2003

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Recommended Citation

Camera, G., R. Reed and C. Waller (2003). Can monetizing trade lower welfare? An example. *Economics Letters* 81(2), 33-40. doi: 10.1016/S0165-1765(03)00168-X

Can Monetizing Trade Lower Welfare? An Example

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May 2003

Abstract

In decentralized trade individuals self-insure against consumption risk via costly diversification of skills. Although money acts as consumption insurance, it may lead to a moral hazard problem. If the problem is severe, monetizing trade can lower welfare relative to barter. JEL Codes: E4, E5, Keywords: Money, Specialization, Lotteries

1 Introduction

In a decentralized barter economy agents tend to self-insure against consumption risk by producing a broad set of goods to increase the probability of creating a double coincidence match. In short, they diversify their production skills rather than specialize. The use of money allows agents to consume in more states of the world (single coincidence matches). Thus, money acts as a form of social consumption

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insurance. However, a moral hazard problem arises since agents diversify less when money is introduced which lowers the amount of self-insurance. If the moral hazard problem is severe enough, then a monetary equilibrium may have higher consumption risk and lower welfare than under barter.

We develop a model in which money is 'essential' (see Wallace, 1998) to see how monetizing trade affects decisions to self-insure against consumption risk. We find that for some parameterizations, the moral hazard problem is severe enough that introducing money can *lower* welfare relative to barter. In short, individuals specialize too much because they do not internalize that more diversification makes it easier for others to trade as well.

2 Environment

The environment follows Kiyotaki and Wright (1993). Time is continuous and the discount rate is $r > 0$. There is a $[0, 1]$ continuum of infinitely-lived agents and perishable good types uniformly distributed along a unit circle. Agent i derives utility from consumption of commodities in the interval $[i, i + x]$ where $x \in [0, 1]$ $\forall i$. Consuming quantity q of any good in the consumption set provides utility $u(q) > 0$ and zero otherwise. Additionally $u'(q) > 0$, $u''(q) < 0$, and $u(0) = 0$.

Following Camera, Reed and Waller (2002), each agent is assigned a production location k and chooses to produce goods in the interval $[k - y/2, k + y/2]$ where $y \in [0, 1]$. The agent chooses the magnitude of y to maximize her expected discounted lifetime utility from consumption. A larger y means the individual 'diversifies' more by acquiring more production skills. The choice of y is made at the beginning of life, is permanent, and requires an initial investment cost $e(y)$, with $e'(y) > 0$ and $e''(y) > 0$. One can think of $e(y)$ as the cost of acquiring the skill needed to produce each type of good. Producing some quantity q of any good imposes a utility loss on the agent of $c(q) \geq 0$, where $c(0) = 0$, $c'(q) > 0$, $c''(q) \geq 0$. Consuming own production yields zero utility. We assume that there exists $0 < \bar{q} < \infty$ such that $u(\bar{q}) = c(\bar{q})$, and $u'(0) > c'(0)$.

A fraction $M \in [0, 1]$ of individuals initially receives one unit of indivisible fiat money. As in Camera et al. (2002) and in "model-S" of Rupert et al. (2001), individuals cannot store more than one unit of money but barter is allowed in all double coincidence matches. Agents engage in "small" trades, by using lotteries over monetary payments as in Berentsen, Molico and Wright (2001).[2]

2.1 Exchange

Agents meet bilaterally and randomly according to a Poisson process with arrival rate α. Only one transaction per period can be carried out and trading histories are private information. Random matching generates consumption risk since agents only consume when an appropriate barter or single-coincidence match occurs.

Let $p(y)$ denote the probability agent i produces a consumption good for a randomly encountered agent and $p(Y)$ the probability a randomly encountered agent

[2] This allows us to capture in a simple way the notion of 'divisible' money without having to solve for a distribution of money holdings. As shown by Berentsen and Rocheteau (2002) doing so generates results comparable to those with full divisibility and a degenerate distribution of money.

produces a consumption good for her. Following Camera et al, the ex-ante probability that a randomly encountered agent can produce i's consumption good is

$$p(Y) = \begin{cases} x+Y & \text{if } Y \leq 1-x \\ 1 & \text{otherwise.} \end{cases}$$

Similarly,

$$p(y) = \begin{cases} x+y & \text{if } y \leq 1-x \\ 1 & \text{otherwise.} \end{cases}$$

The ex-ante probability of double coincidence of wants in a match is $p(Y)p(y)$.

The sequence of events for an agent is as follows. At the beginning of time she chooses y. Subsequently she searches, meeting other agents pairwise and randomly over time. Contingent on a match, she bargains and trades after which she searches anew. We study stationary symmetric equilibria.

2.2 Bargaining

Consider a match where trade is feasible. In barter matches agents engage in symmetric Nash bargaining. In single coincidence matches buyers make take-it-or-leave-it offers to sellers.

Let Q denote the quantity of output received by agent i in a barter match and q the quantity she produces. It is easy to demonstrate that in double coincidence matches, agents will choose to barter rather than trade money.[3] The quantities traded

[3] See Rupert et al (2001) and Camera et al (2002). If barter matches were asymmetric then agents would resort to money as a means of trade (see Berentsen and Rocheteau (2003)).

satisfy

$$\max_{q,Q}[u(q) - c(Q)][u(Q) - c(q)]$$

$$\text{s.t. } u(q) > c(Q) \text{ and } u(Q) > c(q).$$

The solution yields $q = Q = q^*$ where q^* satisfies $u'(q^*) = c'(q^*)$.

In a single coincidence match, the trade can be executed if the buyer has money but the seller does not. In this case the buyer makes a take-it-or-leave-it offer to the seller, asking her to produce Q_m and offering her money with probability τ such that the seller is indifferent. Lotteries effectively allow the buyer to ask for a smaller quantity of goods by reducing τ below one. This makes money 'divisible' and gives it greater value. The buyer solves:

$$\max_{Q_m,\tau}[u(Q_m) - \tau(V_m - V_0)]$$

$$s.t. \;\; \tau(\mathcal{V}_m - \mathcal{V}_0) = c(Q_m) \text{ and } \;\tau \leq 1$$

where V_m is the value of holding money for the buyer and V_0 is the value of having no money. Also, $\mathcal{V}_m$ and $\mathcal{V}_0$ are the seller's value of holding or not holding money, respectively. Define $\lambda \in \mathbb{R}^+$ as the Lagrange multiplier associated with the constraint on τ. The solution satisfies

$$\begin{aligned} u'(Q_m) &= \frac{(V_m - V_0) + \lambda}{(\mathcal{V}_m - \mathcal{V}_0)} c'(Q_m) \qquad (1) \\ \tau(\mathcal{V}_m - \mathcal{V}_0) &= c(Q_m) \\ \lambda(1 - \tau) &= 0, \quad \lambda \geq 0 \end{aligned}$$

Let τ^* denote the optimal τ. If the constraint $\tau \leq 1$ binds then $\tau^* = 1$, $\lambda > 0$, $c(Q_m) = \mathcal{V}_m - \mathcal{V}_0$. In a symmetric equilibrium, $V_m - V_0 = \mathcal{V}_m - \mathcal{V}_0$ implying $u'(Q_m) >$

$c'(Q_m)$. If the constraint does not bind, $\lambda = 0$ and

$$\tau^* = \frac{c(Q_m)}{\mathcal{V}_m - \mathcal{V}_0} \leq 1.$$

As a result, $u'(Q_m) = c'(Q_m)$ with $Q_m = q^*$. Thus, lotteries support surplus maximizing quantities whenever $\tau^* < 1$. Therefore, in a symmetric equilibrium all monetary trades involve $Q_m \leq q^*$ so trades are 'small'. Our numerical analysis chooses parameterizations such that $\tau^* < 1$ for most values of M so our welfare results are not being driven by inefficient trades.

2.3 Value Functions

Under the bargaining procedures specified, the value functions are given by:

$$\begin{aligned} \rho V_0(y,Y) = \quad & p(Y)p(y)[u(Q) - c(q)] \\ \rho V_m(y,Y) = \quad & p(Y)p(y)[u(Q) - c(q)] \\ & +(1-M)p(Y)(1-p(y))[u(Q_m) - \tau(V_m(y,Y) - V_0(y,Y))] \end{aligned} \tag{2}$$

where $\rho = r/\alpha$. Rearranging yields the value of holding money:

$$V_m(y,Y) - V_0(y,Y) = \frac{(1-M)p(Y)(1-p(y))u(Q_m)}{\rho + \tau(1-M)p(Y)(1-p(y))}. \tag{3}$$

2.4 Individual's choice of diversification

An individual chooses y prior to the distribution of money in the economy taking Y as given. Knowing that $q = Q = q^*$ is independent of y but Q_m and τ^* depend on y via (3) the agent chooses y to maximize:

$$\begin{aligned} W(y,Y) &= (1-M)\rho V_0(y,Y) + M\rho V_m(y,Y) - \rho e(y) \\ &= p(Y)p(y)[u(q^*) - c(q^*)] - \rho e(y) + \frac{\rho M(1-M)p(Y)(1-p(y))u(Q_m)}{\rho + \tau^*(1-M)p(Y)(1-p(y))}. \end{aligned}$$

The partial derivative of $W(y,Y)$ with respect to y is given by:

$$\mathcal{D}(y,Y) = p(Y)[u(q^*) - c(q^*)] - \rho e'(y) - \frac{\rho^2 M(1-M)p(Y)(1-p(y))u(Q_m)}{[\rho + \tau^*(1-M)p(Y)(1-p(y))]^2}$$
$$+\frac{\rho M(1-M)p(Y)(1-p(y))}{\rho + \tau^*(1-M)p(Y)(1-p(y))}\left[u'(Q_m) - \frac{c'(Q_m)[V_m(y,Y) - V_0(y,Y)]}{\mathcal{V}_m - \mathcal{V}_0}\right]\frac{\partial Q_m}{\partial y}$$

where

$$\frac{\partial \tau^*}{\partial y} = \frac{c'(Q_m)}{\mathcal{V}_m - \mathcal{V}_0} \cdot \frac{\partial Q_m}{\partial y}.$$

If $\lambda > 0$, then $\tau^* = 1$, $\partial \tau^*/\partial y = 0$, $c(Q_m) = \mathcal{V}_m - \mathcal{V}_0$ and $\partial Q_m/\partial y = 0$ since $\mathcal{V}_m - \mathcal{V}_0$ does not depend on y. If $\lambda = 0$, then using (1) the term in square brackets is zero. Hence,[4]

$$\begin{aligned}\mathcal{D}(y,Y) &= p(Y)[u(q^*) - c(q^*)] - \rho e'(y) - \frac{\rho^2 M(1-M)p(Y)(1-p(y))u(Q_m)}{[\rho + \tau^*(1-M)p(Y)(1-p(y))]^2} \quad (4)\\ &= \mathcal{D}(y,Y)|_{M=0} - \frac{\rho^2 M(1-M)p(Y)(1-p(y))u(Q_m)}{[\rho + \tau^*(1-M)p(Y)(1-p(y))]^2}\end{aligned}$$

The first term in (4) is the value of additional barter matches gained from producing a wider variety of goods. The second term is the 'annuity cost' of the initial investment in skills. The third term is the reduction in the value of money from greater diversification. By diversifying more, agents raise the probability of trading, obtain more self-insurance and have less consumption risk. This lowers the insurance value of having money which acts as a disincentive to diversify. When $M = 0$, the choice of diversification in the barter economy comes from the first two terms and is denoted $\mathcal{D}(y,Y)|_{M=0}$. Since $\mathcal{D}(y,Y)|_{M=0} - \mathcal{D}(y,Y) > 0$, the agent specializes more

[4] Without lotteries (4) determines y given that $\tau = 1$ for all parameterizations. This is done in the numerical section to compare welfare with and without lotteries.

in the monetary economy, i.e., she self-insures less when money exists. This is the moral hazard effect associated with monetizing trade.

For a given value of Y, $\partial\mathcal{D}(y,Y)/\partial y < 0$. Thus, if $\mathcal{D}(y,Y)$ has an extremum on $[0, 1-x]$, then it is unique and a maximum. The agent's choice of y satisfies:

$$\begin{array}{lcl} \mathcal{D}(0,Y) < 0 & \Rightarrow & y = 0 \\ \mathcal{D}(0,Y) \geq 0 \geq \mathcal{D}(1-x,Y) & \Rightarrow & 0 < y < 1-x \\ \mathcal{D}(1-x,Y) > 0 & \Rightarrow & y = 1-x \end{array} \cdot$$

In a symmetric steady state,

$$\begin{array}{lcl} y^* = 0, & & \mathcal{D}(0,0) < 0 \\ y^* = 1-x, & \text{if} & \mathcal{D}(1,1) > 0 \\ \mathcal{D}(y^*,y^*) = 0 & & \text{otherwise} \end{array} \cdot \tag{5}$$

If $\lambda > 0$, $\tau^* = 1$, and $Q_m = q_m$ satisfies

$$c(q_m) = \frac{(1-M)p(y)(1-p(y))u(q_m)}{\rho + (1-M)p(y)(1-p(y))} \tag{6}$$

while for $\lambda = 0$, $Q_m = q_m = q^*$ and[5]

$$\tau^* = \frac{\rho c(q^*)}{(1-M)p(y)(1-p(y))[u(q^*) - c(q^*)]}. \tag{7}$$

Definition of Equilibrium. *A symmetric, stationary, monetary, equilibrium is a list* $\{V_0, V_m, y, q, q_m, \tau\}$ *satisfying (2), (5),* $q = q^*$*, and* q_m *satisfies (6) if* $\tau^* = 1$ *and* $q_m = q^*$ *if* τ^* *satisfies (7).*

[5] Since $\tau^* \leq 1$ is required, we must have $M \leq 1 - \frac{\rho c(q^*)}{p(y)(1-p(y))[u(q^*)-c(q^*)]}$ which is satisfied for sufficiently small values of M and ρ for all values of y.

3 Welfare

How is steady-state welfare affected by the introduction of money? As demonstrated above, the existence of money induces agents diversify less. But if *all* agents diversify less, the number of matches in which a double or single coincidence occurs may fall so much that trading volume actually falls. In short, while the use of money expands the number of trades (welfare improving), agents actions *collectively* reduce the probabilities that trades occur (welfare reducing). If this latter effect dominates, then introducing money can reduce welfare.

It is difficult to determine analytical conditions such that welfare falls. Thus, we resort to numerical simulations to calculate welfare. We use the functional forms

$$u(q) = \frac{q^{\sigma}}{\sigma},\ 0 < \sigma < 1,\ c(q) = q,\ e(y) = \frac{ay^2}{2}$$

which, for $\tau^* < 1$, yield

$$\begin{array}{lll} y^* = 0, & & \sigma = 1 \\ y^* = \frac{(1-\sigma)[2x - M(1-M)(2x-1)]}{((2+a\rho)\sigma - 2) + 2(1-\sigma)M(1-M)} & \text{if} & 1 > \sigma > \frac{2-M(1-M)}{2+a\rho(1-x)-M(1-M)} \\ y^* = 1 - x & & 0 < \sigma \leq \frac{2-M(1-M)}{2+a\rho(1-x)-M(1-M)} \end{array} .$$

Figure 1 plots welfare for barter, denoted W_B, with lotteries, W_L, and without lotteries, W_m, where $\sigma = .67, x = .3, \rho = 1,\ a = 1.5$. For these parameters, the moral hazard problem is small so introducing money improves welfare relative to barter and the use of lotteries raises welfare even more. At a sufficiently high value of M, $\tau^* = 1$ and the two graphs coincide. Thus, the use of money raises welfare and having it be 'divisible' enhances welfare.

In contrast, in Figure 2 we set $\sigma = .5, x = .2, \rho = 1$, $a = 1.4$. In this case, the moral hazard problem is large and introducing money causes diversification to fall so much that welfare falls relative to barter, with and without lotteries. By specializing too much, trading probabilities fall dramatically and in equilibrium, consumption risk increases relative to barter. Surprisingly, welfare falls more with lotteries than without. Lotteries raise the insurance value of money because they allow an agent to hold onto money for a longer period of time on average. Thus, lotteries exacerbate the moral hazard problem causing agents to diversity even less and welfare to fall further.

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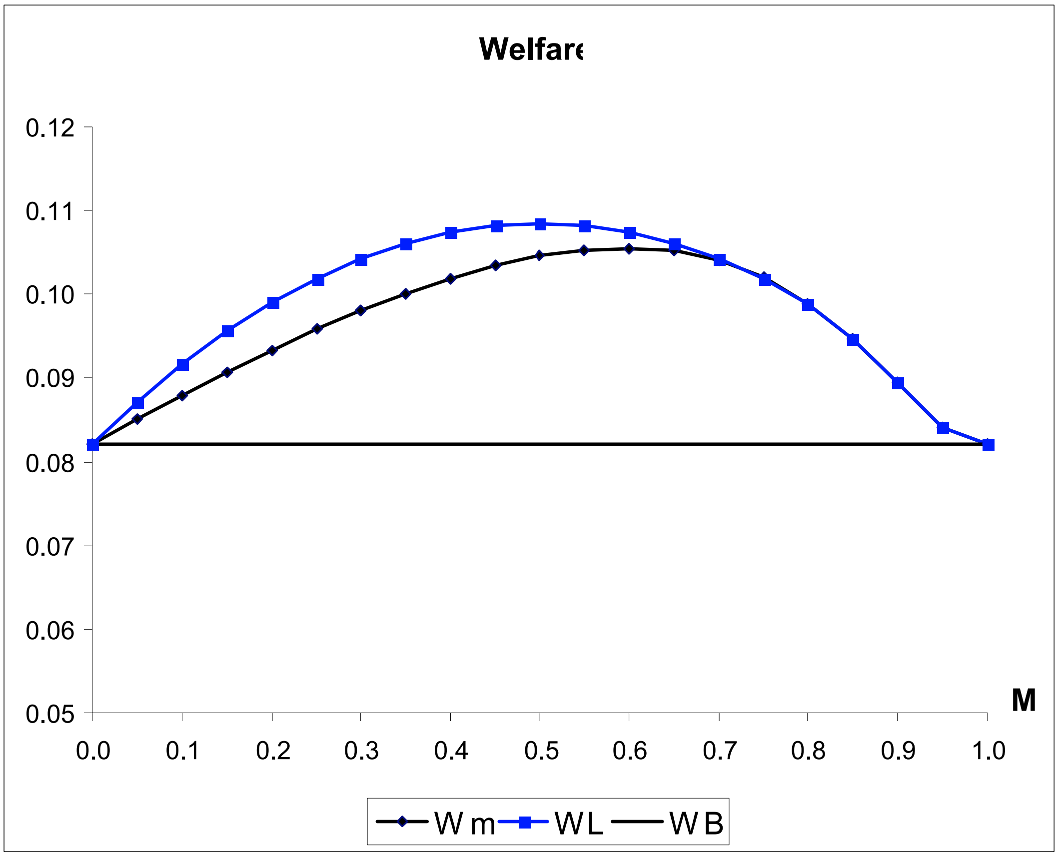

Welfare
0.12
0.11
0.10
0.09
0.08
0.07
0.06
0.05
M
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
W m
W L
W B

Figure 1:

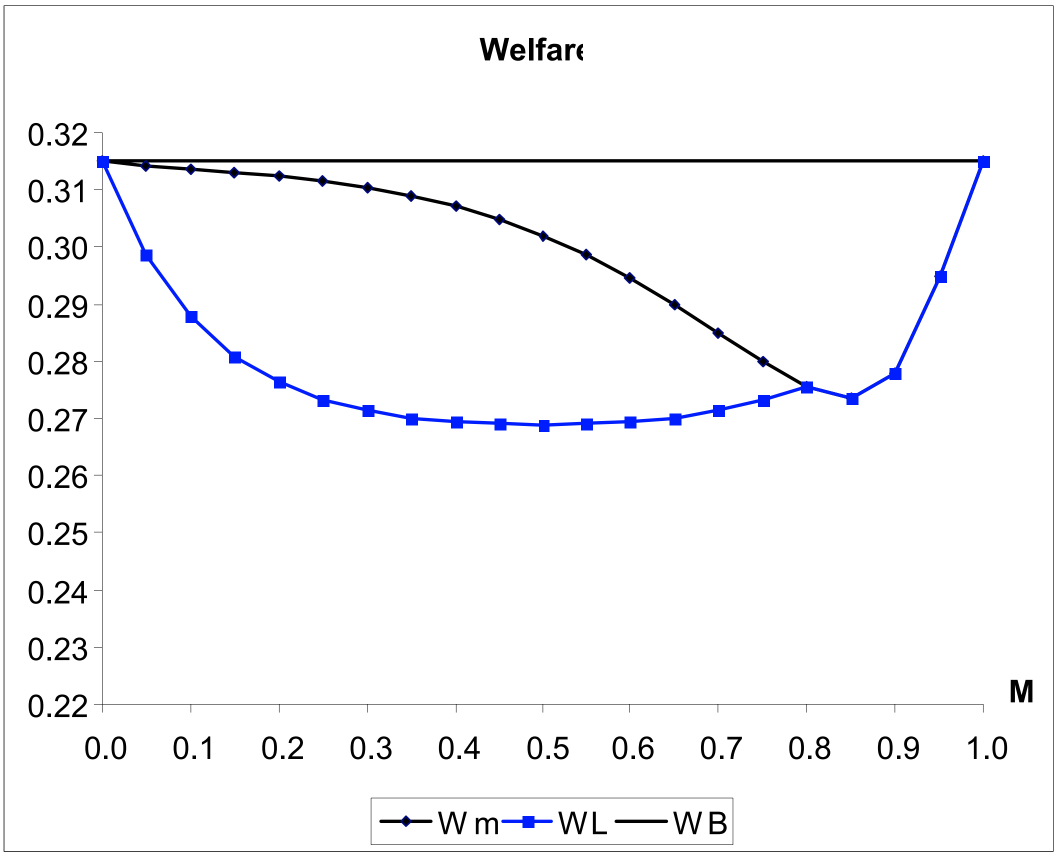
Welfare
0.32
0.31
0.30
0.29
0.28
0.27
0.26
0.25
0.24
0.23
0.22
M
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
W m
WL
WB

Figure 2: