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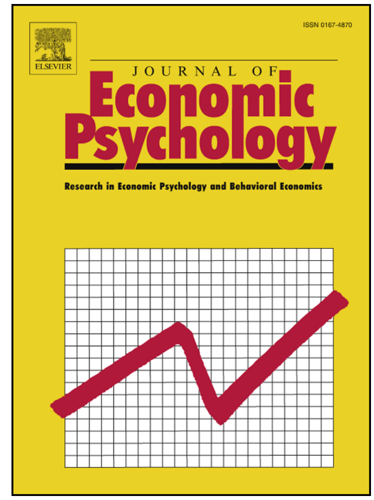

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A Theory of Focal Points in 2x2 Games[1]

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A Theory of Focal Points in 2x2 Games

Abstract

Since the classic work of Schelling, the notion of a focal point has been widely applied to explain coordinated behavior. However, focal points remain largely outside the formal apparatus of game theory. This paper develops a model of play in 2x2 games where payoff differences determine what strategy players will perceive as "salient" and choose to play. The model uniquely predicts which outcome will emerge for virtually the entire class of 2x2 normal form games. For the subset of such games involving coordination and asymmetric payoffs, payoff differences identify focal outcomes and strategies in the same way shared social knowledge produces coordination in Schelling's symmetric games. The model characterizes situations when Nash equilibria are likely to be played, even in a one-shot interaction, and predicts which equilibrium will obtain in games containing more than one. It identifies other circumstances in which a non-equilibrium outcome will predominate. Finally, the theory specifies when a player is likely to select a strategy based solely on consideration of her own payoffs, and when and why the same player will be prompted to act strategically. Experimental results are presented that test the predictions of the model.

"The concepts of 'focal point' and 'salience' have become part of the tool-kit of game theory…Despite that, focal points have never been integrated into the formal structure of the theory. Although theorists often invoke notions of salience when dealing with otherwise intractable problems of equilibrium selection, they do so in a spirit of faute de mieux."

(Gold and Sugden (2006, p. 24)

1. Introduction

The dilemma of how two people can coordinate on the same strategy despite the presence of multiple equilibria is among the oldest and most fundamental in game theory. Firms, nations, and individuals frequently face situations where coordination is among the most desirable outcomes, and sometimes such coordination is achieved. However, traditional game theory offers little guidance on how to *systematically* predict when coordination will occur.

Two different streams of literature have addressed this problem. One stream, originating with Schelling (1960), examines behavior in pure matching games in which players receive a fixed positive payoff whenever their strategy selections match and a payoff of zero otherwise.[2] A simple example of such a game is one where players each choose between strategies labeled Heads and Tails, receive $1 if their choices match (i.e., are HH or TT), and receive $0 otherwise. A large body of experimental evidence demonstrates that people do much better than chance at coordinating in these types of games. Specifically, it appears that the strategy Heads comes immediately to mind more often than Tails, resulting in the focal point outcome Heads-Heads. Schelling (1960) attributed this better-than-chance performance to one of the strategy options

[2] Subsequent experimental work on behavior in pure coordination games has been conducted by Mehta, Starmer and Sugden (1994). There has also been a significant amount of theoretical work attributing "focality" to a notion of group rationality and team reasoning, rather than to shared cognition or culture. See, for example, Sugden (1995), Bacharach & Bernasconi (1997), Gold & Sudgen (2006), and Colman et al. (2014).

appearing more salient in the minds of the players because of commonly shared culture or psychology.[3] The shared salience of one of the strategy labels results in that outcome being perceived as a focal point. Traditional game theory cannot explain the coordination observed in symmetric matching games since the theory gives no special status to a label of Heads versus Tails.

Although salient labels may be powerful coordination devices in games with symmetric payoffs, they lose much of their potency in games with payoff asymmetries. For instance, Crawford et al. (2008) conducted experiments involving games with salient labels but asymmetric payoffs for coordination. They conclude that, "when payoffs are symmetric across players, salient labels yield high coordination rates, but that when payoffs are even minutely asymmetric and the salience of labels conflicts with the salience of payoff differences, salient labels may lose much of their effectiveness and coordination rates may be very low."[4] This observation that payoff salience dominates label salience in asymmetric games provides a foundation for our approach to the study payoff-based focal points in asymmetric games.

A second stream of literature, initiated by theorists John Harsanyi and Reinhardt Selten (1988), examines coordination in asymmetric games. Harsanyi and Selten (1988) proposed two deductive criteria, payoff dominance and risk dominance, that agents might employ to decide which equilibrium to aim for in coordination problems with payoff asymmetries. Beginning with papers by Cooper et al. (1990) and Van Huyck et al. (1990), experimentalists set out to determine which of the criteria better described actual behavior. The results of these and subsequent studies have not been encouraging. Neither selection criteria proves consistently

[3] For a much more recent discussion, see Sugden (2011).

[4] Similarly, Agranov et al (2012) report results from experiments in which an "Announcer" informing players regarding the differences in cooperative payoffs in 'Battle of the Sexes' type games can make focal outcomes "re-emerge" by describing the payoffs in vague or ambiguous terms that mask differences in what players receive.

successful at explaining behavior observed in games involving multiple equilibria. Subsequent experiments (e.g., Haruvy and Stahl (2004) and Leland (2012)) suggest that a correspondence between the predictions of either selection criteria and observed behavior is coincidental.

Although deductive approaches to equilibrium selection such as payoff-dominance and risk-dominance systematically fail to predict behavior in games, they have been integrated into the formal structure of game theory. Somewhat paradoxically, the intuitive approach to equilibrium selection pioneered by Schelling, based on salience and focal points, has been successful in explaining coordinated behavior, but remains outside the formal apparatus of game theory.[5]

In this paper we attempt to partially address this dilemma. As noted earlier, focal points are commonly conceived of as arising from perceptions or knowledge shared by players in a game. In the context of symmetric games, the perception might be that a red colored square attracts more of players' attention relative to three other squares colored white, or that the label "Heads" is perceived as salient relative to "Tails." When we move to the world of asymmetric games, perhaps the most obvious shared characteristic of the game is that it is being played by players who care about their payoffs. As such, at least part of what makes strategies in asymmetric games appear focal to players, and at least part of what produces game outcomes that are focal, must have to do with the structure of the payoffs players face.

In both our definition and in Schelling's definition, a focal point is a strategy profile that is salient for both players and a Nash equilibrium. That is, a focal point is mutually salient and a mutual best-response. In Schelling's case of symmetric games, mutual salience of strategy labels is determined by shared culture or shared knowledge. In our setting of asymmetric games, mutual salience is determined by a shared intuition of how payoff differences are perceived.

[5]In this regard Colman (2006) notes, the "… focal point concept is almost universally acknowledged by game theorists as an important device for equilibrium selection, but it has not been assimilated into formal game theory".

With this in mind, we examine a model of play in 2x2 games in which focality of outcomes and strategies is determined by the payoff structure of the game. We consider games played by players who focus on larger differences in payoffs across strategies, paralleling work by Bordalo et al. (2012) in choices under risk in which agents focus on larger differences in payoffs across lotteries. Our proposed model assumes that in a 2x2 game, players attend to salient payoff differences across strategies. A player's 'salient strategy' is defined as that player's strategy yielding the larger payoff associated with the larger difference in payoffs. A player perceives his payoffs to be more salient than those of the other player if his largest payoff difference across strategies is at least as large as the other player's largest payoff difference. Otherwise, he perceives the other player's strategy to be more salient. If a player's own payoffs are most salient, he myopically focuses on those payoffs and chooses his salient strategy. Otherwise, he focuses on the other player's salient payoffs which prompts him to think about what the other player will do. This motivates him to best respond to his belief about what the other player will do. Under these assumptions, the model predicts systematic Nash equilibrium play, equilibrium selection, and out-of-equilibrium play, and it enables players to be systematically strategic in some situations and myopic in others.

In this paper, we develop and apply the model of salience-based choice noted in the preceding paragraph to formalize the notion of a focal point in games where focality is determined by the payoff structure of the game, and to predict behavior in 2x2 games. The model applies to games with a unique equilibrium and to games with multiple equilibria. We derive formal testable propositions from the model which we investigate in an experiment that we conducted involving choices in 2x2 games. We then apply both the aggregate and subject-level data to distinguish our model from prominent alternative explanations for coordination in 2x2 games.

More specifically, we build on Crawford et al.'s (2008) observation that the structure of payoff differences can account for observed instances of coordination and coordination failures.[6] To motivate this idea, consider the following coordination game in which Player 1's task is to decide whether to play strategy U(p) or strategy D(own) against a Player 2 referred to as "Other" who chooses between strategy L(eft) and strategy R(ight).

If Other chooses L and you choose U	You receive	11.00	and other receives 10.00
D	You receive	4.10	and other receives 3.00
If Other chooses R and you choose U	You receive	4.00	and other receives 3.10
D	You receive	4.10	and other receives 3.10

The payoff matrix for this game is shown on the left in Figure 1 and denoted I. From Player 1's perspective, what grabs attention in this game is the payoff of 11 associated with the larger payoff difference to Player 1 (11.00 - 4.10) versus (4.10 – 4.00). The salience of the payoff of 11 confers salience on strategy U (indicated by italics). That is, U is Player 1's salient strategy. The salient payoff to Player 2 conditional on Player 1 choosing U or D will be the outcome 10.00 - the better outcome associated with the larger payoff difference (10.00-3.10) versus (3.10-3.00). The salience of the payoff of 10 confers salience on strategy L. That is, L is Player 2's salient strategy. The outcome of the game, (11,10) (in bold) is the one where the choices of salience-based strategies U and L coincide. This *salience coincident* outcome is also the payoff dominant Nash equilibrium. This is a situation where the game outcome is a *focal point*, in that it is the salience coincident outcome (it is arrived at through each player choosing her salient strategy),

[6] It is important to emphasize that we are not implying that payoff differences are the only determinants of focality in asymmetric games, but we hope to convince readers that they are an important source of focality that can be integrated into the formal apparatus of game theory.

and it is a best response for both players (neither has an incentive to revise her strategy, holding the other player's strategy fixed).

Figure 1

To consider how this outcome mirrors the outcome in a symmetric payoff setting, imagine a 2x2 game where each player chooses between strategies labeled "Star Spangled Banner" (SSB) and "Oh Canada" (OC), are each paid a dollar if their choices match and are paid nothing otherwise. For two players who are patriotic Americans, it seems plausible that SSB is perceived as the salient strategy in which case the outcome is salience coincident and a Nash equilibrium.

In game II in Figure 1, the payoff that stands out to Player 1 is again 11, recommending the choice of strategy U. However, in this game, for Player 2 the larger payoff (10) associated with the larger payoff difference (10-3.10) confers salience on strategy R. Here the choice of strategies associated with salient payoffs results in the non-equilibrium outcome UR. In this game, the salience coincident outcome is not a focal point since Player 2 will wish to revise her choice.

The analogous outcome in the (SSB,OC) game described above might occur if one player is American and the other is Canadian and neither knows the nationality of the other. Here the outcome resulting from players choosing the strategy they perceive as salient, (SSB, OC), is salience coincident but not focal since both receive a payoff of zero.

In the games described above agents play myopically in the sense that each is struck by the salience of one of his or her own payoffs and chooses the strategy corresponding to that payoff without reflecting on what the other player might choose. But now consider game III depicted on the right in Figure 1. Here, once again, the payoff that is salient to Player 1 is the 11 he receives by choosing U when Player 2 chooses L. For Player 2, the difference in her payoffs conditional on Player 1 choosing U (7-6) is smaller than if Player 1 chooses D (6-3) so the comparison of own-payoff differences favors D. Nevertheless, intuitively, the payoff in the game that will grab Player 2's attention may instead be Player 1's salient payoff of 11 since Player 1's largest payoff difference is greater than Player 2's largest payoff difference. To the extent that Player 1's payoffs are more salient to Player 2 in this game than Player 2's own payoffs, Player 2 will realize the strategic aspect of the interaction, anticipate Player 1's choice of U, and best respond to that choice by choosing L, producing the Nash equilibrium UL. This is a second instance where the outcome produced by the trait players share – sensitivity to large differences in payoff – is a focal point.

An (SSB,OC) game analog to game III might be one where the Canadian Player 2 knows Player 1 is American making SSB salient for both, resulting in the salience coincident outcome SSB which is, once again, focal.

Finally, consider the game in the lower center of Figure 1. Here, payoff differences conditional on the other player choosing one or the other of his or her strategies are equal for each player so sensitivity to larger payoff differences provides no guidance as to what either player should choose. In such cases, we assume strategy choices are random. The symmetric payoff game analog here might be one in which both players are French and neither song is salient or of special significance.

It is interesting to note that the strategy choices that follow from agents choosing as described above will, in many cases, be identical to the choices Level-1 players would make in a cognitive hierarchy model (e.g., as in Camerer et al. 2004). Level-1 players choose the strategy that maximizes their expected utility assuming the opponent is a Level-0 player choosing strategies randomly. In games I, II and IV this results in the same strategy choices and outcomes as salience-based choice.[7] Predictions of the two models diverge only in games where one player perceives one of the opponent's payoffs as salient. In game III, for example, a Level-1 row player will behave like a player choosing based on salience. The Level-1 column player, on the other hand, will choose R as 0.5U(6)+0.5U(6) > 0.5U(7)+ 0.5U(3). This results in the game outcome UR for Level-1 players rather than the outcome UL predicted for salience-based players.

In Section 2, we propose a formal model of this type of salience-based play in asymmetric 2x2 games. For this purpose, we extend the intuition from models of binary risky choice involving cross-lottery comparisons (e.g., Tversky (1969), Gonzalez-Vallejo (2002)) that emphasize the roles of regret, gist, similarity and salience perception associated with attribute differences across alternatives (e.g., Loomes and Sugden (1982), Reyna and Brainerd (1991), Rubinstein (1988), Leland (1994, 1998) and Bordalo et al (2012)) to model and predict behavior in games. Our approach is also consistent with models of binary intertemporal choice based on similarity and focusing (Leland (2002), Rubinstein (2003), Scholten & Read (2010), Koszegi & Szeidl (2013)).

Specifically, we assume that players choose strategies associated with their own salient payoff if that payoff appears more salient to them than their opponent's salient payoff, and best respond to their opponent's salient strategy if the opponent's salient payoff appears more salient. We then demonstrate the following properties of 2x2 games with salience-based players:

1) For games where each player has a salient strategy, there is exactly one salience coincident (SC) outcome (as in games I, II and III.)

[7] As pointed out by one of the reviewers of this paper, it is sometimes assumed that Level-0 players instead play the focal strategy to the game if one exists. However, as noted in the introductory quote, the precise definition of focal strategies in asymmetric payoff games is ill-defined so we focus here on the random response assumption.

2) For games containing a Nash equilibrium, if the Nash equilibrium is the SC outcome, then that will be the outcome of the game (as in games I and III).
3) For games containing multiple pure-strategy Nash equilibria, when one's salient payoff is more salient than the other's salient payoff and these salience perceptions are shared by both players, the outcome will be a Nash equilibrium (there will be no coordination failure as in game III).
4) For games containing multiple pure-strategy equilibria, when each player perceives her own payoff to be at least as salient as her opponent's, the outcome will always be the SC outcome (as in game I and II), and will be an equilibrium if and only if that equilibrium is SC (as in game I).

We refer to the resulting theory as the payoff salience model (PSM). Under the PSM, properties 1 – 4, make strong predictions about which equilibrium will be selected and when out-of-equilibrium play and coordination failures will occur. The framework also provides a natural definition of a focal point for games where focality is determined by endogenously defined salient payoffs.

While the restriction to 2x2 games may seem limiting, we note that our approach is related to models of risky and intertemporal choice that posit comparisons of attributes across alternatives and, as such, focus on binary choices[8]. Interestingly, just as comparative models of risky and intertemporal choice explain choice anomalies, we will show that our comparative, salience-based model of strategy choice explains important puzzles in game theory, such as how and when players achieve coordination, by predicting equilibrium selection and out-of-equilibrium play. The PSM also makes predictions about when players will systematically play

[8] Di Guida et al (2013) presents an analysis of results from an experimental study exploring aspects of focality with a larger number of strategies.

myopically (by focusing on their own payoffs) and when they will play strategically (by focusing on their opponent's payoffs). This contrasts with classical game theory (where players are always strategic) and it contrasts with cognitive hierarchy models (where players have a fixed level of strategic sophistication).

Individual 2x2 games also serve as the building blocks for larger games and they are the games with classic stories that are used in game theory lore (e.g., battle of the sexes, stag hunt, chicken) and so have a natural interpretation about the types of strategic interactions they represent. Moreover, as Shubik (2012) notes, many decisions involve binary choice and much of human interaction is dyadic. The 2x2 game may thus serve as a natural starting point for considering how the salience of strategies and the focality of game outcomes will inform our understanding of equilibrium selection and coordination failure more generally.

2. A Model of Focal Points with Endogenous Salient Payoffs

In this section we summarize the main theoretical results of our model that we will test experimentally in Section 3. A formal presentation of the definitions and proofs pertaining to our model is provided in the Appendix.

The basic idea of the model we propose is that players focus on their own payoffs in a game when their own payoffs are most salient and they focus on their opponent's payoffs when those payoffs are most salient, where the salience of payoffs is endogenously predicted by the model. The differential focus on one's own versus another's payoffs is important because it leads naturally to the idea that (i) when a player's own payoffs are most salient, he chooses myopically (e.g., by selecting his strategy containing is largest salient payoffs), and that (ii) when the other player's payoffs are most salient, the player chooses strategically (by considering what the other player will do and selecting a best response). In other words, the salience of payoffs might moderate the degree of strategic thinking in which a player engages. Thus, the strategic sophistication of a player is also endogenous in our model. In contrast, standard approaches do not allow for endogenous changes in the strategic behavior of players. In classical game theory, players are always strategic, and in models of Level-k thinking, each player has a fixed level of strategic sophistication. Finally, the notion that behavior is myopic when one's own payoffs are salient and is more strategic when other's payoffs are salient is consistent with recent research on salience effects for decisions under risk and over time. For risk, Bordalo et al. (2012) argue that behavior is risk-averse when a lottery's downside is salient and is more risk-seeking when a lottery's upside is salient. In an experiment for choices over time, Fisher and Rangel (2014) find that behavior is relatively impatient when the delay to receiving a reward is salient and is more patient when the magnitude of the reward is salient. Our approach thus naturally relates to this prior work on salience in individual choice and suggests that, at least qualitatively, shifts in behavior due to changes in salience perception provides a unifying behavioral principle across the domains of risk, time, and strategic interactions.

In presenting the intuition behind the model, consider any 2x2 game of the form in Table 1 in which $x_1(s_1, s_2)$ denotes the payoff to Player 1 when he plays strategy s_1 and Player 2 plays s_2. To capture the behavior of Player 1, if influenced by salient payoffs, consider strategy s_0 (defined more precisely in the appendix):

Strategy s_0:

(i) *Choose Player 1's salient strategy if his own payoffs are at least as salient as his opponent's.*

(ii) *Best respond to Player 2's strategy if her payoffs are more salient than Player 1's payoffs.*

In (i), Player 1's salient strategy is defined as the strategy containing Player 1's largest salient payoff, which is defined in more detail in the appendix. Strategy s_0 dictates that when Player 1's own salient strategy is at least as salient to him as Player 2's salient strategy, he chooses his own salient strategy. However, if Player 2's salient strategy is more salient to Player 1 than Player 1's own salient strategy, Player 1 is prompted to think strategically, in which case he best responds to Player 2.

As we state more precisely in the appendix, there may be a parameter, $k \geq 0$, which reflects the decision maker's sensitivity to his own versus other's payoff salience. In our analysis of our experimental data, we implicitly set $k = 1$. Of the fourteen games we analyze, this specification predicts unique strategies for each player for thirteen of the games. The remaining game has equal payoff differences for both strategies for both players and so does not have a unique salient strategy for either player.

A nice property of the parameter k is that, under strategy s_0 a player becomes increasingly myopic as k goes to zero and becomes increasingly strategic as k grows large. Thus, accounting for the parameter k may make the model econometrically testable when applied to predict empirical frequencies with which strategies are played. In this sense, k may be thought of as a continuous analog to levels of strategic sophistication in a cognitive hierarchy model of boundedly rational behavior.

To the extent both players employ Strategy $\boldsymbol{s_0}$ with the corresponding notation and definitions applied analogously for Player 2, there will always be a *salience coincident* outcome defined by the salient strategies selected by the two players. If neither player has any incentive to deviate from the strategy profile that produced this outcome (i.e., to the extent the outcome is a mutual best response), this outcome is a focal point. However, if the salience coincident outcome is not a mutual best response, salience based reasoning leads to a coordination failure.

Definition (Salience Coincident Outcome): *A strategy profile* $\{s_1, s_2\}$ is *salience coincident if* s_1 *is salient for Player 1 and* s_2 *is salient for Player 2.*

Based on the above definition, we formally define a focal point for payoff-salient games (that is for games where each player has a salient payoff):

Definition (Focal Point): *A focal point is a strategy profile that is both salience coincident and a mutual best-response.*

In essence, a focal point is a strategy profile that is salient for both players and a Nash equilibrium. In other words, a focal point is mutually salient and a mutual best-response. This notion for payoff-based focal points is qualitatively similar to Schelling's notion of label-based focal points which also consists of a strategy profile that is salient for both players and a Nash equilibrium. The main difference is that in Schelling's case, what is salient for both players is driven by salient strategy labels based on a shared culture, whereas in our case, it is driven by salient strategies based on a shared perception of payoff differences.

One implication of our definition is that there may be strategy profiles that are salience coincident but *not* focal in that they result in failure to coordinate on an equilibrium. Another implication is that focal points are endogenous to the model since both salience coincident outcomes and Nash equilibria are endogenous. Applying the preceding definitions, we offer the following results:

Proposition 1: *Every 2x2 payoff-salient game has a unique salience coincident outcome.*

Proposition 1 implies that a focal point is unique in any 2x2 payoff salient game, whenever one exists. However, while payoff-salient games always have a salience coincident outcome as in games I and II in Figure 1, this outcome is not necessarily an equilibrium as reflected in Game II, and thus existence of focal points is not guaranteed. We next provide our main result in this section:

Proposition 2 (Nash equilibrium with salience-based players): *For any 2x2 game, if a Nash equilibrium is a focal point, then the outcome for salience-based players will be that equilibrium.*

Proposition 2 applies to all 2x2 games where each player has a unique salient strategy, including coordination games (such as the stag hunt and battle-of-the-sexes) where there are multiple Nash equilibria, as well as non-coordination games (such as dominance solvable games) where there is a unique equilibrium.

Proposition 2 demonstrates that in certain cases, the usual strong rationality assumptions about players' behavior can be weakened to assumptions which require much less strategic sophistication, while still ensuring that a Nash equilibrium will obtain. In particular, since at least one player chooses his salient strategy in the proof of Proposition 2, the usual assumption that both players best-respond and arrive at a Nash equilibrium is a *conclusion* in Proposition 2 rather than an assumption. Moreover, Proposition 2 predicts *which* equilibrium will obtain when there are multiple equilibria and it predicts *when* equilibrium behavior will emerge even in one-shot games.

2.1 Coordination in 2x2 games

Proposition 2 established sufficient conditions under which a Nash equilibrium will be played for the entire class of 2x2 games with salience-based players. We can establish a stronger result if we restrict attention to payoff-salient games with multiple pure-strategy equilibria (which include the entire class of 2x2 coordination games such as the stag-hunt as well as the class of anti-coordination games such as battle of the sexes and the game of chicken).

Proposition 3 (Coordination for salience-based players): *Consider any 2x2 payoff-salient game with multiple pure strategy equilibria. If one player's payoffs are perceived as more salient than the other's and both players follow strategy* $\boldsymbol{s_0}$*, both players will always select a unique equilibrium (i.e., there will be no coordination failure).*

The result in Proposition 3 is somewhat surprising since it implies that for a large and important class of 2x2 games, salience-based players will always coordinate on a unique Nash equilibrium. Proposition 3 tells us that any 2x2 payoff-salient game with multiple equilibria will result in an equilibrium even if players are less sophisticated than perfectly rational agents. As before, since one player plays his salient strategy in each case in the proof of Proposition 3, the usual assumption that both players always best respond and arrive at an equilibrium is a *conclusion* in Proposition 3, rather than an assumption. Salience perceptions guide players to an equilibrium. Proposition 3 also predicts which equilibrium will obtain. Moreover, the result in Proposition 3 does not rely on any of the conditions commonly identified as promoting coordination (e.g., repeat play, (one-way) communication between players, salient strategy labels). Instead, it obtains because the players share a common perceptual apparatus. In contrast to rational agents who share common knowledge of their rationality, our model involves salience-based players who share a common sensitivity to the magnitude of payoff differences.

Our final result considers the case where each player focuses only on her own payoffs:

Proposition 4: *Consider any 2x2 payoff-salient game where each player perceives his own payoffs to be more salient than his opponent's payoffs. Then:*

(i) *The outcome for salience-based players is salience coincident*

(ii) *Salience-based players select an equilibrium if and only if that equilibrium is a focal point.*

3. Testing the model on experimental data

We have presented a model of boundedly rational behavior that applies to both coordination (multiple-equilibria) games and non-coordination (single-equilibrium games) in which salience coincident outcomes and focal points arise endogenously through the payoff structure of the game. We refer to this model as the *payoff-salience model* (PSM). An implication of the PSM is that there will be certain game payoff structures for which strategy choice will be sensitive to the cardinal values of the payoffs players face and others where only the ordinal ranking matters. We refer to the former as payoff-variant games and the latter as payoff-invariant games. To illustrate, consider battle-of-the-sexes game BOS 1 and matching game M 1 in Figure 2 each involving three possible outcomes we can think of as (H(igh), M(edium) and L(ow) to Player 1 and h(igh), m(edium) and l(ow) to Player 2. In BOS 1with salience-based players, for M =2 and m=2 as shown, P1 will choose U and P2 will choose L resulting in the outcome UL. If we now increase M to 9, P1 will choose D resulting in the outcome DL. BOS 1 is an example of a game with a payoff-variant game structure. In contrast, in game M 1, players' strategy choices will be D and R, the outcome of the game will be DR, and this result will not change for variations in the values of M and/or m. It is an example of a payoff-invariant game.

To test these predictions, in (citation withheld) we presented subjects with twenty two games, twenty of which were 2x2s each involving the three possible outcomes H, M, and L to Player 1 and h, m, and l to player 2.[9] Fourteen of these were 2x2 coordination games, each with two pure strategy equilibria (6 battle of the sexes games, 5 matching games, and 3 stag-hunt games), three were cycle games with no Nash equilibria, and three games were solvable by

[9] Two other games, examined how players choose when their opponent has more than two strategies available.

iterative dominance. Some games were predicted to be payoff-variant whereas others were predicted to be payoff-invariant. [10] These predictions were tested by varying the values of the M(edium) and/or m(edium) payoffs.

Seventy-eight students at the University of Trento in Italy participated in one of four sessions with approximately 20 subjects per session. Half of the subjects were assigned the role of Player 1 and the other half the role of Player 2. Participants sat at computer terminals separated by partitions. At the beginning of the experiment, students were asked to read the instructions shown below (written in Italian in the experiment) while the laboratory administrator read them aloud:

> For each of the following screens you are randomly paired with another participant in the experiment referred to as "Other." For each screen, you are presented with a choice between two options U or D. Simultaneously, Other is presented with a choice between two options L and R. You and Other will receive payoffs depending on the decision you make and the decision Other makes. Information about what you and Other will receive depending on the choices you both make will be provided in a table like the one shown below. For this table, if Other chooses his or her option L and you choose your option U, you receive 5 and Other receives 2. If instead you choose your option D, you receive 5 Euros and Other received 4 Euros. If Other chooses his or her option R and you choose your option U, you receive 7 Euros and Other receives 7 Euros. If you instead choose your option D, you receive 1 Euro and Other receives 2 Euros.
>
> If other choose L and you choose U You receive 5.00 and other receives 2.00

[10] In (citation withheld due to double blind review) we derive these predictions from a version of the PSM in which players only best respond when their own payoff differences are equal and, therefore, uninformative. In the present paper, we make the more plausible assumption that a player best responds when the other player's payoff differences are more salient than his own, and we extend the PSM to provide an axiomatic approach to focal points, equilibrium selection, and out-of-equilibrium play. Our formal results (Propositions 1, 2, 3, and 4) all originate in this paper and emerge as a consequence of our novel definitions of a salience coincident outcome and a focal point. These propositions make novel predictions regarding the behavior of salience-based players which we test experimentally in this section.

D You receive 5.00 and other receives 4.00

If other choose R and you choose U You receive 7.00 and other receives 7.00

D You receive 1.00 and other receives 2.00

For each question please indicate the option you would rather have. At the end of the experiment, one of these situations will be selected at random and you and the person with whom you are matched will be paid according to the decisions you both made. As such, in addition to the 5 Euro participation fee you can earn an additional amount between 0 and 13 Euros. Please do not talk with the other participants for the duration of the experiment. If you do not understand something, raise your hand and one of the experimenters will come to your aid.

Subjects then proceeded with the experiment at their own pace. Subjects registered their choices by clicking their mouse on the desired strategy and were then asked to confirm their choice.
The order in which the games were presented to subjects was randomized and payoffs corresponding to the U or D and L or R label in each game alternated at random.

Here we test the predictions of the PSM regarding equilibrium selection and out of equilibrium play using the subset of games from (citation withheld) involving coordination. We considered three types of incentive structures in our experimental games involving multiple equilibria – Pareto-ranked equilibria in which one equilibrium is preferred to but riskier than the other (Stag-Hunt games); conflicting interest games where each player prefers a different equilibrium (Battle-of-the-sexes games); and games with identical equilibria that can be arrived at if players match each other’s strategy choices (matching games).

3.1 Predicting Equilibrium Selection in 2x2 Games

Proposition 2 provides a sufficient condition under which a Nash equilibrium will be played by salience-based players for any 2x2 game. Consider again the Battle of the Sexes game BOS 1 in the upper left in Figure 2. Player 1 will select strategy U to the extent that $10 is his salient payoff. He will also choose U as a best-response, if he instead perceives Player 2's payoff of $9 as salient. Consistent with these possibilities, 90% of subjects in the role of Player 1 chose U. Player 2 will choose L if she perceives $9 as salient or if she perceives Player 1's payoff of $10 as salient. In the experiment, 97% of subjects in the role of Player 2 do so. The resulting outcome of the game ($10, $9) is both salience coincident (indicated in bold font) and a Nash equilibrium (indicated by an *). The same logic applies in the remaining four games in Figure 2. A significant majority of Player 1s (79%, 92%, 85%, and 85%) and Player 2s (67%, 95%, 72%, and 85%) either chose the strategy corresponding to their own salient payoff (if that strategy is perceived to be most salient) or played their best response (if the other player's strategy is perceived to be more salient). As predicted by Proposition 2, the game outcomes that result from these choices are, in every game, salience coincident and a Nash equilibrium.

Figure 2

The overall response patterns subjects exhibit make clear that the occurrence of the salience coincident outcome in these games is the result of players consistently choosing the strategy corresponding to their salient payoff. If Player 1s and 2s choose their own-payoff salient or best response strategies across all five games in Figure 2, the choice patterns observed for games (BOS 1, M 1, M 2, M 3, SH 1) should be (U, D, D, D, U) for Player 1s and (L, R, R, R, L) for Player 2. These are, in fact, the majority (59%) and modal (44%) patterns observed, respectively. Moreover, nearly 80% of all patterns observed are no more than one response away from the predicted ones, and no other pattern occurs more that 13% of the time (and those involve only 1 deviation from the predicted patterns).

Although each game contains two pure-strategy equilibria, the PSM uniquely predicts which equilibrium is selected. Choices in games like BOS 1 and SH 1 could be focal for additional reasons. Di Guida and Devetag (2013), for example, propose that an “obvious or intuitive” solution to such games would be to select “the strategy supporting an outcome that is attractive for both players.” However, this heuristic would not provide guidance in the remaining games. The observation that the number of subjects choosing strategies consistent with either salience or obtaining the best joint outcome is larger for BOS 1 and SH 1 than the other games in Figure 2 (in BOS 1 and SH 1 at least 90% of subjects choose strategies consistent with the focal outcome) may suggest that both heuristics come into play when applicable. Our claim is only that focality based on payoff differences is important, not that it is the only source of focality.

It is worth noting that equilibrium selection might be expected to be particularly challenging in games such as M 1, M 2, and M 3, since both pure strategy equilibria offer the same payoffs to each player. In these games, it is somewhat remarkable that minor variations in payoffs *out of equilibrium* can serve as a coordination device such that one of the two equilibria with identical payoffs will be selected by most subjects. A similar observation that a non-equilibrium outcome may act as a coordination device was proposed in Bosch-Domenech and Vriend (2008).

3.2 Predicting out-of-equilibrium play

Proposition 4 applies to 2x2 salient games with multiple (pure strategy) Nash equilibria. For the case when each player’s own salient payoffs are more salient to him than the payoffs of his opponent, two key observations in Proposition 4 are (i) a Nash equilibrium will be played if it is salience coincident and (ii) a Nash equilibrium will be played only if it is salience coincident.

The sufficient condition (i) is implied by Proposition 2 and data supporting this prediction is presented in Figure 2. Figure 3 provides behavioral evidence regarding the necessary condition (ii) in Proposition 4 – that in a 2x2 salient coordination or anti-coordination game, where each player perceives his own payoffs as more salient than the payoffs of his opponent, the outcome will only be a Nash equilibrium (*) if that equilibrium is salience coincident.

The salience coincident outcome (in bold) is not an equilibrium in any of the four games[11] depicted in Figure 3. In each case, the majority of Player 1s (64%, 90%, 64%, and 95%) and a majority of Player 2s (79%, 90%, 59% and 97%) choose the strategy associated with their own-salient payoff. As a result, in every game the predicted outcome is the outcome which is salience coincident, as predicted by Proposition 4.

Figure 3

Consistent with the prediction that equilibria do not obtain in the games in Figure 3 because players are choosing strategies corresponding to their largest salient payoffs, the modal choice patterns exhibited by Player 1s (U, D, U, U occurring 33%) and Player 2s (R, L, R, R, occurring 44%) are those implied by the PSM and 80% of all the response patterns observed are no more than one response away from the predicted patterns. Note that focality determined by an outcome being attractive to both players would not produce the majority result in any of these games.

3.3 Salience-Based Play or Level-1 Behavior?

[11] These games are asymmetric analogs to symmetric games where players disagree on which strategy label is salient.

The experimental results presented to this point are consistent with the hypothesis that players base their choices on salient payoffs. However, as suggested in the Introduction, they are also consistent with the hypothesis that respondents are Level-1 players in a Level k (Stahl and Wilson 1994; 1995) or Cognitive hierarchy model (Camerer et al. 2004). Given the philosophical differences between these models and the PSM, it is somewhat surprising that their predictions considerably overlap. Level-k models attempt to explain behavior deviating from standard game theoretic predictions by modeling the agent as possessing limited and fixed strategic sophistication. A salience-based player, in contrast, is myopic in some situations and strategic in others. In the absence of there being something particularly distinctive about a counterpart's payoffs, the salience-based player focuses exclusively on what is in it for him.

The endogenous variation in strategic sophistication in the PSM provides a way of distinguishing between the two models. In the PSM, when each player perceives his own payoffs as more salient than those of his opponent, the predictions of the models correspond to those that would follow if players are Level-1 boundedly rational. However, when an opponent's payoffs are more distinct, the salience-based player focuses on his opponent's payoffs, which prompts her to think strategically and conform to Nash equilibrium play. It is in these circumstances that the predictions of Level-1 bounded rationality and the PSM diverge.

Figure 4

To attempt to discriminate between explanations of play in 2x2 games, consider the games in Figure 4. In Matching game M 5 and the Stag Hunt SH 2, Player 1 has no salient payoffs and the expected value of strategies U and D are equal assuming Player 2 is Level-0. In the Battle of the Sexes games BOS 5 and BOS 6, Player 2 has no salient payoffs and expected values for strategies L and R are equal in both games assuming Player 1 is a Level-0 opponent. Thus, for games M 5 and SH 2, Player 1 would play randomly if Player 1 is Level 1, and in games BOS 5 and BOS 6, Player 2 would play randomly if Player 2 is Level 1. In contrast, for games M 5 and SH 2, the PSM predicts Player 1 to best respond to Player 2's salient strategy and play D in both games. For games BOS 5 and BOS 6, the PSM predicts Player 2 to best respond to Player 1's salient strategy and play L in BOS 5 and R in BOS 6. In every case, a majority of players lacking own-salient payoffs chose the strategy that best responds to their opponent's salient strategy as predicted by the PSM.

At the level of individual response patterns, the predicted Player 1 response pattern for games M 5 and SH 2, pattern, DD, occurs 31% of the time. The pattern DU occurs 28% of the time and 31% of subjects exhibit UD. In contrast, UU, the response pattern opposite the predicted one occurs only 10% of the time. These observations suggest that the responses were not entirely random, given the systematic prevalence of DD over UU, consistent with PSM. However, the prevalence of DU and UD response patterns indicates that there could be some degree of random choice in these games, consistent with Level 1.

Player 2 response patterns more clearly favor the PSM over Level-1. The predicted choice pattern for Player 2s in BOS 5 and BOS 6, is LR. This pattern occurs 44% of the time. Patterns LL and RR, departing from the predicted pattern by one choice, occur 33% and 15% of the time, respectively. The pattern RL, opposite that predicted by the PSM, occurs only 8% of the time. In game SH 3, both players lack salient payoffs, and the behavior of both players appears random, as predicted by both the PSM and Level 1.

Analysis of the response patterns players exhibit across the thirteen games for which the PSM predicts systematic strategy choices provides additional support for the PSM over Level 1 thinking. Figure 5 shows the number of Player 1s and Player 2s exhibiting the response pattern predicted by the PSM along with the number of subjects who deviate from that pattern by one or more response. For Player 1s, four subjects (10%) exhibit the pattern of choices predicted by the PSM, another 10 (26%) exhibit patterns deviating from the predicted pattern by 1 response and another 6 (15%) deviate from the predicted pattern by 2 responses. Among Player 2s, only one (3%) exhibits a perfect pattern but an additional 13 (33%) deviate by only 1 response and 10 (26%) by two responses. Among the 51% of Player 1s and 62% of Player 2s exhibiting these perfect or near perfect response patterns, only 9/(20x2) or 23% of Player 1s choices on M 5 and SH 2 and 11/(24x2) 23% of Player 2s choices on BOS 5 and BOS 6 depart from best-response predictions of the PSM. These percentages are quite different from the 50% predicted by L-1 for these players in these games. It is only among choice patterns that differ by more than 2 responses from the patterns predicted by the PSM and for Level 1 players (i.e., that look more random overall) that responses on M 5 and SH 2 for Player 1 and BOS 5 and BOS 6 for Player 2 begin to look random (22/38 or 58% and 14/30 or 47%, respectively.)

Figure 5

3.4 Predictive Performance Relative to Deductive Equilibrium Selection Criteria

As noted in the Introduction, attempts to address the coordination problem in asymmetric games have focused on the potential role of deductive criteria like payoff dominance and risk dominance. It is worth considering how these criteria compare with the PSM in terms of explanatory power. For this purpose, consider Figure 6 in which games are organized by type

(Battle of the Sexes (BOS), then Matching (M), then Stag-hunt (SH) games), with the predictions of the PSM for Player 1 (2) listed for each game, along with the percentage of respondents choosing the option predicted by PSM.

Figure 6

The PSM predicts the majority choice for both players in thirteen of the fourteen games. In the fourteenth game (payoff matrix SH 3in Figure 4), the differences in payoffs for both strategies of both players are equal and PSM predicts random choice. The payoffs to strategies U and to L are \$6 and \$0 while payoffs to D and to R are \$3 and \$3. So exactly 50% of the 78 subjects (16 P1s and 23 P2s) choose the strategy yielding either \$6 or \$0 and 50% (23 P1s and 16 P2s) choose the strategy yielding \$3 or \$3.

In Figure 6, entries below the "% Choosing" lines show the predicted strategy choices for Player 1 and Player 2 assuming they use either a payoff dominance (PD), risk dominance (RD), or the Level 1 criterion. Payoff dominance only makes predictions regarding the outcomes of the Stag-hunt games SH 1 and SH 2, and only correctly predicts the majority choices in game SH 1. Risk dominance fairs better, correctly predicting the majority choices for Players 1 and 2 in BOS 5 and 6, M 1 through M 3 and in SH 1 and SH 2. It makes no predictions in the other games and, to the extent it is an equilibrium selection criterion, it cannot explain why majority choices in the games in Figure 3 all correspond to disequibria outcomes. The Level 1 model makes unique predictions in nine of the fourteen games and correctly predicts the majority choice in these nine games. In contrast, PSM makes unique predictions for thirteen of the fourteen games and accurately predicts both players' majority choices and hence, equilibrium selection and out-of-equilibrium play, for all of the thirteen games.

4. Conclusion

There is little doubt that notions of salience and focal points are important to understanding behavior in games. However, as Gold and Sugden (2006) point out, they have not been integrated into the formal structure of game theory. Here, we have attempted to address this situation. In particular, we provided an axiomatic approach to characterizing focal points and to predicting equilibrium selection and out-of-equilibrium play in 2x2 games. The model applies to both coordination games (with multiple equilibria) and non-coordination games (with a unique equilibrium). We have shown that in the context of asymmetric games, the assumptions that players perceive certain payoffs as salient, and that the salient payoffs are those associated with larger payoff differences, lead naturally to definitions of salient strategies and a salience coincident outcome. If the salience coincident outcome is a Nash equilibrium, then the outcome is focal and players coordinate on that equilibrium. This result obtains even if players do not communicate, payoffs are not symmetric and the outcomes have no special labels. If the salience coincident outcome is not Nash, the game can result in coordination failure. Experimental results for games in Figures 2 and 3 confirm these predictions regarding equilibrium selection and out-of-equilibrium play.

In our analysis, we have shown an equivalence between two forms of bounded rationality – players driven by salient payoffs, and those employing Level-1 reasoning both take the same actions in 2x2 games in which a player's own payoffs are at least as salient as the payoffs of the opponent. In contrast, players driven by salient payoffs and those playing rationally in the sense of classical game theory both take the same actions in 2x2 games in which a player's own payoffs are less salient than the payoffs of the opponent.

The payoff salience model also provides a natural way to address the question of *when* a player thinks strategically. This question is not addressed in traditional game theory (where players are always assumed to be strategic), nor in Level-k models (where players have a fixed level of strategic sophistication). In contrast, the strategy proposed for salience-based players predicts myopic play when one's own payoffs are perceived as salient but predicts strategic thinking (in the sense of anticipating and best-responding to an opponent) if it is the opponent's payoffs that are salient. This sensitivity to theoretically irrelevant variations in payoff differences is echoed in models of salience and similarity-based choice under risk and time. The individual who plays myopically when her own-payoff differences are salient but strategically when it is the opponent's payoff differences that stand out might, by related reasoning, exhibit risk-seeking behavior when the upside of a lottery is salient but risk aversion when its downside is salient or exhibit patient behavior when payoff differences are salient and impatient behavior when the time dimension is salient (Bordalo et al., 2012; Fisher & Rangel, 2014). In other words, the same bounds on rationality that determine whether play in games appears myopic or strategic may also determine whether choices are risk seeking versus risk-averse or patient versus impatient.

Finally, the PSM establishes a logical connection between salience-based play in the symmetric games considered by Schelling and in games with asymmetric payoffs. For Schelling games (where payoffs in each cell are the same for both players and payoff differences are equal) agents do not care which strategy they play, as long as it matches the one played by their counterpart. Such games are not payoff salient so coordination can only be determined by the perceived salience of strategy labels. For games involving payoff asymmetries without salient labels, coordination can only be determined by the perceived salience of the payoffs in the game. Here, the PSM is consistent with the observation in Crawford et al. (2008) that games with even small payoff asymmetries (which make the games payoff salient) are no longer solved based on salient labels. However, in either type of game, if the salience coincident outcome is a Nash equilibrium, then the coordination problem is solved – by a commonly shared intuition about salient labels in Schelling games or by commonly perceived salient payoffs in asymmetric games. Thus, the model of payoff-based focal points presented here for games with asymmetric payoffs is a natural analog to the label-based focal points in games with symmetric payoffs. In addition, the two properties of a focal point in Definition 7 (that it is salience coincident and a mutual best response) apply in both types of games.

Appendix: A Model of Focal Points with Endogenous Salient Payoffs

In our model, we assume players will perceive the larger of their payoffs, associated with the larger difference in their payoffs (holding the opponent's strategy fixed), as their *own salient payoff*. Each player's *salient strategy* is the strategy that might yield that player's own salient payoff. When each player has a salient strategy, we have a *payoff salient game*. Players choose their salient strategy to the extent they perceive it as more salient than their opponent's salient strategy and they best respond to their opponent's salient strategy otherwise. More formally, consider the following definitions regarding Player 1 (the corresponding notation and definitions apply analogously for Player 2) engaged in a general 2x2 game of the form in Table 1 in which $x_1(s_1, s_2)$ denotes the payoff to Player 1 when he plays strategy s_1 and Player 2 plays s_2.

Definition 1 (Own Salient Payoffs): Player 1 perceives $x_1(s_1, s_2)$, as *salient* if it is the largest payoff associated with the largest payoff difference. That is, $x_1(s_1, s_2)$ is salient if (i) and (ii) hold:

(i) *(ordinal salience)* $x_1(s_1, s_2) > x_1(s_1', s_2)$.

(ii) *(cardinal salience)* $| x_1(s_1, s_2) - x_1(s_1', s_2)| > | x_1(s_1, s_2') - x_1(s_1', s_2')|$.

Condition (i) in Definition 1 is consistent with a perceptual system that naturally searches for the largest payoffs in the game the player can receive. Condition (ii) in Definition 1 is consistent with the intuition that larger differences in payoffs are more salient than smaller differences.

Definition 2 (Payoff-Salient Game): *A game is payoff salient if there exists a salient payoff for each player.*

Definition 2 applies to numerical instantiations of all 2x2 strict ordinal games.

Definition 3 (Salient Strategy): *Strategy s_1 is salient for Player 1 if it contains her salient payoff.*

Definition 1 makes predictions about *which* of a player's payoffs will be salient to her in a given 2x2 game. We next define a measure of the *degree* of salience of a player's own payoffs, as the largest absolute difference in her payoffs, holding the other player's strategy fixed:

Definition 4 (Degree of Own-payoff salience): *For Player 1, the degree of own-payoff salience is given by:* $\sigma_1(s_1, s_1') = \max\{| x_1(s_1, s_2) - x_1(s_1', s_2)|, | x_1(s_1, s_2') - x_1(s_1', s_2')|\}$.

Similarly, we measure the salience of Player 2's payoffs, as perceived by Player 1, as the largest absolute difference in Player 2's payoffs, holding Player 1's strategy fixed.

Definition 5 (Degree of Other-payoff salience): *For Player 1, the degree of other-payoff salience is given by:* $\sigma_1(s_2, s_2') = \max\{| x_2(s_1, s_2) - x_2(s_1, s_2')|, | x_2(s_1', s_2) - x_2(s_1', s_2')|\}$.

Own-payoff and other-payoff salience are defined analogously for Player 2. For simplicity, in our analysis, we have implicitly set $\sigma_1(s_i, s_i') = \sigma_2(s_i, s_i') = \sigma(s_i, s_i')$ for $i = 1{,}2$. We thus assume that players have a shared perceptual apparatus for identifying salient payoffs.

To capture the behavior of Player 1, if influenced by salient payoffs, consider strategy s_0:

Strategy $\boldsymbol{s_0}$: *Choose the salient strategy for Player 1 if* $\sigma(s_1, s_1') \geq \sigma(s_2, s_2')$.

Best respond to Player 2's salient strategy if $\sigma(s_1, s_1') < \sigma(s_2, s_2')$.

Strategy s_0 dictates that when Player 1's own salient strategy is at least as salient to him as Player 2's salient strategy, he chooses his own salient strategy. However, if Player 2's salient strategy is more salient to Player 1 than Player 1's own salient strategy, Player 1 is prompted to think strategically, in which case he best responds to Player 2.

In principle, there may be a parameter, $k \geq 0$, which reflects the decision maker's sensitivity to own versus other's payoff salience (e.g., Player 1 chooses his salient strategy if $\sigma(s_1, s_1') \geq k\sigma(s_2, s_2')$ and otherwise best-responds). In our analysis, we implicitly set $k = 1$. Under strategy s_0 a player becomes increasingly myopic as k goes to zero and becomes increasingly strategic as k grows large. Thus, accounting for the parameter k may make the model econometrically testable when applied to predict empirical frequencies with which strategies are played. In this sense, k may be thought of as a continuous analog to levels of strategic sophistication in a cognitive hierarchy model of boundedly rational behavior.

To the extent both players employ Strategy $\boldsymbol{s_0}$ with the corresponding notation and definitions applied analogously for Player 2, there will always be a *salience coincident* outcome defined by the salient strategies selected by the two players. If neither player has any incentive to deviate from the strategy profile that produced this outcome (i.e., to the extent the outcome is a mutual best response), this outcome is a focal point. However, if the salience coincident outcome is not a mutual best response, salience based reasoning leads to a coordination failure.

Definition 6 (Salience Coincident Outcome): *A strategy profile* $\{s_1, s_2\}$ is *salience coincident if* s_1 *is salient for Player 1 and* s_2 *is salient for Player 2.*

Based on Definition 6, we now formally define a focal point for payoff-salient games:

Definition 7 (Focal Point): *A focal point is an outcome that is both salience coincident and a mutual best-response.*

Applying the preceding definitions, we offer the following results:

Proposition 1: *Every 2x2 payoff-salient game has a unique salience coincident outcome.*

Proof: *In a payoff salient game, each player has a unique salient strategy since the absolute difference in payoff differences must be strictly positive for the game to be payoff-salient. Thus, the strategy profile containing the salient strategies for each player is unique.* ■

Proposition 2 (Nash equilibrium with salience-based players): *For any 2x2 game, if a Nash equilibrium is a focal point, then the outcome for salience-based players will be that equilibrium.*

Proof: *Without loss of generality, refer to the generic 2x2 game in Table 1 and suppose that* $\{s_1, s_2\}$ *is both a Nash equilibrium and is salience coincident. Then* $x_1(s_1, s_2) > x_1(s_1', s_2)$ *and* $x_2(s_1, s_2) > x_2(s_1, s_2')$. *First, consider the case where* $\sigma(s_1, s_1') > \sigma(s_2, s_2')$. *Then a salience-based Player 1 plays salient strategy* s_1, *and a salience-based Player 2 best responds to Player 1's salient strategy. Thus Player 2 plays* s_2. *Analogously, if* $\sigma(s_1, s_1') < \sigma(s_2, s_2')$, *a salience-based Player 2 plays salient strategy* s_2, *and Player 1 best responds by playing* s_1. *Finally, if* $\sigma(s_1, s_1') = \sigma(s_2, s_2')$, *Players 1 and 2 play their own salient strategies.* ■

Proposition 3 (Coordination for salience-based players): *Consider any 2x2 payoff-salient game with multiple pure strategy equilibria. If* $\sigma(s_1, s_1') \neq \sigma(s_2, s_2')$, *salience-based players will always select a unique equilibrium (i.e., there will be no coordination failure).*

Proof: *For 2x2 games, pure-strategy Nash equilibria occur along a diagonal of the matrix. Without loss of generality, refer to the generic 2x2 game in Table 1 and suppose that* $\{s_1, s_2\}$ *and* $\{s_1', s_2'\}$ *are both Nash equilibria. Proposition 2 implies that if either* $\{s_1, s_2\}$ *or* $\{s_1', s_2'\}$ *is salience coincident, the outcome of the game played by salience-based players will be an equilibrium. Next, suppose that* $\{s_1, s_2'\}$ *is salience coincident. If* $\sigma(s_1, s_1') > \sigma(s_2, s_2')$, *then Player 1 plays* s_1 *and Player 2 best responds to* s_1, *and thus plays* s_2. *If* $\sigma(s_1, s_1') < \sigma(s_2, s_2')$, *then Player 2 plays* s_2' *and Player 1 best responds by playing* s_1'. *Proceeding analogously, we find that a Nash equilibrium is played if* $\{s_1', s_2\}$ *is salience coincident, provided that* $\sigma(s_1, s_1') \neq \sigma(s_2, s_2')$. ■

Proposition 4: *Consider any 2x2 payoff-salient game. If* $\sigma(s_1, s_1') = \sigma(s_2, s_2')$, *then:*

(iii) *The outcome for salience-based players is salience coincident*

(iv) *Salience-based players select an equilibrium if and only if that equilibrium is a focal point.*

Proof: *We return to the generic 2x2 game in Table I. Suppose* $\{s_1, s_2\}$ *is salience coincident. Since* $\sigma(s_1, s_1') = \sigma(s_2, s_2')$, *Player 1 plays* s_1 *and Player 2 plays* s_2. *Proceeding analogously for each of the other cells in Table 1, we find that the (unique) salience coincident outcome will be played. Part (ii) of Proposition 4 is implied by (i).* ■

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Figure 1

Game Outcomes as a Result of Salience-Based Reasoning

I

	L	R
U	**11.00 10.00**	4.00 3.10
D	4.10 3.00	4.10 3.10

II

	L	*R*
U	11.00 11.00	**4.00 10.00**
D	4.10 3.10	4.10 10.00

III

	L	R
U	**11.00** 7.00	4.00 6.00
D	4.10 3.00	4.10 6.00

IV

	L	R
U	11.00 11.00	3.00 7.00
D	7.00 3.00	7.00 7.00

Table 1. A generic 2x2 game

	s_2	s_2'
s_1	$x_1(s_1, s_2), x_2(s_1, s_2)$	$x_1(s_1, s_2'), x_2(s_1, s_2')$
s_1'	$x_1(s_1', s_2), x_2(s_1', s_2)$	$x_1(s_1', s_2'), x_2(s_1', s_2')$

Figure 2

Testing Sufficient Conditions for Equilibrium Play – Between Subject Results

BOS 1

	L	R	
U	**10.00 9.00** *	1.00 1.00	35 90%
D	2.00 9.00	2.00 10.00 *	4 10%
	38 97%	1 3%	

M 1

	L	R	
U	11.00 11.00 *	3.00 3.50	8 21%
D	3.50 3.00	**11.00 11.00** *	31 79%
	13 33%	26 67%	

SH 1

	L	R	
U	**6.00 6.00** *	0.00 1.00	36 92%
D	1.00 0.00	1.00 1.00 *	3 8%
	37 95%	2 5%	

M 2

	L	R	
U	11.00 11.00 *	3.00 3.50	6 15%
D	10.50 3.00	**11.00 11.00** *	33 85%
	11 28%	28 72%	

M 3

	L	R	
U	11.00 11.00 *	3.00 10.50	6 15%
D	10.50 3.00	**11.00 11.00** *	33 85%
	6 15%	33 85%	

BOS denotes Battle-of-the-Sexes Game; M denotes Matching Game; SH denotes Stag-Hunt Game; Salience Coincident outcomes are in bold font; Nash Equilibria are indicated by (*).

Figure 3

Testing Necessary Conditions for Equilibrium Play – Between Subject Results

BOS 2

	L	R	
U	10.00 2.00 *	**1.00 1.00**	25 64%
D	2.00 2.00	2.00 10.00 *	14 36%
	8 21%	31 79%	

BOS 3

	L	R	
U	10.00 9.00 *	1.00 1.00	4 10%
D	**9.00 9.00**	9.00 10.00 *	35 90%
	35 90%	4 10%	

BOS 4

	L	R	
U	10.00 9.00 *	**1.00 1.00**	25 64%
D	1.00 1.00	9.00 10.00 *	14 36%
	16 41%	23 59%	

M 4

	L	R	
U	11.00 11.00 *	**10.50 10.50**	37 95%
D	3.00 3.00	11.00 11.00 *	2 5%
	1 3%	38 97%	

BOS denotes Battle-of-the-Sexes Game; M denotes Matching Game;

Salience Coincident outcomes are in bold font; Nash Equilibria are indicated by (*).

Figure 4

Level-1 versus Salience-Based Play

Random Choice vs. Best Response – Between Subject Results

M 5

	L	R	
U	11.00 11.00 *	5.00 10.50	16 41%
D	5.00 3.00	11.00 11.00 *	23 59%
	5 13%	34 87%	39

SH 2

	L	R	
U	6.00 6.00 *	0.00 5.90	15 38%
D	3.00 0.00	3.00 5.90 *	24 62%
	10 26%	29 74%	39

SH 3

	L	R	
U	6.00 6.00 *	0.00 3.00	16 41%
D	3.00 0.00	3.00 3.00 *	23 59%
	23 59%	16 41%	39

BOS 5

	L	R	
U	10.00 5.50 *	1.00 1.00	33 85%
D	2.00 5.50	2.00 10.00 *	6 15%
	30 77%	9 23%	39

BOS 6

	L	R	
U	10.00 5.50 *	1.00 1.00	2 5%
D	9.00 5.50	9.00 10.00 *	37 95%
	16 41%	23 59%	39

BOS denotes Battle-of-the-Sexes Game; M denotes Matching Game; SH denotes Stag-Hunt Game; Nash Equilibria are indicated by (*).

Figure 5

Comparison of Individual Response Patterns to PSM Predictions

Deviations From Predicted Choice Pattern	Number of Player 1s Exhibiting Pattern	Player 1 Errors on M 5	Errors on SH 2	Total Errors on M5 and SH2
0	4	0	0	0
1 Off	10	4	0	4
2 Off	6	3	2	5
3 Off	8	2	6	8
4 Off	6	3	4	7
5 Off	3	2	1	3
6 Off	1	1	1	2
9 Off	1	1	1	2

Deviations From Predicted Choice Pattern	Number of Player 2s Exhibiting Pattern	Player 2 Errors on BOS 5	Errors on BOS 6	Total Errors on BOS 5 and BOS 6
Perfect Pattern	1	0	0	0
1 Off	13	0	4	4
2 Off	10	4	3	7
3 Off	4	0	3	3
4 Off	4	2	2	4
5 Off	2	1	1	2
6 Off	4	2	2	4
7 Off	1	0	1	1

Figure 6

Predictive Performance of the PSM Compared to Traditional Equilibrium Selection Criteria

	BOS 1	BOS 2	BOS 3	BOS 4	BOS 5	BOS 6	M 1	M 2	M 3	M 4	M 5	SH 1	SH 2	SH 3
Player 1														
PSM predicts	U	U	D	U	U	D	D	D	D	U	D	U	D	---
% Choosing	90%	64%	90%	64%	85%	95%	79%	85%	85%	95%	59%	92%	62%	41%
PD predicts	---	x	x	x	---	---	---	---	---	x	---	U	*U*	---
RD predicts	U	x	x	x	U	D	D	D	D	x	D	U	D	---
Level 1 predicts	U	U	D	U	U	D	D	D	D	U	---	U	---	---
Player 2														
PSM predicts	L	L	R	L	L	R	R	R	R	L	R	L	R	---
% Choosing	97%	79%	90%	59%	77%	59%	67%	72%	85%	97%	87%	95%	74%	59%
PD predicts	---	x	x	x	---	---	---	---	x	x	---	L	*L*	---
RD predicts	L	x	x	x	L	R	R	R	R	x	R	L	R	---
Level 1 predicts	L	L	R	L	---	---	R	R	R	L	R	L	R	---

“x” denotes a non-unique and incorrect prediction. In particular, it denotes a game with ‘out-of-equilibrium’ play which violates both Payoff Dominance (PD) and Risk Dominance (RD) that predict that players will coordinate on an equilibrium; “---” denotes a non-unique prediction.

Highlights for “A Theory of Focal Points in 2x2 Games”

- A theory of focal points for asymmetric 2x2 games is proposed.
- Salient payoff differences endogenously identify focal outcomes.
- The model predicts equilibrium selection and out-of-equilibrium play
- Experimental results confirming predictions of the model are discussed