

2012

# The Herodotus Paradox

Michael R. Baye  
*Indiana University*

Dan Kovenock  
*Chapman University*, [kovenock@chapman.edu](mailto:kovenock@chapman.edu)

Casper G. de Vries  
*Erasmus University*

Follow this and additional works at: [http://digitalcommons.chapman.edu/economics\\_articles](http://digitalcommons.chapman.edu/economics_articles)

 Part of the [Economic History Commons](#), and the [Economic Theory Commons](#)

---

## Recommended Citation

Baye, Michael R., Dan Kovenock, and Casper G. De Vries. "The Herodotus Paradox." *Games and Economic Behavior* 74.1 (2012): 399-406.

This Article is brought to you for free and open access by the Economics at Chapman University Digital Commons. It has been accepted for inclusion in Economics Faculty Articles and Research by an authorized administrator of Chapman University Digital Commons. For more information, please contact [laughtin@chapman.edu](mailto:laughtin@chapman.edu).

---

# The Herodotus Paradox

## **Comments**

NOTICE: this is the author's version of a work that was accepted for publication in *Games and Economic Behavior*. Changes resulting from the publishing process, such as peer review, editing, corrections, structural formatting, and other quality control mechanisms may not be reflected in this document. Changes may have been made to this work since it was submitted for publication. A definitive version was subsequently published in *Games and Economic Behavior*, volume 74, issue 1, in 2012. DOI: [10.1016/j.geb.2011.07.004](https://doi.org/10.1016/j.geb.2011.07.004)

## **Copyright**

Elsevier

# The Herodotus Paradox

Michael R. Baye  
Dan Kovenock  
Casper G. de Vries

CESIFO WORKING PAPER NO. 3135  
CATEGORY 12: EMPIRICAL AND THEORETICAL METHODS  
JULY 2010

*An electronic version of the paper may be downloaded*

- *from the SSRN website:* [www.SSRN.com](http://www.SSRN.com)
- *from the RePEc website:* [www.RePEc.org](http://www.RePEc.org)
- *from the CESifo website:* [www.CESifo-group.org/wp](http://www.CESifo-group.org/wp)

# The Herodotus Paradox

## Abstract

The Babylonian bridal auction, described by Herodotus, is regarded as one of the earliest uses of an auction in history. Yet, to our knowledge, the literature lacks a formal equilibrium analysis of this auction. We provide such an analysis for the two-player case with complete and incomplete information, and in so doing identify what we call the “Herodotus Paradox.”

JEL-Code: C72, D44.

*Michael R. Baye*  
*Department of Business Economics and*  
*Public Policy*  
*Kelley School of Business*  
*Indiana University*  
*USA - Bloomington IN 47405*  
*mbaye@indiana.edu*

*Dan Kovenock*  
*University of Iowa*  
*Economics Department*  
*W284 John Pappajohn Bus Bldg*  
*USA - Iowa City, IA 52242-1994*  
*dan-kovenock@uiowa.edu*

*Casper G. de Vries*  
*Tinbergen Institute and Erasmus University Rotterdam*  
*Department of Economics and Business*  
*P.O. Box 1738*  
*NL – 3000 DR Rotterdam*  
*The Netherlands*  
*cdevries@few.eur.nl*

July 2010

We are grateful to Chaim Fershtman, Vladimir Karamychev, Michael Rauh, and Eric Schmidbauer for helpful conversations.

# 1 Introduction

Cassady (1967) and Milgrom and Weber (1982) note that Herodotus' account of 500 B.C. Babylonian bridal auctions is one of the earliest recorded uses of an auction in history. According to the translation by Rawlinson (1885)<sup>1</sup>, Herodotus wrote:

“Once a year in each village the maidens of age to marry were collected ... Then a herald called up the damsels one by one, and offered them for sale. He began with the most beautiful... when the herald had gone through the whole number of the beautiful damsels, he should then call up the ugliest...and offer her to the men, asking who would agree to take her with the smallest marriage-portion... The marriage-portions were furnished by the money paid for the beautiful damsels...”

We examine a simultaneous-move (sealed bid) version of Herodotus' Babylonian bridal auction with two suitors and two maidens. Suitor  $i \in \{1, 2\}$  values the more beautiful of the two maidens at  $v_i^B > 0$  and the less attractive maiden at  $v_i^L < v_i^B$ . The higher bidder wins the more beautiful maiden, and pays the auctioneer the amount bid by the lower bidder. The auctioneer then transfers this entire amount to the lower bidder as a “sweetener” along with the less fair of the two maidens. Notice that, as in Herodotus' original account, the “sweetener” is a transfer from the winner to the loser, such that the auctioneer's revenues are zero in the mechanism.

The payoff to suitor  $i$  when he bids  $x_i \in [0, \infty)$  and suitor  $j$  bids  $x_j \in [0, \infty)$  is:

$$u_i(x_i, x_j; v_i^B, v_i^L) = \begin{cases} v_i^B - x_j & \text{if } x_i > x_j \\ \frac{1}{2}(v_i^B - x_j + v_i^L + x_i) & \text{if } x_i = x_j \\ v_i^L + x_i & \text{if } x_i < x_j \end{cases} \quad (1)$$

We assume that, in the case of a tie, the auctioneer allocates the maidens to the bidders based on the flip of a fair coin. Notice that in the case of complete information and symmetric values ( $v_i^B = v_B > v_i^L = v_L$ ), this is a constant-sum game.

In Section 2 we examine this auction with complete information. We show that the standard technique used to construct symmetric mixed-strategy equilibria (which involves identifying an atomless mixed-strategy in which each player's expected payoff is constant on

---

<sup>1</sup> An online version of this translation by Rawlinson is available in Ch. 196 at the following website: [http://www.shsu.edu/~his\\_ncp/Herobab.html](http://www.shsu.edu/~his_ncp/Herobab.html).

its support and verifying that a player cannot improve his payoff by submitting a bid outside of the support) yields an apparent paradox: There exists a continuum of symmetric mixed-strategy equilibria in which each player earns an arbitrarily high payoff, despite the fact that the game is constant-sum and bids are pure transfers between the two players. We show that this paradox stems from the fact that the standard techniques for constructing mixed-strategy equilibria in games of complete information are not sufficient to guarantee that the resulting mixed-strategy is a symmetric Nash equilibrium. In particular, the standard steps identify a symmetric mixed-strategy,  $F^*$ , such that, for any bid contemplated by one player, his expected payoff is constant on its support and cannot be improved by deviating to a bid outside of the support, so that  $x_i$  is a best response to  $F^*$ . Notice that if an outside arbiter assigns each player a pure strategy that is an independent draw from such an  $F^*$ , and each player's particular assignment is private information, then neither suitor has an incentive to unilaterally deviate from his assigned bid, as the expected payoff from submitting *any* such assigned bid cannot be improved by submitting any alternative bid. The paradox stems from the fact that this "assignment equilibrium" is not a Nash equilibrium because expected utility with respect to the joint distribution induced by  $F^*$  does not exist. In other words, even though  $F^*$  is an "assignment equilibrium,"  $F^*$  is not a best response to  $F^*$  because players cannot determine their (*ex ante*) expected utility from employing  $F^*$  as a strategy.

We also show that, while nonexistence of expected utility in the Babylonian bridal auction stems from the unboundedness of payoffs, there are many economic games (including the war of attrition, the sad loser auction, and a variant of the Babylonian bridal auction) with unbounded payoffs that do not lead to existence problems. The primary take-away from our analysis is that it is important to add an additional step to the standard analysis for constructing mixed-strategy Nash equilibria in games with unbounded payoffs: One must verify that players' utilities are integrable with respect to the joint distribution of putative equilibrium mixed strategies. To the best of our knowledge, this point has not been identified in the literature; in fact, we stumbled upon it purely because of the Herodotus paradox.

Section 3 shows that similar issues arise in games of incomplete information, thereby demonstrating that the issues are not purely an artifact of mixed-strategies. In auctions with incomplete information, the standard method used to derive symmetric pure strategies in fact yields *interim* equilibria, and these equilibria may not comprise an *ex ante* equilibrium because of the failure of *ex ante* expected utility to exist. We provide an example in the

context of the Babylonian bridal auction that leads to the Herodotus paradox with incomplete information: Conditional on the maidens being unveiled (so that their values are private information to the suitors), there exists a continuum of (interim) equilibria in which players earn an arbitrarily high payoff—even though all moments of the assumed value distribution are finite. Yet, none of these paradoxical equilibria are *ex ante* equilibria. As before, the failure of these equilibria to constitute *ex ante* equilibria arises because the *ex ante* expected utility arising from the (paradoxical) interim equilibria cannot be computed in the first place.

## 2 The Babylonian Bridal Auction with Complete Information

Suppose first that players are symmetric and have complete information, so that  $v_i^B \equiv v_B$  and  $v_i^L \equiv v_L$  in equation (1). One may readily verify that  $x^* = (v_B - v_L) / 2$  is a symmetric pure-strategy equilibrium, and that this is the unique symmetric pure-strategy equilibrium<sup>2</sup>. In this equilibrium, each suitor earns a payoff of  $EU = \frac{1}{2}(v_B + v_L)$ .

### 2.1 A Paradox

The standard approach for finding a symmetric mixed-strategy equilibrium typically involves three steps: (1) identify a “candidate” continuous distribution function  $F$  such that each player’s expected payoff is constant on its support, given that the rival’s bid is determined by  $F$ , (2) verify that “candidate”  $F$  is indeed a well-defined continuous cumulative distribution function, and (3) show that neither player can unilaterally increase his payoff by submitting a bit outside of the support of  $F$ , given that the rival’s bid is determined by  $F$ .

Following this approach, suppose the rival suitor bids according to a continuous  $F$  on  $[m, u]$  (so that the probability of a “tie” is zero). Then the expected payoff to a player that submits a bid of  $x \in [m, u]$  against his rival’s  $F$  is

$$\begin{aligned} EU(x) &= \int_m^x (v_B - s) dF(s) + \int_x^u (v_L + x) dF(s) \\ &= \int_m^x (v_B - s) f(s) ds + \int_x^u (v_L + x) f(s) ds \end{aligned}$$

---

<sup>2</sup> Uniqueness follows from Proposition 1 in Baye, Kovenock and de Vries (forthcoming).

Using step (1) and letting  $f$  denote the density of  $F$ , constancy of expected payoffs requires that for all  $x \in [m, u]$ :

$$\begin{aligned} 0 &= \frac{d}{dx}EU(x) = (v_B - x)f(x) - (v_L + x)f(x) + \int_x^u f(s) ds \\ &= (v_B - v_L - 2x)f(x) + 1 - F(x) \end{aligned}$$

which implies  $v_B - v_L - 2x < 0$  for  $x \in [m, u)$ . The solution to this differential equation is

$$F(x) = 1 - \left( \frac{c}{v_B - v_L - 2x} \right)^{\frac{1}{2}}$$

where  $c < 0$  is a constant determined in step (2). In particular, moving to step (2), for this to be a well-defined continuous distribution function requires

$$F(m) = 1 - \left( \frac{c}{v_B - v_L - 2m} \right)^{\frac{1}{2}} = 0,$$

which implies  $c = v_B - v_L - 2m < 0$  (and hence,  $m > (v_B - v_L)/2$ ). Thus,

$$F(x) = 1 - \left( \frac{v_B - v_L - 2m}{v_B - v_L - 2x} \right)^{\frac{1}{2}}$$

Next, setting  $F(u) = 1$

$$F(u) = 1 - \left( \frac{v_B - v_L - 2m}{v_B - v_L - 2u} \right)^{\frac{1}{2}} = 1$$

implies  $u = \infty$ .

Hence, the candidate symmetric equilibrium entails each suitor  $i$  submitting a bid,  $x_i$ , based on a cumulative distribution function

$$F^*(x_i) = 1 - \left( \frac{v_B - v_L - 2m}{v_B - v_L - 2x_i} \right)^{\frac{1}{2}} \text{ on } [m, \infty), \quad (2)$$

where  $m \in (\frac{v_B - v_L}{2}, \infty)$  is arbitrary. It is easily verified that  $F^*$  is a well-defined atomless probability distribution with density

$$f^*(x_i) = \frac{dF^*}{dx_i} = \left( \frac{v_B - v_L - 2x_i}{v_B - v_L - 2m} \right)^{\frac{1}{2}} \left( \frac{2m + v_L - v_B}{(v_B - v_L - 2x_i)^2} \right) > 0.$$

This completes step (2).

The expected payoff to a player that submits a bid of  $x \in [m, \infty)$  against his rival's  $F^*$  is

$$\begin{aligned} EU(x) &\equiv \int_m^x (v_B - s) dF^*(s) + \int_x^\infty (v_L + x) dF^*(s) \\ &= v_L + m, \end{aligned} \quad (3)$$



which is constant on  $[m, \infty)$ . To complete step (3), note that for  $x' < m$ ,  $EU(x') = v_L + x' < EU(m)$ , which means a suitor cannot improve his payoff by submitting a bid below  $m$ , given that the rival's bid is based on  $F^*$ .

Thus, applying the usual reasoning—that is, steps (1) through (3)—one would conclude that  $F^*$  is a symmetric mixed-strategy Nash equilibrium in which each player earns a finite expected payoff of  $EU^* = v_L + m < \infty$ . Notice that, since  $m > (v_B - v_L)/2$  is arbitrary, there is a continuum of such equilibria and, for arbitrarily large  $m$ , each player's payoff is arbitrarily large (but finite) in any such equilibrium. This is what we call the Herodotus Paradox with complete information: The bid transferred to the loser exactly equals the amount paid by the winner (the auctioneer earns zero profit with probability one), so it would seem that sum of the two suitors' payoffs is constant ( $v_B + v_L$ ). Yet, application of steps (1) through (3) leads to the conclusion that players can earn an arbitrarily high expected payoff in a symmetric mixed-strategy Nash equilibrium. That is, the mixed strategies derived using standard arguments appear to lead to a “utility pump.”

## 2.2 A Closer Look at the Paradox

The Herodotus paradox suggests that the standard arguments used to derive mixed-strategy equilibria may be incomplete. Notice that the standard arguments imply that if one suitor's bid is a random draw from  $F^*$ , then the other suitor is indifferent between submitting any bid  $x \in [m, \infty)$  and strictly prefers such a bid to bidding  $x' < m$ . Based on this, it is tempting to conclude that—so long as one suitor randomizes based on  $F^*$ —the other suitor can do no better than to also choose a bid at random from  $F^*$ , since it places all mass on  $[m, \infty)$ . Using equations (1) and (3), this reasoning would seem to imply that the equilibrium expected payoff to a player that randomizes against  $F^*$  by (independently) using  $F^*$  himself is (letting  $\mu_{x_i, x_j}^*$  denote the product measure induced by  $F^*(x_i)$  and  $F^*(x_j)$ )

$$\begin{aligned}
u_i(F^*, F^*) &\equiv \int_{[m, \infty) \times [m, \infty)} u_i(x_i, x_j; v_i^B, v_i^L) d\mu_{x_i, x_j}^* \\
&= \int_m^\infty \int_m^\infty u_i(x_i, x_j; v_i^B, v_i^L) dF^*(x_i) dF^*(x_j) \tag{4}
\end{aligned}$$

$$= \int_m^\infty \int_m^\infty u_i(x_i, x_j; v_i^B, v_i^L) dF^*(x_j) dF^*(x_i) \tag{5}$$

$$\begin{aligned}
&= \int_m^\infty EU(x_i) dF^*(x_i) \\
&= v_L + m
\end{aligned}$$

It turns out that this reasoning fails in the Babylonian bridal auction with complete information:  $F^*$  is not a best response to  $F^*$ —not because there is a profitable deviation, but because  $u_i(F^*, F^*)$  does not exist. Consequently, the conditions of Fubini's Theorem (Chung, 1974, p. 59-60) are not satisfied; indeed, in the case at hand the integrals in equations (4) and (5) are *not* equal.<sup>3</sup> In short,  $u_i(F^*, F^*) \neq \int_m^\infty EU(x_i) dF^*(x_i)$ , and it is erroneous to use the fact that  $EU(x_i) = v_L + m$  is constant for  $x_i \in [m, \infty)$  and  $EU(x_i) < v_L + m$  for  $x_i < m$  to conclude that  $F^*$  is a best response to  $F^*$ . The Herodotus paradox—that the sum of the putative equilibrium payoffs can exceed  $v_B + v_L$  by an arbitrarily large amount—stems from the fact that the putative strategies do not comprise a Nash equilibrium in the first place.

To establish these assertions, first recall that the expectation of a random variable  $X$  does not exist if  $E[X^+] = E[X^-] = \infty$ , where  $X^+ \equiv \max(0, X)$  and  $X^- \equiv \max(-X, 0)$  (see Chung, 1974, p. 40). We will demonstrate that  $u_i(F^*, F^*)$  does not exist by taking  $X \equiv u_i(x_i, x_j; v_i^B, v_i^L)$  and showing that  $E[X^+] = E[X^-] = \infty$ .

Note that  $E[X^+]$  and  $E[X^-]$  both exist since they are, by definition, expectations of non-negative real numbers. Hence, Fubini's Theorem (see Chung, 1974, p. 60) implies that the integral of  $X^+$  and  $X^-$  with respect to the product measure induced by  $F^*$  can be written as a double integral that is invariant to the order of integration. Thus, for the case at hand,

$$\begin{aligned}
E[X^+] &\equiv E[\max(u_i, 0)] = \int_m^\infty \int_m^\infty \max((v_B - x_j), 0) I_{x_j < x_i} dF^*(x_i) dF^*(x_j) \\
&\quad + \int_m^\infty \int_m^\infty \max((v_L + x_i), 0) I_{x_i < x_j} dF^*(x_i) dF^*(x_j)
\end{aligned}$$

---

<sup>3</sup> Indeed, one can show that  $\int_m^\infty \int_m^\infty u_i(x_i, x_j; v_i^B, v_i^L) dF^*(x_i) dF^*(x_j) = v_B - m$  while  $\int_m^\infty \int_m^\infty u_i(x_i, x_j; v_i^B, v_i^L) dF^*(x_j) dF^*(x_i) = v_L + m$ .

Note that the first term is finite, since the integrand is bounded above by  $v_B$ . The second term is

$$\begin{aligned}
& \int_m^\infty \int_m^\infty \max((v_L + x_i), 0) I_{x_i < x_j} dF^*(x_i) dF^*(x_j) \\
&= \int_m^\infty \int_m^\infty (v_L + x_i) 1_{(x_i < x_j)} f^*(x_i) f^*(x_j) dx_i dx_j \\
&= v_L \int_m^\infty \int_m^\infty 1_{(x_i < x_j)} f^*(x_i) f^*(x_j) dx_i dx_j + \int_m^\infty \int_m^\infty x_i 1_{(x_i < x_j)} f^*(x_j) f^*(x_i) dx_j dx_i \\
&= v_L \int_m^\infty \int_m^\infty 1_{(x_i < x_j)} f^*(x_i) f^*(x_j) dx_i dx_j + \int_m^\infty x_i f^*(x_i) \int_m^\infty 1_{(x_i < x_j)} f^*(x_j) dx_j dx_i \\
&= v_L \int_m^\infty \int_m^\infty 1_{(x_i < x_j)} f^*(x_i) f^*(x_j) dx_i dx_j + \int_m^\infty x_i f^*(x_i) (1 - F^*(x_i)) dx_i
\end{aligned}$$

Once again the first term is finite. The second term is

$$\begin{aligned}
& \int_m^\infty x_i f^*(x_i) (1 - F^*(x_i)) dx_i \\
&= \int_m^\infty x_i \left( \frac{v_B - v_L - 2x_i}{v_B - v_L - 2m} \right)^{\frac{1}{2}} \left( \frac{2m + v_L - v_B}{(v_B - v_L - 2x_i)^2} \right) \left( \frac{v_B - v_L - 2m}{v_B - v_L - 2x_i} \right)^{\frac{1}{2}} dx_i \\
&= \int_m^\infty x_i \left( \frac{2m + v_L - v_B}{(v_B - v_L - 2x_i)^2} \right) dx_i \\
&= (2m + v_L - v_B) \int_m^\infty \frac{x_i}{(v_B - v_L - 2x_i)^2} dx_i \\
&= \infty
\end{aligned}$$

Hence, we conclude that  $E[X^+] = \infty$ .

Similarly,

$$\begin{aligned}
E[X^-] &\equiv E[\max(-u_i, 0)] = \int_m^\infty \int_m^\infty \max(-(v_B - x_j), 0) I_{x_j < x_i} dF^*(x_i) dF^*(x_j) \\
&\quad + \int_m^\infty \int_m^\infty \max(-(v_L + x_i), 0) I_{x_i < x_j} dF^*(x_i) dF^*(x_j) \\
&= \int_m^\infty f^*(x_j) \max(-(v_B - x_j), 0) \int_m^\infty I_{x_j < x_i} f^*(x_i) dx_i dx_j \\
&= \int_m^\infty f^*(x_j) \max(-(v_B - x_j), 0) (1 - F^*(x_j)) dx_j \\
&= \int_m^{v_B} 0 dx_j + \int_{v_B}^\infty (x_j - v_B) (1 - F^*(x_j)) f^*(x_j) dx_j \\
&= -v_B \int_{v_B}^\infty (1 - F^*(x_j)) f^*(x_j) dx_j + \int_{v_B}^\infty x_j (1 - F^*(x_j)) f^*(x_j) dx_j
\end{aligned}$$

The first term is once again bounded. The second term is

$$\begin{aligned}
& \int_{v_B}^{\infty} x_j (1 - F^*(x_j)) f^*(x_j) dx_j \\
&= \int_{v_B}^{\infty} x_j \left( \frac{v_B - v_L - 2m}{v_B - v_L - 2x_j} \right)^{\frac{1}{2}} \left( \frac{v_B - v_L - 2x_j}{v_B - v_L - 2m} \right)^{\frac{1}{2}} \left( \frac{2m + v_L - v_B}{(v_B - v_L - 2x_j)^2} \right) dx_j \\
&= \int_{v_B}^{\infty} x_j \left( \frac{2m + v_L - v_B}{(v_B - v_L - 2x_j)^2} \right) dx_j \\
&= \infty
\end{aligned}$$

Since  $E[X^+] = E[X^-] = \infty$ ,  $E[X] \equiv u_i(F^*, F^*)$  does not exist, and thus:

**Theorem 1**  *$F^*$  is not a symmetric Nash equilibrium to the Babylonian bridal auction.*

To summarize, existence of a symmetric mixed strategy Nash equilibrium requires that  $F^*$  be an (*ex ante*) best response to  $F^*$ . The  $F^*$  constructed based on standard arguments (the three steps described above) does not satisfy this condition—not because there is a profitable deviation, but because the expectation cannot be computed in the first place. The larger point is that in auctions and contests with “spillovers” and payoff functions that are unbounded, it is important to add to steps (1) through (3) a fourth step: one must (4) verify that each player’s utility is integrable with respect to the joint distribution of putative equilibrium mixed strategies— $F^*$  in this case.<sup>4</sup> In the interest of full disclosure, we stumbled upon the relevance of the fourth step purely because of the Paradox; had  $F^*$  not led to a paradox, we would have presumed it was a legitimate equilibrium to the Babylonian bridal auction.

It is important to stress that, in games where payoff functions are bounded, if steps (1) through (3) are satisfied then step (4) is automatically satisfied. However, there are a number of important games in economics where payoff functions are unbounded and steps (1) through (3) lead to a symmetric mixed-strategy with unbounded support. Probably the best known example is the war of attrition, where the analogue of equation (1) is

$$u_i(x_i, x_j; v) = \begin{cases} v - x_j & \text{if } x_i > x_j \\ \frac{1}{2}(v - x_j - x_i) & \text{if } x_i = x_j \\ -x_i & \text{if } x_i < x_j \end{cases}$$

---

<sup>4</sup> A random variable is integrable if its expectation exists and is finite; see Chung (1974, p. 40). If one is willing to admit equilibria in which players earn infinite equilibrium payoffs, one can replace this fourth step with a step that merely verifies existence of expected utility with respect to the product measure induced by  $F^*$ .

and the analogue to equation (2) is

$$F_{War}^*(x) = 1 - \exp(-x/v) \text{ on } [0, \infty)$$

Notice, however, that with  $X \equiv u_i(x_i, x_j; v)$ ,  $E[X^+] < \infty$  and  $E[X^-] < \infty$ . Hence in the War of Attrition, utility is integrable with respect to the joint distribution induced by  $F_{War}^*$ . In other words, step (4) is satisfied in the war of attrition, so  $F_{War}^*$  is indeed a symmetric mixed-strategy Nash equilibrium.

As a second example, consider a variant of the Babylonian bridal auction in which the low bidder receives, in addition to the transfer from the winning bidder, a matching dowry from an outside party. In this case,

$$u_i(x_i, x_j) = \begin{cases} v_B - x_j & \text{if } x_i > x_j \\ \frac{1}{2}(v_B - x_j + v_L + 2x_i) & \text{if } x_i = x_j \\ v_L + 2x_i & \text{if } x_i < x_j \end{cases} \quad (6)$$

One may directly verify that  $x^* = (v_B - v_L)/3$  is a symmetric pure-strategy equilibrium. In addition, there also exists a continuum of “candidate” symmetric mixed-strategy equilibria that satisfy steps (1) through (3), which is as follows: For every  $m \in (\frac{v_B - v_L}{3}, \infty)$ ,

$$F_{Dow}^*(x) = 1 - \left( \frac{v_B - v_L - 3m}{v_B - v_L - 3x} \right)^{\frac{2}{3}} \text{ on } [m, \infty).$$

Straightforward calculations reveal that, since  $X \equiv u_i(x_i, x_j)$ ,  $E[X^+] < \infty$  and  $E[X^-] < \infty$ . Thus, step (4) is also satisfied. It follows that  $F_{Dow}^*$  is a symmetric mixed-strategy equilibrium in which each player earns an expected payoff of  $EU^* = 2m + v_L \in (\frac{2}{3}v_B + \frac{1}{3}v_L, \infty)$ . Since  $m > (v_B - v_L)/3$  is arbitrary, there is a continuum of such equilibria in which each player’s expected payoff is arbitrarily large. However, the Babylonian bridal auction with a matching dowry does not exhibit a Herodotus Paradox, since these arbitrarily high payoffs come out of the hide of the outside party that pays the matching dowry. Interestingly, for any given  $m \in (\frac{v_B - v_L}{3}, \infty)$ ,  $\int_m^\infty x dF_{Dow}^* = \infty$ , so that the expected bid of each player is unbounded. This illustrates that unbounded expected bids do not imply the failure of a candidate equilibrium that satisfies steps (1) through (3) to, in fact, be a Nash equilibrium.<sup>5</sup>

---

<sup>5</sup> The complete information version of the Riley-Samuelson Sad Loser auction is another example of a game that has a symmetric mixed-strategy Nash equilibrium where players’ expected bids are unbounded.

## 2.3 Discussion

This analysis illustrates that there are games where mixed-strategies satisfy the standard three steps (constancy of payoffs on the support, well-defined probability distribution, and no profitable deviation outside of the support) but yet do not comprise a Nash equilibrium. Nonetheless, in these instances a mixed strategy constructed using steps (1) through (3) may be viewed as an *assignment* equilibrium: If an outside arbiter assigns each player a pure strategy that is an independent draw from a mixed-strategy satisfying steps (1) through (3) and each player's particular assignment is private information, then given the assigned pure strategy each player's expected payoff exists and is given by  $EU(x_i)$ .

For the case of the Babylonian bridal auction with complete information, if an outside arbiter assigns each suitor a bid  $x_i$ , it is common knowledge that each player's bid assignment is an independent draw from  $F^*$ , and each suitors's particular assignment is private information, then given the assigned bid each suitor's expected payoff exists and is given by  $EU(x_i) = m + v_L \in (\frac{1}{2}v_B + \frac{1}{2}v_L, \infty)$ . Moreover, neither suitor has an incentive to deviate from  $x_i$ , as the expected payoff from submitting *any* such assigned bid is constant on  $[m, \infty)$  and cannot be improved by submitting any alternative bid. Thus, steps (1) through (3) essentially yield an equilibrium to this assignment game. The equilibrium to this assignment game is not a Nash equilibrium to the original game, however, since players cannot compute their (*ex ante*) utility from participating in this assignment game. Indeed, the Herodotus paradox is related to a similar observation made by Bhattacharyya and Lipman (1995) in the context of a speculative bubble game. They show that two symmetric traders can enjoy positive expected gains to exchange, despite the fact that the gain to one trader in any realization exactly equals the loss to the other trader. Similar to the Herodotus paradox, this result stems from the fact that the underlying game does not have an *ex ante* equilibrium (owing to a failure of the integrability of expected utility), but does have an *interim* equilibrium.

In contrast, the symmetric equilibrium mixed strategies in the war of attrition and the Babylonian bridal auction with a matching dowry satisfy steps (1) through (4), so these mixed strategies are both an equilibrium to the original game (the mixed strategies are mutual best responses) and an equilibrium to the assignment game (any  $x_i$  assigned is a best response to the rival's mixed strategy).

### 3 The Babylonian Bridal Auction with Incomplete Information

This section shows that the Paradox identified above is neither an artifact of mixed-strategies nor the assumption that the players have complete information. To see this, suppose that it is common knowledge that both suitors value the lesser maiden at  $v_i^L \equiv 0$ , but that the suitors' valuations of the fairer maiden are privately observed random variables,  $v_i^B \equiv v_i$ , which are independently and identically distributed with an exponential distribution function,  $G(v) = 1 - \exp(-v)$  on  $[0, \infty)$  with associated density  $g(v) = \exp(-v)$ . Note that, since for any  $k = 1, 2, 3, \dots$ ,

$$E[v^k] = \int_0^\infty v^k \exp(-v) dv = (k-1)!$$

all moments of the assumed value distribution are bounded, including  $E[v] = 1$ . Thus, each suitor's (*ex ante*) expected value of the most beautiful of the two maidens is unity.<sup>6</sup>

Under these assumptions, equation (1) simplifies to

$$u_i(x_i, x_j; v_i) = \begin{cases} v_i - x_j & \text{if } x_i > x_j \\ \frac{1}{2}(v_i - x_j + x_i) & \text{if } x_i = x_j \\ x_i & \text{if } x_i < x_j \end{cases}$$

#### 3.1 A Paradox

The standard approach to solving for a symmetric (pure-strategy) equilibrium in this incomplete information environment is to: (1) assume that the rival follows a monotonically increasing bid function,  $x_j(v_j)$ , that maps the rival's valuation into a bid; (2) determine the bid,  $x_i$ , that solves the first-order condition for maximizing  $i$ 's expected payoff given that he knows his own valuation is  $v_i$  but not the specific valuation of the rival; (3) impose symmetry of the two players' bid functions and solve for a candidate symmetric equilibrium bid function; (4) verify that it is monotonically increasing; and (5) verify that a player cannot profitably deviate from the symmetric bid function.

In the case at hand, step (1) implies that the expected payoff to suitor  $i$  who knows his

---

<sup>6</sup> Results similar to those described below obtain for other distributions, including the unit Pareto distribution where  $G(v) = 1 - v^{-1}$  on  $[1, \infty)$  and  $E[v] = \infty$ .

own valuation is  $v_i$  but not that of the rival is

$$\begin{aligned} EU(x_i, x_j; v_i) &= \int_0^{x_j^{-1}(x_i)} [v_i - x_j(v_j)] g(v_j) dv_j + \int_{x_j^{-1}(x_i)}^{\infty} x_i g(v_j) dv_j \\ &= \int_0^{x_j^{-1}(x_i)} [v_i - x_j(v_j)] \exp(-v_j) dv_j + \int_{x_j^{-1}(x_i)}^{\infty} x_i \exp(-v_j) dv_j \end{aligned}$$

Applying step (2), we differentiate with respect to  $x_i$  and set the marginal expected payoff equal to zero:

$$\begin{aligned} \frac{d}{dx_i} EU(x_i, x_j; v_i) &= \frac{1}{x'_j(x_j^{-1}(x_i))} [v_i - x_j(x_j^{-1}(x_i))] \exp(-x_j^{-1}(x_i)) \\ &\quad - \frac{1}{x'_j(x_j^{-1}(x_i))} x_i \exp(-x_j^{-1}(x_i)) + \exp(-x_j^{-1}(x_i)) \\ &= 0 \end{aligned}$$

Applying step (3) with  $x_i(v) = x_j(v) = x(v)$  implies

$$\frac{v - x(v)}{x'(v)} \exp(-v) - \frac{x(v)}{x'(v)} \exp(-v) + \exp(-v) = 0$$

or

$$x'(v) = 2x(v) - v$$

The solution to this first-order ordinary differential equation is

$$x(v) = K \exp(2v) + \frac{v}{2} + \frac{1}{4} \tag{7}$$

Turning to step (4), note that  $x(v)$  is strictly increasing for all  $K \geq 0$ , so there is a continuum of such candidate equilibria with  $K \geq 0$ .

Finally, turning to step (5), note first that the support of the random variable  $v_i$  is  $[0, \infty)$ . Consequently for any  $K \geq 0$ , the rival's bid  $x_j \in [K + \frac{1}{4}, \infty)$  in the putative equilibrium. This implies that player  $i$  cannot gain by deviating to a bid below  $K + \frac{1}{4}$  since doing so yields player  $i$  a payoff of  $x_i < K + \frac{1}{4}$  with probability one, which is dominated by bidding  $x_i = K + \frac{1}{4}$  and earning a payoff of  $K + \frac{1}{4}$  (with probability one the rival suitor's valuation is strictly positive and hence  $x_j > K + \frac{1}{4}$  with probability one). If both suitors play the putative equilibrium strategies, then the expected payoff to a suitor whose valuation of the



maiden is  $v_i$  is

$$\begin{aligned}
EU(x(v_i), x(v_j); v_i) &= \int_0^{v_i} [v_i - x(v_j)] g(v_j) dv_j + \int_{v_i}^{\infty} [x(v_i)] g(v_j) dv_j \\
&= \int_0^{v_i} \left[ v_i - \left( K \exp(2v_j) + \frac{v_j}{2} + \frac{1}{4} \right) \right] \exp(-v_j) dv_j \\
&\quad + \left( K \exp(2v_i) + \frac{v_i}{2} + \frac{1}{4} \right) \int_{v_i}^{\infty} \exp(-v_j) dv_j \\
&= \int_0^{v_i} \left[ v_i - \left( K \exp(2v_j) + \frac{v_j}{2} + \frac{1}{4} \right) \right] \exp(-v_j) dv_j \\
&\quad + \left( K \exp(2v_i) + \frac{v_i}{2} + \frac{1}{4} \right) \exp(-v_i) \\
&= \left( v_i - \frac{1}{4} \right) (1 - \exp(-v_i)) - \int_0^{v_i} \left( \frac{v_j}{2} + K \exp(2v_j) \right) \exp(-v_j) dv_j \\
&\quad + \left( K \exp(2v_i) + \frac{v_i}{2} + \frac{1}{4} \right) \exp(-v_i) \\
&= \left( v_i - \frac{1}{4} \right) (1 - \exp(-v_i)) \\
&\quad - \left( -\frac{1}{2} ((v_i + 1) \exp(-2v_i) - 2K + (2K - 1) \exp(-v_i)) \exp(v_i) \right) \\
&\quad + \left( K \exp(2v_i) + \frac{v_i}{2} + \frac{1}{4} \right) \exp(-v_i) \\
&= K + v_i - \frac{3}{4} + \exp(-v_i)
\end{aligned}$$

where  $K \geq 0$ .

To complete step (5), we establish that there is no incentive for player  $i$  to deviate to an alternative bid  $x_i \in [K + \frac{1}{4}, \infty)$  by verifying that a suitor with valuation  $v_i$  cannot gain by bidding as though his valuation is  $q \neq v_i$ . Let  $EU(x(q), x(v_j); v_i)$  denote the payoff a player obtains by bidding as though his valuation is  $q$  when it is in fact  $v_i$ . Then

$$\begin{aligned}
EU(x(q), x(v_j); v_i) &= \int_0^q [v_i - x(v_j)] g(v_j) dv_j + \int_q^{\infty} x(q) g(v_j) dv_j \\
&= \int_0^q \left[ v_i - \left( K \exp(2v_j) + \frac{v_j}{2} + \frac{1}{4} \right) \right] \exp(-v_j) dv_j \\
&\quad + \left( K \exp(2q) + \frac{q}{2} + \frac{1}{4} \right) \int_q^{\infty} \exp(-v_j) dv_j \\
&= K + v_i - \frac{3}{4} + \exp(-q) + (q - v_i) \exp(-q)
\end{aligned}$$

The gain from such a deviation,  $\Delta(q) \equiv EU(x(q), x(v_j); v_i) - EU(x(v_i), x(v_j); v_i)$ , is

$$\begin{aligned} \Delta(q) &= K + v_i - \frac{3}{4} + \exp(-q) + (q - v_i) \exp(-q) - \left( K + v_i - \frac{3}{4} + \exp(-v_i) \right) \\ &= \exp(-q) + (q - v_i) \exp(-q) - \exp(-v_i) \\ &= \exp(-q) - \exp(-v_i) + (q - v_i) \exp(-q) \\ &\leq 0 \end{aligned}$$

where the inequality follows by letting  $h(x) \equiv \exp(-x) = -h'(x)$  and noting that, since  $h(x)$  is convex,

$$h(v) - h(q) \geq h'(q)(v - q)$$

Thus, applying the usual reasoning—steps (1) through (5) in this case—one would conclude that the strategies identified in equation (7) comprise a symmetric Bayesian-Nash equilibrium.

Since steps (1) through (5) hold for any  $K \geq 0$ , there is a continuum of symmetric pure-strategy equilibria in which a player whose valuation is  $v_i \in [0, \infty)$  earns a (finite) expected payoff of  $EU = K + v_i - \frac{3}{4} + \exp(-v_i) \in [v_i - \frac{3}{4} + \exp(-v_i), \infty)$ . This is the Herodotus paradox with incomplete information: Even though all moments of the value distribution are bounded, and conditional on values any payment by one suitor is a pure transfer to the other, application of steps (1) through (5) leads to the conclusion that players can earn an arbitrarily high (but finite) expected payoff in a symmetric pure-strategy equilibrium.

### 3.2 A Closer Look at the Paradox

Note that the equilibrium strategies that obtain from steps (1) through (5), and which are summarized for the case at hand in equation (7), correspond to an *interim* equilibrium in that each suitor knows his own valuation of the maidens, but not the valuation of the rival suitor. Expressed differently, the paradoxical equilibrium corresponds to a situation where the two maidens are unveiled, and each suitor knows his own valuation of the most beautiful maiden but not how much the rival suitor values her. Once the maidens are unveiled (that is, conditional on each player's private information), there is a well-defined (interim) equilibrium in which each player can earn an arbitrarily high (interim) expected payoff.

Given the prospect of achieving such a blissful state once the maidens are unveiled, would the two players have an incentive to attend the auction in the first place to learn the private

information required to play the interim equilibrium strategies? That is, do the putative strategies comprise an *ex ante* equilibrium? The answer, as it turns out, is that when  $K > 0$  the suitors' (*ex ante*) payoffs suffer from the same integrability problem that arose in the case of complete information, and hence they are incapable of determining whether or not to participate in the auction prior to the maidens being unveiled.

To formally establish that *ex ante* expected utility does not exist when  $K > 0$ , note that

$$\begin{aligned}
E[X^+] &\equiv E[\max(u_i, 0)] = \int_0^\infty \int_0^\infty \max[(v_i - x(v_j)), 0] I_{v_j < v_i} dG(v_i) dG(v_j) \\
&\quad + \int_0^\infty \int_0^\infty \max(x(v_i), 0) I_{v_i < v_j} dG(v_i) dG(v_j) \\
&= \int_0^\infty \int_0^\infty \max\left[v_i - K \exp(2v_j) - \frac{v_j}{2} - \frac{1}{4}, 0\right] I_{v_j < v_i} \exp(-v_j) dv_j \exp(-v_i) dv_i \\
&\quad + \int_0^\infty \int_0^\infty \left(K \exp(2v_i) + \frac{v_i}{2} + \frac{1}{4}\right) I_{v_i < v_j} \exp(-v_j) dv_j \exp(-v_i) dv_i \\
&= \int_0^\infty \int_0^{v_i} \max\left[\left(v_i - \left(K \exp(2v_j) + \frac{v_j}{2} + \frac{1}{4}\right)\right), 0\right] \exp(-v_j) dv_j \exp(-v_i) dv_i \\
&\quad + \int_0^\infty \int_{v_i}^\infty \left(K \exp(2v_i) + \frac{v_i}{2} + \frac{1}{4}\right) \exp(-v_j) dv_j \exp(-v_i) dv_i
\end{aligned}$$

Note that the first term is non-negative, and second term, for  $K > 0$ , is

$$\begin{aligned}
S &= \lim_{a \rightarrow \infty} \int_0^a \int_{v_i}^a \left(K \exp(2v_i) + \frac{v_i}{2} + \frac{1}{4}\right) \exp(-v_j) dv_j \exp(-v_i) dv_i \\
&= \lim_{a \rightarrow \infty} \left[ (a-1)K + \left(\frac{1}{4}a + \frac{1}{2}\right) \exp(-2a) + \left(K - \frac{3}{4}\right) \exp(-a) + \frac{1}{4} \right] \\
&= \frac{1}{4} + K \lim_{a \rightarrow \infty} (a-1) \\
&= \infty
\end{aligned}$$

Similarly,

$$\begin{aligned}
E[X^-] &\equiv E[\max(-u_i, 0)] = \int_0^\infty \int_0^\infty \max[-(v_i - x(v_j)), 0] I_{v_j < v_i} dG(v_i) dG(v_j) \\
&\quad + \int_0^\infty \int_0^\infty \max(-x(v_i), 0) I_{v_i < v_j} dG(v_i) dG(v_j) \\
&= \int_0^\infty \int_0^\infty \max\left[K \exp(2v_j) + \frac{v_j}{2} + \frac{1}{4} - v_i, 0\right] I_{v_j < v_i} \exp(-v_j) dv_j \exp(-v_i) dv_i + 0 \\
&\geq \int_0^\infty \int_0^{v_i} \left(K \exp(2v_j) + \frac{v_j}{2} + \frac{1}{4} - v_i\right) \exp(-v_j) dv_j \exp(-v_i) dv_i \\
&= \int_0^\infty \left(K + \left(\frac{v_i}{2} - \frac{3}{4}\right) \exp(-2v_i) - \left(K + v_i - \frac{3}{4}\right) \exp(-v_i)\right) \exp(v_i) \exp(-v_i) dv_i \\
&= \lim_{a \rightarrow \infty} \left( (a-1)K - \frac{1}{4}(a-1) \exp(-2a) + \left(K + a + \frac{1}{4}\right) \exp(-a) - \frac{1}{2} \right) \\
&= \lim_{a \rightarrow \infty} \left( (a-1)K - \frac{1}{2} \right) \\
&= \infty
\end{aligned}$$

(Note that the integral on the right-hand side of the inequality exists, since it is the sum of two integrals, one finite and the other with a non-negative integrand; see Chow and Teicher, 1978, p. 85). Since  $E[X^-] = E[X^+] = \infty$ , we conclude that *ex ante* expected utility does not exist when  $K > 0$ .

It is important to stress that the nonexistence of *ex ante* expected utility for  $K > 0$  does not stem from the nonexistence of interim expected utility; indeed for any value  $v_i \in [0, \infty)$  observed when the maidens are unveiled, each player's expected (interim) payoff is finite and given by  $EU(v_i) = K + v_i - 3/4 + \exp(-v_i)$ . Nor does nonexistence stem from pathological properties of the value distribution; indeed, all moments of the value distribution are bounded (and in fact,  $E[v_i] = 1$ ). And, while each player's expected bid is infinite when  $K > 0$ , we show in the next section that this is not sufficient for the failure of interim equilibria to be *ex ante* equilibria.<sup>7</sup>

---

7

$$\begin{aligned}
E[x(v)] &= \int_0^\infty \left(K \exp(2v) + \frac{v}{2} + \frac{1}{4}\right) \exp(-v) dv \\
&= K \int_0^\infty \exp(v) dv + \frac{1}{4} + \frac{1}{2} \int_0^\infty v \exp(-v) dv \\
&= K \lim_{v \rightarrow \infty} e^v - K + \frac{3}{4} \\
&= \infty
\end{aligned}$$

To summarize:

**Theorem 2** *The specified Babylonian bridal auction with incomplete information has a unique symmetric pure-strategy  $ex\ ante$  equilibrium that corresponds to  $K = 0$  in equation (7). There is a continuum of symmetric pure-strategy interim equilibria that corresponds to  $K \geq 0$  in equation (7).*

### 3.3 Discussion

This analysis highlights that the standard methods used to derive symmetric pure-strategy equilibria in auctions with incomplete information, in fact, yield *interim* equilibria. For the Babylonian bridal auction with incomplete information, there is a continuum of interim equilibria that give rise to the Herodotus paradox: Players can earn an arbitrarily high (interim) expected payoff even though all moments of the value distribution are finite and any payment received by one player is a pure transfer from the other. These paradoxical equilibria arise because the paradoxical *interim* equilibria are not *ex ante* equilibria. Again, this failure of the interim equilibria to be *ex ante* equilibria arises because the *ex ante* payoff arising from the interim equilibria (for  $K > 0$ ) cannot be computed in the first place. Expressed differently, for any of the interim equilibria with  $K > 0$ , a player that does not already know his own private value of the more beautiful maiden cannot evaluate his expected payoff from attending the auction to observe the unveiled maidens. This implies that if one considers a two-stage game where, in the first stage, players decide whether to enter a Babylonian bridal auction based on knowledge of the value distribution (but not knowing the exact beauty of the maidens) and, upon entering, learn their private value of the maidens and submit bids, then the Herodotus paradox does not arise (the only equilibrium is the  $K = 0$  equilibrium). But, if one is modeling an environment where players already possess private information, one cannot avoid the Herodotus paradox.

As before, we stress that there are a number of important games of incomplete information in economics where steps (1) through (5) lead to unbounded bid functions, including the War of Attrition (cf. Bishop, Cannings, and Maynard Smith, 1978; Riley, 1980) and the Sad Loser Auction (Riley and Samuelson, 1981). However, it is straightforward to show that integrability conditions hold for these two games, and thus the interim equilibria are also *ex ante* equilibria.

As an additional example, consider the variant of the Babylonian bridal auction introduced in Section 2.2 in which the low bidder receives, in addition to the transfer from the winning bidder, a matching dowry from an outside party. As above, it is common knowledge that both players value the lesser maiden at zero, but their private values of the fairer maiden are independent draws from a unit exponential distribution. It is straightforward to show that (interim) symmetric equilibrium bid functions are given by

$$x(v) = K \exp(3v/2) + \frac{2}{9} + \frac{v}{3}$$

where  $K \geq 0$  is arbitrary, so that there is again a continuum of symmetric pure-strategy equilibria. One may verify that each player's expected bid is unbounded:  $E[x(v)] = \infty$ . Yet, the resulting (interim) expected utility is finite and given by

$$EU(v) = v - \frac{2}{9} + \exp(-v) + 2K$$

Moreover, integrability conditions are satisfied, and *ex ante* expected utility is

$$EU = \frac{17}{18} + 2K.$$

Thus, there exist continua of both *interim* and *ex ante* symmetric equilibria in which players achieve arbitrarily high payoffs. However, these equilibria do not exhibit a Herodotus Paradox, since the increased payoffs to the players come out of the hide of the outside party that pays the matching dowry. These examples illustrate that unbounded expected interim equilibrium bids *do not* imply the failure of an interim equilibrium to be an (*ex ante*) Bayesian-Nash equilibrium.

## 4 Conclusion

The Babylonian bridal auction is an example of a game in which standard techniques for identifying symmetric equilibria lead to a continuum of “assignment equilibria” in mixed strategies under complete information and a continuum of interim equilibria in pure strategies under incomplete information. Across both of these continua, players' equilibrium payoffs are arbitrarily large (but finite). We show, however, that in each case all but one of the resulting equilibria fail to generate *ex ante* equilibria because the corresponding candidate equilibria do not satisfy integrability. In fact, *ex ante* expected utility does not exist with respect to the

joint distribution induced by the candidate equilibrium strategies. Consequently, under both complete and incomplete information, *ex ante* symmetric equilibrium payoffs are uniquely pinned down.

Our analysis of the Herodotus paradox illustrates that, in both complete and incomplete information games with unbounded payoffs (such as the Babylonian bridal auction or the war of attrition), it is potentially important to add an additional step to the standard methods of deriving symmetric equilibria. This step is to verify the integrability of utility with respect to the joint distribution induced by the players' strategies.

## References

- [1] Baye, M. R.; Kovenock, D. and de Vries, C. G. "Contests with Rank-Order Spillovers." *Economic Theory*, forthcoming.
- [2] Bhattacharyya, S. and Lipman, B.L., "Ex Ante versus Interim Rationality and the Existence of Bubbles." *Economic Theory*, 1995, 6(3), 469-494.
- [3] Bishop, D.T., C. Cannings, and J. Maynard Smith, "The War of Attrition with Random Rewards." *Journal of Theoretical Biology*, 1978, 3, 377-388.
- [4] Cassady, R. Jr. *Auctions and Auctioneering*, University of California Press, 1967.
- [5] Chow T. and H. Teicher, *Probability Theory: Independence, Interchangeability, Martingales* Springer Verlag, New York 1978.
- [6] Chung, K., *A Course in Probability Theory*, Second Edition. New York: Academic Press, 1974.
- [7] Rawlinson, G. (editor and translator), *The History of Herodotus, Volume 1*. New York: D. Appleton and Company, 1885.
- [8] Riley, J., "Strong Evolutionary Equilibrium and the War of Attrition." *Journal of Theoretical Biology*, 1980, 82, 383-400.
- [9] Riley, J. and W. Samuelson, "Optimal Auctions." *American Economic Review*, 1981, 71, 381-392.
- [10] Milgrom, P. R. and R. J. Weber, "A Theory of Auctions and Competitive Bidding." *Econometrica*, 1982, 50, September, pp. 1089-1122.



# CESifo Working Paper Series

for full list see [www.cesifo-group.org/wp](http://www.cesifo-group.org/wp)

(address: Poschingerstr. 5, 81679 Munich, Germany, [office@cesifo.de](mailto:office@cesifo.de))

---

- 3073 Marcel Boyer and Donatella Porrini, Optimal Liability Sharing and Court Errors: An Exploratory Analysis, June 2010
- 3074 Guglielmo Maria Caporale, Roman Matousek and Chris Stewart, EU Banks Rating Assignments: Is there Heterogeneity between New and Old Member Countries? June 2010
- 3075 Assaf Razin and Efraim Sadka, Fiscal and Migration Competition, June 2010
- 3076 Shafik Hebous, Martin Ruf and Alfons Weichenrieder, The Effects of Taxation on the Location Decision of Multinational Firms: M&A vs. Greenfield Investments, June 2010
- 3077 Alessandro Cigno, How to Deal with Covert Child Labour, and Give Children an Effective Education, in a Poor Developing Country: An Optimal Taxation Problem with Moral Hazard, June 2010
- 3078 Bruno S. Frey and Lasse Steiner, World Heritage List: Does it Make Sense?, June 2010
- 3079 Henning Bohn, The Economic Consequences of Rising U.S. Government Debt: Privileges at Risk, June 2010
- 3080 Rebeca Jiménez-Rodríguez, Amalia Morales-Zumaquero and Balázs Égert, The VARYing Effect of Foreign Shocks in Central and Eastern Europe, June 2010
- 3081 Stephane Dees, M. Hashem Pesaran, L. Vanessa Smith and Ron P. Smith, Supply, Demand and Monetary Policy Shocks in a Multi-Country New Keynesian Model, June 2010
- 3082 Sara Amoroso, Peter Kort, Bertrand Melenberg, Joseph Plasmans and Mark Vancauteren, Firm Level Productivity under Imperfect Competition in Output and Labor Markets, June 2010
- 3083 Thomas Eichner and Rüdiger Pethig, International Carbon Emissions Trading and Strategic Incentives to Subsidize Green Energy, June 2010
- 3084 Henri Fraise, Labour Disputes and the Game of Legal Representation, June 2010
- 3085 Andrzej Baniak and Peter Grajzl, Interjurisdictional Linkages and the Scope for Interventionist Legal Harmonization, June 2010
- 3086 Oliver Falck and Ludger Woessmann, School Competition and Students' Entrepreneurial Intentions: International Evidence Using Historical Catholic Roots of Private Schooling, June 2010

- 3087 Bernd Hayo and Stefan Voigt, Determinants of Constitutional Change: Why do Countries Change their Form of Government?, June 2010
- 3088 Momi Dahan and Michel Strawczynski, Fiscal Rules and Composition Bias in OECD Countries, June 2010
- 3089 Marcel Fratzscher and Julien Reynaud, IMF Surveillance and Financial Markets – A Political Economy Analysis, June 2010
- 3090 Michel Beine, Elisabetta Lodigiani and Robert Vermeulen, Remittances and Financial Openness, June 2010
- 3091 Sebastian Kube and Christian Traxler, The Interaction of Legal and Social Norm Enforcement, June 2010
- 3092 Volker Grossmann, Thomas M. Steger and Timo Trimborn, Quantifying Optimal Growth Policy, June 2010
- 3093 Huw David Dixon, A Unified Framework for Using Micro-Data to Compare Dynamic Wage and Price Setting Models, June 2010
- 3094 Helmuth Cremer, Firouz Gahvari and Pierre Pestieau, Accidental Bequests: A Curse for the Rich and a Boon for the Poor, June 2010
- 3095 Frank Lichtenberg, The Contribution of Pharmaceutical Innovation to Longevity Growth in Germany and France, June 2010
- 3096 Simon P. Anderson, Øystein Foros and Hans Jarle Kind, Hotelling Competition with Multi-Purchasing: Time Magazine, Newsweek, or both?, June 2010
- 3097 Assar Lindbeck and Mats Persson, A Continuous Theory of Income Insurance, June 2010
- 3098 Thomas Moutos and Christos Tsitsikas, Whither Public Interest: The Case of Greece's Public Finance, June 2010
- 3099 Thomas Eichner and Thorsten Upmann, Labor Markets and Capital Tax Competition, June 2010
- 3100 Massimo Bordignon and Santino Piazza, Who do you Blame in Local Finance? An Analysis of Municipal Financing in Italy, June 2010
- 3101 Kyriakos C. Neanidis, Financial Dollarization and European Union Membership, June 2010
- 3102 Maela Giofré, Investor Protection and Foreign Stakeholders, June 2010
- 3103 Andrea F. Presbitero and Alberto Zazzaro, Competition and Relationship Lending: Friends or Foes?, June 2010

- 3104 Dan Anderberg and Yu Zhu, The Effect of Education on Martial Status and Partner Characteristics: Evidence from the UK, June 2010
- 3105 Hendrik Jürges, Eberhard Kruk and Steffen Reinhold, The Effect of Compulsory Schooling on Health – Evidence from Biomarkers, June 2010
- 3106 Alessandro Gambini and Alberto Zazzaro, Long-Lasting Bank Relationships and Growth of Firms, June 2010
- 3107 Jenny E. Ligthart and Gerard C. van der Meijden, Coordinated Tax-Tariff Reforms, Informality, and Welfare Distribution, June 2010
- 3108 Vilen Lipatov and Alfons Weichenrieder, Optimal Income Taxation with Tax Competition, June 2010
- 3109 Malte Mosel, Competition, Imitation, and R&D Productivity in a Growth Model with Sector-Specific Patent Protection, June 2010
- 3110 Balázs Égert, Catching-up and Inflation in Europe: Balassa-Samuelson, Engel’s Law and other Culprits, June 2010
- 3111 Johannes Metzler and Ludger Woessmann, The Impact of Teacher Subject Knowledge on Student Achievement: Evidence from Within-Teacher Within-Student Variation, June 2010
- 3112 Leif Danziger, Uniform and Nonuniform Staggering of Wage Contracts, July 2010
- 3113 Wolfgang Buchholz and Wolfgang Peters, Equity as a Prerequisite for Stable Cooperation in a Public-Good Economy – The Core Revisited, July 2010
- 3114 Panu Poutvaara and Olli Ropponen, School Shootings and Student Performance, July 2010
- 3115 John Beirne, Guglielmo Maria Caporale and Nicola Spagnolo, Liquidity Risk, Credit Risk and the Overnight Interest Rate Spread: A Stochastic Volatility Modelling Approach, July 2010
- 3116 M. Hashem Pesaran, Predictability of Asset Returns and the Efficient Market Hypothesis, July 2010
- 3117 Dorothee Crayen, Christa Hainz and Christiane Ströh de Martínez, Remittances, Banking Status and the Usage of Insurance Schemes, July 2010
- 3118 Eric O’N. Fisher, Heckscher-Ohlin Theory when Countries have Different Technologies, July 2010
- 3119 Huw Dixon and Hervé Le Bihan, Generalized Taylor and Generalized Calvo Price and Wage-Setting: Micro Evidence with Macro Implications, July 2010

- 3120 Laszlo Goerke and Markus Pannenberg, 'Take it or Go to Court' – The Impact of Sec. 1a of the German Protection against Dismissal Act on Severance Payments -, July 2010
- 3121 Robert S. Chirinko and Daniel J. Wilson, Can Lower Tax Rates be Bought? Business Rent-Seeking and Tax Competition among U.S. States, July 2010
- 3122 Douglas Gollin and Christian Zimmermann, Global Climate Change and the Resurgence of Tropical Disease: An Economic Approach, July 2010
- 3123 Francesco Daveri and Maria Laura Parisi, Experience, Innovation and Productivity – Empirical Evidence from Italy's Slowdown, July 2010
- 3124 Carlo V. Fiorio and Massimo Florio, A Fair Price for Energy? Ownership versus Market Opening in the EU15, July 2010
- 3125 Frederick van der Ploeg, Natural Resources: Curse or Blessing?, July 2010
- 3126 Kaisa Kotakorpi and Panu Poutvaara, Pay for Politicians and Candidate Selection: An Empirical Analysis, July 2010
- 3127 Jun-ichi Itaya, Makoto Okamura and Chikara Yamaguchi, Partial Tax Coordination in a Repeated Game Setting, July 2010
- 3128 Volker Meier and Helmut Rainer, On the Optimality of Joint Taxation for Non-Cooperative Couples, July 2010
- 3129 Ryan Oprea, Keith Henwood and Daniel Friedman, Separating the Hawks from the Doves: Evidence from Continuous Time Laboratory Games, July 2010
- 3130 Mari Rege and Ingeborg F. Solli, The Impact of Paternity Leave on Long-term Father Involvement, July 2010
- 3131 Olaf Posch, Risk Premia in General Equilibrium, July 2010
- 3132 John Komlos and Marek Brabec, The Trend of BMI Values by Centiles of US Adults, Birth Cohorts 1882-1986, July 2010
- 3133 Emin Karagözoğlu and Arno Riedl, Information, Uncertainty, and Subjective Entitlements in Bargaining, July 2010
- 3134 John Boyd, Gianni De Nicolò and Elena Loukoianova, Banking Crises and Crisis Dating: Theory and Evidence, July 2010
- 3135 Michael R. Baye, Dan Kovenock and Casper G. de Vries, The Herodotus Paradox, July 2010