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Search, Dealers, and the Terms of Trade*

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Abstract

I study a search-theoretic model with pairwise meetings where dealers arise endogenously. The extent of intermediation depends on its cost, trade frictions, and the dealers' ability to negotiate favorable terms of trade. Under Nash bargaining, there is a unique equilibrium where dealers buy and hold the low-storage-cost good and, depending on their relative bargaining power, resell it at a premium or a discount. The distribution of the terms of trade is non-degenerate unless storage cost and frictions vanish. Due to an externality created by intermediation, the efficient allocation can be achieved only if dealers can charge a positive markup.

Key Words: Search, Intermediation, Prices, Bargaining

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1. Introduction

I construct a search theoretic environment that allows endogenous determination of the number of trade facilitators and the negotiated terms of trade. In it, mediated exchange emerges as a natural response to market frictions. I study how the incidence of intermediation responds to economic incentives linked to frictions, intermediation costs, the availability of different goods, and the ability to negotiate favorable terms of trade.¹ I also complement work on matching models of exchange, by pointing to the implications that the absence or the type of pricing mechanism have for existence and efficiency of equilibria. I do so by proving existence of equilibria for a simple transaction pattern, of which I characterize the terms of trade, extent of intermediation, and study the efficiency properties.

The economy is modeled in section 2, along the lines of Kiyotaki and Wright (1989). This is a natural starting point because the model's frictions make the role of intermediation explicit: certain agents choose to undertake the role of dealers, costly storing a commodity they don't consume to resell it to others. I relax the assumption of fixed terms of trade (as in Shi, 1995, and Trejos-Wright, 1995), but also of exogenous distribution of agents specialized in each consumption-production activity (as in Wright, 1995). I study the fundamental transaction pattern, where some agents engage in a sequence of indirect trades involving only the lowest-storage-cost good. Several transaction patterns have been shown to exist in this class of models (e.g. Kehoe et al., 1993). I focus on the fundamental pattern for several reasons. To study the link between absence (or choice) of price mechanisms and existence of equilibria, I can restrict attention to a single trade pattern; investigating more than one provides little additional insight. Focusing on fundamental equilibria allows me to provide an especially clear illustration of the subject of interest by resolving an issue raised by Wright (1995). He proves the non-existence of fundamental equilibria when agents choose their specialty production, and the terms of trade are fixed at par. In fact, I prove existence of a continuum of 'prices' consistent with a fundamental strategy. Finally, the fundamental strategy is often considered the most "natural" when trade requires costly storage of goods. This has been suggested by studies of similar synthetic and experimental economies (Marimon et al., 1990, Brown, 1996, Duffy and Ochs, 1999).²

I develop the analysis in section 3, assuming that the negotiated terms of trade satisfy a Nash bargaining protocol. In section 4, I prove that equilibria exist where dealers arise endogenously only if their bargaining position is not extreme. When dealers are weak bargainers, they may sell at a discount but charge a markup on their sales when they are strong negotiators. Impatient consumers are willing

¹ A number of studies have focused on intermediation in bilateral search markets with fixed prices. Examples include Rubinstein and Wolinsky (1987), Yavas (1994), Li (1998) and Shevchenko (1999).

² In particular, Duffy and Ochs emphasize how subjects show a strong tendency to play fundamental strate-

to pay a premium, and producers offer discounts, to someone capable of quickly satisfying their effective demand. This obtains even if search frictions vanish. I find that in equilibrium there is terms of trade dispersion, which, however, may disappear as frictions and intermediation costs vanish (as in Camera and Corbae, 1999). The extent of intermediation responds in an intuitive way to changes in fundamentals. For example, as storage costs fall, more dealers arise. I also show existence of a continuum of terms of trades supporting the fundamental exchange pattern, for arbitrary price mechanisms. Due to a trading externality generated by intermediation, however, an equilibrium is efficient only if dealers can sell at a premium and buy at a discount. Section 5 concludes.

2. Environment

Consider a version of model A in Kiyotaki and Wright (1989), with a unit mass of ex-ante identical agents. They can produce and consume one non-storable autarkic good, good 0, with net utility $a \geq 0$. At the beginning of life (only), agents can acquire a market production opportunity $i \in \mathbf{N} \equiv \{1, 2, 3\}$, losing the ability to produce good 0. This allows the agent to specialize in the costly production of market good i , that she cannot consume and whose production requires consumption of some amount of market good $i-1$. Market production opportunities can be freely discarded to revert to autarkic production. Let a producer of $i+1$ be identified as agent (of type) i . Consumption of q_i units of good i generates instantaneous utility $u(q_i) > 0$ only to agent i (zero otherwise); $u(q)$ is strictly increasing, concave, continuously differentiable, $u(0)=0$, and future utility is discounted by $\beta \in (0, 1)$. Production of q_{i+1} generates disutility γq_{i+1} . Anyone can store one unit of good i suffering disutility c_i per period, $0 < c_1 \leq c_2 \leq c_3$. Storage and market production are mutually exclusive, and once good i is stored it must be traded as an indivisible unit. Assume $u(q_i) - \gamma q_i > 0$ increasing on $q_i \in (0, 1]$, let $q = \{q_1, q_2, q_3\}$, and normalize $\gamma = u'(1)$. Autarkic producers stay out of the market. At each date market producers are paired randomly and anonymously. Objects stored and type are observable in a match, trading histories are private information and agents cannot commit themselves to future actions. If two matched agents agree to trade, they bargain over the terms of trade. Production and consumption occur at the beginning of the following period. Let $U_i = u(q_i) - \gamma q_{i+1}$ denote the net instantaneous utility derived by agent i , if she produces q_{i+1} in exchange for q_i .³

gies (over others) irrespective of marketability conditions in the experimental economy.

³ Some of these features are extreme but make the model simple enough to generate clear results without sacrificing the rigorousness of the analysis. In particular, preferences and technologies motivate the existence of gains from specialization and trade, while storage bound and indivisibility limit the state space (see the difficul-

3. Stationary Symmetric Fundamental Equilibrium

Consider outcomes with no autarkic production where agents only accept own consumption or the lowest storage-cost good.⁴ I focus on stationary symmetric rational expectations equilibria where agents adopt Nash strategies. The terms of trade are reached via bilateral negotiations assumed to satisfy Nash bargaining. Strategies are based on the correct evaluation of the gains from trade and are chosen to maximize the expected lifetime utility from consumption. In equilibrium production and trading decisions are individually optimal, given the correctly perceived strategies of others and distribution of objects, and are time-invariant and identical for individuals of identical type.

3.1 The initial choice of productive activity

Individuals initially simultaneously choose a production opportunity, taking as given the strategies of others. Each market production is chosen by someone if it is weakly preferred to autarky and to the remaining others. Because of the link between production and types, I interpret the choice of production as a choice over types. Let $p'_i \in [0,1]$ denote the probability that, at the beginning of life, the average individual chooses to produce good $i=0,1,..,3$, given the choices $\{p'_i\}$ of all others. For any $j \neq i$, $p'_i = 1$ ($\in [0,1]$) if i is strictly (weakly) preferred to every j , and $p'_i = 0$ if there is some j that is strictly preferred to i . Define a search equilibrium as an outcome where market production is strictly preferred to autarky, i.e. $p_0 = 0$. Since there must be positive demand for every market commodity, it requires ex-ante indifference among all market production.⁵ Thus, let $p = \{p_1, p_2, p_3\}$ and $p_2 = 1 - (p_1 + p_3)$; $0 < p_i < 1$ is the population proportion of agents $i=1,2,3$, who produce good $i+1$ and consume i .

Agent i would never store own production, due to costs, and would consume good i as soon as possible, due to discounting (conditions are provided later). She could, however, choose to store good $i+2$ if this allows her to obtain good i more frequently or at better terms of trade. Thus, at every date agent i can be either a producer of $i+1$, or a dealer storing one good $i+2$. Let $p_{i,j}$ denote the proportion of agents i who can offer some good j . Then

$$\sum_{j \in N} p_{i,j} = 1. \tag{1}$$

Let $V_{i,j}$ be agent i 's lifetime utility when she can offer good j , and V_a be the autarkic lifetime util-

ties due to complex distributions of holdings in Camera and Corbae, 1999, or Shevchenko, 1999).

⁴ No additional insight is obtained by considering equilibria where some choose autarky.

⁵ Assume $p_i = 0$ for some market good i and $p_j > 0$ for $j \neq i$. Then, autarky is at least weakly preferred to market production $i+1$, since market good i is not produced. A similar conclusion holds if $p_i = 1$ for some i .

ity. Let $E(V_i) \equiv \sum_{j \in N} p_{i,j} V_{i,j}$ be the expected lifetime utility of agent i , unconditional on her current inventory. It may be interpreted as the ex-ante average utility for agent i , a function of the endogenous distribution of inventories $\{p_{i,j}\}$. I say that an agent is ex-ante indifferent between becoming a type i or h when the two expected lifetime utilities, unconditional on inventory, are identical but larger than the value of autarky.⁶ In a search equilibrium agents must be indifferent across production types, when taking as given $\{p, \pi, q\}$, and must strictly prefer market production to autarky; i.e. for all $i, h=1,2,3$

$$E(V_i) = E(V_h) > V_a \quad (2)$$

3.2 Trading Strategies and Distributions

Focus on meetings that may lead to mutually beneficial exchange, between i and $j \in \{i-1, i+1\}$. With probability $p_j p_{j,h}$ agent i meets agent j who offers good h . They choose to trade taking as given q . In a rational expectations equilibrium choices are based on the correct forecast of the negotiated terms of trade (later discussed). Agent i in equilibrium always accepts to trade for good i , and refuses good $i+1$. What must be studied is whether she enters a trade in which she expects to produce q_{i+1} in exchange for one unit of good $i+2$ (modulo 3). Let $\pi'_i \in \{0,1\}$ denote the trading strategy of agent i when she takes as given the strategies $\pi = \{\pi_1, \pi_2, \pi_3\}$ of all others, q, p and $\{p_{i,j}\}$. Thus, π'_i is the probability of accepting one unit of good $i+2$ in exchange for q_{i+1} .⁷ If the fundamental trading strategy is played agents $i=1,3$ only accept own consumption, while agent 2 accepts also one unit of good 1, i.e. $\pi = \pi^* \equiv \{0, 1, 0\}$. Hence, when π^* is an equilibrium

$$p_{i,i} = p_{1,3} = p_{3,2} = 0 < p_{1,2} = p_{3,1} = 1 \quad (3)$$

for all i , while $p_{2,3}$ and $p_{2,1}$ are positive and must satisfy the steady-state law of motion

$$p_{2,1} p_1 p_{1,2} = p_{2,3} p_3 p_{3,1}$$

Its left hand side reflects how frequently agents 2 sell the intermediated good, and the right hand side how frequently they buy the intermediated good from its producers. Using the law of motion and (1)

$$p_{2,1} = \frac{p_3}{p_1 + p_3} \quad \text{and} \quad p_{2,3} = \frac{p_1}{p_1 + p_3}. \quad (4)$$

3.3 Value functions

⁶ While other measures are possible (for instance measuring expected utility conditional on type *and* current inventory) this measure is easy to work with, and it has been previously proposed (Wright, 1995).

⁷ The focus on pure strategies is without loss in generality, due to divisible production. In equilibrium, agent i 's partner (agent j) must want good $i+1$ (if j could produce it she would not accept it, and if $j=i+2$ she would not

Suppose there exists a search equilibrium where $\pi=\pi^*$ and (1)-(4) hold, for some q . Then, the stationary value function for agent i who can offer good j , must satisfy the standard equations

$$V_{1,2}=\beta\{(p_1+p_3)V_{1,2}+p_2[p_{2,1}\max\{U_1+V_{1,2},V_{1,2}\}+p_{2,3}\max_{\pi'_1}\{\pi'_1(V_{1,3}-\gamma q_2)+(1-\pi'_1)V_{1,2}\}]\} \quad (5)$$

$$V_{2,1}=-c_1+\beta\{p_1\max\{u(q_2)+V_{2,3},V_{2,1}\}+p_2V_{2,1}+p_3V_{2,1}\} \quad (6)$$

$$V_{2,3}=\beta\{p_1V_{2,3}+p_2V_{2,3}+p_3\max_{\pi'_2}[\pi'_2(V_{2,1}-\gamma q_3)+(1-\pi'_2)V_{2,3}]\} \quad (7)$$

$$V_{3,1}=\beta\{p_1\max_{\pi'_3}\{\pi'_3(V_{3,2}-\gamma q_1)+(1-\pi'_3)V_{3,1}\}+p_2[p_{2,3}\max\{U_3+V_{3,1},V_{3,1}\}+p_{2,1}V_{3,1}]+p_3V_{3,1}\} \quad (8)$$

Equation (5) describes the expected flow return to agent 1, as a producer. With probability p_1+p_3 she meets someone who does not trade with her, given the proposed π . With probability $p_2p_{2,1}$ she meets a dealer who has one unit of her consumption good. If she chooses to trade, she expects to receive $q_1=1$ and to produce q_2 , with net utility U_1 and continuation utility $V_{1,2}$. With probability $p_2p_{2,3}$ agent 1 meets a type 2 who is a producer, and can choose to accept and store one unit of good 3 ($\pi'_1=1$).⁸ Expressions (6)-(8) have a similar interpretation, and illustrate why the terms of trade may differ across matches. Agents 1 and 3 produce and consume simultaneously and never suffer storage costs. Agent 2, though, produces before consuming, and endures storage costs. Thus, the harder it is to sell her inventory, the greatest is her burden. She can make these losses up by requesting discounts to producers of the intermediated good, $q_3 < q_1=1$, or charging a premium to its consumers, $q_2 > q_1=1$.

3.4 Equilibrium Strategies and Negotiated Terms of Trade

Individuals choose to trade only if they expect to obtain positive surplus. I say that, given π^* , q is feasible if in equilibrium it leaves positive surplus to those who buy a consumption good. Specifically, feasibility requires $U_i > 0$ for $i=1,3$ (which, in turn, implies $V_{1,2}, V_{3,1} > 0$), i.e.

$$q_2 < \frac{u(q_1)}{\gamma} \quad \text{and} \quad q_3 > u^{-1}(\gamma q_1) \quad (9)$$

while in the case of agent 2

$$u(q_2)+V_{2,3}-V_{2,1} > 0. \quad (10)$$

Given q and $\pi=\pi^*$, individual optimality of the trading strategy requires

$$\pi'_1=0 \quad \text{if} \quad V_{1,2} > V_{1,3}-\gamma q_2 \quad (11)$$

offer good $i+2$). Thus, if i is indifferent to the trade, j could slightly reduce her request q_{i+1} to get the trade.

⁸ Out of equilibrium actions must take into account the proposed q . For example when agent 1 considers this transaction, she expects to be required to produce q_2 in exchange for one good 3.

$$\pi'_2=1 \quad \text{if} \quad V_{2,3} < V_{2,1} - \gamma q_3 \quad (12)$$

$$\pi'_3=0 \quad \text{if} \quad V_{3,1} > V_{3,2} - \gamma q_1. \quad (13)$$

Contingent on having chosen to trade with someone who can offer good j , producer i and her partner bargain over the terms of trade, i.e. i 's acquisition of Q_j in exchange for Q_{i+1} . Due to storage restrictions, no dealer would request more than one unit to store. Thus, in a match between agents 2 and 3, bargaining involves determination of a quantity Q_3 to be produced by agent 2 in exchange for $Q_1=1$. In a match between agent 1 and a dealer with $Q_1=1$, bargaining involves determination of Q_2 in exchange for the dealer's inventory. The negotiated terms of trade are assumed to satisfy the solution of a Nash bargaining problem with non-zero threat points and bargaining weight $\theta \in (0,1)$ to the dealer.⁹ Given q and $Q_1=q_1=1$, $\{Q_2, Q_3\}$ solves

$$\max_{Q_2} [V_{2,3} + u(Q_2) - V_{2,1}]^\theta [u(1) - \gamma Q_2]^{(1-\theta)} \quad \text{and} \quad \max_{Q_3} [V_{2,1} - \gamma Q_3 - V_{2,3}]^\theta [u(Q_3) - \gamma]^{(1-\theta)}$$

subject to (9)-(10) and (12), since surpluses must be positive; $\{Q_2, Q_3\}$ satisfies Nash bargaining if

$$u(Q_2) + V_{2,3} - V_{2,1} = [u(1) - \gamma Q_2] \frac{\theta u'(Q_2)}{(1-\theta)\gamma} \quad (14)$$

$$V_{2,1} - \gamma Q_3 - V_{2,3} = [u(Q_3) - \gamma] \frac{\theta \gamma}{(1-\theta)u'(Q_3)} \quad (15)$$

The left hand sides show the trade surplus to the dealer. Her customer's trade surplus is on the right hand side, 'weighted' by relative bargaining power, and the ratio of marginal utility from consumption to marginal cost of production. Note that by construction the total surplus in each match is maximized when trades occur at par, $Q_2=Q_3=1$, since $u'(1)=\gamma$.¹⁰ In a symmetric equilibrium (9)-(10), (12), and $Q=q$ must also be satisfied. Since $q_1=1$, then q_2 is the real price offered by the dealer (the "ask" price) and q_3 the real price paid by the dealer (the "bid" price) to, respectively, consumers and producers of the intermediated good. Thus, $q_2/q_3 - 1$ can be taken to measure the real markup.

A rational expectations symmetric stationary equilibrium is a set of value functions, strategies, distributions, and terms of trade, such that agents maximize expected lifetime utility, the distribution is stationary, and the terms of trade satisfy the assumed pricing rule, are feasible, and consistent with storing restrictions. That is, $\{V_{ij}\}$ satisfy (5)-(8), $\{p', \pi'\} = \{p, \pi\}$ satisfy (2) and (11)-(13), $\{p_{ij}\}$ satisfy

⁹ While I do not model explicitly the bilateral bargaining process, this may be seen as implementing a solution to a more structured game (see for example Trejos and Wright, 1995).

¹⁰ The two total surpluses are $u(Q_2) - \gamma Q_2 + u(1) - V_{2,1} + V_{2,3}$ and $u(Q_3) - \gamma Q_3 + \gamma + V_{2,1} - V_{2,3}$.

(3) and (4), and $q=Q$ satisfies (9)-(10), $q_1=1$, and (14)-(15).

4. Existence and Characterization

To prove existence of the fundamental equilibrium I check that, given $\{p, \pi, q\}$, (i) no agent prefers autarky, (ii) the strategy π^* is individually optimal, and (iii) there is at least a fixed point $\{Q_2, Q_3\} = \{q_2, q_3\}$ to the map defined by (14)-(15), given $q_1=1$. Requirement (i) is satisfied for $a > 0$ sufficiently small, which I henceforth assume. The strategy π^* is individually optimal if (11)-(13) hold. Given a feasible q , the value functions associated to out-of-equilibrium actions, are

$$V_{1,3} = \{-c_3 + \beta p_3 [u(q_1) + V_{1,2}]\} / [1 - \beta(p_1 + p_2)],$$

when agent 1 has accepted (and now stores) one unit of commodity 3, and

$$V_{3,2} = \{-c_2 + \beta p_2 p_{2,3} [u(q_3) + V_{3,1}]\} / [1 - \beta(p_1 + p_2 p_{2,1} + p_3)],$$

when agent 3 has one unit of commodity 2. A lemma follows (all proofs in appendix).

Lemma 1. Let $\pi = \pi^*$. Given q, p , and $\{p_{ij}\}$, then π^* is individually optimal iff

$$c_3 > \beta(p_3 - p_2 p_{2,1}) U_1 - \gamma q_2 (1 - \beta) \quad (16)$$

$$c_1 < \beta p_1 u(q_2) - \gamma q_3 [1 - \beta(1 - p_1)] \quad (17)$$

in which case (9) is sufficient for q to be feasible.

Given q , dealers of good 1 can endogenously arise if the return to intermediation is sufficiently large. Two components affect it. One is the storage cost c_1 , that can't be too large. The other is linked to the extent of trade frictions, captured by β , and the demand for the intermediated commodity, specified by p_1 . Great impatience and infrequent sales of intermediated goods (low β and p_1) contribute to increase the expected length of storage, hence raise the dealer's losses and reduce her expected utility from future consumption. Thus agent 2 would elect to not store good 1 if β or p_1 are too low, even in the absence of storage costs.

Given $\pi = \pi^*$, stationary $\{p_{ij}\}$, and a feasible q , I can now characterize the endogenous distribution p . Since all market commodities must be produced in a search equilibrium, I look for a mixed strategy $p' = p = p^*$. The following lemma shows that p^* is unique: in equilibrium every individual selects ex-ante to become type i with identical probability p_i^* .

Lemma 2. Let $\{p_{ij}\}$ satisfy (3)-(4), $\pi = \pi^*$, and q be feasible. There is a unique p^* that satisfies (2),

where $p_1^* = \frac{\beta U_1 + c_1}{\beta(U_1 + U_2 + U_3)}$, $p_3^* = p_1^* \frac{U_3}{U_1}$, $p_2^* = 1 - (p_1^* + p_3^*)$, and $p_i^* \in (0,1) \forall i$.

The initial choice of type is a function of storage cost and discount factor, but also of expected terms of trade and the transaction pattern. As storage costs grow the return to intermediation falls and indifference requires more frequent matches with consumers of the intermediated good. This decreases the duration of storage and increases the dealer's frequency of consumption. Thus, given q , p_1^* and p_3^* increase with c_1 , and p_2^* falls. Similar considerations apply to a decrease in β or q_2 : more impatience and less consumption make intermediation less attractive. Note that p_i^* converges to $1/3$ for all i (as in Kiyotaki and Wright, 1989) as $c_1 \rightarrow 0$, and only if all trades occur at par, $q_i = 1 \forall i$. The next lemma describes the properties of the equilibrium terms of trade, q^* .

Lemma 3. Let $\pi = \pi^*$, and $p = p^*$. If a feasible q^* is a fixed point of (14)-(15), then it has the following properties: (i) it cannot exist unless $\theta \in (\theta_L, \theta_H)$, $0 < \theta_L < \theta_H < 1$, and if it exists it is unique; (ii) both q_2^* and q_3^* fall in c_1 and rise in β , while q_2^* rises and q_3^* falls in θ ; (iii) if $\theta \in (\theta_M, \theta_H)$ then $q_2^* > q_3^*$ and $q_3^* < 1$, $\theta_L < \theta_M = 1/3$; (iv) trades occur at par if and only if $\theta = \theta_M$, $c_1 \rightarrow 0$, and $\beta \rightarrow 1$; (v) q_2^*/q_3^* increases in c_1 and decreases in β if q_3^* is small, and the opposite occurs if q_3^* is large.

Suppose $q = q^*$ is an equilibrium. A lower storage cost (or lower frictions) increases the return to a dealer. This creates stronger incentives for intermediation, p_2 rises, and allows dealers to offer better terms of trade to producers, q_3 rises. These factors spur production of good 1, p_3 rises. The resulting fall in p_1 , however, lowers the dealer's probability of consumption. She counteracts it by asking for more, q_2 grows. The relative size of q_2 and q_3 depends on the bargaining powers. When the dealer's bargaining position is weak, $\theta_L < \theta \leq \theta_M$, she might sell intermediated goods at a discount (if the storage cost is small) and may also be able to buy them at discount (if θ is not too low). When the dealer's bargaining position is strong, $\theta_M < \theta < \theta_H$, she always buys at a discount, $q_3 < 1$, and sells at a premium, $q_2 > q_3$. This results even if dealers have the weaker negotiating position (since $\theta_M = 1/3$) or if discounting and storage costs are negligible. The markup charged responds in intuitive ways to changes in parameters. In particular, if dealers have been asking steep discounts to producers, q_3 low, the markup is eased as frictions and costs fall ($q_2/q_3 - 1$ falls).

If an equilibrium exists, it is generally characterized by terms of trade dispersion, due to heterogeneity in consumption frequencies. Since $p_i \neq p_j$ unless $c_1=0$, the terms of trade need to adjust to support ex-ante indifference between economic activities. The total surplus is maximized in each match only if $\theta=\theta_M$, as frictions and intermediation cost vanish. In that case every transaction occurs at par. This is so because, when $q_i=1$, p becomes a uniform distribution as $c_1 \rightarrow 0$, in which case dealers transact twice as frequently as anyone else. This provides a rationale for why the dealers' relative bargaining power, $\theta_M/(1-\theta_M)$, must be half that of their customers. Discounting must be negligible since dealers always produce prior to consumption. Conditions sufficient for existence of an equilibrium are next provided, using $\beta_L \in (0,1)$ and $c_H, c_L > 0$ (functions of the model's parameters).

Proposition 1. Let $a > 0$ small, $c_3 > c_L$ and $\theta_L < \theta < \theta_H$. If $\beta_L < \beta < 1$ and $c_1 < c_H$, then a unique search equilibrium $\{p^*, \pi^*, q^*\}$ exists. The equilibrium $\{p^*, \pi^*\}$ is also supported by pricing mechanisms that generate terms of trade close to q^* .

The model illustrates why the absence of a pricing mechanism has consequences for existence of equilibria. When the distribution is endogenous, it is the inability of bargaining over better terms of trade that rules out the fundamental equilibrium (as in Wright, 1995). Indeed, when negotiations are allowed, there is a continuum of terms of trade that support it. Its existence depends also on the pricing mechanism adopted. For instance, take-it-or-leave-it offers, $\theta=0,1$, cannot support it since surpluses from trade must be bounded away from zero. Under Nash bargaining and $\theta > \theta_H$, the dealer charges steep prices to her customers; thus, very few would choose to be one, $p_1, p_3 \approx 0$. The opposite is true if $\theta < \theta_L$, when few would choose to be intermediaries, $p_2 \approx 0$. In both of these cases some agents either get very little surplus frequently or some surplus very infrequently.

When is a decentralized allocation efficient? I answer the question by considering the outcome due to a planner who, taking as given the exchange arrangement, chooses the terms of trade to maximize social welfare $W(q_2, q_3) = \sum_{i=1}^3 p_i E[V_i(q_2, q_3)]$, i.e. the ex-ante expected utility of an agent.¹¹

Proposition 2. Let $\{p^*, \pi^*\}$ be an equilibrium. The welfare-maximizing q is such that $q_3 < 1 < q_2$.

¹¹ Thus, I am not asking whether the fundamental trading pattern is the best way to organize exchange. Storage of other goods by other agents might increase the average consumption frequency and be welfare improving.

The planner would let dealers charge a markup because they provide a positive trade externality. Intermediation facilitates consumption through indirect trades. An insufficient number of dealers reduces the consumption frequency of some (the dealer's customers), while an excessive number diminishes the dealers' ability to consume. Dealers also economize on societal use of resources by holding the cheapest inventory. Thus, in general there is a 'desirable' interior intermediation level that generates maximum average lifetime utility, for the given pattern of exchange. To gain intuition, abstract from the cost saving aspect by letting $c_1 > 0$ small, and consider β close to 1 and $\theta = \theta_M$. Here the decentralized equilibrium involves the exchange of the surplus-maximizing quantity $q_i = 1$, but there is underprovision of intermediation, with agents (almost) equally distributed across economic tasks. The planner can raise the average frequency of consumption by inducing a greater number of dealers to arise through discounts on their inventory acquisitions and extra consumption on their purchases.

5. Concluding Remarks

I have illustrated a general equilibrium model with pairwise trades, where dealers arise endogenously to mediate the sale of a good whose price is negotiated. I have done so by relaxing the assumptions of exogenous distribution and fixed prices in a prototypical search-theoretic model. I have proved existence of a unique equilibrium for a simple transaction pattern, when the negotiated terms of trade satisfy Nash bargaining. In equilibrium extent of intermediation and the terms of trade are fully flexible and respond in intuitive ways to changes in economic fundamentals. For example, more dealers arise as their relative bargaining position strengthens or storage costs fall. The distribution of the terms of trade is non-degenerate, but trades may occur at par if frictions and storage costs vanish. I have shown that the choice of pricing mechanism has implications for efficiency of equilibria. Due to an externality created by intermediation, the efficient allocation can be achieved only if dealers are able to charge a markup.

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Appendix

Proof of Lemma 1. Use $\pi=\pi^*$ in (5)-(8) to rearrange the inequalities in (11) and (12) as (16) and (17), respectively; (13) is always satisfied. When $q_2 < q^H = u(1)/\gamma$ and $q_3 > q^L = u^{-1}(\gamma)$ then (9) is satisfied, and (10) holds since $U_2 > 0$ (true when $\pi_2 = 1$). Thus, if π^* is an equilibrium, $q_2 < q^H$ and $q_3 > q^L$ guarantee feasibility of q . Note that $q^H > 1 > q^L > 0$ because of the assumptions made. ■

Proof of Lemma 2. Let $\{p_{ij}\}$ satisfy (3)-(4), $\pi=\pi^*$, $q_2 < q^H$, $q_3 > q^L$, and $a > 0$ small. Use (5)-(8) in (2) for $i=1,2,3$. Then, $V_{1,2} - V_{3,1} = 0$, if $p_3^* = p_1 \frac{U_3}{U_1} > 0$; $V_{1,2} - E[V_2] = 0$ if $p_1^* = \frac{\beta U_1 + c_1}{\beta(U_1 + U_2 + U_3)} \in (0,1)$, since $U_1 > 0$ and $c_1 < \beta U_2$ when $\pi=\pi^*$ (by Lemma 1). Thus, $p_2^* = 1 - (p_1^* + p_3^*) \in (0,1)$ since $p_1^* + p_3^* < 1$ if $c_1 < \frac{\beta U_1 U_2}{U_1 + U_3}$, holding since (17) holds (seen by substituting p_1^* in it). It is obvious that $p_1^* + p_3^* > 0$. ■

Proof of Lemma 3. Let $a > 0$ small, $\pi = \pi^*$, $p = p^*$, given q . $Q = q$ must be feasible, thus restrict attention to $Q_2 < q^H$, $Q_3 > q^L$, $Q_1 = 1$. Q satisfies (14)-(15) if $f(Q_2, \theta) = g(Q_3, \theta) = \Delta(Q_2, Q_3, c_1) > 0$, where $\Delta(\cdot) \equiv V_{2,1} - V_{2,3}$, $f(\cdot) \equiv u(Q_2) - [u(1) - \gamma Q_2] \frac{\theta u'(Q_2)}{(1-\theta)\gamma}$, and $g(\cdot) \equiv \gamma Q_3 + [u(Q_3) - \gamma] \frac{\theta \gamma}{(1-\theta)u'(Q_3)}$; f_{Q_2} and g_{Q_3} are positive (and equal only at $Q_3 = Q_2 = 1$), $g_{\theta} > 0 > f_{\theta}$, $f(Q_i, \theta) < g(Q_i, \theta) = \infty$ as $Q_i \rightarrow \infty$, and $f(Q_i, \theta) = -\infty < g(Q_i, \theta) = 0$ as $Q_i \rightarrow 0$. $\exists \theta_H \in (0, 1)$ such that $f(q^H, \theta) > g(q^L)$ only if $\theta < \theta_H$. Thus, a root $Q_2 = h(Q_3, \theta)$ to $f(Q_2, \theta) - g(Q_3, \theta) = 0$ exists only if $\theta < \theta_H$. The root is unique, and h is a strictly increasing, continuous and invertible function. If $\theta > \theta_M = 1/3$ then f lays below g for all feasible Q_3 , i.e. $q^L < Q_3 < \bar{Q}_3 \equiv h^{-1}(q^H)$, and are tangent at $Q_2 = Q_3 = 1$ when $\theta = \theta_M$. If $\theta \in (\theta_M, \theta_H)$ then $Q_2 = h(Q_3, \theta) > Q_3$ for all feasible Q_3 . If $\theta \in (0, \theta_M)$ then $Q_2 < Q_3$ if Q_3 is close to 1, since f is concave in Q_2 . Finally, if $Q_2 = h(Q_3, \theta)$ then Q_2 increases and Q_3 decreases in θ , since $g_{\theta} > 0 > f_{\theta}$, while $U_2 \geq 0$.

Let $\theta < \theta_H$, $Q_3 \in (q^L, \bar{Q}_3)$, $Q_2 = h(Q_3, \theta)$; $\Delta(Q_3, \theta, c_1) = \frac{-c_1 + \beta[p_1 u(Q_2) + p_3 \gamma Q_3]}{1 - \beta p_2}$ positive if (17) holds.

$\exists \theta_L \in (0, \theta_M)$ such that $\Delta(Q_3, \theta, c_1) \leq g(Q_3)$ if $\theta \leq \theta_L$, since if $\theta = 0$ then $\Delta(\cdot) - g(\cdot) \propto -c_1 - \gamma Q_3(1 - \beta) < 0$. Thus let $\theta_L < \theta < \theta_H$. Rewrite (17) as

$$c_1 < c_1(Q_3) \equiv \frac{\beta U_1 U_2 - \gamma Q_3 (1 - \beta)(U_1 + U_2 + U_3)}{U_1 + U_3}$$

where $c_1(Q_3) > 0$ if $\beta > \beta(Q_3) \equiv \frac{U_1 \gamma Q_3 + \gamma Q_3 (U_2 + U_3)}{U_1 u(h(Q_3)) + \gamma Q_3 (U_2 + U_3)}$, $\beta(Q_3) \leq 1$ iff $Q_3 \leq \bar{Q}_3$. Define $c_H = c_1(q^L)$ and

$\beta_L = \beta(q^L)$. If $Q_3 = \bar{Q}_3$ (17) is violated $\forall c_1 > 0$, which implies $\Delta(\bar{Q}_3, \theta, c_1) < \gamma \bar{Q}_3 < g(\bar{Q}_3, \theta)$. If $Q_3 = q^L$ (17) is satisfied iff $\beta \in (\beta_L, 1)$ and $c_1 \in (0, c_H)$, which implies $\Delta(q^L, \theta, c_1) > \gamma q^L = g(q^L)$ (since $\theta > \theta_L$). Let

$\beta \in (\beta_L, 1)$ and $c_1 \in (0, c_H)$. Since $g_{Q_3} > 0$ and $\Delta(\bar{Q}_3, \theta, c_1) < g(Q_3, \theta) < \Delta(q^L, \theta, c_1)$, by the intermediate value

theorem it follows that there exists a fixed point $q_3^* \in (q^L, \bar{Q}_3)$ such that $g(q_3^*, \theta) = \Delta(q_3^*, \theta, c_1)$. The

fixed point is unique. If not, this would imply that $d^2 \Delta(Q_3, \theta, c_1) / dQ_3^2$ must switch sign at least twice as Q_3 increases. This is not possible since it can be shown that the second partial in Q_3 is monotone.

Thus, let $q^* = \{1, h(q_3^*, \theta), q_3^*\}$. Fix Q . $\Delta_{c_1} < 0$ since a maximum of $d\Delta(Q_3, \theta, c_1) / dc_1$ is proportional to

$-1 + \beta p'_1 U_2 < 0$. Since $g(Q_3, \theta)$ is independent of β and c_1 , then q_3^* (and q_2^*) fall in c_1 . $\Delta_{\beta} > 0$ hence q_3^* (and q_2^*) increase in β .

Let $\theta \in (\theta_M, \theta_H)$. I show $q_3^* < 1$ by letting $c_1 = 0$ and $\beta = 1$, since q_3^* falls in c_1 and rises in β . Suppose

$Q_3=1 < Q_2=h(1)$. Then $\Delta(1,\theta,0)-g(1,\theta)$ is $p_1u(Q_2)+p_3\gamma+p_2[\gamma+U_3\theta/(1-\theta)]-\gamma-U_3\theta/(1-\theta)<0$, since the least stringent case is $Q_2=1$ rearranged as $\theta/(1-\theta)>1/2$ (holding since $\theta>\theta_M$). Hence, $\Delta(Q_3,\theta,0)<g(Q_3,\theta)$ when $Q_3=1$. Since $g_{Q_3}>0$ and $g_\theta>0$ then $q_3^*<1$, and $q_2^*>q_3^*$ even if $\beta\rightarrow 1$ and $c_1\rightarrow 0$. Now let $\theta=\theta_M$, $c_1>0$ and $\beta<1$: $Q_2=h(1)=1$ since $f(1,\theta_M)=g(1,\theta_M)$. Then, $\Delta(1,\theta_M,c_1)=g(1,\theta_M)$ is $-2c_1=[u(1)+\gamma](1-\beta)$, satisfied only in the limit, as $c_1\rightarrow 0$ and $\beta\rightarrow 1$.

In equilibrium $f(Q_2,\theta)-g(Q_3,\theta)=0$, a constant. Taking its total differential it's easy to see that $dQ_2/dQ_3>0$ for all $\theta\in(\theta_L, \theta_H)$, $dQ_2/dQ_3<1$ if Q_3 is sufficiently close to q^L , and $dQ_2/dQ_3>1$ if Q_3 is sufficiently close to \bar{Q}_3 . Since $d(Q_2/Q_3)/dQ_3=d(Q_2/Q_3)Q_3-Q_2$, we conclude that Q_2/Q_3 is increasing in c_1 and decreasing in β , if Q_3 is sufficiently close to q^L .

Proof Proposition 1. Let $p=p^*$, $q=q^*$; (16) is satisfied if $c_3>c_L\equiv u(1)-\gamma h(q^L)$. Using lemmas 1-3, π^* is an equilibrium for $a>0$ small, by continuity. Since (16)-(17) are strict inequalities, there are q in a neighborhood of q^* capable of supporting $\{p^*, \pi^*\}$, generated by arbitrary pricing mechanisms.■

Proof of Proposition 2. In equilibrium $W(q_3,q_2)=E[V_i(q_3,q_2)]=E[V_j(q_3,q_2)] \forall i\neq j$, hence let $W(\cdot)$ be given by $E[V_1(q_3,q_2)]=V_{1,2}(q_3,q_2)=\frac{\beta U_3}{1-\beta}(\frac{U_1}{U_1+U_3}-p_1)$. Note that $W(q_L,q_2)=0\geq W(q_3, q_H)$; U_3 is concave in q_3 with maximum at $q_3=1$; p_1 is convex in q_3 with minimum at $q_3=1$ but it decreases in q_2 ; $U_1/(U_1+U_3)$ falls in q_2 and q_3 . Thus, there is an interior maximum $W(q_3,q_2)>W(1,1)$. It is easy to verify that $W(q_3,q_2)$ falls in q_3 and rises in q_2 , when $q_3=q_2=1$. Thus, $q_2>1>q_3$ maximizes $W(q_3,q_2)$.■