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The implied jump risk of LIBOR rates

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Abstract

This paper examines implied parameters from options on LIBOR futures. Jump-diffusion models are found to offer superior in-sample and out-of-sample performance when compared to their pure diffusion counterpart. The need to incorporate stochastic jump magnitudes into LIBOR dynamics is also documented. In addition, empirical evidence reveals that the jump component in LIBOR rates is important for pricing their derivatives. Furthermore, variation in jump risk often coincides with Federal Open Market Committee (FOMC) decisions and a small subset of macroeconomic announcements.

JEL classification: G120; G130
Keywords: LIBOR; Jump-diffusion; Federal reserve; Macroeconomic announcements

1. Introduction

In the equity options literature, jump-diffusion models are often justified by improved out-of-sample forecasts and hedging performance. Furthermore, they are usually better at reproducing the skewness and kurtosis in observed data. This paper offers two contributions to the term structure literature. First, we find evidence that jump risk is priced by market participants, and that jump-diffusion models are

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required to adequately explain LIBOR dynamics. Second, our implied parameters reveal that jump risk corresponds with the release of economic information.

Since the seminal paper of Merton (1976), jump-diffusion models have become increasingly common in the equity derivatives literature. More recently, Bakshi et al. (1997) demonstrate the importance of incorporating jumps into equity option pricing models.

Amin and Morton (1994) provide one of the first empirical examinations of the Heath et al. (1992) term structure model. Their findings suggest Eurodollar option prices are consistent with alternatives to standard diffusion models. Furthermore, Das (2002) argues that the excess kurtosis in interest rate data is consistent with a jump-diffusion process. In the context of currency markets, Bates (1996) reports that jump-diffusion dynamics are necessary to explain the ‘‘volatility smile’’ in option contracts. Moreover, Ederington and Lee (1993)’s study indicates that the volatility of foreign exchange futures contracts is almost entirely explained by macroeconomic announcements.

In general, empirical research documents that macroeconomic announcements influence interest rate dynamics. For example, Balduzzi et al. (2001) as well as Fleming and Remolona (1999) find that large price movements in US Treasury bonds coincide with macroeconomic announcements. Jones et al. (1998) also report that the release of employment and inflation data produces significant bond price impacts.

Although Balduzzi et al. (2001) do not explicitly assume an underlying interest rate process, their important empirical study suggests that a jump-diffusion term structure model is warranted. However, previous studies typically employ absolute price changes to proxy for volatility, and these statistics have limitations. First, these measures cannot determine whether jump risk is priced by market participants since they cannot decompose price movements into diffusion and jump components. Consequently, it remains an open question whether term structure models actually require additional factors when their objective is to price interest rate sensitive securities. Second, the probability and magnitude of jumps cannot be examined. Specifically, it is important to emphasize that the uncertainty regarding future interest rates is not equivalent to price adjustments. While Balduzzi et al. (2001) demonstrate that price adjustments are generally completed within the trading day, variation in interest rate volatility and jump risk may persist. [1] Thus, implied parameters contain valuable information that cannot be inferred from the underlying price data.

In particular, implied jump parameters capture the possibility of future interest rate movements. Indeed, they may change before, during, and after an event. Consequently, we infer diffusion and jump parameters on a daily basis to investigate their response to the release of economic information.

Empirically, we find evidence consistent with jump risk being priced by the market. Furthermore, our analysis demonstrates that jump risk responds to Federal

[1] For example, fluctuations in equity option prices may be caused by volatility movements (as well as time-varying risk premiums in incomplete markets) even when the underlying equity price remains constant.

Open Market Committee (FOMC) decisions and a small subset of macroeconomic announcements. Statistical tests indicate that a jump-diffusion model with constant jump sizes has smaller in-sample and out-of-sample pricing errors in comparison to a pure diffusion LIBOR model. Moreover, performance improves considerably when the jump magnitudes are stochastic. In addition, random jump sizes enable the negative skewness and excess kurtosis in the data to be captured more effectively. Consequently, jump-diffusion LIBOR models have both economic and statistical motivations.

The remainder of this paper begins with Section 2 describing our three LIBOR models. The empirical implementation is described in Section 3. In Section 4, we discuss the empirical results while Section 5 contains our conclusions.

2. A jump-diffusion LIBOR model

Jarrow and Madan (1995) as well as Björk et al. (1997) extend the original Heath et al. (1992) model (abbreviated HJM hereafter) to incorporate discontinuities. Although Sandmann and Sondermann (1997) demonstrate that forward rates in a pure diffusion HJM model become infinite, Brace et al. (1997) propose a stable log-normal LIBOR model by modifying instantaneous HJM dynamics. [2]

As in the existing literature, a marked point process produces discontinuities, with details in Brémaud (1981). Let $\lambda(t)$ denote their "arrival rate" while N_t records the number of jumps within the time interval $[0, t]$. With the expected value of N_t under the empirical measure being $\int_0^t \lambda(s)\,\mathrm{d}s$, a compensated jump martingale

$$M_t = N_t - \int_0^t \lambda(s)\,\mathrm{d}s \tag{1}$$

is formed with zero expected value. With LIBOR rates being martingales under the forward measure, [3] they evolve as

$$\begin{aligned}\frac{\mathrm{d}L(t,T)}{L(t-,T)} &= -\lambda(t)\psi_2(t,T)\,\mathrm{d}t + \psi_1(t,T)\,\mathrm{d}W_t + \psi_2(t,T)\,\mathrm{d}N_t \\ &= \psi_1(t,T)\,\mathrm{d}W_t + \psi_2(t,T)\,\mathrm{d}M_t, \end{aligned} \tag{2}$$

where W_t denotes a Brownian motion (referred to as the diffusion process), and M_t denotes a compensated jump martingale $N_t - \int_0^t \lambda(s)\,\mathrm{d}s$ under the forward measure, while δ is the tenor of the LIBOR rate. For instance, $\delta = 1/4$ corresponds to the three-month LIBOR rate.

[2] Musiela and Rutkowski (1997) offer an excellent description of LIBOR models and the forward measure invoked for pricing their associated derivatives. Glasserman and Kou (2003) provide a discontinuous LIBOR model although connections between its implied parameters and underlying economic events are not investigated.

[3] A discontinuous LIBOR model results from risk neutral jump-diffusion HJM forward rates. These dynamics produce risk neutral LIBOR rates which are then transformed to be a martingale under the forward measure. Details are available from the authors upon request.

The coefficient $\psi_1(t,T)$ is necessarily positive and measures the diffusion volatility. In contrast, $\psi_2(t,T)$ may be either positive or negative, indicating an increase or decrease in LIBOR rates respectively.

The model in Eq. (2) with random arrival times for jumps of unknown magnitudes is most appropriate for modeling LIBOR rates. In contrast to the federal funds rate set by the Federal Reserve at "known" timepoints (eight scheduled annual meetings), LIBOR rates are determined on a "continuous" (daily) basis by market forces. Therefore, unlike the federal funds rate, LIBOR rate discontinuities are not constrained to be multiples of 25 basis points. Moreover, LIBOR rates may adjust to market conditions before and after FOMC decisions, implying the occurrence of jumps is random.

3. Data and methodology

Daily closing prices for Eurodollar futures contracts and their American options traded on the Chicago Mercantile Exchange (CME) from January 1996 through the end of June 2001 are examined, with contracts maturing in March, June, September, and December of each year. This sample period consists of several interesting events, notably the crises in Asia (1997), Russia and Long Term Capital Management (1998), Brazil (1999), as well as the collapse of the dotcom bubble (2000). Exogenous shocks of this nature reinforce the need to have discontinuities arrive randomly. The data was purchased from The Institute for Financial Markets Data Center.

We denote the Eurodollar futures price as $F(0,T)$. The initial LIBOR term structure is inferred from futures contracts as [4] $L(0,T) = \frac{100-F(0,T)}{100}$ where $L(0,T)$ denotes the forward three-month annualized LIBOR rate effective from time T to time T plus three months. We are careful to only infer parameters from option prices matched with futures contracts of the same maturity to avoid any maturity bias in our results. However, all available option maturities are utilized in our empirical study.

Similar to Das (2002), we investigate the impact of Federal Open Market Committee (FOMC) decisions on jump risk. FOMC decisions are classified into three categories; reductions, increases, and neutral movements in the federal funds rate. During our sample period, 10 reductions, 6 increases, and 27 neutral positions are recorded. [5]

As in Balduzzi et al. (2001), the impact of macroeconomic events on the term structure is examined. Dates for the release of macroeconomic data are obtained from Bloomberg. We investigate nine macroeconomic announcements: durable goods orders, housing starts, initial jobless claims, producer price index, retail sales (less autos), unemployment rate, change in nonfarm payroll, construction spending,

[4] Specifically, $L(0,T) \equiv L(0,T,T+\delta)$ where δ is fixed at three months (1/4 year) but omitted for notational simplicity.

[5] The FOMC holds eight meetings each year with no distinction between 1-day and 2-days meetings being apparent in the results.

and the National Association of Purchasing Managers Index (abbreviated NAPM hereafter).

3.1. Parameter inference

Our approach is related to Balduzzi et al. (2001) as we investigate the effects of FOMC meetings and macroeconomic announcements on LIBOR futures rates. However, as in Bakshi et al. (1997), we utilize an option-theoretic approach to ascertain the information contained in implied parameters. Investigating implied parameters is a common empirical technique. For example, Bates (1991) reports that a large downward movement was anticipated by the market before the October 1987 crash.

The implied parameters in our models are obtained by minimizing the mean squared percentage errors between observed market and theoretical option prices. Our theoretical option prices are determined by multinomial lattices described in the next subsection. We restrict our study to a parsimonious version of Eq. (2) with a constant intensity function. However, both constant and stochastic jump magnitudes are analyzed.

We begin with a simplified version of Eq. (2) that has constant parameters,

$$\text{JD}: \quad \frac{\mathrm{d}L(t,T)}{L(t-,T)} = \psi_1\,\mathrm{d}W_t + \psi_2\,\mathrm{d}M_t. \tag{3}$$

Eq. (3) is the most parsimonious model available for decomposing LIBOR rates into diffusion and jump components. For simplicity and ease of reference, these dynamics are abbreviated JD. This formulation is generalized below in Eq. (4) to have stochastic jump magnitudes, with a corresponding JDS abbreviation:

$$\text{JDS}: \quad \frac{\mathrm{d}L(t,T)}{L(t-,T)} = \psi_1\,\mathrm{d}W_t + \psi_2^{\mathrm{S}}\,\mathrm{d}M_t. \tag{4}$$

To distinguish between the jump processes in Eqs. (3) and (4), ψ_2 is denoted ψ_2^{S} to emphasize its stochastic nature which has the following distribution:

$$\psi_2^{\mathrm{S}} = \begin{cases} \mu + \gamma & \text{with probability } \frac{1}{2}, \\ \mu - \gamma & \text{with probability } \frac{1}{2}, \end{cases}$$

as in Das (1999). This binomial assumption implies the mean and variance of the jump sizes equal μ and γ^2, respectively.

We also calibrate the standard lognormal diffusion model

$$\text{PD}: \quad \frac{\mathrm{d}L(t,T)}{L(t,T)} = \phi\,\mathrm{d}W_t, \tag{5}$$

which imposes the constraint that λ equals zero. The PD abbreviation is applied to the pure diffusion LIBOR model in Eq. (5) hereafter.

A minimum of four observations each day is required to infer all the JDS parameters in Eq. (4). Therefore, only days with 4 or more available option prices are considered. Imposing a more stringent requirement of five observations per day does not have a material effect on our estimates, but merely results in fewer days with implied parameters. In total, 968 daily sets of parameter estimates are computed from an average of 6.4 options per day, ranging from 4 to 17.

Implied parameters are constrained by the conditions, ψ_1, $\phi > 0$ to ensure the diffusion volatility estimate is positive, and $0 < \lambda < 1$ since λ represents the probability of a discontinuity. Observe that in the JD model, the quantity $\sqrt{\hat{\lambda}}\hat{\psi}_2$ represents jump risk, while the quantity $\sqrt{\hat{\lambda}(\hat{\mu}^2 + \hat{\gamma}^2)}$ serves as the proxy when stochastic jump magnitudes in the JDS formulation are estimated.

3.2. Multinomial lattices for pricing

A call option's terminal value is derived from receiving a long position in the futures contract at time τ, provided the futures price $F(\tau, T)$ is above the strike price K. The payoff of this option at maturity is defined as $\max\{0, F(\tau, T) - K\}$ for $T \geqslant \tau$ where T and τ represent the maturity of the futures contract and its option respectively. Similarly, a put option grants the owner the ability to receive a short position in the futures contract. The underlying LIBOR term structure serves as the discount rate. [6]

Since there is no analytical formula for valuing the American CME options, we employ a numerical lattice. The JD model in Eq. (3) is discretized over a Δ time increment to construct a multinomial tree with four branches that describes LIBOR rates under the forward measure, with details in Appendix A. This multinomial lattice provides a methodology to price futures contracts and their American options, with early exercise possible at each node. [7] Appendix B demonstrates that the multinomial lattice for the JD model in Appendix A is not equivalent to a mixture of diffusions, except in a circumstance that is inconsistent with our empirical results.

In addition to the quadrinomial model with four branches required for the JD formulation, an enhanced six-branch hexanomial model that allows for stochastic jump magnitudes is implemented. Following Das (1999), two additional branches are required to estimate the parameters of the JDS model, as illustrated in Appendix C.

For all multinomial lattices, ten intervals is sufficient for convergence, yielding pricing errors of the order 10^{-4}. Smaller intervals increase computational time significantly without offering any additional insights or material change to the results.

4. Empirical results

This section analyzes the implied risk parameters with respect to the release of FOMC decisions and macroeconomic data. As discussed in the introduction, the

[6] More precisely, the dollar-denominated payoff per option is $2500 \times \max\{0, F(\tau, T) - K\}$ for calls and $2500 \times \max\{0, K - F(\tau, T)\}$ for puts when prices are expressed in percentages.

[7] Implementing the procedure without the possibility of early exercise reveals the American option price premia ranges from 0.05% to 10% relative to its European counterpart.

previous literature considers realized price changes. However, implied jump parameters capture the market's assessment of future interest movements and may change before, during, and after an event.

Summary statistics for the implied parameters are reported below in Table 1 for each year as well as the entire sample period.

Overall, the parameters estimates in all three implementations appear relatively stable. As the sample period contains more interest rate reductions than increases, the estimates $\hat{\mu}$ and $\hat{\psi}_2$ are negative, with the other implied parameters also assuming reasonable values.

In terms of in-sample fit, the JD and JDS models are superior to their PD counterpart according to their root mean squared errors (RMSE) and Akaike information criteria (AIC). Furthermore, the addition of random jump sizes in the JDS formulation yields a significant improvement in model fit. These results are confirmed in out-of-sample tests reported in the next subsection.

When comparing the JD and JDS implementations, the $\hat{\gamma}$ estimates attest to the variability of jump sizes. In addition, the estimated jump arrival rates $\hat{\lambda}$ are smaller with random jump sizes. Thus, constraining the jump magnitudes to be constant in the JD model biases their frequency upwards. This disparity likely results from the JDS formulation not requiring as large a compensating factor, $\lambda\mu\Delta$ instead of $\lambda\psi_2\Delta$, to achieve a martingale under the forward measure. Indeed, along with $\hat{\lambda}$ being smaller, their estimated expectation $\hat{\mu}$ is less than $\hat{\psi}_2$ when jump magnitudes are stochastic.

More importantly, average jump risk in the JDS model is 0.083 while its average diffusion risk volatility is 0.090. Thus, total risk is 0.122 in comparison with 0.114 for the PD implementation. [8] Hence, the PD model underestimates risk by ignoring the jump component.

Augmented Dickey–Fuller tests performed on the implied parameters as reported in Table 1 reveal that no unit roots are present in the time series of implied parameters, consistent with the stationarity required for our ensuing regression procedures to be well specified.

4.1. Out-of-sample tests

It is important to test the predictive ability of jump-diffusion dynamics to determine their contribution in asset pricing applications. For this purpose, theoretical option prices at $t+1$, $t+3$, and $t+7$ are constructed from implied parameters at date t. Observed ex-post prices are then compared with these theoretical prices to generate out-of-sample RMSE values for the percentage deviation in option price forecasts.

Average RMSE values over the various sample periods are reported in Table 2 along with their associated standard deviations.

[8] The total risk is computed as $\sqrt{(0.083)^2 + (0.090)^2} = 0.122$.

Table 1

Annual mean values for implied parameters and their standard deviations (in parentheses)

Year	Obs	Jump-diffusion with stochastic jumps, JDS model							Jump-diffusion with constant jumps, JD model						Pure diffusion, PD model		
		$\hat{\psi}_1$	$\hat{\mu}$	$\hat{\gamma}$	$\hat{\lambda}$	$\sqrt{\hat{\lambda}(\hat{\mu}^2+\hat{\gamma}^2)}$	RMSE	AIC	$\hat{\psi}_1$	$\hat{\psi}_2$	$\hat{\lambda}$	$\sqrt{\hat{\lambda}}\hat{\psi}_2$	RMSE	AIC	$\hat{\phi}$	RMSE	AIC
1996	200	0.086	−0.102	0.279	0.073	0.084	0.024	−9.335	0.097	−0.055	0.152	−0.007	0.090	−7.447	0.108	0.155	−6.260
		(0.023)	(0.198)	(0.205)	(0.057)	(0.041)	(0.023)		(0.029)	(0.221)	(0.106)	(0.052)	(0.108)		(0.027)	(0.133)	
1997	122	0.064	−0.050	0.238	0.059	0.072	0.049	−5.201	0.069	−0.073	0.128	−0.007	0.074	−5.672	0.079	0.169	−4.250
		(0.014)	(0.222)	(0.211)	(0.072)	(0.053)	(0.039)		(0.018)	(0.253)	(0.096)	(0.056)	(0.095)		(0.022)	(0.160)	
1998	192	0.095	−0.193	0.274	0.112	0.105	0.087	−4.318	0.107	−0.196	0.155	−0.052	0.168	−3.747	0.129	0.366	−1.940
		(0.043)	(0.194)	(0.205)	(0.097)	(0.051)	(0.069)		(0.054)	(0.202)	(0.115)	(0.048)	(0.166)		(0.060)	(0.157)	
1999	175	0.084	−0.204	0.194	0.058	0.065	0.086	−3.876	0.092	−0.193	0.110	−0.036	0.215	−2.735	0.106	0.426	−1.555
		(0.022)	(0.134)	(0.176)	(0.053)	(0.028)	(0.052)		(0.026)	(0.213)	(0.103)	(0.035)	(0.180)		(0.024)	(0.160)	
2000	154	0.063	−0.248	0.177	0.025	0.051	0.090	−3.910	0.067	−0.179	0.079	−0.026	0.167	−3.304	0.076	0.371	−1.881
		(0.010)	(0.155)	(0.161)	(0.019)	(0.023)	(0.099)		(0.012)	(0.200)	(0.092)	(0.028)	(0.160)		(0.013)	(0.167)	
2001	125	0.156	−0.121	0.372	0.116	0.127	0.052	−5.476	0.174	−0.111	0.247	−0.047	0.140	−4.382	0.196	0.242	−2.808
		(0.028)	(0.173)	(0.206)	(0.081)	(0.042)	(0.036)		(0.036)	(0.144)	(0.072)	(0.059)	(0.124)		(0.032)	(0.096)	
Total	968	0.090	−0.157	0.253	0.074	0.083	0.065	−5.477	0.100	−0.137	0.143	−0.030	0.145	−4.583	0.114	0.293	−3.157
		(0.038)	(0.192)	(0.203)	(0.075)	(0.048)	(0.064)		(0.046)	(0.216)	(0.111)	(0.050)	(0.152)		(0.050)	(0.183)	
ADF test		−3.3*	−8.1***	−9.3***	−6.5***	−6.2***			−3.8**	−8.9***	−7.6***	−7.9***			−3.5**		

The implied parameters are obtained by minimizing the mean squared percentage pricing errors. As a test of the model's in-sample fit, the average root mean squared error (RMSE) along with its standard deviation below in parentheses are reported, as well as the Akaike information criteria (AIC). Overall, the jump-diffusion model with stochastic jump magnitudes offers the best in-sample fit. Results from augmented Dickey–Fuller (ADF) tests of the unit root hypotheses using four lags indicate parameter stationarity. For these tests, *, **, and *** denote MacKinnon one-sided p-values that are significant at the 10%, 5%, and 1% level, respectively.

Table 2
Root mean squared errors (RMSE) for out-of-sample percentage deviation in option price forecasts from January 3, 1996 to June 29, 2001

Year	Forecast	Jump-diffusion with stochastic jumps (JDS)			Jump-diffusion with constant jumps (JD)			Pure diffusion, no jumps (PD)			Comparison, JDS versus JD		Comparison, JDS versus PD	
		$t+1$	$t+3$	$t+7$	$t+1$	$t+3$	$t+7$	$t+1$	$t+3$	$t+7$	Percent	t-stat	Percent	t-stat
1996	Mean	0.108	0.162	0.197	0.170	0.230	0.270	0.225	0.275	0.316	70.4%	0.062	86.4%	0.117
	Std. dev.	(0.101)	(0.147)	(0.170)	(0.137)	(0.182)	(0.193)	(0.159)	(0.190)	(0.198)		(9.136)***		(13.426)***
1997	Mean	0.148	0.226	0.271	0.168	0.256	0.287	0.249	0.329	0.372	59.8%	0.020	68.0%	0.101
	Std. dev.	(0.130)	(0.164)	(0.193)	(0.146)	(0.186)	(0.214)	(0.186)	(0.213)	(0.250)		(2.672)***		(7.771)***
1998	Mean	0.185	0.266	0.312	0.254	0.322	0.364	0.406	0.450	0.500	63.0%	0.070	92.2%	0.221
	Std. dev.	(0.138)	(0.178)	(0.233)	(0.169)	(0.189)	(0.224)	(0.154)	(0.172)	(0.197)		(6.630)***		(20.602)***
1999	Mean	0.211	0.305	0.341	0.311	0.381	0.419	0.467	0.517	0.538	72.0%	0.100	93.7%	0.256
	Std. dev.	(0.182)	(0.292)	(0.290)	(0.216)	(0.263)	(0.274)	(0.163)	(0.191)	(0.214)		(7.859)***		(17.696)***
2000	Mean	0.209	0.238	0.287	0.270	0.311	0.368	0.418	0.456	0.495	69.5%	0.061	90.9%	0.209
	Std. dev.	(0.220)	(0.155)	(0.201)	(0.206)	(0.199)	(0.250)	(0.168)	(0.177)	(0.195)		(4.533)***		(13.558)***
2001	Mean	0.163	0.212	0.243	0.204	0.255	0.294	0.278	0.307	0.337	62.4%	0.042	92.8%	0.116
	Std. dev.	(0.155)	(0.161)	(0.177)	(0.127)	(0.143)	(0.163)	(0.114)	(0.124)	(0.142)		(3.253)***		(9.913)***
Total	Mean	0.170	0.235	0.276	0.232	0.295	0.337	0.345	0.394	0.431	66.7%	0.062	88.1%	0.175
	Std. dev.	(0.162)	(0.197)	(0.222)	(0.180)	(0.206)	(0.231)	(0.185)	(0.202)	(0.220)		(13.820)***		(32.406)***

Based on price forecasts for $t+1$, $t+3$, and $t+7$ using implied parameters computed at date t, percentage price deviations with observed option prices are reported. The column "Percent" denotes the percentage of days in which the jump-diffusion model with stochastic jumps (JDS) has a lower RMSE than either the jump-diffusion model with constant jumps (JD) or the pure diffusion model (PD). The first row in each column labeled "t-stat" corresponds to the average difference, for the $t+1$ forecast, in the mean RMSE between the two models being compared while the quantity in parentheses provides its t-statistic. *** denotes significance at the 1% level. As in the previous table, the jump-diffusion model with stochastic jump magnitudes is found to offer superior performance.

The out-of-sample forecasts indicate that the JD and JDS processes are superior to the PD model across all horizons. In particular, the average RMSE values are lowest for the JDS model, followed by its special case in the JD implementation, and finally the PD formulation. This pattern is consistent for every year in the sample period. Furthermore, the percentage of days in which the JDS model outperforms both its rivals is substantial.

To summarize, when considering the implications of various LIBOR models on option pricing, a jump-diffusion process is required. Moreover, out-of-sample forecasts are greatly improved by allowing for stochastic jump magnitudes. As seen later in this section, these conclusions are confirmed when the implied skewness and kurtosis of the JD and JDS processes are examined.

4.2. Implied parameters and economic information

To evaluate whether implied risk parameters correlate with FOMC decisions and also macroeconomic announcements, the following regressions are conducted:

$$Y_t = \beta_0 + \beta_1 X_{1,t} + \beta_2 X_{2,t} + \beta_3 X_{3,t} + \beta_4 Y_{t-1} + \sum_{j=1}^{9} \delta_j Z_{j,t} + \epsilon_t. \tag{6}$$

Under this specification, the dependent variable Y_t represents either $\hat{\phi}$, $\hat{\psi}_1$, $\sqrt{\hat{\lambda}}\hat{\psi}_2$, or $\sqrt{\hat{\lambda}(\hat{\mu}^2 + \hat{\gamma}^2)}$. The explanatory variables $X_{j,t}$ for $j = 1, 2, 3$ are dummy variables corresponding to FOMC decisions. These indicators equal 1 for a neutral stance, rate increase, and rate decrease respectively while the regression residual denoted ϵ_t represents a mean zero normal random variable. Similarly, the explanatory variables $Z_{j,t}$ for $j = 1, 2, \ldots, 9$ are indicator functions corresponding to macroeconomic announcements.

Results of the least squares regression (6) using only FOMC decisions as explanatory variables ($\delta_j = 0$) are reported in Table 3. Table 4 reports the regression results using only macroeconomic announcements ($\beta_j = 0$). Finally, the combined regression in Eq. (6) using both FOMC decisions and macroeconomic announcements is summarized in Table 5.

Two sets of analyses are conducted. First, indicator functions are defined for the event itself. Second, a window of 14 days before and after a FOMC decision, and 2 days before and after a macroeconomic announcement, is highlighted. These windows capture information impounded in market prices around the event. For macroeconomic events, 2 days is chosen since a larger window results in too much overlap between the release dates, rendering the investigation unable to distinguish between the effects of different macroeconomic announcements. Justification for these horizons stems from them both containing an equal [9] and significant number of events.

[9] The indicator functions equal one an average of 25.5% and 26.0% of the sample period for FOMC decisions and macroeconomic announcements respectively. A variety of other window lengths are examined with similar results.

Table 3
Regression results from Eq. (6) involving FOMC decisions ($\delta_j = 0$)

Coefficients	Jump-diffusion with stochastic jumps, JDS model				Jump-diffusion with constant jumps, JD model				Pure diffusion, PD model	
	Event date		±14 Days		Event date		±14 Days		Event date	±14 Days
	$\hat{\psi}_1$	$\sqrt{\hat{\lambda}(\hat{\mu}^2+\hat{\gamma}^2)}$	$\hat{\psi}_1$	$\sqrt{\hat{\lambda}(\hat{\mu}^2+\hat{\gamma}^2)}$	$\hat{\psi}_1$	$\sqrt{\hat{\lambda}}\hat{\psi}_2$	$\hat{\psi}_1$	$\sqrt{\hat{\lambda}}\hat{\psi}_2$	$\hat{\phi}$	$\hat{\phi}$
Constant, β_0	0.005***	0.036***	0.009***	0.043***	0.014***	−0.017***	0.022***	−0.013***	0.005***	0.008***
p-Value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
FOMC neutral, β_1	0.000	−0.005	−0.002**	−0.003	−0.008	−0.009	−0.005***	−0.003	−0.002	−0.003**
p-Value	(0.859)	(0.550)	(0.017)	(0.339)	(0.110)	(0.328)	(0.004)	(0.334)	(0.448)	(0.011)
FOMC increase, β_2	0.002	0.008	−0.002	−0.010**	0.009	0.009	−0.004	−0.007	−0.002	−0.002
p-Value	(0.732)	(0.654)	(0.247)	(0.025)	(0.394)	(0.659)	(0.164)	(0.174)	(0.798)	(0.145)
FOMC decrease, β_3	0.014***	0.009	0.004***	0.028***	0.005	−0.027*	0.012***	−0.018***	0.013***	0.003**
p-Value	(0.001)	(0.465)	(0.004)	(0.000)	(0.496)	(0.064)	(0.000)	(0.000)	(0.002)	(0.040)
Lagged volatility β_4	0.939***	0.571***	0.905***	0.448***	0.860***	0.397***	0.779***	0.371***	0.959***	0.937***
p-Value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
R^2 (adjusted)	0.888	0.325	0.889	0.377	0.745	0.158	0.756	0.170	0.927	0.927
F-statistic	1919.1***	117.3***	1937.0***	147.1***	706.2***	46.3***	747.1***	50.4***	3047.9***	3064.4***
p-Value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

The independent variables Y_t are the risk proxies $\hat{\phi}$, $\hat{\psi}_1$, $\sqrt{\hat{\lambda}}\hat{\psi}_2$, and $\sqrt{\hat{\lambda}(\hat{\mu}^2+\hat{\gamma}^2)}$. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively. Quantities in parentheses are the corresponding p-values of the coefficients. Two time periods are analyzed. The first set of indicator functions denote the decision date itself, while the second considers 14 days before and after the event.

Table 4
Regression results from Eq. (6) involving macroeconomic announcements ($\beta_j = 0$)

Coefficients	Jump-diffusion with stochastic jumps, JDS model				Jump-diffusion with constant jumps, JD model				Pure diffusion, PD model	
	Event date		±2 Days		Event date		±2 Days		Event date	±2 Days
	$\hat{\psi}_1$	$\sqrt{\hat{\lambda}(\hat{\mu}^2+\hat{\gamma}^2)}$	$\hat{\psi}_1$	$\sqrt{\hat{\lambda}(\hat{\mu}^2+\hat{\gamma}^2)}$	$\hat{\psi}_1$	$\sqrt{\hat{\lambda}}\hat{\psi}_2$	$\hat{\psi}_1$	$\sqrt{\hat{\lambda}}\hat{\psi}_2$	$\hat{\phi}$	$\hat{\phi}$
Constant	0.004***	0.036***	0.005***	0.034***	0.012***	−0.016***	0.013***	−0.004	0.003***	0.004***
p-Value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.224)	(0.008)	(0.003)
Construction	0.007*	0.012	0.005**	−0.001	0.007	−0.015	0.005	−0.014*	0.016***	0.008***
p-Value	(0.080)	(0.317)	(0.040)	(0.861)	(0.355)	(0.286)	(0.286)	(0.091)	(0.000)	(0.003)
Durable good orders	0.004	−0.016**	0.000	0.001	0.004	0.004	0.002	0.005	0.001	0.001
p-Value	(0.124)	(0.041)	(0.805)	(0.708)	(0.357)	(0.657)	(0.512)	(0.188)	(0.825)	(0.380)
Nonfarm payroll	0.010	−0.058	0.000	0.000	0.013	0.005	−0.001	0.009	0.011	0.001
p-Value	(0.444)	(0.140)	(0.949)	(0.997)	(0.576)	(0.914)	(0.916)	(0.652)	(0.431)	(0.880)
Housing starts	0.001	−0.006	−0.001	−0.004	0.001	0.000	−0.003	0.001	0.000	−0.001
p-Value	(0.728)	(0.345)	(0.450)	(0.238)	(0.765)	(0.996)	(0.200)	(0.728)	(0.997)	(0.431)
Initial jobless claims	0.003***	−0.001	0.000	0.003	0.002	−0.008*	0.001	−0.026***	0.002**	0.000
p-Value	(0.008)	(0.770)	(0.681)	(0.317)	(0.387)	(0.055)	(0.628)	(0.000)	(0.043)	(0.701)
NAPM	0.003	−0.025**	−0.003	0.001	0.000	0.012	−0.004	0.013	−0.003	−0.004*
p-Value	(0.457)	(0.034)	(0.146)	(0.847)	(0.976)	(0.370)	(0.390)	(0.104)	(0.458)	(0.068)
Producer price index	0.002	−0.004	0.002	0.002	0.004	−0.003	0.004*	−0.004	0.001	0.003*
p-Value	(0.481)	(0.637)	(0.133)	(0.719)	(0.409)	(0.724)	(0.096)	(0.417)	(0.675)	(0.076)
Retail sales	−0.003	0.004	−0.002	0.000	−0.001	0.002	−0.002	0.003	0.000	−0.001
p-Value	(0.232)	(0.523)	(0.187)	(0.928)	(0.771)	(0.815)	(0.340)	(0.533)	(0.930)	(0.523)
Unemployment rate	−0.011	0.061	−0.001	−0.002	−0.016	−0.011	0.001	−0.003	−0.010	−0.001
p-Value	(0.397)	(0.115)	(0.932)	(0.907)	(0.495)	(0.819)	(0.925)	(0.870)	(0.456)	(0.895)
Lagged volatility	0.946***	0.579***	0.940***	0.568***	0.864***	0.393***	0.858***	0.320***	0.966***	0.962***
p-Value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
R^2 (adjusted)	0.890	0.332	0.887	0.323	0.744	0.154	0.744	0.195	0.929	0.927
F-statistic	785.7***	48.9***	761.0***	46.9***	282.1***	18.5***	281.8***	24.4***	1258.6***	1222.9***
p-Value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

The independent variables Y_t are the risk proxies $\hat{\phi}$, $\hat{\psi}_1$, $\sqrt{\hat{\lambda}}\hat{\psi}_2$, and $\sqrt{\hat{\lambda}(\hat{\mu}^2+\hat{\gamma}^2)}$. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively. Quantities in parentheses are the corresponding p-values of the coefficients. Two time periods are analyzed. The first set of indicator functions denote the day of the macroeconomic announcement itself while the second considers 2 days before and after the event. As in the previous table, the coefficient for the lagged volatility indicates that diffusion risk is far more predictable than its jump counterpart.

Table 5
Regression results from Eq. (6) involving both FOMC decisions and macroeconomic announcements

Coefficients	Jump-diffusion with stochastic jumps, JDS model				Jump-diffusion with constant jumps, JD model				Pure diffusion, PD model	
	Event date		±14/±2 Days		Event date		±14/±2 Days		Event date	±14/±2 Days
	$\hat{\psi}_1$	$\sqrt{\hat{\lambda}(\hat{\mu}^2+\hat{\gamma}^2)}$	$\hat{\psi}_1$	$\sqrt{\hat{\lambda}(\hat{\mu}^2+\hat{\gamma}^2)}$	$\hat{\psi}_1$	$\sqrt{\hat{\lambda}}\hat{\psi}_2$	$\hat{\psi}_1$	$\sqrt{\hat{\lambda}}\hat{\psi}_2$	$\hat{\phi}$	$\hat{\phi}$
Constant	0.004***	0.036***	0.009***	0.044**	0.013***	−0.016***	0.023***	0.000	0.003***	0.008***
p-Value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.952)	(0.002)	(0.000)
FOMC neutral	−0.002	−0.002	−0.002**	−0.002	−0.009*	−0.008	−0.005***	−0.005	−0.004	−0.003**
p-Value	(0.387)	(0.791)	(0.026)	(0.406)	(0.061)	(0.435)	(0.006)	(0.176)	(0.124)	(0.012)
FOMC increase	−0.002	0.020	−0.002	−0.010**	0.007	0.010	−0.004	−0.005	−0.004	−0.003*
p-Value	(0.718)	(0.274)	(0.257)	(0.026)	(0.539)	(0.624)	(0.133)	(0.406)	(0.476)	(0.096)
FOMC decrease	0.012***	0.010	0.004***	0.029***	0.004	−0.026*	0.013***	−0.015***	0.012***	0.003**
p-Value	(0.003)	(0.411)	(0.004)	(0.000)	(0.572)	(0.083)	(0.000)	(0.001)	(0.005)	(0.043)
Construction	0.007*	0.013	0.005**	−0.005	0.007	−0.014	0.004	−0.014	0.015***	0.007***
p-Value	(0.090)	(0.271)	(0.046)	(0.522)	(0.302)	(0.333)	(0.359)	(0.102)	(0.000)	(0.004)
Durable good orders	0.004*	−0.016**	0.001	0.002	0.005	0.004	0.002	0.007	0.001	0.001
p-Value	(0.091)	(0.039)	(0.646)	(0.646)	(0.328)	(0.692)	(0.327)	(0.130)	(0.678)	(0.279)
Nonfarm payroll	0.010	−0.058	−0.002	−0.003	0.014	0.005	−0.005	0.009	0.011	0.000
p-Value	(0.434)	(0.139)	(0.761)	(0.882)	(0.566)	(0.911)	(0.612)	(0.650)	(0.414)	(0.952)
Housing starts	0.001	−0.007	−0.001	−0.006*	0.001	0.000	−0.003*	0.002	0.000	−0.001
p-Value	(0.788)	(0.299)	(0.344)	(0.077)	(0.810)	(0.975)	(0.100)	(0.568)	(0.957)	(0.359)
Initial jobless claims	0.003***	−0.001	0.000	0.000	0.002	−0.007*	0.000	−0.025***	0.002*	−0.001
p-Value	(0.013)	(0.682)	(0.963)	(0.950)	(0.384)	(0.074)	(0.844)	(0.000)	(0.050)	(0.609)
NAPM	0.003	−0.026**	−0.003	0.007	0.000	0.012	−0.003	0.012	−0.003	−0.004*
p-Value	(0.433)	(0.025)	(0.161)	(0.337)	(0.973)	(0.402)	(0.483)	(0.143)	(0.530)	(0.080)
Produced price index	0.002	−0.003	0.002	0.001	0.004	−0.003	0.004*	−0.004	0.001	0.003*
p-Value	(0.430)	(0.669)	(0.154)	(0.721)	(0.367)	(0.723)	(0.087)	(0.375)	(0.608)	(0.088)
Retail sales	−0.003	0.005	−0.002	0.000	−0.001	0.002	−0.002	0.003	0.000	−0.001
p-Value	(0.255)	(0.503)	(0.213)	(0.970)	(0.801)	(0.828)	(0.331)	(0.450)	(0.974)	(0.564)
Unemployment rate	−0.011	0.061	0.001	0.000	−0.016	−0.011	0.005	−0.003	−0.010	0.001
p-Value	(0.399)	(0.115)	(0.863)	(0.992)	(0.493)	(0.813)	(0.610)	(0.899)	(0.455)	(0.911)

(*continued on next page*)

Table 5 (*continued*)

Coefficients	Jump-diffusion with stochastic jumps, JDS model				Jump-diffusion with constant jumps, JD model				Pure diffusion, PD model	
	Event date		±14/±2 Days		Event date		±14/±2 Days		Event date	±14/±2 Days
	$\hat{\psi}_1$	$\sqrt{\hat{\lambda}(\hat{\mu}^2+\hat{\gamma}^2)}$	$\hat{\psi}_1$	$\sqrt{\hat{\lambda}(\hat{\mu}^2+\hat{\gamma}^2)}$	$\hat{\psi}_1$	$\sqrt{\hat{\lambda}}\hat{\psi}_2$	$\hat{\psi}_1$	$\sqrt{\hat{\lambda}}\hat{\psi}_2$	$\hat{\phi}$	$\hat{\phi}$
Lagged volatility	0.941***	0.578***	0.902***	0.440***	0.861***	0.393***	0.773***	0.302***	0.961***	0.936***
p-Value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
R^2 (adjusted)	0.891	0.331	0.889	0.375	0.745	0.155	0.755	0.203	0.929	0.928
F-statistic	609.5***	37.7***	596.5***	45.5***	217.7***	14.6***	230.3***	19.9***	977.2***	954.1***
p-Value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

The independent variables Y_t are the risk proxies $\hat{\phi}$, $\hat{\psi}_1$, $\sqrt{\hat{\lambda}}\hat{\psi}_2$, and $\sqrt{\hat{\lambda}(\hat{\mu}^2+\hat{\gamma}^2)}$. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively. Quantities in parentheses are the corresponding *p*-values of the coefficients. Two time periods are analyzed. The first set of indicator functions denote the day of the FOMC decision or the macroeconomic announcement while the second considers 14 days/2 days before and after the FOMC decision/macroeconomic announcement respectively. The results below closely parallel the earlier results in Tables 3 and 4, indicating that the information sets are not substitutes.

In addition, a period only before and on the event date itself is examined. However, results of this ex ante analysis are nearly identical to those of the enlarged pre- and post-window. Consequently, for brevity, we only report results for the wider horizon.

Table 3 reports that the regressions yield stronger results over the 14-day window than the event date itself. The impact of federal funds rate reductions is most influential although neutral FOMC stances also impact the diffusion coefficients over the wider horizon. Interestingly, the JDS model is the only formulation whose parameters respond to FOMC increases in the fed funds rate. Overall, the adjusted R^2 statistics indicate that a significant portion of the variance in jump risk is attributable to FOMC decisions. Thus, the contribution of monetary policy to jump risk appears substantial.

The coefficients for the lagged dependent variables indicate that the diffusion parameters ($\hat{\phi}$ and $\hat{\psi}_1$) are highly autocorrelated, much more so than those of jump risk. Movements in the diffusion volatility also appear more predictable (higher R^2), indicating that time-varying jump risk is a likely source of variability in LIBOR-based option prices. As a robustness check, we include Y_{t-2} in the linear regressions. Results indicate that the coefficients remain largely the same with statistical significance preserved at similar levels. Furthermore, the two lagged coefficients approximately sum to the single one-lag value.

In general, macroeconomic announcements exert far less influence than FOMC decisions on LIBOR rates. Indeed, nonfarm payrolls, housing starts, retail sales, and the unemployment rate have no statistical effect on the implied parameters while the role of the producer price index is very faint.

To better understand the relative contribution of FOMC decisions and macroeconomic announcements to the implied risk measures, a combined regression with both their occurrences is performed. The same 14- and 2-day windows for FOMC decisions and macroeconomic announcements are examined. [10]

The results are similar to those reported in Tables 3 and 4. Specifically, FOMC decisions to reduce the fed funds rate, construction, initial jobless claims, durable goods orders, and NAPM all remain significant in at least one of the LIBOR models. The estimated coefficients are also broadly similar. For example, the coefficient for a decrease in the fed funds rate under the JDS model in Table 5 is 0.029, compared with 0.028 in Table 3.

Overall, FOMC and macroeconomic announcements contain different sets of information, with each contributing something distinct to the evolution of LIBOR rates.

To summarize, variation in implied jump risk is attributable to FOMC decisions and, in a much more limited manner, macroeconomic announcements.

[10] A 2-day pre- and post-event date window was also studied for FOMC decisions. Results for this analysis are weaker but broadly similar.

Table 6
Skewness and kurtosis conditioned on implied parameter estimates from jump-diffusions model with and without stochastic jump magnitudes

Year	Jump-diffusion with stochastic jump magnitudes, JDS model		Jump-diffusion with constant jump magnitudes, JD model	
	Skewness	Kurtosis	Skewness	Kurtosis
1996	−0.446	10.832	−0.254	4.790
1997	−0.209	11.958	−0.347	5.886
1998	−1.453	13.793	−0.880	6.409
1999	−1.050	10.125	−0.792	6.419
2000	−1.521	14.401	−0.829	7.175
2001	−0.390	5.824	−0.230	3.407
Total	−0.889	11.355	−0.576	5.744

These computations follow from Eqs. (7) and (8) which appear in Das and Sundaram (1999). For constant jump sizes with $\gamma = 0$, these formulae reduce to Eqs. (9) and (10) respectively. The results below indicate that stochastic jump magnitudes are required to capture the negative skewness and excess kurtosis in LIBOR rates.

4.3. Skewness and kurtosis

To further ascertain whether jumps with stochastic jump magnitudes are essential in modeling the LIBOR dynamics, we examine the higher-order moments implied by their parameter estimates. Das and Sundaram (1999) provide the skewness and kurtosis for the JDS process,

$$\text{Skewness} = \frac{\lambda(\mu^3 + 3\mu\gamma^2)}{(\psi_1^2 + \lambda\gamma^2 + \lambda\mu^2)^{3/2}}, \tag{7}$$

$$\text{Kurtosis} = 3 + \frac{\lambda(\mu^4 + 6\mu^2\gamma^2 + 3\gamma^4)}{(\psi_1^2 + \lambda\gamma^2 + \lambda\mu^2)^2}. \tag{8}$$

With constant jump sizes ($\gamma = 0$) under the JD model, these equations reduce to

$$\text{Skewness} = \frac{\lambda\psi_2^3}{(\psi_1^2 + \lambda\psi_2^2)^{3/2}}, \tag{9}$$

$$\text{Kurtosis} = 3 + \frac{\lambda\psi_2^4}{(\psi_1^2 + \lambda\psi_2^2)^2}. \tag{10}$$

A comparison between Eq. (7) versus (9) as well as Eq. (8) versus (10) indicates the importance of admitting stochastic jump sizes into the model's formulation. [11]

[11] Bakshi et al. (2003) provide a procedure that enables higher-order moments to be computed directly from option prices without any assumption for the underlying LIBOR model. However, this technique requires a multitude of prices at different strike prices. In our context, the data does not permit the application of this model-independent approach to calculating skewness and kurtosis. Furthermore, observe that our implied volatility estimates are inferred from market prices across several strike prices. Thus, they reflect aggregate information and cannot be paired with strike prices to yield a smile pattern.

If the stochastic jump process is over-identified, then the additional γ parameter is unnecessary, and similar values of skewness and kurtosis across the two models are expected. However, the results in Table 6 suggest that stochastic jump magnitudes are required to capture the negative skewness and excess kurtosis in LIBOR data. Hence, a JDS formulation is warranted and offers superior performance when compared with the JD model.

5. Conclusions

This paper develops jump-diffusion models for LIBOR rates and verifies their empirical validity with data from the Chicago Mercantile Exchange on Eurodollar futures and their options. Our study finds that jump risk comprises a significant portion of LIBOR-based market prices. In addition, jump-diffusion models outperform their pure diffusion counterpart in terms of smaller in-sample and out-of-sample pricing errors.

Using implied diffusion and jump parameters, we also provide empirical evidence that Federal Open Market Committee (FOMC) decisions as well as a small subset of macroeconomic announcements cause salient changes in jump risk. Moreover, statistical performance is greatly improved when jump magnitudes are allowed to be random, a necessary extension to capture the significant negative skewness and excess kurtosis in LIBOR data. Consequently, jump-diffusion models for LIBOR rates have both economic and statistical motivations.

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Appendix A. Multinomial lattice for LIBOR rates

Discretizing the JD model in Eq. (3) for a Δ time increment results in a multinomial tree with four branches. This lattice describes LIBOR rates under the forward measure.

$$\frac{L(\Delta,T)}{L(0,T)} = \begin{cases} \exp\{-\lambda\psi_2\Delta + \psi_1\sqrt{\Delta}\} & \text{with probability } \frac{1-\lambda\Delta}{2}, \\ \exp\left\{-\lambda\psi_2\Delta - \psi_1\sqrt{\Delta}\right\} & \text{with probability } \frac{1-\lambda\Delta}{2}, \\ \exp\{-\lambda\psi_2\Delta + \psi_1\sqrt{\Delta} + \psi_2\} & \text{with probability } \frac{\lambda\Delta}{2}, \\ \exp\{-\lambda\psi_2\Delta - \psi_1\sqrt{\Delta} + \psi_2\} & \text{with probability } \frac{\lambda\Delta}{2}. \end{cases}$$

Appendix B demonstrates that this formulation is not identical to a mixture of diffusions while Appendix C extends the lattice to include stochastic jump magnitudes. Appendix D then proves that these lattices yield martingale dynamics.

Appendix B. Jump-diffusion versus mixture of distributions

The material in this appendix confirms that estimation of the multinomial model in Appendix A is derived from a jump-diffusion process and not a mixture of two diffusions. Specifically, the latter has the following multinomial representation:

$$\frac{L(\Delta,T)}{L(0,T)} = \begin{cases} \exp\left\{-\psi_2'\sqrt{\Delta}+\psi_1'\sqrt{\Delta}\right\} & \text{with probability } \frac{1}{4}, \\ \exp\{-\psi_2'\sqrt{\Delta}-\psi_1'\sqrt{\Delta}\} & \text{with probability } \frac{1}{4}, \\ \exp\left\{\psi_2'\sqrt{\Delta}+\psi_1'\sqrt{\Delta}\right\} & \text{with probability } \frac{1}{4}, \\ \exp\left\{\psi_2'\sqrt{\Delta}-\psi_1'\sqrt{\Delta}\right\} & \text{with probability } \frac{1}{4}, \end{cases}$$

where ψ_1' and ψ_2' denote two distinct diffusion coefficients. Observe that the above has no λ term corresponding to the jump intensity. In particular, to reproduce the equally-weighted probabilities associated with each of the four movements above, the constraint $\lambda\Delta = 1/2$ would be required. Returning to the branches of the jump-diffusion multinomial tree in Appendix A, and imposing the constraint $\lambda\Delta = 1/2$, we have the following equations:

$$-\lambda\psi_2\Delta + \psi_1\sqrt{\Delta} = -\frac{\psi_2}{2} + \psi_1\sqrt{\Delta}, \tag{11}$$

$$-\lambda\psi_2\Delta - \psi_1\sqrt{\Delta} = -\frac{\psi_2}{2} - \psi_1\sqrt{\Delta}, \tag{12}$$

$$-\lambda\psi_2\Delta + \psi_1\sqrt{\Delta} + \psi_2 = -\frac{\psi_2}{2} + \psi_1\sqrt{\Delta} + \psi_2 = \frac{\psi_2}{2} + \psi_1\sqrt{\Delta}, \tag{13}$$

$$-\lambda\psi_2\Delta - \psi_1\sqrt{\Delta} + \psi_2 = -\frac{\psi_2}{2} - \psi_1\sqrt{\Delta} + \psi_2 = \frac{\psi_2}{2} - \psi_1\sqrt{\Delta}. \tag{14}$$

By setting $\psi_1 \equiv \psi_1'$ and $\frac{\psi_2}{2} \equiv \psi_2'\sqrt{\Delta}$, we obtain equivalence with the mixture of diffusions. However, for a jump size $\psi_2 = 2\psi_2'\sqrt{\Delta}$, the diffusion coefficient is required to be very large. In addition, the constraint that $\lambda\Delta = 1/2$ is not satisfied by our empirical results.

Appendix C. Stochastic jump sizes

Following Das (1999), a hexanomial model with six branches is implemented to allow for random jump sizes in the JDS formulation of Eq. (4). This formulation requires an additional parameter as the jump magnitude ψ_2^S is distributed with mean μ

and variance γ^2. In particular, ψ_2^S has a binomial distribution with jump magnitudes of $\mu+\gamma$ and $\mu-\gamma$ being equally-likely, implying $p = 1/2$.

$$\frac{L(\varDelta,T)}{L(0,T)} = \begin{cases} \exp\left\{-\lambda\mu\varDelta + \psi_1\sqrt{\varDelta}\right\} & \text{with probability } \frac{1-\lambda\varDelta}{2}, \\ \exp\left\{-\lambda\mu\varDelta - \psi_1\sqrt{\varDelta}\right\} & \text{with probability } \frac{1-\lambda\varDelta}{2}, \\ \exp\left\{-\lambda\mu\varDelta + \psi_1\sqrt{\varDelta} + \mu + \gamma\right\} & \text{with probability } \frac{\lambda\varDelta}{4}, \\ \exp\left\{-\lambda\mu\varDelta - \psi_1\sqrt{\varDelta} + \mu + \gamma\right\} & \text{with probability } \frac{\lambda\varDelta}{4}, \\ \exp\left\{-\lambda\mu\varDelta + \psi_1\sqrt{\varDelta} + \mu - \gamma\right\} & \text{with probability } \frac{\lambda\varDelta}{4}, \\ \exp\left\{-\lambda\mu\varDelta - \psi_1\sqrt{\varDelta} + \mu - \gamma\right\} & \text{with probability } \frac{\lambda\varDelta}{4}. \end{cases}$$

With $\gamma = 0$, this hexanomial tree reduces to the four branch quadrinomial representation described in Appendix A with the notation $\psi_2 = \mu$.

Appendix D. Martingale property

For completeness, we demonstrate that the JDS LIBOR dynamics in Appendix C are martingales under the forward measure. This result implies that JD LIBOR rates with $\gamma = 0$ (constant jump magnitudes) also evolve as martingales under the forward measure.

Let $\varDelta L(t,T)$ be the change in LIBOR over a time interval $\varDelta$. Using the hexanomial tree in Appendix C, the discrete representation of $\varDelta L(t,T)/L(t_-,T)$ is given by

$$\frac{\varDelta L(t,T)}{L(t_-,T)} = \varDelta \begin{cases} -\lambda\mu\varDelta + \psi_1\sqrt{\varDelta} & \text{with probability } \frac{1-\lambda\varDelta}{2}, \\ -\lambda\mu\varDelta - \psi_1\sqrt{\varDelta} & \text{with probability } \frac{1-\lambda\varDelta}{2}, \\ -\lambda\mu\varDelta + \psi_1\sqrt{\varDelta} + \mu + \gamma & \text{with probability } \frac{\lambda\varDelta}{4}, \\ -\lambda\mu\varDelta - \psi_1\sqrt{\varDelta} + \mu + \gamma & \text{with probability } \frac{\lambda\varDelta}{4}, \\ -\lambda\mu\varDelta + \psi_1\sqrt{\varDelta} + \mu - \gamma & \text{with probability } \frac{\lambda\varDelta}{4}, \\ -\lambda\mu\varDelta - \psi_1\sqrt{\varDelta} + \mu - \gamma & \text{with probability } \frac{\lambda\varDelta}{4}. \end{cases}$$

Showing the expectation of the above rate of change is zero proceeds according to the following steps. First, we note that the probabilities sum to one. The common $-\lambda\mu\varDelta$ terms in each of the exponents are aggregated across the six branches to produce $-\lambda\mu\varDelta$. Second, since the first two branches have identical probabilities, as well as the last four branches, the expected value of the diffusion term is zero:

$$\frac{1-\lambda\varDelta}{2}[\psi_1\sqrt{\varDelta} - \psi_1\sqrt{\varDelta}] + \frac{\lambda\varDelta}{4}[\psi_1\sqrt{\varDelta} - \psi_1\sqrt{\varDelta} + \psi_1\sqrt{\varDelta} - \psi_1\sqrt{\varDelta}] = 0. \tag{15}$$

Third, a similar procedure with γ for the bottom four branches also yields zero, whereas the contribution from the expected jump size μ is

$$\mu\left[\frac{\lambda\varDelta}{4} + \frac{\lambda\varDelta}{4} + \frac{\lambda\varDelta}{4} + \frac{\lambda\varDelta}{4}\right] = \mu\lambda\varDelta. \tag{16}$$

Therefore, the sum of all the non-zero expected values is $-\lambda\mu\Delta + \lambda\mu\Delta = 0$ and the proof is complete. To summarize, LIBOR dynamics implemented according to the lattices in Appendices A and C are martingales.

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