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# Sporadic Mass Ejection in Red Supergiants

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## SPORADIC MASS EJECTION IN RED SUPERGIANTS

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### ABSTRACT

We have applied a general mechanism first proposed by J. A. Burke to red supergiants for determining the spin-down rate and angular momentum loss of rotating stars. This model relies principally on sporadic mass ejection, which is assumed to be the result of turbulent elements accelerating material in cool supergiant atmospheres. Mass is preferentially expelled in the forward direction of rotation, resulting in a rapid loss of angular momentum on time scales of  $10^4$ – $10^6$  years in the supergiant evolutionary phase. Such rotational breaking will occur if the turbulent elements have characteristic sizes a few percent of the stellar radius, and rms velocities one-third the escape speed of the star. This model predicts the formation of a cool silicate disk or torus around the star because of the preferred expulsion of material near equatorial regions of the supergiant.

*Subject headings:* stars: circumstellar shells — stars: mass loss — stars: rotation

### I. INTRODUCTION

Sharp-line observations in late-type giants and supergiants are probably the best indication that evolved stars are very likely slow rotators. Typical equatorial rotational velocities for type I K and M supergiants are generally believed to be less than  $10 \text{ km s}^{-1}$  (Allen 1973). In contrast to O and B supergiants that have equatorial velocities that range  $100$ – $300 \text{ km s}^{-1}$  (Rosenhdal 1970), post-main-sequence stars in general appear to have reduced their rotational speeds considerably. This in part is due to increased radius, and possibly to the dissipation of angular momentum through stellar winds and magnetic field interaction during main-sequence evolution that occurs on time scales of  $10^{10}$  yr (Weber and Davis 1967).

In this paper we apply recent observations to a mechanism first suggested by Burke (1969, 1972), which involves the sporadic ejection of material from rotating stars, and the effect which mass expulsion has on removing angular momentum. In view of the large body of information that has recently been assembled concerning mass loss in K and M giants and supergiants, and spectral observations of macroscopic mass motions in late type stellar atmospheres, it would be of interest here to examine the suitability of this mechanism as it relates to the rotational properties of late type M stars, and large-scale velocity motions that are believed present in cool giant atmospheres.

### II. ANGULAR MOMENTUM LOSS AND STELLAR ROTATION

Following Burke (1969, 1972), we assume that material is expelled isotropically at the stellar surface

and is described by a Maxwellian velocity distribution with rms velocity  $v_{\text{rms}}$ . We adopt  $v_{\text{rms}}$  as the mean turbulent velocity of elements present at the stellar surface. Furthermore, assuming that the star is rotating, elements or globules moving in the direction of rotation are preferably ejected from the atmosphere, since they more easily attain velocities comparable to the escape speed of the star than do elements moving in the opposite direction. Accordingly, mass expulsion occurs preferentially in an equatorial plane in the forward direction of rotation.

Intuitively, one would expect that if such a mechanism were operative, it would apply only to rapid rotators such as O and B supergiants. Late type red supergiants would seem unlikely candidates owing to their suspected rather low rotational speeds. However, if elements of material in the atmosphere attain rms speeds  $\sim 0.3 v_{\text{esc}}$ , then globules in the high-velocity tail of the Maxwellian distribution will have speeds exceeding the escape velocity of supergiants. Even at the low rotational speeds, less than  $10 \text{ km s}^{-1}$ , encountered in red supergiants, and even if the escaping material is only a small fraction of the total mass of the atmosphere involved in the sporadic activity, the fact that mass loss occurs principally in the direction of rotation could significantly affect stellar rotation. This is due to the exponential nature of the velocity distribution.

From Burke (1969, 1972) we can write the mass loss rate as

$$\dot{M} = (8/3)I_m s \bar{p} v_{\text{esc}} R, \quad (1)$$

where  $I_m$  is a parameter that is a function of  $u_r = v_{\text{rot}}/v_{\text{rms}}$  and  $u_e = v_{\text{esc}}/v_{\text{rms}}$ , for which the values corre-

sponding to the rotational and escape speeds of a rotating star can be obtained from Burke (1969). In equation (1),  $s$  represents a linear size of the sporadic ejection elements,  $\bar{\rho}$  is the average density of the medium in which the elements are emitted, and  $R$  is the stellar radius.

For a working model, we postulate the existence of turbulent elements of scale size  $s \approx 5\%$  of the stellar radius  $R$ ; this corresponds to approximately 1600 elements. If the characteristic velocity  $v_{rms} \sim 30 \text{ km s}^{-1}$ , consistent with late type supergiant spectra, then approximately 3% of the elements will have velocities comparable to the  $\sim 100 \text{ km s}^{-1}$  escape speed of supergiants. An additional  $5 \text{ km s}^{-1}$  imposed on the motion of the elements by rotation will significantly increase the number of globules having velocities exceeding the escape speed, owing to the exponential nature of the Maxwellian velocity distribution. Accordingly, material moving in the direction of rotation is preferentially ejected. It should be noted that Burke (1972) requires the minimum free path of the ejected elements to be comparable to the radius of the star for substantial angular momentum loss to occur.

Using observational values of  $\dot{M}$  from infrared measurements of Gehrz and Woolf (1971) and circumstellar line observations of Sanner (1976), adopting a mean atmospheric density  $\bar{\rho} \approx 10^{-10} \text{ g cm}^{-3}$  appropriate for cool giant atmospheres, and using values of radii and masses for red supergiants (Allen 1973), we obtain  $I_m$  from equation (1).

One can estimate the maximum theoretical values for  $v_{rot}$  by assuming that the original angular momentum on the main sequence has been conserved, at least to a first order approximation, during the evolutionary course of the star off the main sequence. Mean rotational velocities on the main sequence for 20  $M_{\odot}$  O and B supergiants are of the order of  $200 \text{ km s}^{-1}$ , with

a maximum upper limit of  $400 \text{ km s}^{-1}$ . Conserving angular momentum as the star enters the red supergiant region, such stars would have surface rotational velocities of  $\lesssim 5 \text{ km s}^{-1}$ . This agrees with the observed upper limit of  $10 \text{ km s}^{-1}$  (Allen 1973).

Since  $I_m$  is a function of  $u_r = v_{rot}/v_{rms}$  and  $u_e = v_{esc}/v_{rms}$ , knowing its value from equation (1), and knowing  $v_{esc}$  and  $v_{rot}$ , enables us to find the  $v_{rms}$  necessary for such sporadic loss to occur.

In Table 1, supergiant observations by Gehrz and Woolf (1971) and Sanner (1976) are given, together with the respective masses and radii (Allen 1973), effective temperatures (Lang 1975) and computed luminosities. The escape speeds and Kelvin time scales  $\tau_K$  are also given for reference.

In Table 2, we show the main-sequence rotational speeds (mean and maximum), and the corresponding rotational velocities in the red supergiant region that these particular stars would have assuming conservation of angular momentum during post-main-sequence evolution. The deduced turbulent velocities  $v_{rms}$  are given. Decreasing the cell size by an order of magnitude, and correspondingly increasing the number of elements, tends to result in velocities that are large compared to the observed turbulent velocities of supergiants. From the computed values of  $u_e$ ,  $u_r$  one can estimate the fractional mass  $\Delta M/M$  lost during a characteristic spin-down time  $\tau$  of the supergiant (Burke 1972, eq. [7]). Writing the time  $\tau$  in the form

$$\tau = (\Delta M/M)(M/\dot{M}) \tag{2}$$

and using the observed value for  $M/\dot{M}$ , we may obtain  $\tau$ . From Burke (eq. [6]) we know the ratio  $\tau f/\tau_K$ , and hence, using our calculated  $\tau_K$ , we obtain  $f$ , the fraction of the stellar luminosity that drives the ejection process. These quantities are displayed in Table 2. Whether one uses the mean or the upper

TABLE 1  
SUPERGIANT OBSERVATIONS

| Star              | Type       | $\dot{M}(\times 10^{-6} M_{\odot}\text{yr}^{-1})^*$ | $M_*(M_{\odot})\dagger$ | $R_*(R_{\odot})\dagger$ | $T_{\text{eff}}(\text{K})\ddagger$ | $L_*(L_{\odot})\S$ | $v_{\text{esc}}(\text{km s}^{-1})$ | $\tau_K(\text{yr})$ |
|-------------------|------------|---|-------------------------|-------------------------|------------------------------------|--------------------|------------------------------------|---------------------|
| $\mu$ Cep.....    | M2 Ia      | 10 GW<br>1 S  | 20                      | 800                     | 3500                               | $8.6 \times 10^4$  | 100                                | 180                 |
| RW Cep.....       | K5 Ia-0    | 5 GW  | 16                      | 400                     | 4410                               | $5.4 \times 10^4$  | 125                                | 366                 |
| RW Cyg.....       | M2 Ia,b    | 5 GW  | 20                      | 800                     | 3500                               | $8.6 \times 10^4$  | 100                                | 180                 |
| TV Gem.....       | M1 Ia,b    | 1.2 GW  | 18                      | 630                     | 3680                               | $6.5 \times 10^4$  | 100                                | 245                 |
| $\alpha$ Ori..... | M2 Ia,b    | 0.7 GW<br>0.17 GW                                   | 20                      | 800                     | 3500                               | $8.6 \times 10^4$  | 100                                | 180                 |
| BC Cyg.....       | M3.5 Ia    | 7 GW  | 22                      | 900                     | 3295                               | $8.6 \times 10^4$  | 100                                | 195                 |
| BI Cyg.....       | M3 Ia,b    | 7 GW  | 22                      | 900                     | 3360                               | $9.3 \times 10^4$  | 100                                | 180                 |
| S Per.....        | M4 Ia      | 27 GW   | 25                      | 1000                    | 3230                               | $9.7 \times 10^4$  | 100                                | 199                 |
| UY Sct.....       | M4 Ia,b    | 6 GW  | 25                      | 1000                    | 3230                               | $9.8 \times 10^4$  | 100                                | 199                 |
| 119 Tau.....      | M2 Ib      | 0.24 GW   | 20                      | 800                     | 3500                               | $8.6 \times 10^4$  | 100                                | 180                 |
| 6 Gem.....        | M1 Ia      | 2.1 S   | 18                      | 630                     | 3680                               | $6.5 \times 10^4$  | 100                                | 245                 |
| $\Psi^1$ Aur..... | K5-M0 Ia,b | 0.48 S  | 16                      | 450                     | 4160                               | $5.4 \times 10^4$  | 115                                | 325                 |
| $\alpha$ Sco..... | M1 Ia,b    | 0.1 S   | 18                      | 630                     | 3680                               | $6.5 \times 10^4$  | 100                                | 245                 |
| VV Cep.....       | M2 Ia,b    | 0.13 GW   | 20                      | 800                     | 3500                               | $8.6 \times 10^4$  | 100                                | 180                 |

\* GW = Gehrz and Woolf 1971. S = Sanner 1976.

†  $M_*$  and  $R_*$  from Allen 1973.

‡  $T_{\text{eff}}$  from Lang 1974.

§  $L_*$  computed from  $T_{\text{eff}}$  and  $R_*$ .

TABLE 2  
ROTATIONAL SPEEDS

| Star            | Main Sequence<br>$v_{\text{rot}}(\text{km s}^{-1})$<br>(average-max) | Red Giant<br>$v_{\text{rot}}(\text{km s}^{-1})$<br>(average-max) | $v_{\text{rms}}(\text{km s}^{-1})$ | $\Delta M/M$         | $\tau f/\tau_K$ | $\tau(\text{yr})$ | $f$                  |
|-----------------|--|--|------------------------------------|----------------------|-----------------|-------------------|----------------------|
| $\mu$ Cep GW*   | 200-420  | 2.0-4.2  | 32.5                               | $7.0 \times 10^{-3}$ | 1.5             | $1.4 \times 10^4$ | $1.9 \times 10^{-2}$ |
| $\mu$ Cep S     | 200-420  | 2.0-4.2  | 28.9                               | $5.2 \times 10^{-3}$ | 9.0             | $1.0 \times 10^5$ | $1.6 \times 10^{-2}$ |
| RW Cep GW       | 220-410  | 3.5-6.5  | 41.9                               | $6.8 \times 10^{-3}$ | 1.0             | $2.2 \times 10^4$ | $1.7 \times 10^{-2}$ |
| RW Gem GW       | 200-420  | 2.0-4.2  | 31.2                               | $6.2 \times 10^{-3}$ | 2.6             | $2.5 \times 10^4$ | $1.9 \times 10^{-2}$ |
| TV Gem GW       | 200-420  | 2.5-5.0  | 29.8                               | $8.9 \times 10^{-3}$ | 8.5             | $1.3 \times 10^5$ | $1.5 \times 10^{-2}$ |
| $\alpha$ Ori GW | 200-420  | 2.0-4.2  | 28.6                               | $7.7 \times 10^{-3}$ | 18.5            | $2.2 \times 10^5$ | $1.5 \times 10^{-2}$ |
| $\alpha$ Ori S  | 200-420  | 2.0-4.2  | 27.0                               | $6.5 \times 10^{-3}$ | 57.2            | $7.6 \times 10^5$ | $1.3 \times 10^{-2}$ |
| BC Cyg GW       | 190-413  | 2.0-4.3  | 31.4                               | $9.9 \times 10^{-3}$ | 3.8             | $3.1 \times 10^4$ | $2.4 \times 10^{-2}$ |
| BI Cyg GW       | 190-410  | 2.0-4.3  | 31.4                               | $9.9 \times 10^{-3}$ | 3.8             | $3.1 \times 10^4$ | $2.2 \times 10^{-2}$ |
| S Per GW        | 190-410  | 2.0-4.3  | 33.9                               | $1.0 \times 10^{-2}$ | 1.4             | $9.2 \times 10^3$ | $3.0 \times 10^{-2}$ |
| UY Sct GW       | 190-410  | 2.0-4.3  | 31.0                               | $8.6 \times 10^{-3}$ | 4.5             | $3.6 \times 10^4$ | $2.5 \times 10^{-2}$ |
| 119 Tau S       | 200-420  | 2.0-4.2  | 27.2                               | $8.1 \times 10^{-3}$ | 51.2            | $6.8 \times 10^5$ | $1.4 \times 10^{-2}$ |
| 6 Gem S         | 200-420  | 2.5-5.0  | 30.8                               | $8.9 \times 10^{-3}$ | 5.1             | $7.6 \times 10^4$ | $1.6 \times 10^{-2}$ |
| $\Psi^1$ Aur S  | 220-410  | 3.0-5.5  | 33.8                               | $5.3 \times 10^{-3}$ | 7.2             | $1.8 \times 10^5$ | $1.3 \times 10^{-2}$ |
| $\alpha$ Sco S  | 200-420  | 2.5-5.0  | 26.8                               | $4.7 \times 10^{-3}$ | 43.1            | $8.5 \times 10^5$ | $1.6 \times 10^{-2}$ |
| VV Cep S        | 200-420  | 2.0-4.2  | 26.7                               | $4.4 \times 10^{-3}$ | 48.8            | $6.8 \times 10^5$ | $1.3 \times 10^{-2}$ |

\* GW = Gehrz and Woolf 1971. S = Sanner 1976.

value for the supergiant rotation does not greatly affect these results for  $\Delta M/M$ ,  $\tau f/\tau_K$ ,  $\tau$ , and  $f$ . These four columns in Table 2, therefore, represent an average.

### III. RESULTS AND DISCUSSION

The turbulent velocities we find necessary to drive the observed rate of mass loss are in the range  $v_{\text{rms}} = 27\text{--}34 \text{ km s}^{-1}$ . These values are consistent with the observed values of  $\sim 30 \text{ km s}^{-1}$ , and hence yield a workable model. The number of cells in the model is about 1600. On average,  $4 \times 10^{-3}$  to  $10^{-2}$  of the original stellar mass is lost during a time scale  $\tau$ , where  $\tau$  is in the range  $10^4\text{--}9 \times 10^5$  years. During this phase, the star loses a considerable fraction of its angular momentum, this being the result of sporadic emission or bursts of material possibly associated with propagating shock waves. About 1-3% of the total stellar luminosity is generally required to drive the mass loss. It is further found that a smaller turbulent scale size  $s$  does not greatly affect the time scale  $\tau$ . The turbulent velocity  $v_{\text{rms}}$  increases slightly, but the fraction of energy required to drive the mass loss  $f$  decreases significantly. Similar sensitivities result in the opposite sense when the scale size  $s$  is increased slightly. During a time scale  $\tau$ , the number of ejected elements is  $\mathfrak{N} = (\Delta M/M)/(\mu/M)$ , where  $\mu$  is the mean mass of the elements. Therefore, one needs an element to be ejected on average every  $\tau/\mathfrak{N}$  years. This time scale is rather small ( $\sim 10$  days) for relatively large mass loss rates, i.e.,  $\sim 10^{-5} M_{\odot} \text{ yr}^{-1}$ . Note that while an element needs to be ejected every 10 days, there will be many more elements moving with a velocity less than the escape speed. Since  $\tau/\mathfrak{N} \propto s^3$ , then for a particular value of  $\dot{M}$ , a smaller scale size  $s$  yields a smaller interval between bursts. However, the observed mass loss rates are more likely not constant. We would expect a correlation between  $\dot{M}$  and  $v_{\text{rms}}$  or  $s$ . It would be important to have observations that simultaneously

measure  $\dot{M}$ , cell size, and turbulent velocities. A direct correlation among these three quantities would support the model developed here.

An increasingly large body of observational evidence now supports the theory that mass expulsion in late type stars is driven by large-scale shock waves in extended cool atmospheres. Willson (1976) has suggested that strong shocks in M giants that have velocities in the range  $v = 50 \text{ km s}^{-1}$  explain the observed luminosity variations in long-period variables. These shocks drive stellar winds from the immediate vicinity of the star to distances at which the continuous cooling of material allows the formation of grains and silicates, and radiation pressure can further expand the circumstellar shell (Salpeter 1974). Chiu *et al.* (1977), using high-resolution Ca II K line spectrophotometry of  $\alpha$  Boo (K2 IIIb), find a variable mass loss rate of  $10^{-9}$  to  $8 \times 10^{-9} M_{\odot} \text{ yr}^{-1}$ , which varies on a time scale comparable to the typical luminosity variations observed in K and M supergiants. The expansion velocity deduced from these observations suggests  $v_{\text{rms}} = 13 \text{ km s}^{-1}$ , which would almost definitely constitute supersonic motion in an extended cool atmosphere.

Additionally, Brooke, Lambert, and Barnes (1974) have found from interferometric spectra of  $\alpha$  Ori (M2 Ia,b) that large-scale motions in the stellar atmosphere are evident. They attribute their results to sporadic bursts of material with estimated mass loss rates as high as  $4 \times 10^{-3} M_{\odot} \text{ yr}^{-1}$ , consistent with the model here. The notion that strong atmospheric bursts of material are responsible for the observed mass loss rates in M giants and also determine the temperature and ionization structure of the cool giant atmosphere can also be inferred from spectroscopic data of long-period variables and eclipsing systems such as  $\xi$  Aur.

The star R Aql, for example, is a Mira variable that ranges from M5 to M8 with a visual period of 293 days

(Kukarkin *et al.* 1969). Using the bolometric data obtained from Flower (1977), we estimate the mean radius at maximum and at minimum spectral phase, for which we have, at M5,  $R = 1.9 \times 10^2 R_{\odot}$ ,  $T_e = 3220$  K, and at M8,  $R = 4.8 \times 10^2 R_{\odot}$ ,  $T_e = 2480$  K. The difference between the radii found at extreme spectral phases M5 and M8, divided by half the pulsation period  $\Pi$ , should yield an average radial expansion velocity  $\bar{v}_{\text{rad}} \approx \Delta R / (\Pi/2)$ . For R Aql,  $\bar{v}_{\text{rad}} \approx 80 \text{ km s}^{-1}$ , that is, a velocity well in excess of the mean sound speed of the envelope, and corresponds to a propagating shock wave Mach  $\sim 8$ . Spectral phase observations of M giants by Lockwood and Wing (1971) yield velocities that are typically  $\bar{v}_{\text{rad}} = 50\text{--}100 \text{ km s}^{-1}$ . Calculations using bolometric magnitudes for the mean spectral classification of late type stars from Tsuji (1978) produce similar results. Accordingly, the spectral phase variation of long-period variables during a pulsation cycle appears in conflict with models that assume the star undergoes a substantial radial redistribution of mass, since actual material speeds would be far in excess of the sound speed of the medium, and unreasonable on physical grounds. Opacity and temperature changes in the stellar atmosphere could account for the observed variations in luminosity (Wallerstein 1977).

We emphasize that the time scale between bursts computed here, 10 days  $\lesssim \tau/\mathfrak{M} \lesssim 6$  years, is consistent with the observed time scale of irregular variations associated with late type supergiants such as  $\alpha$  Ori.

The rather small time scale for spin-down of  $\tau = 10^4\text{--}9 \times 10^5$  years suggests that this phase is transitory. We therefore might expect that, if this mechanism is operative, approximately one-half of the angular momentum is rapidly dissipated from the star and leads to a further decrease in rotational speed. This would be consistent with the results of Kraft (1967*a, b*), who finds that late type stars lose an appreciable fraction of their angular momentum in a relatively short period of time in the late giant phase. However, more recent work concerning the redistribution of angular momentum in late type giants by Sofia and

Endal (1979) indicates that rapid loss of angular momentum does not occur if more current evolutionary models are considered. This important point remains open to further investigation.

The observational detection of large cells comparable to the stellar radius in cool supergiants could be a manifestation of this mechanism, the general size being indicative of the rotation rate. It should also be noted that the size of the globules obtained from this model may require incorporation of spherical geometry effects in a more detailed treatment. A more convincing observation would be the apparent shape which a cloud of dust or ejected grains might assume around a star. The appearance of a disklike or torus feature in the infrared would suggest an equatorial ejection of material in the direction of rotation, consistent with this model. Very large array (VLA) observations would be very useful for searching for this effect. A possible confirmation of this model could be indicated by the results of Van Blerkom (1978), who finds that profiles of SiO and H<sub>2</sub>O maser lines in cool supergiants suggest a ring system as the best interpretation for fitting the observed line profiles.

#### IV. SUMMARY

We might summarize the general properties of this model as follows:

1. Sporadic activity induces mass loss in supergiants.
2. Sporadic mass loss and rotation also lead to angular momentum loss (without incorporating magnetic fields).
3. As a result of points 1 and 2 above, a disk or torus composed of silicate grains is formed preferentially at equatorial regions of the star. VLA observations might prove useful in discerning such a feature.

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