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Game-Theoretic Foundations of Monetary Equilibrium*

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Abstract

Monetary theorists have advanced an intriguing notion: we exchange money to make up for a lack of enforcement, when it is difficult to monitor and sanction opportunistic behaviors. We demonstrate that, in fact, monetary equilibrium cannot generally be sustained when monitoring and punishment limitations preclude enforcement—external or not. Simply put, monetary systems cannot operate independently of institutions—formal or informal—designed to monitor behaviors and sanction undesirable ones. This fundamental result is derived by integrating monetary theory with the theory of repeated games, studying monetary equilibrium as the outcome of a matching game with private monitoring.

Keywords: Social norms, repeated games, cooperation, payment systems.

JEL codes: E4, E5, C7

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1 Introduction

Why do societies rely on money? The traditional view is that monetary systems overcome barter and intertemporal trade frictions [10]. A recent view is that money makes up for a lack of enforcement in society: money has value if, by exchanging it, we outperform equilibria based on rules of *voluntary* behavior [1, 7, 9].

To develop this idea, imagine a group of strangers facing repeated opportunities to aid someone else, at a cost. Payoffs are maximized if everyone helps. However, monitoring is difficult, direct reciprocation is impossible, no one can self-commit to actions, and no coercion is possible: any help is *voluntary.* Establishing a norm of mutual support requires trust that help today will be later returned by strangers, which calls for enforcement of defections [6, 8]. Monetary trade requires none of this—argue monetary theorists: we exchange help for money instead of promises of future help *because* we cannot monitor and cannot sanction, individually or collectively, opportunistic behavior. What incentives can monetary systems provide that norms of voluntary behavior cannot reproduce?

We answer this question by adopting analysis techniques from the literature on repeated matching games to study monetary exchange. Such methodological innovation allows us to demonstrate that money cannot generally make up for a lack of enforcement (external or not) in society.

In fact, monetary equilibrium collapses if monitoring and punishment limitations hinder individual and group enforcement. In particular, without enforcement of spot trades monetary exchange cannot be sustained in large anonymous economies, i.e., the economies that are the bread and butter of monetary models.

In the model, a stable population of anonymous players is randomly divided in pairs in each period. In every encounter one subject can provide a benefit to the other by sustaining a small cost (= make a voluntary transfer). This interaction is infinitely repeated [3, 4]. Since players cannot build reputations and cannot adopt relational contracts, there is an incentive to behave opportunistically and avoid making transfers. May the introduction of symbolic objects (=tokens) support an outcome that is socially preferred?

Monetary theorists have offered a positive answer by imposing *quid-pro-quo* constraints: any transfer requires a concurrent payment, or else it fails.[1] In a simultaneous-moves game this amounts to assuming away any temptation to defect (= give nothing) by imposing mechanical punishment (= get nothing), so if money has value, monetary trade is incentive-compatible by design. *Quid-pro-quo* is a form of external enforcement: it converts the social dilemma into a coordination game by restricting the outcome set.

[1]E.g., see the survey in [10]; the "no-commitment trading mechanism" in [9], which embeds a technology that filters out outcomes that are not mutually desirable; the trading mechanism in [1].

What if we do not restrict outcomes in a match? Agents might not voluntarily deliver their "quid," even if they get the "quo." In sequential equilibrium, such opportunistic behavior must be deterred with proper dynamic incentives. The result that money sustains exchange without enforcement—external or not—is thus overturned. All we need is a sufficiently large economy with poor monitoring; Folk-theorem type results for matching games reveal that even if everyone sanctions a defection by forever defecting, such community enforcement cannot deter opportunistic behavior in large groups [6, 8].

Our finding that money cannot single-handedly make up for a lack of enforcement in society is unique. It is meaningful because it provides a theoretical foundation for the notion that monetary exchange cannot operate as a stand-alone institution to overcome trade frictions. The option to exchange symbolic objects for goods does not *per se* remove opportunistic temptations, so monetary trade must be supported by enforcement institutions, formal (=external) or informal. This leads us to hypothesize that the monitoring difficulties due to the growth in size of human settlements over the course of history, might have provided a push towards adoption of monetary exchange in those communities equipped with effective enforcement institutions, and not in societies that lacked such institutions—as current thinking would instead suggest.

The paper proceeds as follows. Section 2 presents the model and reports the main Theorem, which is proved in Section 3. Section 4 offers some final remarks.

2 A model of intertemporal exchange

Consider an economy populated by $N = 2n \geq 4$ infinitely-lived agents who face a social dilemma [3, 4]. An exogenous matching process partitions the population into n pairs in each period $t = 0, 1, \ldots$. Pairings are random, equally likely, independent over time, and last only one period. Let $o_i(t) \neq i$ be agent i's opponent (or partner) in period t.

In each pair $\{i, o_i(t)\}$, a coin flip assigns the role of *buyer* to one agent, and *seller* to the other. Hence, in each period an agent is equally likely to either be a seller meeting a buyer, or a buyer meeting a seller. The buyer has no action to take. The seller can choose C or D; C is interpreted voluntarily transferring a good; Figure 1 reports the payoff matrix, where $g - d - l > 0$ and $-l \leq 0 \leq d < g$.[2]

	Seller C	Seller D
Buyer	$g,\ d-l$	$d\ ,\ d$

Figure 1: Interaction in a match

Notes: Row player is a buyer, column player is a seller. Payoffs to (buyer, seller).

The outcome C is called *gift-giving*: the buyer earns surplus $g - d$

[2]E.g., sellers have a perishable good, and buyers derive greater utility than sellers from consuming goods.

and the seller's surplus loss is $-l$. The outcome D is called *autarky*, as it generates no trade surplus. Define the (socially) efficient outcome in a match as the one in which, giving equal weight to players, total surplus is maximized. Gift-giving is efficient, because $g-d-l > 0$, but is *not* mutually beneficial, because buyers benefit at the expense of sellers. Autarky is the unique Nash equilibrium of a one-shot interaction.

Now consider infinite repetition of such interaction. It is assumed that, in each t, each agent in $\{i, o_i(t)\}$, for $i = 1, \ldots, N$, observes only the outcome in their match (=private monitoring). The identity of $o_i(t)$ and the outcome in other pairs are unobservable, so players cannot recognize past opponents if they meet them again (= anonymity). These assumptions imply that agents can neither build a reputation nor engage in relational contracting—a standard assumption in monetary theory.

Payoffs in the repeated game are the sum of period-payoffs, discounted by a common factor $\beta \in [0, 1)$.[3] In the repeated game, the efficient outcome corresponds to the one in which total surplus is maximized in each match, and in each period. We call this outcome "gift-giving" because it involves an infinite sequence of unilateral transfers.

Consider a strategy described by a two-state automaton with states "active" and "idle." The agent takes actions only as a seller. At the start

[3]Equivalently, let the economy be of *indefinite* duration where β is the time-invariant probability that, after each period the economy continues for one additional period, while with probability $1 - \beta$, the economy ends.

of any date, if seller i is active, he selects C, and otherwise D. Agent i is active on date $t = 0$, and in all $t \geq 1$ (i) if agent i is active, then i becomes idle in $t + 1$ only if the seller in $\{i, o_i(t)\}$ chooses D. Otherwise, agent i remains active; (ii) There is no exit from the idle state. If everyone adopts this strategy, then the entire group participates in enforcing defections, and gift-giving is a sequential equilibrium if N is sufficiently small [1]. Otherwise, community enforcement does not represent a sufficient deterrent [6, 8]. So, let us add fiat money to study if its use can solve such enforcement problems.

2.1 The game with money

A random fraction $m = \dfrac{M}{N} \in (0, 1)$ of agents is initially endowed with one indivisible, intrinsically worthless token. As is standard in the literature, token holdings are observable, and cannot exceed one [1]. The introduction of tokens expands action sets: in addition to others choices he may have, a player with a token must also decide to either *keep* the token or to *give* it to his opponent. The left panel in Figure 2 illustrates the game in a *monetary match*, defined as a meeting where only the buyer has a token. All other meetings are called *non-monetary*. Players simultaneously choose actions.

Adding tokens does not eliminate *any* outcomes possible when $M = 0$: to see this, consider strategies that ignore tokens, which brings us back to Figure 1. Adding tokens expands the strategy set. Consider a strategy

Buyer \ Seller	C	D
give	$g, (d-l)^{\star}$	$d, d^{\star}$
keep	$g^{\star}, d-l$	$d^{\star}, d$

Buyer \ Seller	C	D
give	$g, (d-l)^{\star}$	$d^{\star}, d$
keep	$d^{\star}, d$	$d^{\star}, d$

Figure 2: Monetary match with and without *quid-pro-quo* constraint

Notes: Left panel: stage game in a *monetary match*. Payoffs (row, column) are not affected by the buyer's action because tokens have no intrinsic value; the * by a player's payoff denotes token possession at the end of the interaction. The right panel shows how outcomes change when the *quid-pro-quo* constraint is imposed, making unilateral transfers impossible.

that can support monetary exchange. Following [6], we represent it using a two-state automaton.

Definition 1 (**Monetary trade strategy**). *At the start of any period t, agent i can be "active" or "idle:" when active and in a monetary match, i transfers his inventory to $o_i(t)$; otherwise, i makes no transfer. The agent starts active on $t = 0$; in all $t \geq 1$*

- *If agent i is active, then i becomes idle in $t+1$ only if $\{i, o_i(t)\}$ is a monetary match where someone makes no transfer. Otherwise, agent i remains active.*
- *There is no exit from the idle condition.*

We call *monetary trade* the outcome that results when everyone adopts the strategy in Definition 1. Under monetary trade transfers occur only in monetary matches–the seller selects "C" and the buyer selects "give." These actions are simultaneous and voluntary. There are no transfers in all other matches because a seller selects "C" only if the buyer has a token, and a buyer with a token selects "give" only if the seller has no token.

Monetary trade has two components: a rule of desirable behavior (= equilibrium) and a rule of *punishment* to be followed if a departure from desirable behavior is observed (= off equilibrium). Players start by making transfers in all monetary matches, but stop forever after observing a deviation. Such switch to a "punishment mode" is absent from monetary models, which impose *quid-pro-quo* constraints: every transfer requires a concurrent payment, and unilateral transfers are impossible.[4] This is not an innocuous assumption. It removes from the outcome set any outcome that is not mutually beneficial, which changes the nature of the game, from a social dilemma to a pure coordination game with Pareto-ranked outcomes (Figure 2). Ruling out opportunistic temptations in this manner amounts to assuming an institution for the enforcement of spot trades. Our model lifts this restriction—money does not embody an enforcement technology—hence players must rely on informal enforcement to sustain intertemporal exchange.

2.2 Monetary equilibrium

Conjecture that monetary trade is an equilibrium. Consider an agent with $j = 0, 1$ tokens at the start of a period. Define the probability m_0 that someone without money randomly meets a buyer with a token, and the probability $1 - m_1$ that someone with money randomly meets a seller with-

[4]The monetary trade strategy in the literature is: after any history, the agent in a monetary match makes an unconditional transfer, and no transfer otherwise. Autarky is the outcome in any match where a departure from this strategy occurs; see [1, 3, 9].

out a token. Since being a seller or a buyer in a meeting is equally likely, and independent of money holdings, we have

$$\begin{aligned} m_0 &:= \frac{1}{2} \times \frac{M}{N-1} \quad \text{for } M \leq N-1, \\ 1 - m_1 &:= \frac{1}{2} \times \frac{N-M}{N-1} \quad \text{for } M \geq 1. \end{aligned}$$

Recursive arguments imply that the start-of-period equilibrium payoff v_j satisfies

$$\begin{aligned} v_0 &= m_0 \,(d - l + \beta v_1) + (1 - m_0)\,(d + \beta v_0), \\ v_1 &= m_1 \,(d + \beta v_1) + (1 - m_1)\,(g + \beta v_0)\,. \end{aligned} \tag{1}$$

Lemma 1. *We have* $v_1 > v_0$ *always, and* $v_0 \geq \dfrac{d}{1-\beta}$ *if*

$$\beta \geq \beta_m := \frac{l}{(1-m_1)(g-d) + m_1 l} \in (0, 1).$$

Proof of Lemma 1. In Appendix □

$\beta \geq \beta_m$ is necessary for existence of monetary equilibrium: payoffs must be above that ensured by permanent autarky $\frac{d}{1-\beta}$, which is always an equilibrium. However, it is not sufficient because players can suffer involuntary losses.

Theorem 1 (Existence of monetary equilibrium). *There exists* $(l^*, \beta^*) \in [0, g-d) \times (\beta_m, 1)$ *such that for* $(l, \beta) \in [l^*, g-d) \times [\beta^*, 1)$ *monetary trade is a sequential equilibrium. Monetary trade is not a sequential equilibrium as* $N \to \infty$.

Given the lack of enforcement technologies, monetary exchange must rely on some form on *community* enforcement of defections. Voluntary transfers are made today only if there are sufficient incentives (i) to avoid community enforcement in the future (the requirement on β) and (ii) to participate in community enforcement if someone deviates (the requirement l). Hence, in the absence of enforcement technologies, monetary exchange is self-sustaining only if punishment can spread quickly in the economy. But this is impossible if the group is too large. Before turning to the proof of this theorem, it is helpful to contrast its findings with the standard result in the monetary literature.

Proposition 1 (**Monetary equilibrium with external enforcement**). *Assume quid-pro-quo. Monetary equilibrium is supported for all $\beta \in [\beta_m, 1)$ and for any N.*

Here, no agent can sustain (in)voluntary losses, so opportunistic behavior is assumed away. That is, some enforcement has been introduced, e.g., an institution that enforces private property rights. So, there is no need for community enforcement and we consider a simple history-independent strategy: in a monetary match, players make transfers (money or goods) conditional on receiving a concurrent transfer; otherwise sellers choose D, and buyers do not make transfers. Monetary trade is incentive compatible if the seller's loss is small relative to the benefit expected from spending the token in the future. All we need is sufficiently patient players, $\beta \geq \beta_m$; see [1].

The rest of the paper constructs the proof of Theorem 1 by studying the incentives to make voluntary transfers in equilibrium (=*cooperate*) and to punish off equilibrium (=*defect*). We start by studying how punishment spreads off equilibrium, and proceed by calculating off-equilibrium payoffs.

3 Off-equilibrium punishment and payoffs

Consider the start of an arbitrary period t in which the economy is or goes off the (monetary) equilibrium path. Following well-established terminology in repeated games, we refer to active agents as *cooperators* as opposed to *defectors*, who are idle. Suppose the population is partitioned into $N-k$ cooperators and $k = 1, \ldots, N$ defectors. For $k \geq 2$ the economy is off the equilibrium path. Let $k = 1$ denote the case in which someone defects for the first time in a monetary match, moving the economy off equilibrium. Let

$$k \in \kappa := (1, \ldots, N)^{\mathsf{T}}$$

denote the *state* of the economy at the start of a generic date and define the $N-$dimensional column vector e_k with 1 in the k^{th} position and 0 everywhere else (T = transpose).[5]

It can be shown that, in this case, the probability distribution of defec-

[5] To be precise, $k = 1$ denotes the state of the economy after matching takes place in equilibrium, when someone defects in a monetary match. This slight abuse in notation is made for convenience. Also note that we use $y_j \in y := (y_1, \ldots, y_N)$ to denote a generic element of vector y.

tors $t \geq 1$ periods forward is given by $e_k^\intercal Q^t$ where

$$Q = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & Q_{22} & Q_{23} & Q_{24} & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & Q_{33} & Q_{34} & Q_{35} & Q_{36} & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & Q_{N-1,N-1} & Q_{N-1,N} \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 1 \end{pmatrix}$$

is an $N \times N$ transition matrix with elements $Q_{kk'}$ satisfying $Q_{kk} < 1$ for all $k < N$.

When everyone follows the strategy in Definition 1, the upper-triangular matrix Q describes how contagious punishment spreads from period to period, i.e., how the economy transitions from a state with $k \in \kappa$ to $k' \in \kappa$ defectors. This type of community enforcement has four main properties [5, Theorem 1]. First, it is *irreversible* and *contagious*: If someone defects today, then tomorrow there cannot be less defectors than today. The number of additional defectors depends on the random matching outcome.[6] Since defection is an absorbing state, the number of defectors expected on any date is greater if we start with more defectors, and can only increase over time. A single defection eventually leads to 100% defections, an absorbing state that is reached in finite time almost surely.

Suppose there are $k \geq 2$ defectors. Let $v_j(k)$, $j = 0, 1$, be the payoff to a generic defector i at the start of t when $k = 2, \dots, N$. To construct $v_j(k)$, we must compute earnings/losses that i expects in t, and so we

[6] $Q_{12} = 1$ by definition. The first line of Q represents the case when someone defects in a monetary match, in equilibrium.

must consider all possible encounters in which i may take part. Indeed, i's opponent in period t, $o_i(t)$, may or may not be a cooperator, and may or may not have money. We also must compute i's continuation payoffs in each possible encounter $\{i, o_i(t)\}$. Such payoffs depend on money holdings of i, and the number of defectors k' in the continuation game, which depends on the outcome in the match $\{i, o_i(t)\}$ and all other matches. For this reason it is convenient to proceed by considering the probability of each possible encounter $\{i, o_i(t)\}$ that is *conditional* on reaching a specific $k' \geq k$, where $Q_{kk'}$ is the probability of reaching k' starting from $k \geq 2$.

We construct $v_0(k)$, the payoff to defector i when he has no money at the start of date t. Consider any outcome in t leading to $k' \geq k$ defectors in the continuation game. Focus on a match between defector i and $o_i(t)$. Conditional on k' being the state on $t+1$ and k being the state on t, let $\alpha^0_{kk'}$ denote the probability that, on date t, $o_i(t)$ is a cooperating buyer with money when i has no money (the 0 superscript).[7] Hence, with probability $\alpha^0_{kk'}$ agent i gains a token and earns d; with complementary probability $1-\alpha^0_{kk'}$ agent i either meets a defecting buyer who has money or is not in a monetary match, and in either of these circumstances i does not receive a

[7] $\alpha^0_{kk} = 0$ because, if the state does not change ($k' = k$), then it must be the case that no idle seller meets a buyer who cooperates. The probability $\alpha^0_{kk'}$ depends on the distribution of money across defectors.

token and earns d. Using a recursive formulation for $k = 2, \dots, N$ we have

$$\begin{aligned} v_0(k) &= \sum_{k'=k}^{N} Q_{kk'} \{\alpha^0_{kk'}[d + \beta v_1(k')] + (1 - \alpha^0_{kk'})[d + \beta v_0(k')]\} \\ &= d + \beta \sum_{k'=k}^{N} Q_{kk'} v_0(k') + \beta \sum_{k'=k}^{N} Q_{kk'} \alpha^0_{kk'} [v_1(k') - v_0(k')]. \end{aligned} \quad (2)$$

We have $v_0(N) = \dfrac{d}{1-\beta}$ because $\alpha^0_{Nk'} = 0$ for all k' (it is impossible to meet a cooperator when everyone is a defector). In addition, since $v_1(k) \geq v_0(k)$ for all k (which we show later), we also have $v_0(k) \geq v_0(N) = v_a$ for all $k = 2, \dots, N$.

The payoff to someone who has no money and defects in equilibrium is

$$v_0(1) = d + \beta v_1(2),$$

which follows from the fact that $Q_{12} = 1$ and $\alpha^0_{12} = 1$ by definition (if a seller is the first player to defect, $k = 1$, then he must defect in a monetary match, in which case he surely meets a cooperating buyer and we transition to $k' = 2$).

Now, we construct $v_1(k)$, i.e., the payoff to defector i when he has money at the start of t. Consider any outcome in t leading to $k' \geq k$ defectors in the continuation game starting on $t+1$. Conditional on reaching k' and k being the current state, let $\alpha^1_{kk'}$ denote the probability that $o_i(t)$ is a cooperating seller without money.[8] Hence, with probability $\alpha^1_{kk'}$ agent i keeps his token and earns g; with complementary probability $1 - \alpha^1_{kk'}$, defector i either

[8]Here $\alpha^1_{kk} = 0$ because, by definition, no defector meets a cooperator in this case.

meets a defecting seller who has no money, or is not in a monetary match, and in either case, i keeps the token and earns d. Consequently for $k = 2, \ldots, N$ we have

$$\begin{aligned} v_1(k) &= \sum_{k'=k}^{N} Q_{kk'}\{\alpha^1_{kk'}[g + \beta v_1(k')] + (1 - \alpha^1_{kk'})[d + \beta v_1(k')]\} \\ &= \sigma_k g + (1 - \sigma_k)d + \beta \sum_{k'=k}^{N} Q_{kk'} v_1(k'). \end{aligned} \tag{3}$$

The second line above follows from observing that—when there are k defectors in the economy—the *unconditional* probability that i (who has a token) is in a monetary match with a cooperating seller is

$$\sigma_k := \sum_{k'=k}^{N} Q_{kk'}\alpha^1_{kk'} = \sum_{k'=k+1}^{N} Q_{kk'}\alpha^1_{kk'} \quad \text{for } k = 2, \ldots, N. \tag{4}$$

We have $\sigma_k \geq \sigma_h$ if $h \geq k$ (with more defectors, meeting cooperators is less likely).

It is convenient to define $\sigma_1 = 1$ and the vector

$$\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_{N-1}, 0)^\intercal.$$

For $k \geq 2$, each element σ_k defines the probability that buyer i is in a monetary match and meets a cooperator, given that i is one of k defectors at the start of a period. It should be clear that $0 = \sigma_N < \sigma_{k'} < \sigma_k < \sigma_1 = 1$ for $2 \leq k < k' \leq N - 1$. Also, define

$$v_1(1) = g + \beta v_1(2),$$

as the payoff to a buyer who is in monetary match *in equilibrium*, and defects. Define

$$\phi_k = (1-\beta)e_k^\intercal (I-\beta Q)^{-1}\sigma \quad \text{for } k=1,\dots,N.$$

Following [5], it can be interpreted as the expected number of cooperators without money that a defector with money meets in the continuation game, normalized by $(1-\beta)^{-1}$.

Lemma 2. *We have*

$$v_1(k) = \frac{1}{1-\beta}[\phi_k g + (1-\phi_k)d] \quad \textit{for } k=1,\dots,N, \tag{5}$$

with $v_1(k)$ *non-increasing in* k *and* $\lim_{\beta\to 1^-} \frac{\phi_k}{1-\beta} < \infty$.

The proof immediately follows from [5, Theorem 2]. Having characterized payoffs in and out of equilibrium, we can now study deviations in and out of equilibrium.

3.1 Equilibrium deviations

Suppose everyone has been active until period t and in period $t+1$ agent i deviates, reverting to play the monetary trade strategy on $t+2$. Agent i meets cooperator $o_i(t)$ who may or may not have money.

In a non-monetary match, player i does not deviate by making a transfer. Doing so is suboptimal because i has a loss but no future gain (continuation payoffs do not change because his opponent remains active). Hence, consider a monetary match in equilibrium. Two cases may occur:

- i has no money and $o_i(t)$ is a buyer with money. If i deviates by choosing D, then i receives money and his opponent becomes idle. Such deviation is suboptimal if

$$d + \beta v_1(2) \leq d - l + \beta v_1. \tag{6}$$

- i has money and $o_i(t)$ is a seller without money. If i keeps his token, then he obtains g and $o_i(t)$ becomes idle. Such deviation is suboptimal if

$$g + \beta v_1(2) \leq g + \beta v_0. \tag{7}$$

Lemma 3 (**No deviations in equilibrium**). *There exists a value $\beta^* < 1$ such that (6)-(7) hold for all $\beta \in [\beta^*, 1)$.*

Proof of Lemma 3. In Appendix. □

The proof of the Lemma reveals that the key participation constraint is the buyer's. In equilibrium, if the buyer pays a seller — which occurs if β is sufficiently large — then it is also true that the seller serves the buyer. The reverse, however, is not true.

Lemma 4 (**Large economies**). *Monetary trade is not an equilibrium as $N \to \infty$.*

Proof of Lemma 4. In Appendix. □

Intuitively, in large economies buyers prefer to avoid paying because they can immediately consume, cannot be immediately punished, *and* can spend their money in the future. This destroys the value of money. The conclusion is that monetary equilibrium cannot generally be sustained when monitoring and punishment limitations preclude adequate enforcement—external or not. This is true for the same reason it is true for social norms: in equilibrium individuals *voluntarily* sustain a loss to provide a benefit to others only if group punishment is a significant threat. In the next section, we study the credibility of the threat, by considering the incentives to punish off-equilibrium.

3.2 Off-equilibrium deviations

Community enforcement is credible if actions in the punishment phase are individually optimal. Deviating in non-monetary matches is suboptimal (the deviator has a loss and cannot slow down contagion). However, a defector might wish to deviate in a monetary match, to slow down the contagious spread of punishment. We study this case.

Suppose there are $k \geq 2$ defectors, and agent i is one of them. Let $\hat{v}_j(k)$ denote i's payoff when he has $j = 0, 1$ tokens. Deviating is suboptimal if

$$\hat{v}_j(k) \leq v_j(k).$$

To derive these continuation payoffs, consider that the expected payoffs from not punishing depend on the probabilities of meeting a defector with and without money.

Consider defector i when he does not have money. Conditional on k'

being the state next period and k currently, let $\mu^0_{kk'}$ denote the probability that $o_i(t)$ is a buyer with money and $\delta^0_{kk'}$ the conditional probability that $o_i(t)$ is a defecting buyer with money, so

$$\alpha^0_{kk'} + \delta^0_{kk'} = \mu^0_{kk'}, \qquad \text{and} \qquad \sum_{k'=k}^{N} Q_{kk'}\mu^0_{kk'} = m_0 \qquad \text{for all } k \geq 2.$$

Similarly, consider defector i when he has money. Let $\mu^1_{kk'}$ denote the conditional probability that $o_i(t)$ is a seller without money, and $\delta^1_{kk'}$ the probability that $o_i(t)$ is a defecting seller without money. Hence, we have

$$\alpha^1_{kk'} + \delta^1_{kk'} = \mu^1_{kk'}, \qquad \text{and} \qquad \sum_{k'=k}^{N} Q_{kk'}\mu^1_{kk'} = 1 - m_1 \quad \text{for all } k \geq 2.$$

Using a recursive formulation, we have

$$\begin{aligned}
\hat{v}_0(k) = \quad & \sum_{k'=k}^{N} Q_{kk'}\alpha^0_{kk'}[d - l + \beta v_1(k'-1)] + \sum_{k'=k}^{N} Q_{kk'}\delta^0_{kk'}[d - l + v_0(k')] \\
& + \sum_{k'=k}^{N} Q_{kk'}(1 - \mu^0_{kk'})[d + \beta v_0(k')], \\
\hat{v}_1(k) = \quad & \sum_{k'=k}^{N} Q_{kk'}\alpha^1_{kk'}[g + \beta v_0(k'-1)] + \sum_{k'=k}^{N} Q_{kk'}\delta^1_{kk'}[d + \beta v_0(k')] \\
& + \sum_{k'=k}^{N} Q_{kk'}(1 - \mu^1_{kk'})[d + \beta v_1(k')].
\end{aligned}$$

To derive these expressions note that i deviates *only* in a monetary match. If i has no money, then we consider $\hat{v}_0(k)$. The first line accounts for the cases in which i is in a monetary match (i.e., a seller). Here, the agent earns $d-l$ because he cooperates instead of punishing. Player i might also receive money, but this depends on whether his opponent (who is a buyer with money) cooperates or defects. If his opponent is a cooperator, then i's continuation payoff is $v_1(k'-1)$, because this cooperator does not become a defector (i cooperates) and gives money to i. Otherwise, we have $v_0(k')$ because there is no impact on the number of future defectors and i does not receive money.

The second line defines matches in which i is not in a monetary match. The probability of not meeting a buyer with money can be decomposed as

$$1 - m_0 = \sum_{k'=k}^{N} Q_{kk'}(1 - \mu^0_{kk'}),$$

because, conditional on transitioning from k to k', the probability of not meeting a buyer with money is $1-\mu^0_{kk'}$. In all these matches player i earns d and does not receive money so the continuation payoff is $v_0(k')$, depending on the realization of k'.

If i has money, instead, then we consider $\hat{v}_1(k)$. Here agent i deviates only when he is in a monetary match (i.e., a buyer), which is reported in the first line. Deviating means that i transfers money instead of keeping it. If his opponent is a cooperator, then i earns g and the continuation payoff is $v_0(k'-1)$ because his opponent does not observe a defection; otherwise i earns d and gets $v_0(k')$ continuation payoff. The second line refers to the matches that are not monetary.

Now, we use the definitions of $v_j(k)$, for $k = 2, \ldots, N$, to derive inequalities that guarantee that single-period deviations are not profitable, out of equilibrium. Deviating from punishment in a monetary match is suboptimal for defector i, when i is a buyer, if $\hat{v}_1(k) \leq v_1(k)$, which is rewritten as

$$\sum_{k'=k}^{N} Q_{kk'} \alpha^1_{kk'} [v_0(k'-1) - v_1(k')] \leq \sum_{k'=k}^{N} Q_{kk'} \delta^1_{kk'} [v_1(k') - v_0(k')], \quad (8)$$

by manipulating expressions $v_1(k)$ and $\hat{v}_1(k)$ and using the fact that $\alpha^1_{kk'} + \delta^1_{kk'} = \mu^1_{kk'}$.

Lemma 5 (Buyers punish). *Inequality (8) holds for all $\beta \in (0,1)$.*

Proof of Lemma 5. In Appendix. □

Out of equilibrium, it is never optimal for a buyer to deviate from the punishment strategy. The reason is simple. Suppose buyer i today pays the seller, when in fact he should not. Paying the seller may slow down the growth in the number of defectors but agent i cannot benefit from it until he re-acquires money. Hence, since future payoffs are discounted, it is a dominant strategy to not pay out of equilibrium.

Deviating from punishment in a monetary match is suboptimal for defector i, when i is a seller, if $\hat{v}_0(k) \leq v_0(k)$. Using $\alpha^0_{kk'} + \delta^0_{kk'} = \mu^0_{kk'}$, the inequality is

$$\beta \sum_{k'=k}^{N} Q_{kk'} \alpha^0_{kk'} [v_1(k'-1) - v_1(k')] \leq l \sum_{k'=k}^{N} Q_{kk'} \mu^0_{kk'}. \quad (9)$$

Lemma 6 (Sellers punish). *There exists $0 \leq l^* < g - d$ such that if $l \in [l^*, g-d)$, then inequality (9) holds for all $\beta \in (0,1)$.*

Proof of Lemma 6. In Appendix □

Out of equilibrium, cooperating as a seller may slow down the growth in defectors. This benefits the seller because he acquires money and may be able to spend it tomorrow. To remove the incentive to deviate, the seller's loss from making a unilateral transfer must be sufficiently high.

4 Final remarks

Can money overcome trade frictions when voluntary trading arrangements cannot? It can—according to current thinking—because monetary systems operate independently of institutions designed to monitor behaviors and sanction undesirable ones. Conceptually, money exists *because* of a lack of enforcement in society. This view hinges on a routine assumption that money embodies a technology, which prevents players from suffering losses—a technology to enforce property rights, perhaps.

We have proved that without some enforcement hard-wired into the model, monetary exchange is afflicted by the same problems that undermine norms of voluntary behavior. The analysis can be extended to prove that when rules of voluntary behavior support intertemporal exchange, monetary exchange may not. The take-away is that monetary systems *do* need institutions designed to monitor behaviors and sanction undesirable ones. Money and enforcement are complementary institutions, not substitutes, after all.

So what explains the widespread use of money in society? It may be a consequence of the *kind* of enforcement it requires, which is perhaps less costly, less cognitively demanding, or behaviorally more effective compared to that needed to sustain alternative trading arrangements. Recent experimental work on indefinitely repeated social dilemmas suggests that opportunistic temptations are not easily deterred by community enforcement, whereas costly personal punishment is quite effective [2]. Moreover, when external enforcement of monetary spot trades is available, fiat monetary exchange endogenously emerges and empirically outperforms non-monetary outcomes even if rules of voluntary behavior can theoretically sustain intertemporal exchange [3, 4].

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Appendix

Proof of Lemma 1. The definition of v_j immediately implies

$$v_0 \quad = \quad \frac{[(1-m_1)g + m_1 d]m_0\beta + (d - m_0 l)(1 - m_1\beta)}{(1-\beta)[1+\beta(m_0 - m_1)]},$$

$$v_1 \quad = \quad \frac{[(1-m_1)g + m_1 d][1-(1-m_0)\beta] + (d - m_0 l)(1 - m_1)\beta}{(1-\beta)[1+\beta(m_0 - m_1)]},$$

hence we have

$$v_1 - v_0 = \frac{(1-m_1)(g-d) + m_0 l}{1+\beta(m_0 - m_1)} > 0.$$

This difference is positive because $m_0 - m_1 \geq -1$, and it reflects the difference in expected trade surpluses. With probability $1 - m_1$ someone with a token earns surplus $g - d$ and with probability m_1 earns no surplus. Someone without a token earns a surplus $-l$ with probability m_0 and no surplus otherwise. The denominator is an adjusted discount factor.

The outcome corresponding to infinite repetition of the static Nash equilibrium (every seller always chooses D) is always an equilibrium of the repeated game. Call this equilibrium "autarky" since every player is idle in each period and obtains payoff $v_a := \frac{d}{1-\beta}$. Note $v_0 \geq \frac{d}{1-\beta}$, iff $\beta \geq \beta_m = \frac{l}{(1-m_1)(g-d)+m_1 l}$, with $\beta_m < 1$ since $g - d > l > 0$. □

Proof of Lemma 3. Start by considering deviations by a buyer in a monetary match, i.e., (7), which is satisfied if $v_0 - v_1(2) \geq 0$. Using the definition of $v_1(k)$ in (5) we have

$$v_1(2) = \frac{d}{1-\beta} + \frac{1}{1-\beta}\phi_2(g-d).$$

Hence,

$$v_0 - v_1(2) = \frac{(1-\beta)v_0 - d}{1-\beta} - \frac{\phi_2(g-d)}{1-\beta}.$$

Using the definition of v_0 in (1) we have

$$(1-\beta)v_0 - d \equiv \frac{\beta m_0(1-m_1)(g-d) - m_0 l(1-\beta m_1)}{1+\beta(m_0-m_1)} > 0 \qquad \text{if} \quad \beta > \beta_m,$$

$$\lim_{\beta\to 1^-} \frac{(1-\beta)v_0 - d}{1-\beta} = \infty.$$

By [5, Theorem 3] $\lim_{\beta\to 1^-} \frac{\phi_2}{1-\beta} < \infty$. Hence, by continuity $\beta^* \in (\beta_m, 1)$ exists such that $v_0 - v_1(2) \geq 0$ for $\beta \in [\beta^*, 1)$.

Now, consider deviations by a seller in a monetary match. Inequality (6) is satisfied if $\beta[v_1 - v_1(2)] \geq l$. From the definition of v_0 and v_1 in (1) we have

$$\begin{aligned} \beta(v_1 - v_0) &\equiv \beta\frac{(1-m_1)(g-d)+m_0 l}{1+\beta(m_0-m_1)} \\ &\geq \beta_m\frac{(1-m_1)(g-d)+m_0 l}{1+\beta_m(m_0-m_1)} \equiv l \quad \text{for } \beta \geq \beta_m. \end{aligned}$$

Now fix $\beta \in [\beta^*, 1)$ so that $v_0 \geq v_1(2)$. Consequently $v_1 - v_1(2) \geq v_1 - v_0 \geq l/\beta$, i.e., inequality (6) is also satisfied when $\beta \in [\beta^*, 1)$. □

Proof of Lemma 4. We have to simply show that buyers do not wish to pay in economies that are "large." To define a large economy, let $M = bN$ for $b \in (0,1)$ and let $N \to \infty$. That is, we fix a per-capita money supply and let the economy grow large.

Consider a defector who is a buyer in a monetary match, out of equilibrium when there are $k \geq 2$ defectors. Since the number of defectors is finite, the unconditional probability that the buyer is in a monetary match with a seller who is a cooperator is $\lim_{N\to\infty} \sigma_k = \frac{1}{2}(1-b)$. Hence, we have

$$\begin{aligned}\lim_{N\to\infty} \phi_2 &= \lim_{N\to\infty} (1-\beta)e_2^\intercal (I-\beta Q)^{-1}\sigma = (1-\beta)\sum_{j=2}^{\infty}(I-\beta Q)^{-1}_{2j} \lim_{N\to\infty} \sigma_j \\ &= \frac{1}{2}(1-b)(1-\beta)\sum_{j=2}^{\infty}(I-\beta Q)^{-1}_{2j} = \frac{1}{2}(1-b),\end{aligned}$$

where $(I-\beta Q)^{-1}_{2j}$ denotes element in row 2 column j of matrix $(I-\beta Q)^{-1}$; $(I-\beta Q)^{-1}_{21} = 0$ because Q is upper triangular.

Recall that a buyer does not deviate in equilibrium if (7) holds, i.e., if $v_0 - v_1(2) \geq 0$. But this is violated for all $\beta \in (0,1)$ as $N \to \infty$. To see this note that

$$\begin{aligned}\lim_{N\to\infty} [v_0 - v_1(2)] &= \lim_{N\to\infty} \left[\frac{(1-\beta)v_0 - d}{1-\beta} - \frac{\phi_2(g-d)}{1-\beta}\right] \\ &= -\frac{[2-\beta(1+b)][(g-d)(1-b)+bl]}{2(1-\beta)(2-\beta)} < 0.\end{aligned}$$

□

Proof of Lemma 5. We prove by contradiction that $v_1(k) \geq v_0(k)$, for all $k = 2, \ldots, N$.

Suppose $v_1(h) < v_0(h)$ for some $2 \leq h \leq N$. Use the definition of $v_0(k)$ in (2) and notice that $v_j(k) \geq v_j(k+1)$ for $j = 0, 1$ and all $k = 2, \ldots, N-1$.[9]

[9]Suppose there exists a k such that $v_j(k) < v_j(k+1)$ for $j = 0, 1$. This means that in the economy starting with one more defector, any defector is more likely to meet a cooperator. But this cannot be true, as it contradicts the properties of the transition matrix Q as in [5, Theorem 1].

We have

$$
\begin{aligned}
v_0(h) &= d + \beta \sum_{k'=h}^{N} Q_{hk'} \alpha^0_{hk'} v_1(k') + \beta \sum_{k'=h}^{N} Q_{hk'} (1 - \alpha^0_{hk'}) v_0(k') \\
&< d + \beta v_1(h) \sum_{k'=h}^{N} Q_{hk'} \alpha^0_{hk'} + \beta v_0(h) \sum_{k'=h}^{N} Q_{hk'} (1 - \alpha^0_{hk'}) \\
&< d + \beta v_0(h) \sum_{k'=h}^{N} Q_{hk'} \alpha^0_{hk'} + \beta v_0(h) \sum_{k'=h}^{N} Q_{hk'} (1 - \alpha^0_{hk'}) \\
&= d + \beta v_0(h),
\end{aligned}
$$

which provides the desired contradiction because $v_0(k) \geq \dfrac{d}{1-\beta}$ for all $k = 2, \ldots, N$.

Consider inequality (8). We prove that it holds whenever $v_1(k) \geq v_0(k-1)$, for all $k = 2, \ldots, N$. Using the definition of $v_0(k)$ in (2) we have

$$
\begin{aligned}
v_0(k) &= d + \beta \sum_{k'=k}^{N} Q_{kk'} v_0(k') + \beta \sum_{k'=k}^{N} Q_{kk'} \alpha^0_{kk'} (v_1(k') - v_0(k')) \\
&= d + \beta \sum_{k'=k}^{N} Q_{kk'} v_0(k') + \beta \sum_{k'=k+1}^{N} Q_{kk'} \alpha^0_{kk'} (v_1(k') - v_0(k')) \\
&\leq d + \beta \sum_{k'=k}^{N} Q_{kk'} v_0(k') + \beta \sum_{k'=k+1}^{N} Q_{kk'} (v_1(k') - v_0(k')) \\
&= d + \beta Q_{kk} v_0(k) + \beta \sum_{k'=k+1}^{N} Q_{kk'} v_1(k') \\
&\leq d + \beta Q_{kk} v_0(k) + \beta (1 - Q_{kk}) v_1(k+1).
\end{aligned}
$$

To derive the second line we have used the fact that $\alpha^0_{kk'} = 0$ when $k' = k$. For the third line note that $\alpha^0_{kk'} \leq 1$. The fourth line follows

$$
\sum_{k'=k+1}^{N} Q_{kk'} v_1(k') \leq v_1(k+1) \sum_{k'=k+1}^{N} Q_{kk'} = v_1(k+1)(1 - Q_{kk}).
$$

Since

$$
v_1(k+1) \geq \frac{1 - \beta Q_{kk}}{\beta(1 - \beta Q_{kk})} v_0(k) - \frac{d}{\beta(1 - Q_{kk})},
$$

we have

$$\begin{aligned} v_1(k) - v_0(k-1) &\geq \frac{1-\beta Q_{k-1,k-1}}{\beta(1-Q_{k-1,k-1})} v_0(k-1) - \frac{d}{\beta(1-Q_{k-1,k-1})} - v_0(k-1) \\ &= \frac{1-\beta}{\beta(1-Q_{k-1,k-1})} v_0(k-1) - \frac{d}{\beta(1-Q_{k-1,k-1})} \\ &\geq \frac{1-\beta}{\beta(1-Q_{k-1,k-1})} \frac{d}{1-\beta} - \frac{d}{\beta(1-Q_{k-1,k-1})} = 0. \end{aligned}$$

The last line follows from $v_0(k) \geq v_0(N) = \dfrac{d}{1-\beta}$ for all $k = 2, \ldots, N$. □

Proof of Lemma 6. Consider inequality (9). We derive an expression for $v_1(k-1) - v_1(k)$ when $k = 2, \ldots, N$. From the definition of $v_1(k)$ in (5), for all $k = 2, \ldots, N$

$$v_1(k-1) - v_1(k) = (g-d)\frac{1}{1-\beta}(\phi_{k-1} - \phi_k). \tag{10}$$

By [5, Theorem 3] we have $\lim_{\beta \to 1^-} \dfrac{\phi_k}{1-\beta} < \infty$. Hence, (9) holds whenever l is sufficiently large. To ensure that the parameter set is nonempty we take a second step.

By [5, Theorem 3] we have $\phi_k - \phi_{k+1} \leq \phi_{k-1} - \phi_k$ for all $k = 2, \ldots, N$. So, we have

$$v_1(k-1) - v_1(k) \leq v_1(1) - v_1(2) = (g-d)\frac{1}{1-\beta}(\phi_1 - \phi_2).$$

We wish to prove that $\dfrac{\beta(\phi_1 - \phi_2)}{1-\beta} < 1$, hence a value $l^* < g - d$ exists, which satisfies (9). From equation (5) in Lemma 2 we have

$$v_1(1) = \frac{1}{1-\beta}[\phi_1 g + (1-\phi_1)d].$$

By definition of $v_1(1)$, we also have

$$\begin{aligned} v_1(1) &= \sigma_1 g + (1-\sigma_1)d + \beta v_1(2) \\ &= \sigma_1 g + (1-\sigma_1)d + \frac{\beta}{1-\beta}[\phi_2 g + (1-\phi_2)d], \end{aligned}$$

where we have used equation (5) for $v_1(2)$. Hence, we must have

$$\frac{1}{1-\beta}[\phi_1 g + (1-\phi_1)d] = \sigma_1 g + (1-\sigma_1)d + \frac{\beta}{1-\beta}[\phi_2 g + (1-\phi_2)d],$$

which is rewritten as

$$\frac{\phi_1 - \beta\phi_2}{1-\beta} = \sigma_1.$$

Recall that $\sigma_1 = 1$ (if there is only one defector, then the defector meets a cooperator with certainty) and $\beta(\phi_1 - \phi_2) < \phi_1 - \beta\phi_2$, hence $\frac{\beta(\phi_1 - \phi_2)}{1-\beta} < 1$.

Finally, note that inequality (9) is the most stringent when $k = 2$, and in this case it can be rewritten as

$$(g-d)\frac{1}{m_0}\frac{\beta}{1-\beta}\sum_{k'=2}^{N} Q_{2k'}\alpha^0_{2k'}(\phi_{k'-1} - \phi_{k'}) \leq l,$$

which is achieved by substituting on the right hand side $\sum_{k'=2}^{N} Q_{2k'}\mu^0_{2k'} = m_0$ and by substituting in the difference $v_1(k'-1) - v_1(k')$ the expressions $v_1(k)$ given in Lemma 2. Now define

$$\gamma \;:=\; \sup_{\beta\in(0,1)} \sum_{k'=2}^{N} Q_{2k'}\alpha^0_{2k'}\frac{\beta}{1-\beta}(\phi_{k'-1} - \phi_{k'}).$$

Note that $\gamma < m_0$ because $\frac{\beta(\phi_{k'-1} - \phi_{k'})}{1-\beta} < 1$ for all β. Hence, defining

$$l^* := (g-d)\frac{\gamma}{m_0},$$

inequality (9) holds for all $\beta \in (0,1)$ if l is in $[l^*, g-d)$. □