6-25-2024

Influencers: The Power of Comments

Cristina Nistor

Matthew Selove

Influencers: The Power of Comments

Influencers: The Power of Comments*

Cristina Nistor and Matthew Selove†

June 17, 2024

Abstract

Many customers choose products based on information from social media influencers. Companies can pay these influencers to promote their products. We develop a model in which customers read an influencer's sponsored post for a mix of entertainment and product information, and those who purchase the product can leave comments for future customers. We derive conditions in which a large celebrity influencer endorses all products, whereas a micro-influencer adopts a policy of endorsing only high quality products. In equilibrium, the micro-influencer screens for high quality products so his followers do not waste time reading informative comments about low quality products. By contrast, the celebrity influencer attracts so many uninformative comments his followers do not use his comments as a source of product information, and the value of his endorsement arises solely from generating product awareness.

*Helpful (and informative) comments were provided by Ron Berman, Nanda Kumar, Chenxi Liao, Birger Wernerfelt, Juanjuan Zhang, and participants at the Marketing Science conference, the QME conference, the Marketing Theory Symposium, the UT Dallas FORMS conference, and the SICS conference.

†nistor@chapman.edu, selove@chapman.edu

1 Introduction

Firms can pay social media influencers to endorse their products on blogs, Instagram, and other social media websites. Although regulators require influencers to disclose when they are paid for an endorsement, advertisers have nonetheless found that these endorsement deals can generate a significant increase in product purchases (Nielsen 2016, 2020). For example, each year clothing retailer Nordstrom partners with dozens of social media influencers to promote its summer Anniversary Sale, which *AdWeek* calls "the Super Bowl for fashion influencers," an event that causes stockouts of many popular clothing items and rivals the December holidays in terms of sales volume for the retailer (Pearl 2018). Such influencer marketing campaigns represent a rapidly growing share of firms' advertising budgets. Firms spent about $16.4 billion on influencer marketing in 2022, which is expected to increase to $21.1 billion in 2023 (Influencer Marketing Hub 2023). These endorsement deals also generate significant income for top influencers. In 2019, soccer star Cristiano Ronaldo earned about $47 million from sponsored Instagram posts, which was greater than his annual salary for playing soccer (McCarthy 2019).

Influencers can be broadly divided into two categories. Celebrity influencers such as sports, music, and reality television stars often have millions of followers on social media websites like Instagram. By contrast, micro-influencers have fewer than one million followers, and in many cases have only several thousand followers.

Influencer marketing firms like TapInfluence and ApexDrop help arrange sponsorship deals with influencers. These firms claim that the smaller micro-influencers have greater authenticity, meaning they only endorse products they truly use and enjoy (Cruz 2018). There is widespread anecdotal evidence from advertising managers that micro-influencers typically refuse paid endorsement deals if they believe the product is a poor fit for their followers (Baklanov 2021).[1] By contrast, celebrity influencers often endorse products they do not even use (Nephew 2020).

[1] An interior design influencer with 80 thousand followers stated, "When brands approach me, I'd like to know that they respect me, my audience, and what I put out in the world. I can help brands reach my audience authentically and turn that engagement into new relationships, fans, followers, and customers – but it has to resonate with my audience, and I know them best" (Carufel 2021).

Another key difference is micro-influencers have greater engagement, meaning their followers are more likely to write comments sharing their own opinions and experiences (Cruz 2018). Popular business articles advise firms that want to advertise with a micro-influencer to read comments by the influencer's followers to determine if they are a good fit for the advertiser's product (Bowman 2018; Sortino 2022). Academic research has also shown that social media followers are more likely to comment on posts by smaller influencers (Bentley et al. 2021), and that such comments contain valuable information about customer needs and opinions (Schweidel and Moe 2014).

Even casual browsing of the Instagram pages of celebrity influencers and micro-influencers reveals striking differences in the types of products they endorse and the comments their followers make. For example, Cristiano Ronaldo, who has over 600 million followers on Instagram, recently made a sponsored post endorsing an Italian online university called eCampus University (Bertacche and Remondini 2019). The overwhelming majority of comments on this post consist only of emojis or short messages expressing support for Ronaldo himself, with no substantive information about the school he is endorsing. As a contrasting example, a micro-influencer with the user name mississippivegan, who has about 150 thousand Instagram followers, recently made a sponsored post with a recipe using Muir Glen tomatoes. Most of the comments on this post provide a description of his followers' own experiences cooking this dish and even suggesting improvements to the recipe. (See Appendix A for screenshots of these posts by Ronaldo and mississippivegan.)

Standard reputation models imply that larger firms make greater effort to protect their brand (e.g., Kreps and Wilson 1982; Diamond 1989; Chu and Chu 1994; Cabral and Hortaçsu 2010). Therefore, it is somewhat surprising that smaller influencers, who have relatively few followers, seem to be more protective of their brand than celebrity influencers with millions of followers.

This paper develops a formal model that helps explain why smaller influencers are more selective about which types of products they endorse and also generate more informative comments from their followers. In our model, an influencer who has

either high or low fraction of followers who are interested in a product category can endorse a seller's product. The influencer first chooses an endorsement policy, which can involve either endorsing all products or endorsing only a high quality product. The influencer and seller then observe the product's quality level. If product quality is consistent with the influencer's endorsement policy, the seller and the influencer agree to an endorsement deal in which they split the expected profits from a sponsored post.

A first group of followers read the sponsored post, can purchase the product, and can also leave a comment. Customers who purchase the product leave informative comments that provide both entertainment utility and status utility to commenters, whereas those who read the post purely for entertainment value leave uninformative comments that provide commenters only with entertainment utility. A second group of followers then read the post and can purchase the product, with the key difference that each of these customers can learn whether the product is a good fit for her by reading comments by the first group of customers.

We derive conditions in which a celebrity influencer (with a small fraction of followers interested in the product category) endorses all products, whereas a micro-influencer (with a large fraction of followers interested in the product category) endorses only high quality products. Intuitively, a sponsored post by a celebrity influencer leads to so many uninformative comments that the only value from this sponsorship results from generating product awareness with a large number of potential customers. For such an influencer, expected profits are maximized by endorsing any product with an expected value to customers that exceeds its marginal cost. As long as the expected surplus generated by selling low quality products is positive, a celebrity influencer endorses all products. By contrast, a sponsored post by a micro-influencer generates a higher percentage of informative comments, allowing customers from the second group to learn whether the product is a good fit for them before they make purchase decisions. For such an influencer, endorsing low quality products generates higher reading costs for followers without generating many additional product sales because informative comments usually reveal that the

product is a poor fit for his followers. Therefore, a micro-influencer may want to avoid endorsing low quality products, so his followers do not waste time reading comments about products that are unlikely to be a good fit. Thus, our model predicts that celebrity influencers have an endorsement policy that varies across product categories depending on marginal production costs for the product being endorsed, whereas micro-influencers endorse only high quality products unless their followers have very low reading cost.

Popular business articles contain advice and analysis consistent with key results of our model. For example, social media marketing firm Emplifi advises brands to work with celebrity influencers if the goal of their advertising campaign is "broad reach" and "massive exposure." By contrast, Emplifi advises brands to work with micro-infuencers if their goal is "high engagement" and promotions that "feel more like genuine recommendations" (Emplifi 2023). This advice is consistent with our results showing that celebrity influencers generate widespread product awareness, whereas micro-influencers are selective about which products they endorse and generate comments that provide useful information about the product.

In addition to our key results on the endorsement policies of large and small influencers, we also present four model extensions. First, we develop a more detailed model of the costs and benefits of commenting, and we present results for this model extension. Second, we show that a micro-influencer may want to specialize in endorsing products for which he has the expertise to identify quality levels, whereas a celebrity influencer is willing to endorse products outside of his expertise. Third, we derive conditions in which reputation concerns compel a micro-influencer not to deviate from his policy of endorsing only high quality products. Fourth, we derive conditions in which an influencer deletes uninformative comments from his post to make it easier for followers to find informative comments.

Section 2 discusses related literature. Section 3 presents the model set-up. Section 4 presents formal results, numerical examples, and the first two model extensions. Section 5 concludes. The appendix contains examples of sponsored Instagram posts, the other two model extensions, and proofs of all formal results.

2 Related Literature

Game theory research on influencer marketing has studied the optimal product variety when influencers provide product information (Kuksov and Liao 2019), the optimal affiliation between a firm and influencers (Pei and Mayzlin 2021), how firms compete for influencers considering the overlap in the influencers' reach (Katona 2020), how influencers compete with each other for paid sponsorships (Fainmesser and Galeotti 2021), and differences between influencer marketing and targeted advertising (Berman et al. 2023). Our model focuses instead on how an influencer's followers affect the informativeness of his comment section and which products he endorses.

Previous research has used a variety of methods to study why consumers write online comments or product reviews. Toubia and Stephen (2013) conduct a field experiment to compare intrinsic motivation with image-related utility for Twitter users. Brodie et al. (2013) find in an online ethnographic study that creating content or being part of a community increases satisfaction for some customers. He et al. (2018) conduct experiments showing that expressing a like or dislike increases task enjoyment. Iyer and Katona (2016) and Campbell et al. (2017) build formal models in which customers derive status-based utility from social communication. Based on this earlier research, our model explicitly includes both intrinsic utility and status utility from commenting, and we show how the resulting comments by followers affect an influencer's optimal endorsement policy.

Our paper has a conceptual connection with research by Wernerfelt (1994, 1996) on efficient provision of products and information. In our model, celebrity influencers generate mass awareness but do not provide information about product fit, which results in the seller incurring production costs even for customers for whom the product is a poor fit. By contrast, micro-influencers generate information about product fit, which allows the seller to avoid unnecessary production costs but also results in a cost of followers reading comments. Thus, we show the optimal endorsement policy for a celebrity influencer depends on product production cost, whereas the endorsement

policy for a micro-influencer depends on followers' reading costs.

Our paper also has a conceptual connection with literature on mainstream and niche product designs (Johnson and Myatt 2006; Bar-Isaac et al. 2012). In our model, the micro-influencer appeals to a small niche audience who are interested in a particular product category, whereas the celebrity influencer appeals to a large mainstream audience of which only a small fraction are interested in any particular type of product. We explore the equilibrium endorsement policy and the commenting behavior of followers for each type of influencer.

Previous research has developed models in which a firm can affect the informativeness of online comments, either by writing messages pretending to be customers reviewing their product (Mayzlin 2006), or by allowing product information created by the consumers to be posted on the seller's website (Chen and Xie 2008). Our paper includes a model extension in which the influencer can increase the informativeness of their comment section by deleting some uninformative comments, and we compare the incentives for comment deletion by large versus small influencers.

Recent empirical evidence shows that followers' engagement with an influencer depends on the size of the influencer's reach. Followers click the like button on posts by large and small influencers at similar rates, but have a higher rate of commenting on posts by smaller influencers, suggesting deeper engagement with smaller influencers (Bentley et al. 2021). Followers are also more likely to perceive a post by a larger influencer as an attempt to persuade rather than an honest attempt to share product knowledge (Chu et al. 2021). However, more entertaining content can increase the effectiveness of influencer ad campaigns for some types of products (Yang et al. 2021). Our paper proposes a formal model to explain why firms use celebrity influencers to build mass product awareness (due to their large number of followers, many of whom are not interested in the product category), whereas an endorsement by a small micro-influencer is more informative as followers of small influencers post comments that help future consumers in their search for product information.

Sponsored influencer posts, as a form of advertising, are particularly susceptible to deceptive practices. Game theory models have investigated how consumers respond

to deceptive advertising (Wu and Geylani 2020) and how publishers and advertisers respond to fraudulent online ad impressions (Choi and Sayedi 2023). Conceptual research has identified both influencers and followers as potentially deceptive in sharing product information (Nistor et al. 2018). Yalcin et al. (2020) present anecdotal evidence that the role of influencers in sharing information about product attributes ranges from pure advertising to education of followers. Cain et al. (2010) present experimental evidence that consumer do not sufficiently discount the content of a recommendation even when they know the post or ad is sponsored. Our model implies one reason a consumer may trust a sponsored influencer post is that she can read informative comments by other followers to determine whether the product is truly a good fit for her. We show that larger influencers are in a sense less trustworthy because they endorse lower quality products. However, there is no explicit deception in our model because customers are aware of the influencer's endorsement policy.

3 Model

An influencer can make a social media post endorsing a seller's product. His followers can read and comment on this post and can purchase the product.

To begin the game, the influencer chooses an endorsement policy. He can choose either to endorse all product types, or to endorse only a high quality product. This choice of endorsement policy is publicly observed and binding. We later present a model extension in which reputation concerns compel the influencer not to deviate from his chosen endorsement policy.

A monopolist sells a product that has high quality with probability α and low quality with probability $1-\alpha$, where $0 < \alpha < 1$. The product meets the needs of a fraction q of potential customers. For a high quality product $q = \overline{q}$, and for a low quality product $q = \underline{q}$, where $0 < \underline{q} < \overline{q} < 1$. Thus, for any given customer, the product has either good or bad fit, and product quality is defined as the probability that the product has good fit for a given customer. Nature chooses product quality.

The seller and the influencer observe product quality.[2] The influencer's followers do not directly observe quality. If the product has low quality and the influencer has a policy of only endorsing high quality products, the game ends with zero payoff for all players. Otherwise, the game proceeds to the next stage.

The seller and the influencer agree to an endorsement deal in which the seller makes a fixed payment to the influencer, such that the two parties proportionally split the expected profits from an endorsement. Because the influencer receives a fixed proportion of expected profits, he sets his endorsement policy to maximize expected profits. After making the fixed payment, the seller sets prices to maximize profits. Thus, any fixed payment that proportionally splits expected profits implies both parties take actions to maximize total expected profits, and the influencer's equilibrium endorsement policy does not depend on whether he has high or low bargaining power.[3]

The seller sets the product's initial price.

Followers of the influencer then read the influencer's post endorsing the product. Reading the post makes customers aware that the product exists. Customers may know in advance that the company will launch some type of food or some type of clothing, for example, but they do not know the particular type of food or clothing until they read the influencer's post. Reading the post also reveals the product's current price. Customers then have the opportunity to buy the product.

There are two types of customers. There is a total mass M_Z of customers who are not interested in the product and derive zero value from it (these followers may read the influencer's post purely for entertainment value), and a mass M_V who are interested in the product and derive value one from consuming the product if it is a good fit. Customers derive zero value from consuming a product that is a poor fit.

[2] A model extension considers the case in which the influencer cannot observe quality for products outside of his expertise.

[3] Formally, the fixed payment is chosen such that the seller receives a fraction ϕ and the influencer receives a fraction $1-\phi$ of expected profits, where $\phi \in (0,1)$. For any value of ϕ, each party chooses actions to maximize expected total profits, so the equilibrium endorsement policy and prices do not depend on ϕ. The split of expected surplus could arise from an endogenous bargaining process, for example (Binmore et al. 1986).

These customers arrive in two sequential groups. Those who have time to read the post soon after the influencer writes it are part of the first group, whereas those who have time to read the post at a later date (after the first group has already made product purchases) are part of the second group. Similar to previous theoretical research on advertising (e.g, Milgrom and Roberts 1986; Miklós-Thal and Zhang 2013), we allow the firm to adjust the product's price after the initial group of consumers make their purchases. The firm initially sets price P_1, and customers in the first group can purchase at this price. After the first group of customers makes their purchase decisions and comments, the firm can change its price to a different price P_2, and customers in the second group can purchase at this new price. The firm chooses endogenous prices, which we will show are equal to willingness to pay of customers who are interested in the product in each period. This setup is equivalent to allowing the firm to offer a limited-time discount code to encourage the first group of followers to try the product, which is a common feature of Instagram influencer ads (Chitrakorn 2020; Markerly 2022; Grin 2022).[4]

Each customer has probability β of belonging to the first group of customers who can read the post immediately after the influencer posts, and probability $1-\beta$ of belonging to the second group who can read the post at a later date, where $0 < \beta < 1$. For example, there is a mass βM_Z of followers who derive zero value from the product in the first group, and a mass $(1-\beta)M_Z$ of such followers in the second group. We will show that, in equilibrium, followers in both groups receive the same expected utility, so there would be no incentive for a follower from the first group to delay reading the post to join the second group, for example.

Customers in the first group do not learn whether the product is a good fit for them until after they have purchased it. Customers who purchase the product can then make an informative comment describing their experience with the product,

[4]In principle, we could assume product prices are exogenous and derive similar results. With exogenous prices, quality expectations would affect product demand instead of affecting prices. We could also assume prices are endogenous and fixed across the two periods, in which case quality expectations would affect both price and demand. In all of these cases, equilibrium profits depend on followers' quality expectations given the influencer's endorsement policy.

for example, confirming whether the product performs as promised on a particular vertical attribute, or providing additional information about a horizontal attribute of the product. If they do not purchase the product, they can write an uninformative comment such as an emoji or general message of support for the influencer.

Previous research has shown customers write comments both for the intrinsic utility from commenting and for the status utility from having other members of their community read their comment (Toubia and Stephen 2013; He et al. 2018). For modeling parsimony, in the main version of the model, we let the intrinsic utility from commenting exceed the cost of commenting, so each follower from the first group writes a comment. We later present a model extension with more detailed costs and benefits of commenting, including status utility from posting a comment that is read by the second group of followers, and we adapt our results to this richer model of commenting.

Customers in the second group who read the influencer's post observe the second period price. Second period customers can then read comments. Reading an informative comment reveals to a customer whether the product is a good fit for her. Thus, customers in the second group may choose to continue reading comments until they learn whether the product is a good fit before they buy the product. For simplicity of exposition, wet let there be a continuum of customers of each type, so the first group effectively produces an infinite number of comments, and the probability that the next comment will be informative does not change as a customer in the second group continues reading uninformative comments.

In this model set-up, the influencer's endorsement policy provides product quality information, and followers' comments provide product fit information. For example, if an influencer endorses only high quality products, his endorsement of a brand of canned tomatoes would inform followers that these tomatoes have high quality, that is, they have a high probability of satisfying a follower's needs. Reading comments would then provide a follower with product fit information, that is, inform her whether the tomatoes in fact serve her particular needs. The intuition is that followers

can write comments about their own unique experiences, allowing later followers to learn whether the product satisfies the needs of other similar customers. In principle, we could derive similar results while allowing the influencer and followers to provide other types of information as well. For example, the influencer could also provide information about product fit or offer advice that enhances the value of the product (Ronaldo provides scholarship information for eCampus University, and mississippivegan provides a recipe using Muir Glen tomatoes). The current set-up allows us to explore differences in the informativeness of comments on posts by micro-influencers and celebrity influencers, and the effect of these comments on the influencer's endorsement policy. We show that a micro-influencer attracts a higher fraction of informative comments, which implies followers spend time reading comments, and the influencer needs to consider followers' cost of reading comments when deciding his endorsement policy.

To summarize, the game timing is as follows:

1. The influencer decides his endorsement policy.

2. Nature chooses the product quality level, which the firm and the influencer observe. The game ends if the influencer's policy is not to endorse products with the given quality level. Otherwise, the firm and the influencer agree to a fixed payment the firm makes to the influencer such that they proportionally split the expected profits from an endorsement.

3. The firm sets the product's first period price.

4. Followers in the first group read the influencer's post. These followers then learn the product's current price, have the option to buy the product, and leave comments.

5. The firm sets the product's second period price.

6. Followers in the second group read the influencer's post. These followers learn

the product's current price. They can then read comments and have the option to buy the product.

Each customer maximizes her expected utility, with all variables defined such that a customer's marginal utility of wealth is normalized to one. For example, in the first period, if a customer purchases the product at price P_1 and finds it is a good fit, then her utility is $1 - P_1$, but if she purchases the product and it turns out to have poor fit, her utility is $-P_1$. Similarly, in the second period, if a customer who is interested in the product reads one comment, discovers the product has good fit, and purchases the product, then her utility is $1 - R - P_2$. If a second period customer reads a comment and finds the product has poor fit, so she chooses not to purchase it, her utility is $-R$ due to the reading cost.

The firm and the influencer each maximize their expected profits. Let D_1 and D_2 denote product demand from the first and second group of customers, respectively, and let C denote the product's marginal production cost, where $0 < C < [\alpha\overline{q} + (1-\alpha)\underline{q}]$. Total profits are $\pi = (P_1 - C)D_1 + (P_2 - C)D_2$. Recall that the fixed endorsement payment is chosen such that the firm and influencer split expected profits proportionally.

Table 1. Model notation

α	Probability that the product has high quality
$\underline{q}, \overline{q}$	Probability that low and high quality product meets customer i's needs
P_1	Price of the product in period 1
P_2	Price of the product in period 2
M_Z	Followers who derive zero value from the product
M_V	Followers who derive value one from product with good fit
β	Probability that any given customer arrives in the first group
R	Cost of reading each comment
C	Marginal production cost for the product

4 Results

We derive the optimal endorsement policy and equilibrium profits. Our solution concept is perfect Bayesian equilibrium, which requires that players have correct beliefs on the equilibrium path, and that no player has a profitable deviation after any possible history.

4.1 Fraction of informative comments

If followers in the first group purchase the product, then the first group make M_V informative comments and M_Z uninformative comments.[5] Letting λ denote the fraction of comments that are informative, we have the following.

$$\lambda = \frac{M_V}{M_Z + M_V} \tag{1}$$

Thus, an influencer generates a higher fraction of informative comments if most of his followers read the post due to their interest in the product category rather than their interest in the influencer himself. Furthermore, marketing analysts have observed that micro-influencers typically have followers who are interested in a niche product or service, whereas celebrity influencers have a more general audience. For example, a recent article in *Brandwatch* states, "As micro-influencers serve a specific niche, their audience is more likely to be specifically interested in recommendations in this area" (Vogl 2022). Similarly, an article in *Forbes* states, "Micro-influencers have the potential to engage audiences around topics that are specific to a particular interest" (Ehlers 2021).

To connect these real world observations with our model, imagine two vegan chefs on Instagram. One influencer has a small following and is only somewhat entertaining. He attracts people who care passionately about vegan food (most of his followers are in the M_V group). The other influencer is very entertaining. He attracts even

[5] We allow for the possibility that the first group do not purchase the product, but we show that, in equilibrium, the firm sets first period price such that they do purchase.

more of the people who are interested in vegan food, but he also attracts a large segment of followers (the M_Z group) who do not care much about vegan food and like him mostly for his entertaining commentary. Thus, the larger influencer generates a smaller fraction of informative comments. Similarly, we could compare a soccer player with a small following of passionate soccer fans with another soccer player with many followers who like him partly for his glamour and appearance. In each case, the larger influencer has a smaller proportion of fans who have a strong interest in his key product.[6]

Although celebrity influencers have only a small fraction of followers interested in a given product (the M_V group makes up a small fraction of their followers), the absolute number of followers in both groups is typically larger for a celebrity than for a micro-influencer (the absolute size of both the M_V group and the M_Z group is larger for the celebrity). In our model, profits are multiplied by the term M_V, so an endorsement by a celebrity influencer may generate much higher profits than an endorsement by a micro-influencer, even if the average quality of products endorsed by the celebrity is lower. In reality, the typical payment for a sponsored post by a micro-influencer is about \$10 per thousand followers (Shopify 2022). For example, an influencer with 100 thousand followers would be paid about \$1000 for each sponsored post. By contrast, the payment for a sponsored post by Cristiano Ronaldo is closer to \$5 per thousand followers, given that he has about 600 million followers and is paid about \$3 million per sponsored post (Geyser 2023). Thus, the endorsement payment for a celebrity may be smaller on a per-follower basis but much larger in absolute terms.

4.2 Customer behavior in the second period

We now derive the optimal strategy for customers in the second period. At this stage of the game, customers have observed the influencer's endorsement policy and the

[6] A previous version of this paper explicitly modeled the influencer's entertainment level and made the shift in the type of followers who read the post endogenous as entertainment increases. For simplicity of exposition, we no longer include the entertainment level as a formal variable in the model.

firm's first and second period prices. Let $\widetilde{q}$ denote customer beliefs about product quality. In equilibrium, these beliefs must be accurate given the strategies of all other players, and in the derivations that follow, we also consider quality beliefs if the seller deviates to a price off the equilibrium path.

Lemma 1. *In period two, the utility-maximizing strategy for a customer who is interested in the product is as follows. If* $(1-\widetilde{q})P_2 \leq \frac{R}{\lambda}$ *and* $\widetilde{q} \geq P_2$*, the customer purchases the product without reading any comments. If* $(1-\widetilde{q})P_2 > \frac{R}{\lambda}$ *and* $\widetilde{q} \geq \widetilde{q}P_2 + \frac{R}{\lambda}$*, the customer reads comments until finding an informative comment, and purchase the product if it is a good fit. Otherwise, the customer does not read any comments and does not purchase.*

Each customer who reads the post in the second period faces the following dynamic optimization problem. She can either purchase immediately, or incur cost R to read a comment which has probability λ of revealing whether the product is a good fit for her. If the customer reads an informative comment, she purchases the product only if it is a good fit. If she reads an uninformative comment, then the cost R of reading that comment is now a sunk cost. Therefore, she faces the same dynamic optimization problem as before, and if reading the first comment was optimal, then it is also optimal to read another comment after finding an uninformative comment (see the proof in the appendix for additional detail).

The strategy of purchasing immediately leads to a probability $1-\widetilde{q}$ of paying price P_2 for a product that is a poor fit. The strategy of reading comments avoids this cost of purchasing a product that is a poor fit but also incurs an additional expected cost $\frac{R}{\lambda}$ of reading comments. The customer chooses a strategy based on which of these costs is lower. Finally, if both of these strategies produce negative expected utility, the customer simply does not read comments and does not purchase.

4.3 Equilibrium endorsement policy and product prices

We now compute the equilibrium endorsement policy and product prices. In order to study the distinct strategies of celebrity influencers and micro-influencers, we focus our analysis on low and high values of λ. In particular, the fraction of followers who are interested in the product is $\underline{\lambda}$ for a celebrity influencer and $\overline{\lambda}$ for a micro-influencer, where $0 < \underline{\lambda} < \overline{\lambda}$. We will shown how this difference in the proportion of different types of followers affects the influencer's equilibrium endorsement policy, the seller's equilibrium price, and the commenting, reading, and purchase behavior of followers.

4.3.1 Celebrity influencer

We now derive the equilibrium outcome for a celebrity influencer. The proof in the appendix contains the precise conditions on $\underline{\lambda}$ required for the following lemma to hold.

Lemma 2. *If $\underline{\lambda}$ is sufficiently small, the equilibrium given an endorsement by a celebrity influencer is as follows. Both high and low quality firms set $P_1 = P_2 = \alpha\overline{q} + (1-\alpha)\underline{q}$ if the influencer has a policy of endorsing all products. The firm sets $P_1 = P_2 = \overline{q}$ if the influencer has a policy of endorsing only high quality products. Customers purchase the product without reading any comments.*

The intuition for this result is the following. In the first period, the equilibrium price is just low enough to induce customers to purchase given quality beliefs that are based on the influencer's endorsement policy. In the second period, if a sufficiently small faction of comments are informative, Lemma 1 implies that, due to the high expected cost of finding an informative comment, there is no feasible price level and quality beliefs for which customers read comments. Furthermore, a high price cannot serve as a quality signal in either period because a low quality firm would have an incentive to mimic this high price. Therefore, customers base their quality beliefs only on the influencer's endorsement policy and do not change these quality beliefs if the seller deviates to an out-of-equilibrium price. Thus, for a celebrity influencer, the

only price equilibrium is a pooling equilibrium in which both high and low quality firms set the same price in both periods, and customers do not read any comments.

We now derive the influencer's equilibrium endorsement policy. We let $\underline{\lambda}$ be low enough that Lemma 2 holds, so followers do not read comments on a celebrity influencer's post in equilibrium. The influencer and seller proportionally split the profits from the endorsement deal, so the influencer will choose whichever endorsement strategy leads to the highest expected profits.

When choosing an endorsement policy, a celebrity endorser faces the following trade-off. A selective policy of endorsing high quality products implies the influencer may not make a sponsored post. In particular, the probability of making a sponsored post is one if the influencer endorses all product types and α if he endorses only high quality products. However, a policy of endorsing high quality products also allows the seller to set a higher equilibrium price because followers expect higher quality conditional on an endorsement. In particular, the equilibrium price in both periods is $(\alpha\overline{q} + (1-\alpha)\underline{q})$ if the influencer endorses all products and $\overline{q}$ if he endorses only high quality products. The optimal policy considers both of these effects of the endorsement policy.

Based on Lemma 2, if a celebrity influencer endorses all products, expected profits are:

$$E[\pi] = \big(\alpha\overline{q} + (1-\alpha)\underline{q} - C\big)M_V \tag{2}$$

Lemma 2 also implies, if a celebrity influencer endorses only high quality products, expected profits are:

$$E[\pi] = \alpha\big(\overline{q} - C\big)M_V \tag{3}$$

In both of these equations, the first part of the profit expression is expected profits per follower who is interested in the product, which is then multiplied by the number of followers M_V who are interested in the product. For either endorsement policy, an influencer with a larger number of followers interested in the product generates higher expected earnings.

By comparing these profit functions, we find that the change in profits from endorsing all products instead of only high quality products is $(1-\alpha)(\underline{q}-C)M_V$. Therefore, if $\underline{q} > C$, profits are higher if the influencer endorses all products. For a celebrity influencer, followers do not read comments and do not learn whether the product is a good fit prior to purchase. Therefore, the firm sets the price equal to the expected value that a customer receives from the product. As long as a customer's expected value from a low quality product exceeds the product's marginal cost, endorsing all products allows the seller to capture additional surplus generated by selling low quality products, and expected profits are maximized by endorsing both high and low quality products.

Proposition 1. *A celebrity influencer endorses all products if $\underline{q} > C$, and endorses only high quality products if $\underline{q} < C$.*

4.3.2 Micro-influencer

We now derive the equilibrium for a micro-influencer with a high fraction of informative comments $\overline{\lambda}$. In particular, we focus on cases in which the expected cost of reading comments to find an informative comment, given by $\frac{R}{\overline{\lambda}}$, is sufficiently small. The proof in the appendix contains the precise conditions on $\frac{R}{\overline{\lambda}}$ for this lemma to hold.

Lemma 3. *If $\frac{R}{\overline{\lambda}}$ is sufficiently small, the equilibrium given an endorsement by a micro-influencer is as follows. In the first period, both high and low quality firms set $P_1 = \alpha\overline{q} + (1-\alpha)\underline{q}$ if the influencer has a policy of endorsing all products, and the firm sets $P_1 = \overline{q}$ if the influencer has a policy of endorsing only high quality products. In the second period, both high and low quality firms set $P_2 = 1 - \frac{R}{[\alpha\overline{q}+(1-\alpha)\underline{q}]\overline{\lambda}}$ if the influencer endorses all products, and the firm sets $P_2 = 1 - \frac{R}{\overline{q}\overline{\lambda}}$ if the influencer endorses only high quality products. In period two, customers read comments until they find an informative comment and purchase only if the product is a good fit.*

The intuition for this result is the following. In the first period, the firm sets

the same price as it would for a celebrity influencer, that is, the maximum price customers are willing to pay given their quality beliefs based on the influencer's endorsement policy. In the second period, given the high fraction of informative comments, Lemma 1 implies the firm has two possible pricing strategies that could be optimal. One strategy is to set a low price so that customers purchase immediately without reading comments, and the other strategy is to set a higher price so that customers read comments before deciding whether to purchase. The strategy of setting a high price leads to higher profits for a high quality firm than for a low quality firm because, after reading the comments, a larger fraction of customers find the product is a good fit if the firm has high quality $\overline{q}$ rather than low quality $\underline{q}$. Therefore, under some conditions, a high price could serve as a signal of high quality in the second period. However, if the reading cost is sufficiently low, a low quality firm prefers to pool with the high quality firm by setting a high price in period two. In this pooling equilibrium, deviating to a lower out-of-equilibrium price leads customers to believe that the firm has low quality. By contrast, if the firm deviates to an out-of-equilibrium price higher than the one stated in Lemma 3, customers do not update their quality beliefs, and their beliefs are still based only on the influencer's endorsement policy. See the proof in the appendix for additional detail.

We now derive the influencer's equilibrium endorsement policy. We let $\frac{R}{\overline{\lambda}}$ be low enough that Lemma 3 holds and, in equilibrium, second period followers read comments on a post by a micro-influencer.

A micro-endorser must consider how his endorsement policy affects the probability of accepting an endorsement offer, the product price in each period, and also the probability of customers purchasing the product in the second period. The probability of making a sponsored post is one if the influencer endorses all product types and α if he endorses only high quality products. In the first period, the equilibrium price is $(\alpha\overline{q} + (1-\alpha)\underline{q})$ if the influencer endorses all products and $\overline{q}$ if he endorses only high quality products. Both of these effects are the same for celebrity influencers and micro-influencers.

However, for a micro-influencer, in the second period the product price is $1-\frac{R}{\overline{q}\overline{\lambda}}$ if he endorses only high quality products and $1-\frac{R}{[\alpha\overline{q}+(1-\alpha)\underline{q}]\overline{\lambda}}$ if he endorses all products. The influencer's endorsement policy affects the price in the second period because the maximum price at which followers are willing to read comments is higher if there is a high probability the product has good fit. Furthermore, the fraction of customers who purchase the product in the second group is $\overline{q}$ if the influencer endorses only high quality products and $[\alpha\overline{q}+(1-\alpha)\underline{q}]$ if he endorses all products. Thus, for a micro-influencer, a selective endorsement policy helps customers avoid the cost of reading comments about low quality products and results in both a higher product price and a higher fraction of customers purchasing the product in the second period. Note that prices are endogenous in our model, so high quality expectations result in higher equilibrium prices. Alternatively, in a model with fixed prices, higher quality expectations would lead to an increase in the number of products sold. In either case, the seller's profits conditional on an endorsement are higher if followers do not expect to waste time reading comments about low quality products.

The influencer will choose whichever endorsement strategy leads to the highest expected profits. Based on Lemma 3, if a micro-influencer endorses all products, expected profits are:

$$E[\pi]=\beta\big(\alpha\overline{q}+(1-\alpha)\underline{q}-C\big)M_V+ \\ (1-\beta)[\alpha\overline{q}+(1-\alpha)\underline{q}]\left[1-C-\frac{R}{[\alpha\overline{q}+(1-\alpha)\underline{q}]\overline{\lambda}}\right]M_V \tag{4}$$

Lemma 3 also implies, if a micro-influencer endorses only high quality products, expected profits are:

$$E[\pi]=\alpha\beta\big(\overline{q}-C\big)M_V+\alpha(1-\beta)\overline{q}\left[1-C-\frac{R}{\overline{q}\overline{\lambda}}\right]M_V \tag{5}$$

By comparing these two profit functions and rearranging terms, we find that the incremental profits from endorsing all products (instead of just high quality products)

is the following:

$$(1-\alpha)\left[\underline{q} - \beta C - (1-\beta)\underline{q}C - \frac{R(1-\beta)}{\overline{\lambda}}\right]M_V \tag{6}$$

Endorsing low quality products affects profits in two ways. First, it allows the seller to capture surplus from the difference between the expected value of these products and their marginal cost, considering that, in the second period, the firm incurs production cost only for customers who find the product is a good fit. Second, endorsing low quality products imposes a cost on second period customers by inducing them to spend time reading comments about these products, which results in a lower equilibrium price in the second period. In other words, each additional product a micro-influencer endorses leads to additional costs for his followers that result from the time they spend reading comments. If this reading cost is too high relative to the surplus from consuming low quality products, then the influencer prefers to endorse only high quality products.

Proposition 2. *A micro-influencer endorses all products if* $\underline{q} > \beta C + (1-\beta)\underline{q}C + (1-\beta)\left(\frac{R}{\overline{\lambda}}\right)$*, and endorses only high quality products if this inequality is reversed.*

Thus, our model implies a key difference between influencer types is that micro-influencers try to protect the time of their followers, so these followers do not waste time reading comments about low quality products. Because celebrity influencers have so many uninformative comments that followers do not read the comments in any case, these influencers do not face this same concern, and big celebrity influencers may be more inclined to endorse all types of products.

For modeling parsimony and clarity, we have focused our analysis on low values of λ for which followers never read comments and high values of λ for which they always read comments in equilibrium. If the fraction of informative comments lies in an intermediate range, other types of equilibria are possible. For example, in some cases with intermediate values of λ, price can act as a quality signal, and followers in the second period read comments only for high quality products with a

high price, whereas they purchase low quality products immediately at a lower price without reading comments. In this case, reading costs would not affect the influencer's endorsement decision because, in equilibrium, followers do not read comments on low quality products.

4.4 Numerical example

We now present a numerical example to help provide more intuition for the influencer's equilibrium endorsement policy. Table 2 presents the parameter values used for the numerical example. These numerical values satisfy all of the model's conditions, including the conditions of Lemmas 2 and 3 and Propositions 1 and 2.

Based on results from the previous sections, we derive the equilibrium outcome for each type of influencer and each endorsement policy. For a celebrity influencer, the fraction of informative comments is $\underline{\lambda} = 0.01$, so on average a customer would need to read 100 comments to find an informative comment. For a micro-influencer, with $\overline{\lambda} = 0.18$, a customer needs to read only 5.6 comments on average to find an informative comment. For these parameter values, customers in the second period read comments for a micro-influencer but not for a celebrity influencer.

Table 2. Parameter values used in the numerical example

$\alpha = 0.2$	Probability that the product has high quality
$\underline{q} = 0.07$	Probability that low quality product meets customer i's needs
$\overline{q} = 0.7$	Probability that high quality product meets customer i's needs
$\beta = 0.2$	Probability that any given customer arrives in the first group
$R = 0.02$	Cost of reading each comment
$C = 0.06$	Marginal production cost for the product
$\overline{\lambda} = 0.18$	Fraction of informative comments for micro-influencer
$\underline{\lambda} = 0.01$	Fraction of informative comments for celebrity influencer

If a celebrity influencer endorses all products, the equilibrium price in both periods is 0.196, which given a marginal production cost of 0.06 implies total profits are

$(0.196 - 0.06)M_V = 0.136M_V$. If the celebrity influencer endorses only high quality products, then there is a probability 0.2 that he endorses the product, the price in both periods is 0.7, and total expected profits are $0.2(0.7 - 0.06)M_V = 0.128M_V$. Thus, for a celebrity influencer, the strategy of endorsing all products, with low average quality and low equilibrium price, generates 6% higher profits than the strategy of being more selective and endorsing only high quality products with higher equilibrium price (profits are $0.136M_V$ versus $0.128M_V$).

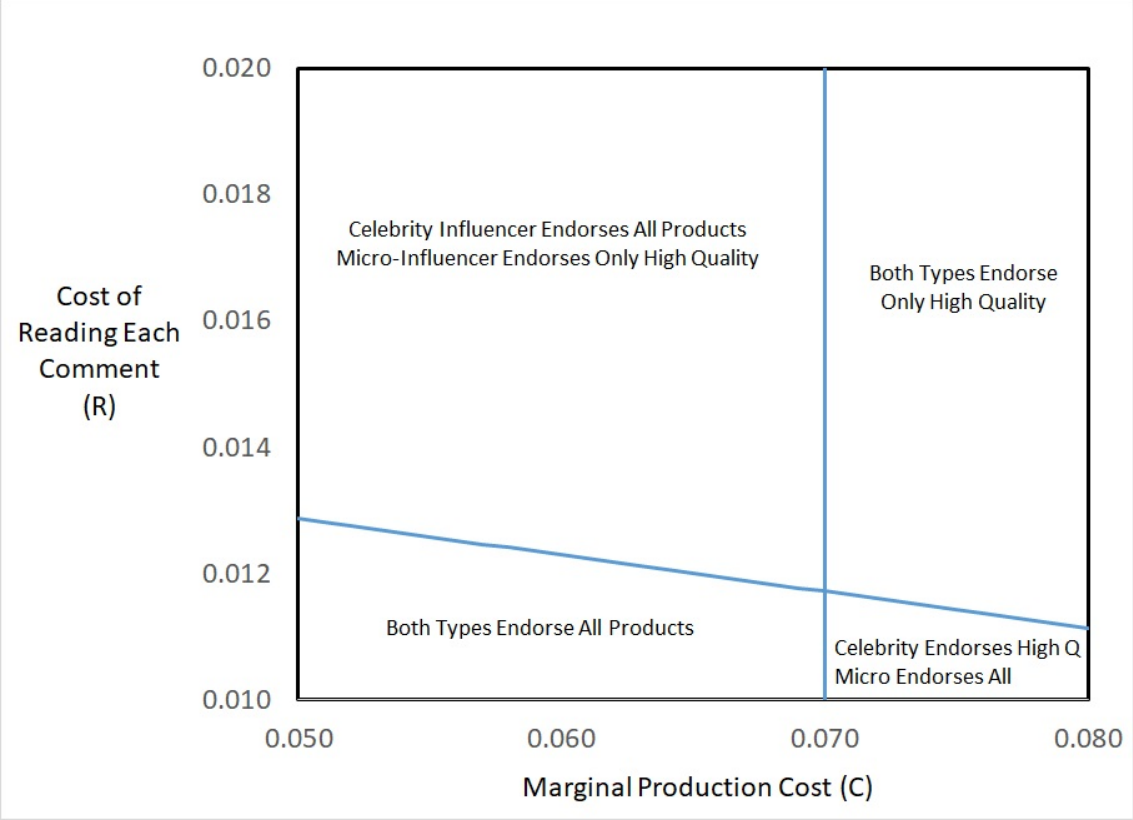
If a micro-influencer endorses all products, the first period price is 0.196; the second period price is 0.433, and second period customers read comments before purchasing, which results in a fraction 0.196 of these second period customers finding the product is a good fit and purchasing. Total profits are $[0.2 * (0.196 - 0.06) + 0.8 * 0.196 * (0.433 - 0.06)]M_V = 0.086M_V$. If a micro-influencer endorses only high quality products, there is probability 0.2 that he endorses the product. In this case, the first period price is 0.7. The second period price is 0.841, and second period customers read comments before deciding whether to purchase, which results in a fraction 0.7 of these customers finding the product is a good fit and purchasing. Total profits are $0.2[0.2 * (0.7 - 0.06) + 0.8 * 0.7 * (0.841 - 0.06)]M_V = 0.113M_V$. Thus, for a micro-influencer, profits are 32% higher if the influencer is selective and endorses only high quality products, rather than endorsing all products (profits are $0.113M_V$ versus $0.086M_V$).

To illustrate comparative statics for our results, Figure 1 presents the equilibrium endorsement policies using the same numerical values as above. When both costs (reading cost R and production cost C) are high, both influencer types endorse only high quality products. When both costs are low, both types endorse all products. When the production cost is relatively low but the cost of reading comments is relatively high, the celebrity influencer endorses all products, whereas the micro-influencer endorses only high quality products. Finally, there is a region in the bottom-right of the figure in which a celebrity influencer endorses only high quality products because production cost is so high that it is not profitable to sell a low quality

product to all customers, whereas a micro-influencer endorses all products because the reading cost is low enough that customers can easily learn whether the product is a good fit. Thus, the celebrity influencer's endorsement strategy depends on the marginal production cost of the product, whereas the micro-influencer's endorsement strategy depends mostly on the marginal cost of reading comments.

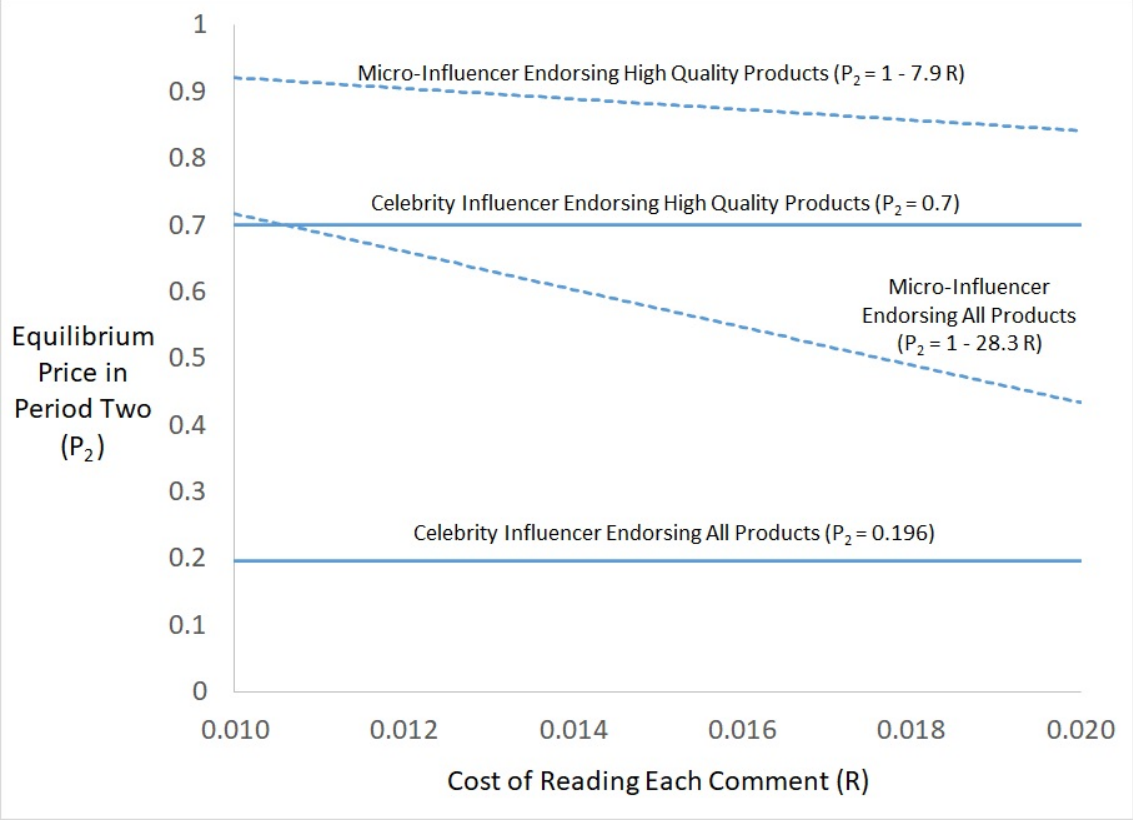
For example, our results imply Ronaldo may want to endorse a low quality online university which offers remote learning and has low production costs, but he should be more selective if endorsing a traditional in-person university with higher costs. Furthermore, our results imply a micro-influencer like mississippivegan should be selective in his endorsement policy because his audience of healthy eaters place high value on their time, whereas a micro-influencer who produces content for children who play video games, for example, may endorse low quality products because the cost of his followers' time is lower.

Figure 1. Influencer's equilibrium endorsement policy



As illustrated by this numerical example, our model implies that celebrities endorse all products in categories with low marginal production costs and endorse only high quality products in categories with higher production costs. By contrast, micro-influencers always endorse high quality products if the cost of reading comments is high. Surveys show that Millennial and Generation Z shoppers spend an average of about 18 minutes reading online product reviews, including comments on social media ads, before deciding whether to purchase a product (Kats 2020; Ryan 2023). With many competing options for how to spend time on social media, followers would prefer not to spend this much time reading comments on a product that is likely to be a poor fit, which helps explains why micro-influencers only endorse products they genuinely expect will appeal to their followers.

Figure 2. Second period price as a function of reading costs, for each influencer type and endorsement policy



To provide further insight into how reading costs affect the influencer's endorsement policy, Figure 2 presents the second period price for the numerical example as a function of reading costs. For a celebrity influencer, followers do not read comments, and the price is based only on quality expectations given the influencer's endorsement policy. If the celebrity endorses only high quality products, there is probability 0.7 that the product has good fit for any particular follower, so the equilibrium price is 0.7. By contrast, if the celebrity endorses all products, there is probability 0.196 of good fit, so the equilibrium price is 0.196. These prices are the maximum amount followers are willing to pay given their quality expectations.

For a micro-influencer, followers read comments, and an increase in reading costs implies the seller must set a lower product price. To understand the intuition for how reading costs affect price, imagine a follower reads many endorsement posts by a micro-influencer, and consider the number of comments the follower reads on average (including comments on products she does not purchase) before finding a product she decides to purchase. If each endorsed product has probability $\widetilde{q}$ of good fit, a follower reads an average of $\frac{1}{\widetilde{q}\lambda}$ comments before she purchases a product. Our model reflects the expected outcome for a single product in this type of game. If the micro-influencer endorses only high quality products, a follower reads an average of $\frac{1}{0.7*0.18} = 7.9$ comments before she purchases a product, which implies the equilibrium second period price is $1 - 7.9R$. By contrast, if the micro-influencer endorses all products, a follower reads an average of $\frac{1}{0.196*0.18} = 28.3$ comments before she purchases a product, and the equilibrium second period price is $1 - 28.3R$. These equilibrium prices represent the highest price for which followers are willing to spend time reading comments to learn if the product is a good fit.

Thus, the equilibrium price is more sensitive to reading costs if a micro-influencer endorses all products instead of only high quality products, and the influencer must

anticipate these reading costs when choosing an endorsement policy. If reading costs are sufficiently high, the influencer prefers to endorse only high quality products. For this numerical example, if the production cost C is 0.06, a celebrity endorses all products, whereas for this same value of C, a micro-influencer endorses all products if the reading cost R is less than 0.0123 but endorses only high quality products if the reading cost exceeds this value.

4.5 Model extension: Costs and benefits of commenting

This model extension explicitly accounts for the costs and benefits of commenting. People write comments for the intrinsic utility of commenting and for the status utility of having other members of their community read their comments (Toubia and Stephen 2013; He et al. 2018). Our model allows for both types of utility. Similar to the word-of-mouth model by Campbell et al. (2017), the status utility from writing a comment depends on how many other followers read the comment.

Each follower can write an uninformative comment, which provides the commenter with utlity $e - k_u + NS_u$, where e is the intrinsic entertainment utility of commenting, k_u is the cost of writing an uninformative comment, N denotes the expected number of people who will read the comment, and S_u is the status utility the commenter derives for each person who reads the uninformative comment. Followers who are interested in the product and purchase it may, alternatively, write an informative comment, which provides utility $e - k_i + NS_i$, where k_i is the cost of writing an informative comment, and S_i is the status utility the commenter derives for each person who reads an informative comment. We let $k_i > e > k_u > 0$, and $S_i > S_u > 0$.

For a celebrity influencer, under the conditions of Lemma 2, followers in the second period do not read comments, which implies $N = 0$. Therefore, followers write only uninformative comments, and the equilibrium product price and endorsement policy

for a celebrity influencer are the same as in the main version of the model.

We now derive the equilibrium for a micro-influencer. Under the conditions of Lemma 3, each follower in the second period who is interested in the product reads comments until finding an informative comment, which on average requires reading $\frac{1}{\lambda}$ comments. In this case, there are $\beta(M_V + M_Z)$ total followers who write comments in the first period and $(1-\beta)M_V$ followers interested in the product who each expect to read $\frac{1}{\lambda}$ comments in the second period, which implies that, for first period commenters, the expected number of people who will read their comment is $N = \frac{(1-\beta)M_V}{\beta(M_V+M_Z)\lambda}$. Given that $\lambda = \frac{M_V}{M_V+M_Z}$, we have $N = \frac{(1-\beta)}{\beta}$. Therefore, first period commenters who purchase the product will leave an informative comment if $(S_i - S_u)\frac{1-\beta}{\beta} > k_i - k_u$. This condition ensures the incremental status utility from writing an informative (rather than uninformative) comment exceeds the additional cost of writing an informative comment, so an equilibrium with informative comments exists.

If this condition holds, the results remain the same as in the main model, except for two key differences for a micro-influencer. First, the firm's first period price accounts for the incremental utility a customer gains from buying the product and writing an informative comment, given by $(S_i - S_u)\frac{1-\beta}{\beta} - (k_i - k_u)$. The second difference is that, in the main version of the model, followers in both groups receive the same expected utility, whereas in this extension, followers in the first group receive an additional equilibrium utility of $\frac{(1-\beta)S_u}{\beta}$ from having their comments read. However, in this model extension, we let second-period followers have a better outside option for how to spend their time soon after the influencer posts, so it would be costly for them to read the post earlier as part of the first group. In particular, if they face an additional cost of at least $\frac{(1-\beta)S_u}{\beta}$ if they decided to read the post earlier, these followers prefer wait and read the post as part of the second group.

These results are stated formally in the following proposition.

Proposition 3. *For a celebrity influencer, the results are the same as in the main model. For a micro-influencer, if* $\frac{R}{\overline{\lambda}}$ *is sufficiently small,* $(S_i - S_u)\frac{1-\beta}{\beta} > k_i - k_u$*, and second period followers have a cost of at least* $\frac{(1-\beta)S_u}{\beta}$ *from reading the post earlier, the equilibrium is as follows. In the first period, both high and low quality firms set* $P_1 = \alpha\overline{q} + (1-\alpha)\underline{q} + (S_i - S_u)\frac{1-\beta}{\beta} - (k_i - k_u)$ *if the influencer has a policy of endorsing all products, and the firm sets* $P_1 = \overline{q} + (S_i - S_u)\frac{1-\beta}{\beta} - (k_i - k_u)$ *if the influencer has a policy of endorsing only high quality products. In the second period, both high and low quality firms set* $P_2 = 1 - \frac{R}{[\alpha\overline{q} + (1-\alpha)\underline{q}]\overline{\lambda}}$ *if the influencer endorses all products, and the firm sets* $P_2 = 1 - \frac{R}{\overline{q}\overline{\lambda}}$ *if the influencer endorses only high quality products. In period two, customers read comments until they find an informative comment and purchase only if the product is a good fit. The equilibrium endorsement policy is to endorse all products if* $\underline{q} + (S_i - S_u)(1-\beta) - \beta(k_i - k_u) > \beta C + (1-\beta)\underline{q}C + (1-\beta)\left(\frac{R}{\overline{\lambda}}\right)$*, and to endorse only high quality products if this inequality is reversed.*

4.6 Model extension: Influencer specialization

In this model extension, the influencer can either specialize in endorsing products for which he has expertise, or he can also endorse products outside his expertise. As stated by an article in *Forbes*, "Micro-influencers have the potential to engage audiences around topics that are specific to a particular interest. This type of micro-influencer tends to be an expert in their area and has followers because of their unique, specialized interests." (Ehlers 2021). We show that, in some cases, a micro-influencer makes an endogenous decision to endorse products only in his area of expertise.

Similar to the literature on expert certification (e.g., Morrison and White 2005; Lerner and Tirole 2006), we model expertise as the ability to evaluate a product's quality level. Suppose there is probability γ the influencer can observe the product's quality level, and probability $1 - \gamma$ he cannot observe quality, where $0 < \gamma < 1$.

When nature determines the product's quality level, it also independently determines whether the influencer has the expertise to evaluate quality. The influencer and firm can observe whether the influencer has expertise in that particular product, but followers cannot. For example, a chef may be an expert in certain types of food but not others, and followers may be unaware of which types of food fall in each category.

When the influencer chooses his endorsement policy, he now has three options: (1) endorse all products, (2) specialize in endorsing only products for which he has expertise and quality is high, (3) endorse products for which he has expertise and quality is high and also all products for which he lacks expertise. All other model assumptions remain the same.

We now derive the influencer's optimal endorsement policy for this model extension. When results from previous sections imply the influencer endorses all products, expertise is irrelevant, and the influencer continues to endorse all products in the current model extension. For example, if $\underline{q} > C$, Proposition 1 implies a celebrity influencer endorses all products, including those for which he lacks expertise.

Now suppose results from the previous section imply the influencer endorses only high quality products. The decision to endorse products for which the influencer lacks expertise is then formally equivalent to the decision to endorse a low quality product in the original model if the low quality product had quality level $\widehat{q} \equiv [\alpha\overline{q} + (1-\alpha)\underline{q}]$. We can therefore use results from the previous sections to derive the optimal endorsement policy.

Recall that in the initial model set-up $\widehat{q} > C$. Therefore, Proposition 1 implies the celebrity influencer always endorses products for which he lacks expertise. By contrast, Proposition 2 implies a micro-influencer may not want to endorse products for which he lacks expertise because he wants to avoid imposing additional reading costs on his followers. These results are formally stated in the following proposition.

Proposition 4. *A celebrity influencer endorses all products if* $\underline{q} > C$*, and endorses high quality products for which he has expertise and also all products for which he lacks expertise if* $\underline{q} < C < \widehat{q}$*. A micro-influencer endorses all products if* $(1-\beta)\left(\frac{R}{\overline{\lambda}}\right) < \underline{q}1 - \beta C - (1-\beta)\underline{q}C$*, endorses high quality products for which he has expertise and also all products for which he lacks expertise if* $\underline{q} - \beta C - (1-\beta)\underline{q}C < (1-\beta)\left(\frac{R}{\overline{\lambda}}\right) < \widehat{q}1 - \beta C - (1-\beta)\widehat{q}C$*, and endorses only high quality products for which he has expertise if* $\widehat{q}1 - \beta C - (1-\beta)\widehat{q}C < (1-\beta)\left(\frac{R}{\overline{\lambda}}\right)$.

Thus, whereas the celebrity influencer endorses products for which he lacks expertise, the micro-influencer may avoid endorsing such products so that his followers do not waste time reading comments about low quality products.

4.7 Discussion of other model extensions

The appendix contains two other extensions of our model. The first extension relaxes the assumption that the influencer's endorsement policy is binding. We then derive results for an infinitely repeated version of the game. Intuitively, reputation concerns can compel a micro-influencer to maintain a policy of endorsing only high quality products. If he ever deviates and endorses a low quality product, the game then moves to a bad equilibrium in which followers expect the influencer to begin endorsing all products. The second model extension allows the influencer to delete some of the uninformative comments. We show that a micro-influencer would like to delete uninformative comments to make it easier for followers to find informative comments. However, in some cases, a celebrity influencer prefers to keep uninformative comments on his post so followers purchase the product without reading comments.

5 Conclusion

Our paper creates a model of sponsored influencer and follower communication about a product. An influencer decides which types of products to endorse, and followers can read the influencer's post and leave comments for future customers. We derive conditions in which a less entertaining influencer, on average, endorses higher quality products. In equilibrium, a large celebrity influencer who generates widespread product awareness but uninformative comments endorses all products, whereas a smaller micro-influencer whose followers write informative comments endorses only high quality products. A key difference is that smaller influencers avoid endorsing low quality products so their followers do not waste time reading comments about a product that is unlikely to be a good fit.

Influencer advertising is one of the fastest growing areas of marketing, with campaigns spanning industries ranging from beauty and fitness to vaccination campaigns (Tiffany 2021; Ravindranath 2021). However, managers have found that micro-influencers often refuse paid endorsement deals (Carufel 2021; Baklanov 2021). Our results can help managers target influencers who are willing to form a partnership. If a firm wants to create deep engagement with followers who share product information, then it should produce a high quality product and work with micro-influencers with expertise in the product. For example, the state of Alaska ran a marketing campaign that paid "chefs and culinary influencers" to promote recipes that use Alaskan salmon, and these micro-influencers encouraged followers to share their own cooking tips and seafood recipes (Albright and Moran 2022; Alaska Seafood 2022). By contrast, if a firm's goal is simply to create widespread product awareness, it should work with large celebrity influencers, who are generally willing to endorse almost any firm that pays their endorsement fee. For example, reality television star Kim Kardashian was paid $250 thousand to make an Instagram post promoting the EthereumMax

cryptocurrency token (Calia and Corba 2022).

Future research could empirically test some of the model's predictions. Our model predicts that small influencers accept an endorsement deal only if the product has high quality and fits their expertise, whereas celebrity influences are willing to endorse low quality products unrelated to their expertise. Our model also predicts that smaller influencers generate deeper engagement with more informative comments, and are more likely to have a policy of deleting irrelevant comments.

Future research could model other types of information provided by influencers, for example, advice that enhances the value of a product being endorsed. It would also be interesting to develop dynamic models of influencer reputation to study how endorsement policies change as an influencer's following grows (Nistor et al. 2024). Classic models of reputation formation have studied conditions in which larger or smaller firms have a stronger incentive to invest in their reputation (e.g., Board and Meyer-ter-Vehn 2013). In the social media context, a new influencer may initially endorse only high quality products in their area of expertise, but later begin endorsing products with lower quality levels and products outside their area of expertise after they develop a sufficiently large following.

Funding and competing interests

All authors certify that they have no affiliations with or involvement in any organization or entity with any financial interest or non-financial interest in the subject matter or materials discussed in this manuscript. The authors have no funding to report.

References

Alaska Seafood (2022). AlaskaSeafoodHacks communications toolkit. https://www.alaskaseafood.org/wp-content/uploads/00_Alaska-Seafood-Hacks-Campaign-Communications-Toolkit.pdf.

Albright, A. and D. Moran (2022). Biden's rescue money doled out to racehorse owners, influencers. *Bloomberg*. https://www.bloomberg.com/news/features/2022-05-16/states-american-rescue-plan-arp-pandemic-aid-goes-to-influencers-racetracks.

Baklanov, N. (2021). Influencer outreach survey: Best ways to contact influencers. *HypeAuditor* . https://hypeauditor.com/blog/influencer-outreach-survey/ .

Bar-Isaac, H. (2003). Reputation and survival: Learning in a dynamic signalling model. *The Review of Economic Studies 70*(2), 231–251.

Bar-Isaac, H., G. Caruana, and V. Cuñat (2012). Search, design, and market structure. *American Economic Review 102*(2), 1140–60.

Bentley, K., C. Chu, C. Nistor, E. Pehlivan, and T. Yalcin (2021). Social media engagement for global influencers. *Journal of Global Marketing*, forthcoming.

Berman, R., A. Öry, and X. Zheng (2023). Influence or advertise: The role of social learning in influencer marketing. Working paper.

Bertacche, M. and C. Remondini (2019). Now there's a degree designed for those who want to become an influencer. *Bloomberg*. https://www.bloomberg.com/news/articles/2019-10-12/ronaldo-promoted-italian-school-now-offers-influencers-degree.

Binmore, K., A. Rubinstein, and A. Wolinsky (1986). The Nash bargaining solution in economic modelling. *The RAND Journal of Economics 17*(2), 176–188.

Board, S. and M. Meyer-ter-Vehn (2013). Reputation for quality. *Econometrica 81*(6), 2381–2462.

Bowman, A. (2018). Influencer marketing: Proven strategies to find influencers who fit your brand. *Crowdspring*. https://www.crowdspring.com/blog/influencer-marketing-strategies/.

Brodie, R. J., A. Ilic, B. Juric, and L. Hollebeek (2013). Consumer engagement in a virtual brand community: An exploratory analysis. *Journal of Business Research 66*(1), 105–114.

Cabral, L. and A. Hortaçsu (2010). The dynamics of seller reputation: Evidence from eBay. *The Journal of Industrial Economics 58*(1), 54–78.

Cain, D. M., G. Loewenstein, and D. A. Moore (2010). When sunlight fails to disinfect: Understanding the perverse effects of disclosing conflicts of interest. *Journal of Consumer Research 37*(5), 836–857.

Calia, M. and J. Corba (2022). Kim Kardashian pays over $1 million to settle SEC charges linked to a crypto promo on her Instagram. *CNBC*. https://www.cnbc.com/2022/10/03/kim-kardashian-settles-sec-charges-instagram-crypto-promotion.html.

Campbell, A., D. Mayzlin, and J. Shin (2017). Managing buzz. *The RAND Journal of Economics 48*(1), 203–229.

Carufel, R. (2021). Why don't influencers want to work with your brand? Here's what they want to see from brands. *Agility PR Solutions* . https://www.agilitypr.com/pr-news/public-relations/why-dont-influencers-want-to-work-with-your-brand-heres-what-they-want-to-see-from-brands/ .

Chen, Y. and J. Xie (2008). Online consumer review: Word-of-mouth as a new element of marketing communication mix. *Management Science 54*(3), 477–491.

Chitrakorn, K. (2020). Influencer discounting: Needs must. *Vogue Business*. https://www.voguebusiness.com/companies/influencer-discounting-needs-must.

Choi, W. J. and A. Sayedi (2023). Open and private exchanges in display advertising. *Marketing Science*, forthcoming.

Chu, C., C. Nistor, E. Pehlivan, and T. Yalcin (2021). Less is more: The effect of influencer reach on consumer engagement and persuasion knowledge. Working paper.

Chu, W. and W. Chu (1994). Signaling quality by selling through a reputable retailer: An example of renting the reputation of another agent. *Marketing Science 13*(2), 177–189.

Cruz, F. (2018). Influencers: Micro & macro. `https://www.tapinfluence.com/influencers-micro-macro/`.

Diamond, D. W. (1989). Reputation acquisition in debt markets. *Journal of Political Economy 97*(4), 828–862.

Ehlers, K. (2021). Micro-influencers: When smaller is better. *Forbes*. `https://www.forbes.com/sites/forbesagencycouncil/2021/06/02/micro-influencers-when-smaller-is-better/`.

Emplifi (2023). Micro-influencers vs. macro-influencers: Who's best for your marketing efforts? `https://emplifi.io/resources/blog/micro-influencers-vs-macro-influencers`.

Fainmesser, I. P. and A. Galeotti (2021). The market for online influence. *American Economic Journal: Microeconomics 13*(4), 332–72.

Geyser, W. (2023). 20 of Instagram's highest paid stars in 2024. *Influencer Marketing Hub*. `https://influencermarketinghub.com/instagram-highest-paid/`.

Grin (2022). Influencer discount codes and affiliate links: How and when to use them in your campaigns. `https://grin.co/blog/influencer-discount-codes-affiliate-links-how-and-when-to-use-them-in-your-campaigns/` .

He, D., S. Melumad, and M. T. Pham (2018). The pleasure of assessing and expressing our likes and dislikes. *Journal of Consumer Research 46*(3), 545–563.

Influencer Marketing Hub (2023). The state of influencer marketing 2023: Benchmark report. https://influencermarketinghub.com/influencer-marketing-benchmark-report/.

Iyer, G. and Z. Katona (2016). Competing for attention in social communication markets. *Management Science 62*(8), 2304–2320.

Johnson, J. P. and D. P. Myatt (2006). On the simple economics of advertising, marketing, and product design. *American Economic Review 96*(3), 756–784.

Katona, Z. (2020). Competing for influencers in a social network. Working paper.

Kats, R. (2020). How are consumers spending some of their time? Reading reviews. Lots of Reviews. *Insider Intelligence*. https://www.insiderintelligence.com/content/how-are-consumers-spending-some-of-their-time-reading-reviews-lots-of-reviews.

Kohn, M. G. and S. Shavell (1974). The theory of search. *Journal of Economic Theory 9*(2), 93–123.

Kreps, D. M. and R. Wilson (1982). Reputation and imperfect information. *Journal of Economic Theory 27*(2), 253–279.

Kuksov, D. and C. Liao (2019). Opinion leaders and product variety. *Marketing Science 38*(5), 812–834.

Lerner, J. and J. Tirole (2006). A model of forum shopping. *American Economic Review 96*(4), 1091–1113.

Markerly (2022). Do influencers make money from discount codes? https://markerly.com/blog/do-influencers-make-money-from-discount-codes/.

Mayzlin, D. (2006). Promotional chat on the Internet. *Marketing Science 25*(2), 155–163.

McCarthy, N. (2019). The highest earners on Instagram. *Forbes*. https://www.forbes.com/sites/niallmccarthy/2019/10/28/the-highest-earners-on-instagram-infographic/.

Miklós-Thal, J. and J. Zhang (2013). (De)marketing to manage consumer quality inferences. *Journal of Marketing Research 50*(1), 55–69.

Milgrom, P. and J. Roberts (1986). Price and advertising signals of product quality. *Journal of Political Economy 94*(4), 796–821.

Morrison, A. D. and L. White (2005). Crises and capital requirements in banking. *American Economic Review 95*(5), 1548–1572.

Nephew, R. (2020). Why do influencers promote products that they don't use? *Medium*. https://medium.com/plus-marketing/why-do-influencers-promote-products-that-they-dont-use-887ba80c09cc.

Nielsen (2016). Sales effect study: Influencer marketing. https://www.tapinfluence.com/tp_resource/nielsen-case-study/.

Nielsen (2020). Case study: Measuring the effectiveness of influencer marketing beyond likes and shares. https://www.nielsen.com/uk/en/insights/case-study/.

Nistor, C., M. Selove, and J. M. Villas-Boas (2024). Influencers: To grow or to monetize. *Management Science*, forthcoming.

Nistor, C., T. Yalcin, and E. Pehlivan (2018). Duplicity in alternative marketing communications. *Markets, Globalization and Development Review 3*(2).

Pearl, D. (2018, July). How Nordstrom's anniversary sale became the 'Super Bowl' for fashion influencers. *AdWeek*. https://www.adweek.com/brand-marketing/how-nordstroms-anniversary-sale-became-the-super-bowl-for-fashion-influencers/.

Pei, A. and D. Mayzlin (2021). Influencing social media influencers through affiliation. *Marketing Science*, forthcoming.

Ravindranath, M. (2021). Social media "micro-influencers" join effort to get America vaccinated. *Politico*. https://www.politico.com/news/2021/01/30/vaccine-rollout-influencers-463917.

Ryan, E. (2023). 11 online review statistics you need to know. *Website Builder Expert*. https://www.websitebuilderexpert.com/building-online-stores/online-review-statistics/.

Schweidel, D. A. and W. W. Moe (2014). Listening in on social media: A joint model of sentiment and venue format choice. *Journal of Marketing Research 51*(4), 387–402.

Selove, M. (2019). Dynamic pricing with fairness concerns and a capacity constraint. *Quantitative Marketing and Economics 17*(4), 385–413.

Shopify (2022). Influencer pricing: How much should you pay. `https://www.shopify.com/blog/influencer-pricing`.

Sortino, K. (2022). How to reach influencers as a brand or small business to promote your product. `https://www.prezly.com/academy/how-to-reach-influencers-as-a-brand-or-small-business-to-promote-your-product`.

Tiffany, K. (2021). America's health will soon be in the hands of very minor Internet celebrities. *The Atlantic*. `https://www.theatlantic.com/technology/archive/2021/02/vaccine-influencers-covid-anti-vax-instagram/618052/`.

Toubia, O. and A. T. Stephen (2013). Intrinsic vs. image-related utility in social media: Why do people contribute content to twitter? *Marketing Science 32*(3), 368–392.

Vogl, M. (2022). Is bigger always better? When to opt for micro-influencers. *Brandwatch*. `https://www.brandwatch.com/blog/when-to-choose-micro-influencer/`.

Wernerfelt, B. (1994). An efficiency criterion for marketing design. *Journal of Marketing Research 31*(4), 462–470.

Wernerfelt, B. (1996). Efficient marketing communication: Helping the customer learn. *Journal of Marketing Research 33*(2), 239–246.

Wu, Y. and T. Geylani (2020). Regulating deceptive advertising: False claims and skeptical consumers. *Marketing Science 39*(4), 788–806.

Yalcin, T., C. Nistor, and E. Pehlivan (2020). Sustainability influencers: Between marketers and educators. *Business Forum 28*(1).

Yang, J., J. Zhang, and Y. Zhang (2021). First law of motion: Influencer video advertising on TikTok. Working paper.

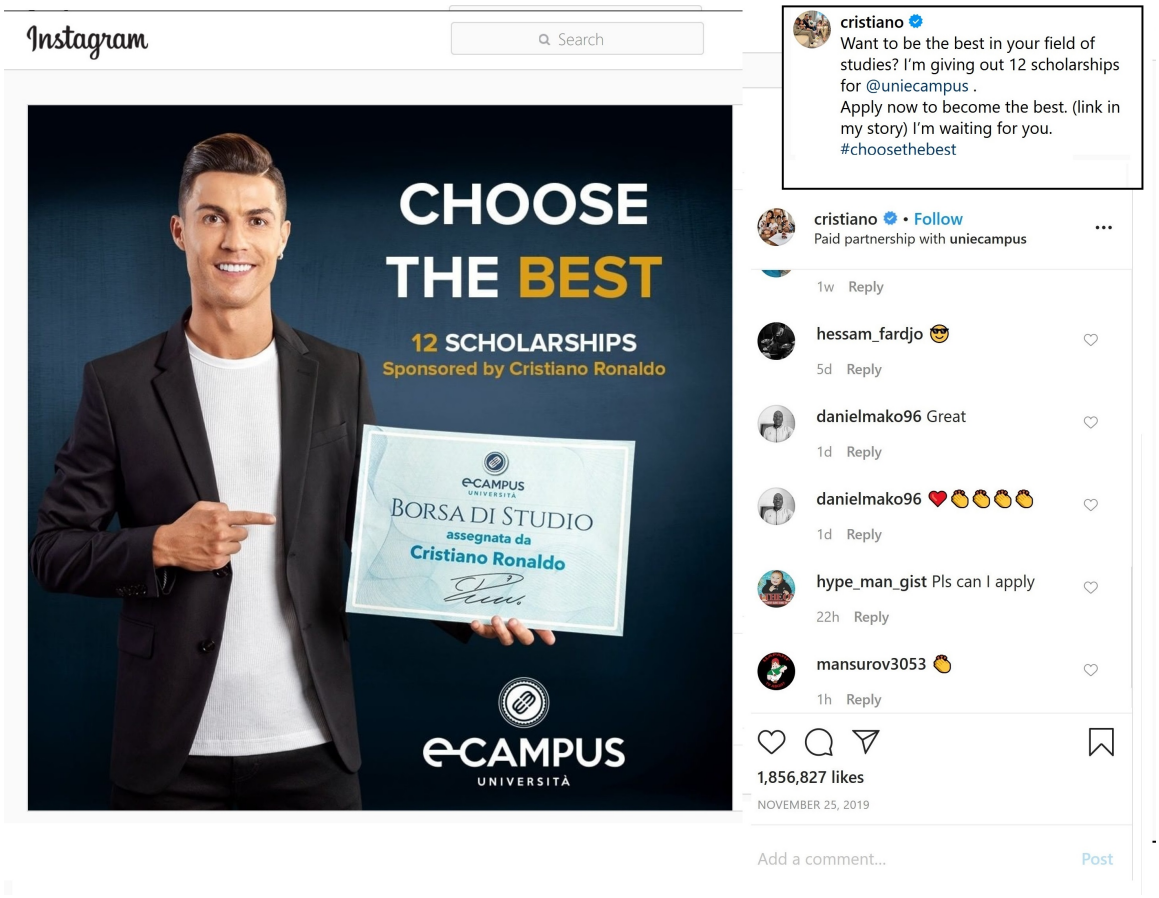
Supplemental Appendices

Influencers: The Power of Comments

Appendix A presents real world examples of endorsements by a celebrity influencer and a micro-influencer. Appendix B contains two model extensions. Appendix C presents formal proofs of all results.

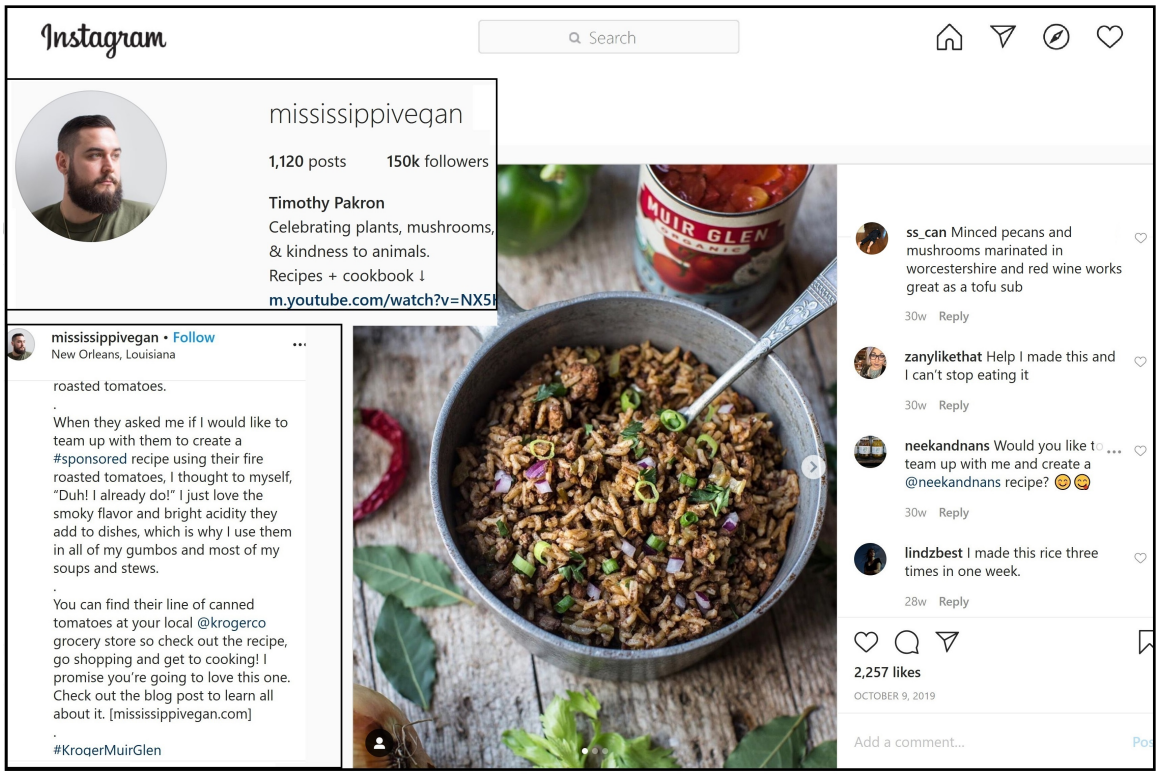
Appendix A: Influencer Examples

Cristiano Ronaldo, Sponsored Post on Instagram



Retrieved May 15, 2020, https://www.instagram.com/p/B5SFl7YK0Px/

Vegan Chef, Sponsored Post on Instagram



Retrieved May 15, 2020, https://www.instagram.com/p/B3ar1oKngcJ/

Appendix B: Model extensions

This appendix contains two model extensions. These extensions allow for influencer reputation and the influencer to delete uninformative comments.

Model extension: Influencer reputation

This model extension relaxes the assumption that the influencer's endorsement policy is binding, and we derive conditions, for an infinitely repeated version of the game, in which reputation effects compel the influencer not to deviate from his chosen endorsement policy.

The influencer first announces a non-binding endorsement policy that specifies whether he endorses all products or only high quality products. There is then a new product the influencer can endorse in each period $t \in \{1, 2, \ldots\}$. Similar to previous reputation models (e.g., Kreps and Wilson 1982; Bar-Isaac 2003; Selove 2019), all players learn at the end of period t about any deviation from the influencer's endorsement policy, for example, through news stories or online word-of-mouth. We show that, under some conditions, this threat of loss of reputation provides an incentive for the influencer to maintain a policy of endorsing only high quality products.

The game timing in each period t is the following:

1. Nature chooses the quality level for the period t product, which the influencer observes.

2. The influencer decides whether to endorse the product. If he does not endorse the product, the subgame for period t ends with no payoffs for the period. Otherwise, the firm and the influencer agree to a fixed payment the firm makes

to the influencer in period t such that they proportionally split the expected profits.

3. The firm sets the period t product price for the first group of customers.

4. The first group of customers read the influencer's post, learn the product's current price, have the option to buy the product, and leave comments.

5. The firm sets the period t product price for the second group of customers.

6. The second group of customers read the influencer's post and learn the product's current price. They can then read comments and have the option to buy the product.

In a one-shot version of this game, with a nonbinding endorsement policy, the influencer would endorse all products. The influencer could not credibly promise to endorse only high quality products, because he would then have an incentive to endorse the product and earn positive profits even if it has low quality. Therefore, the only possible equilibrium of a one-shot version of this game involves the influencer endorsing all products and customer beliefs that the endorsement does not convey any quality information. We focus on an equilibrium of our repeated game in which, if the influencer ever deviates from his announced endorsement policy, players revert to this one-shot equilibrium in all subsequent periods. This punishment strategy provides the strongest possible incentive for the influencer not to deviate because it implies, if the influencer ever violates an announced policy of endorsing only high quality products, then followers will always believe he endorses all products and never again believe his claim that the products he endorse all have high quality.

We focus our analysis on a micro-influencer, although we could also perform similar derivations of reputation effects for a celebrity influencer. Consider a proposed

equilibrium in which the influencer endorses only high quality products, and suppose the fraction of informative comments ($\overline{\lambda}$) is high enough that the conditions of Lemma 3 hold. If the influencer deviates and endorses a low quality product in period t, Lemma 3 implies he will generate the following profits for the period:

$$\left[\beta\left(\overline{q} - C\right) + (1-\beta)\underline{q}\left(1 - C - \frac{R}{\overline{q}\overline{\lambda}}\right)\right] M_V \tag{7}$$

Profits for this equilibrium deviation involve customers beliefs that quality is high ($\overline{q}$), but in the second period only a low fraction of customers ($\underline{q}$) find the product is a good fit because quality is actually low.

This deviation from the proposed equilibrium causes all future periods to revert to the equilibrium in which the influencer endorses all products instead of only high quality products. The resulting change in expected profits in each future period is given by expression (6). If Proposition 2 implies the influencer should endorse only high quality products, this expression is negative, that is, profits are lower when the influencer endorses all products. Letting δ denote the influencer's discount factor, this threat of lost future profits is sufficient to prevent the influencer from endorsing a low quality if the following condition holds:

Condition 1.

$$\begin{aligned}&\left[\beta\left(\overline{q} - C\right) + (1-\beta)\underline{q}\left(1 - C - \frac{R}{\overline{q}\overline{\lambda}}\right)\right] \leq \\ &\frac{\delta(1-\alpha)}{1-\delta}\left[-\underline{q} + \beta C + (1-\beta)\underline{q}C + \frac{R(1-\beta)}{\overline{\lambda}}\right]\end{aligned} \tag{8}$$

Intuitively, when this condition holds, the short-term profits from endorsing a low quality product in any period t are less than the discounted value of the lost future profits that result from imposing additional reading costs on followers by moving to an equilibrium in which the influencer endorses all products. The following proposition

states this result formally.

Proposition 5. *If Condition 1 holds, a micro-influencer can sustain a policy of endorsing only high quality products, even if its endorsement policy is not binding.*

Model extension: Deleting uninformative comments

We now allow the influencer to delete some of the uninformative comments on his post.

Previous research has developed models in which a seller can write biased reviews of their product which, in equilibrium, results in less informative comments (Mayzlin 2006), or can make informative comments easier to find by allowing customer reviews on the seller's website (Chen and Xie 2008). Similarly, we allow the influencer to choose a policy that affects the informativeness of comments. At the same time he sets his endorsement policy, the influencer can decide either to keep all of the comments on his post or to delete some uninformative comments. If a micro-influencer has a policy of deleting uninformative comments, the fraction of informative comments on his post increases to $\overline{\lambda}^*$, where $\overline{\lambda}^* \in (\overline{\lambda}, 1]$. If a celebrity influencer deletes uninformative comments, the fraction of informative comments increases to $\underline{\lambda}^*$, where $\underline{\lambda}^* \in (\underline{\lambda}, 1]$. Note we allow for the possibility that the influencer is not able to delete all uninformative comments, so the fraction of informative comments could remain less than one.

Equations (4) and (5) show that a micro-influencer's equilibrium profits are strictly increasing in $\overline{\lambda}$. Therefore, by reducing reading costs for second period customers, a policy of deleting uninformative comments increases profits for a micro-influencer.

For a celebrity influencer, we let $\underline{\lambda}^*$ be high enough that the conditions of Lemma 3 hold for this fraction of informative comments. Therefore, if a celebrity influencer has a policy of deleting uninformative comments, customers in the second period read

comments on his post, and profits can then be derived using the same equations as for a micro-influencer in the original version of the model.

By comparing equation (2) with equation (4), and comparing (3) with (5), we see that deleting uninformative comments affects profits for the celebrity influencer in two ways. First, by allowing second period customers to learn whether the product is a good fit, this policy allows the seller to avoid the cost of producing the product for customers for whom it is a bad fit. Second, it causes customers in the second period to incur costs of reading comments about the product. The net effect can go in either direction, that is, a policy of deleting uninformative comments could increase profits if production costs are relatively large, but could also reduce profits if the reading costs are relatively large.

These results are formalized in the following proposition.

Proposition 6. *For a micro-influencer, deleting uninformative comments increases profits. For a celebrity influencer, deleting uninformative comments increases the profits from a given endorsement policy if and only if* $(1 - \widetilde{q})C > \frac{R}{\underline{\lambda}^*}$*, where* $\widetilde{q}$ *is expected quality given the influencer's endorsement policy.*

Thus, a celebrity influencer may or may not want to delete uninformative comments. Intuitively, if production costs are low relative to the cost of reading comments, it is more profitable to have all customers purchase the product without reading comments, so a celebrity influencer prefers to keep uninformative comments on his post.

Appendix C: Proofs

Proof of Lemma 1

We consider all feasible search strategies for followers and derive conditions in which each strategy is optimal. One possible strategy is to continue reading comments until finding an informative comment and then purchase the product if it is a good fit. Given that a fraction λ of comments are informative, in expectation one has to read $\frac{1}{\lambda}$ comments to find an informative comment. Furthermore, given that there is probability $\widetilde{q}$ the product has good fit for a given follower, the expected utility of this strategy is $\widetilde{q}(1-P_2)-\frac{R}{\lambda}$. Another possible strategy is to purchase immediately without reading any comments, which generates expected utility $\widetilde{q}-P_2$. A third possible strategy is not reading any comments and not purchasing, which generates utility 0.

Among these three strategies, reading comments before deciding whether to purchase generates the highest expected utility if $(1-\widetilde{q})P_2 > \frac{R}{\lambda}$ and $\widetilde{q}(1-P_2)-\frac{R}{\lambda} \geq 0$, purchasing immediately generates the highest expected utility if $(1-\widetilde{q})P_2 \leq \frac{R}{\lambda}$ and $\widetilde{q}-P_2 \geq 0$, and not reading comments or purchasing generates the highest expected utility otherwise.

Another feasible strategy would be to read a finite number of comments and stop searching if an informative comment has not yet been found. However, reading an uninformative comment has no effect on the probability λ that the next comment will be informative or on the probability $\widetilde{q}$ that the product has good fit. Furthermore, reading an uninformative comment has no effect on the expected utility of stopping search and making a purchase decision immediately, which generates utility $\max\{0, \widetilde{q}-P_2\}$. Therefore, the optimal threshold level of utility needed to stop search does not change over time (Kohn and Shavell 1974). After reading

an uninformative comment, the follower still faces the same dynamic optimization problem they faced before reading the comment. Thus, if it is optimal to read the first comment, it must also be optimal to continue reading comments after finding an uninformative comment, and reading a finite number of uninformative comments and then stopping search cannot be strictly better than continuing to search until an informative comment is found. QED

Proof of Lemma 2

We first compute the equilibrium in the second period. Based on Lemma 1, followers read comments only if $\frac{R}{\lambda} < (1-\widetilde{q})P_2$ and $\frac{R}{\lambda} < \widetilde{q}(1-P_2)$. The second inequality implies $P_2 < 1$, so together these inequalities imply $\frac{R}{\lambda} < 1-\widetilde{q}$. Therefore, if the fraction of informative comments λ is small enough that $\frac{R}{\lambda} > 1-\underline{q}$, there cannot be an equilibrium in which followers read comments. Under this condition, followers may purchase the product immediately without reading comments if the price is low enough given their quality beliefs. Furthermore, a high price cannot serve as a high quality signal because a low quality firm would have an incentive to mimic this high price. Thus, in equilibrium, followers base their quality beliefs only on the influencer's endorsement policy, and the firm sets price $\overline{q}$ if the influencer endorses only high quality products and price $\alpha\overline{q} + (1-\alpha)\underline{q}$ if the influencer endorses all products. Similar logic implies the firm also follows this same price strategy in the first period. QED

Proof of Lemma 3

We first consider the case in which the influencer endorses only high quality products, which implies $\widetilde{q} = \overline{q}$. In the second period, based on Lemma 1, the highest possible price for which followers would purchase immediately is $\frac{R}{(1-\overline{q})\lambda}$. Setting this price

and selling to all followers generates second period profits $(1-\beta)(\frac{R}{(1-\overline{q})\lambda}-C)M_V$. Alternatively, for small values of $\frac{R}{\lambda}$, the firm can set a higher price, $1-\frac{R}{\overline{q}\lambda}$, in which case followers read comments before deciding whether to purchase. Setting this price and selling to a fraction $\overline{q}$ of followers generates second period profits $(1-\beta)\overline{q}(1-\frac{R}{\overline{q}\lambda}-C)M_V$. If $\frac{R}{\lambda}$ is sufficiently small, profits from the latter strategy are higher, and the firm prefers to set a higher price so followers read comments.

In the first period, the firm can set price $\overline{q}$, which generates first period profits $\beta(\overline{q}-C)M_V$ and results in followers writing informative comments. Alternatively, the firm can set a higher price, which results in zero first period profits and no informative comments, in which case second period profits will be $(1-\beta)(\overline{q}-C)M_V$. If $\frac{R}{\lambda}$ is sufficiently small, the strategy of selling to first period followers so there are informative comments that are read by followers in the second period is more profitable.

We now consider the case in which the influencer endorses all products. We will show that, under some conditions, the intuitive criterion implies a strategy of setting a low price so customers do not read comments is perceived as a low quality signal. For this strategy, the maximum second period price is $\frac{R}{(1-\underline{q})\lambda}$, which results in second period profits $(1-\beta)(\frac{R}{(1-\underline{q})\lambda}-C)M_V$. Alternatively, the firm can set a higher price $1-\frac{R}{[\alpha\overline{q}+(1-\alpha)\underline{q}]\lambda}$, which results in followers reading comments, so a fraction $\overline{q}$ buy the product if quality is a high and a fraction $\underline{q}$ buy if quality is low. If $\frac{R}{\lambda}$ is low enough that $\underline{q}\Big[1-\frac{R}{[\alpha\overline{q}+(1-\alpha)\underline{q}]\lambda}-C\Big]>\frac{R}{(1-\underline{q})\lambda}-C$, a low quality firm prefers to pool with the high quality firm and set a high price so that followers read comments.

Furthermore, we now derive conditions for which a potential pooling equilibrium in which both firm types set price $\frac{R}{(1-[\alpha\overline{q}+(1-\alpha)\underline{q}])\lambda}$ and customers purchase immediately cannot satisfy the intuitive criterion. If $\frac{R}{\lambda}$ is sufficiently small, a high quality firm would prefer to deviate to price $1-\frac{R}{\overline{q}\lambda}$ if customers believe this higher price signals

high quality. Furthermore, because a higher fraction of customers purchase after search if the firm has high quality than if it has low quality, there must exist a price such that the high quality firm prefers to deviate and the low quality firm does not if such a price deviation signals high quality. Thus, if a high quality firm prefers to deviate from the pooling equilibrium to signal high quality, a pooling equilibrium on the low price cannot satisfy the intuitive criterion.

In addition, if $\frac{R}{\lambda}$ is sufficiently small, it is more profitable to set first period price $\alpha\overline{q} + (1-\alpha)\underline{q}$ so that followers buy the product and write informative comments, rather than setting a higher price so they do not buy or write informative comments. QED

Proof of Proposition 1

The profit functions for a celebrity influencer given by (2) and (3) follow from Lemma 2. If $\underline{q} > C$, the profits in (2) are greater than the profits in (3), which implies profits are maximized by endorsing all products. If $\underline{q} < C$, the profits in (3) are greater than in (2), which implies profits are maximized by endorsing only a high quality product. Because the seller and influencer split the profits proportionally, the influencer chooses the endorsement policy that maximizes expected profits, which is a policy of endorsing all products. QED

Proof of Proposition 2

The profit functions given by (4) and (5) follow from Lemma 3. If $(1-\beta)\left(\frac{R}{\lambda}\right) > \underline{q} - \beta C - (1-\beta)\underline{q}C$, the profits in (5) are greater than the profits in (4), which implies for a micro-influencer profits are maximized by endorsing only high quality products. If this inequality is reversed, the profits in (4) are greater, and the influencer maximizes profits by endorsing all products. Because the seller and influencer split the

profits proportionally, the influencer chooses whichever endorsement policy maximizes expected profits. QED

Proof of Proposition 3 For a celebrity influencer, the proof of Lemma 2 shows that, even if first period followers who buy the product write informative comments, followers in the second period do not read the comments. Therefore, $N = 0$, which implies all followers prefer to write uninformative comments, including those who purchase the product. The rest of the analysis for a celebrity influencer is the same as in Lemma 2 and Proposition 1.

For a micro-influencer, the proof of Lemma 3 shows that followers in the second period read comments if $\frac{R}{\lambda}$ is sufficiently small. As shown in the body of the paper, each commenter in the first period then expects $\frac{1-\beta}{\beta}$ second period followers to read her comment. Given this value of N, followers who purchase the product prefer to write an informative comment if $(S_i - S_u)\frac{1-\beta}{\beta} > k_i - k_u$. Furthermore, first period followers derive utility $e - k_u + S_u\frac{1-\beta}{\beta}$ if they do not buy and the product and they write an uninformative comment, and those who are interested in the product are willing to pay $\widetilde{q} + (S_i - S_u)\frac{1-\beta}{\beta} - (k_i - k_u)$ to reflect the utility from consuming the product, the incremental status utility from writing an informative comment, and the incremental cost of writing an informative comment. The equilibrium utility for first period followers, regardless of whether they buy the product, is then $e - k_u + S_u\frac{1-\beta}{\beta}$. Second period followers pay the same price as in the main model and derive equilibrium utility $e - k_u$. Therefore, second period followers wait to read in the second period if their cost of reading the post earlier exceeds the incremental utility of first period followers. The equilibrium endorsement policy then follows from similar analysis to the proof of Proposition 2.

Proof of Proposition 4 A policy of endorsing only high quality products for which the influencer has expertise leads to probability $\gamma\alpha$ of an endorsement, and conditional

on an endorsement, expected quality is $\overline{q}$. A policy of endorsing high quality products for which the influencer has expertise and also all products for which he lacks expertise leads to probability $\gamma\alpha+(1-\gamma)$ of an endorsement, and conditional on an endorsement, expected quality is $\frac{\gamma\alpha\overline{q}+(1-\gamma)\widehat{q}}{\gamma\alpha+(1-\gamma)}$. For a celebrity influencer, similar derivations to those in the original model show that the latter policy generates higher profits if $\widehat{q} > C$. For a micro-influencer, similar derivations to those in the original model show that the latter policy generates higher profits if $\widehat{q}-\beta C-(1-\beta)\widehat{q}\,C > (1-\beta)\left(\frac{R}{\overline{\lambda}}\right)$. Finally, a policy of endorsing all products leads to probability one of an endorsement, and conditional on an endorsement, expected quality is $\widehat{q}$. Similar derivations to those in the original model yield conditions in which this policy generates the greatest profits. QED

Proof of Proposition 5 Lemma 3 implies the profits from deviating and endorsing a low quality product are given by expression (7). Lemma 3 also implies the reduction in each period's future profits from moving to an equilibrium in which the influencer endorses all products is given by expression (6). Under Condition 1, the discounted value of this loss in future profits is greater than the one time benefit from deviating and endorsing a low quality product. Therefore, the influencer can sustain the equilibrium in which he endorses only high quality products, and the threat of moving to the bad equilibrium in which he endorses all products prevents him from deviating. QED

Proof of Proposition 6 For a micro-influencer, equations (4) and (5) show that profits are strictly increasing in $\overline{\lambda}$. For a celebrity influencer, comparing equation (2) with equation (4) and substituting $\overline{\lambda} = \underline{\lambda}^*$ shows that deleting comments increases the profits from endorsing all products if $(1-[\alpha\overline{q}+(1-\alpha)\underline{q}])C > \frac{R}{\underline{\lambda}^*}$, and comparing (3) with (5) and substituting $\overline{\lambda} = \underline{\lambda}^*$ shows that deleting comments increases the profits from endorsing only high quality products if $(1-\overline{q})C > \frac{R}{\underline{\lambda}^*}$. QED