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TIME-DEPENDENT IONIZATION EQUILIBRIUM AND LINE RADIATION UNDER FLARELIKE CONDITIONS

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ABSTRACT

The results of calculations of time-dependent ionization equilibrium and line emission are presented and compared with the values obtained under the assumption that steady-state conditions prevail. In the models considered, it is assumed that the electron density is constant ($= 10^9 \text{ cm}^{-3}$) and that the temperature increases by a factor of 10 from $3 \times 10^6 \text{ }^\circ\text{K}$ on timescales ranging from 100 to 300 s and decays back to $3 \times 10^6 \text{ }^\circ\text{K}$ on a timescale ranging from 600 to 1400 s. Ions of oxygen and silicon are considered, and it is found that the spectrum is softer during the rise and harder during the fall than predicted by the steady-state approximation.

I. INTRODUCTION

In recent years observations of X-ray line emission have provided a wealth of information about the excitation conditions in solar flares. However, almost all the model calculations against which these observations can be compared are based on the assumption that the ionization balance depends only on the temperature at a given instant, so that the steady-state approximation is valid (Tucker and Koren 1971; Jordan 1969; Cox and Tucker 1969 and references cited therein). In this paper, we present the results of some model calculations where the time dependence of the ionization equilibrium was taken into account.

The characteristics of the ionization equilibrium can be understood qualitatively by comparing the timescales for changes in the temperature, ionization, and recombination. Consider a typical flare consisting of three phases. In phase I the plasma is in the steady-state preflare condition with a temperature $\sim 3 \times 10^6 \text{ }^\circ\text{K}$ and a density $N \sim 10^9 \text{ cm}^{-3}$. In phase II the plasma is heated suddenly to a temperature $T \sim 10^7 \text{ }^\circ\text{K}$ in a time $t_H \sim 100 \text{ s}$. In phase III the plasma cools on a timescale $t_C \sim 700 \text{ s}$.

The ionization time for a hydrogenic ion of nuclear charge Z is (Cox and Tucker 1970)

$$t_I \sim 50T_7^{-1/2}N_9^{-1}(Z/10)^4 \exp [1.6(Z/10)^2/T_7] \text{ seconds}$$

$$(T_7 = T/10^7 \text{ }^\circ\text{K}, \quad N_9 = N/10^9 \text{ cm}^{-3}).$$

Thus for $Z \lesssim 10$, the ionization time is less than the heating time and the steady-state calculations should apply for phase II. For $Z \gtrsim 10$, $t_I \gtrsim t_H$, and the ionization will be less than in the steady-state models.

The recombination time is

$$t_R \sim 1000T_7^{1/2}/N_9(Z/10).$$

The situation here is just opposite to that of phase II. For those elements with $Z \gtrsim 10$, $t_R \lesssim t_C$ and the steady-state calculations apply, whereas for those with $Z \lesssim 10$, $t_R \gtrsim t_C$ and the ionization will be greater than in a steady-state model.

Therefore, for a given temperature, the spectrum will be softer than in the steady state during the rise and harder during the decay of a flare, and temperature estimates based on the overall spectral shape may be significantly in error. On the other hand, if the temperature can be determined by the ratio of two lines of the same ion, then observation of the overall spectrum could yield information about the departure from steady-state conditions. The amount of the departure depends on the heating and cooling times, the temperature, and the density. If t_H , t_C , and T can be determined, one could set limits on the density of the plasma. Note that low densities produce the largest effects. The intensities of the ratio of the intercombination to forbidden lines in helium-like ions yield upper limits to the density (Gabriel and Jordan 1970; Blumenthal, Drake, and Tucker 1972). Thus, the two effects are complementary and serve to set lower and upper limits on the density in flare plasmas. On the other hand, if the density is known, the departures from steady-state conditions would help to establish the heating and cooling laws for the plasma.

In § II we discuss the basic equations and the method used in the calculations. The results are presented in § III; their significance is discussed in § IV.

II. CALCULATIONS

The X-ray line emissivities were calculated from the formula

$$P(Z, zT) = 1.86 \times 10^{-19} T_6^{-1/2} N_e N_{Z,z} \langle \Omega(Z, z) \rangle (E(Z, z)/I_H) e^{-E(Z,z)/kT} \text{ ergs cm}^{-3} \text{ s}^{-1},$$

where T_6 is the electron temperature in millions of degrees, N_e is the electron number density, and $N_{Z,z}$ is the number density of ion species Z, z . $\langle \Omega(Z, z) \rangle$ is an average value of the collision strength which is approximately equal to its value at incident electron energies = $1.5 E(Z, z)$, where $E(Z, z)$ is the excitation energy of the first excited state. I_H is the ionization potential of hydrogen. Values of $\langle \Omega(Z, z) \rangle$ and $E(Z, z)$ were taken from the compilation of Tucker and Koren (1971). The line emission from hydrogenic and helium-like ions of oxygen and silicon were considered. The line emission due to the radiative de-excitation of excited states formed by recombination was also considered. It was found to be negligible except during the very late stages which are not of interest for the present investigation. The abundances relative to hydrogen were assumed to be $N(O)/N(H) = 3 \times 10^{-4}$ and $N(Si)/N(H) = 5 \times 10^{-5}$.

To calculate the ionization equilibrium, we solved the coupled ionization balance equations with an assumed heating and cooling equation:

$$dN_z/dt = N_e(\alpha_{z+1}N_{z+1} - \alpha_z N_z + C_{z-1}N_{z-1} - C_z N_z), \dots, \quad dT/dt = f(t).$$

Here α_z denotes the total recombination coefficient (dielectronic and radiative) for the process $z \rightarrow z - 1$, and c_z denotes the collisional ionization coefficient for the process $z \rightarrow z + 1$. Other sources of ionization (fast particles, photons) were assumed to be negligible. The dielectronic recombination coefficients were calculated from the formula given by Burgess (1965) multiplied by a correction factor as given by Shore (1969). The necessary energy levels and oscillator strengths were taken from Wiese, Smith, and Glennon (1966). The formula given by Seaton (1959) was used to calculate the radiative recombination coefficients. The collisional ionization coefficients were calculated as in Cox and Tucker (1969). If the timescale t_T for significant changes in the temperature is long compared to the timescales for recombination t_R and ionization t_I , then for t_R or $t_I \ll t$ and t_T , the ions will have had time to reach a quasi-steady state in which $dN_z/dt = 0$, and $N_{z+1}/N_z = C_z/\alpha_{z+1}$. In this limit the ionization equilibrium is independent of the electron density and depends only on the instantaneous temperature. When the above inequalities are not satisfied, the equations do not decouple, and the ionization equilibrium depends on the electron density and the history of the system.

The equations were solved by a Runge-Kutta routine. The electron density was

assumed to be constant and equal to 10^9 cm^{-3} . The models were therefore determined by the assumed temperature dependence. Three cases are reported on here:

- (1) $T_6 = 3(1 + t/10)$, $t < 100 \text{ s}$; $T_6 = 30 e^{-(t-100)/600} + 3$, $t > 100 \text{ s}$;
- (2) $T_6 = 3(1 + t/30)$, $t < 300 \text{ s}$; $T_6 = 30 e^{-(t-300)/400} + 3$, $t > 300 \text{ s}$;
- (3) $T_6 = 3(1 + t/10)$, $t < 100 \text{ s}$; $T_6 = 30(1 + t/2)^{-2/5} + 3$, $t > 100 \text{ s}$.

III. RESULTS

The results of the calculations are presented in figures 1–4. In these figures we have plotted the temperature, ionization equilibrium, line intensities of the strongest lines from the hydrogenic and helium-like ions, and the ratio of the line intensities as a function of time. The solid curves refer to the time-dependent values; the dashed curves, to the steady-state values.

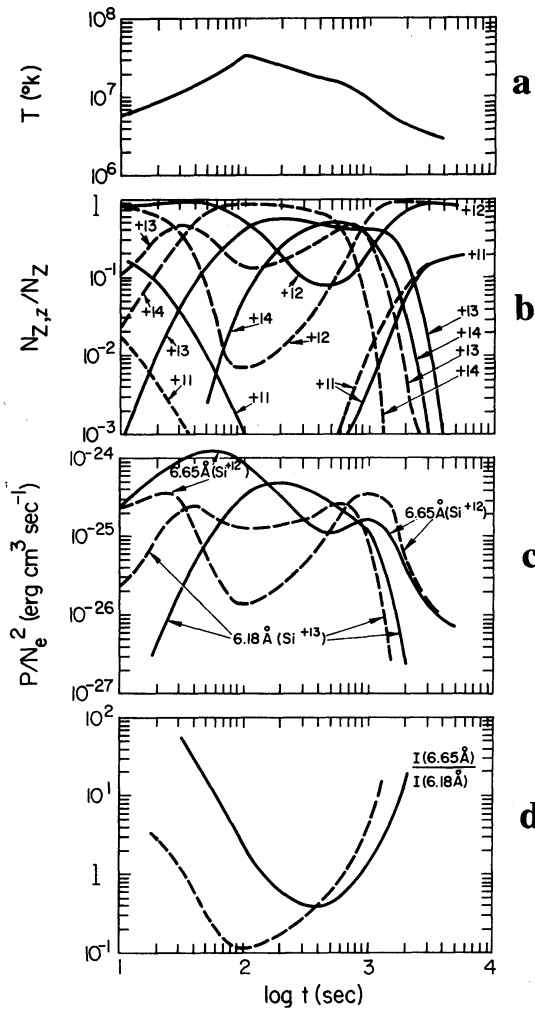


FIG. 1

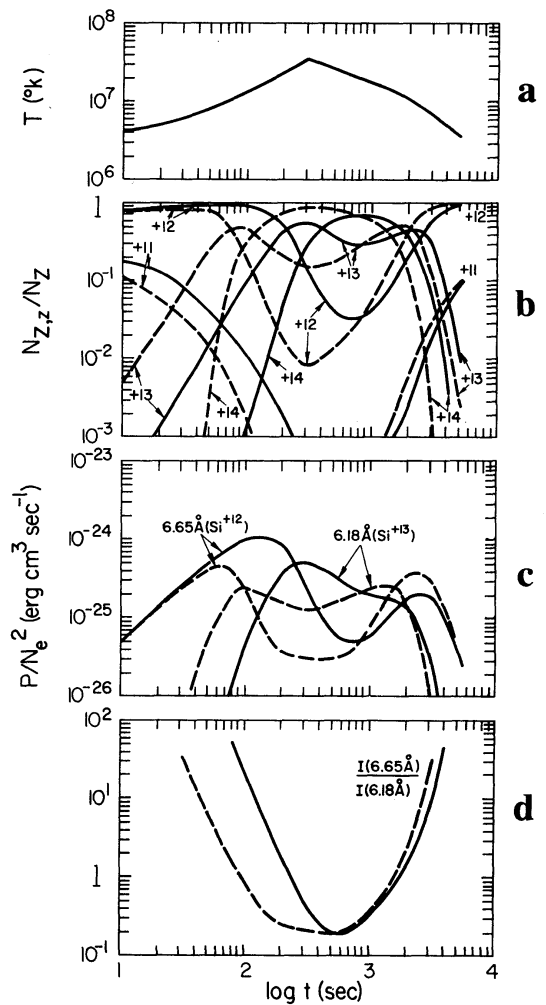


FIG. 2

FIG. 1.—Results for silicon, Model 1. The solid curves refer to time-dependent calculations; the dashed curves, to steady-state ones: (a) temperature as a function of time, (b) ionization equilibrium as a function of time, (c) line intensities from strongest lines from hydrogenic and helium-like ions as a function of time, (d) the ratio of the intensities of these lines as a function of time.

FIG. 2.—Same as fig. 1 for Model 2.

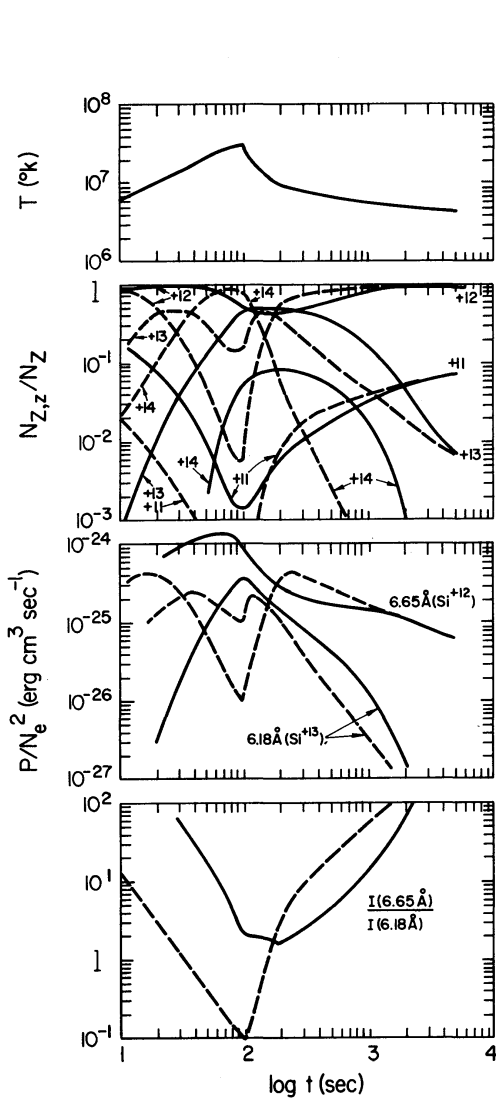


FIG. 3

FIG. 3.—Same as fig. 1 for Model 3.

FIG. 4.—Results for oxygen, Model 3.

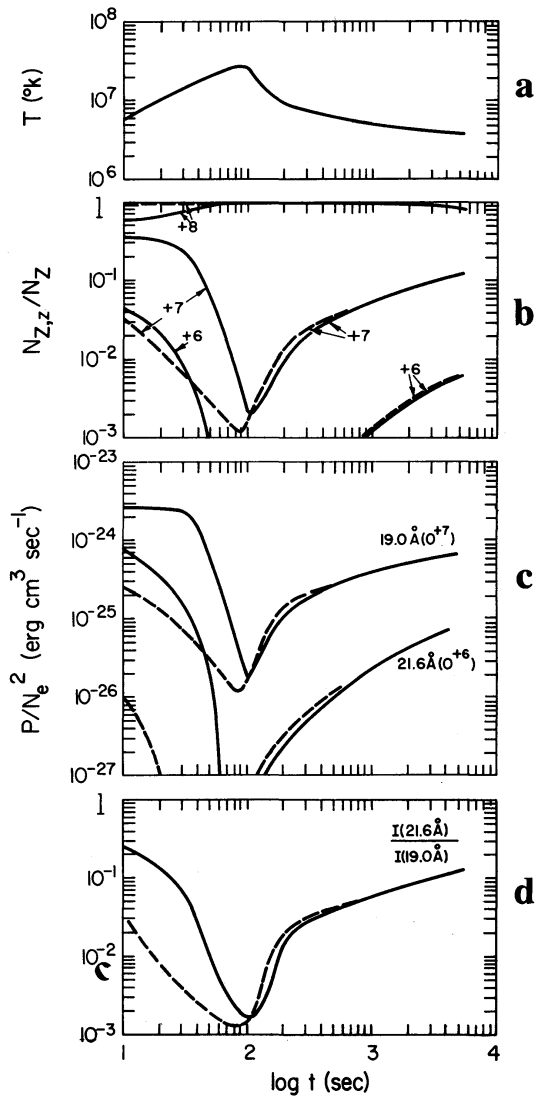


FIG. 4

Figure 1 shows the results for silicon for a model in which the temperature rises linearly from 3×10^6 ° to 3.3×10^7 ° K in 100 s and drops exponentially with a time constant of 600 s until it returns to 3×10^6 ° (model 1). Initially almost all the silicon is in the +12 form. As the temperature rises, the ionization shifts to a higher stage but because of the long ionization times the shift is not so fast as in the steady-state case. After about 100 s the ionization begins to take its toll and the abundance begins to drop. By this time the temperature is dropping, so the steady-state abundance of Si^{+12} is increasing; because of the lag in ionization the abundances of the time-dependent and steady-state cases are changing in opposite directions during this time. Finally after about 500 s, ions begin to recombine, and the calculations start the approach to the steady-state conditions which is essentially complete at $t = 3000$ s (fig. 1b). The behavior of the abundances of Si^{+12} is reflected in the intensity of the Si^{+12} line at 6.65 Å (fig. 1c). For the first few hundred seconds it is more intense than in the steady state

and afterwards is less intense. The course of Si^{+13} with time is a little more complicated because in the steady state the abundance of Si^{+13} peaks around 10^7 °. Thus as the temperature rises to 3.3×10^7 ° and then decays back to 3×10^6 °, the abundance will show two maxima since the temperature of maximum abundance was traversed twice. However, in the time-dependent calculations these maxima are completely smeared out because of the lag in the ionization during the rise and the recombination during the decay. The general behavior of the system is conveniently summarized by the ratio of the intensities of the 6.65 and 6.18 Å lines of Si^{+12} and Si^{+13} , respectively (fig. 1*d*). $I(6.65)/I(6.18)$ is greater than in the steady state for the first few hundred seconds (long ionization time), crosses the steady-state value around $t = 400$ s, and is less thereafter (long recombination time).

Figure 2 shows the results for the case where the temperature rises linearly from 3×10^6 ° to 3.3×10^7 ° K in 300 s and drops exponentially thereafter with a time constant of 1400 s until it levels off around 3×10^6 ° again (Model 2). The behavior is qualitatively the same as for Model 1; quantitatively the departures from the steady-state calculations are less because of the longer time constants involved.

In figure 3 the results for a different kind of cooling law are shown (Model 3). In this case the temperature rises to a maximum in 100 s as in Model 1, but decreases after the maximum according to the law $T \propto (1 + t/2)^{-2/5}$, in a manner roughly similar to what one might expect if conduction losses were important in the cooling of the plasma (Culhane, Vesecky, and Phillips 1970). This law is characterized by a faster rate of decrease in the temperature than for Models 1 and 2 immediately after maximum, and by a slower rate later on. The departures from the steady-state results are again large during the rise, but are not too great during the decay. The primary difference from the results of Model 1 is that the intensity of the Si^{+13} line is weaker during the decay, since the temperature is not above 10^7 ° for very long.

Figure 4 shows the results of calculations for oxygen for Model 3 temperature dependence. The departures from steady state during the rise are less pronounced than for silicon. During the decay the calculations always remain fairly close to the steady-state value, even though the recombination time is longer than the decay time.

As mentioned above, a constant electron density $N_e = 10^9 \text{ cm}^{-3}$ has been assumed in all the results discussed here. These results can be scaled to other densities simply by scaling the time constants inversely as the electron density. Thus, for example, Model 1 can equally well describe a case where the density is 10^{10} cm^{-3} , the temperature rises in 10 s and falls in 60 s. We have done additional calculations for $N_e = 10^8 \text{ cm}^{-3}$ and $N_e = 10^{10} \text{ cm}^{-3}$, with the expected result that the departures from steady-state conditions are less when the density is greater and vice versa.

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