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C. D. Aliprantis
Purdue University

Gabriele Camera
Chapman University, camera@chapman.edu

D. Puzzello
University of Kentucky

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Matching and Anonymity*

C. D. Aliprantis, G. Camera, and D. Puzzello

Department of Economics, Purdue University, West Lafayette, IN 47907–2056, USA

ABSTRACT: This work introduces a rigorous set-theoretic foundation of deterministic bilateral matching processes and studies systematically their properties. In particular, it formalizes a link between matching and informational constraints by developing a notion of anonymity that is based on the agents' matching histories. It also explains why and how various matching processes generate different degrees of "informational isolation" in the economy. We illustrate the usefulness of our approach to modeling matching frameworks by discussing the classical turnpike model of Townsend.

Keywords and Phrases: Bilateral matching, frictions, anonymous trading, spatial interactions

JEL Classification Numbers: C78, E00

1 Introduction

A large segment of economics is concerned with the study of allocations when markets are not functioning well. Market frictions are often seen as involving scarcity of information or geographical separation or inadequate institutions, and have been modeled in a variety of ways. A well-established research program has made these frictions explicit motivating their presence by modeling trade as occurring in small groups, often pairwise matches. The central assumption is that some *technology* exists that exogenously selects agents from the population and matches them together.[1]

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[1] We interpret a technology broadly as a process that transforms or rearranges some "input" (the population) into some "output" (spatially separated matches). Our use of the word technology is similar in spirit to that of other scholars, for example see the "market technology" of Hahn [14] , the "communication technology" of Townsend [26], or the "enforcement technology" of Kocherlakota [19].

Matching frameworks have been used to answer basic questions in a variety of settings. For instance, how market frictions affect equilibrium output and unemployment, as in [8], the cyclical behavior of job creation and destruction, as in [22], or the relative value of currencies in an international setting, as in [6]. A limitation of these frameworks is that the matching technology is superficially formalized and mostly descriptive. One is often confronted with hazy explanations as to how (and to what extent) the assumed frictions are an implication of the mechanism by which agents interact with each other. In short, a unifying theoretical structure is missing. This prevents a clear understanding of the exact connection between the constraints imposed by the meeting technology, the types of obstacles faced by market participants, the trades they can execute or the information they can access, and the possible allocations. A more structured formalization of these links can improve the formulation of models whose central trait is markets with impaired functioning. Indeed, a comprehensive theory of exchange should clarify how the constraints assumed to be in place originate from the underlying physical environment.

In this paper, we take a step toward advancing the theoretical foundations of matching frameworks by considering physical environments with technologies that *exogenously* pair agents deterministically. The major contributions are the development of an *explicit* set-theoretic representation of bilateral matching technologies and the formalization of their method of operation. We describe different matching processes and explain how they can facilitate (or obstruct) the interactions among agents. Especially, we focus on the informational aspects since matching frameworks are used to motivate the existence of spatial separation as well as more general obstacles to information flows.[2] Indeed, we introduce a map between properties of the matching process and the degrees of informational openness (that is, the degrees of anonymity) that are consistent with the physical description of the environment.

Our work complements two strands of literature that concern matching environments. One strand includes a growing research on network games and network formation (for a survey see [16]), a line of work dealing with endogenous matching and its allocative consequences, as in [12] and [23] to cite a few, and recent efforts on endogenizing matching frictions and "matching functions" in models with spatially separated agents, as in [21]. A second strand of literature comprises research directed at building solid mathematical foundations for random matching models; examples include studies on random meetings between agents drawn from countable or uncountable populations, as in [3], [11], or [2], and work on the exact law of large numbers for random pairwise matching, [9] and [10].

This paper—that focuses on exogenous matchings and abstracts from their allocative implications—is more closely related to this second strand of literature. However, our approach differs from the foundations of matching studies cited above in that we remove

[2] Examples include studies of the role of money in differential information economies, as in [15], the sustainability of credit when agents are matched repeatedly, as in [7], and cooperation when agents change their partners over time, as in [17].

all stochastic elements. We can thus offer a basic conceptual framework that complements both strands of literature not only by contributing to developing a common language and basic notions for the mechanics of matching, but also by allowing us to explore some links between matching dynamics and possible information flows.

The layout of the paper is as follows. In Section 2 we familiarize the reader with our notation. Then, in Section 3 we describe the technical procedure that we use to pair agents in any population, during a single period. To do so we define a notion of matching technology—which we call a *bilateral matching rule*—and then present a theorem that establishes the structure of matching on any population. Subsequently, in Section 4, we discuss matching over time—as a sequence of matching rules—and then characterize matching processes according to the levels of informational isolation they impose on the economy. To do so, we develop a taxonomy of anonymity that is based on the agents' matching histories. Then, in Section 5, we demonstrate how to construct economies where the matching process provides each agent with an infinite sequence of deterministic pairings, while imposing an extreme degree of informational isolation. Finally, in Section 6 we offer some concluding remarks.

2 Preliminaries

If A is any set, then the symbol $|A|$ denotes the cardinality of A. As usual, $|A| = \aleph_0$ means that A is a countable set and $|A| = \mathfrak{c}$ indicates that the cardinality of A is the continuum. If a set A is a union of a pairwise disjoint family of sets $\{A_i\}_{i \in I}$, i.e., $A = \bigcup_{i \in I} A_i$ and $A_i \cap A_j = \emptyset$ if $i \neq j$, then we denote this by the symbol $A = \bigsqcup_{i \in I} A_i$. Throughout the paper we denote by X a non-empty set representing the population in the economy. We assume these agents are infinitely lived and time is discrete.

3 Bilateral Matching Rules

In this section, we study how to pair agents in a period. Naturally, the first step we must take is to formalize a general notion of a pairwise matching *technology.* To do so we introduce the concept of bilateral matching rule.

Definition 1. *A **bilateral matching rule** for the population X is a function $\phi\colon X \to X$ satisfying $\phi^2(x) = x$ for all $x \in X$, i.e., $\phi^2 = I$, the identity mapping on X.*

According to the above definition, if $\phi\colon X \to X$ is a bilateral matching rule, then the function ϕ is invertible—and so ϕ is a permutation of X since ϕ is a surjective function that is also one-to-one. However, ϕ belongs to the special class of permutations whose inverses coincide with themselves, i.e., $\phi^{-1} = \phi$; these functions are known in mathematics as "involutions." This simply says that any way of pairing agents in the population must be such that the partner of an agent's partner is the agent himself.

Thus, if ϕ is a matching rule and agent x is matched to agent $\phi(x)$, then we call $\phi(x)$ the **partner** of x. Symmetrically, $x = \phi(\phi(x))$ is the partner of $\phi(x)$ so that we can call the set $\{x, \phi(x)\}$ a **match**. For concreteness, we think of matches as distinct pairs of agents that are spatially separated.

Here are two simple examples of matching rules.

a. Let $X = \mathbb{N} = \{1, 2, \ldots\}$, the set of natural numbers, and define $\phi\colon X \to X$ by

$$\phi(2a) = 2a - 1 \quad \text{and} \quad \phi(2a-1) = 2a\,.$$

b. Let $X = (0, \infty)$ and define $\phi\colon X \to X$ by $\phi(x) = \frac{1}{x}$.

Of course, a trivial bilateral matching rule is the identity mapping of X, that is, $\phi(x) = x$ for all $x \in X$, a case in which agents are basically unmatched (they are matched to themselves). For this reason we provide the following definition of a technology that pairs every agent to some other agent.

Definition 2. *A bilateral matching rule $\phi\colon X \to X$ is said to be **exhaustive** if $\phi(x) \neq x$ holds for all agents $x \in X$, i.e., whenever no agent is matched under ϕ to herself.*

Thus, we see that the matching rule in example (a) above is exhaustive, but it is not in example (b), because we have the fixed point $x = 1$. Having introduced a notion of matching technology, we now wish to establish not only the feasibility of bilateral matching on any population X, but also the structure of matching. This is done in the following theorem.

Theorem 3. *If $\phi\colon X \to X$ is a bilateral matching rule, then there exist three pairwise disjoint subsets A, B, and C of X such that:*

1. *$X = A \sqcup B \sqcup C$.*
2. *$\phi(a) = a$ for each $a \in A$.*
3. *$\phi(B) = C$ (or, equivalently, $\phi(C) = B$).*

Proof. Let $\phi\colon X \to X$ be a bilateral matching rule. Assume first that ϕ is exhaustive. So, in this case $A = \emptyset$. We shall establish the existence of the sets B and C using Zorn's lemma (a rigorous discussion of Zorn's Lemma is in [13]).

To this end, let $\mathcal{C}$ denote the collection of all non-empty subsets B of X such that $B \cap \phi(B) = \emptyset$. Notice that for each $x \in X$ the set $B = \{x\}$ belongs to $\mathcal{C}$. Indeed, if $B \cap \phi(B) = \{x\} \cap \{\phi(x)\} \neq \emptyset$, then $x = \phi(x)$, which contradicts the fact that ϕ is exhaustive. Clearly the set $\mathcal{C}$ is partially ordered by the inclusion relation $\supseteq$.

Next, we claim that the partially ordered set $\mathcal{C}$ satisfies the condition of Zorn's lemma. That is, we claim that every chain of $\mathcal{C}$ has an upper bound in $\mathcal{C}$. To see this, let $\{B_j\}_{j \in J}$

be a chain of $\mathcal{C}$, that is, for any pair of indices $i, j \in J$ we either have $B_i \supseteq B_j$ or $B_j \supseteq B_i$. Let $B = \bigcup_{j\in J} B_j$, and we claim that $B \in \mathcal{C}$. To establish this claim, assume by way of contradiction that $B \cap \phi(B) \neq \emptyset$. Fix some $b \in B \cap \phi(B)$ and let $a \in B$ be such that $b = \phi(a)$. Choose $i, j \in J$ such that $a \in B_i$ and $b \in B_j$. Since either $B_i \supseteq B_j$ or $B_j \supseteq B_i$ is true, we can assume without loss of generality that $a, b \in B_j$. In particular, we have $b = \phi(a) \in B_j \cap \phi(B_j) = \emptyset$, which is impossible. This contradiction shows that $B \cap \phi(B) = \emptyset$, and so $B \in \mathcal{C}$.

According to Zorn's lemma there exists a maximal element in $\mathcal{C}$, say B^*. We claim that $B^* \sqcup \phi(B^*) = X$. To see this, assume by way of contradiction that $B^* \sqcup \phi(B^*) \neq X$. So, there exists some $x \in X$ such that $x \notin B^* \sqcup \phi(B^*)$. Now consider the set $B' = B^* \cup \{x\}$. Clearly, the set B' properly contains B^* and we claim that $B' \cap \phi(B') = \emptyset$. Indeed, if

$$y \in B' \cap \phi(B') = \big[B^* \cup \{x\}\big] \cap \big[\phi(B^*) \cup \{\phi(x)\}\big] = B^* \cap \{\phi(x)\}\,,$$

then we have $y = \phi(x) \in B^*$. This implies, $x = \phi\big(\phi(x)\big) = \phi(y) \in \phi(B^*)$, contrary to $x \notin B^* \sqcup \phi(B^*)$. Thus, $B' \cap \phi(B') = \emptyset$ must be the case, which contradicts the maximality property of the set B^*. Therefore, $B^* \sqcup \phi(B^*) = X$. This shows that in this case the desired conclusion is true with $A = \emptyset$, $B = B^*$, and $C = \phi(B^*)$.

Now consider the general case. That is, assume that $\phi\colon X \to X$ is an arbitrary bilateral matching rule.

Let $A = \{x \in X\colon\ \phi(x) = x\}$ and put $X_1 = X \setminus A$. If $x \in X_1$, then notice that $\phi(x) \in X_1$. Otherwise, $\phi(x) \in A$ implies $x = \phi^2(x) = \phi\big(\phi(x)\big) = \phi(x)$ or $x \in A$, which is impossible. It follows that $\phi\colon X_1 \to X_1$ is an exhaustive bilateral matching rule, and so by the previous part there exist two disjoint sets B and C with $B \sqcup C = X_1$ and $\phi(B) = C$. Now notice that the sets A, B, and C satisfy the desired properties. ■

In short, Theorem 3 establishes that—given any matching technology—we can always match the population X in the following way. First, delete the set A of fixed points of ϕ from the set X. The agents in A are unmatched, and we pair *everyone* in the set $X \setminus A$. Indeed, the theorem establishes that $X \setminus A$ can be divided into two sets B and C having the same cardinality, which is why we can have ϕ map B onto C. For instance, in example (b) above we can have $A = \{1\}$, $B = (0, 1)$ and $C = (1, \infty)$. Clearly, the partition $X = A \sqcup B \sqcup C$ is not unique, since B and C need not be uniquely determined (unlike the set A). For instance, we can have $A = \{1\}$, $B = \big(0, \frac{1}{2}\big] \sqcup (1, 2)$ and $C = \big(\frac{1}{2}, 1\big) \sqcup [2, \infty)$. In either case $|B| = |C|$.

Theorem 3 also tells us how—given any population—one can construct bilateral matching rules on X. It involves two steps. First we must select a partition $X = A \sqcup B \sqcup C$ such that $|B| = |C|$. That is, we select who is to remain unmatched. Then, we can select a permutation $\phi : B \sqcup C \to B \sqcup C$ such that $\phi^{-1} = \phi$, and we use the identity function on the set A. For example, if our population is $X = [0, 1]$, then we can partition it using the sets $A = \big[\frac{1}{2}, \frac{3}{4}\big]$, $B = \big[0, \frac{1}{2}\big)$, and $C = \big(\frac{3}{4}, 1\big]$. If we take any permutation $f\colon B \to C$, then the function $\phi\colon X \to X$, defined by

$$\phi(x) = \begin{cases} x & \text{if } x \in A \\ f(x) & \text{if } x \in B \\ f^{-1}(x) & \text{if } x \in C\,, \end{cases}$$

is a bilateral matching rule for the population X.

Of course, it may be impossible to pair everyone to someone else. The obvious case is that of a finite population with an odd number of agents. The following lemma expands on this a bit.

Lemma 4. *For a set of agents X we have the following.*

1. *If X is a finite set with an odd number of agents, then X does not admit any exhaustive bilateral matching rule.*
2. *If X is a non-empty compact convex subset of a Hausdorff locally convex space, then X does not admit any continuous exhaustive bilateral matching rule.*

Proof. (1) Assume that X is a finite set with an odd number of agents and let $\phi\colon X \to X$ be a bilateral matching rule. If $X = A \sqcup B \sqcup C$ is a partition with respect to ϕ, as described in Theorem 3, then we have $|X| = |A|+|B|+|C| = |A|+2|B|$. Since $|X|$ is an odd number, we get $|A| \neq 0$, so ϕ cannot be an exhaustive bilateral matching rule.

(2) If X is a non-empty compact convex subset of some Hausdorff locally convex space, then according to the Brouwer–Schauder–Tychonoff fixed point theorem every continuous function $\phi\colon X \to X$ must have a fixed point; see [1, Corollary 16.52, p. 550]. ■

Now that we have formalized bilateral matching in a period, we move on to discussing the mechanics of matching over time.

4 Bilateral Matching Processes

We start by formalizing a general notion of bilateral matching over time. To do so we introduce the concept of a bilateral matching *process*.

Definition 5. *A **bilateral matching process** on a population X, a matching process for short, is a sequence $\Phi = (\phi_0, \phi_1, \phi_2, \ldots)$ such that:*

a. *For each $t \geq 1$ the function $\phi_t\colon X \to X$ is a bilateral matching rule for X.*

b. *The function $\phi_0\colon X \to X$ is the identity mapping, i.e., $\phi_0(x) = x$ for each $x \in X$.*

*The agent $\phi_t(x)$ is called the **partner** of agent x and x is the partner of $\phi_t(x)$ in period t.*

For convenience, it is assumed that agents do not know the matching process but they are familiar with its properties.[3] Also, we interpret $t = 0$ as an initial date before the matching process starts in period $t = 1$. Of course, this *does not* imply that a bilateral matching process necessarily pairs every agent to someone else at every date $t \geq 1$. However, this is often assumed for practical purposes (see, for instance, [25]), in which case we the say the matching process Φ is **exhaustive**.

As noted earlier, bilateral matching frameworks are often used to motivate the existence of obstacles to trade or information flows. It is this second aspect that we are mostly interested in studying, now that we have a sequence of matches. In short, we want to characterize the possible interactions among agents, for a given matching process. To this end, it may be helpful to think of matches as spatially separated encounters. This simply means that agents can directly interact only within matches. For concreteness, suppose that agents in a match observe their respective identities and can voluntarily exchange any information on past matches that is available to them. That is, although agents cannot observe any information related to matches in which they were not directly involved, they can exploit their partners to indirectly acquire or relay this information to someone else.[4]

Naturally, to discuss information flows we need to formalize the type of experience—direct or indirect—that agents may have of past matches. This depends on the agents' history of encounters generated by the matching process Φ. For this reason we start by defining the following concepts. Consider an agent $a \in X$. For each $t \geq 0$ we denote by $P_t(a)$ the set of all partners of a in periods up to and including period t. That is,

$$P_t(a) = \{\phi_0(a), \phi_1(a), \phi_2(a), ..., \phi_t(a)\}\,.$$

Since $\phi_0(a) = a$ we have $P_0(a) = \{a\}$, hence $a \in P_t(a)$ holds for all $t \geq 0$ and all $a \in X$.

We now can introduce some terminology that we will later exploit to describe the possible interactions among agents.

Definition 6. *We say that two agents a and b:*

1. *Share a* ***common partner****, if there exist periods $t_1 < t_2 < t_3$ and an agent c different than a and b such that:*

$$\begin{aligned} \phi_{t_1}(a) &= b\,, \\ \phi_{t_2}(b) &= c\,, \\ \phi_{t_3}(c) &= a\,. \end{aligned}$$

[3] As a referee points out, it could be argued that if the agents do not have complete knowledge of the matching process, then from their point of view the matching process need not be deterministic.

[4] Of course this information-exchange process need not be voluntary— as in [19] where "societal memory" allows information to flow effortlessly and unimpeded. Also, we emphasize that observability of identities is not needed for our results. It is assumed to stress how obstacles to information exchange are independent of the knowledge of identities but rather hinge on the matching process.

2. *Share an* ***indirect partner****, if there exist periods* $t_1 < t_2 < t_3 < \cdots < t_k$ *and agents* $a_1, a_2, \ldots, a_{k-2}$ *different than* a *and* b*, where* $k \geq 4$ *such that:*

$$\begin{aligned}
\phi_{t_1}(a) &= b\,, \\
\phi_{t_2}(b) &= a_1\,, \\
\phi_{t_3}(a_1) &= a_2\,, \\
&\vdots \\
\phi_{t_{k-1}}(a_{k-3}) &= a_{k-2}\,, \\
\phi_{t_k}(a_{k-2}) &= a\,.
\end{aligned}$$

This helps us define the way in which information (or other economic resources) may flow across agents, over time. For example, suppose a and b meet, at some date. Keep in mind that matches are spatially separated, so that agents belonging to different matches at a date t cannot communicate. Now suppose b wants to transfer information to a after the match breaks. This is possible if a meets c, after b has met both a and c. In this case b and a share the common partner c; she can transfer information from b to a after their match has ended. Of course, this does not mean that c will necessarily communicate something about b to agent a, when the two meet. It simply means this information transfer is possible.

Communication from b to a can also take place via a sequence of matches, by means of the indirect partner a_{k-2} of agent b. Here, a meets a_{k-2}. Although agent a_{k-2} has never met b, she has met someone who was in (direct or indirect) contact with b.

At this point it should be clear that to formalize the *possible* information flows we must keep track not only of the matchings of an agent's common partners but also of her indirect partners, and so on. To do so we introduce some general notation, drawing from [19]. Specifically, we denote by $\Pi_t(a)$ the set of all of a's past and current partners (including a herself), the past partners of a's current partner, the partners that a's partner in period $t-1$ met until period $t-2$, and so on. This set of agents is given by the recursive formula

$$\Pi_0(a) = P_0(a) = \{a\}\,, \quad \text{and} \quad \Pi_t(a) = \Pi_{t-1}(a) \bigcup \Pi_{t-1}\big(\phi_t(a)\big) \ \text{ for } \ t = 1, 2, \ldots\ .$$

From the above and an easy inductive argument it follows that $\Pi_t(a)$ is a finite set since it includes a finite set of dates and partners. In particular, we note that $\Pi_t(a)$ does not include the agents that a's partners have met *after* meeting a and until the current period t. Also, it should be clear that $P_t(a) \subseteq \Pi_t(a)$ holds for all $a \in X$ and all $t \geq 0$.

We now have all the necessary machinery to characterize the information that can be made available to agents in a match. We do so by defining an exact map between properties of the matching technology and the degrees of informational isolation that are consistent with the physical description of the environment. In this way we are able to formalize a general notion of *anonymity* in terms of the matching process in place.

Definition 7. *A bilateral matching process Φ on a population X is said to be:*

1. ***Eventually weakly anonymous****, if for each agent a there exists a period t (depending on a) such that:*
   (a) *the partners of a after period t are all distinct, and*
   (b) $P_t(a) \cap \{\phi_{t+1}(a), \phi_{t+2}(a), \ldots\} \subseteq \{a\}$.
2. ***Weakly anonymous****, if for each agent a and each $t \neq \tau$ with $\phi_t(a) \neq a$ we have*
$$\phi_t(a) \neq \phi_\tau(a)\,.$$
3. ***Anonymous****, if for each agent a that satisfies $\phi_{t+1}(a) \neq a$ in some period $t \geq 1$ we have*
$$P_t(a) \cap P_t\big(\phi_{t+1}(a)\big) = \varnothing\ .$$
4. ***Strongly anonymous****, if for each agent a that satisfies $\phi_{t+1}(a) \neq a$ in some period $t \geq 1$ we have*
$$\Pi_t(a) \cap \Pi_t\big(\phi_{t+1}(a)\big) = \varnothing\ .$$

Our notion of anonymity is developed as follows. First, we formalize a matching process Φ on the population X. Given Φ, we can then trace every agent's matching history, i.e., we can define a precise map between the matching process and the information (about past matches) that is potentially available to him at each date. The degree of anonymity that can result in the economy is then measured by comparing the information sets of matched agents. Thus, different matching processes imply various degrees of anonymity, i.e., various levels of informational isolation in the economy.

In a model with eventually weak anonymity, enduring relationships are possible but they will eventually break down without the possibility to be reconstituted. For example, this type of matching is very common in the job-search literature (where ongoing worker-firm matches are randomly terminated, as in [22]) as well as in other contexts (as in the study of markets with decentralized price formation processes of [5]).

Weakly anonymous matching processes rule out the possibility of ongoing relationships. In short, no two agents can directly interact with each other over time since a match between a and b lasts only one period, after which a will never meet b again. Technically, in every $t \geq 1$ and for each a we have $\phi_{t+1}(a) \notin P_t(a)$. Matching processes with this property have been commonly adopted in some monetary models (e.g., [18]). The reason is that weak anonymity impedes direct credit arrangements (such as the direct redemption of IOUs). This is taken to be a sufficient justification for the use of money in the process of exchange, since current sales cannot be based on future direct repayment.

However, under weak anonymity the door is open to the possibility that a and b, although never meeting again, may share a common partner c. Stronger degrees of anonymity progressively remove all direct and indirect links, past and future, between agents, as we demonstrate next.

Lemma 8. *If the bilateral matching process Φ on the population X is:*

a. anonymous, then no pair of agents will share any common partner over their lifetimes.

b. strongly anonymous, then no pair of agents will share any common or indirect partner over their lifetimes.

Proof. (a) Let Φ be an anonymous bilateral matching process and assume by way of contradiction that two agents a and b share a common partner. This means that there exist three periods $t_1 < t_2 < t_3$ and an agent c such that:

$$\text{(i) } \phi_{t_1}(a) = b, \quad \text{(ii) } \phi_{t_2}(b) = c, \quad \text{and} \quad \text{(iii) } \phi_{t_3}(c) = a\,.$$

Clearly, we have

$$t_1 < t_2 \leq t_3 - 1\,. \tag{$\star$}$$

Now note that (iii) yields $a = \phi_{t_3}(c) = \phi_{(t_3-1)+1}(c) \neq c$ and so by the anonymity of Φ, we get $P_{t_3-1}(c) \cap P_{t_3-1}(\phi_{(t_3-1)+1}(c)) = \emptyset$ or

$$P_{t_3-1}(c) \cap P_{t_3-1}(a) = \emptyset\,. \tag{$\star\star$}$$

Using (ii) we obtain $b = \phi_{t_2}(c)$ and a glance at ($\star$) guarantees that $b \in P_{t_3-1}(c)$. Next, observe that (i) in conjunction with ($\star$) implies $b \in P_{t_3-1}(a)$. So $b \in P_{t_3-1}(c) \cap P_{t_3-1}(a)$ contrary to ($\star\star$). This contradiction establishes the validity of (a).

(b) Assume that Φ is a strongly anonymous bilateral matching process and suppose first by way of contradiction that two agents a and b share an indirect partner. This means that there exist periods $t_1 < t_2 < t_3 < \cdots < t_k$ and agents $a_1, a_2, \ldots, a_{k-2}$ different than a and b, where $k \geq 4$ such that:

$$\begin{aligned} \phi_{t_1}(a) &= b\,, \\ \phi_{t_2}(b) &= a_1\,, \\ \phi_{t_3}(a_1) &= a_2\,, \\ &\vdots \\ \phi_{t_{k-1}}(a_{k-3}) &= a_{k-2}\,, \\ \phi_{t_k}(a_{k-2}) &= a\,. \end{aligned}$$

Clearly, we have

$$t_1 < t_2 < t_3 < \cdots < t_{k-1} \leq t_k - 1\,. \tag{$\dagger$}$$

From $a = \phi_{t_k}(a_{k-2}) = \phi_{(t_k-1)+1}(a_{k-2}) \neq a_{k-2}$ and the strong anonymity of Φ, it follows that $\Pi_{t_k-1}(a_{k-2}) \cap \Pi_{t_k-1}(\phi_{(t_k-1)+1}(a_{k-2})) = \emptyset$ or

$$\Pi_{t_k-1}(a_{k-2}) \cap \Pi_{t_k-1}(a) = \emptyset \ . \qquad (\dagger\dagger)$$

Now note that $a_{k-2} \in \Pi_{t_k-1}(a_{k-2})$ is trivially true. On the other hand, it is not difficult to see that $a_{k-2} \in \Pi_{t_k-1}(a)$. But then we have $a_{k-2} \in \Pi_{t_k-1}(a_{k-2}) \cap \Pi_{t_k-1}(a)$, contrary to ($\dagger\dagger$).

Finally, to establish that no pair of agents share a common partner in their life times, use part (a) in conjunction with the fact that strong anonymity implies anonymity. (See also the proof of Lemma 9.) ■

Lemma 8 states that in an anonymous matching process any two agents a and b cannot interact over time by means of a common partner c. However, a possibility still exists that a and b may share some experience of past events by means of an indirect partner d. For example, the matching process assumed in the Turnpike model of Townsend [25] has this property (we work this out explicitly later on).

To rule out any possibility of information sharing we need a strongly anonymous matching process. Our formulation reflects an assumption found in [19] (assumption (A2)), which imposes the most severe restriction on information flows among agents. It rules out the possibility that an arbitrary agent a may meet former partners or anyone who has been in direct or indirect contact with any of a's former partners. In short, strong anonymity imposes a restriction on the pattern of matching that insures total information isolation in every meeting.

Strong anonymity is clearly a feature of the overlapping generations model of Samuelson [24]—which can be interpreted as a deterministic matching model—since the agents live for two periods. Although we do not know of any infinitely-lived agent models that make strong anonymity an *explicit* feature of the physical environment, similarly severe informational frictions seem to be *implicitly* assumed in many models. Such assumptions are usually motivated by the presence of spatial separation, random meetings and inability to recognize the partners' features (identities, etc). [5] Of course, our formalization demonstrates these justifications are unnecessary to achieve informational isolation. While this is perhaps obvious, it is certainly less obvious whether any strongly anonymous matching process in fact exists. For this reason, we devote Section 5 to demonstrating a general method for constructing a deterministic matching process that insures complete informational isolation. Before doing so, however, it may be helpful to discuss the relationship between different degrees of anonymity and to provide an example, by characterizing a well-known deterministic matching model.

[5] For instance see the work [20] on long-term exchange relationships and anonymous market exchange as well as several recent monetary models such as [4].

4.1 Order of Implications of Anonymity

We start by discussing the order of implications.

Lemma 9. *We have the following implications:*

$$\begin{aligned} \textit{Strong Anonymity} &\implies \textit{Anonymity} \\ &\implies \textit{Weak Anonymity} \\ &\implies \textit{Eventual Weak Anonymity} \end{aligned}$$

In general, no reverse implication is true.

Proof. Let $\Phi = (\phi_0, \phi_1, \phi_2, \phi_3, \ldots)$ be a bilateral matching process on a set of agents X and fix some agent $a \in X$.

Assume first that Φ is strongly anonymous. If $\phi_{t+1}(a) \neq a$, then from

$$P_t(a) \cap P_t\big(\phi_{t+1}(a)\big) \subseteq \Pi_t(a) \cap \Pi_t\big(\phi_{t+1}(a)\big) = \emptyset\,,$$

it follows that $P_t(a) \cap P_t\big(\phi_{t+1}(a)\big) = \emptyset$. This shows that Φ is an anonymous bilateral matching process.

Now suppose that Φ is anonymous. Assume by way of contradiction that for some $1 \leq t < \tau$ and some agent a we have $\phi_t(a) \neq a$ and $\phi_t(a) = \phi_\tau(a)$. Let $t^* = \tau - 1$ and $b = \phi_{t^*+1}(a) = \phi_\tau(a) \neq a$. Clearly, $t \leq t^*$. Now note that $b \in P_{t^*}(b)$ and that $b = \phi_\tau(a) = \phi_t(a) \in P_{t^*}(a)$, contrary to $P_{t^*}(a) \cap P_{t^*}(b) = \emptyset$. This contradiction shows that anonymity implies weak anonymity.

The fact that weak anonymity implies eventual weak anonymity is obvious. To see that no reverse implication holds true, see the example in Section 4.2. ■

As expected, the more restrictive anonymity subsumes a less restrictive one. In general, although the opposite implication is not true, there are cases in which less restrictive anonymity implies more stringent anonymity. For example divide a population into two sets with the same cardinality and call them "sellers" and "buyers." In each period match every seller to a different buyer, i.e., impose weak anonymity. This yields anonymity since the agents cannot share any common partner as sellers only meet buyers (and vice-versa). More specifically,

Lemma 10. *Let Φ be a weakly anonymous matching process on X. Assume that there exists a partition $X = B \sqcup C$ of X such that $\phi_t(B) = C$ for each $t \geq 1$. Then the matching process Φ is anonymous.*

Proof. By symmetry we have $\phi_t(C) = B$ for each $t \geq 1$. This implies ϕ_t is an exhaustive matching rule for each $t \geq 1$. Assume by way of contradiction that there exists some a such that $P_t(a) \cap P_t(b) \neq \emptyset$ holds true for some $t \geq 1$, where $b = \phi_{t+1}(a)$. Without loss of generality, we can assume that $a \in B$; and so $b = \phi_{t+1}(a) \in C$. Clearly, $b \neq a$.

Since $\phi_t(a) \in C$, $\phi_t(b) \in B$ and $B \cap C = \emptyset$, it follows from $P_t(a) = \{a, \phi_1(a), \ldots, \phi_t(a)\}$, $P_t(b) = \{b, \phi_1(b), \ldots, \phi_t(b)\}$, and $P_t(a) \cap P_t(b) \neq \emptyset$ that there exists some $1 \leq \tau \leq t$ such that either $a = \phi_\tau(b)$ or $b = \phi_\tau(a)$. In either case, we have $\phi_\tau(a) = b = \phi_{t+1}(a)$. However, the latter conclusion contradicts the weak anonymity of Φ. Thus, Φ is anonymous. ■

At this point we are ready for an example.

4.2 An Example: Anonymity in Townsend's Turnpike

Townsend [25] proposes a model where agents undergo an infinite deterministic sequence of bilateral matches. This model is often used in monetary theory and has been adapted and exploited in experimental economics, as its structure restricts substantially the interactions that are possible among agents. Here, we demonstrate that the scope of these restrictions is limited, in that the matching process is anonymous but not strongly so.

The model has countably many agents. We can interpret this economy as having countably many spatially separated trading posts located at the integer points along the real line. Each agent is assumed to be located into one of the countably many of trading posts. The bilateral matching process is such that "any two agents are paired at most once during their lifetimes" (i.e., it satisfies weak anonymity), and "they share no common third agent as a trading partner" (i.e., it satisfies anonymity). In each period every agent travels on a line (the 'turnpike'), either east or west, moving by one position (see [25] for an illustration) so that every trading post has two agents, called "odd" and "even," depending on the direction in which they move.

Define the population by $X = \mathbb{Z} \setminus \{0\} = \{\ldots, -2, -1, 1, 2, \ldots\}$, i.e., the set of integers deprived of the zero. Without loss of generality we can identify the west-moving agents with even numbers and the others with odd numbers. The exhaustive bilateral matching process Φ is defined as follows. We let $\phi_0 = I$. Now let $B = \{\ldots, -4, -2, 2, 4, \ldots\}$, the set of all even integers, and $C = \{\ldots, -3, -1, 1, 3, \ldots\}$, the set of all odd integers. For $t \geq 1$ the matching rule $\phi_t \colon B \to C$ is defined by:

$$\phi_1(a) = \begin{cases} a-1 & \text{if } \ 0 < a \in B \\ a+1 & \text{if } \ 0 > a \in B\,, \end{cases}$$

and

$$\phi_t(a) = \phi_1(a) - 4(t-1) \ \text{ if } \ t > 1\,.$$

The following table describes this matching process where the even agents in the first row are paired to odd agents (directly below them) in periods $t = 1, 2, 3, 4$.

t	⋯ -12	-10	-8	-6	-4	-2	2	4	6	8	10	12 ⋯
1	⋯ -11	-9	-7	-5	-3	-1	1	3	5	7	9	11 ⋯
2	⋯ -15	-13	-11	-9	-7	-5	-3	-1	1	3	5	7 ⋯
3	⋯ -19	-17	-15	-13	-11	-9	-7	-5	-3	-1	1	3 ⋯
4	⋯ -23	-21	-19	-17	-15	-13	-11	-9	-7	-5	-3	-1 ⋯

It is a routine matter to verify that Φ is weakly anonymous. By Lemma 10, it is also anonymous. It is not strongly anonymous since the Townsend's Turnpike model allows for *indirect* links among agents. To see why, consider the first three periods. In $t = 1$ agent -4 meets -3. In $t = 2$ agent -7 meets -4, while -3 meets 2. In $t = 3$ agents 2 and -7 meet and both have links (direct or indirect) to agent -4. In terms of the formalization we have earlier developed:

$$\begin{aligned}
\Pi_2(2) &= \Pi_1(2) \cup \Pi_1\big(\phi_2(2)\big) = \Pi_0(2) \cup \Pi_0\big(\phi_1(2)\big) \cup \Pi_0\big(\phi_2(2)\big) \cup \Pi_0\big(\phi_1(\phi_2(2))\big) \\
&= P_0(2) \cup P_0(1) \cup P_0(-3) \cup P_0(-4) \\
&= \{2\} \cup \{1\} \cup \{-3\} \cup \{-4\} = \{-4, -3, 1, 2\} \ ,
\end{aligned}$$

and

$$\begin{aligned}
\Pi_2(\phi_3(2)) &= \Pi_2(-7) = \Pi_1(-7) \cup \Pi_1\big(\phi_2(-7)\big) \\
&= \Pi_0(-7) \cup \Pi_0\big(\phi_1(-7)\big) \cup \Pi_0\big(\phi_2(-7)\big) \cup \Pi_0\big(\phi_1(\phi_2(-7))\big) \\
&= P_0(-7) \cup P_0(-8) \cup P_0(-4) \cup P_0(-3) \\
&= \{-7\} \cup \{-8\} \cup \{-4\} \cup \{-3\} = \{-8, -7, -4, -3\} \ .
\end{aligned}$$

Thus, $\Pi_2(2) \cap \Pi_2(\phi_3(2)) = \{-3, -4\} \neq \emptyset$, hence Φ is not strongly anonymous.

5 The Mathematics of Anonymous Matchings

In this section, we demonstrate how to construct economies with complete informational isolation, given any infinite population X.

It is obvious that one can always introduce enough anonymity in an economy by having agents face a *finite* sequence of matches. One can simply pair agents to someone else for only a limited number of dates, after which the agents are left permanently unmatched. Such a scheme effectively represents the overlapping generations model of Samuelson [24]. That model can be thought of as one of an infinitely-lived economy with an infinite population X. Then, we can let $X = \bigsqcup_{n=1}^{\infty} A_n$ for $n = 0, 1, 2, \ldots$, where A_n have identical cardinality, and adopt this matching process. Every agent in A_n for $n = 1, 2, \ldots$ is paired to an agent in A_{n-1} in dates $t = n$ and to an agent A_{n+1} in date $t = n + 1$. In all other dates, the agents in A_n are unmatched.

Achieving strong anonymity when agents face an *infinite* sequence of meetings is substantially more complicated. It requires a completely different matching process, which we now discuss. We start by choosing an initial countable partition $X = \bigsqcup_{n=1}^{\infty} A_n$ of X such that all the A_n have the same cardinality. Based on this initial partition, we construct recursively a sequence of "matching blocks" for each period as shown in the following table. (The square brackets indicate the partition sets in each period.)

$$
\begin{array}{cl}
\text{Period} & \qquad \text{Partition of the population } X \\
0 & X = [A_1] \sqcup [A_2] \sqcup [A_3] \sqcup [A_4] \sqcup [A_5] \sqcup [A_6] \sqcup \cdots \\
1 & X = \big[A_1 \sqcup A_2\big] \sqcup \big[A_3 \sqcup A_4\big] \sqcup \big[A_5 \sqcup A_6\big] \sqcup \cdots \\
2 & X = \big[A_1 \sqcup A_2 \sqcup A_3 \sqcup A_4\big] \sqcup \big[A_5 \sqcup A_6 \sqcup A_7 \sqcup A_8\big] \sqcup \cdots \\
3 & X = \big[A_1 \sqcup A_2 \sqcup A_3 \sqcup A_4 \sqcup A_5 \sqcup A_6 \sqcup A_7 \sqcup A_8\big] \sqcup [A_9 \sqcup \cdots A_{16}] \sqcup \cdots \\
\vdots & \vdots \\
t & X = \displaystyle\bigsqcup_{n=1}^{\infty} \big[A_{(n-1)2^t+1} \sqcup A_{(n-1)2^t+2} \sqcup \cdots \sqcup A_{n2^t}\big] \\
 & \quad = \displaystyle\bigsqcup_{n=1}^{\infty} \bigsqcup_{k=1}^{2^t} A_{(n-1)2^t+k} \\
 & \quad = \displaystyle\bigsqcup_{n=1}^{\infty} B_n^t = \big[B_1^t \sqcup B_2^t\big] \sqcup \big[B_3^t \sqcup B_4^t\big] \sqcup \cdots \\
 & \quad = \displaystyle\bigsqcup_{n=1}^{\infty} \big[B_{2n-1}^t \sqcup B_{2n}^t\big] = \bigsqcup_{n=1}^{\infty} B_n^{t+1} \\
 & \vdots \; .
\end{array}
\tag{✠}
$$

Notice that we let $B_n^t = \bigsqcup_{k=1}^{2^t} A_{(n-1)2^t+k}$. These pairwise disjoint sets that have all the same cardinality will be used to construct the matchings blocks in period t. Precisely, a **matching block** in period t is any set of agents of the form

$$B_n^{t+1} = B_{2n-1}^t \sqcup B_{2n}^t \,.$$

Given these matching blocks, we can define a strongly anonymous matching process Φ on the population X. The idea is to construct a sequence of matching rules on each matching block B_n^{t+1} by pairing each agent in B_{2n-1}^t to an agent in B_{2n}^t. This can be extended to a matching rule for the entire population, as follows.

For each $n \geq 1$ and each $t \geq 1$, let $f_n^t \colon B_{2n-1}^t \to B_{2n}^t$ be a one-to-one and surjective function. Also, let $(f_n^t)^{-1} \colon B_{2n}^t \to B_{2n-1}^t$ be the inverse of f_n^t. Clearly, the function

$\phi_{t,n}\colon B_n^{t+1} \to B_n^{t+1}$ defined by

$$\phi_{t,n}(x) = \begin{cases} f_n^t(x) & \text{if } x \in B_{2n-1}^t \\ (f_n^t)^{-1}(x) & \text{if } x \in B_{2n}^t \end{cases} \qquad (\dagger)$$

is an involution on B_n^{t+1}. Next, for each $t \geq 1$ we define a matching rule $\phi_t\colon X \to X$ as follows: If $x \in X$ choose the unique n such that $x \in B_n^{t+1}$ and then let $\phi_t(x) = \phi_{t,n}(x)$. This gives rise to the matching process $\Phi = (\phi_0, \phi_1, \phi_2, \ldots)$, where $\phi_0 = I$.

Definition 11. *Any matching process Φ on X that is obtained from (✠) and the preceding procedure will be referred to as a* ***block recursive*** *matching process on X.*

Clearly, for each $t \geq 1$ the function $\phi_t\colon X \to X$ is an exhaustive matching rule on X. Moreover, from $B_n^{t+1} = B_{2n-1}^t \sqcup B_{2n}^t$ and the definition of $\phi_{t,n}$ given in (†), it follows that $\phi_t(B_n^{t+1}) = B_n^{t+1}$. This implies that for each n the set B_n^{t+1} is ϕ_t-invariant, i.e., $\phi_t(B_n^{t+1}) \subseteq B_n^{t+1}$. More generally, we have the following.

Lemma 12. *For each $t \geq 0$ and each $\tau = 0, 1, \ldots, t$ the sets B_n^{t+1}, $n \geq 1$, are ϕ_τ-invariant.*

Proof. We shall use induction on t. For $t = 0$ the conclusion is obvious. Therefore, for the induction step, assume that the conclusion is true for some $t \geq 0$. For each n we have $B_n^{t+2} = B_{2n-1}^{t+1} \sqcup B_{2n}^{t+1}$ and by our induction hypothesis for each $i = 1, \ldots, t$ the functions $\phi_i\colon B_{2n-1}^{t+1} \to B_{2n-1}^{t+1}$ and $\phi_i\colon B_{2n}^{t+1} \to B_{2n}^{t+1}$ are exhaustive bilateral matching rules. It easily follows that for each $i = 1, \ldots, t$ the function $\phi_i\colon B_n^{t+2} \to B_n^{t+2}$ is itself an exhaustive bilateral matching rule on the set B_n^{t+2}. Also, by the discussion preceding the lemma we have $\phi_{t+1}(B_n^{t+2}) \subseteq B_n^{t+2}$. This completes the induction and the proof of the lemma. ■

The invariance under ϕ_τ for $0 \leq \tau \leq t$ of the sets of agents B_n^{t+1} is the essential property in achieving strong anonymity. It guarantees that, at each date t, the matching technology pairs the agents in B_n^t to those in $X \setminus B_n^t$. This implies two basic properties. First, all prior partners of any agent in B_n^t remain in this set. Second, at date t agents in B_n^t are matched to agents *outside* of B_n^t. In particular, it follows that matched agents cannot share common or indirect partners, i.e, they are informationally isolated. These properties are formalized below.

Theorem 13. *Any block recursive matching process is strongly anonymous.*

Proof. Let $a \in X$ be an arbitrary agent and fix $t \geq 1$. Let k be the unique natural number such that $a \in B_k^{t+1}$. According to Lemma 12, we have $\phi_i(a) \in B_k^{t+1}$ for each $i = 0, 1, \ldots, t$. This easily implies $\Pi_t(a) \subseteq B_k^{t+1}$. Now according to the definition of the bilateral matching rule ϕ_{t+1} either we have $b = \phi_{t+1}(a) \in B_{k-1}^{t+1}$ or $b = \phi_{t+1}(a) \in B_{k+1}^{t+1}$.

In particular, as above, either $\Pi_t(b) \subseteq B_{k-1}^{t+1}$ or $\Pi_t(b) \subseteq B_{k+1}^{t+1}$. Since $B_{k-1}^{t+1} \cap B_k^{t+1} = \emptyset$, $B_{k+1}^{t+1} \cap B_k^{t+1} = \emptyset$ and $\Pi_t(a) \subseteq B_k^{t+1}$, we infer that $\Pi_t(a) \cap \Pi_t(b) = \emptyset$. Consequently, $(\phi_0, \phi_1, \phi_2, \ldots)$ is a strongly anonymous bilateral matching process. ■

To sum up, the first step in constructing an infinite sequence of strongly anonymous matchings on an infinite population X is to specify a countable partition of X. The partition must be such that all sets of the partition have the same cardinality. For example, consider:

$$\begin{aligned} X &= (0,1] = \bigsqcup_{n=1}^{\infty} \left(\tfrac{1}{n+1}, \tfrac{1}{n}\right] \\ X &= \mathbb{N} = \{1,2,\ldots\} = \bigsqcup_{n=1}^{\infty} \{n\} \\ X &= \mathbb{N} = \{1,2,\ldots\} = \bigsqcup_{n=1}^{\infty} \{2n-1, 2n\} \\ X &= [0,\infty) = \bigsqcup_{n=1}^{\infty} [n-1, n)\,. \end{aligned}$$

Having specified an initial partition, the second step is to construct matching blocks recursively, as in (✠), taking care to match agents as in (†). For example, suppose $X = \mathbb{N}$ and we choose the partition $X = \mathbb{N} = \bigsqcup_{n=1}^{\infty} \{n\}$. Then, a block recursive matching process corresponds to the matrix shown below (it describes how the even agents, in the first row, are paired to odd agents in periods $t = 1, 2, 3, 4$).

t	2	4	6	8	10	12	14	16	18	20	⋯
1	1	3	5	7	9	11	13	15	17	19	⋯
2	5	7	1	3	13	15	9	11	21	23	⋯
3	9	11	13	15	1	3	5	7	25	27	⋯
4	17	19	21	23	25	27	29	31	1	3	⋯
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋯

In this case consider $n = 1, 2$ and periods $t = 0, 1, 2$. Then, in $t = 0$ we have $B_1^0 = \{2\}$, and $B_2^0 = \{4\}$. In $t = 1$ we have $B_1^1 = B_1^0 \sqcup B_2^0 = \{2, 4\}$, $B_2^1 = B_3^0 \sqcup B_4^0 = \{1, 3\}$ and $\phi_{1,1}(x)\colon B_1^1 \to B_2^1$. In $t = 2$, $B_1^2 = \{1, 2, 3, 4\}$, $B_2^2 = \{5, 6, 7, 8\}$ and $\phi_{2,1}(x)\colon B_1^2 \to B_2^2$. Thus, we see that agent 2 and her partners have nothing in common at dates $t = 1, 2$.

We note that it is also easy to construct non-exhaustive matching processes that are strongly anonymous. We only need to modify the construction of the block recursive matching process, as follows. For each $t \geq 1$ and each $n \geq 1$ let F_{2n}^t and F_{2n-1}^t be (possibly empty) subsets of B^t_{2n} and B^t_{2n-1} respectively having the same cardinality. Then, let $f^t_n\colon B^t_{2n-1}\setminus F^t_{2n-1} \to B^t_{2n}\setminus F^t_{2n}$ be a one-to-one and surjective function, and let $(f^t_n)^{-1}$ be its inverse. Finally, define

$$\phi_{t,n}(x) = \begin{cases} x & \text{if } x \in F^t_{2n-1} \cup F^t_{2n} \\ f^t_n(x) & \text{if } x \in B^t_{2n}\setminus F^t_{2n} \\ (f^t_n)^{-1}(x) & \text{if } x \in B^t_{2n-1}\setminus F^t_{2n-1}\,, \end{cases}$$

and then apply the same procedures used above.

6 Concluding remarks

We have taken a step toward developing a theoretical foundation for economic frameworks where agents interact via some exogenous matching process. We have laid the groundwork by formalizing—via an explicit set-theoretic representation—the method of operation of technologies that generate deterministic pairings. This has allowed us to chart a method by which different meeting processes can facilitate or obstruct the exchange of economic resources and, in particular, of information. We have also introduced a general approach of how to construct environments, where a technology used to pair agents infinitely often imposes severe constraints on the interactions among agents. The matching framework we have presented can contribute to provide more solid mathematical foundations for general classes of matching processes as in our companion paper [2].

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