1978

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Recommended Citation

Leiter, D., Kafatos, M. (1978) Penrose Pair Production in Massive, Extreme Kerr Black Holes, *Astrophysical Journal*, Vol. 226:32-36. doi: 10.1086/156581

THE ASTROPHYSICAL JOURNAL, 226:32–36, 1978 November 15

PENROSE PAIR PRODUCTION IN MASSIVE, EXTREME KERR BLACK HOLES

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Received 1978 March 20; accepted 1978 May 4

ABSTRACT

We show that in the case of very massive ($M > 10^8\ M_\odot$) Kerr black holes with extreme ranges of angular momentum $1 \geq a/M > 0.998$, the possibility of photon-induced Penrose pair production (PPP) in the ergosphere is very plausible. The pairs that escape the ergosphere are very high energy and on the order of the GeV rest mass of the Penrose accreted protons which participate in the photon-induced pair production process. Application of this model to the case of quasi-stellar objects (QSOs) is shown to lead to reasonable predictions about their masses, lifetimes, and luminosities. The physical mechanism is very general, and also can be applied to other types of active galactic nuclei.

Subject headings: black holes — galaxies: nuclei — quasars

I. INTRODUCTION

In recent publications, Piran and Shaham (1977*a*, *b*) studied the mechanism of Penrose collision processes in the context of "inverse" Compton scattering in the ergosphere of a spinning black hole ($M = 15\ M_\odot$) with "canonical" ranges of angular momentum density ($a/M = 0.998$). Thorne (1974) has shown that black holes will have a strong tendency to spin-down to this value asymptotically. Piran and Shaham showed that such processes offered plausible models for recently observed γ-ray bursts.

II. PENROSE PAIR PRODUCTION

In the present work, we wish to point out that by applying arguments similar to those of Piran and Shaham (1977*a*, *b*) to the case of a very massive black hole, $M \sim 10^8\ M_\odot$, with "extreme Kerr" ranges (i.e., values of a/M which are larger than the canonical value but still less than unity) of angular momentum density, $(1 - a/M) < 2 \times 10^{-3}$, the possibility of photon-induced Penrose pair production (PPP) in the ergosphere of such a spinning black hole is shown to be very plausible. The pairs that are produced by this mechanism are of very high energy (on the order of the rest mass of the Penrose accreted protons, participating in the photon-induced pair production process). They are driven through the event horizon by the recoil effects of the extremely blueshifted photons, $E_\gamma \sim m_p$, and can escape to infinity (in a manner similar to that of the previously mentioned inverse Compton scattering) with energies on the order of 1 GeV. This leads us to suggest that the case of massive ($M \geq 10^8\ M_\odot$), extreme ($a/M > 0.998$) Kerr black holes (surrounded by accreting matter in the centers of galaxies) might be a plausible model to explain the nature of the vast, fluctuating, energy production associated with quasars. It may also be applicable to other types of active galactic nuclei (e.g., Seyfert galaxies, giant elliptical radio galaxies, etc.), the only difference being the detailed nature of the energetics involved in each specific case.

We present our analysis in the manner of Bardeen, Press, and Teukolsky (1974) who replace the Boyer-Lindquist (1967) coordinate form of the Kerr (1963) metric by a continuous distribution of local physical observers, tied to local nonrotating frames (LNRF) connected to orthonormal tetrads which are chosen to cancel out the "frame dragging effects" inside the ergosphere of the spinning black hole. These local frames rotate with the black hole in such a way that physical processes described in them take on a much simpler form than those of other physical observers (whose coordinates will be dragged in a complicated nonlocal manner by the spinning black hole). In this context we will analyze the case of the collision of an infalling photon with a proton moving in a "marginally bound" or "marginally stable" orbit, deep within the ergosphere. Generalizing on the specific discussion of Penrose processes given by Piran and Shaham (1977*a*), we expect the initial photon energy, outside the ergosphere, to depend on the specific choice of boundary conditions chosen for the spinning black hole. The boundary conditions will involve the nature and parameters of the flow of matter and radiation into the hole via the associated accretion disk. However, when the photon arrives at the proton, deep within the ergosphere, its energy will be extremely blueshifted by the gravitational field. Considering the case of the extreme Kerr metric located in the equatorial region near the horizon r_+, the photon energy E_γ, in the LNRF, is related to the photon energy E_{ph} and orbital angular momentum l_{ph} at infinity by the relationship (cf. Piran and Shaham 1977*a*)

$$E_\gamma \approx [(E_{ph} - \omega l_{ph})/(r - r_+)]\,, \tag{1}$$

where $r_+ = [M + (M^2 - a^2)^{1/2}]$ defines the event horizon and $\omega = a/2Mr_+$. For a collision occurring at the coordinate $r = M(1 + \epsilon)(\epsilon \ll 1)$ in the ergosphere of a spinning black hole with $a/M = (1 - \alpha^3/4)$ $(\alpha \ll 1)$ it was shown by Piran and Shaham that equation (1) can be written as

$$E_\gamma \approx e^{-\nu}[E_{\rm ph} - \omega l_{\rm ph}] \approx [8/(2\epsilon^2 - \alpha^3)]^{1/2}[E_{\rm ph} - \omega l_{\rm ph}] , \tag{2}$$

which implies that for ϵ, α, $\ll 1$ the photon energy blueshift can be made very large. For this collision to produce a pair, the photon energy E_γ would have to be at least as large as $2m_e$. However, in order for the pair production process to be a Penrose one, the proton must be driven into a "negative energy orbit" (and pass through the event horizon, thus allowing the pair to escape to infinity with energies on the order of the rest mass of the proton [GeV]), with a subsequent change in the angular momentum of the black hole. Proton velocity boosts $V \gtrsim c/2$ are needed to cause protons in "marginally bound" ($r_{\rm mb}$) or "marginally stable" ($r_{\rm ms}$) orbits, inside the ergosphere, to be injected into the required "negative energy orbits" (Bardeen, Press, and Teukolsky 1974). For this reason, the PPP process must occur at a radius where the photon energy blueshift is on the order of, or larger than, m_p, so that the subsequent recoil of the proton can be sufficiently energetic to drive the proton through the event horizon as required by the Penrose process energetics. Hence we will require that the value of ϵ and α in equation (2) be chosen such that $E_\gamma \approx m_p$. Suppose we consider the case of protons which are marginally bound; then we have that (Piran and Shaham 1977*a*) $\epsilon_{\rm mb} \sim \alpha^{3/2}$, while for marginally stable protons we have $\epsilon_{\rm ms} \sim \alpha$. To see what the black hole angular momentum requirements are, for the PPP process to function, we set $E_\gamma \approx m_p$ and $\epsilon \sim \epsilon_{\rm mb}$; $\epsilon_{\rm ms}$ in equation (2). We find the associated values of $\alpha_{\rm mb}$ and $\alpha_{\rm ms}$ as

$$\alpha_{\rm mb} \sim 2[(E_{\rm ph} - \omega l_{\rm ph})/m_p]^{2/3} , \tag{3a}$$

$$\alpha_{\rm ms} \sim 2[(E_{\rm ph} - \omega l_{\rm ph})/m_p] . \tag{3b}$$

It is plausible that for massive black holes ($M > 10^6\ M_\odot$) that $(E_{\rm ph} - \omega l_{\rm ph})/m_p \lesssim 10^{-2}$ holds. From this we can draw the following general conclusion: If the PPP is to occur in the orbital range $r_{\rm mb} < r < r_{\rm ms}$ with the input energy ratio value of $(E_{\rm ph} - \omega l_{\rm ph})/m_p \leq 10^{-2}$, we must be dealing with an extreme Kerr spinning black hole whose angular momentum density lies in a region such that $(1 - a/M) \sim \frac{1}{4}\alpha^3$ with the range given by $2 \times 10^{-6} \leq (1 - a/M) \leq 2 \times 10^{-4}$.

The reason that this is required is that the region where proton targets tend to collect, for a given value of a/M, is of a proper coordinate thickness $D(r_{\rm ms} - r_{\rm mb}) \approx M \ln(1/\alpha^{1/2})$, where $\alpha \ll 1$.

However, this region must be deep within the ergosphere, and must also be a region of large photon blueshift; hence α must be very small, leading to the extremely large angular momentum density ranges indicated above. In PPP processes, the associated escape energies of the PPP pairs will be on the order of m_p, in a 40° angle range with respect to the equator (Piran and Shaham 1977*a*). This is because they will tend to follow a photon-like trajectory as they leave the black hole, and the kinematic analysis for escaping photons given by Piran and Shaham (1977*a*) should hold, to a good approximation. The PPP pairs give an energy output. Suppose this energy output gives a net luminosity at infinity of the value L; then the PPP process can generate this output by converting the mass of the "Penrose injected" protons into the total energy of the "Penrose ejected" pairs. If we write the value of the luminosity L at infinity as a function of the effective dM/dt, converted into photon energy via PPP, we have

$$L = \beta dM/dt , \tag{4}$$

where β is the efficiency of conversion of Penrose accreted proton masses into outgoing photon energy, via the Penrose ejected $e_\pm$ pairs (here natural gravitational units with $c = 1$, $G = 1$ are used). However, in the LNRF, the mass rate $dM/d\tau$ is given by

$$dM/d\tau = (dt/d\tau)(dM/dt) ; \tag{5}$$

and for the LNRF observer, $d\tau = e^\nu dt$.

On the other hand, $V \gtrsim \frac{1}{2}$ for the Penrose injected protons on negative energy trajectories. This is because the blueshifted photon energies are $E_\gamma \geq m_p$ just before the collision occurs in the LNRF. Hence we expect the recoil proton to have velocity boosts $V \geq \frac{1}{2}$. This is the same kind of assumption made by Piran and Shaham (1977*a*) except in terms of electron velocity boosts in their associated photon-electron collisions. Therefore, the LNRF proton accretion rate through a spherical surface at r, associated with a local proton number density n_p, and flux $n_p V \geq n_p/2$ is

$$\frac{dM}{d\tau} = m_p(n_p/2)4\pi r^2 = e^{-\nu}\left(\frac{dM}{dt}\right) = e^{-\nu}L/\beta . \tag{6}$$

On the other hand, in the equatorial region, for collisions at $r = m(1 + \epsilon)$ in the ergosphere of spinning black holes with $a/M = 1 - \alpha^3/4$,

$$e^{-\nu} \approx \frac{1}{r - r_+} \approx [8/(2\epsilon^2 - \alpha^3)]^{1/2} . \tag{7}$$

Hence the local value of $dM/d\tau$ is larger than dM/dt, due to the same gravitational mechanism which causes the corresponding photon blueshift mentioned earlier.

From (6) and (7), we find n_p, the proton number density, involved in the PPP process, as:

$$n_p(r, \alpha) \approx \left[\left(\frac{L}{\beta 2\pi r^2 m_p}\right)\left(\frac{8}{2\epsilon^2 - \alpha^3}\right)^{1/2}\right], \tag{8}$$

where m_p is the mass of the proton, and $c = 1$ the speed of light. Most of the proton targets for PPP will be in the $r = r_{\rm mb}$ to $r = r_{\rm ms}$ range, and the n_p there are what counts for PPP. Converting (8) back to cgs units (noting that this causes a factor of c^3 to appear in the denominator of [8]), and using the astrophysically convenient measure of M (mass of black hole); $\dot{M}$ (the PPP proton mass injection rate); $r \approx M$ (Kerr event horizon for mass M); and L (the PPP induced luminosity); we find that (8) can be written in a more physically revealing form as

$$n_p(r_{\rm mb}, \alpha) \approx \left[(2.8 \times 10^{12})\left(\frac{\alpha^{-3/2}\dot{M}_1}{(M_8)^2}\right)\right] \text{ protons cm}^{-3}, \tag{9a}$$

$$n_p(r_{\rm ms}, \alpha) \approx \left[(2.0 \times 10^{12})\left(\frac{\alpha^{-1}\dot{M}_1}{(M_8)^2}\right)\right] \text{ protons cm}^{-3}, \tag{9b}$$

where we made the substitution $M = (10^8\, M_\odot)M_8$; $\dot{M} = (M_\odot\, {\rm yr}^{-1})\dot{M}_1$, $r = (1.5 \times 10^{13}\,{\rm cm})M_8$; $L = [(6 \times 10^{46}\,{\rm ergs\,s}^{-1})\beta\dot{M}_1]$, and we choose to let M_8 and $\dot{M}_1$ be given in astrophysical units of $10^8\, M_\odot$ and $M_\odot\,{\rm yr}^{-1}$, respectively. Now the mean free path for the PPP process is $\lambda = 1/n_p\sigma$, where σ is the corresponding pair production cross section, at the value of the $E_\gamma \sim$ (GeV) blueshifted photon energy, in the LNRF. Hence the mean free path for the PPP process is

$$\lambda_{\rm mb} \approx (3 \times 10^{-13})\left[\left(\frac{\alpha^{3/2}}{\sigma}\right)[(M_8)^2/\dot{M}_1]\right] \text{ cm}, \tag{10a}$$

$$\lambda_{\rm ms} \approx (5 \times 10^{-13})\left[\left(\frac{\alpha}{\sigma}\right)[(M_8)^2/\dot{M}_1]\right] \text{ cm}. \tag{10b}$$

We expect a reasonable condition for PPP to work is that λ be on the order of the thickness of the target domain, $D(r_{\rm ms} - r_{\rm mb}) \approx 1.5 \times 10^{13} M_8 \ln(1/\alpha^{1/2})$. We base our estimates on the proper thickness $D(r_{\rm ms} - r_{\rm mb})$, since we find that if we use the formula for "spatial distance," Δl given in Landau and Lifshitz (1958) but applied to the LNRF observer's proper frame, where $d\phi = \omega dt$, then

$$\Delta l = g_{rr} dr\,.$$

When this formula is applied to *radial* distances, then $D(r_{\rm ms} - r_{\rm mb}) = \int_{r_{\rm mb}}^{r_{\rm ms}} g_{rr} dr$. This is for the Kerr metric, inside the ergosphere, where the LNRF observer is located. Hence we require, for PPP to occur efficiently in the ergosphere, that $D(r_{\rm ms} - r_{\rm mb}) \geq \lambda$. This yields

$$(M_8/\dot{M}_1) \leq 0.5 \times 10^{26}\sigma[\ln(1/\alpha^{1/2})/\alpha^{3/2}] \quad (r_{\rm mb}\text{ case}); \tag{11a}$$

$$(M_8/\dot{M}_1) \leq 0.3 \times 10^{26}\sigma[\ln(1/\alpha^{1/2})/\alpha] \quad (r_{\rm ms}\text{ case}). \tag{11b}$$

For $e_\pm$ production at blueshifted energy of $E_\gamma \sim m_p$, then (cf. Lang 1974) $\sigma \approx 10^{-26}\,{\rm cm}^2$, so for this case PPP will occur efficiently if

$$(M_8/\dot{M}_1) \leq 0.5\,\frac{\ln(1/\alpha^{1/2})}{\alpha^{3/2}} \quad (r_{\rm mb}\text{ case}), \tag{12a}$$

$$(M_8/\dot{M}_1) \leq 0.3[\ln(1/\alpha^{1/2})]/\alpha \quad (r_{\rm ms}\text{ case}), \tag{12b}$$

with α given by equation (3).

III. LIFETIME OF PENROSE PAIR PRODUCTION PHASE

Thorne (1974) has shown how accretion of matter and radiation causes extreme black holes to spin-down to "canonical" values of $a/M \approx 0.998$ in finite periods of time. Thorne showed that the asymptotic behavior of spinning Kerr metrics, with an infall of matter and radiation, will tend to evolve to the "canonical" limit of $a/M \approx 0.998$. Even though this asymptotic limit is relatively insensitive to perturbations on the model, the "initial response" of the system will be very sensitive to elements which affect the photon flux through the event horizon. Our value is associated with such an "initial response" period. Although the details of the spin-down effect with PPP occurring remain to be worked out, it is interesting to estimate the lifetime of a "PPP powered QSO" using Thorne's calculation as a limiting model. Assuming initial conditions of $M = M_i$ and $a/M = 1.0$, Thorne showed that the extreme initial spin-down period was characterized by $0.999960 \leq a/M \leq 1.000000$, for $1.001 \geq M/M_i \geq 1.000$ where $M_i = (10^8 M_\odot)M_8$. This means that during this extreme period $\Delta M < 10^{-3}M_i = (10^5 M_\odot)M_8$; hence we can estimate a lower bound on the time interval τ, of this period, assuming PPP is present in the dynamics, as

$$\tau(\text{yr}) = \Delta M/\dot{M} < (10^5\ \text{yr})(M_8/\dot{M}_1)\,, \tag{13}$$

with $(a/M)_{\text{avg}} \approx 0.999980$, or recalling that $a/M = (1 - \alpha^3/4)$, with $(\alpha)_{\text{avg}}{}^3 \approx 8 \times 10^{-5}$, over the spin-down period τ. This value of $(\alpha)_{\text{avg}}{}^3$ is to be understood as an upper bound to the actual value, when PPP and "inverse" Compton scattering processes are included in the dynamics, since they will tend to reduce the number of photons accreted by the hole in favor of negative energy Penrose injected protons from the r_{ms}, r_{mb} regions (Piran 1978). So if we use $(\alpha)_{\text{avg}}{}^3 \approx 8 \times 10^{-5}$ in (11), we have the conditions

$$(M_8/\dot{M}_1) \lesssim 88 \quad (r_{\text{mb}}\ \text{case})\,; \tag{14a}$$

$$(M_8/\dot{M}_1) \lesssim 11 \quad (r_{\text{ms}}\ \text{case})\,, \tag{14b}$$

which represents the range of $(M_8/\dot{M}_1)$ associated with PPP being feasible during the initial spin-down period of an extreme, massive black hole, of magnitude given by (12). The actual value of $(\alpha)_{\text{avg}}{}^3$ remains to be calculated from a generalization of Thorne's work, but it is interesting to estimate the $(M_8/\dot{M}_1)$ value necessary for $\tau < 10^6$ yr. This is a characteristic time period which is consistent with the concept of QSO "aging," given their apparent redshift distribution in the universe. This implies that from (13), $(M_8/\dot{M}_1) = 10$, which is consistent with (14b), for r_{ms} scattering points, and uses the value $(\alpha)_{\text{avg}}{}^3 \approx 8 \times 10^{-5}$; while r_{mb} scattering will be less efficient in this case. It is known that the observed ranges of $\dot{M}_1$ for QSOs lie in the region $0.06 \leq \dot{M}_1 \leq 60$ (these values are for luminosities in the range $10^{45} \lesssim L \lesssim 10^{48}$ ergs s^{-1} and $\beta \approx 0.3$), so the associated mass range of "PPP powered QSOs" is $0.6 \leq M_8 \leq 600$ or $6 \times 10^7 M_\odot \leq M \leq 6 \times 10^{10} M_\odot$, which is consistent with various theoretical estimates from other sources (cf. Eardley and Press 1975). Using (3a) and $(\alpha)_{\text{avg}}{}^3 < 8 \times 10^{-5}$, we see that the required energy of the incoming photons, emitted by the accretion disk—which is assumed to be surrounding the PPP powered QSO—is $(E_{\text{ph}} - \omega l_{\text{ph}}) < 10$ MeV. This photon energy requirement is reasonable for a variety of accretion disk scenarios and suggests that it is plausible that PPP in extreme, massive, Kerr metrics, may be a viable model for QSO energy output. The physics of accretion disks have been examined by Novikov and Thorne (1973). For very massive black holes, the "inner region" of the accretion disk could be emitting photon energies $E \approx 100$ MeV when this region is optically thin. This is certainly possible for very massive black holes $M \approx 10^8 M_\odot$. Optically thin disks have instabilities associated with them which might account for the fluctuations in QSO output, via induced fluctuations in the photon-induced PPP processes. Based on the bremsstrahlung cooling times in the inner disk, such fluctuations could range from 1 light-day to 1 lt-yr, depending on the mass values used.

Thorne (1974) showed that it is not possible for a black hole, accreting matter and radiation, to spin-up above the "canonical" value $a/M \approx 0.998$ or $\alpha^3 \approx 8 \times 10^{-3}$. For this reason it is possible that the extreme values of $(\alpha)_{\text{avg}}{}^3 \ll 8 \times 10^{-3}$, required for PPP-powered QSOs to work, may come from "primordial turbulence." The well-known concept of primordial turbulence offers an astrophysical scenario for the existence of massive black holes in galactic nuclei, which considers them to be the primordial $M \approx 10^7 M_\odot$ "seeds" around which galaxies condense (Ryan 1972; Ozernoy and Chibisov 1971; Harrison 1970). The astrophysical scenario associated with primordial turbulence has also been used in a different context to explain the possible existence of microscopic black holes (Hawking 1967).

We finally mention the fact that, since our PPP model for QSOs involves the production of matter and antimatter (i.e., electron-positron pairs), we might expect that there would be some evidence of redshifted 0.5 MeV pair-annihilation radiation coming from the region of ejection near the equatorial region where pairs could be effectively brought to rest by severe synchrotron and Compton losses. The observation of the resultant, redshifted, pair-annihilation γ-rays (emanating from QSOs) would be strong evidence for the PPP mechanism, as a source of energy, to explain the magnitude of QSO luminosity and the compactness of their size.

IV. CONCLUSIONS

We have shown that photon-induced Penrose pair production in the ergospheres of massive Kerr black holes with $a/M \lesssim 1$ is a physically possible process. This might provide a natural explanation for the energy production in QSOs and active galactic nuclei if the extreme values of $a/M \lesssim 1$ required by the process could be justified via some other mechanism (e.g., primordial turbulence). We have shown that application of this process to QSOs yields reasonable predictions about their masses, lifetimes, and luminosities, and suggest that rapid fluctuations in the inner region, of the associated optically thin accretion disk, might cause the PPP process to vary in its output, in a manner which could explain the fluctuations associated with QSOs. For these reasons, we feel that a more detailed analysis of this process should be carried out, as well as an experimental search for the copious, diffuse 0.5 MeV γ-ray output which may be emanating from QSOs, if they are indeed being powered by the PPP process.

We thank Dr. T. Piran for useful comments and suggestions.

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