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STATISTICAL TIME-DEPENDENT MODEL FOR THE INTERSTELLAR GAS

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ABSTRACT

We present models for temperature and ionization structure of low, uniform-density ($n \simeq 0.3 \text{ cm}^{-3}$) interstellar gas in a galactic disk which is exposed to soft X-rays from supernova outbursts occurring randomly in space and time. The structure was calculated by computing the time record of temperature and ionization at a given point by Monte Carlo simulation. The calculation yields probability distribution functions for ionized fraction x , temperature T , and their various observable moments. These time-dependent models predict a bimodal temperature distribution of the gas with structure in x, T that agrees with various observations. Cold regions in the low-density gas may have the appearance of clouds in 21-cm absorption. The time-dependent model, in contrast to the steady-state model, predicts large fluctuations in ionization rate and the existence of cold ($T \simeq 30^\circ \text{ K}$), ionized ($x \simeq 0.1$) regions.

Subject headings: atomic processes — interstellar matter — supernovae — X-rays

I. INTRODUCTION

The density, temperature, and ionization structure of the interstellar medium are studied by comparing various observations that measure quantities averaged over lines of sight in the galaxy, viz., 21-cm emission ($\langle n_{\text{H}} \rangle$), 21-cm absorption ($\langle n_{\text{H}}/T \rangle$), pulsar dispersion measures ($\langle n_e \rangle$), low-frequency absorption and radio recombination lines ($\langle n_e^2 T^{-3/2} \rangle$, approximately), and diffuse H α and H β emission ($\langle n_e^2 T^{-1/2} \rangle$, approximately). The brackets denote averages over paths, $\langle A \rangle \equiv \int A dl / \int dl$; it is not generally possible to measure different quantities over identical paths. Many of the relevant observations have been discussed by Dalgarno and McCray (1972), Silk (1973a), and Falgarone and Lequeux (1973); to summarize briefly, observations indicate $\langle n_e \rangle \simeq 0.025 \text{ cm}^{-3}$, a mean temperature of the cold gas of $\sim 70^\circ \text{ K}$, and $10^{-8} \leq n_e^2 T^{-3/2} \leq 10^{-5} \text{ cm}^{-6} (\text{deg. K})^{-3/2}$, with the greater values occurring toward the galactic center. These observations indicate a high average (over space and time) hydrogen ionization rate, $10^{-15} \leq \zeta_{\text{H}} \leq 10^{-13} \text{ s}^{-1}$, and the source of this ionization has not been positively identified.

Many observers have found it instructive to interpret their results in terms of a simple and elegant model due to Field, Goldsmith, and Habing (1969; hereafter called FGH) in which the interstellar medium is heated and ionized by a nonthermal source (low-energy cosmic rays) which is constant in space and time. According to this model, the interstellar medium is divided into two distinct phases, "clouds" and "intercloud medium," which are in pressure equilibrium with each other. The model makes strong predictions, and therefore it is not surprising that various

modifications to the original model have been proposed to make it agree better with more detailed observations as they have become available. Perhaps the most significant of these is the indication from radio recombination-line and 21-cm absorption observations that the ionization rate in clouds is much less than that in the intercloud medium (Silk 1973a, b).

The absence of an identified ionizing source has led several authors to suggest that supernovae may account for much of the heating and ionization, either by cosmic rays (Hayakawa, Nishimura, and Takayanagi 1961; Spitzer and Tomasko 1968; FGH) or by a burst of ultraviolet radiation or soft X-rays (Werner, Silk, and Rees 1970; Bottcher *et al.* 1970; McCray and Schwarz 1971). Dalgarno and McCray (1972) give arguments that if supernovae make a significant contribution, the resulting heating and ionization may not be well approximated by a steady source. If so, the interstellar medium may not be in pressure equilibrium, because the thermal relaxation time may be short compared with the dynamical equilibration time.

The question of whether a time-dependent description of the interstellar medium is required rests essentially on the nature and distribution of the sources and on the propagation properties of the ionizing agents (radiation or particles). We must compare the time scales for fluctuations of the ionization source with the relaxation time scales of the gas. The gas cooling time scale is given by

$$\tau_c \equiv T/|dT/dt| = \frac{3}{2} kT/n\Lambda(x, T) \simeq 2T/n_e \text{ years} \quad \text{for } 10^2 \text{ }^\circ \text{ K} < T < 10^4 \text{ }^\circ \text{ K}, \quad (1)$$

where $\Lambda(x, T)$ is the rate coefficient for cooling by radiative transitions (Dalgarno and McCray 1972). The recombination time scale is given by

$$\tau_r \equiv n_e \left/ \left| \frac{dn_{\text{H}}}{dt} \right| \right| = [n_e \alpha(T)]^{-1} \simeq 10^3 T^{1/2}/n_e \text{ years}, \quad (2)$$

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where $\alpha(T)$ is the rate coefficient for radiative recombination of hydrogen to excited states (Spitzer 1968).

The fluctuation time scale of the ionization rate is estimated as follows. Assume a galactic disk of volume V_g , area A_g , and thickness $2H$, containing impulsive point sources—"supernovae"—which affect the gas out to a distance D . The average time τ_{exp} between exposures of a given point in the gas to the effects of an outburst is given approximately by

$$\begin{aligned}\tau_{\text{exp}} &\simeq \tau_{\text{SN}} V_g / (4\pi D^3/3) & (D < H) \\ &\simeq \tau_{\text{SN}} A_g / (\pi D^2) & (D > H),\end{aligned}\quad (3)$$

where τ_{SN} is the inverse of the mean rate of supernova outbursts in the Galaxy.

If $\tau_{\text{exp}} \gtrsim \tau_C, \tau_R$, then the ionization and heating may not be well approximated by a time-independent description. For example, assume $A_g = \pi(10 \text{ kpc})^2$, $H = 100 \text{ pc}$, $n_e = 0.03 \text{ cm}^{-3}$, $T = 10^4 \text{ K}$, and $\tau_{\text{SN}} = 100 \text{ years}$. Then we estimate the ratio

$$\begin{aligned}\eta &\equiv \tau_C / \tau_{\text{exp}} \simeq 0.4(D/100 \text{ pc})^3 & \text{for } D < H, \\ \eta &\simeq 0.6(D/100 \text{ pc})^2 & \text{for } D > H,\end{aligned}\quad (4)$$

and conclude that if such "supernovae" are the heat source for the interstellar gas, and if $D \leq 150 \text{ pc}$, a time-dependent description is required.

Suppose the ionizing agent is soft X-rays with energy $\epsilon_x < 200 \text{ eV}$. Then, the major X-ray absorption is due to He, and the stopping length D for the X-rays is given by (cf. Brown and Gould 1970)

$$D \approx 8(\epsilon_x/100 \text{ eV})^3/n \text{ pc}. \quad (5)$$

So, for example, if $n = 0.3 \text{ cm}^{-3}$ and $\epsilon_x = 200 \text{ eV}$, we find $D \approx 200 \text{ pc}$, which is not sufficient for the time-independent description to be satisfactory.

Suppose instead that low-energy cosmic rays are the main ionizing source. No satisfactory theory for the propagation of cosmic rays from a point source exists. The main problem is to understand the effects of wave-particle interactions in the presence of a magnetic field (Skilling 1971; Kulsrud and Cesarsky 1971). In view of this uncertainty, we simply characterize the diffusion of cosmic rays by an energy-independent mean free path δ , and estimate the influence radius D of a supernova cosmic ray outburst by

$$D \simeq (L\delta/3)^{1/2}, \quad (6)$$

where $L \simeq 300(\epsilon_{\text{CR}}/1 \text{ MeV})/n \text{ pc}$ is the stopping length of the cosmic ray of energy ϵ_{CR} . For example, if $\epsilon_{\text{CR}} = 2 \text{ MeV}$ and $n = 0.3 \text{ cm}^{-3}$, we have $D = 25\delta^{1/2} \text{ pc}$; and unless $\delta \gtrsim 100 \text{ pc}$, a time-dependent description is required.

In the steady-state model, the temperature and ionization are functions of a single parameter ζ/n , where ζ is the steady ionization rate per particle (Hjellming, Gordon, and Gordon 1969). In the time-dependent model ζ may fluctuate widely, but we may

estimate, for comparison with the steady-state model, an average ionization rate

$$\bar{\zeta} = E_{\text{SN}} f / \epsilon_{\text{ion}} \tau_{\text{SN}} N_g, \quad (7)$$

where E_{SN} is the energy in ionizing radiation or particles, ϵ_{ion} is the energy per ionization, $N_g \approx 10^{66}$ is the number of atoms in the galactic interstellar gas, and f is a factor ≤ 1 that accounts for the escape of ionizing particles from the Galaxy. The steady-state model requires $\zeta \gtrsim 10^{-15} \text{ s}^{-1}$ to account for the observed ionization of the gas. If supernovae are the source of the ionization, the above equation gives an estimate for the ionizing energy per supernova, $E_{\text{SN}} \gtrsim 10^{50} \text{ ergs}$. We anticipate that the time-dependent model will require roughly the same E_{SN} if it is to yield an interstellar medium with similar observed properties.

The time-dependent model is characterized essentially by two parameters. The first, $\bar{\zeta}/n$, is equivalent to that of the steady-state model. The second parameter, $\eta = \tau_C / \tau_{\text{exp}} \propto D^3$, is a measure of how much the conditions at a given point in the Galaxy should fluctuate. In the steady-state model, the physics of the propagation of ionizing agents from sources is suppressed by the implicit assumption $\eta \rightarrow \infty$.

Several effects complicate the development of a correct time-dependent description. Each point in the Galaxy is affected not only by nearby supernova events but also by more distant ones which have a cumulative effect. Also, the effect of each outburst depends on the immediate state of the gas. Moreover, the fact that the interstellar medium is not in local pressure equilibrium implies a need to treat dynamical effects as well as thermal effects in a self-consistent way.

In this paper, we present a particular version of a time-dependent model for the interstellar gas in detail, in order to illustrate essential differences between steady-state and time-dependent heating that may in principle be tested by observations. The main approximation we have made is to assume a galactic disk of constant density n , which is tantamount to neglecting dynamical effects. In this disk sudden bursts of monochromatic X-rays from point sources ("supernovae") occur with a specified mean rate at randomly chosen locations in the disk at random times.

Observations of the diffuse X-ray background for $\epsilon_x > 200 \text{ eV}$ put an upper limit on the energy output per supernova in such X-rays (Silk 1973a, b). This limit is far below that required for supernova X-ray bursts to account for the heating and ionization of the interstellar medium. However, if $\epsilon_x < 200 \text{ eV}$, absorption by local gas would make the galactic disk opaque to extragalactic X-rays, and there is no longer any observational constraint on the soft X-ray output of supernovae.

Detailed calculations (Gerola, Iglesias, and Gamba 1973; Schwarz 1973) of the effect of an X-ray outburst on the interstellar medium already exist. It should also be possible to construct a similar time-dependent model in which the ionizing source is low-energy cosmic-ray outbursts, with similar results. However, because of the difficulties concerning the propagation

of such cosmic rays, we have chosen to treat only the X-ray case in detail.

Because the sources of ionization and heating are presumed to fluctuate in space and time, it is not possible to predict the temperature T and ionized fraction $x = n_e/n$ of any particular point in the interstellar medium. Rather, the model predicts probability distributions for T and x .

Although our constant-density model is drastically different from the FGH model or variations thereof, the results of our calculations embody many of the observed characteristics of the gas. For example, the model yields a gas with a bimodal temperature distribution with peaks at $T \simeq 30^\circ \text{K}$ and $T \simeq 8000^\circ \text{K}$, which may appear as a two-component medium consisting of cool "clouds" embedded in a hot "intercloud medium," as observed.

II. THE MODEL

We assume a galactic disk of radius R_g , thickness $2H$, and volume $V_g = \pi R_g^2 H$, filled with gas of constant density n and cosmic abundances (cf. Dalgarno and McCray 1972). We fix our attention on a given point P in the galactic plane. We include a small amount of steady ionization and heating by the observed soft X-ray background. Supernova events are assumed to occur randomly in space and time with a mean rate per unit volume $(\tau_{\text{SN}} V_g)^{-1}$ which is assumed uniform throughout the disk. (These specific assumptions could easily be relaxed, but they seem to be a reasonable first approximation for the distribution of Type II supernovae.) For each supernova event at distance D from P , we calculate the discontinuous changes of T , x . Between supernova events we calculate the time-dependent cooling and recombination of the gas. By allowing this process to continue long enough, it is possible to construct a probability distribution function $P(x, T)$ which is proportional to the fraction of the time record that the gas spends within a given range of the parameters (x, T) . Then we invoke a sort of ergodic hypothesis which states that the probability distribution of time-fluctuating variables at one point in space is the same as their probability distribution over a large number of equivalent points at one instant of time. Spatial averages of the various observable quantities are constructed by taking the appropriate moments of the distribution function.

For purposes of computation, we consider concentric shells each of thickness ΔD surrounding P . The volume in a shell is then given by $\Delta V = 4\pi D^2 \Delta D$ for $D < H$ and by $\Delta V \approx 4\pi D \Delta D H$ for $D \gg H$. In a time interval Δt , the probability y of an event in a shell at distance D is given by

$$y = \Delta V \Delta t / (V_g \tau_{\text{SN}}). \quad (8)$$

We choose Δt small enough to ensure that the probability of having more than one supernova event in Δt near enough ($D < D_{\text{max}}$) to affect P is negligible. In order to construct the time record at P , we make a Monte Carlo simulation of the supernova events.

This is accomplished by using a random number generator which selects a value $0 \leq y \leq 1$. Given y , we calculate from equation (8) the distance D of the event. We compute then the discontinuous change $(\Delta x, \Delta T)$ of (x, T) at P due to the event.

In general, this change will be a function of D , the local conditions (x, T) at P , and the condition of the intervening matter. In the case of a burst of radiation, Gerola *et al.* (1973) and Schwarz (1973) have calculated the initial ionization structure of the "fossil Strömgren sphere" produced by a radiation burst in a homogeneous gas which is initially cold and neutral.

For simplicity we have considered a burst of monochromatic soft X-rays (energy ϵ_x), and we have generated solutions for different initial conditions of the gas. Figures 1a and 1b show the results of these calculations for three different photon energies; the effect of different initial conditions for $\epsilon_x = 150 \text{ eV}$ is also shown. Note that the solutions are characterized by a fully ionized core region with $T \gtrsim 10^5 \text{ }^\circ \text{K}$, surrounded by a thick transition region in which the temperature and ionization profiles are well approximated by $\Delta T \propto \exp(-R/R_0)$ and $\Delta x \propto \exp(-R/R_1)$, where R_0 and R_1 are scale lengths that depend on ϵ_x and n . The thickness of the transition region decreases rapidly with decreasing ϵ_x ($R_0, R_1 \propto \epsilon_x^{-3}$, typically). We have found that if the X-rays are too soft ($\epsilon_x \lesssim 70 \text{ eV}$) the transition region is small relative to the fully ionized core, and it is impossible to construct a model that appears like the observed interstellar medium. Therefore, we have concentrated our attention on models with $70 \text{ eV} \leq \epsilon_x \leq 200 \text{ eV}$, the upper limit being dictated by the observational constraint of the soft X-ray background.

As a further simplification, we have ignored possible variations in the ionized fraction between the source at distance D and the point P . This is a reasonable approximation because a fully ionized region will substantially recombine before another supernova event affects P [$\tau_{\text{rec}}(x > 0.1) \ll \tau_{\text{exp}}(D_{\text{max}})$]. But unless the intervening gas is substantially ionized (say, $x > 0.1$), its ionization level does not significantly alter the effect of an outburst on P .

Subsequent to the discontinuous changes ΔT , Δx caused by the supernova event, the gas cools and recombines. The hydrogen recombination equation is given by

$$\frac{dx}{dt} = -\alpha^{(2)} x^2 n + (1 - x)[1 + \phi(x)]\zeta, \quad (9)$$

where $\alpha^{(2)}$ is the recombination coefficient to the excited states and $\phi(x)$ is the number of secondary ionizations (Jura 1971). The steady ionization rate ζ is uncertain because it depends on the flux of X-rays and cosmic rays at low energies ($\epsilon_x < 250 \text{ eV}$ and $\epsilon_{\text{CR}} < 100 \text{ MeV}$) where observations are most difficult and spatial fluctuations are more likely. We have estimated ζ due to soft X-rays in the range $100 \text{ eV} < \epsilon_x < 250 \text{ eV}$ (Yentis, Novick, and Vanden Bout 1972a, b; Bunner *et al.* 1971) to be in the range $10^{-18} \text{ s}^{-1} < \zeta < 2 \times 10^{-17} \text{ s}^{-1}$. Yentis *et al.* also

FIG. 1a

FIG. 1b

FIG. 1a.—Variation of temperature ΔT , produced by a monochromatic burst of soft X-rays of energy ϵ_x , as a function of distance to the supernova. Curve label means: B: $\epsilon_x = 100$ eV, $x(\text{initial}) = 10^{-3}$, $T(\text{initial}) = 100^\circ \text{K}$; A1: $\epsilon_x = 150$ eV, $x(\text{initial}) = 10^{-3}$; A2: $\epsilon_x = 150$ eV, $x(\text{initial}) = 10^{-1}$; C: $\epsilon_x = 200$ eV, $x(\text{initial}) = 10^{-3}$. For all the curves, $n = 0.3 \text{ cm}^{-3}$, $E(\text{total energy emitted by the supernova}) = 7 \times 10^{50} \text{ ergs}$.

FIG. 1b.—Variation of ionized fraction x . Labels same as fig. 1a.

observe a “local” component of soft X-rays of uncertain (possibly terrestrial) origin, which could contribute an ionization rate $\zeta \approx 10^{-16} \text{ s}^{-1}$ (Silk 1973b). The observed cosmic rays contribute an ionization rate $\zeta \lesssim 10^{-17} \text{ s}^{-1}$. Unless otherwise noted ζ was taken to be 10^{-17} s^{-1} . Note that this value of ζ is much smaller than that needed to account for the observed ionization. The average ionization rate $\bar{\zeta}$ due to supernova outbursts will be considerably greater.

The energy balance equation is given by

$$\frac{3}{2} k \frac{dT}{dt} = -\Lambda(x, T)n + A - B - C + \zeta \bar{E}, \quad (10)$$

where the terms A , B , C arising from recombinations and dx/dt are given by (Kafatos 1973)

$$A = \frac{3}{2} k T \alpha^{(1)} n x^2, \quad B = \langle E \sigma v \rangle n x^2, \quad C = \frac{3}{2} k T \langle \sigma_1 v \rangle n x^2,$$

where the angular brackets denote integrations over the Maxwellian distribution, $\alpha^{(1)}$ is the total recombination coefficient, and σ_1 is the cross-section for recombinations to the ground state. These terms introduce a net heating effect. The total heat per ionization $\bar{E}$ was taken from Jura (1971). The cooling term Λ is made up of two contributions, Λ_e , the cooling arising from excitation of atoms and ions by electron impact, and Λ_H , the cooling arising from excitation by neutral-hydrogen impact. These rates were taken from Dalgarno and McCray (1972). Since the cooling due to collisional excitation of trace elements is fairly insensitive to their ionization stage (Schwarz 1973), we have assumed that the trace elements have fixed ionization distribution as follows: $\text{C}^+ = 1$, $\text{S}^+ = 1$, $\text{Si}^+ = 1$, $\text{Fe}^+ = 1$, $\text{N}^0 = 0.55$, $\text{N}^+ = 0.45$, $\text{Ne}^+ = 0.22$, and the O^0 , O^+ abundances tied to the H^0 , H^+ abundances by charge exchange (Field and Steigman

1971). The hydrogen $\text{L}\alpha$ cooling provides a strong barrier against $T \gtrsim 20,000^\circ \text{K}$.

After each interval Δt , the program again looks for a supernova event inside D_{max} then resumes cooling and recombination. In order to ensure convergence to a smooth probability distribution, the random number generator was called by the program a large number of times, typically 10^4 to 10^5 times in each run. Typical segments of the time records $T(t)$, $x(t)$ are shown in figures 2a and 2b. Most of the times that the random number generator is called, nothing occurs because the supernova event is beyond D_{max} . The distance of each significant event is indicated in figure 2a. Notice that nearby events which cause a large ΔT , Δx are rare, as expected.

The joint probability distribution $P(x, T)$ is calculated by dividing the space $[10 \leq T \leq 2 \times 10^4, 2 \times 10^{-4} \leq x \leq 1]$ into equal logarithmic intervals, typically 50×50 . $P(x, T) \Delta \log x \Delta \log T$ is then the fraction of the time record that the gas spends in the range $(\log T, \log x; \log T + \Delta \log T, \log x + \Delta \log x)$ normalized so that $\int d \log x \int d \log T P(x, T) = 1$. Similarly, other distribution functions are calculated from the time records.

In previous discussions of time-dependent models (Bottcher *et al.* 1970; Dalgarno and McCray 1972; Gerola *et al.* 1973) a simple analytical formula for the probability distribution $P(T)$ was given. This formula followed from the assumptions that the initial conditions of the gas subsequent to a supernova event are independent of the location of the source and of the state of the gas before the event, and that the time interval between outbursts obeys Poisson statistics. The resulting bimodal distribution of $P(\log T)$ was suggestive of the possibilities of the time-dependent description of the interstellar medium, but the above assumptions precluded any definite comparison with observations.

FIG. 2a (top).—Time evolution of temperature at a given point P in the galactic disk as a function of time. The numbers on the curve indicate the distance (in parsecs) from P at which each supernova occurs $\tau_{SN} = 30$ years; $n = 0.3 \text{ cm}^{-3}$.

FIG. 2b (bottom).—Time evolution of the ionization fraction x .

The strong dependence of the effect of a supernova event on distance from the source, and the fact that the random process is non-Markovian in the sense that the effect of an event depends on the previous history of the gas, could be properly accounted for only by a numerical simulation.

III. RESULTS

Various models have been calculated with parameters as listed in table 1. We present results for a gas of uniform density $n \leq 0.3 \text{ cm}^{-3}$, appropriate to the "intercloud" medium in the solar neighborhood (Dalgarno and McCray 1972; Mebold 1972; Macchetto and Panagia 1973). The supernova rate τ_{SN} (30 years, 100 years) is in the range indicated by optical (Tammann 1970) and radio (Caswell 1970; Ilovaisky and Lequeux 1972) observations. The total energy per supernova E_{SN} and the energy of the monochromatic X-rays ϵ_X were chosen respecting the observational constraints mentioned at the end of § I.

One may imagine the time history of the gas at

point P , as shown in figures 2a and 2b to be represented by a point which moves around the (x, T) -plane. The probability density $P(x, T)$, which is proportional to the fraction of the time that P spends in any part of this plane, is shown for model 1 of table 1 in figure 3. The contour lines are lines of equal probability density in the (x, T) -plane. The logarithm of the probability density at each contour line is shown on the figure.

The contour map shows not only the bimodal temperature distribution indicated by earlier studies, but also the correlation of ionized fraction with temperature. The main features of this diagram—the peak centered at $x \approx 0.03$, $T \approx 3000^\circ$ and the ridge at $T \approx 30^\circ$, $3 \times 10^{-3} \leq x \leq 0.2$ —can be understood qualitatively in terms of the relaxation timescales of the gas, given by equations (1) and (2). Suppose, for example, that the gas at P has cooled and recombined so that its state is represented by point A in figure 3. If it is exposed to a (relatively rare) nearby supernova event, it will suddenly become highly ionized ($x \approx 1$) and very hot ($T \gtrsim 10^4^\circ \text{ K}$). As a result of rapid cooling by allowed transitions (cf. Kafatos 1973), the gas will quickly relax to point B .

The further relaxation of the gas is represented by the dashed curve BCA of figure 3 (cf. Bottcher *et al.* 1970). Since $\tau_c(10^4^\circ \text{ K}) > \tau_c(10^3^\circ \text{ K})$, the gas lingers for a while at 10^4° K before it cools to 10^3° K . The relatively rapid cooling for $10^2^\circ \text{ K} \leq T \leq 10^3^\circ \text{ K}$ explains the low probability density in this range. Since

FIG. 3.—Probability density $P(x, T)$ for model 1 of table 1. The contour lines are equal probability density lines. Curve SS is the steady-state relation, from Dalgarno and McCray (1972), CL indicates the clouds, and ICI the intercloud medium, in the FGH model. The numbers in the light solid curves are the logarithm of the probability. Dashed curves are the evolutionary tracks referred to in the text.

TABLE 1
PARAMETERS OF THE CALCULATED MODELS

Supernova Model	Total Energy Emitted E_{SN} (ergs)	Photon Energy ϵ_x (eV)	Density n (cm^{-3})	Inverse SN Rate τ_{SN} (years)	Observations*
1.....	7×10^{50}	150	0.3	30	
2.....	7×10^{50}	100	0.3	30	
3.....	7×10^{50}	200	0.3	30	
4.....	7×10^{50}	70	0.3	30	
5.....	10^{50}	150	0.3	30	
6.....	7×10^{50}	150	0.3	100	
7.....	7×10^{50}	150	0.1	30	
8.....	7×10^{50}	150	0.3	30	Depletion = 3
9.....	7×10^{50}	150	0.3	30	Depletion = 10
10.....	7×10^{50}	150	0.3	30	$\zeta = 10^{-18} \text{ s}^{-1}$

* Unless otherwise noted, $\zeta = 10^{-17} \text{ s}^{-1}$ and no depletion.

$\tau_C < \tau_R$ for $T \geq 10^2 \text{ }^\circ \text{K}$, the gas cools along segment BC of curve BCA without significant recombination. But when the temperature has dropped below $10^2 \text{ }^\circ \text{K}$, $\tau_C > \tau_R$ and the gas now recombines along segment CA without substantial further cooling.

The evolutionary time scale along segment CA is considerably longer than the time scale along BC , so that there is now an appreciable chance of exposure to another supernova event during this evolution. A relatively distant (say, $D \approx 80 \text{ pc}$) supernova event, which is more likely than a nearby one, may heat the gas $T \geq 10^4 \text{ }^\circ \text{K}$ without fully ionizing it, according to figures 1 and 2, and perhaps discontinuously change the state of the gas from some point along CA to point D . Then the gas will relax along curve DA . Since $\tau_C \propto n_e^{-1}$, the gas would remain near point D longer than it would remain near point B , so that the high-temperature peak of $P(x, T)$ is found in the vicinity of D . Another reason the peak is found for $x \approx 10^{-2}$ is that relatively nearby bursts (say $50 \leq D \leq 150 \text{ pc}$) are the most important and for these $0.001 \leq \Delta x \leq 0.1$ (cf. figs. 1a, 1b, and 3).

The effect of the steady ionization rate ζ and the heating due to recombinations is noticeable at low temperatures. Figure 2 shows that the gas temperature sometimes increases with time. This occurs because the electron-ion cooling decreases with recombination while the heating remains approximately constant. The effect can also be seen in figure 3, where the lowest temperatures are found with $x \approx 0.1$.

In the steady-state theory, where heating balances cooling and ionization balances recombination, x and T are both single-valued functions of the parameter ζ/n . Therefore, regardless of the values of ζ/n , the theory yields a unique relationship $x(T)$. This relationship is rather insensitive to the nature of the ionizing flux (cosmic rays or X-rays), but at low temperatures it depends somewhat on the chemical composition of the gas. The curve marked SS in figure 3 is taken from Dalgarno and McCray (1972) with $\epsilon_x = 200 \text{ eV}$ and no depletion of trace elements.

If there are spatial fluctuations in ζ or n , any given point in the gas must still lie on curve SS . For example, if ζ is constant and the gas pressure is also assumed constant in space, the gas must be in one of two states,

dense "clouds" or in the lower density "intercloud medium," represented by points Cl and ICl , respectively, on curve SS . This is the two-phase model of FGH.

In our time-dependent model we have abandoned the assumption of pressure equilibrium, and have assumed a constant density gas. As discussed in §1, the distribution for $P(x, T)$ should then be characterized by two parameters, ζ/n and τ_C/τ_{exp} . The condition of the gas is no longer characterized by a unique point on the (x, T) -curve, but by the contour map of $P(x, T)$.

It is evident from figure 3 that the gas divides itself into distinct hot and cold components. But in contrast to the steady-state theory, our model has cool regions with the same density as the hot region. In the time-dependent model presented here the relative fractions of the gas that are expected to be in the hot and cool phases are completely determined by the choice of ζ/n and τ_C/τ_{exp} ; but in the steady-state model, additional considerations are needed to determine the fractions in clouds and intercloud medium.

Notice that (x, T) of most of the gas does not lie on the curve SS . Generally, the time-dependent model predicts that the high-temperature regions will be less ionized ($x < 0.1$) than the steady-state intercloud medium. Further, the time-dependent model predicts a significant probability (few percent) for regions of cool ($T \approx 30 \text{ }^\circ \text{K}$), ionized ($x \geq 0.05$) gas. Such regions are forbidden in the steady-state theory and modifications thereof (e.g. Mészáros 1973).

Given the time record of figure 2, we also construct probability distributions for various observable parameters: $P(x)$, figure 4; $P(T)$, figure 5; $P[(1-x)T^{-1}]$, figure 6; $P(x^2T^{-3/2})$, figure 7; $P(x^2T^{-1/2})$, figure 8; each normalized such that $\int P(A) d \log A = 1$. The noise in the probability distributions is a result of the finite time record of the Monte Carlo program. (This noise was smoothed by hand in fig. 3 for the sake of visual clarity.) The arrow in each figure indicates the algebraic mean value of the function

$$\bar{A} = \int AP(A) d \log A.$$

Note that these averages differ from the "eyeball average," which would correspond to the geometric mean of the function $\int \log AP(A) d \log A$.

FIG. 4.—Probability density for the ionized fraction x for model 1 of table 1. The arrow indicates the algebraic mean value of the function.

We are modeling the galactic interstellar medium by a superposition of fossil Strömgren spheres (FSS) of different ages, each of a certain characteristic radius D ($100 \leq D \leq 200$ pc), where the state of each point within that radius is controlled primarily by the same event. The average elapsed time necessary to erase the memory of that event at any given point is roughly τ_{exp} . Therefore, a spatial average over a distance $L = kD$ is roughly equivalent to a time average over a record $T = k\tau_{\text{exp}}$. Observations ordinarily measure the column density of some parameter over a finite path, which yields a spatial average along that path. If the parameter has large random spatial fluctuations, one would not expect averages over two different finite paths of the same length to be equal. The averages would converge as the path length increases. Similarly, if a parameter has large temporal fluctuations at one point, one would not expect averages taken over different finite periods of equal duration to be equal. The size of the FSS plays the role of a correlation length.

FIG. 5.—Probability density for the temperature T for model 1 of table 1.

FIG. 6.—Probability density for $(1-x)/T$ for model 1 of table 1.

The large fluctuations in x and T inherent in the time-dependent model preclude a specific prediction about the state of a particular point in the interstellar medium. For example, referring to figure 5, one sees that there is a substantial chance of finding a point whose temperature is less than the average temperature by a factor ~ 100 . The same is true for the other moments. The time-dependent model only has power to predict averages taken over a large number of different points in the Galaxy.

If we consider observations of quantities over paths ~ 100 pc ($k \sim 1$), we would expect (x, T) to be in any part of figure 3 where $P(x, T)$ is large, and the variables represented in figures 4–8 might have values anywhere the probabilities are large, subject to the correlations of figure 3. Further, quantities observed in different directions might differ greatly. This might be the case with the ultraviolet observations of nearby hot stars taken with the OAO-3 *Copernicus* satellite, Rogerson *et al.* (1973). On the other hand, radio observations which measure column densities of quantities over large distances (e.g., 2 kpc, $k \sim 20$ for

FIG. 7.—Probability density for $x^2/T^{3/2}$ for model 1 of table 1.

FIG. 8.—Probability density for $x^2/T^{1/2}$ for model 1 of table 1.

the Crab nebula; 10 kpc, $k \sim 100$ for the galactic center), should yield values that approach the computed averages. Of course, structural features of the Galaxy which we have ignored, such as variations in gas density and supernova rate in the galactic center and in the spiral arms, may also introduce large variations in quantities measured in different directions.

Table 2 presents results computed for the models of table 1. The column headings are the “observable” average quantities $\langle A \rangle = \int AP(A)d \log A \equiv \int Adl / \int dl$ over a large path $\int dl$. Note the large difference (factor ~ 40) between the average temperature $\langle T \rangle$ (col. [2]) and the harmonic mean temperature $\langle n_H \rangle / \langle n_H T^{-1} \rangle$ (col. [3]). This is a consequence of the bimodal temperature distribution (fig. 5). The harmonic mean temperature favorably weighs the cool regions. This effect is also characteristic of the two-phase model of FGH, provided that roughly equal column densities of hydrogen are assumed to be in the hot and cool phases. Column (4) shows that for most of our models, the hot and cool fractions are comparable. If the fractions differ substantially, as in model 9, the difference between $\langle T \rangle$ and $\langle T^{-1} \rangle^{-1}$ decreases. The observed absorption features are ordinarily interpreted as clouds. In our constant density model, we expect the 21-cm absorption to appear as distinct features corresponding to individual cold FSS of dimension ~ 100 –200 pc. The quantity $\langle n_e^2 T^{-1/2} \rangle$ of column (7) is roughly proportional to the recombination rate and hence gives a measure of H α and H β emission. The quantity $\langle n_e^2 T^{-3/2} \rangle$ of column (8) is roughly that measured by low-frequency radio continuum absorption, and by (low frequency) radio recombination-line emission. Almost all of the contribution to $\int n_e^2 T^{-3/2} dl$ comes from the small fraction of cool, highly ionized gas with $T \simeq 30^\circ$ K and $x \simeq 0.05$, as can be seen by comparing column (8) with column (9).

If steady-state ionization prevailed, we should have the relation

$$\zeta n_H = n_e^2 \alpha(T),$$

where the recombination rate $\alpha(T) \propto T^{-1/2}$, roughly, so that $\zeta \propto n_e^2 / n_H T^{-1/2}$. Hughes, Thompson, and Colvin (1971) have suggested that the mean ionization rate could be measured by dividing the 21-cm absorption measure $\int n_H T^{-1} dl$ by the low-frequency continuum absorption measure $\int n_e^2 T^{-3/2} dl$. They therefore list an observed quantity which we call “ ζ ”, defined by

$$“\zeta” = 1.27 \times 10^{-15} \nu^{2.1} T^{-0.3} \tau_{\text{ff}} / \int \tau dv. \quad (11)$$

We have calculated the analogous quantity “ ζ ” for our models in column (10).

We emphasize that it can be deceptive to associate the value of “ ζ ” with the mean ionization rate $\bar{\zeta}$. In our models the major contribution to $\langle n_e^2 T^{-3/2} \rangle$ comes from the rare cold ionized regions and the major contribution to $\langle n_H T^{-1} \rangle$ comes from the fairly common cold neutral regions. Therefore “ ζ ” has nothing to do with local conditions. Despite this, we note that the calculated “ ζ ” is close to the average ionization rate (eq. [7]), $\bar{\zeta} = 10^{-14} \text{ s}^{-1}$ for model 1, assuming $N_0 = 10^{66}$, $\epsilon_{\text{ion}} = 50 \text{ eV}$, $f = 1$. We consider this agreement fortuitous. In contrast to the time-dependent model, the steady-state model has the dominant contribution to both the low-frequency continuum absorption and the 21-cm absorption coming from the same regions (the clouds). In that case “ ζ ” should be a good measure of the local ionization rate in clouds.

Table 2 shows the dependence of the time-dependent models on the various parameters. Generally, the topology of the $P(x, T)$ contour map is the same for all models. The main effect of changing parameters is to shift the relative probability in the low-temperature ridge and the high-temperature peak but not to change much the location of either. For example, it is impossible to construct a time-dependent model in which most of the gas has intermediate temperature ($10^2 \text{ }^\circ\text{K} < T < 2 \times 10^3 \text{ }^\circ\text{K}$).

A comparison of models 1–4 shows that the results are not very sensitive to the assumed photon energy. Column (4) shows that the relative fractions of hot and cool gas are insensitive to ϵ_x for $\epsilon_x > 100 \text{ eV}$; but when ϵ_x decreases below 100 eV, the high-temperature fraction begins to decrease. The ionization level of the gas (cols. [6], [9], and [11]) and hence the low-frequency absorption (col. [8]) increases with decreasing ϵ_x .

Comparisons of models 1 and 5 show the dependence on the assumed supernova energy E_{SN} . As E_{SN} is increased, the ionization level increases but the temperature is fairly constant. Model 6 shows that a change of supernova rate has much the same effect, which is expected since the average ionization rate $\bar{\zeta} \propto E_{\text{SN}} / \tau_{\text{SN}}$. A value $E_{\text{SN}} / \tau_{\text{SN}} \gtrsim 3 \times 10^{48} \text{ ergs per year}$ is required if supernovae are to keep a good fraction of the intercloud medium hot. Comparison of model 7 with model 1 shows that the ionization and temperature are more sensitive to gas density n than to E_{SN} or τ_{SN} . Although we have not constructed models for the gas above the plane, this strong density

TABLE 2
RESULTS FOR THE MODELS OF TABLE 1

SN Model (1)	$\langle T \rangle$ (2)	$\langle T^{-1} \rangle^{-1}$ (3)	Percent for $T < 500$ (4)	$\langle n_H T^{-1} \rangle$ (5)	$\langle n_e \rangle$ (6)	$\langle n_e^2 T^{-1/2} \rangle$ (7)	$\langle n_e^2 T^{-3/2} \rangle$ (8)	Percent for $x > 5 \times 10^{-2}$, $T < 100$ (9)	" ζ " (10)	Percent for $x > 0.1$ (11)	Percent for $x < 0.1$, $T > 500$ (12)
1.....	2×10^3	58	60	5×10^{-3}	1.25×10^{-2}	2.7×10^{-5}	4.3×10^{-7}	2.6	7×10^{-15}	7.2	34
2.....	2.2×10^3	50	63	5.6×10^{-3}	1.6×10^{-2}	5.5×10^{-5}	8.2×10^{-7}	4	1.2×10^{-14}	8.3	30
3.....	1.6×10^3	58	61	4.9×10^{-3}	1.2×10^{-2}	1.8×10^{-5}	2.7×10^{-7}	1.6	4.4×10^{-15}	5	35
4.....	1.2×10^3	39	80	7.2×10^{-3}	1.8×10^{-2}	6.7×10^{-5}	9.8×10^{-7}	4.3	1.1×10^{-14}	8.6	14
5.....	1.9×10^3	65	63	4.5×10^{-3}	6.4×10^{-3}	10^{-5}	1.7×10^{-7}	< 1	3×10^{-15}	1.5	36
6.....	10^3	49	77	6×10^{-3}	5×10^{-3}	9.6×10^{-6}	1.5×10^{-7}	< 1	2.1×10^{-15}	1.4	22
7.....	4.5×10^3	146	26	6×10^{-4}	1.3×10^{-2}	10^{-5}	9.6×10^{-8}	8	1.3×10^{-14}	39	42
8.....	3.4×10^3	178	34	1.5×10^{-3}	2.4×10^{-2}	3.6×10^{-5}	9.4×10^{-8}	2	4.9×10^{-15}	16	51
9.....	5.4×10^3	2.6×10^3	4	9.8×10^{-5}	4.5×10^{-2}	4.7×10^{-5}	8.2×10^{-9}	0	6.7×10^{-15}	52	46
10.....	2×10^3	37	59	7.7×10^{-3}	1.2×10^{-2}	2.7×10^{-5}	4.8×10^{-7}	2.4	5×10^{-15}	7.1	35
Observations.....	$> 750^a$ $> 600^f$	$60-80^a$ $62-80^f$	50^a $36-45^f$ 75^f	8×10^{-3b} 3×10^{-3b}	2.5×10^{-2c}	8×10^{-5d}	2×10^{-7e} 4×10^{-8g} 10^{-6g} 5×10^{-6j} 5×10^{-6h}	2×10^{-15f} 10^{-13h} 2.5×10^{-15i} 10^{-14d}			

SOURCE.—(a) Radhakrishnan *et al.* 1972; (b) Radhakrishnan and Goss 1972; (c) Falgarone and Lequeux 1973; (d) Reynolds *et al.* 1971; (e) Bridle 1970; (f) Hughes *et al.* 1971; (g) Ellis and Hamilton 1966; (h) Gordon and Cato 1972; (i) Hjellming *et al.* 1969; (j) Dulk and Slee 1972.

dependence suggests that the probability of cool regions will decrease above the plane. In other words, we expect that the scale height of the cool component is smaller than that of the hot component, as observed by Radhakrishnan *et al.* (1972). Models 8 and 9 show that reduced cooling due to depletion of trace elements causes a marked reduction in the fraction of cool gas. Comparison of model 10 with model 1 shows the influence of the steady ionizing background. The background is too weak to have a substantial effect on the ionization level or on the relative fraction of hot and cold gas, but it does noticeably affect the temperature of the cool gas, as can be seen from column (3).

IV. DISCUSSION

The last row of table 2 shows the average values of observed quantities. Some of these are found in the references listed in the footnotes to table 2, and the rest were deduced from them.

We expect the existence of two temperature phases of the neutral gas as observed (Radhakrishnan *et al.* 1972; Hughes *et al.* 1971). From figure 3 we see that the temperature of the hot component should be in the range 10^3 – 10^4 °K. The average temperature $\langle T \rangle$ is not observed, but in some instances a lower limit to the temperature of the hot component can be found, and this is listed at the bottom of column (2) of table 2. The calculated harmonic mean temperature (col. [3]) essentially weighs the cold component. Radhakrishnan *et al.* (1972) and Hughes *et al.* (1971) measure the spin temperature for the cold absorption features; this is shown in column (3). The fraction of matter in the cold phase is roughly 50 percent, although this fraction depends on location in the Galaxy; the upper value taken from Hughes *et al.* (1971) refers to the solar neighborhood, and the authors claim this is perhaps representative of the whole Galaxy; the second value refers to a line of sight close to the galactic plane. The diameter of an individual FSS introduces a natural scale length in the time-dependent model, which may be compared with observations. The scale size of our absorption features is therefore roughly 100–200 pc. This, taken with the fact that typically 60 percent of the matter is in the cool state (e.g., models 1, 2, 3), yields an average of 3–6 features per kpc. Radhakrishnan and Goss (1972) give a minimum value of 2.5 features per kpc. The value of $\langle n_H T^{-1} \rangle$ can be deduced from the observations if the total path length through the cold features is known. We expect this to be 600 pc if one looks 1 kpc away. Combining this with the value of $n_H T^{-1} = 1.5 \times 10^{19}$ atoms cm^{-2} (deg. K) $^{-1}$ kpc $^{-1}$ (low resolution, 10 kHz wide filter) and $n_H T^{-1} = 6 \times 10^{18}$ (high resolution, 1 kHz wide filter) given by Radhakrishnan and Goss (1972), we obtain the two values listed in column (5). Essentially all the absorption comes from the component with $T < 500^\circ \text{K}$ according to our model. The maximum value of $n_H T^{-1}$ in our theory is 1.6×10^{-2} (model 1). Therefore, we expect a maximum integrated optical depth

$\int \tau(v) dv$ of 2.75 km s^{-1} for one feature. A number of cold features observed have higher values, and this is not surprising since higher optical depths would be obtained if higher densities were introduced in our calculations.

The value of $\langle n_e \rangle$ is obtained using dispersion measures of seven pulsars with known distances. Typically 5–8 percent of the matter has $n_e > 0.03 \text{ cm}^{-3}$ in our theory (col. [11] of table 2).

The quantity $\langle n_e^2 T^{-1/2} \rangle$ is deduced from diffuse H β observations in the direction of four pulsars (Reynolds *et al.* 1971). The upper limit to $n_e^2 T^{-1/2}$ (model 1) is $3 \times 10^{-3} \text{ cm}^{-6} (\text{deg. K})^{-1/2}$, yielding an emission measure E.M. of $30 \text{ cm}^{-6} \text{ pc}$ at 10^4 °K. Therefore the H α observations, which typically yield E.M. $\sim 300 \text{ cm}^{-6} \text{ pc}$ at 10^4 °K, do not refer in our picture to a $n \leq 0.3 \text{ cm}^{-3}$ intercloud medium. These observations could be selectively picking up low-intensity H II regions.

The quantity $\langle n_e^2 T^{-3/2} \rangle$ can be inferred either from low-frequency absorption measurements or from low-frequency radio recombination lines. This value calculated by Bridle (1970) refers to $T \simeq 100^\circ \text{K}$; a scale height of the absorbing gas of 200 pc was used to derive the same quantity from Ellis and Hamilton (1966) (the first number refers to observations at $b \simeq -60^\circ$; the second at $b \simeq -5^\circ$). The observations of 20 nonthermal sources at 80 MHz carried out by Dulk and Slee (1972) yield an average of (E.M.) $T^{-1.35}$ and an average distance. The value quoted is obtained if $T \simeq 100^\circ \text{K}$. Dickel (1973), though, claims that many of the sources observed by Dulk and Slee have no noticeable decrease in flux density at 80 MHz. The value of $\langle n_e^2 T^{-3/2} \rangle$ from Gordon and Cato (1972) was determined from observations of the H 157 α recombination line in the galactic plane in a zone bounded by galactic radii of approximately 3 and 9 kpc. Cesarsky and Cesarsky (1973), by comparing the H 92 α recombination line and free-free absorption observed in the direction of the remnant 3C 391, obtain a most probable T of the emitting region of 20°K ; the corresponding emission measure is E.M. = $4 \text{ cm}^{-6} \text{ pc}$. This observation may be the first one to pick out a low- T ($T \approx 20^\circ \text{K}$), high- x ($x \approx 0.1$) region, which is forbidden in any steady-state theory. Observations of this kind are very useful in distinguishing between time-dependent and steady-state theories.

The ionization rate is measured in a variety of ways. Hughes *et al.* (1971) compare 21-cm absorption measurements with free-free absorption; Reynolds *et al.* (1971) combine H β and 21-cm emission measurements; Gordon and Cato (1972) combine radio recombination line and 21-cm absorption measurements; Hjellming *et al.* (1969) combine pulsar dispersion measures for three pulsars and 21-cm absorption.

The parameters obtained with several of our constant density models agree remarkably well with observations of quantities, such as pulsar dispersion measure, low-frequency continuous absorption, and 21-cm absorption, which are sensitive to the conditions in different regions of the interstellar medium.

We now discuss the implications of this agreement.

Intrinsic to the time-dependent model are large fluctuations in the various quantities. For example, figure 7 shows that particular regions may have with high probability values of $n_e^2 T^{-3/2}$ which differ from the average value by 10^2 . Therefore, this model predicts that the low-frequency absorption measure will be extremely patchy, as is observed by Dulk and Slee (1972). Similarly, the theory predicts large fluctuations in T (fig. 5) and $n_H T^{-1}$ (fig. 6), as observed. Of course, such fluctuations also occur in the two-phase models. However, the two-phase model of FGH predicts that n_e is approximately the same in clouds and intercloud medium and that " ζ " is constant everywhere. Our models predict large local fluctuations in n_e and " ζ ", as observed.

In the steady-state model, the fraction of the gas in clouds and the scale size of the clouds is indeterminate. Since the cool clouds in the steady-state model have roughly 100:1 density contrast with the intercloud medium, the scale size of clouds that would give the same absorption features would be much smaller, say 2 pc instead of 200 pc.

The time-dependent model predicts a clumpy distribution as well for the Na^0 and Ca^+ absorption features. The density of these atoms is proportional to their recombination rate (Habing 1969), which goes roughly as $n_e^2 T^{-1/2}$. These lines should therefore originate primarily from the cool partially ionized regions. Further, one might expect a good correlation between radio recombination-line emission features and Na and Ca absorption features.

The constant-density assumption is not an essential feature of the time-dependent model; in fact, the pressure gradients and nonuniform cooling rates built into the model will ensure that large density fluctuations will develop in the gas (Schwarz, McCray, and Stein 1972). We have made the extreme assumption of constant density to illustrate that much of the observed structure of the interstellar medium can be due to fluctuations in heating and ionization rather than fluctuations in density. Of course, it should be possible to generalize the time-dependent model by assuming a distribution function for the density structure of the gas. At this point, such a further adventure in parameter fitting does not seem warranted.

In contrast to the time-dependent model, the steady-state model makes the definite prediction of a cloud intercloud density contrast of at least 40:1 and more likely 100:1. In view of that, we should examine the evidence for density fluctuations of the gas in the galactic plane. This evidence comes from various sources: 21-cm emission, interstellar extinction, optical and ultraviolet absorption, and molecular radio lines. Recent studies (cf. Heiles 1974) do not support the standard cloud model. The evidence from 21-cm emission in the galactic plane is ambiguous. For optically thin gas, the integrated brightness temperature gives a measure of column density, but clumpiness in $T_B(\nu)$ may indicate density fluctuations or velocity gradients (Burton 1971). The small-scale ($10'$) structure of 21-cm emission in the galactic plane (Kerr and

Westerhout 1965) is fairly smooth and gives no indication of large density contrasts. The finite resolution of the telescope may not be sufficient to detect large density fluctuations on a small scale (say ~ 1 pc in diameter), and confusion of many emission features may obscure the density contrasts. The high-resolution 21-cm emission maps at high latitudes (Heiles 1974; Verschuur 1974) do show considerable structure. The fact that the fine structure disappears when integrated over velocity suggests that velocity fields, not density contrasts, are responsible for this structure. The large-scale structure is clearly controlled by magnetic fields, and may not be representative of the gas in the plane.

The patchy nature of interstellar reddening and extinction and the observation of interstellar molecules clearly indicates a wide range of gas densities (say from 0.1 cm^{-3} to more than 10^5 cm^{-3}). However, there is no clear indication that 100:1 is the favored density contrast.

Of course, many of the 21-cm absorption features are probably due to regions of enhanced density. But at the very least we have shown that cool regions in the intercloud medium may have the appearance of clouds. Heiles (1974) in fact suggests that the intercloud medium may have structure.

As noted above, observations in different directions show large variations in " ζ ". One could imagine variants of the steady-state theory (e.g., Lea and Silk 1973) which incorporate spatial fluctuations in ζ due to the random distribution of steady sources. Such models allow for a much wider range of observable parameters. However, it is still possible in principle to discriminate time-dependent models from such generalized steady-state models by looking for regions of cool ($T \lesssim 100^\circ \text{ K}$), ionized gas, forbidden by any version of the steady-state theory. Perhaps such evidence already exists. One puzzling result from the OAO-C *Copernicus* ultraviolet satellite is the high ratio $N(\text{C}^0)/N(\text{C}^+) \approx 10^{-3}$ in the direction of several unreddened stars (Rogerson *et al.* 1973). This result could be understood if the low-density ($n \approx 0.3 \text{ cm}^{-3}$) gas in the solar neighborhood had $T \approx 30$, $x \approx 0.05$, which gives a high recombination rate for C^+ (Kafatos *et al.* 1974). Another possibility is the study of cool regions that are emitters of radio recombination lines (Cesarsky and Cesarsky 1973). If such regions can be shown to have low density, they would provide evidence for time-dependent ionization.

Radically different assumptions—the steady-state assumption of pressure equilibrium between clouds and intercloud medium, and the time-dependent assumption of no pressure equilibrium—each produce models for the interstellar gas with many of the observed characteristics. The observed bimodal temperature distribution is indicative of the shape of the radiative cooling curve of the gas, and is not specific to the model. Modern observations further show that the structure of the interstellar gas is influenced greatly by magnetic fields. It therefore appears that the problem of interpreting observations of the gas will require consideration of several effects of comparable importance.

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