

2011

Banking in a Matching Model of Money and Capital

Valerie R. Bencivenga
University of Texas - Austin

Gabriele Camera
Chapman University, camera@chapman.edu

Follow this and additional works at: http://digitalcommons.chapman.edu/economics_articles

 Part of the [Economics Commons](#)

Recommended Citation

Bencivenga V., and G. Camera (2011). Banking in a matching model of money and capital. *Journal of Money, Credit, and Banking* 43(7), 449-476. doi: 10.1111/j.1538-4616.2011.00446.x

This Article is brought to you for free and open access by the Economics at Chapman University Digital Commons. It has been accepted for inclusion in Economics Faculty Articles and Research by an authorized administrator of Chapman University Digital Commons. For more information, please contact laughtin@chapman.edu.

Banking in a Matching Model of Money and Capital

Comments

This is the accepted version of the following article:

Bencivenga V., and G. Camera (2011). Banking in a matching model of money and capital. *Journal of Money, Credit, and Banking* 43(7), 449-476

which has been published in final form at DOI: [10.1111/j.1538-4616.2011.00446.x](https://doi.org/10.1111/j.1538-4616.2011.00446.x).

Copyright

Wiley

Banking in a Matching Model of Money and Capital¹

Valerie R. Bencivenga
Dept. of Economics
University of Texas at Austin

Gabriele Camera
Dept. of Economics
Purdue University

Abstract

We introduce banks in a model of money and capital with trading frictions. Banks offer demand deposit contracts and hold primary assets to maximize depositors' utility. If banks' operating costs are small, banks reallocate liquidity eliminating idle balances and improving the allocation. At moderate costs, idle balances are reduced but not eliminated. At larger costs, banks are redundant. A central bank policy of paying interest on bank reserves can reverse inflation's distortionary effects, and increase welfare, but only when costs are small. The threshold levels of banks' costs increase with inflation, suggesting inflation and banks' utilization are positively associated.

JEL classification: E41; E50

Keywords: Money; Credit; Banking; Monetary Policy

1 Introduction

This paper introduces banks into a model of money and capital based on Lagos and Wright (2005) and Aruoba and Wright (2003). Banks have a natural role to play in this model because money balances not used in transactions carry an opportunity cost. But because agents experience random shocks to trading opportunities and preferences, the value of money balances is heterogeneous and potentially very large. Therefore, as long as bank operating costs (transaction costs) are not too large, banks improve the return on saving and increase welfare, by pooling agents' deposits in order to provide liquidity insurance.

Of course, there is a long literature starting with Diamond and Dybvig (1983) investigating this role of banks in various monetary models; see Bencivenga and Smith (2003) for references. The novel aspect here is an analysis of liquidity insurance in a setting in which money has a role to play due to the structure of markets and of information, and banks cannot obviate the role of money by reallocating deposits of the consumption good.² In this setting, banks can improve welfare if the resource costs of operating banks are not too big, and if inflation is not too high relative to these costs. However, banks cannot always achieve rates of return on deposits that are socially optimal. Under some circumstances, a central bank policy of paying interest on bank reserves may improve welfare further, although not to the level attained by a social planner.

When choosing their portfolio of assets in the centralized market (the second of two markets opening sequentially each period), agents view the illiquid investment technology (capital formation) as an alternative to money for transforming current consumption into consumption in the next centralized market. The return on this investment is, in general, higher than the rate of return on money. However, the value of money balances is random when agents

choose their portfolio, as a result of shocks to both trading opportunities and the marginal utility of consumption, which are realized in the intervening decentralized market, and which are independent across agents and over time. Faced with random liquidity needs, it may be welfare-improving for agents to create a third instrument for saving in the centralized market—agents may form coalitions (banks) that accept deposits of the consumption good, and hold the portfolio of money and capital investment that maximizes expected utility of depositors.³

When (banks') operating costs are small, banks improve the allocation, by making it unnecessary for agents to carry money into the decentralized market. Banks hold the entire money supply as reserves, to meet withdrawal demand of buyers who engage in decentralized trade, and they hold remaining assets in the form of investment in capital, to benefit depositors who withdraw in the next centralized market (residual claimants). As in Diamond and Dybvig (1983), banks eliminate idle money balances. In such an equilibrium, a policy of interest on reserves improves welfare even further. If lump-sum taxes are feasible, then the central bank can implement the Friedman rule by paying interest on reserves, which allows efficient decentralized trades (although there is a distortion in the centralized market, due to banks' operating costs, and therefore a slight departure from the allocation at the Friedman rule).

At higher levels of operating costs, banks improve the allocation, but by less. There is a threshold level of costs, which depends on the inflation rate, above which banks reduce, but do not eliminate, idle money balances—only the most liquidity-constrained buyers optimally incur the costs of bank services. Once costs pass this threshold, a policy of interest on reserves does not improve welfare. When operating costs are even higher relative to inflation, the value to agents of this limited reduction in idle money balances is too small to justify the costs. Above this second threshold, banks are redundant, and the equilibrium is as in Aruoba and Wright

(2003). In this model, the usefulness of banks—the value to agents of pooling their saving, in order to create bank deposits as an additional financial asset—is an endogenous outcome, dependent on the capital stock and the price level, as well as transaction costs, the money growth rate, and the distributions of trading opportunities and the marginal utility of consumption in decentralized trade, i.e., on the distribution of liquidity needs.

As is standard in many representative-agent monetary models, the allocation without banks is efficient when nominal interest rates are zero, so there is scope for Diamond-Dybvig banks to increase welfare only when nominal interest rates are positive, and banks cannot restore full efficiency. Banks can reallocate liquidity across depositors, but because money is used to carry out decentralized trade, buyers making withdrawals for this purpose demand redemptions in money, which constrains banks to hold reserves that are subject to the inflation tax. Therefore, the higher the inflation rate, the larger are the threshold levels of bank operating costs that at first, prevent banks from completely eliminating idle money balances, and then make banks redundant.

These results suggest that when enforcement problems are severe (which limits loans), as in developing countries with weak legal systems, we should see heavy utilization of banks in high-inflation environments. At lower inflation rates, much smaller costs would be predicted to render the liquidity provided by banks too expensive to be utilized. In a developing country context, relevant costs include costs of accessing banks, such as the time needed to visit a bank branch (time away from work), as well as costs of maintaining a network of bank branches. For example, this model can explain why bank services were large relative to GDP in Brazil even during its years of acute inflation, while in India, it has taken decades for banks to penetrate rural

villages, even though inflation has been much lower. In general, we should observe a positive correlation between utilization of banks and inflation.

In an equilibrium in which agents do not carry money into the decentralized market, a policy of paying interest on reserves (or equivalently, allowing banks to hold reserves in the form of interest-bearing government debt) is an incentive-compatible way for the central bank to give money injections only to those agents with immediate liquidity needs. This mechanism breaks down when banks' operating costs are sufficiently large that all agents hold precautionary money balances. The threshold level of costs is increasing in the inflation rate. Thus, this model suggests that in low-cost environments, we should observe the central bank paying interest on reserves only at low inflation rates. For various reasons, we would expect bank operating costs to be larger in developing countries, suggesting we might see central banks paying interest on reserves even at higher inflation rates. However, government reliance on seigniorage is not modeled here, and that motive for inflation would suggest high reserve requirements and zero or low interest rates paid on reserves, in order to permit a budget deficit to be financed at the lowest possible inflation rate.

Another policy implication of the model concerns currency unions. A currency union can be thought of as a coalition of agents committed to harmonizing their financial policies in order to improve agents' access to liquidity. If financial integration involves substantial resource costs, our results suggest that a currency union might be socially desirable only for coalitions of high-inflation countries. In this respect, our analysis mirrors some of the concerns of Kiyotaki and Moore (2002), who demonstrate that a currency union can lead to too little specialization, and an inferior allocation of consumption.

Section 2 of the paper describes the environment. Section 3 presents the solution to the planner's problem. Sections 4 and 5 solve agents' problems and partially characterize stationary equilibrium. Section 6 sets up the case where banks are redundant because transaction costs are large. Section 7 analyzes the allocative effects and welfare benefits of banks arising when costs are zero or small. Section 8 analyzes central bank policy of paying interest on reserves. Section 9 discusses the case of moderate costs, when banks improve welfare, but an interest-on-reserves policy would yield no further improvement. Section 10 concludes.

2 Environment

The model is based on Lagos and Wright (2005). Time is discrete, and indexed $t = 0, 1, \dots$. Each period, two markets open in sequence. In the decentralized market (market 1), agents experience bilateral matches in which a perishable "specialized good" is produced and consumed. Labor is the sole factor of production of the specialized good. All agents meet and trade in the centralized market (market 2), in which a nonstorable "general good" is produced, consumed, and invested in capital formation. The general good is produced using capital and labor, as in Aruoba and Wright (2003).

At the start of each date, agents are identical. Upon entering market 1, each agent becomes a buyer or a seller, or is idle, and each buyer has either high or low marginal utility of consumption, as a result of a "type" shock. Realizations of agents' type shocks are independent across agents and over time. Every buyer is in a bilateral match with a seller.

The two primary assets are fiat money and capital. Capital is produced using an investment technology; one unit of the general good invested in market 2 on date $t - 1$ yields one unit of capital at the beginning of market 2 on date t . The central bank changes the money supply at an exogenous, constant rate via lump-sum money transfers to agents.

Transactions in market 1 require money.⁴ The type shock causes heterogeneity in agents' liquidity needs in market 1, and therefore it is a liquidity shock. Agents faced with random liquidity needs would like to pool their saving at the end of date $t - 1$, in order to reduce unused money balances in market 1 of date t , and increase investment, which has a higher yield. Our environment allows agents to deposit their saving in banks in market 2 of date $t - 1$, and then pay a fixed cost ϕ to withdraw money from their deposit in market 1 on date t , once their type shock is realized. Remaining deposits are paid out in market 2 of date t , in the general good. Thus a bank is a coalition of agents that offers demand deposit contracts, and invests deposits in the primary assets to maximize expected utility of depositors. Banks' potential to improve welfare derives from their ability to reallocate liquidity and improve the composition of aggregate saving.⁵ In this model, banks do not make loans because, by assumption, they cannot solve the enforcement problems that arise in recovering loans.

In market 2, investment undertaken on the previous date yields capital. Competitive firms employ capital and labor to produce the general good according to a neoclassical technology. Capital fully depreciates during use. Agents supply labor, earn wage income, and receive interest income on their existing deposits. Banks rent capital to firms and pay depositors in market 2 out of capital income. Equivalently, depositors are paid in capital, which is then rented to firms.

We use subscripts to indicate the date, and whether the agent is a buyer, a seller, or idle (b , s , or n), in market 1. An additional subscript indicates the market (1 or 2). A superscript indicates whether the agent's match in market 1 involves a buyer with low or high marginal utility of consumption (L , H). For example, h_{2st}^L gives hours worked in market 2 on date t by an

agent who was a seller in market 1, in a match with a low marginal utility buyer. When possible, subscripts and superscripts are omitted.

2.A Market 1

In market 1, an agent is matched with probability α , and is idle otherwise. A matched agent is equally likely to be a buyer or a seller. A buyer derives utility $\theta^i u(x)$ from consuming x units of specialized goods, where θ^i ($i = L, H$) is the marginal utility component of the type shock, which is observable by the seller. Assume $\theta^H > \theta^L$, and $\theta^i = \theta^H$ with probability σ . The notation $\sigma^H = \sigma$ and $\sigma^L = 1 - \sigma$ will be convenient. The function u satisfies the Inada conditions. A seller can produce h units of specialized goods by working h hours, at utility cost $c(h)$, where $c(0) = 0$, and $c', c'' > 0$. Letting h_{1t}^i denote the output of a seller matched with a buyer of type i in market 1 on date t , market-clearing requires $h_{1t}^i = x_{1t}^i$. An idle agent can neither produce nor consume.

We will study the model in real terms, dividing nominal magnitudes on date t by the price level in market 2, denoted p_{2t} . The expected value of entering market 1 on date t with real money balances $m_t = M_t / p_{2t}$ and real bank deposit $d_t = D_t / p_{2t}$ is given by

$$V(m_t, d_t) = (1 - \alpha)W(m_{nt}, d_{nt}) + \frac{\alpha}{2} \sum_{i=L, H} \sigma^i \left[\theta^i u(x_{1t}^i) - c(h_{1t}^i) + W(m_{bt}^i, d_{bt}^i) + W(m_{st}^i, d_{st}^i) \right]$$

where $W(m_{jt}^i, d_{jt}^i)$ is the value of carrying real money balances m_{jt}^i and real deposit d_{jt}^i into market 2 ($j = n, s, b$ and $i = L, H$). We conjecture (and later verify) that agents optimally enter

the period with the same portfolio of money and deposits (m_t and d_t), due to quasi-linear preferences in market 2.

Agents can withdraw money from their bank deposit in market 1, but if they do, a real transaction cost ϕ is deducted from their deposit. Let r_{1t} and r_{2t} denote the real interest rates paid by banks on withdrawals in markets 1 and 2 on date t . Under the conjecture that in equilibrium $r_{2t} \geq r_{1t}$, only buyers make withdrawals. A buyer's budget constraint in market 1 is

$$(1) \quad \frac{p_{1t}^i x_{1t}^i}{p_{2t}} \leq m_t + g_t^i (d_t - \phi)(1 + r_{1t}) \quad i = L, H$$

where p_{1t}^i is the price of specialized goods in transactions involving buyers of type i ($i = L, H$), and g_t^i denotes the proportion of buyer i 's deposit, net of the fixed cost, that is withdrawn, i.e.,

$$(2) \quad 0 \leq g_t^i \leq 1 \quad i = L, H .$$

Buyers and sellers are price takers. Buyers choose x_{1t}^i and g_t^i , and sellers choose h_{1t}^i , to maximize $V(m_t, d_t)$. Money balances carried into market 2 are given by

$$(3) \quad \begin{aligned} m_{nt} &= m_t + \tau_t \\ m_{st}^i &= m_t + \frac{p_{1t}^i x_{1t}^i}{p_{2t}} + \tau_t \quad i = L, H ; j = n, s, b \\ m_{bt}^i &= m_t + g_t^i (d_t - \phi)(1 + r_{1t}) - \frac{p_{1t}^i x_{1t}^i}{p_{2t}} + \tau_t \end{aligned}$$

where τ_t is a real money transfer received (or paid) in market 2. Deposits are given by

$$(4) \quad \begin{aligned} d_{nt} &= d_{st}^i = d_t \\ d_{bt}^i &= (1 - g_t^i)(d_t - \phi) + \chi_t^i \phi \end{aligned}$$

where the indicator function χ_t^i “rebates” the transaction cost to buyers who choose not to withdraw money in market 1 ($\chi_t^i = 1$ if $g_t^i = 0$, and equals zero otherwise).

2.B Market 2

At the start of market 2 on date t , each agent is indexed by the realization of his shock in market 1, and if he was a seller, the buyer’s marginal utility. Preferences are $U(x_{2jt}^i) - h_{2jt}^i$, where x_{2jt}^i is consumption of the general good, and h_{2jt}^i is hours worked at the real wage rate w_t , $j = n, b, s$, and $i = L, H$. The function U satisfies the Inada conditions.

Expected lifetime utility from entering market 2 with real balances m_{it}^j and real deposit d_{it}^j is

$$W(m_{jt}^i, d_{jt}^i) = U(x_{2jt}^i) - h_{2jt}^i + \beta V(m_{t+1}, d_{t+1}) .$$

Agents choose non-negative values x_{2jt}^i , h_{2jt}^i , m_{t+1} , and d_{t+1} to maximize W subject to the nominal budget constraint

$$M_{t+1} + p_{2t}d_{t+1} + p_{2t}x_{2jt}^i = p_{2t}w_t h_{2jt}^i + M_{jt}^i + p_{2t}d_{jt}^i(1 + r_{2t}) .$$

The left-hand side gives consumption of the general good, plus saving in the form of money and deposits of the general good. The right-hand side gives wage income, plus money balances and deposits (including interest earned). Divide by p_{2t} , and use $\pi_{t+1} = \frac{p_{2,t+1}}{p_{2t}}$, to obtain

$$(5) \quad h_{2jt}^i = \frac{m_{t+1}\pi_{t+1} + d_{t+1} + x_{2jt}^i - m_{jt}^i - d_{jt}^i(1+r_{2t})}{w_t}.$$

From this point on we assume $h_{2jt}^i \geq 0$; this means assuming $U'(x_2)$ is sufficiently large that even the wealthiest agents in market 2 choose to work.

2.C Banks

On each date t , banks offer a one-period deposit contract consisting of real net interest rates $r_{1,t+1}$ and $r_{2,t+1}$ to be paid on withdrawals of money in market 1, and the general good in market 2, on date $t+1$.⁶ Banks cannot observe realizations of agents' shocks, and therefore, $r_{1,t+1}$ and $r_{2,t+1}$ are not contingent on the agent's current shock or history.

A bank's objective is to maximize expected utility of its depositors, subject to several constraints. First, we have the balance sheet constraint faced by the bank. In market 2 of date t , each agent deposits d_{t+1} general goods in a bank, and the bank does three things with these goods. Per depositor, the amount k_{t+1} is allocated to investment in capital formation. The transaction costs borne by buyers who make withdrawals in market 1 are resource costs of operating banks. Therefore, the bank must set goods aside to cover the operating costs in market 1 of date $t+1$. Per depositor, this amount is l_{t+1} , given by

$$(6) \quad l_{t+1} = \frac{\alpha}{2} \sum_{i=L,H} \sigma^i (1 - \chi_{t+1}^i) \phi$$

Clearly, $l_{t+1} = 0$ if there are no withdrawals ($\chi_{t+1}^i = 1, i = L, H$). Finally, denote by Z_{t+1} the nominal amount of reserves of money needed per depositor to meet withdrawal demand in market 1 of $t + 1$. Letting $z_{t+1} = \frac{Z_{t+1}}{p_{2,t+1}}$, the bank must allocate $z_{t+1}\pi_{t+1}$ of deposits to acquiring reserves on date t , in order to have real balances of z_{t+1} in market 1 of date $t + 1$.

Hence, the balance sheet constraint of a bank is

$$(7) \quad d_{t+1} = k_{t+1} + l_{t+1} + z_{t+1}\pi_{t+1} .$$

Second, a bank's real money balances in market 1 on date $t + 1$ must be sufficient to pay interest at the rate $r_{1,t+1}$ on withdrawals in market 1:

$$(8) \quad z_{t+1} = (1 + r_{1,t+1}) \frac{\alpha}{2} \sum_{i=L,H} \sigma^i g_{t+1}^i (d_{t+1} - \phi) .$$

In market 1, interest is paid only on the fraction g_{t+1}^i of the buyer's deposit (net of the transaction cost) that he withdraws. This constraint assumes banks do not carry reserves from market 1 to market 2. At inflation rates above the Friedman rule, this assumption holds because the opportunity cost of holding money balances is positive, and at the Friedman rule, it is without loss of generality.

Third, a bank's capital income must be sufficient to pay interest at the rate $r_{2,t+1}$ on deposits kept until market 2 of date $t + 1$. This constraint is

$$(9) \quad \rho_{t+1} k_{t+1} = (1 + r_{2,t+1}) \left\{ \left(1 - \frac{\alpha}{2}\right) d_{t+1} + \frac{\alpha}{2} \sum_{i=L,H} \sigma^i \left[(1 - g_{t+1}^i) (d_{t+1} - \phi) + \chi_t^i \phi \right] \right\} .$$

The real value of a unit of capital, ρ_{t+1} , is its marginal product on date $t+1$. Sellers and idle agents have intact deposits $(1 - \frac{\alpha}{2})$ of agents). Buyers, whose proportions are $\frac{\alpha}{2} \sigma^i$ ($i = L, H$), have the fractions $1 - g_{t+1}^i$ of their deposits remaining (net of the transaction cost, for buyers who made withdrawals in market 1). Depositors who withdraw in market 2 are residual claimants on the bank.

2.D Production and market clearing in market 2

In market 2 on date t , real per capita output of the general good is given by

$$(10) \quad y_t = f(h_{2t}, k_t)$$

where h_{2t} is per capita labor input, k_t is the per capita capital stock, and f is a constant returns to scale production technology employed by competitive firms. Per capita labor input is given by

$$h_{2t} = \frac{\alpha}{2} \sum_{i=L,H} \sigma^i (h_{2bt}^i + h_{2st}^i) + (1 - \alpha) h_{2nt} .$$

Factor markets are competitive, and we can think of wages and rent on capital as being paid in the general good. This gives us the standard factor pricing relationships

$$(11) \quad \rho_t = \frac{\partial f(h_{2t}, k_t)}{\partial k_t} \quad \text{and} \quad w_t = \frac{\partial f(h_{2t}, k_t)}{\partial h_{2t}} .$$

Goods market clearing in market 2 requires

$$(12) \quad y_t = k_{t+1} + l_{t+1} + x_{2t}$$

where x_{2t} is per capita consumption in market 2 (a weighted average, defined analogously to h_{2t}).

The per capita nominal money supply at the beginning of date t , denoted \bar{M}_t , is held as money balances M_t by agents, and as reserves Z_t by banks. Dividing by p_{2t} , the money market clearing condition in real terms is $\bar{m}_t = m_t + z_t$. The constant gross rate of growth of the money supply is $\mu > 0$. We focus on stationary monetary economies, where $m_t = m_{t+1} = m$ and $z_t = z_{t+1} = z$, and therefore $\pi = \mu$. As described earlier, the money stock changes via lump sum transfers to agents in market 2. The real per capita money transfer is given by

$\tau_t = \bar{m}_t(\mu - 1)$, which, in a stationary allocation, becomes

$$(13) \quad \tau = (m + z)(\pi - 1).$$

3 The planner's allocation

A brief discussion of the planner's problem is useful at this point, for two reasons. One is to motivate our focus on stationary equilibrium, and the other is to describe the efficient allocation as a reference against which to compare monetary allocations.

The planner's problem is to choose non-negative x_{1t}^H , x_{1t}^L , x_{2t} , h_{2t} , and k_t , for all t , to solve

$$\max \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\alpha}{2} \sum_{i=L,H} \sigma^i [\theta^i u(x_{1t}^i) - c(x_{1t}^i)] + U(x_{2t}) - h_{2t} \right\}$$

subject to $x_{2t} = f(h_{2t}, k_t) - k_{t+1}$ and $k_{t+1} \in [0, f(h_{2t}, k_t)]$. This problem separates into two problems, one static and the other dynamic. The static problem is to maximize the surplus of matches in market 1:

$$\max_{x_{1t}^i \geq 0} [\theta^i u(x_{1t}^i) - c(x_{1t}^i)], \quad i = L, H \quad \forall t.$$

The solution $x_1^i = x_1^{i*}$ is time-invariant, and satisfies $\theta^i u'(x_1^{i*}) = c'(x_1^{i*})$, with $x_1^{H*} > x_1^{L*} > 0$.

The dynamic problem is

$$\max_{h_{2t} \geq 0, k_{t+1} \geq 0} \sum_{t=0}^{\infty} \beta^t [U(f(h_{2t}, k_t) - k_{t+1}) - h_{2t}]$$

subject to $k_{t+1} \in [0, f(h_{2t}, k_t)]$, which has first-order conditions:

$$U'(x_{2t}) f_{h_2}(h_{2t}, k_t) - 1 = 0$$

$$-U'(x_{2t}) + \beta U'(x_{2,t+1}) f_k(h_{2,t+1}, k_{t+1}) = 0.$$

In stationary equilibrium, $f_k(h_2, k) = \frac{1}{\beta}$, $U'(x_2) = \frac{1}{f_{h_2}(h_2, k)}$, and $x_2 = f(h_2, k) - k$. These

three equations imply unique solutions for x_2 , h_2 , and k , and that the rate of return on capital is

$$\rho = 1/\beta.$$

4 Agents' optimization problems in market 1

A seller matched with a buyer of type $i = L, H$ chooses $h_{1t}^i \geq 0$ to solve

$$\max \left[-c(h_{1t}^i) + W(m_{st}^i, d_{st}^i) \right].$$

To obtain $W(m_{st}^i, d_{st}^i)$, use h_{2st}^i from (5), and m_{st}^i and d_{st}^i from (3)-(4). Imposing market-clearing yields

$$W(m_{st}^i, d_{st}^i) = U(x_{2jt}^i) - \frac{x_{2jt}^i}{w_t} + \frac{m_t + d_t(1+r_{2t}) + \tau_t}{w_t} + \frac{p_{1t}^i x_{1t}^i}{p_{2t} w_t} - \frac{m_{t+1} \pi_{t+1} + d_{t+1}}{w_t} + \beta V(m_{t+1}, d_{t+1}).$$

The seller's problem then becomes $\max_{x_{1t}^i \geq 0} \left[-c(x_{1t}^i) + \frac{p_{1t}^i x_{1t}^i}{p_{2t} w_t} \right]$, with first-order condition

$$(14) \quad -c'(x_{1t}^i) + \frac{p_{1t}^i}{p_{2t} w_t} = 0.$$

A buyer of type $i = L, H$ chooses $x_{1t}^i \geq 0$ and g_t^i to solve

$$\max \left[\theta^i u(x_{1t}^i) + W(m_{bt}^i, d_{bt}^i) \right]$$

subject to (1)-(2), where

$$W(m_{bt}^i, d_{bt}^i) = U(x_{2bt}^i) - \frac{x_{2bt}^i}{w_t} + \frac{m_t + g_t^i(d_t - \phi)(1+r_{1t}) + \left[(1-g_t^i)(d_t - \phi) + x_{1t}^i \phi \right] (1+r_{2t}) + \tau_t}{w_t} - \frac{p_{1t}^i x_{1t}^i}{p_{2t} w_t} - \frac{m_{t+1} \pi_{t+1} + d_{t+1}}{w_t} + \beta V(m_{t+1}, d_{t+1}).$$

The multipliers associated with the two constraints in (2) are $\underline{\lambda}_{gt}^i$ and $\bar{\lambda}_{gt}^i$ ($i = L, H$). Since (1)

may bind, the first-order condition for the choice of x_{1t}^i is

$$(15) \quad \frac{p_{2t}}{p_{1t}^i} \theta^i u'(x_{1t}^i) \geq \frac{1}{w_t}$$

where the left-hand side gives the marginal utility of expenditure. Substituting (14) into (15) yields

$$(16) \quad \frac{\theta^i u'(x_{1t}^i)}{c'(x_{1t}^i)} \geq 1 .$$

From (16) and the definition of x_1^{i*} we immediately have the following lemma.

Lemma 1: If (1) does not bind, then $x_{1t}^i = x_1^{i*}$ ($i = L, H$).

Using (16), the first-order conditions for g_t^i ($i = L, H$) can be written as

$$(17) \quad \frac{d_t - \phi}{w_t} \left\{ (1 + r_{1t}) \left[\frac{\theta^i u'(x_{1t}^i)}{c'(x_{1t}^i)} - 1 \right] - (r_{2t} - r_{1t}) \right\} + \underline{\lambda}_{gt}^i - \bar{\lambda}_{gt}^i = 0 .$$

Intuitively, if $\phi = 0$, agents do not carry money into market 1, and going into market 1, all money will be held by banks as reserves. However, if $\phi > 0$, then the magnitudes of ϕ and π determine which buyers (if any) make withdrawals from their deposits, and in what amounts (regardless of whether or not agents carry money into market 1). At this point we consider stationary allocations in which H buyers, at least, and possibly all buyers, make withdrawals in market 1. I.e., we consider stationary allocations in which $0 \leq g^L < g^H \leq 1$, and some or all money is held by banks as reserves. It is intuitive that $g^L < g^H$, or equivalently, that $x_1^H \geq x_1^L$, and this will be verified in equilibrium. From inspection of (17) we have the following Lemma.

Lemma 2: In a stationary allocation in which $0 \leq g^L < g^H \leq 1$

$$(18) \quad \frac{\theta^L u'(x_1^L)}{c'(x_1^L)} \leq \frac{1 + r_2}{1 + r_1} \leq \frac{\theta^H u'(x_1^H)}{c'(x_1^H)}$$

where the first inequality is strict iff $g^L \geq 0$ binds, and the second inequality is strict iff $g^H \leq 1$ binds.

The multiplier $\bar{\lambda}_g^H$ gives the value of relaxing the constraint that in market 1, an H buyer's withdrawal is limited to his deposit (minus ϕ), i.e., he cannot borrow. The multiplier $\underline{\lambda}_g^L$ gives the value of relaxing the constraint that an L buyer cannot lend. Whether buyers' consumption levels in market 1 are efficient depends on whether these constraints bind, which in turn depends on $r_{1,t+1}$ and $r_{2,t+1}$.

5 Agents' optimization problem in market 2

Using (5) in $W(m_{jt}^i, d_{jt}^i)$, we have

$$W(m_{jt}^i, d_{jt}^i) = U(x_{2t}) - \frac{x_{2t}}{w_t} + \frac{m_{jt}^i + d_{jt}^i(1+r_{2t})}{w_t} + \max_{m_{t+1}, d_{t+1}} \left[-\frac{m_{t+1}\pi_{t+1} + d_{t+1}}{w_t} + \beta V(m_{t+1}, d_{t+1}) \right].$$

The first-order condition with respect to x_{2jt}^i implies that the optimal level of consumption in market 2 depends on the wage, and is the same for all agents:

$$(19) \quad U'(x_{2jt}^i) = \frac{1}{w_t} \Rightarrow x_{2jt}^i = x_{2t} \quad j = n, b, s; i = L, H.$$

Now consider an agent's choices of $m_{t+1} \geq 0$ and $d_{t+1} \geq 0$. The value function V is differentiable (see Aliprantis, Camera, and Ruscitti, 2009, for a proof), and therefore the first-order conditions are

$$(20) \quad \begin{aligned} -\frac{\pi_{t+1}}{w_t} + \beta \frac{\partial V(m_{t+1}, d_{t+1})}{\partial m_{t+1}} + \lambda_{mt} &= 0 \\ -\frac{1}{w_t} + \beta \frac{\partial V(m_{t+1}, d_{t+1})}{\partial d_{t+1}} + \lambda_{dt} &= 0 \end{aligned}$$

where $\lambda_{mt} \geq 0$ and $\lambda_{dt} \geq 0$ are multipliers on the non-negativity constraints on m and d .

Writing $W(m_{jt}^i, d_{jt}^i)$ as

$$W(m_{jt}^i, d_{jt}^i) = W(0, 0) + \frac{m_{jt}^i + d_{jt}^i(1+r_{2t})}{w_t}$$

we have

(21)

$$\begin{aligned} V(m_t, d_t) &= W(0, 0) + (1-\alpha) \left[\frac{m_{nt} + d_{nt}(1+r_{2t})}{w_t} \right] \\ &\quad + \frac{\alpha}{2} \sum_{i=L,H} \sigma^i \left[\theta^i u(x_{1t}^i) - c(h_{1t}^i) + \frac{m_{bt}^i + d_{bt}^i(1+r_{2t})}{w_t} + \frac{m_{st}^i + d_{st}^i(1+r_{2t})}{w_t} \right]. \end{aligned}$$

Using (3)-(4) in (21) yields

(22)

$$\begin{aligned} V(m_t, d_t) &= W(0, 0) + \frac{m_t + \tau_t}{w_t} + \frac{1+r_{2t}}{w_t} \left\{ \left(1 - \frac{\alpha}{2}\right) d_t + \frac{\alpha}{2} \sum_{i=L,H} \sigma^i (1 - g_t^i) [(d_t - \phi) + \chi_t^i \phi] \right\} \\ &\quad + \left(\frac{1+r_{1t}}{w_t} \right) \left(\frac{\alpha}{2} \right) \sum_{i=L,H} \sigma^i g_t^i (d_t - \phi) + \frac{\alpha}{2} \sum_{i=L,H} \sigma^i \left[\theta^i u(x_{1t}^i) - c(h_{1t}^i) + \frac{p_{1t}^i}{p_{2t} w_t} (h_{1t}^i - x_{1t}^i) \right]. \end{aligned}$$

The second and third terms give wealth at date t , inclusive of the money transfer and the return on deposits left intact until market 2. The fourth term gives the return on deposits withdrawn in

market 1. The last term is the expected surplus from trade in market 1. If (1) is binding, then

using (14), and $\frac{\partial x_{1t}^i}{\partial m_t} = \frac{p_{2t}}{p_{1t}^i}$ from (1), we have

$$\frac{\partial V(m_t, d_t)}{\partial m_t} = \frac{1}{w_t} \left\{ 1 + \frac{\alpha}{2} \left[\sum_{i=L,H} \sigma^i \frac{\theta^i u'(x_{1t}^i)}{c'(x_{1t}^i)} - 1 \right] \right\}.$$

Using this in (20), the Euler equation for m in a stationary allocation is

$$(23) \quad \frac{\pi}{\beta} = 1 + \frac{\alpha}{2} \left[\sum_{i=L,H} \sigma^i \frac{\theta^i u'(x_{1t}^i)}{c'(x_{1t}^i)} - 1 \right] + \frac{w}{\beta} \lambda_m.$$

Since the term in square brackets is non-negative (see (16)), a standard result immediately follows.

Lemma 3: $\pi \geq \beta$ must hold in any stationary allocation with $m > 0$. If $\pi = \beta$, then $x_1^i = x_1^{i*}$ ($i = H, L$).

From (14), and conjecturing that (1) is binding, we have $\frac{\partial x_{1t}^i}{\partial d_t} = g_t^i (1 + r_{1t}) \frac{p_{2t}}{p_{1t}^i}$, and

$$\frac{\partial V(m_t, d_t)}{\partial d_t} = \left(\frac{1 + r_{1t}}{w_t} \right) \left(\frac{\alpha}{2} \right) \sum_{i=L,H} \sigma^i g_t^i \frac{\theta^i u'(x_{1t}^i)}{c'(x_{1t}^i)} + \frac{1 + r_{2t}}{w_t} \left(1 - \frac{\alpha}{2} \sum_{i=L,H} \sigma^i g_t^i \right).$$

Using this in (20), the Euler equation for d in a stationary allocation is

$$(24) \quad \frac{1}{\beta} = 1 + r_2 + \frac{\alpha}{2} \sum_{i=L,H} \sigma^i g_t^i \left[(1 + r_1) \frac{\theta^i u'(x_{1t}^i)}{c'(x_{1t}^i)} - (1 + r_2) \right] + \frac{w}{\beta} \lambda_d.$$

Since $\lambda_d = 0$ when $d > 0$, Lemmas 1 and 2 immediately imply the following.

Lemma 4: In a stationary allocation with $d > 0$, $x_1^i = x_1^{i*}$, $i = L, H$ iff $1 + r_1 = 1 + r_2 = 1/\beta$.

Definition: A stationary equilibrium is a set of time-invariant quantities $(x_1^i, x_{2j}^i, h_1^i, h_{2j}^i, \text{ and } y)$, prices $(p_1^i, p_2, w, \rho, r_1, \text{ and } r_2)$, and asset holdings $(m_j^i, d_j^i, m, d, z, k, \text{ and } l)$, $j = n, b, s$, and $i = L, H$, that solve agents', banks', and firms' maximization problems, and clear all markets (goods, money, capital, and labor) on all dates. In particular, agents' first-order conditions (14), (15), (17), (19), and (20); the factor-pricing relationships (11); the market-clearing conditions (12), (13), and $x_1^i = h_1^i$; and banks' resource constraints (7), (8), and (9) must be satisfied in every period.

6 No withdrawals of deposits in market 1 (large ϕ)

We first look at the benchmark case where the transaction cost ϕ is so large that buyers do not make withdrawals in market 1. In this case, banks hold no monetary reserves. Banks invest all deposits in capital formation, and the equilibrium allocation is equivalent to one without banks, in which agents make direct use of the investment technology. The allocation is similar to that in Aruoba and Wright (2003), the key difference being that heterogeneity in market 1 implies consumption of L buyers may be unconstrained if inflation is sufficiently low.

To see this, consider a stationary equilibrium with $m, d > 0$ and $g^i = 0$ for $i = L, H$. From (24), $1 + r_2 = 1/\beta$. It follows that the allocation in market 2 corresponds to the planner's allocation, because banks' resource constraint is now $\rho k = d(1 + r_2)$, and the balance sheet constraint becomes $d = k$, implying $\rho = 1/\beta$. However, in market 1, buyers' consumption levels depend on π , and from Lemma 3, buyers' consumption is efficient only at the Friedman rule ($\pi = \beta$).

To study the allocation at higher inflation rates note that efficient consumption differs across buyers' types (Lemma 1). At the Friedman rule, L and H buyers leave market 1 with different money balances, because they both enter market 1 with money balances equal to m , but their expenditure differs. Since $x_1^{H*} > x_1^{L*}$, there is a range of inflation rates at which $x_1^H < x_1^{H*}$ and $x_1^L = x_1^{L*}$. To find this range, note that if $x_1^L = x_1^{L*}$, then using (23), x_1^H solves

$$\frac{\pi}{\beta} = 1 + \frac{\alpha}{2} \sigma \left[\frac{\theta^H u'(x_1^H)}{c'(x_1^H)} - 1 \right].$$

Using $g_t^i = 0$, (1) becomes

$$(25) \quad m = w x_1^i c'(x_1^i)$$

if buyer i is constrained. If $\pi > \beta$, then (25) must hold for H buyers, at least. Clearly, as π rises,

x_1^H falls and m falls (w is unaffected), until $\pi = \pi^*$, where $\pi^* = \beta + \frac{\beta\alpha}{2} \left(\frac{\theta^H}{\theta^L} - 1 \right)$. If $\pi > \pi^*$, L

buyers are also constrained. Then (25) holds for both H and L buyers, implying $x_1^H = x_1^L = x_1$,

since all agents bring m into market 1. To derive π^* , note that (23) becomes

$$\frac{\pi}{\beta} = 1 + \frac{\alpha}{2} \left\{ \frac{u'(x_1)}{c'(x_1)} [\sigma \theta^H + (1 - \sigma) \theta^L] - 1 \right\}.$$

We can solve for $\frac{u'(x_1)}{c'(x_1)}$, by using the fact that when $\pi = \pi^*$, we have

$$1 + \frac{\alpha}{2} \sigma \left[\frac{\theta^H u'(x_1)}{c'(x_1)} - 1 \right] = 1 + \frac{\alpha}{2} \left\{ \frac{u'(x_1)}{c'(x_1)} [\sigma \theta^H + (1 - \sigma) \theta^L] - 1 \right\},$$

and then we substitute the result into (23). Note that π^* increases with $\theta^H - \theta^L$, and $\pi^* \rightarrow \beta$ as $\theta^H \rightarrow \theta^L$; without heterogeneity, consumption of buyers is unconstrained only at the Friedman rule.

7 No transaction costs ($\phi = 0$)

Now bank deposits dominate money in the portfolio carried by agents from one period to the next, since money can be withdrawn at no cost in market 1. The entire money supply is held by banks as reserves, to meet withdrawal demand during decentralized trade. Remaining bank assets are held as capital investment, on behalf of agents who hold deposits into market 2. Clearly, banks eliminate unused money balances, which increases the return on agents' saving, and improves welfare, except at the Friedman rule.

To derive the equilibrium allocation, note that banks' constraint giving feasible combinations of r_1 and r_2 is obtained using (6)-(9). When $\phi = 0$, (6) implies $l = 0$, and (7) becomes $d = k + \pi z$. (8) becomes

$$z = (1 + r_1) \frac{\alpha}{2} \sum_{i=H,L} \sigma^i g^i d,$$

and therefore (9) can be written as

$$(26) \quad 1 + r_2 = \rho \left[\frac{1 - \pi(1 + r_1) \frac{\alpha}{2} \sum_{i=H,L} \sigma^i g^i}{1 - \frac{\alpha}{2} \sum_{i=H,L} \sigma^i g^i} \right].$$

At the Friedman rule, (23) requires $x_{1t}^i = x_1^{i*}$, $i = L, H$, and Lemma 2 implies $r_1 = r_2$.

(24) becomes $1 + r_2 = 1/\beta$, implying $1 + r_1 = 1/\beta$ as well. Substitution into (26) yields $\rho = 1/\beta$. Thus, the first-order conditions and constraint faced by the planner are replicated, and the planner's allocations in both markets are replicated, by the deposit contract $1 + r_1 = 1 + r_2 = 1/\beta$. But at the Friedman rule, agents can alternatively achieve the efficient allocation by carrying money balances and using the investment technology directly (see (23) when $\pi = \beta$). When nominal interest rates are zero, there is no opportunity cost of holding idle money balances, and therefore, trading frictions do not create distortions.

We now show that banks improve welfare for $\pi > \beta$, proceeding in two steps. First, we show the existence of a deposit contract that supports a welfare-superior allocation compared to an economy without banks. This contract is given by $1 + r_1 = 1/\pi$ and $1 + r_2 = 1/\beta$. The corresponding allocation is the allocation that would be achieved if agents knew the realizations of their type shocks when choosing their savings. Second, we show this deposit contract is optimal in the sense that banks cannot find an alternative contract that achieves a superior allocation in market 1.

Lemma 5: In stationary equilibrium, the deposit contract $1 + r_1 = 1/\pi$ and $1 + r_2 = 1/\beta$ supports an allocation characterized by $\rho = 1/\beta$, $d > m = 0$, and $\frac{\theta^i u'(x_1^i)}{c'(x_1^i)} = \frac{\pi}{\beta}$, $i = L, H$. This allocation is superior to an allocation without banks.

Proof of Lemma 5: Suppose banks offer the contract $1 + r_1 = 1/\pi$ and $1 + r_2 = 1/\beta$. Conjecture that this contract leads to an equilibrium in which agents carry deposits, but no money, into market 1 ($d > m = 0$).

First, we check that the contract is feasible. $1 + r_1 = 1/\pi$ is simply the gross real rate of return on reserves of money. Hence, banks certainly can offer this return on withdrawals in market 1. $1 + r_2 = 1/\beta$ is the gross real rate of return on capital, and so banks can offer this return on withdrawals in market 2. By (26), if $1 + r_1 = 1/\pi$, then $1 + r_2 = \rho$, implying $\rho = 1/\beta$.

Second, we check that $d > 0$ and $m = 0$ are optimal, i.e., agents prefer to save entirely in the form of bank deposits. Start by conjecturing that $d > 0$. This means (24) must hold with $\lambda_d = 0$. Substituting $1 + r_2 = 1/\beta$ and $1 + r_1 = 1/\pi$ into (24), we note that (24) holds

with $\lambda_d = 0$ if $\frac{\theta^i u'(x_1^i)}{c'(x_1^i)} = \frac{1 + r_2}{1 + r_1}$, $i = L, H$. Clearly, $d > m = 0$ requires $g^L > 0$, and therefore,

$\frac{\theta^L u'(x_1^L)}{c'(x_1^L)} = \frac{1 + r_2}{1 + r_1}$ must hold (Lemma 2). Hence, $\frac{\theta^H u'(x_1^H)}{c'(x_1^H)} = \frac{1 + r_2}{1 + r_1}$, also. This means all

buyers' withdrawals in market 1 are unconstrained; given the deposit contract, buyers are indifferent between withdrawing and spending an additional dollar in market 1, and leaving that marginal dollar in the bank until market 2. Now we prove that $m = 0$ is optimal by developing a contradiction. Suppose $m > 0$. This would imply (23) holds with $\lambda_m = 0$. (23) implies

$$\frac{\pi}{\beta} - 1 > \frac{\alpha}{2} \left[\sum_{i=L,H} \sigma^i \frac{\theta^i u'(x_{1t}^i)}{c'(x_{1t}^i)} - 1 \right] = \frac{\alpha}{2} \left(\frac{\pi}{\beta} - 1 \right)$$

which, in turn, implies $m = 0$, giving us the desired contradiction.

Third, we must check that the deposit contract being considered improves the allocation relative to an economy without banks. The allocation in market 2 is efficient since $\rho = 1/\beta$, as in the solution to the planner's problem. Consider market 1. In the economy without banks, prior knowledge of the realization of agents' type shocks would improve the allocation. This would allow agents who will be sellers and idle next period to hold no money, and it would allow

agents who will be H buyers in market 1 to save more money than agents who will be L buyers. Buyers would equate the discounted marginal utility of holding a dollar to the cost of acquiring the dollar, leading them to acquire money so that a deterministic version of the Euler equation

(23) holds, i.e., $1 = \frac{\beta \theta^i u'(x_1^i)}{\pi c'(x_1^i)}$, $i = L, H$. Sellers and idle agents would invest in capital

formation such that the discounted marginal return equals the cost (one), i.e., $\rho = 1/\beta$. Thus, all money would be held by (future) buyers, and in the optimal ratio across H and L buyers.

Similarly, in the allocation with banks offering the proposed deposit contract, all money ends up in the hands of market 1 buyers (only buyers make withdrawals). Money balances of H and L buyers will be in the ratio buyers would have chosen if they had known their type shocks in

advance, since $\frac{1+r_2}{1+r_1} = \frac{\pi}{\beta}$ is necessary and sufficient to attain this allocation.

Finally, we show that this contract is the best banks can offer. Start by noting that banks should not liquidate capital investment to meet redemption demand in market 1, because buyers cannot consume the proceeds, and the proceeds cannot be used as a medium of exchange in market 1 (money is essential for exchange in market 1). This means banks must meet redemption demand in market 1 with money. In addition, banks should hold reserves of money only in the amount demanded by buyers, since holding money from one period to the next carries an opportunity cost. Letting z^i denote per capita real balances available to meet withdrawal

demand by buyers of type i , $i = L, H$, we see that $z = \frac{\alpha}{2} \sum_{i=H,L} \sigma^i z^i$ gives total reserves (real

balances) banks must hold. Banks choose reserves optimally, to maximize utility of market 1 buyers, given buyers' budget constraints, i.e., banks choose $z^i \geq 0$, $i = L, H$ to maximize

$$-\frac{\pi}{w} \frac{\alpha}{2} \sum_{i=H,L} \sigma^i z^i + \beta \frac{\alpha}{2} \sum_{i=H,L} \sigma^i \theta^i u(x^i)$$

subject to $x^i = \frac{p_2}{p_1^i} z^i$, $i = L, H$. In equilibrium, the first-order conditions for this problem are

$$\frac{\pi}{w} = \beta \theta^i u'(x^i) \frac{p_2}{p_1^i}, \quad i = L, H. \text{ Prices must satisfy } \frac{p_2}{p_1^i} = \frac{1}{c'(x^i)w}, \text{ from (14). Hence, the first-}$$

order conditions become $\frac{\pi}{\beta} = \frac{\theta^i u'(x^i)}{c'(x^i)}$, $i = L, H$. Next, notice that since $m = 0$, L buyers must

withdraw at least some of their deposit, and therefore, Lemma 2 requires $\frac{\theta^L u'(x_1^L)}{c'(x_1^L)} = \frac{1+r_2}{1+r_1}$.

This implies the deposit contract must satisfy $1+r_1 = 1/\pi$ and $1+r_2 = 1/\beta$ because (i) if

$1+r_1 > 1/\pi$, then by (26) we must have $1+r_2 < 1/\beta$ (and $\frac{1+r_2}{1+r_1} > \frac{\pi}{\beta}$) and (ii) if $1+r_1 < 1/\pi$,

then $m > 0$, which is suboptimal. \square

The contract $1+r_1 = 1/\pi$ and $1+r_2 = 1/\beta$ is feasible because it pays the return on money in market 1, and the marginal product of capital in market 2. It improves the allocation, relative to an economy without banks, because banks pool liquidity and redistribute it, in market 1, to agents who need it to consume. The contract is optimal, but it provides only partial insurance against consumption risk. The insurance is partial because although banks can reallocate existing liquidity, they cannot create liquidity beyond the reserves of money that they hold. This is prevented by the same informational and enforcement frictions that make money essential for trade in market 1. The contract is optimal as no other feasible deposit contract can be offered that yields higher expected utility for depositors.

Higher expected utility would result from a contract that did more than reallocate existing liquidity. Such a contract would increase the real rate of return in market 1, i.e., $1 + r_1 > 1/\pi$. Doing this would require banks either to create money, or to deliver consumption directly to buyers (as in consumption loans). Neither is a possibility in this model. This limits the role of banks to reallocating liquidity, which is unlike the allocative role played by banks in the non-monetary banking model in Diamond and Dybvig (1983), where liquidated investment can be directly consumed.

In sum, even if banks have limited information and enforcement abilities, their provision of liquidity insurance improves the allocation, away from the Friedman rule. However, the efficient allocation remains out of reach because banks cannot create money. This raises the question of whether there is a role for central bank injections of money into banks in market 1, through a policy of paying interest on reserves. This possibility is explored next.

8 Interest on reserves when transaction costs are zero

When the central bank pays interest on reserves, the set of resource constraints facing banks is altered, allowing banks to pay a higher nominal interest rate on deposits withdrawn in market 1. Define r_Z as the net real interest rate paid on reserves in market 1. The balance sheet constraint does not change (see the previous section). However, (8), which is the resource constraint for meeting demand for redemptions in market 1, becomes

$$z(1 + r_Z) = (1 + r_1) \frac{\alpha}{2} \sum_{i=H,L} \sigma^i g^i d.$$

Now (26), which links r_1 and r_2 , becomes

$$(27) \quad 1+r_2 = \rho \left[\frac{1 - \pi \left(\frac{1+r_1}{1+r_Z} \right) \left(\frac{\alpha}{2} \right) \sum_{i=H,L} \sigma^i g^i}{1 - \frac{\alpha}{2} \sum_{i=H,L} \sigma^i g^i} \right].$$

Suppose the central bank attempts to increase consumption in market 1 for all buyers, without distorting consumption in market 2. In other words, suppose values for r_1 and r_2 are chosen to loosen the constraints on market 1 consumption, while maintaining the equilibrium

condition that $\frac{\theta^i u'(x_1^i)}{c'(x_1^i)} = \frac{1+r_2}{1+r_1}$, $i = L, H$. Using the fact that $1+r_2 = 1/\beta$ (from the Euler

equation for deposits), and recalling that $\rho = 1/\beta$ (from the planner's problem), (27) implies

$$(28) \quad \frac{1+r_1}{1+r_Z} = \frac{1}{\pi},$$

whereas (26) implied $1+r_1 = 1/\pi$. The central bank can set r_Z to manipulate r_1 . Any value of

r_1 such that $1 \leq \frac{1+r_2}{1+r_1} < \frac{\pi}{\beta}$ improves the allocation in market 1, compared to the allocation in

Lemma 5. The efficient allocation in market 1 requires $1+r_1 = 1+r_2 = 1/\beta$ (by Lemma 4).

Using (28), this requires $1+r_Z = \pi/\beta$.

Why does a policy of paying interest on reserves work? The real rate of return on money is increased to $1/\beta$, the rate of return at the Friedman rule. Each dollar of the money supply pays this rate of return, which is then passed on to buyers, because when $\phi = 0$, no money is held outside of banks going into market 1 (so the central bank policy targets the entire money supply),

and only buyers make withdrawals.¹ Paying interest on reserves does not require the central bank or banks themselves to know any information that is private to agents when they make withdrawals (realizations of type shocks), and it is incentive compatible. As long as $r_2 \geq r_1$, sellers and idle agents do not make withdrawals in market 1, and L buyers have no incentive to withdraw more than is needed to purchase the efficient quantity in market 1. By taking advantage of the liquidity insurance mechanism implemented by banks, a policy of paying interest on reserves effectively allows the central bank to undo the distortionary effects of inflation by taxing agents in market 2 (a negative money transfer), and this policy can be executed at any inflation rate.²

To find the real money transfer required by a central bank policy of paying interest on reserves, consider the money market clearing condition. In stationary equilibrium, banks receive a transfer of real balances equal to $r_Z z$ at the beginning of market 1 (net interest on reserves), while agents receive a transfer of real balances equal to τ in market 2. Recall that the change in aggregate real balances is $\bar{m}(\pi - 1)$ in stationary equilibrium. Thus, in each period,

$\tau + r_Z z = \bar{m}(\pi - 1)$. Using $1 + r_Z = \pi / \beta$ and $z = \bar{m}$ (banks hold the entire money supply), the money transfer in the efficient allocation is a tax given by $\tau = \frac{z \pi (\beta - 1)}{\beta}$.

9 Positive transaction costs ($\phi > 0$)

The assumption that resources are required to operate banks that can be accessed in decentralized markets is consistent with the frictions characterizing such markets. In this model,

¹ We thank an anonymous referee for suggesting this interpretation, and for providing helpful comments on an earlier version of this section.

² If money transfers are restricted to be non-negative, because of enforcement issues, then the efficient allocation cannot be achieved. One way to bypass such enforcement issues, and to achieve a result similar to paying interest on reserves, would be to sell interest-paying government bonds that can be liquidated, at a cost, in market 1 (Boel and Camera, 2006).

a stationary equilibrium when transactions costs are positive will be one of three possible types, depending on the values of ϕ and π .

If bank operating costs (transaction costs) are sufficiently small (relative to inflation), agents carry no money balances into market 1, and all buyers in market 1 withdraw the money they need to undertake their purchases (region (i) in Figure 1). These stationary equilibria closely resemble the stationary equilibrium when $\phi = 0$. If transaction costs are moderate, then agents carry money into market 1, and only H buyers make withdrawals (region (ii) in Figure 1). Here agents undertake a combination of self-insurance and utilization of banks' ability to reallocate liquidity. Finally, if transaction costs are sufficiently large, then the liquidity insurance provided by banks is too costly to utilize, so agents carry money into market 1, and do not make withdrawals (region (iii) in Figure 1). In the following subsections we derive the equilibrium allocations in the regions (i) and (ii), and conditions for the boundaries between regions. (Region (iii) was covered in section 6.)

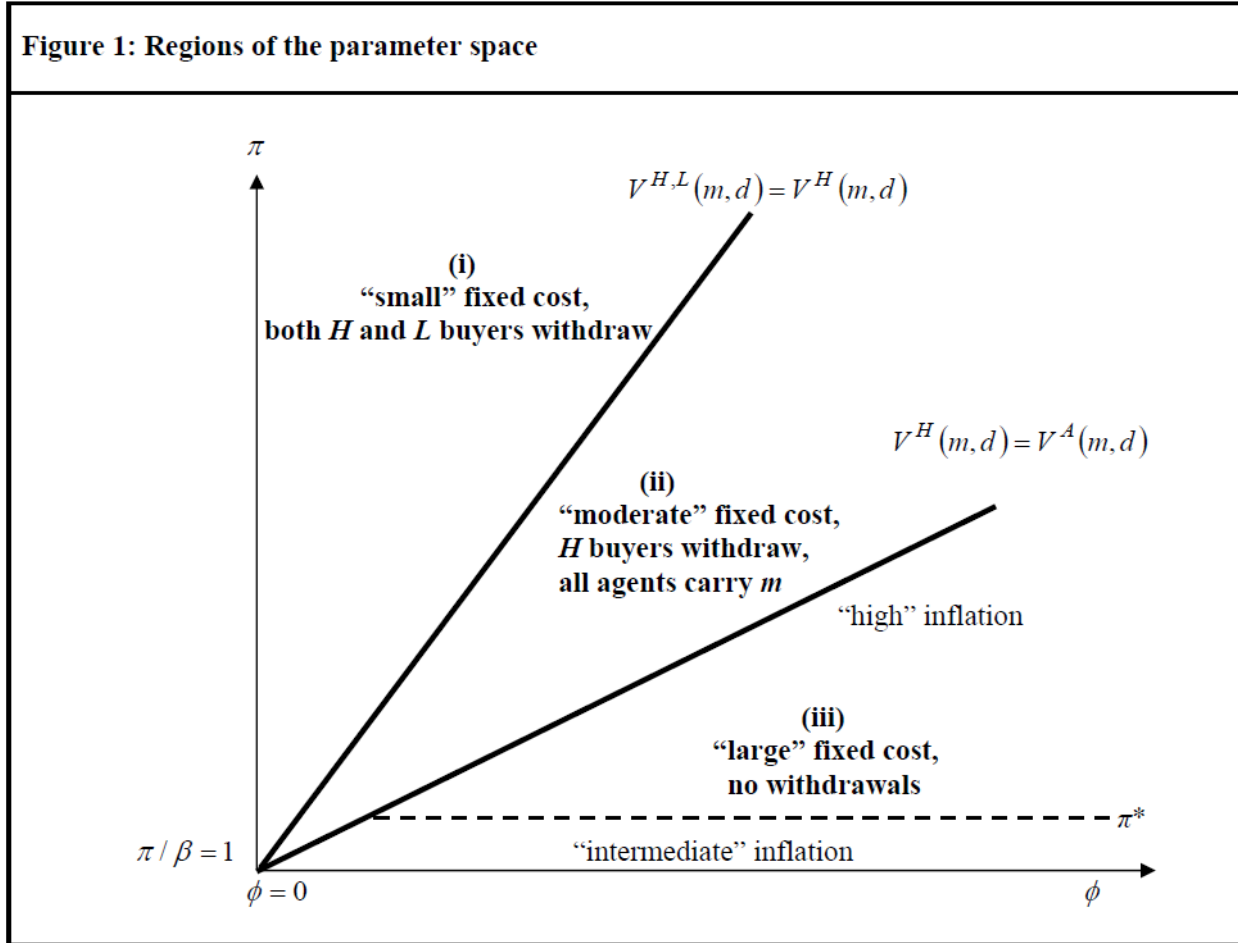
9.A Small transaction costs

For any $\pi > \beta$, there is an interval of values of ϕ that are sufficiently small for expected utility still to be maximized when agents deposit all of their saving in banks. To derive the allocation in this case, we conjecture the existence of a stationary equilibrium in which agents hold deposits, but no money, going into market 1, and all buyers make withdrawals in market 1 ($0 < g^L < g^H \leq 1$). Clearly, the constraint $g^L \geq 0$ cannot be binding, or L buyers would not consume in market 1. The constraint $g^H \leq 1$ is not binding either; if it were, agents could increase the amount they deposit in market 2, and be better off. Since neither constraint is

binding, Lemma 2 implies $\frac{\theta^i u'(x_1^i)}{c'(x_1^i)} = \frac{1+r_2}{1+r_1}$, $i = L, H$. The logic that is the basis for the proof of

Lemma 5 continues to hold, and therefore, the allocation in market 1 is the same as when $\phi = 0$,

characterized by $\frac{\theta^i u'(x_1^i)}{c'(x_1^i)} = \frac{\pi}{\beta}$, $i = L, H$, and $\rho = 1/\beta$. This allocation is sustained by a deposit



contract offering $1 + r_2 = 1/\beta$ and $1 + r_1 = 1/\pi$. To show that this deposit contract and the allocation in market 1 are consistent with banks' constraints, note that transaction costs incurred

making withdrawals in market 1 are $l = \frac{\alpha}{2}\phi$ on a per capita basis, and (26) becomes

$$1+r_2 = \rho \left[\frac{d - \frac{\alpha}{2}\phi - \pi(1+r_1)\frac{\alpha}{2}(\sigma g^H + (1-\sigma)g^L)(d-\phi)}{d - \frac{\alpha}{2}\phi - \frac{\alpha}{2}(\sigma g^H + (1-\sigma)g^L)(d-\phi)} \right].$$

Clearly $1+r_2 = \rho$ iff $1+r_1 = 1/\pi$. However, because of these real operating costs of banks, the allocation in market 2 is less efficient than when $\phi = 0$.

We can infer various features of the equilibrium allocation. Since x_1^i is determined by $\frac{\theta^i u'(x_1^i)}{c'(x_1^i)} = \pi\rho$, $i = L, H$, consumption in market 1 is the same as when $\phi = 0$. Withdrawals in market 1 are the same, and since r_1 is the same, bank reserves z are the same. Banks' balance sheet constraint is

$$d = k + \frac{\alpha}{2}\phi + \pi \frac{\alpha}{2} \sum_{i=H,L} \sigma^i x_1^i c'(x_1^i) w$$

where k , w , x_1^H and x_1^L are known, allowing d to be calculated. Using d , the equilibrium fractions of deposits withdrawn ($g^i, i = L, H$) may be calculated from (25). With constant returns to scale production in market 2, $\rho = 1/\beta$ only if k and h_2 (and therefore y) change in the same proportion, as ϕ increases. Therefore, w remains unchanged, and from (19), x_2 remains unchanged. Average hours (h_2) increase, because agents must work more in order to maintain larger deposits. Deposits are larger, because a fraction of deposits is dissipated in bank operating costs.

9.B Moderate transaction costs

Once the transaction cost exceeds a threshold (which depends on the inflation rate, and is derived later), the expected utility-maximizing deposit contract induces some buyers to incur the

transaction cost, but not all. Banks can still increase expected utility by redistributing liquidity, but the increase is more limited. Agents carry money into market 1, in addition to deposits, and only H buyers, who value consumption in market 1 more, make withdrawals in market 1.

In an equilibrium in which agents carry both money and deposits into market 1, but only H buyers make withdrawals, the incentives for L buyers to make withdrawals from their deposits must be sufficiently weak (as must those of sellers and idle agents). For this reason, the expected utility-maximizing contract offered by banks must be quite different than when the transaction cost is zero or small. Specifically, this contract will have (i) $1 + r_1 < 1/\pi$, so agents who spend a small amount will be content to have carried money balances for this purpose, (ii) $1 + r_2 > 1/\beta > 1 + r_1$, so there is no incentive to withdraw deposits in market 1 unless the money will be used to buy goods in market 1, and (iii) a ceiling on deposit size ($d \leq \bar{d}$), to ensure agents do not overinvest in capital. Then $m > 0$ equals equilibrium expenditure by L buyers (who do not make withdrawals), and $d = \bar{d} > 0$ is determined by the amount of additional money H buyers need as a supplement to m .

We are looking for a stationary equilibrium in which L buyers make no withdrawals in market 1, while H buyers withdraw and spend their entire deposit, and banks offer a deposit contract such that these withdrawal decisions are unconstrained (implying total surplus from trades in market 1 is maximized). I.e., the equilibrium we seek to characterize has $g^L = 0$ and

$g^H = 1$, and $\frac{\theta^i u'(x_1^i)}{c'(x_1^i)} = \frac{1 + r_2}{1 + r_1}$, $i = L, H$, from Lemma 2. For H buyers, this is achieved by

setting the maximum deposit size \bar{d} such that the constraint $g^H \leq 1$ is “just” binding. Note that since agents can make direct use of the investment technology (agents can undertake investment

outside of banks), there need be no utility cost of a deposit ceiling. For L buyers, the constraint $g^L \geq 0$ is also “just” binding; L buyers spend all money they carry into market 1 (otherwise, they would want to redeposit some of it).

When $g^L = 0$ and $g^H = 1$, buyers’ budget constraints are $m + (1 + r_1)(d - \phi) = \frac{p_{1t}^H x_2^H}{p_{2t}}$

and $m = \frac{p_{1t}^L x_1^L}{p_{2t}}$, which, using (25), implies

$$(29) \quad x_1^H c'(x_1^H) - x_1^L c'(x_1^L) = (1 + r_1) \left(\frac{d - \phi}{w} \right).$$

The right-hand-side of (29) must be positive, and hence, $x_1^H > x_1^L$, where x_1^L is pinned down by m , and x_1^H is pinned down by the deposit ceiling, \bar{d} .

Briefly, this deposit contract allows banks still to deliver some benefits from reallocating liquidity. A gap between the marginal utilities of consumption of H and L buyers is necessary for region (ii) to exist; as $\theta^H \rightarrow \theta^L$, region (ii) disappears, because the strategy of this deposit contract to reduce transaction costs—inducing buyers who value consumption in market 1 less to use money balances only—breaks down.

To see why banks need to set a deposit ceiling, note that in equilibrium, (23) must hold

with $\lambda_m = 0$. Using $\frac{\theta^i u'(x_1^i)}{c'(x_1^i)} = \frac{1 + r_2}{1 + r_1}$, (23) becomes

$$1 + r_2 = (1 + r_1) \left[1 + \frac{\alpha}{2} \left(\frac{\pi}{\beta} - 1 \right) \right],$$

which implies $r_2 \geq r_1$. Banks’ constraints (7)-(9) yield

$$(30) \quad 1 + r_2 = \rho \left[\frac{d - \frac{\alpha}{2} \sigma \phi - \pi(1 + r_1) \frac{\alpha}{2} \sigma (d - \phi)}{d \left(1 - \frac{\alpha}{2} \sigma \right)} \right].$$

Recall that we need $1 + r_1 < 1/\pi$ in order to discourage L buyers from withdrawing deposits.

(30) then implies $1 + r_2 > \rho$. The Euler equation for the amount agents invest directly in capital formation (unintermediated investment) implies that $\rho = 1/\beta$ in equilibrium. The Euler equation for deposits, (24), becomes $1 + r_2 > 1/\beta$, indicating that agents want an infinite amount of deposits, which cannot be an equilibrium. Hence, banks must set a maximum deposit size. Capital created by agents directly has the lower rate of return, $\rho = 1/\beta$, and is determined as the difference between the equilibrium level of k and capital investment by banks.

From goods market-clearing, clearly k and y are smaller in region (ii) than in region (i) (at the boundary). Intuitively, the levels of k and y that need to be maintained in region (ii) are smaller, because fewer resources are being used up in transaction costs. From banks' balance sheet constraint, d is also smaller in region (ii). Deposits are smaller because each of reserves, resources set aside to cover transaction costs, and the capital stock is smaller.

9.C Relationship between threshold values of the transaction cost and inflation

Now we characterize the boundaries between regions (i) and (ii), and regions (ii) and (iii). As we know from our analysis of stationary equilibrium in these regions, one possible type of equilibrium involves all buyers bearing the cost of making withdrawals in market 1, which allows agents to pool all of their saving in market 2, and to take full advantage of banks' ability to reallocate liquidity. In a second possible type of equilibrium, all agents carry some money balances into market 1—in the amount L buyers will require for their purchases—but H buyers

bear the cost of making withdrawals in market 1. Banks reduce but do not eliminate idle money balances. The third possibility is that no one is willing to incur the cost of withdrawing money in market 1, and all agents carry money balances into market 1, sufficient for self-insurance against the type shock.

Given parameters of the model, the value function can be evaluated for all three types of equilibria. The boundary between regions (i) and (ii) is derived by equating the value function for the first type of equilibrium, and the value function for the second type of equilibrium. On this boundary, lifetime utilities are equal in the two types of equilibria. The boundary condition can be solved for the magnitude of the transaction cost, as a function of the inflation rate, given all other parameters. Similarly, the boundary between regions (ii) and (iii) is derived from the value functions for the second and third types of equilibria.

Using (21) and (22), the value function in stationary equilibrium is given by

$$(31) \quad V(m, d) = \beta V(m, d) + U(x_2) - \frac{x_2}{w} - \frac{m\pi + d}{w} + \frac{m + \tau}{w} \\ + \frac{1+r_2}{w} \left\{ \left(1 - \frac{\alpha}{2}\right) d + \frac{\alpha}{2} \sum_{i=H,L} \sigma^i (1-g^i) [(d-\phi) + \chi^i \phi] \right\} \\ + \left(\frac{1+r_1}{w} \right) \left(\frac{\alpha}{2} \right) \sum_{i=H,L} \sigma^i g^i (d-\phi) + \frac{\alpha}{2} \sum_{i=H,L} \sigma^i [\theta^i u(x_1^i) - c(x_1^i)]$$

where τ is given by (13).

In region (i), $m = 0$, and $0 < g^L < g^H \leq 1$. Using banks' constraints (7)-(9), (31)

becomes

$$V(m, d)^{H,L} = \beta V(m, d)^{H,L} + U(x_2) - \frac{x_2}{w} + \frac{k(\rho-1)}{w} - \frac{\phi}{w} \frac{\alpha}{2} + \frac{\alpha}{2} \sum_{i=H,L} \sigma^i [\theta^i u(x_1^i) - c(x_1^i)].$$

In region (ii), $m > 0$, $g^L = 0$, and $g^H = 1$. Again using banks' constraints, (31) becomes

$$V(m, d)^H = \beta V(m, d)^H + U(x_2) - \frac{x_2}{w} - \frac{k(\rho-1)}{w} - \frac{\phi}{w} \frac{\alpha}{2} \sigma + \frac{\alpha}{2} \sum_{i=H,L} \sigma^i [\theta^i u(x_1^i) - c(x_1^i)].$$

At the boundary between regions (i) and (ii), $V^{H,L} = V^H$. We know the values of x_2 , w , and ρ are the same in regions (i) and (ii) (given values of all parameters other than ϕ , including π), but x_1^H , x_1^L , and k are different. Therefore $V^{H,L} = V^H$ becomes

$$(32) \quad \frac{\rho-1}{w} [k^{H,L} - k^H] + \frac{\alpha}{2} \sigma \left[\{\theta^H u(x_1^H) - c(x_1^H)\}^{H,L} - \{\theta^H u(x_1^H) - c(x_1^H)\}^H \right] \\ + \frac{\alpha}{2} (1-\sigma) \left[\{\theta^L u(x_1^L) - c(x_1^L)\}^{H,L} - \{\theta^L u(x_1^L) - c(x_1^L)\}^H \right] = \frac{\phi}{w} \frac{\alpha}{2} (1-\sigma)$$

where $k^{H,L}$ and k^H are the equilibrium values of capital in regions (i) and (ii), respectively, and $\{\theta^i u(x_1^i) - c(x_1^i)\}^{H,L}$ and $\{\theta^i u(x_1^i) - c(x_1^i)\}^H$, $i = L, H$, give match surpluses in regions (i) and (ii), respectively. We know $k^{H,L} > k^H$ at the boundary, and that x_1^H and x_1^L are larger in region (i) than in (ii), allowing us to conclude that $\{\theta^i u(x_1^i) - c(x_1^i)\}^{H,L} > \{\theta^i u(x_1^i) - c(x_1^i)\}^H$, $i = L, H$. Since x_1^L is a function of π , this condition gives the threshold value of ϕ as a function of π . Both H and L buyers become more constrained as π / β rises, in both regions (i) and (ii). However, they become more constrained at different rates—they become more constrained faster in region (ii). I.e., in both regions, match surpluses decline as π / β rises, but they decline faster in region (ii). Since inflation has no effect on k , the left-hand-side of (32) increases as π / β rises, and therefore the threshold size of ϕ increases, on the right-hand-side. This gives us the positively-sloped boundary between regions (i) and (ii) in Figure 1.

In region (iii), the value function is given by

$$V(m, d)^A = \beta V(m, d)^A + U(x_2) - \frac{x_2}{w} - \frac{m\pi + d}{w} + \frac{m + \tau + d(1 + r_2)}{w} + \frac{\alpha}{2} \sum_{i=L, H} \sigma^i [\theta^i u(x_1^i) - c(x_1^i)].$$

Using the banks' constraints we obtain

$$V(m, d)^A = \beta V(m, d)^A + U(x_2) - \frac{x_2}{w} + \frac{k(\rho - 1)}{w} + \frac{\alpha}{2} \sum_{i=L, H} \sigma^i [\theta^i u(x_1^i) - c(x_1^i)].$$

The condition defining the boundary between regions (ii) and (iii) is $V^H = V^A$:

$$(33) \quad \frac{\rho - 1}{w} [k^H - k^A] + \frac{\alpha}{2} \sigma \left(\left\{ \theta^H u(x_1^H) - c(x_1^H) \right\}^H - \left\{ \theta^H u(x_1^H) - c(x_1^H) \right\}^A \right) + \frac{\alpha}{2} (1 - \sigma) \left(\left\{ \theta^L u(x_1^L) - c(x_1^L) \right\}^H - \left\{ \theta^L u(x_1^L) - c(x_1^L) \right\}^A \right) = \frac{\phi}{w} \frac{\alpha}{2} \sigma$$

where k^A is the capital stock in region (iii), and $\left\{ \theta^H u(x_1^H) - c(x_1^H) \right\}^A$ and $\left\{ \theta^L u(x_1^L) - c(x_1^L) \right\}^A$ are the match surpluses of H and L buyers, respectively, in region (iii). Recall, that in region (iii), x_1^H and x_1^L must be calculated differently depending upon whether the inflation rate is below or above the value π^* . When inflation is “high,” and π / β rises, both H and L buyers become more constrained faster in region (iii) than in region (ii); match surpluses in both regions decline, but they decline faster in region (iii). We know $k^H > k^A$, and inflation has no effect on k within either region. Therefore, the left-hand-side of (33) increases as π / β rises, and the threshold size of ϕ increases, resulting in a positively-sloped boundary between regions (ii) and (iii) in

Figure 1. When inflation is “intermediate,” the weighted average of $\frac{\theta^i u'(x_1^i)}{c'(x_1^i)}$ is the same for H

and L buyers in the two regions (at a value set by the Euler equation for m). This weighted average is increasing in the inflation rate, and therefore, the wedge between how constrained H

and L buyers are, in region (iii), is increasing in the inflation rate. With the assumptions we have made on u and c , it is also true at “intermediate” inflation rates that match surpluses decline faster in region (iii) than in region (ii). Therefore, the boundary between regions (ii) and (iii) is positively-sloped in this case also.

9.D Interest on reserves when transaction costs are positive

In region (i), where ϕ is small, a policy of paying interest on reserves has the same potential to improve the allocation in market 1 as when $\phi = 0$. It is easy to show that if r_Z is chosen as in the case where $\phi = 0$, interest on reserves allows unconstrained consumption by all buyers in market 1 (and that the Euler equations for deposits and money are both satisfied). However, as discussed earlier, the resources expended as bank operating costs (transaction costs) alter the allocation in market 2.

In region (ii), consumption in market 1 is pinned down by the Euler equation for m , (23). A policy of paying interests on reserves does not change the allocation when H buyers but not L buyers make withdrawals, because the deposit contract requires the rate of return on deposits withdrawn in market 1 to be inferior to the rate of return on money. If this were not the case, then L buyers would also make withdrawals in market 1 (and bring no money into market 1). Although a policy of paying interest on reserves cannot affect the allocation, it shifts the boundary between regions (i) and (ii). The size of region (i) increases, at the expense of region (ii), i.e., the policy extends the range values of ϕ for which the net benefits of the liquidity insurance implemented by banks are positive.

10 Conclusion

We have constructed a model of banking in which demand deposit contracts, which allow the timing of withdrawals to respond to random shocks to trading opportunities and preferences, improve the equilibrium allocation. The potential for banks to improve the allocation arises when banks can reallocate liquidity to those who randomly need it, thereby reducing or eliminating idle money balances, and shifting the composition of saving toward capital formation, which has a higher return. One main contribution of the paper is to explore the potential of banks to improve the allocation, even though operating banks in a decentralized setting involves resource costs (real transaction costs), and even though banks operate subject to limitations on information and enforcement abilities.

As bank operating costs (transaction costs) rise, banks change the nature of the deposit contract they offer, in order to dampen some depositors' incentives to make early withdrawals, while continuing to reallocate liquidity to those who need it most. The potential of banks to improve the allocation is diminished as transaction costs rise, and at some point, it is eliminated. The higher is the inflation rate, the greater is the value to depositors of the reallocation of liquidity implemented by a demand deposit contract, and the larger are the transaction costs agents are willing to incur to have the benefit of access to banks (up to some prohibitive level of costs). A relationship between inflation and the terms of the demand deposit contract exists because, with our assumptions about limited information and enforcement, banks cannot create money, and they cannot set up consumption loans. Thus, although banks can improve the allocation (except at the Friedman rule), banks cannot implement the efficient allocation. The best banks can do is to support the allocation agents would choose if they could perfectly anticipate their trading and marginal utility shocks—and banks can do this only if bank operating costs are not too large.

Because banks cannot create money, a central bank policy of paying interest on reserves (recently implemented by the Federal Reserve) can move the allocation closer to efficiency. This policy works regardless of the inflation rate, without the government or banks needing information about liquidity needs of individual agents, and without any enforcement issues arising, because the demand deposit contract is incentive compatible. However, it works only when transaction costs are sufficiently small that the deposit contract can completely eliminate idle money balances (all money is held by banks), because in that case, interest on reserves gives money injections in proportion to money holdings. This result suggests a central bank policy of paying interest on reserves should be adopted in countries where operating costs of banks are low.

References

- Charalmbos D. Aliprantis, Gabriele Camera, and Francesco Ruscitti (2009). Monetary equilibrium and the differentiability of the value function. *Journal of Economic Dynamics and Control*, 33 (2), 454-462
- Aruoba, S. Boragan and Randall Wright (2003). "Search, Money, and Capital: A Neoclassical Dichotomy." *Journal of Money, Credit & Banking* 35, 1086-1105.
- Aruoba, S. Boragan, Christopher J. Waller, and Randall Wright (2006). "Money and Capital." Manuscript, University of Notre Dame.
- Bencivenga, Valerie and Bruce D. Smith (1992). "Deficits, inflation, and the banking system in developing countries: the optimal degree of financial repression." *Oxford Economic Papers* 44 (Oct.), 767-90.
- Bencivenga, Valerie and Bruce D. Smith (2003). "Monetary Policy and Financial Market Evolution." Federal Reserve Bank of St. Louis *Quarterly Review*, 7-26.
- Bencivenga, Valerie and Gabriele Camera (2008). "Banks, Liquidity Insurance, and Interest on Reserves in a Matching Model of Money." Manuscript.
- Berentsen, Aleksander, Gabriele Camera and Christopher Waller (2007). "Money, credit, and banking." *Journal of Economic Theory* 135 (1), 171-195.
- Boel, Paola and Gabriele Camera (2006). "Efficient monetary allocations and the illiquidity of bonds." *Journal of Monetary Economics* 53 (7), 1693-1715.

Camera, Gabriele and Francesco Ruscitti (2007). “Inflation and intermediation frictions.” Unpublished Manuscript, Purdue University.

Diamond, Douglas, and Philip Dybvig (1983). “Bank runs, deposit insurance, and liquidity.” *Journal of Political Economy* 91, 401–19..

He, Ping, Lixin Huang, and Randall Wright, (2005). “Money and Banking in Search Equilibrium.” *International Economic Review* 46 (2), 637-670

Kiyotaki, Nobuhiro, and John Moore (2002), A Cost of Unified Currency. Unpublished manuscript, Princeton University

Lagos, Ricardo, and Randall Wright (2005). “A Unified Framework for Monetary Theory and Policy Analysis.” *Journal of Political Economy* 113, 463–484.

Schreft, Stacey, and Bruce D. Smith (1998). “The effects of open market operations in a model of intermediation and growth.” *Review of Economic Studies* 65, 519-550 .

¹ We thank two anonymous referees for helpful comments that have substantially improved the paper. We thank Todd Keister for an insightful discussion, and D. Altig, D. Backus, R. Cavalcanti, D. Corbae, M. Feldman, E. Green, N. Kocherlakota, D. Levine, A. Martin, R. Reed, G. Rocheteau, N. Wallace, C. Waller, S. Williamson, and R. Wright, as well as participants in the 2008 Cleveland Fed/JMCB conference on Liquidity in Frictional Markets, 2006 conference on Monetary Economics at the University of Toronto, 2007 SAET meeting, 2007 Money Workshop at the Cleveland Fed, and seminars at NYU Stern and the NY Fed, for helpful comments.

² Our model retains the assumptions in Lagos and Wright (2005) and Aruoba and Wright (2003). Private credit arrangements are ruled out; agents are anonymous when engaging in trade; and agents cannot commit to repaying loans. Claims on investment in any form, including inside money, cannot be verified, and are not accepted in exchange. Banks cannot transfer balances between agents, or solve the enforcement problems that arise in the context of consumption loans.

³ Several other papers have introduced deposit-taking institutions into a Lagos-Wright model. The banks in these papers are very different in their functions from banks here. For example, He, Huang, and Wright (2005) model banks as offering a check-writing technology, and the banks in Berentsen, Camera, and Waller (2007) arrange consumption loans between agents. In Camera and Ruscitti (2007), there is no capital accumulation, and intermediaries make commercial loans.

⁴ Claims on gestating capital cannot be traded since, as in Aruoba, Waller, and Wright (2006), such claims cannot be verified in market 1. Furthermore, although a buyer can identify himself to his bank in market 1, banks cannot transfer claims between depositors, which rules out inside money. The latter assumption can be justified in various ways. For example, it may be prohibitively costly for a buyer to bring the seller in his match to the buyer’s bank.

⁵ In overlapping generations models with random relocation, such as Schreft and Smith (1998) and Bencivenga and Smith (2003), if an agent makes direct use of the investment technology, and then must relocate, his investment is scrapped. Banks similar to those in Diamond and Dybvig (1983) provide liquidity insurance in this environment. The allocation with banks can be replicated by introducing a costless secondary market in claims on investment. However, if agents face a cost to utilizing the secondary market, which banks can avoid, banks form. This is also true in our model, provided the cost of utilizing the secondary market is sufficiently larger than the fixed cost of accessing banks in market 1.

⁶ We do not study more general types of contracts, although the contract when transaction costs are of moderate size can be interpreted as having a nonlinear schedule of interest rates on withdrawals in market 2.