-teaching competencies illustrated in Fig.1, including a measure of usable knowledge that is predictive of student learning (Kersting, et al., 2012).

Hence, we centered the study on the following research questions: (1) Do pre-service teachers develop learning-from-teaching competencies if these are not deliberately focused on during teacher preparation? (2) What is the impact of a methods course that includes structured opportunities to develop such competencies compared to a more typical mathematics methods course? Specifically, does this course facilitate pre-service teachers' development of usable knowledge for teaching mathematics? Does it facilitate the development of student-oriented beliefs about mathematics teaching and learning? Does it help pre-service teachers to structure their practices around student thinking? Does it facilitate the development of evidence-based self-reflection on teaching?

Study Methods

Participants

Sixty-two pre-service teachers, enrolled in a one-year post-bachelor elementary teacher preparation program at a large public west-coast university, were recruited to participate in the study prior to the beginning of the teacher preparation program.

When applying to the program, students could choose between a 9-month credential program and a 15-month Masters' of Arts program. Students in the two programs followed the same curriculum during the 9 months. Masters' of Arts students attended additional courses during the summer prior and after the 9 months. Using a stratified sampling method, applicants from the credential-only and MA programs pools were randomly assigned to attend one of the two math methods courses (LLMT or MMC courses) while they followed the same curriculum for all the other courses. Two study participants left the program; pre and post-test data were thus available for 60 pre-service teachers, 29 from the LLMT and 31 from the MMC group.

Participants attended the fieldwork component of the program while participating in the research project. The fieldwork was not specifically tied to the mathematics methods course and gave prospective teachers opportunities to observe and practice the teaching of all subjects. Fieldwork consisted of two placements: one in a lower-elementary and the other in an upper-elementary classroom. Pre-service teachers were placed in the same classroom for the fall and winter quarter. During the fall, they mainly observed their master teacher, completed brief teaching assignments requested by their methods instructors, and assisted with teaching tasks (e.g., they may lead a small group activity). During the winter, they gradually took over the class, first by teaching mathematics and subsequently all other subjects. The master teacher was always present in the classroom. The extent to which the master teacher was involved in planning and teaching varied across placements. In the spring quarter, pre-service teachers were assigned to a new placement. Some pre-service teachers began their placement in a lower-elementary classroom and then they switched to an upper-elementary; others did the opposite. During the fall/winter placement, almost 60% of pre-service teachers were placed in pairs (with two pre-service teachers placed in the same classroom). During the spring placement, 26% were placed in pairs.

Placements were organized by the program coordinator, regardless of participants' group membership; thus, they varied in terms of teaching approaches and school structures and cultures. Since capturing this variability in objective ways is quite complex and would require systematic observations of the master teachers and their interactions with the student teachers (which the project did not have the resources to complete), we assumed that participants' fieldwork experiences varied quite a bit and that random assignment would address this variability. (Future studies with more resources can identify and match participants in the experimental and control groups.) Although we know that fieldwork experiences constitute relevant settings for teacher variability . Background characteristics of the participants and their placement information are summarized in Table 1.

[INSERT TABLE 1 APPROX. HERE]

Measures

The study included several measures that captured participants' background and the learning-from-teaching competencies outlined above. In addition, observations were conducted to monitor implementation of both courses.

Participants completed a background survey prior to the beginning of the methods course and the other measures both prior to and after completion of the methods course. The knowledge measures were administered during the last day of the math methods course, while the beliefs measure was administered three months later at the end of the school year. Measures were spread out in time mainly to avoid having participants fill out too many surveys during the busy end of the quarter period. Pre-service teachers received a $90 gift card for completing the surveys. The instructional quality and the reflection measures were completed at the end of the math methods course as part of the performance assessment that is required for acquiring a teaching credential. Measures are described in detail below.

Background survey. Participants completed a background survey that asked about their gender, age, ethnicity, college major and minor, prior teaching experience, and number of mathematics content courses and mathematics methods courses taken in college.

Observations and Field Notes. We designed an observation protocol to capture the extent to which the two courses were implemented as planned and were focused on the steps of analysis of practice outlined above. Trained observers attended all sessions for both the MMC and LLMT courses. They collected materials the instructors presented or distributed in class and took notes on all tasks and whole-class discussions.

Classroom Video Analysis Survey. To assess the development of usable knowledge for teaching mathematics, we administered the Classroom Video Analysis (CVA) Survey (Kersting, 2008). The instrument consists of a series of brief video clips (varying between one to three minutes in length) that serve as prompts for teachers' written commentaries on mathematics teaching. Teachers view the clips via an interactive, Web-based platform and are asked to respond to the prompt, "discuss how the teacher and the student(s) interact around the mathematical content." Kersting and her collaborators (2010) argue that the instrument elicits teacher knowledge from the context of viewing and making sense of mathematics teaching as portrayed in the video clips. As such, the instrument measures knowledge that becomes available to teachers as they teach, rather than factual knowledge that they may have acquired, but is not connected to the practice of teaching. Previous studies have investigated the relationships between teachers' performance on the CVA, their instructional quality, and their students' learning and found significant and positive relationships as summarized above (Kersting et al., 2012).

In addition, we see usable knowledge for teaching mathematics as a fundamental component of learning-from-teaching competencies and were thus particularly interested in examining whether the two courses would impact differently the development of this type of knowledge. The video clips used in the present study were drawn from lessons that focused on key ideas within the topic domains of whole numbers and fractions, which are key topics in the elementary-school mathematics curriculum. Previous studies (Kersting, 2008; Kersting et al., 2010) reported an internal item consistency (i.e., Cronbach α) between .89 and .93. In the current study sample, inter-item reliability was lower but still satisfactory, ranging from .71 to .81.

Pre-service teachers watched the clips on a password-protected online portal. For each clip, a brief description of the broader context was provided (i.e., class grade level, mathematics topic, math task, description of the lesson events prior to the selected clip) followed by the prompt: "*Discuss how the teachers and student(s) in the clip interacted around the mathematical content.*" Pre-service teachers were asked to type their commentaries into a textbox with no character or word limit. Responses were saved onto the portal's server and later downloaded by the researchers onto an excel sheet.

Pre-service teachers' written responses to each clip were scored along four dimensions: *mathematics content, suggestions for improvement, student thinking*, and *depth of interpretation* (Kersting, 2010). Each response received a score of 0, 1, or 2, depending on the extent to which it reached the specificity and/or depth defined by each rubric. Two independent raters, blind to the group membership of each teacher, scored all comments. Inter-rater reliability, measured as percent agreement, for all four rubrics was computed initially, at midpoint, and at the end of scoring. It ranged from 84% to 88 % across the four rubrics and time points. In case of disagreements between raters, a third rater reviewed the comments and made the final decision.

The *CVA mathematics content* scoring dimension (CVA-MC) measured the degree to which teachers considered and commented on the mathematics at hand when making sense of the instructional episodes shown in the video clips. Such mathematical considerations could focus on student thinking or understanding, provide the basis for evaluating observed teaching strategies or a rationale for suggestions for improvement, or could further explore the mathematics. A score of 0 was assigned when comments did not include specific math content. Comments were assigned a score of 1 when mathematics was discussed in the response, but only identified or described the observable mathematics in the video. Comments obtained a score of 2 when an analysis of a specific aspect of the mathematical content was present. A mathematical analysis was usually evidenced by the introduction of a mathematical concept(s) or idea not visible in the video.

The *CVA student thinking* (CVA-ST) scoring dimension measured the extent to which commentaries expressed concern for student mathematical thinking/understanding. This concern usually took the form of either a judgment about student understanding or a comment on students' opportunities to learn in the clip. Comments that did not address student thinking were assigned a score of 0. Responses obtained a score of 1 if there was a direct link between the analysis of student thinking/understanding and the mathematics visible in the clip. Responses received a score of 2 if they synthesized, analyzed, or generalized student thinking in a mathematical way or offered mathematical explanation(s) or justification(s) that supported the synthesis/analysis/generalization.

The *CVA suggestions for improvement* (CVA-SI) scoring dimension measured the extent to which comments included suggestions to improve instruction or pedagogy by providing a clear alternative to the instruction shown in the clip. Comments that provided no suggestions or vague suggestions received a score of 0. A score of 1 was assigned to commentaries that included a clear suggestion for improvement addressing the mathematics. Comments were assigned a score of 2 when there was a clear suggestion addressing specific mathematics.

The *CVA depth of interpretation* (CVA-DOI) scoring dimension measured the extent to which a comment included interpretations and justified opinions of teaching or learning portrayed in the clip. A commentary received a score of 0 if it was simply a descriptive account of what was in the clip or if it included a vague or broad interpretation that was not substantiated. Responses obtained a score of 1 when they included a substantiated interpretation in the form of a rationale, evidence, or justification for the interpretation. Finally, responses were assigned a score of 2 when they offered an analysis that included several interpretations along a common theme.

The video clips we used in this study are not publicly released, but similar clips with teacher comments and scoring examples are accessible at: http://www.teknoclips.org/examples/.

Integrating Mathematics and Pedagogy (IMAP) Belief Survey. The second facet of learning-from-teaching competencies we measured involved pre-service teacher beliefs. Specifically, we were interested in student-centered beliefs. Participants completed the web-based IMAP survey (Ambrose, Clement, Philipp, & Chauvot, 2004). The survey portrays complex classroom situations involving students. Respondents are asked to analyze and respond to these situations through a combination of multiple-choice and short-answer questions. Four beliefs were selected to be measured as part of this study: 1) Mathematics is a web of interrelated concepts and procedures (and school mathematics should be too); 2) If students learn mathematical concepts before they learn procedures, they are more likely to understand the procedures when they learn them. If they learn the procedures first, they are less likely ever to learn the concepts; 3) Children can solve problems in novel ways before being taught how to solve such problems. Children in primary grades generally understand more mathematics and have more flexible solution strategies than adults expect; and 4) During interactions related to the learning of mathematics, the teacher should allow the children to do as much of the thinking as possible.

We expected Beliefs 1 and 2 to increase for both LLMT and MMC groups as both courses focus on a balance of procedural and conceptual knowledge and on how conceptual understanding can be fostered. We were interested in examining whether the LLMT curriculum and its focus on the analysis of student thinking might increase Beliefs 3 and 4 more than the MMC curriculum did.

Each belief was measured through a set of questions. For example, pre-service teachers were asked to evaluate student solution strategies to a problem, to analyze the connection between strategies, and to decide which strategies to share in a class discussion and why. In a separate question, they were asked to consider different strategies for multi-digit addition (including solutions that represented the standard algorithm and solutions that were more conceptual in nature) and to select and justify an order for discussing these strategies in the classroom. In another section, they were asked to watch brief video clips of a student-teacher interaction, to discuss the role of teacher, who was portrayed to be very leading, and how they would have structured a lesson, including whether they would have built on students' thinking.

Responses were assigned a score of 0 if interpreted as showing no evidence of the belief and the highest score possible (3 or 4) if they indicated strong evidence of the belief. The survey authors chose to use a maximum score of 4 for Belief 3 because of the wider range of teacher responses and levels of evidence that these provided. The combination of scores obtained in questions targeting a certain belief provided an overall score for that belief.

Two independent raters, blind to group membership, scored all responses. Inter-rater reliability, measured as percent agreement, for all sets of questions was computed initially, at midpoint, and at the end of scoring and ranged from 80% to 96 % across questions and time points. In case of disagreements, a third rater reviewed the response and made the final scoring decision.

Student-Centered Teaching. The third aspect of learning-from-teaching competencies we examined was student-centered classroom practice. This was defined as practice that, by creating opportunities for students to make their thinking visible, allows teachers to build on student thinking during instruction, to gather evidence of student learning as they teach, and to learn about student mathematics understandings over time. The data source were videos collected as part of the Performance Assessment for California Teachers – Teaching Event (PACT-TE), which pre-service teachers completed towards the end of the second quarter of math methods instruction. The PACT-TE consists of a portfolio that includes five tasks: (1) context for learning; (2) planning instruction and assessment; (3) instructing students and supporting learning; (4) assessing student learning; and (5) reflecting on teaching and learning. For the purpose of this study, we selected materials pre-service teachers turned in for task 3: instructing students and supporting learning. Specifically, we reviewed video clips of a lesson pre-service teachers taught and reflection commentaries they wrote about the clip(s). Pre-service teachers were given the following prompt about videotaping their instruction:

> Provide one or two video clips of no more than fifteen minutes total. Select clip(s) that demonstrate how you engage students in understanding mathematical concepts and in participating in mathematical discourse. (You may select conceptual understanding either as the primary focus of instruction or integrate it with the development of your students' understanding of a computation or procedure.) The clip(s) should include interactions

among you and your students and your responses to student comments, questions, and needs.

The video clip(s) only allowed us access to approximately 15 minutes of the pre-service teachers' mathematics teaching. Nonetheless, we felt that was sufficient to uncover teachers' interactions with students. In addition, these 15-minute clip(s) were chosen by pre-service teachers to represent their best practices, thus we felt they exemplified well what pre-service teachers considered their best efforts to include student participation in classroom discourse.

Videos were coded using a coding system developed in the context of a previous study that made use of PACT-TE videos as a source of data (Santagata & Yeh, 2014) and captured the extent to which student thinking was made visible and pursued by the teacher during instruction. The coding system included three levels of sophistication: (1) Low sophistication: Student thinking only minimally visible; (2) Medium sophistication: Student thinking made visible; and (3) High sophistication: Student thinking made visible and pursued (see Table 2). Two independent observers coded each video and disagreements were resolved through discussion. Inter-rater reliability calculated as percentage of agreement between two scorers was 83%. Disagreements were resolved through discussion.

[INSERT TABLE 2 APPROX. HERE]

Self-Reflection on Teaching. Finally, the last aspect of learning-from-teaching competencies we examined was self-reflection. As part of the PACT-Teaching Event, participants completed written comments on the video clip(s) of their teaching. We used responses to the following prompt to assess their abilities to reflect on their own practices:

Describe the strategies you used to monitor student learning during the learning task shown on the video clip(s). Cite one or two examples of what students said and/or did in the video clip(s) or in assessments related to the lesson that indicated their progress toward accomplishing the lesson's learning objectives.

We examined pre-service teachers' evidence-based reflections. We assessed the extent to which they utilized evidence of student learning to assess the effectiveness of their teaching; whether the examples they cited were directly connected to the stated lesson learning goals; and whether they attended to the mathematics content of their lessons. Coding categories are described in Table 3.

[INSERT TABLE 3 APPROX. HERE]

Procedures and Statistical Analyses

First, the effect of random assignment was checked by comparing groups at pre-test. To examine CVA pre-test performance, one-way ANOVAs were run. To examine IMAP pre-test performance, Chi-square analyses were conducted.

Second, to test for treatment effects on CVA performance, repeated-measures analysis of variance was computed controlling for pre-test scores. Subsequently, when necessary, paired T-test and one-way ANOVAs were run to test for pre/post-test differences within groups and group differences at post-test.

To assess changes in pre-service teachers' beliefs as measured by IMAP, for each belief measured, change scores were calculated separately for LLMT and MMC participants. Pre-service teachers were categorized into one of three groups: (1) Pre-service teachers whose belief scores did not increase or decreased; (2) Pre-service teachers whose belief scores increased by one level (small increase); and (3) Pre-service teachers whose belief scores increased two or more levels (large increase). Percentages of pre-service teachers that belonged to each category were then compared between groups and across the four measured beliefs using Chi-square statistics. As a follow up of chi-square results, ordinal regression procedures were run for each belief to test whether group membership predicted score changes at post-test.

Student-centered teaching and self-reflection were measured only at the end of the courses. To compare groups, one-way ANOVAs were used for teaching scores and chi-squares for reflection categories.

Findings

Group Equivalence

Equivalency of groups was checked after random assignment. Groups were compared in terms of average age, prior teaching experiences, and performance on CVA, and IMAP surveys at pre-test. No significant differences were found between groups, except for prior teaching experience. A significantly larger number of LLMT participants (n=19) reported having had prior teaching experiences compared to MMC participants (n=11) [X^2 (2, N= 30) =5.199, p= .023]. The survey question also asked about the type and duration of prior experiences. Since all prior experiences were limited to brief substitute teaching positions and tutoring, we did not think they impacted substantially the type of classroom teaching expertise that we examined in this study, though we cannot exclude this possibility.

Observations and Field Notes

Field notes confirmed the differences in the courses that we have outlined above. The LLMT course provided ample opportunities for systematic analysis of practice centered on the four steps of the Lesson Analysis Framework. In contrast, the MMC course presented only a few instances overall in which pre-service teachers were given opportunities to analyze teaching and these did not include any deliberate feedback on the process of analysis from the instructor.

Comparing Pre-Service Teachers' Usable Knowledge for Teaching Mathematics

As described above, usable knowledge of mathematics teaching was measured by scoring commentaries participants provided in response to 10 video clips of instructional episodes. Each commentary received a score ranging from 0 to 2 on each of four subscales: (1) Math Content; (2) Student Thinking; (3) Depth of Interpretation; and (4) Suggestions for Improvement.

A significant time*condition interaction was found for the total score participants obtained on the CVA survey [$F(1,58) = 14.644$, $p = .000$]. Both groups improved significantly over time, but the LLMT group outperformed the MMC group at post-test [$F(1,59) = 7.361$, $p = .009$]. Group means and standard deviations for the total score and on each subscale by group are reported in Figure 3 below. This indicates that while both courses were successful at improving pre-service teachers' usable knowledge for teaching mathematics, the LLMT course provided some advantages to pre-service teachers. We examine this advantage by presenting the scores obtained by the two groups on each subscale along with sample participant responses.

[INSERT FIG. 3 APPROX. HERE]

Significant interactions were detected for all CVA subscales except for the Math Content sub-scale for which the interaction was not significant (p = .061; See Table 4). The LLMT group outperformed the MMC group at post-test on the Student Thinking [$F(1,59) = 7.262$, $p = .009$] and Suggestions for Improvement [$F(1,59) = 8.665$, $p = .005$] subscales, while group differences on the Depth of Interpretation subscale were borderline not significant [$F(1,59) = 3.614$, $p = .062$]. No significant differences were found at posttest on the Math Content subscale. Within-group paired T-test analyses revealed that both groups improved significantly over time on all subscales, except for the Student Thinking subscale. Only the LLMT group improved significantly over time on this scale [$t(28) = -5.320$, $p = .000$], while the MMC group's pre and post-test averages were almost identical (see Table 5).

In sum, LLMT and MMC courses were both successful at developing aspects of usable knowledge that are related to coherent interpretation of teaching and learning that goes beyond a recollection of distinct features (i.e., Depth of Interpretation subscale) and attention to mathematics content of instruction (i.e., Math Content subscale). A deliberate focus on reflection and analysis made a difference, instead, for the other two aspects of usable knowledge—aspects that, as we will discuss below, are particularly important in our conceptualization of learning from teaching: attention to and analysis of student thinking and considerations of suggestions for improvement.

[INSERT TABLE 4 and 5 APPROX. HERE]

Sample responses from study participants are reported below to clarify these findings. The CVA clips are not publicly released to avoid broad familiarity with the instrument items. To limit the number of clips that are described in publications, we share participants' comments that we have published in earlier work (Yeh & Santagata, 2015) to illustrate group differences. Comments are in response to a clip drawn from a lesson on subtraction focused on developing understanding of regrouping, the exchanging of 1 unit in a place-value position for 10 units in the position to the right. The following comment represent a typical response provided by the study participants at the beginning of teacher preparation when they completed the CVA pre-test.

> This is a really good strategy for the children when working on subtraction problems for the first time. The girl is learning about borrowing and carrying over numbers when you need to subtract. By using both writing visually and 3-D board pieces, a child understands how math works and can use it for real-life experiences.

This comment focuses on the teacher instructional choices—using visuals and manipulatives—as effective for student learning. However, the comment does not include a discussion of how these facilitate student learning and does not attend to the details of how the student engages with the materials. In addition, no teaching improvements are discussed.

The following two comments illustrate typical responses participants provided at the end of teacher preparation when they completed the CVA post-test. The first comment was provided by a MMC participant and represents a typical response from that group.

> I really like the teacher-made worksheet that really helps students make sense of the numbers there. [Since] students see that there are no more pieces in the ones place to take away, students learn to borrow numbers from the tens place. I definitely like the use of concrete material as well (using block manipulatives). The teacher is great at helping the student guide her thinking and comment on what she is doing, as well as guide her thinking. The teacher is patient and waits to see what she gets.

Compared to the pre-test response, this comment displayed more details in the description of the events in the clip and an attempted to interpret what is observed (i.e., Depth of Interpretation subscale). The comment also includes a deeper analysis of the mathematics at the center of this instructional episode by mentioning place value (i.e., Math Content subscale). Nonetheless, the respondent does not attend to the details of the student solution strategy or to her mathematical reasoning. She is pleased with the teacher moves and is satisfied with what she sees as evidence of student learning. No suggestions for improvement are offered.

The second post-test comment was provided by a LLMT participant and illustrates one type of common response from that group.

> It's a good idea for the student to see borrowing with a visual representation, but it seems that the teacher is simply walking her through the steps of the algorithm. I don't feel that the student would have much recall of what is happening. The student understands place

value and that you can split up numbers like 138 into hundreds, tens, and ones but she doesn't seem to understand that tens can be split up into ones, hundreds into tens, etc. In this case, I think physical manipulatives would be more valuable than drawings and I think the student would benefit greatly from working with a partner who understands that tens can be split up into ones.

This response focuses on the details of the student's solution strategy and infers student understanding or lack thereof from these details. The discussion of the teacher's choices is directly related to the close analysis of the student's actions. A distinction is made between mathematical ideas that are evident in the student's actions –understanding that numbers can be split into hundreds, tens, and ones—, and ideas that the student seem to struggle with—not understanding that tens can be split into ones and hundreds into tens. In addition, the comment includes two suggestions for improvement—using physical manipulatives and using more capable peers as supports for learning. In this response, improvements in all four subscales of usable knowledge are evident.

The third comment, also from a LLMT participant, illustrates another common post-test response from this group:

> I really liked this worksheet. It allowed students to work on the problem both in number form and [provides] a visual representation with the base ten blocks. The student did not contribute much to the conversation, and it seemed like the teacher was doing most of the talking. Because the student did not talk much, it was unclear whether she was understanding what she was doing when she was "borrowing" in the subtraction problem.

This response does not focus on a detailed analysis of student thinking and learning; it focuses instead on lack of evidence about whether the student understood the key mathematical ideas focused on in the instructional episode. In this sense, it reflect the respondent's concern for student learning. In the conceptualization of usable knowledge, this type of concern is considered important and indicative of teachers' focus on student thinking and learning. This comment also implicitly suggests that the teacher could do more to make the student thinking visible. In this case, a more sophisticated comment would have elaborated further this suggestions and offered specific ideas for improvement.

Comparing Pre-Service Teachers' Beliefs

As described above, four beliefs were measured: (1) Mathematics is a web of interrelated concepts and procedures; (2) If students learn mathematical concepts before they learn procedures, they are more likely to understand the procedures when they learn them. If they learn the procedures first, they are less likely ever to learn the concepts; (3) Children can solve problems in novel ways before being taught how to solve such problems. Children in primary grades generally understand more mathematics and have more flexible solution strategies than adults expect; and (4) During interactions related to the learning of mathematics, the teacher should allow the children to do as much of the thinking as possible.

Prior to teacher preparation, most participants did not strongly hold all four beliefs. IMAP survey median scores revealed the weakest evidence of Belief 4. The IMAP survey specifically examines participants' beliefs about the role of the teacher in the teaching-learning process and the degree to which they believe teachers should share the mathematics authority in the class with their students. The Belief 4 median score for the entire sample was 0 at the beginning of teacher preparation. Participants had median scores of 1 for Belief 1 and median scores of 1 or 2 for Beliefs 2 and 3 (depending on the group).

Although groups were not significantly different at pre-test, two differences were detected that need to be considered when examining changes in beliefs over time. At pre-test for Belief 2, the LLMT group had a median score of 2, while the MMC group had a median score of 1. Conversely, at pre-test for Belief 3, the LLMT group at pre-test had a median score of 1, while the MMC group had a median score of 2. This suggests that at pre-test, the LLMT group showed stronger evidence than the MMC group of holding the belief that if students learn mathematical concepts before they learn procedures, they are more likely to understand the procedures when they learn them. MMC participants were more likely than the LLMT group to believe that children can solve problems independently before being taught by the teacher.

For Beliefs 1 and 2, both groups' scores changed from pre to post-test. Consistent with our hypothesis, we expected both LLMT and MMC groups to have greater beliefs in the importance of conceptual understanding and the interrelatedness of mathematical concepts and procedures as both courses focused on these ideas. This finding reassures us that both courses were successful at shifting pre-service teachers' beliefs that are essential to many of the core practices centered on conceptual understanding and advocated by the new standards (National Governors Association Center for Best Practices, 2010).

The score for Belief 3 improved for the LLMT group; it did not change for the MMC group. As we discussed earlier, a group difference was detected at pre-test with the MMC showing a stronger alignment to this belief. At post-test, the difference between groups was not significant. The change in median score for the LLMT group, on the other hand, shows that the LLMT curriculum supported a shift toward the belief that children's problem solving strategies can be considered legitimate and sometimes children should be asked to solve problems independently without showing them how to solve them. For a third of LLMT participants, this belief score increased by two levels or more. Alignment to Belief 4 improved for both groups, but a group difference was detected at post-test. We discuss this below. Figure 4 reports median IMAP scores prior to and at the end of teacher preparation by belief and for each group.

[INSERT FIG. 4 APPROX. HERE]

To assess group differences, groups were compared using the change score categories described in the methods section to control for scores at pre-test. We ran Chi-square analyses and none of the group differences were statistically significant. When comparing groups only at post-test, LLMT participants showed stronger evidence of alignment with Belief 4 than their counterparts [X^2 (3, N= 51) = 7.944, p= .047]. As we discussed earlier, both groups at pre-test shared the view that math instruction should be teacher directed. At post-test, while both groups changed this belief towards more student-centered instruction, LLMT participants were more likely to indicate that teachers should give their students opportunities to share their approaches to solving problems during instruction. To further explore these findings, we ran ordinal regressions for all four beliefs and found that for Belief 4, MMC pre-service teachers were less likely than LLMT ones to change (Wald $\chi^2(1) = 4.195$, p =.041).

In sum, the LLMT course offered an advantage towards shifting pre-service teachers' Beliefs 3 and 4. As discussed above, these beliefs are key aspects of learning-from-teaching competencies. We conjecture that they may result in teaching decisions that create opportunities for students to unveil their thinking. This in turn creates available evidence to reflect on what worked and what did not and on instructional improvements.

Comparing Pre-Service Teachers' Classroom Teaching

As described above, classroom practices were examined at one point in time only at the end of the methods courses. Participants were assigned a score ranging from 1 to 3 depending on the extent to which they made visible and built on student thinking during instruction. Data were available for 52 pre-service teachers (27 treatment, 25 control) who gave us permission to access their PACT material. Although the average group score for LLMT participants (*M*=2.15, *SD*= 0.718) was higher than for the MMC participants (*M*=1.84, *SD*= 0.746), the group difference was not significant [$F(1,50)$ =2.302, and p = .135] (See Figure 5). A look at the score frequencies reveals that 36% of MMC participants, compared to 19% of LLMT participants, received a score of 1. In these participants' videos of teaching, student thinking was only minimally visible; interactions were mostly teacher-directed; questions were centered on correct answers; and student limited talk focused on final answers to mathematics problems. Other teachers in this group (i.e., those who obtained a score of 2) were observed to give more opportunities to students to contribute to the unfolding of the lesson and began to inquire into how students arrived at answers.

The average score of 2.15 for the LLMT participants indicates that LLMT teachers gave opportunities to students to contribute to the content of the lesson and asked "how" questions. In addition, a third of them (33.3% compared to 20% in the MMC group) received a score of 3. In these teachers' videos, student thinking was made visible, and interactions built on student input consistently, with frequent questioning that focused on student ideas and explanations.

[INSERT FIG. 5 APPROX. HERE]

Comparing Pre-Service Teachers' Self-Reflections

Data were available for 48 participants (26 treatment and 22 control) who gave us permission to access their Teaching Event material and completed the reflection task following the prompt described above. As mentioned in the methods section, reflection comments were scored according to three dimensions: (1) focus (students or self); (2) mathematics (specific or general); and 3) learning goal (connected or not connected) (see Table 3). Group differences were detected for all three rubrics and were significant for the focus [X^2 (1, *N*= 48) = 6.41, *p*= .011)] and learning-goal [X^2 (1, *N*= 48) = 4.321, *p*= .038)] rubrics, while for the math specificity rubric the difference was borderline not significant [X^2 (1, *N*= 48) = 3.475, *p*= .062)].

While 93% of LLMT participants centered their reflections on their students, only 62% of MMC participants did so. When they focused on student outcomes, pre-service teachers pointed to specific math problems that their students solved as evidence that their lesson was effective:

> Most students in my low/middle group could accurately represent the dimes as ten and the pennies as ones (either by direct modeling, showing numbers, or place value blocks).
>
> These students clearly identified their addition groups as 5 and 2 and formed the addition sentence 'five plus two equals seven'.

These participants responded to the PACT-Teaching Event's explicit request to cite one or two examples of what students said and/or did as evidence of student progress towards the lesson learning goal. Other participants' responses were focused, instead, on their actions as teachers. This is exemplified by the excerpt below:

> In order to help the student meet their learning goal, I gave ample opportunities for them to practice using the appropriate academic language as they explained and justified their responses to the comparison problems.

In addition, 77% of LLMT participants commented on the effectiveness of their lessons with evidence of student progress towards the lesson learning goal(s). For example, one of the participants stated the following learning goal for her lesson:

> The students will solve addition problems representing the joining of two groups and will exercise the writing of addition sentences. Students will identify an addition sentence as suitable for the representation of a certain problem and will create their own addition sentences when representing addition problems.

Then she drew on several instances of her students' addition sentences to show their progress towards the goal. For example, she wrote:

> "...Student U who represented the first addition problem of 7+2=9 by drawing seven circles and two hearts."

Only 33% of MMC participants connected their evidence to the learning goal of the lesson, while the majority mentioned successes that were not connected directly to their stated learning goal(s). For example, one of the MMC participants stated the following learning goal for her lesson:

> "Students will solve the perimeters and areas for all the possible rectangles for a given area to understand the relationship between perimeter and area for rectangles and that rectangles that have the same area can have different perimeters."

Then she commented on her students' use of communication strategies and precise mathematical language without specifically referring to perimeter or area of rectangles or to students' evidence of correct solution of perimeter and area problems.

Finally, while 65% of LLMT participants' evidence of success was related specifically to the learning of mathematics, only 38% of MMC participants discussed math-specific learning (see Figure 6). Mathematics-focused comments explicitly and specifically referred to the mathematics topic focused on in the lesson, such as the following:

> "These students correctly represented the first problem by forming the addition sentence 3+3=6, the second problem by forming the addition sentence 2+4=6, and the third problem by forming the addition sentence 5+1=6."

General comments mentioned only in broad terms what students were learning, such as in the following example:

> During the lesson most of the students were engaged in the lesson because of the different name cards that they had already dealt with. It was important that the students were able to connect what they had previously learned about the scales and connect it with the new things they were learning.

[INSERT FIG. 6 APPROX. HERE]

Summary of Findings

Table 6 summarizes the findings. First, we documented whether knowledge and beliefs changed significantly over time within each group. There were two aspects of knowledge and beliefs that did not change for the MMC group. Specifically, MMC participants on average did not improve in their ability to analyze student mathematical thinking as portrayed in the video clips. In addition, MMC participants' Belief 3 (i.e., Children can solve problems in novel ways before being taught how to solve such problems. Children in primary grades generally understand more mathematics and have more flexible solution strategies than adults expect) did not change over time.

On the right side of the summary table, we documented whether group differences were detected at completion of the methods course. We found that the LLMT course provided a significant advantage to PSTs in terms of their ability to analyze student thinking (as mentioned above, this did not change for the MMC group) and to provide suggestions for improvement—

key aspects of usable knowledge for teaching mathematics at the core of our conceptualization of learning-from-teaching competencies.

In terms of beliefs, LLMT and MMC participants did not differ significantly. Although, for Belief 4 (i.e., During interactions related to the learning of mathematics, the teacher should allow the children to do as much of the thinking as possible), the LLMT group was more likely to change than the MMC group. As discussed above, Belief 4 is a central belief in our conceptualization of learning-from-teaching competencies.

Group differences were also detected in pre-service teachers' self-reflections. When reflecting on the effectiveness of their teaching, LLMT participants, on average, focused significantly more on the student(s) and identified evidence that was connected to the lesson learning goal to a greater extent than the MMC participants did. Focusing on evidence of student learning that is relevant to the lesson learning goal, is important within our conceptualization of learning-from-teaching competencies. We conjecture that evaluation of this evidence may prompt teachers to assess the extent to which their instructional decisions were successful and the modifications and next step that they can take in the future.

[INSERT TABLE 6 APPROX. HERE]

Discussion

This study asks whether spending time during teacher preparation on experiences focused on systematic analysis of practice promotes the development of learning-from-teaching competencies that would not be developed through more typical mathematics methods instruction. The findings suggest that teacher preparation experiences centered on systematic analyses of teaching create opportunities to develop important components of learning-from-teaching competencies. Specifically, we found that when the mathematics methods course did not include a specific focus on analysis of student thinking, pre-service teachers did not learn to pay close attention to student mathematical thinking, interpret evidence of student learning and infer the degree of student understanding. In addition, the experiences included in the LLMT course developed, to a greater degree than the MMC course, future teachers' ability to suggest instructional improvements. This is a core practice in learning from teaching that predicts student learning (Kersting, et al., 2012).

We conjecture that the development of these highly specified practices was facilitated by a combination of activities that were integrated in the mathematics methods course: engagement in cycles of analysis through video-enhanced discussions; collaborative planning, implementation, and reflection on teaching; live observation and co-constructed interpretations and considerations of next steps. Specifically, we argue that these activities simplified the work of teaching by reducing its complexity. They also integrated a variety of supports and representations that revealed the constituent components of a complex system of practice. In Grossman et al.'s (2009) terms, the LLMT activities offered approximations of reflection and analysis practices typically unavailable to teachers. Analysis of students' thinking in the midst of instruction is challenging, particularly for novice teachers. Self-reflection after instruction is not always systematic as teachers have limited time to plan lessons for the next day. The LLMT activities provided opportunities to analyze student thinking, as it occurred in the midst of instruction, and to make hypotheses about the impact of teaching on learning, through the use of video segments that could be reviewed multiple times and discussed. These activities also gave pre-service teachers time to learn to look at samples of student work and attend to details that are important, when reflecting on the effectiveness of instructional choices, or when planning subsequent instruction. In other words, these activities facilitated the decomposition of the essential elements of a systematic reflection and analysis that later can be carried out more easily and in more routinized ways.

Findings related to pre-service teachers' change in beliefs also suggest advantages of the LLMT over the MMC course. While for LLMT participants all the measured beliefs changed over time, for MMC participants Belief 3 (i.e., Children can solve problems in novel ways before being taught how to solve such problems. Children in primary grades generally understand more mathematics and have more flexible solution strategies than adults expect) remained unchanged. We hypothesize that exposure, through video, to instructional episodes in which children are shown solving math problems in innovative ways (without their teacher providing step-by-step instructions) facilitated changes in PSTs' believes, along with opportunities to try out and reflect on lessons centered on children problem solving. We attribute the significant ordinal regression finding for Belief 4 (i.e., During interactions related to the learning of mathematics, the teacher should allow the children to do as much of the thinking as possible) to similar differences in opportunities offered by the two courses. The LLMT participants were exposed to video examples of teaching in which children take a central role and actively construct their learning. They were also asked to unpack student learning goals when planning and student thinking when reflecting on practice.

Finally, findings related to pre-service teachers' reflections on their own teaching are particularly noteworthy. They indicate transfer of knowledge and skills from the teacher preparation setting to practices that have the potential to be generative for continuous improvement, once teachers enter the profession. Similarly to the CVA findings, both MMC and LLMT courses were successful in teaching PSTs to pay attention to the mathematics content in their teaching. The LLMT approach provided an advantage in centering reflections on students and in attending to evidence of student learning connected to the lesson learning goal.

Despite the fact that the PACT-Teaching Event reflection question explicitly asked for examples of evidence of student progress towards the lesson learning goal, 38% of MMC participants described instructional strategies they used instead of focusing on evidence of student learning (compared to 8% of LLMT participants). In addition, over 66% of MMC participants cited evidence that was not relevant to the lesson learning goals. These findings support our interpretation that unless specific activities are designed to focus attention on student thinking and learning, pre-service teachers default to reflecting on the effectiveness of their practices by assessing whether their instructional strategies were theoretically sound. Furthermore, they reflect on the effectiveness of their teaching reporting general impressions, rather than referring specifically to their learning goals for their students.

Group differences were not significant for student-centered teaching, although score frequencies highlight some differences that could be further explored. Our small sample size had low statistical power, which limited our ability to detect significant differences. It is also plausible that differences in usable knowledge, beliefs, and self-reflection did not quite transfer to differences in teaching, or that transfer to teaching takes more time. Examining effects on teaching practices during teacher preparation was complicated by the fact that pre-service teachers were not fully in charge of the class. They may have adapted to requests made by their master teachers or to student resistance to new practices. A more standardized teaching task, perhaps with a small group of pre-selected students, could have provided a better measure of effects on teaching.

This study contributes to the conceptualization of learning-from-teaching competencies we proposed in Figure 1. The focus on systematic reflection and analysis that the LLMT curriculum provided, through the use of the Lesson Analysis Framework, impacted pre-service teachers' self-reflections. Student-centered and learning-goal oriented reflections were more frequent in the LLMT group than the MMC group. This focus also impacted pre-service teachers' usable knowledge and beliefs, as expected by our model of learning-from-teaching competencies. As we discussed above, we conceive of knowledge, beliefs, and reflection as interrelated aspects of learning-from-teaching competencies that are connected by bi-directional arrows. In this study, specific aspects of knowledge and beliefs—those more closely linked to student mathematical thinking and to the consideration of instructional improvements, were impacted by a focus on systematic reflection and analysis. We did not include a measure of dispositions towards continuous improvement. Nonetheless, when commenting on the CVA video clips, many participants provided suggestions for improvement without specific prompting, suggesting a disposition towards improvement. Overall, aspects of knowledge and beliefs that were more closely related to the consideration of mathematics content benefited similarly from the LLMT and MMC curriculum.

As we discussed above, the curriculum focus on systematic reflection and analysis did not impact significantly student-centered classroom practices as one would have expected from our conceptual model. It would be interesting to explore whether combining our approach—centered on reflection and analysis, with other practice-based approaches—largely focused on enactment and routinization of core practices through repeated rehearsal (Kazemi, Ghousseini, Cunard, & Turrou, 2015), would result in stronger impact on classroom practices.

On the other hand, effects on classroom practices should be examined more fully by studying them longitudinally, as participants enter the profession. A central idea in our conceptual model is that the interplay among various components of learning-from-teaching competencies supports teacher learning over time and improves their instructional quality. We have some initial evidence (from case studies we followed for three years after graduation) that indicates that teachers need time to negotiate demands of particular settings before they can implement the kind of teaching they have learned to embrace during teacher preparation. While this is beyond the scope of this manuscript, we briefly discussed this issue in a separate publication (Santagata & Yeh, 2016). We also plan to compare teaching practices of case studies from the two groups in future work.

Notwithstanding these contributions, findings need to be considered with caution for several reasons. The LLMT curriculum was tested in one setting and there might be factors that have contributed to some of the positive outcomes that are specific to the setting rather than the curriculum. For example, the course instructors might have been particularly skilled at certain aspects of the curriculum or the sample of participating pre-service teachers might have been particularly responsive to certain curriculum features. Testing of the curriculum in different settings and of adaptations to local needs are needed to extend outcomes beyond this proof-of-concept study.

While this study provides preliminary evidence that spending time on systematic analysis of teaching during teacher preparation is worthwhile, several additional questions could be explored. These could address, for example, the most effective ways to integrate learning-from-teaching competencies in course activities, thereby saving time to discuss additional mathematics topics. Studies could explore which facilitator practices maximize learning, while using time effectively. Omission of topics like geometry and statistics from the LLMT curriculum may have disadvantaged LLMT participants when they taught these topics. In the current study, we did not include plans for examining this question. In addition, trajectories of sub-groups of participants could also be examined to understand differences in participants' learning beyond average impact and to study how participant learning in other settings, such as fieldwork placements, interacts with learning in the methods course. Testing of the curriculum with limited or no involvement of the researchers is needed to determine the scalability of this approach.

Finally, as mentioned above, to examine fully the question whether systematic analysis of practice during teacher preparation results in learning-from-teaching competencies on the job, a longitudinal study that follows participants into the first few years of teaching is required. Preliminary findings of such an analysis focused on three LLMT participants suggest that teachers continued to learn from their practices over a three-year span and that student-centered reflection was a key process that characterized continuous learning (Santagata & Yeh, 2016).

References

Ambrose, R., Clement, L., Philipp, R., & Chauvot, J. (2004). Assessing prospective elementary school teachers' beliefs about mathematics and mathematics learning: Rationale and development of a constructed-response-format beliefs survey. *School Science and Mathematics, 104*, 56-69.

Ball, D.L., & Forzani, F. (2009). The work of teaching and the challenge for teacher education. *Journal of Teacher Education, 60(*5), 497-511.

Ball, D. L., Sleep, L., Boerst, T. A., & Bass, H. (2009). Combining the development of practice and the practice of development in teacher education. *The Elementary School Journal, 109*(5), 458-474.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special?. *Journal of teacher education*, *59*(5), 389-407.

Bartell, T. G., Webel, C., Bowen, B., & Dyson, N. (2012). Prospective teacher learning: Recognizing evidence of conceptual understanding. *Journal of Mathematics Teacher Education.* Publish first on February 7, 2012.

Black, A. L., & Halliwell, G. (2000). Accessing practical knowledge: How? Why?. *Teaching and Teacher Education, 16*(1), 103-115.

Brouwer, N., & Korthagen, F. (2005). Can teacher education make a difference?. *American Educational Research Journal, 42(*1), 153-224.

Bransford, J.D., Brown, A. L., & Cocking, R. R. (1999). *How people learn*. Washington, DC: National Academy Press.

Carpenter. T., Fennema, E., Franke, M., Levi, L., & Empson, S. (1999). *Children's* Mathematics: *Cognitively Guided Instruction.* Portsmouth, NH: Heinemann.

Cochran-Smith, M. (2004). The problem of teacher education. *Journal of Teacher Education*, *55*(4), 295.

Cochran-Smith, M., & Lytle, S. (2001). Inquiry as stance. In A. Lieberman & L. Miller (Eds.), *Teachers caught in the action: Professional development that matters (*pp. 45–58). New York: Teachers College Press.

Darling-Hammond, L. and Bransford, J., 2005. Preparing Teachers for a Changing World: Report of the Committee on Teacher Education of the National Academy of Education.

Empson, S., & Levi, L. (2011). *Extending Children's Mathematics: Fractions and Decimals*. Portsmouth, NH: Heinemann.

Ermeling, B., Hiebert, J., & Gallimore, R. (2015). "Best practice" – The enemy of better teaching. *Educational Leadership*, *72*(8), 48-53.

Feiman-Nemser, S., & Buchmann, M. (1985). The pitfalls of experience in teacher education. *Teachers College Record, 87*(1), 53-65. http://www.tcrecord.org. ID Number: 677, Accessed: 9/16/2015

Gallimore, R. (1996). Classrooms are just another cultural activity. In D. Speece & B. Keogh (Eds.), *Research on classroom ecologies: Implications for children with learning disability* (pp. 229-250). Mahwah, NJ: Lawrence Erlbaum.

Grossman, P., Compton, C., Igra, D., Ronfeldt, M., Shahan, E., & Williamson, P. (2009). Teaching practice: A cross-professional perspective. *Teachers College Record*, *111*(9), 2055-2100.

Grossman, P., Hammerness, K., & McDonald, M. (2009). Redefining teaching, re-imagining teacher education. *Teachers and Teaching: Theory and Practice, 15*(2), 273-289.

Grossman, P., & McDonald, M. (2008). Back to the future: Directions for research in teaching and teacher education. *American Education Research Journal, 45*(1), 184-205.

Gutiérrez, R. (2013). Why (urban) mathematics teachers need political knowledge. *Journal of Urban Mathematics Education, 6,* 2, 7-19.

Hiebert, J., Gallimore, R., & Stigler, J. W. (2002). A knowledge base for the teaching profession: What would it look like and how can we get one?. *Educational Researcher, 31*(5), 3-15.

Hiebert, J., Morris, A. K., Berk, D., & Jansen, A. (2007). Preparing teachers to learn from teaching. *Journal of Teacher Education, 58(*1), 47-61.

Kersting, N. (2008). Using video clips as item prompts to measure teachers' knowledge of teaching mathematics. *Educational and Psychological Measurement, 68,* 845-861.

Kersting, N. B., Givvin, K.B., Sotelo, F.L., & Stigler, J.W. (2010). Teachers' analyses of classroom video predict student learning of mathematics: Further explorations of a novel measure of teacher knowledge. *Journal of Teacher Education, 61*(1-2), 172-181.

Kersting, N., Givvin, K., Thompson, B., Santagata, R., & Stigler, J. (2012). Measuring usable knowledge: Teachers' analyses of mathematics classroom videos predict teaching quality and student learning. *American Education Research Journal, 49*(3), 568-589.

Leinhardt, G., Young, K. M., & Merriman, J. (1995). Integrating professional knowledge: The theory of practice and the practice of theory. *Learning and instruction, 5(*4), 401-408.

McDonald, M., Kazemi, E., & Kavanagh, S. S. (2013). Core Practices and Pedagogies of Teacher Education A Call for a Common Language and Collective Activity. *Journal of Teacher Education, 64*(5), 378-386.

Morris, A. K., Hiebert, J., & Spitzer, S. M. (2009). Mathematical knowledge for teaching in planning and evaluating instruction: What can preservice teachers learn. *Journal for research in mathematics education*, *40*(5), 491-529.

National Governors Association Center for Best Practices, Council of Chief State School Officers (2010). *Common Core State Standards Mathematics.* National Governors Association Center for Best Practices, Council of Chief State School Officers, Washington D.C.

Putnam, R. T., & Borko, H. (2000). What do new views of knowledge and thinking have to say about research on teacher learning?. *Educational Researcher, 29(*1), 4-15.

Rosaen, C., & Florio-Ruane, S. (2008). The metaphors by which we teach: Experience, metaphor, and culture in teacher education. In M. Cochran□Smith, S. Feiman-Nemser, J., McIntyre, & K. Demers (Eds). *Handbook of research on teacher education: Enduring questions in changing contexts* (3rd ed., pp. 706–731). New York: Routledge.

Santagata, R. (2014). Video and Teacher Learning: Key Questions, Tools, and Assessments Guiding Research and Practice. *Beitraege zur Lehrerbildung,* 32 (2), 196-209.

Santagata, R., & Angelici, G. (2010). Studying the impact of the *Lesson Analysis Framework* on pre-service teachers' ability to reflect on videos of classroom teaching. *Journal of Teacher Education, 61*(4), 339-349.

Santagata, R., Gallimore, R., & Stigler, J. W. (2005). The Use of Videos for Teacher Education and Professional Development: Past Experiences and Future Directions. In C. Vrasidas & G.V.Glass (Eds.) *Current Perspectives on Applied Information Technologies (Volume 2): Preparing Teachers to Teach with Technology. Ch. 9,* pp. 151-167. Greenwich, CT: Information Age Publishing.

Santagata, R., & Guarino, J. (2011). Using Video to Teach Future Teachers to Learn from Teaching. *ZDM The International Journal of Mathematics Education, 43,* 1, 133-145.

Santagata, R., Jovel J., & Yeh, C. (2016). Learning to Analyze Teaching: A Study of Pre-Service Teachers' Group Conversations around Videos of Mathematics Teaching. In P.G. Rossi & L. Fedeli (Eds.). *Integrating Video into Pre-service and In-service Teacher Training.* Advances in Higher Education and Professional Development Book Series. Hershey, PA: IGI Global, Ch.6, Pp.114-126.

Santagata, R., Zannoni, C. & Stigler. (2007). The Role of Lesson Analysis in Pre-Service Teacher Education: An Empirical Investigation of Teacher Learning from a Virtual Video-Based Field Experience. *Journal of Mathematics Teacher Education, 10,*(2), 123-140.

Santagata, R. & Yeh, C. (2014). Learning to teach mathematics and to analyze teaching effectiveness: Evidence from a video- and practice-based approach. *Journal of Mathematics Teacher Education, 17,* 491-514.

Santagata, R. & Yeh, C. (2016). Characterizing Beginning Teachers' Competence and its Role in Professional Decisions and Practices. *ZDM, the International Journal of Mathematics Education, 48,*1, 153-165.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational researcher*, *15*(2), 4-14.

Smagorinsky, P., Cook, L., & Johnson, T. (2003). The twisting path of concept development in learning to teach. *Teachers College Record, 105*(8), 1399–1436.

Stein, M. K., & Smith, M. (2011). Practices for Orchestrating Productive Mathematics Discussions. Reston, VA: National Council of Teachers of Mathematics.

Stigler, J. W., & Hiebert, J. (1999). T*he teaching gap: Best ideas from the world's teachers for improving education in the classroom*. New York, NY: The Free Press.

Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2010). *Elementary and middle school mathematics: Teaching developmentally.* Boston, MA: Allyn & Bacon.

Wilson, S. M., Floden, R. E., Ferrini-Mundy, J. (2001). *Teacher Preparation Research: Current* Knowledge, *Gaps, and Recommendations*. Michigan State University, under the auspices of the Center for the Study of Teaching and Policy, University of Washington.

Table 1.

Participants' Background Characteristics

Background Variables	N	Percent
Gender		
Female	55	88.7
Male	7	11.3
Ethnicity		
Caucasian American	29	46.8
Asian American	26	41.9
Hispanic/Latin American	4	6.5
Multi-ethnic	1	1.6
Other	2	3.2
Fieldwork Placement		
Fall/Winter Paired Fieldwork Placement	37	59.7
Spring Paired Fieldwork	16	26
Credential Program Type		
Credential 9-Month Program	20	32.3
Credential Masters' of Arts 15-Month Program	42	67.7
Teaching Experience		
Prior Teaching Experience	30	51.6
No Prior Experience	32	48.4

Table 2.

Levels and Description of Instructional Quality Codes.

Levels	Description
Low sophistication Student thinking not or only minimally visible	The majority of the interaction in the classroom is directed from the pre-service teacher to the students. All pre-service teacher's questions have known/correct answers. Most student talk focuses on final answer to problems.
Medium Sophistication Student thinking made visible	The interaction in the classroom is mainly directed by the pre-service teacher, but students are given opportunities to contribute to the content of the lesson. The pre-service teacher begins to purposely ask "how" students arrived at an answer after a response is given. Further probing questions that facilitate student elaboration of their thinking are very limited.
High Sophistication Student thinking made visible and pursued	The interaction in the classroom builds on student input consistently. The pre-service teacher solicits students' ideas and explanations frequently.

Table 3.

Coding Dimensions and Categories for Pre-Service Teachers' Reflections on their Own Teaching

Dimensions	**Categories**	
Focus	*Student(s)*	*Self*
	Comment draws on evidence of student learning or difficulty to discuss the effectiveness of the lesson	Comment draws on teacher strategies to discuss the effectiveness of the lesson
Learning Goal	*Connected*	*Not Connected*
	Evidence of lesson effectiveness is directly related to lesson learning goal	It is not clear how evidence of lesson effectiveness is related to lesson learning goal
Mathematics	*Math*	*General*
	Comment includes reference to specific mathematics content	Comment is general and does not mention specific mathematics content

Table 4.

Repeated Measures Analysis of Variance for Classroom Video Analysis (CVA) Survey

Effect	***MS***	***df***	***F***	***p***	**Greenhouse-Geisser**	**Huynh-Feldt**
CVA – Math Content (MC)						
Time	557.471	1	85.278	.000	.000	.000
Time*Condition	23.983	1	3.662	.061	.061	.061
Error	6.537	58				
CVA – Student Thinking (ST)						
Time	85.609	1	21.568	.000	.000	.000
Time*Condition	43.609	1	10.987	.002	.002	.002
Error	3.969	58				
CVA – Suggestions for Improvement (SI)						
Time		1	61.820	.000	.000	.000
Time*Condition	52.904	1	9.662	.003	.003	.003
Error	5.476	58				
CVA – Depth of Interpretation (DoI)						
Time	424.600	1	67.992	.000	.000	.000
Time*Condition	25.666	1	4.110	.047	.047	.047
Error	6.245	58				

Table 5.

Classroom Video Analysis (CVA) Survey Score Means and SD by Subscales and Groups (LLMT n=29; MMC n=31)

	Group	Pre Mean (SD)	Post Mean(SD)
CVA Total	LLMT	12.83 (6.35)	30.31 (11.52)
	MMC	13.94 (6.97)	22.71 (9.74)
CVA MC	LLMT	2.0 (2.12)	7.21 (4.20)
	MMC	3.16 (3.12)	6.58 (3.37)
CVA ST	LLMT	4.14 (2.42)	7.03 (3.21)
	MMC	4.39 (2.38)	4.87 (3.01)
CVA SI	LLMT	1.21 (1.42)	5.90 (3.96)
	MMC	1.23 (1.59)	3.26 (2.58)
CVA DOI	LLMT	5.48 (2.50)	10.17 (3.65)
	MMC	5.16 (2.93)	8.00 (4.62)

Table 6.

Summary of Findings

	Significant Change Over Time		Significant Group Difference at Post-Test and Effect Size
	LLMT	MMC	
Classroom Video Analysis (CVA) Total Score	Yes	Yes	Yes .6895[1]
Mathematics Content (MC)	Yes	Yes	No
Student Thinking (ST)	Yes	No	Yes .6962
Suggestions for Improvement (SI)	Yes	Yes	Yes .7605
Depth of Interpretation (DoI)	Yes	Yes	No
IMAP			
Belief 1 Mathematics is a web of interrelated concepts and procedures	Yes	Yes	No
Belief 2 Learning mathematical concepts before procedures facilitates understanding	Yes	Yes	No
Belief 3 Children can solve problems in novel ways before being taught how to solve such problems.	Yes	No	No
Belief 4 During interactions related to the learning of mathematics, the teacher should allow the children to do as much of the thinking as possible.	Yes	Yes	Yes .086[2]

[1] Cohen's *d* coefficients are reported as measures of effect size for one-way ANOVA analyses.
[2] Nagelkerke Pseudo R-Square, which is often reported as a measure effect size for Ordinal Regression analyses.

Quality of Instruction		
Attending to and building on student thinking	n/a	No
Self-Reflections		
Focus Students versus Self	n/a	Yes .3654[3]
Math Specificity Math specific versus general	n/a	No
Link to Learning Goal Connected versus not connected	n/a	Yes .30

[3] Phi coefficients are used as measures of effect size for Chi-square analyses.

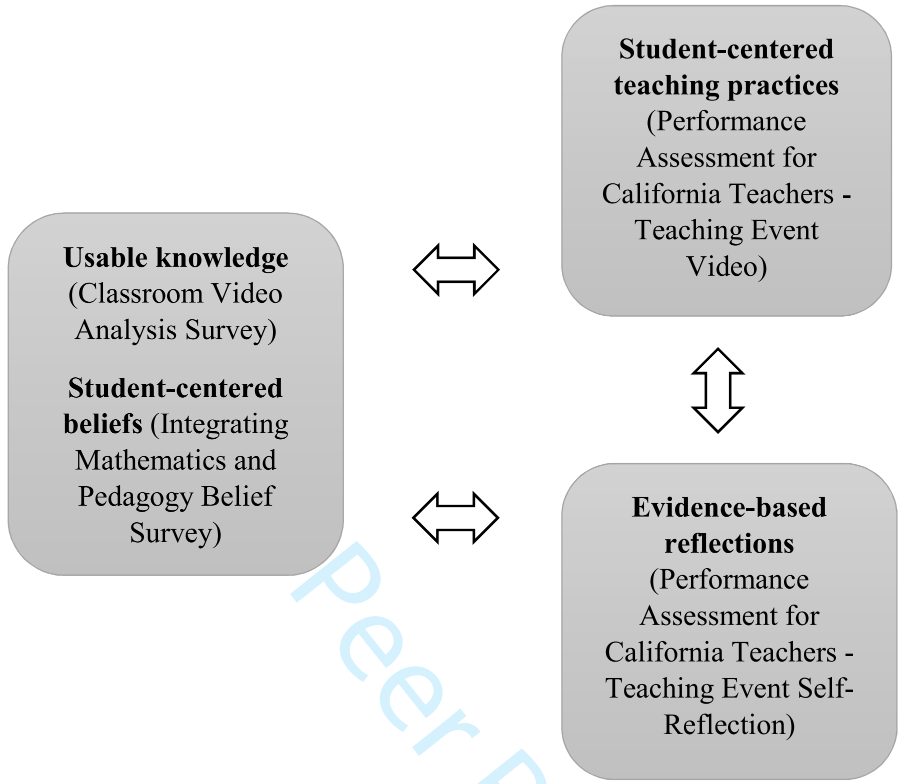

Fig. 1 Learning from teaching competencies and instruments used in this study to measure each aspect

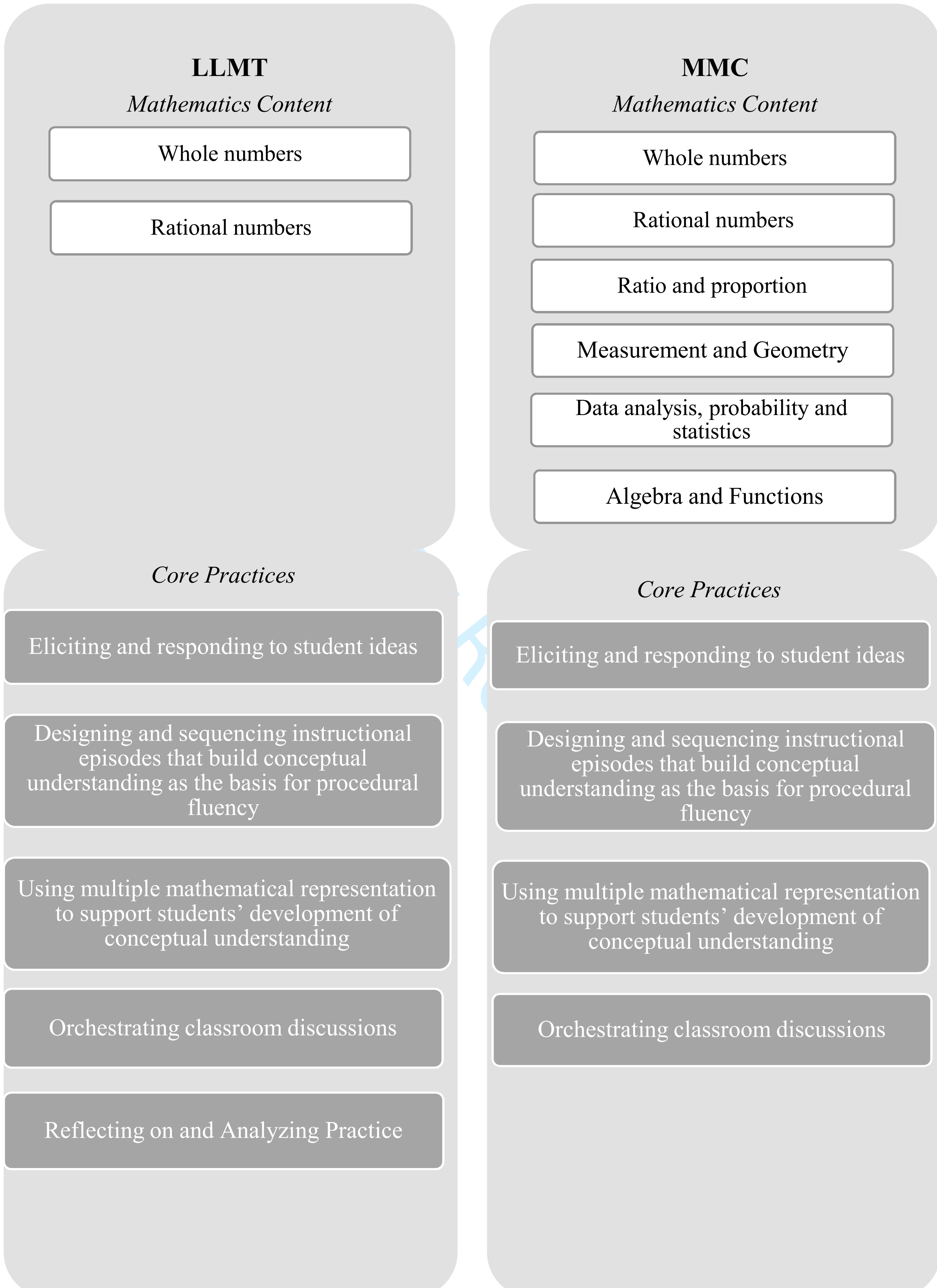

Figure 2. Mathematics Topics and Core Practices Included in the LLMT and MMC courses

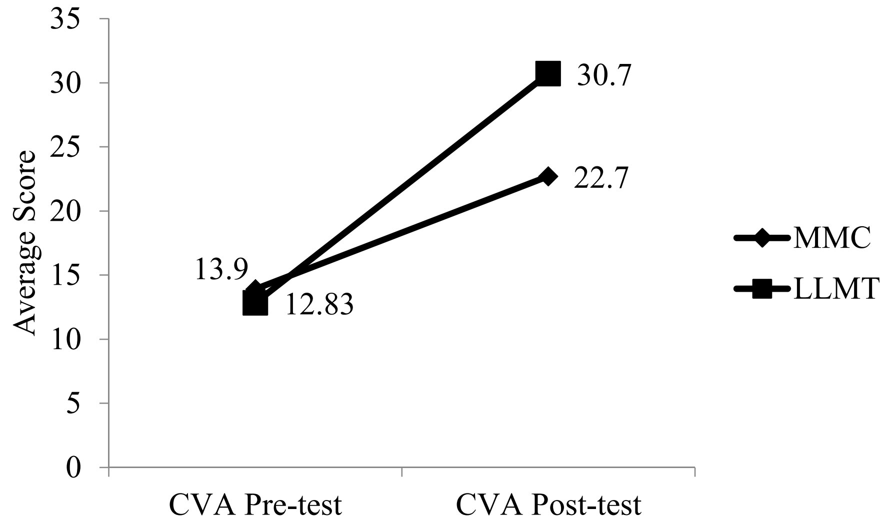

Figure 3. Average score prior to and at the end of teacher preparation on the CVA and by group

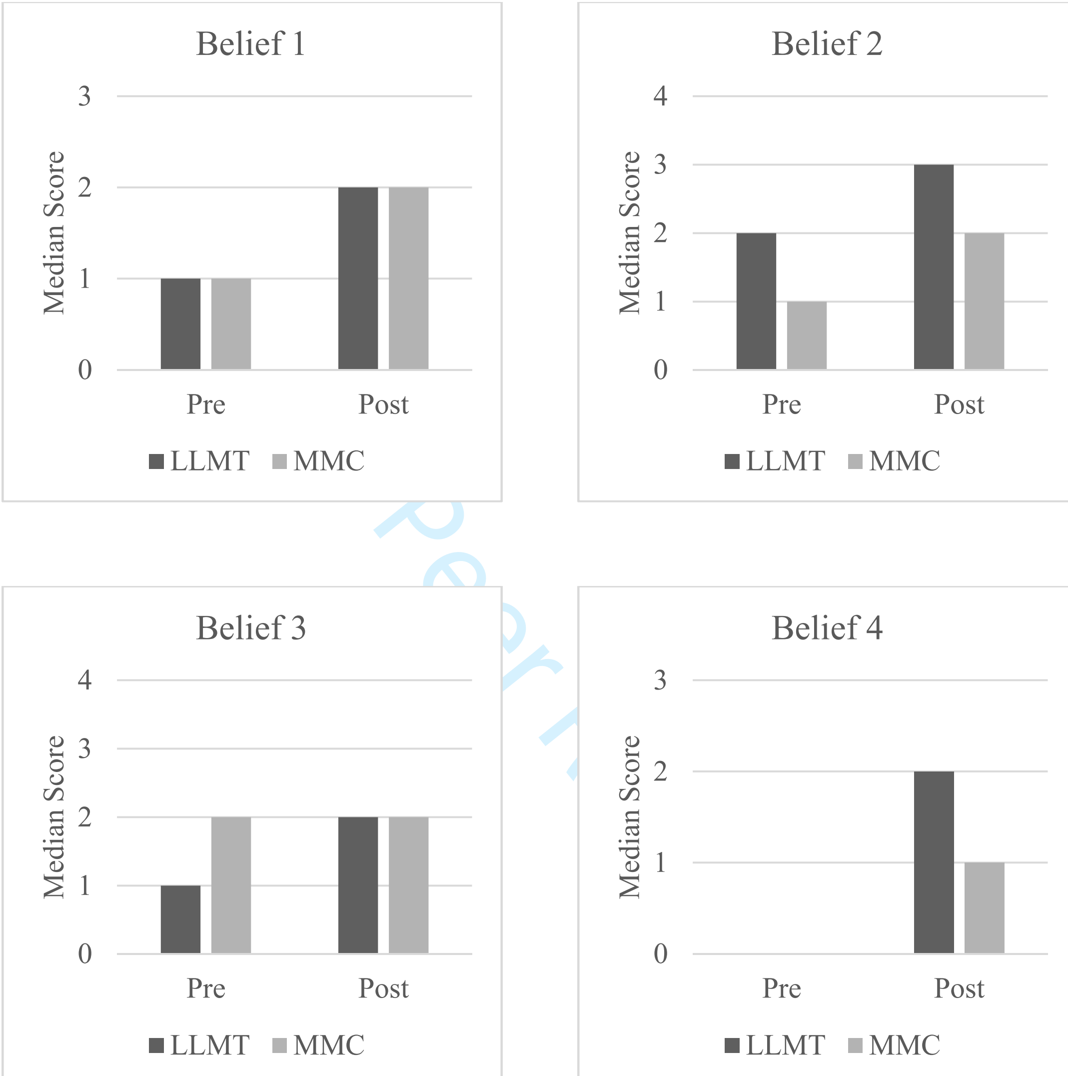

Figure 4. Median scores prior to and at the end of teacher preparation by IMAP belief and group

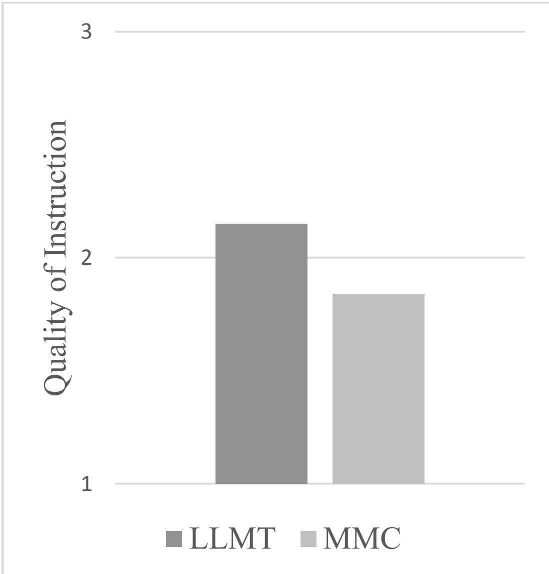

Figure 5. Average score on quality of instruction by group

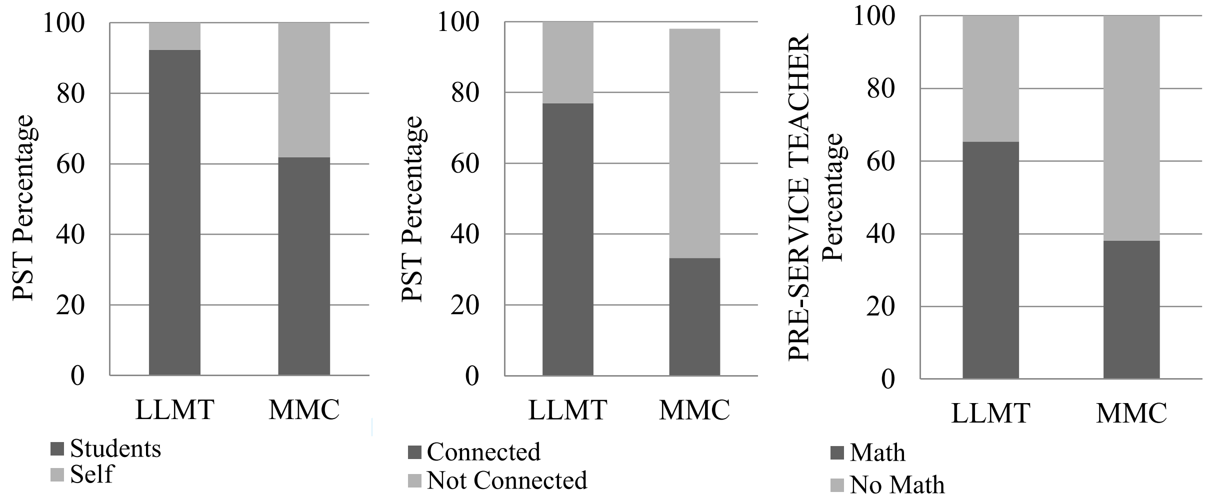

Figure 6. Percentage of pre-service teachers whose reflections on the effectiveness of their teaching centered on different aspects by focus, learning goal, and mathematics.