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Abstract

We endogenize the trade mechanism in a search economy with many homogenous sellers and many heterogenous buyers of unobservable type. We study how heterogeneity and the traders' continuation values–which are endogenous–influence the sellers' choice of trade mechanism. Sellers trade off the probability of an immediate sale against the surplus expected from it, choosing whether to trade with everyone and how quickly. In equilibrium sellers may simply target one buyer type via non-negotiable offers (price posting), or may price discriminate (haggling). We also study when haggling generates trading delays. A price setting externality arises because of a strategic complementarity in the sellers' pricing choices.

Keywords: Search, Prices, Negotiations, Asymmetric Information

JEL: C78, D4, D82, D83

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1 Introduction

A large segment of the macro/labor literature is based on models where market frictions are made explicit, exchange is bilateral, and prices are endogenously formed. These include "workhorse" matching models of the labor market and of monetary economies, where prices are bargained (e.g. Mortensen and Pissarides, 1994, Shi, and Trejos-Wright, 1995), and more recent models where prices are posted (Acemoglu and Shimer, 2000, Burdett, Shi and Wright, 2001, and Moen, 1997). Since the allocations depend on the trading mechanism assumed to be in place, it is natural to ask what pricing mechanisms sellers would select, when given the option.

In this paper, we endogenize the trading mechanism in an informationally opaque market with many sellers and buyers who engage in short-lived trading relationships. We build intuition for the following questions. How does a seller's pricing selection respond to buyers' heterogeneity and his competitors' prices? Is there scope for 'haggling', or is it optimal to charge the same non-negotiable price to every customer? Finally, can coordination failures occur in pricing selections?

To answer these questions we use a random matching economy, in which each seller independently decides how to price a homogeneous good, given that buyers' valuations–high or low–are unobservable. Paired traders play a two-stage game based on the seller's choice between one of two prototypical non-cooperative trading mechanisms. The seller may simply make non-negotiable offers in each stage, à la Fudenberg and Tirole (1983). This strategic bargaining game of imperfect information may generate trading delays. Alternatively, the seller can offer to negotiate if the buyer provides verifiable information on his valuation. Here the seller commits to let the buyer make the first offer and–in case of disagreement–gives him a chance to a second offer. This strategic bargaining game of perfect information generates immediate trade.

In equilibrium, pricing decisions reflect buyers' heterogeneity and also the traders' continuation values–which are endogenous. Sellers trade off the probability of an immediate sale against the surplus expected from it. This implies sellers not only must choose whether to trade with every possible customer, but also how quickly. This depends on the price and the trading mechanism retained.

When sales to some buyer type contribute little to the expected surplus, then sellers target only the other type, via a non-negotiable price that extracts his *entire* surplus. This market resembles one in which sellers 'post prices.' Although not everyone may buy, every purchase occurs at a unique non-negotiable price, and it is immediate. Such outcomes may arise when buyers' valuations are very different, or when some type is predominant.

When 'significant' gains are expected from sales to every buyer, then sellers target both types. This requires sharing surplus with some type, so that the market resembles one in which there is 'haggling,' since sellers trade at different prices with different customers. Whether trading delays occur, however, hinges on *how* sellers choose to price discriminate which–due to unobservable valuations–requires the seller to elicit information. An *indirect* way to do so is to observe the buyer's response to a high initial offer. This may cause wasteful trading delays, but lets the seller extract the surplus of low-value buyers. A *direct* way to elicit information, is to commit to compensating the buyer for supplying it. In our model, this compensation takes the form of letting the buyer make the first offer so that surplus is shared with every buyer, but trade is immediate.

Interestingly, equilibria with or without haggling, or with different price discrimination schemes, may coexist. The reason is traders' option values are endogenous, so there are strategic complementarities in pricing selections. We call this a 'price setting externality': a seller's pricing choice is influenced by the prices expected to prevail on the market, which determine traders' values from searching for a better deal.[2] This may lead to multiplicity of equilibria, and coordination failures. For example prices may be inefficiently high, so that only some buyers consume, or trading delays may systematically occur.

2 Related Literature

We contribute to the literature on endogenous selection of pricing mechanisms in several dimensions. There are studies on trading mechanism choices of a monopolist selling to heterogeneous buyers. For example, Riley and Zeckhauser (1983) find the seller should use a fixed price strategy if commitment is possible, while Wang (1995) focuses on cost differences in selecting bargaining or price posting. Instead, we focus on strategic interaction among many sellers, to emphasize how *price setting externalities* arise due to complementarities in the selection of trading mechanisms.

Another focus has been the issue of commitment to a price. In a model with both search and commitment costs, Bester (1994) illustrates how the seller's benefit from committing to a price, rather than bargaining, depends on the commitment choices of all other sellers. Masters and Muthoo (2000) study the possibility of price *renegotiation* when heterogeneity is match-specific.

[2] See also Rubinstein and Wolinsky (1990), although in our model buyers' trading opportunities are stationary.

Instead, we contrast economies where a commitment technology is or is not available, to show how commitment can be beneficial in eliminating wasteful trading delays.

A third line of research has examined the capacity of different pricing mechanism to better attract buyers, using directed-search models. Peters (1991) shows bargaining is not a stable institution as there is always an incentive for sellers to post ex-ante prices. Using a mechanism design approach, McAfee (1993) proves existence of a unique equilibrium where sellers choose to hold identical auctions–among a vast array of mechanisms–and buyers randomize over which auction they participate in. We depart from these studies by assuming search is random, and *cannot* be directed. This is because we want to focus on the links between buyers' heterogeneity and choice of trading mechanism, abstracting from the trading mechanisms's relative advantages in reducing matching frictions. Hence in our model, the choice of trading mechanism does not affect buyers' arrival rates, but only their willingness to trade.

Studies have also focused on private information issues. For example, Bester (1993) studies competition between pricing mechanisms when goods' quality is private information. Instead, we study the case where buyers' valuations are private information. Moreno and Wooders (2002) consider a model with unobservable buyer valuations, and homogeneous sellers, as we do. Unlike us, they impose bargaining to study trade patterns dynamics and the link between market composition and types of trades realized. Michelacci and Suárez (2002) study a labor market where firms choose between bargaining or price posting given that workers' skills are unobservable. They assume directed search, to examine whether bargaining allows firms to better attract highly skilled workers.

3 Environment

Time is discrete and infinite where $t = 0, 1, 2...$ identifies a period. The economy is comprised of a continuum of agents divided into two sets, *sellers* and *buyers.* The mass of agents in each of these sets is normalized to one. Sellers are endowed with an homogeneous indivisible good from which they derive no utility. Buyers have no endowment and receive some utility from consumption of the sellers' goods, but have heterogenous preferences. A proportion λ of buyers derives utility $u_L > 0$ from consumption of the good, while $1 - \lambda$ buyers have high valuation $u_H > u_L$. Buyers can transfer utility to sellers. The buyer's type $i \in \{L, H\}$ is private information.

At the end of each period sellers choose a trade mechanism characterized by two stages of play, within the period. Traders discount next stage payoffs at rate $\delta \in (0,1)$. We consider two distinct cases. In one of them, *ex-ante* commitment is possible. In the other it is not. Only if *ex-ante* commitment is possible can the seller stand by his choice of trading mechanism. At the beginning of each period t, buyers and sellers are randomly paired. A seller meets a buyer with probability σ, while a buyer meets a seller with probability α. Unmatched agents sit idle in t and undergo matching again in $t+1$. Paired agents attempt to find mutually agreeable terms of trade via the trade mechanism selected by the seller. If this cannot be accomplished by the end of t, the match is dissolved and both agents return to the search pool. If trade and consumption take place the agents exit the market and are replaced by an identical pair. The distribution of buyers' types on the market is thus constant.[3] Agents discount next period utility by $\frac{1}{1+r} = \delta^2$.

4 Symmetric Pure Strategy Equilibria

We focus on equilibria where agents play pure strategies that are invariant functions of t. An agent chooses his strategy taking as given market prices and the strategies adopted by others.

A *trading game*, taking place in t, is a two-stage game characterized by an offer vector $\mathbf{p} = (p_1, p_2)$, p_1 in the first and p_2 in the second stage. The specifics of the trading game depend on the seller's selection of trading mechanism, at the end of $t-1$. This selection, denoted by π, amounts to choosing whether to make unilateral non-negotiable offers, $\pi = 0$, or to engage in bilateral negotiations, $\pi = 1$. If $\pi = 0$ the seller chooses to make an offer in each stage, that the buyer can accept or reject. If $\pi = 1$, bilateral negotiations take place as follows. At the start of the game the seller offers the buyer the possibility to reveal his type, that is to provide costlessly *verifiable* information. The seller commits to letting the buyer who reveals his valuation make the first-stage offer. If the seller rejects this offer, a final counter-offer is made in the second stage by the seller, with probability θ, and by the buyer with probability $1-\theta$. If the buyer does not reveal his type, the seller makes a non-negotiable offer in each stage.[4] Notice that *ex-ante* commitment must be available in order for the seller to be able to credibly propose bilateral negotiations, as he would renege on his beginning-of-game promises absent commitment.[5]

[3] Thus we avoid sorting externalities: the agents' strategies do not influence the distribution of buyers.

[4] Assuming traders go on to the second stage following disagreement is w.l.o.g since matching takes place only at the beginning of t. This implies that disagreement does not change the search pool's composition.

[5] A referee, whom we thank, indicates a practical way to commit. The seller can pay someone to make sales at say, $\mathbf{p}_f$. This agent will pay the seller a large sum, should a sale occur at a different price. Perhaps this is how

In a match where $\mathbf{p}$ is the offer vector, a buyer of type i initiates a purchase at *some* stage of the trading game with probability $\beta_i = 0, 1$. A trade taking place in the first stage at 'price' $p_1 > 0$, is a simultaneous transfer of goods for utility. It gives p_1 period utility to the seller and $u_i - p_1$ period utility to buyer i. Utilities from trades occurring in the second stage are discounted by δ. We let $b_i(\mathbf{p})$ and $s(\mathbf{p})$ denote the beginning-of-period expected utilities to, respectively, a buyer of type i and a seller, from a trade occurring in a match where $\mathbf{p}$ is expected. Thus $b_i(\mathbf{p}) = u_i - p_1$ if trade occurs in the first stage, and $b_i(\mathbf{p}) = \delta(u_i - p_2)$ if trade occurs in the second stage. Similarly $s(\mathbf{p}) = p_1$ if trade occurs in the first stage, and $s(\mathbf{p}) = \delta p_2$ otherwise.

In general negotiated offers depend upon the buyer's valuation. Thus, we let the subscript n stand for "negotiated," so $\mathbf{p}_n^i = (p_{n,1}^i, p_{n,2}^i)$ represents the price vector negotiated by type i. In the absence of negotiations the price vector is type independent, so we denote it $\mathbf{p}_f = (p_{f,1}, p_{f,2})$, indexed f for 'fixed.' The superscript '$*$' identifies equilibrium market strategies and prices.

4.1 Value functions

The problem of a representative agent has a recursive formulation. Thus, we use a dynamic programming approach letting V_i denote the end-of-period value of search to a buyer of type i, and $V = \max\{V_n, V_f\}$ the end-of-period value of search to a seller where V_n and V_f respectively refer to the value from committing to negotiations and not.

In a symmetric pure strategy equilibrium the distribution of offers $\mathbf{p}^*$ is degenerate. Thus, if buyers reveal their valuation to sellers committed to negotiations, in equilibrium

$$\begin{aligned} r\delta V_i &= \alpha\pi^* \max\left\{b_i(\mathbf{p}_n^{i*}) - \delta V_i, 0\right\} + \alpha(1-\pi^*)\max\left\{b_i(\mathbf{p}_f^*) - \delta V_i, 0\right\} \\ r\delta V_n &= \sigma\lambda\beta_L^*(\mathbf{p}_n^{L*})\max\left\{s(\mathbf{p}_n^{L*}) - \delta V, 0\right\} + \sigma(1-\lambda)\beta_H^*(\mathbf{p}_n^{H*})\max\left\{s(\mathbf{p}_n^{H*}) - \delta V, 0\right\} \\ r\delta V_f &= \sigma\max\left\{s(\mathbf{p}_f^*) - \delta V, 0\right\} \end{aligned} \tag{1}$$

These are standard flow return conditions where δV_i and δV denote the beginning-of-period continuation values from avoiding trade. The discount factor δ reminds us that search can take place only at the beginning of a period[6], while V tells us that the seller's choice of trading mechanism can be revised at the end of each period. Clearly all continuation values depend on the equilibrium prices $\mathbf{p}^*$; we do not make this explicit, say by writing $V(\mathbf{p}^*)$, for notational simplicity.

supermarkets commit to a price: the checkout clerk does not have the authority to set prices and gets fired if he does.

[6] For example, if $\pi^* = 0$ the end-of-period lifetime utility of buyer of type i in equilibrium is the sum of two expected payoffs $V_i = \alpha\beta_i^*(\mathbf{p}_f^*)\delta b_i(\mathbf{p}_f^*) + \left[1 - \alpha\beta_i^*(\mathbf{p}_f^*)\right]\delta^2 V_i$.

The first line in (1) tells us that a buyer of valuation i meets a seller with probability α. If the seller negotiates, $\pi^* = 1$, and the buyer reveals his valuation, the offer vector $\mathbf{p}_n^{i*}$ arises. In the absence of negotiations, the buyer expects $\mathbf{p}_f^*$. The value of search to a seller committed to negotiations is described in the second line of (1). He meets buyers with probability σ and if the buyer is of low valuation, with probability λ, $\mathbf{p}_n^{L*}$ results, otherwise $\mathbf{p}_n^{H*}$ results. When the seller chooses to make non-negotiable offers (see the third line in (1)) the vector $\mathbf{p}_f^*$ is match-independent, since valuations are private information. Here the seller's payoff $s(\mathbf{p}_f^*)$ accounts for the likelihood that trade occurs in the first, second stage, or not at all, as we later clarify.

Since traders can reject disadvantageous offers at any stage of the trading game (the max operators in (1)), δV_i and δV are bounded below by zero. Furthermore, δV_i is bounded above by $\frac{\alpha}{r+\alpha} u_i$, when $p_1 = 0$, while δV is bounded above by $\frac{\sigma}{r+\sigma} u_H$, when $p_1 = u_H$ and $\lambda = 1$.

4.2 Optimal Strategies

The discussion above tells us that neither seller nor buyer can do worse than autarky, in equilibrium. If the match generates unfavorable offers to a trader, he can postpone the transaction in the hope of finding better terms of trade. Unfavorable here means that the prices quoted leave the agent strictly negative surplus, defined as the difference between the net period utility from completing the trade and the continuation value from avoiding trade. Thus, a transaction accomplished in the first stage at price p_1, gives $u_i - p_1 - \delta V_i$ surplus to a buyer of type i and $p_1 - \delta V$ to a seller. Transactions accomplished in the second stage at price p_2, give $\delta(u_i - p_2 - V_i)$ surplus to a buyer of type i and $\delta(p_2 - V)$ to a seller, in value discounted to the beginning of the game.

We use this information to discuss the optimal strategies of a buyer of valuation i. This is done by moving backward in the sequence of choices he faces in a trading game. The buyer will want to buy at some stage of a trading game where the offer vector is $\mathbf{p}$, given that market prices are $\mathbf{p}^*$, if the net utility from doing so is no less than his value of search, or

$$\beta_i(\mathbf{p}, \mathbf{p}^*) = \begin{cases} 1 & \text{if } b_i(\mathbf{p}) \geq \delta V_i \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

When the buyer's reservation utility constraint is satisfied, $\min\{u_i - p_1, \delta(u_i - p_2)\} \geq \delta V_i$, he participates in trade, $\beta_i(\mathbf{p}, \mathbf{p}^*) = 1$. This requires a price smaller than the buyer's reservation price, at some stage of the game. The buyer accepts $p_1 \leq u_i - \delta V_i$, and $p_2 \leq u_i - V_i$.

Moving one step back, at the beginning of the trading game, the buyer might be offered the possibility to reveal his valuation in order to negotiate. He will do so if and only if the expected utility is greater than that generated by passively receiving offers from the seller. The participation constraint of a buyer of type i is

$$b_i(\mathbf{p}_n^i) \geq b_i(\mathbf{p}_f). \tag{3}$$

When this inequality holds, it is optimal for a buyer of type i to reveal his valuation to a seller committed to negotiations.

Finally, it is obvious that a buyer will always enter a trading game with any seller. He can always refuse to buy at the proposed price, having no loss, while his best alternative–doing nothing and searching again next period–generates zero surplus.

Now consider a seller's selection of trade mechanism, at the end of a period, in the presence of ex-ante commitment. In doing so the seller considers the market prices $\mathbf{p}^*$ but also the prices $\mathbf{p}$ he expects to arise in a match, based on his choice of mechanism. Feasibility of trade requires that the seller's surplus is non-negative, at some stage of the game. Thus we say that $\mathbf{p}$ is feasible if the seller's reservation utility constraint is satisfied,

$$s(\mathbf{p}) \geq \delta V. \tag{4}$$

This implies that for the seller to willingly trade, the price must be greater than the seller's reservation price at some stage of the game, i.e. $p_1 \geq \delta V$ or $p_2 \geq V$.

Moving one step back, given $\mathbf{p}$ and $\mathbf{p}^*$, the seller chooses between making non-negotiable offers in each stage, or to let a buyer who reveals his valuation free to make the initial offer (and possibly a counter-offer). The best course of action must deliver the highest lifetime utility,

$$\pi(\mathbf{p}, \mathbf{p}^*) = \begin{cases} 1 \text{ if } V_n \geq V_f \\ 0 \text{ otherwise.} \end{cases} \tag{5}$$

In a symmetric equilibrium, the strategies of an individual must reflect those adopted on the market, and prices in every match must be identical to those prevailing on the market:

$$\pi(\mathbf{p}, \mathbf{p}^*) = \pi^*(\mathbf{p}, \mathbf{p}^*), \ \beta_i(\mathbf{p}, \mathbf{p}^*) = \beta_i^*(\mathbf{p}, \mathbf{p}^*), \ \mathbf{p} = \mathbf{p}^*. \tag{6}$$

Definition *An equilibrium is an offer sequence* $\mathbf{p} \in \left\{\mathbf{p}_f, \mathbf{p}_n^i\right\}$, *strategies* $\{\pi(\mathbf{p}, \mathbf{p}^*), \beta_i(\mathbf{p}, \mathbf{p}^*)\}$ *and lifetime utilities* $\{V_i, V_f, V_n\}$ *that are invariant functions of* t *and satisfy (1)-(6).*

We emphasize that stationarity here means that strategies are invariant functions of t. Due to discounting, however, price offers are stage-dependent, as clarified in the next section.

5 The Determination of the Individually Optimal Offers

Consider an economy where ex-ante commitment is available. To start, we define

$$p^i = u_i - \delta V_i \text{ and } p^{i'} = u_i - V_i. \tag{7}$$

Here p^i is the reservation price of a buyer of valuation i, in the first stage of the trading game, as it leaves him zero surplus. The price $p^{i'}$ refers to the second stage. Note that $p^i > p^{i'}$ $\forall V_i > 0$ due to discounting. Let $\mathbf{p}^i = (p^i, p^{i'})$. In the full information case ($\lambda = 0, 1$) the seller would optimally charge $\mathbf{p}^i$. That is $\mathbf{p}^* = \mathbf{p}^H$ if $\lambda = 1$ and $\mathbf{p}^* = \mathbf{p}^L$ if $\lambda = 0$ (see Diamond, 1971). In the remainder of the paper we focus on the case of heterogeneous buyers, $\lambda \in (0, 1)$.

5.1 Negotiable Offers

Suppose a buyer has revealed his valuation i to a seller committed to negotiate. Then, the buyer makes a first-stage offer; if refused the seller makes a counter-offer with probability θ (the buyer makes it otherwise). The following holds.

Lemma 1. *Negotiations with buyer i lead to trade at price $p^i_{n,1} < p^i$ where*

$$p^i_{n,1} = \max\left\{\delta V, \delta V + \theta\delta\left(u_i - V_i - V\right)\right\}.$$

Proof. In appendix.

Negotiated trades are settled immediately due to discounting. The initial offer makes the seller indifferent to attempting a counter-offer. The sale is settled at the seller's reservation price, $p^i_{n,1} = \delta V$, if second-stage trade generates no surplus, $u_i - V_i \leq V$. Otherwise, the buyer increases the offer by a fraction θ of second-stage surplus, $p^i_{n,1} = \delta V + \delta\theta\left(u_i - V_i - V\right)$ hence $p^H_{n,1} > p^L_{n,1}$. Here, the lower the buyer's likelihood to make counter-offers, the higher the price. The need for ex-ante commitment is obvious, because the negotiated price is below the buyer's reservation price, $p^i_{n,1} < p^i$. If $u_i - V_i > V$, $p^i_{n,1}$ corresponds to the Nash bargaining solution where the threat points are the values of search, and θ is the seller's bargaining power.

5.2 Non-Negotiable Offers

Suppose buyer and seller play the unilateral offers trading game and the buyer's valuation is private information. Then, the buyer receives a first offer $p_{f,1}$ whose refusal leads to the seller's final offer $p_{f,2}$, reflecting updated beliefs on the buyer's valuation. The offers must be sequentially rational for $\mathbf{p}_f = (p_{f,1}, p_{f,2})$ to be a perfect Bayesian equilibrium.

To determine $\mathbf{p}_f$ we follow the procedure in Fudenberg and Tirole (1983). Following rejection of $p_{f,1}$, the offer $p_{f,2}$ must maximize the seller's second-stage expected payoff, given $p_{f,1}$ and expectations revised using Bayes' rule. Moving backward, $p_{f,1}$ must maximize the seller's first-stage expected payoff, given $p_{f,2}$. The optimal $\mathbf{p}_f$ depends on these key elements: the probability of meeting a low-value buyer, λ, the disparity in buyers' valuations, $\frac{u_L}{u_H}$, and the *endogenous* continuation values, V and V_i. There are three possible solutions to the seller's pricing problem.

Lemma 2. *Let $\lambda \in (0,1)$. The optimal non-negotiable offer vector is*

$$\mathbf{p}_f = \begin{cases} \mathbf{p}^H & \text{if} \quad u_L - V_L < V \\ \widehat{\mathbf{p}} & \text{if} \quad u_L - V_L \geq V \quad \text{and} \quad 1-\lambda > \frac{u_L}{u_H} \\ \mathbf{p}^L & \text{if} \quad u_L - V_L \geq V \quad \text{and} \quad 1-\lambda \leq \frac{u_L}{u_H} \end{cases} \tag{8}$$

where $\hat{\mathbf{p}} = (\hat{p}, p^{L'})$ and $\hat{p} = (1-\delta)\, u_H + \delta u_L - \delta V_L \in (p^L, p^H)$.

Proof. In Appendix.

There are two key results. First, buyers of low valuation never obtain surplus, so $V_L = 0$. Second, the optimal offer leaves *some* buyer type indifferent to making an immediate purchase. Thus, high valuation buyers might obtain some surplus. The seller sets prices such that buyer $i = H$ buys in the first stage. If $p_{f,1}$ is rejected, the seller's learns that the buyer is of low type. The optimal pricing rule depends on the seller's reservation value in the second stage that, as we will later see, depends also on the pricing strategies adopted by all other sellers.

To understand these results, one must realize that the seller has two distinct strategies. The first is reminiscent of 'price posting'. The seller targets a specific buyer offering $\mathbf{p}^i$ so that type i buys right away. By offering low prices $\mathbf{p}^L$, the sale is immediate but surplus is lost in high-value trades (hence $V_H > 0$). By offering high prices $\mathbf{p}^H$, the seller obtains the entire trade surplus (hence $V_H = 0$) but faces the possibility of a prolonged customer search.

The second strategy, $\hat{\mathbf{p}}$, is reminiscent of 'haggling'. Sellers trade with the first customer met, but the transaction may be delayed. A rejection of the initial high-price offer $\hat{p}$, triggers the price reduction $p^{L'}$ sufficient to entice a purchase from low-value buyers. The initial offer $\hat{p}$ leaves high-value types indifferent to waiting for the price reduction, so its rejection reveals the buyer's true (low) value. This screening device is feasible only if delayed low-value trades generate surplus.

The optimal pricing strategy hinges on distribution of types, but also on the continuation values V and V_i that, we stress, reflect market-prices expectations. These factors influence the seller's opportunity cost of targeting a single buyer type, instead of both, as follows. Offering $\mathbf{p}^H$ makes sense when low value consumption generates so little utility that the seller prefers to search repeatedly for high valuation buyers. Otherwise, a new round of search is never justified and the sale takes place at the first encounter. Risking a one-stage delay by, making an initial high-price sales pitch $\hat{p}$, is worthwhile in markets that are either dominated by high-value buyers or where some buyers like the good a lot. Otherwise, the seller will choose to sell the good at once by offering $\mathbf{p}^L$. In any case, low valuation buyers never earn surplus.

6 Existence of Equilibrium

Having analyzed optimal pricing, we study existence of equilibria starting with a benchmark case.

6.1 The Case of No Commitment

When the promise of negotiations is not credible, $\pi = 0$ since sellers prefer to make non-negotiable offers to buyers of known valuation, hence buyers do not reveal it. It follows

Proposition 1. *Absent commitment, the following are equilibria:*

$$\mathbf{p}^* = \mathbf{p}_f = \begin{cases} \mathbf{p}^H & \Leftrightarrow \quad \frac{u_L}{u_H} < \bar{u}(\lambda) \\ \widehat{\mathbf{p}} & \Leftrightarrow \quad \underline{u}(\lambda) \leq \frac{u_L}{u_H} < 1 - \lambda \\ \mathbf{p}^L & \Leftrightarrow \quad 1 - \lambda \leq \frac{u_L}{u_H} \end{cases} \tag{9}$$

where $0 < \underline{u}(\lambda) < \min\{1 - \lambda, \bar{u}(\lambda)\}$ *and* $\bar{u}(\lambda) < 1 - \lambda$ *when* $\lambda < \underline{\lambda}$.

Corollary 1. *Equilibria with high and low prices may coexist;* $\mathbf{p}^H$ *may coexist with* $\widehat{\mathbf{p}}$ *or* $\mathbf{p}^L$.

Proofs. See the Appendix, where $\bar{u}(\lambda)$, $\underline{u}(\lambda)$ and $\underline{\lambda}$ are also defined.■

The proposition confirms our earlier intuition. Low prices, $\mathbf{p}^L$, arise in markets where many buyers don't care much for the good sold, or where valuations are quite similar. It is otherwise optimal to raise prices to $\widehat{\mathbf{p}}$, to discriminate between buyers' types, or all the way to $\mathbf{p}^H$, targeting only high-value trades. Looking at the expressions $\bar{u}(\lambda)$ and $\underline{u}(\lambda)$, we see that sellers offer high prices on a wider range of the parameter space as agents become more patient, or as the proportion of high-valuation buyers increases. The reason is that sellers' option values V increase and so they will be more inclined to target high-value types.

The crucial finding is the possibility of multiple equilibria, illustrated in Figure 1 and explained as follows.[7] The lifetime utility of high value buyers, hence their reservation price, reflects the prices *expected* to prevail on the market (low market prices imply a low reservation price). There is what we call a *price setting externality,* as a seller's ability to trade at a given price is affected by the price selections made by others. Since pricing decisions are uncoordinated, different prices can prevail on otherwise identical markets. The reason is that traders' continuation values depend on the prices that are expected to prevail.

This opens the door to coordination failures in pricing selections, as is dramatically evident in markets where valuations are moderately different (see Figure 1). In this case $\mathbf{p}^H$ and $\mathbf{p}^L$ coexist but have very different efficiency levels (see later).[8]

6.2 The Case of Commitment

The equilibrium set is richer because bilateral negotiations can take place. It is easy to prove (see the Appendix) that in equilibrium (3) holds. Obviously, low-value buyers prefer to reveal their type in exchange for the ability to make an initial offer. Thus, should a buyer choose not to negotiate, the seller would optimally offer $\mathbf{p}_f = \mathbf{p}^H$. Since $p^H_{n,1} < p^H$, then high value buyers would prefer to engage in bilateral negotiations. We prove the following

Proposition 2. *With commitment, an equilibrium with bilateral negotiations where* $p^i_{n,1} = \delta V + \delta\theta\,(u_i - V_i - V)$ *exists for intermediate values* $\frac{u_L}{u_H}$*, and sufficiently large* δ *and* θ*. Otherwise, only equilibria with non-negotiable offers* $\mathbf{p}_f$ *exist.*

Corollary 2. *Equilibria with and without negotiated offers can coexist.*

Proofs. See the Appendix.■

Commitment enriches the equilibrium set, since sellers *might* have an incentive to share the surplus with buyers who reveal their valuation. In this case *both* traders earn surplus, as $\delta V <$

[7] Technically, when $\underline{u}(\lambda) \leq \frac{u_L}{u_H} < \bar{u}(\lambda)$, then $\mathbf{p}^H$ and $\widehat{\mathbf{p}}$ coexist if $\lambda < \underline{\lambda}$, while $\mathbf{p}^H$ and $\mathbf{p}^L$ coexist if $\lambda \geq \underline{\lambda}$ and $\frac{u_L}{u_H} > 1 - \lambda$. When $\bar{u}(\lambda) \leq \frac{u_L}{u_H}$ then $\widehat{\mathbf{p}}$ is unique if $\lambda < \underline{\lambda}$ and $\bar{u}(\lambda) \leq \frac{u_L}{u_H} < 1 - \lambda$, while $\mathbf{p}^L$ is unique if $\lambda \geq \underline{\lambda}$. Note that $\widehat{\mathbf{p}}$ cannot coexist with $\mathbf{p}^L$ since offering $\widehat{\mathbf{p}}$ makes sense only under minimal risk of trade delays (small λ), the opposite of what is required for $\mathbf{p}^L$ to arise. The numerical illustrations are for $\delta = 0.95$, $\sigma = 0.6$, and $\alpha = 0.7$.

[8] If agents remained in the market indefinitely, trading repeatedly, they would never change state and their reservation prices would not depend on continuation payoffs. Here pricing and trading decisions would be independent of the agent's continuation value, and would only hinge on his period utility. Hence, strategic complementarities are absent, and multiple equilibria are impossible when there are no flows of traders in and out of the market.

$p^i_{n,1} < p^i$. The buyer gets surplus since he makes the first offer, while the seller gets more than his reservation price, even in low-value matches, or otherwise he would turn to offering $\mathbf{p}^H$.

The need for intermediate $\frac{u_L}{u_H}$ reflects our prior discussion of haggling by setting $\mathbf{p}^* = \mathbf{p}_f = \widehat{\mathbf{p}}$. The seller has an incentive to discriminate across buyers, by means of negotiations, only if there is sufficient heterogeneity in buyers' valuations. We emphasize that the bounds on $\frac{u_L}{u_H}$ depend on λ, as in the case of non-negotiable offers (see appendix). The requirement for large δ and θ is also quite intuitive. Greater patience or a greater facility at making counteroffers give the seller a stronger 'bargaining position.' Therefore, the buyer's initial offer rises in δ and θ. This gives the seller a greater incentive to negotiate, as he can acquire information about the buyer's type, quite cheaply (Figure 2 illustrates equilibria when $\theta = 0.8$).

Because reservation values are endogenous and reflect expectations of market prices, equilibrium multiplicity may arise. It is of particular interest to note that each of the two trading mechanisms may emerge in equilibrium. Indeed, it is 'as if,' haggling takes place, as different buyers pay different prices. The reason is that if sellers intend to price discriminate, extracting a higher price from high-value customers, they can overcome private information obstacles in one of two ways. Sellers can elicit information *indirectly*, as part of a trading process where the buyer responds to a sequence of declining offers. Here the seller relinquishes surplus only to high-value buyers, but trading delays are possible. Alternatively, the seller can elicit information *directly* from buyers, compensating them for it, by letting them make the initial offer, and possibly a counter-offer. Here prices increase in the buyer's valuation, delays are avoided, but surplus is shared in every trade.

From an efficiency perspective, eliciting information directly from the buyer appears to be preferable. This eliminates the incidence of trading delays, so that no surplus is dissipated. However the equilibrium course of action hinges on market expectations. If prices are expected to be above $p^{L'}$, for example, sellers have no incentive to reward buyers for information, and no one will negotiate. Thus, one question remains. How do equilibria compare in terms of social efficiency?

6.3 Efficiency

Let average welfare $W(\mathbf{p})$ measure efficiency. Given the equilibrium price $\mathbf{p}^* = \mathbf{p}$

$$W(\mathbf{p}) = \lambda V_L(\mathbf{p}) + (1-\lambda)V_H(\mathbf{p}) + V(\mathbf{p}).$$

Two key components influence $W(\mathbf{p})$. The first is the relative ease of trade for sellers and buyers. In a seller's market, $\sigma > \alpha$, sellers trade more frequently than buyers hence the efficiency criterion W

is 'biased' in favor of sellers. The opposite holds in a buyer's market, $\sigma < \alpha$. Valuation differentials also matter in ranking outcomes. For instance, selling to those who value minimally the good makes little sense, in terms of average welfare, when there are many buyers who like the good a lot.

To disentangle these two separate components affecting W, we first study the case where traders face identical matching probabilities, $\sigma = \alpha$ (details are in the appendix). Here

$$\begin{array}{ll} W(\mathbf{p}^H) < W(\hat{\mathbf{p}}) < W(\mathbf{p}_n) = W(\mathbf{p}^L) & \text{if } \frac{u_L}{u_H} \text{ is large} \\ W(\hat{\mathbf{p}}) < W(\mathbf{p}^H) < W(\mathbf{p}_n) = W(\mathbf{p}^L) & \text{if } \frac{u_L}{u_H} \text{ is moderate} \\ W(\hat{\mathbf{p}}) < W(\mathbf{p}_n) = W(\mathbf{p}^L) < W(\mathbf{p}^H) & \text{if } \frac{u_L}{u_H} \text{ is small} \end{array}$$

Mechanisms eliciting immediate purchases by everyone, $\mathbf{p}^L$ or $\mathbf{p}_n$, are equivalent as they generate identical surplus (the way it is shared does not affect average welfare). They are socially preferred if disparity in valuations is small, as higher prices would only dissipate surplus either form trade delays (as when $\mathbf{p}$=$\hat{\mathbf{p}}$) or no trade (as when $\mathbf{p} = \mathbf{p}^H$). W cannot be a maximum when $\mathbf{p} = \hat{\mathbf{p}}$ as this creates *additional* market frictions, in the form of trading delays. Thus, it is dominated by either $\mathbf{p}_n$ or $\mathbf{p}^L$. However, $\mathbf{p}^H$ maximizes average welfare when low-value trades generate little surplus, as goods should go only to high-value buyers, in such a market.

When σ and α differ by small amounts, the ranking of outcomes is generally similar (see the Appendix). A key difference is that, if high and low valuations are not far apart, $\mathbf{p}^L$ is socially preferred in a sellers' market, but $\mathbf{p}_n$ is preferred in a buyer's market. Intuitively, when buyers get to do a lot of trading, relative to sellers, every buyer should earn some surplus, which can be accomplished through negotiations. In a seller's market, the reverse is true.

We conclude that the possibility to exploit commitment in order to carry out bilateral negotiations, is not necessarily optimal. However, it can be beneficial in one of two ways. In markets where sellers want to discriminate among heterogeneous buyers, the possibility to commit to bilateral negotiations has the potential to eliminate wasteful trading delays, (since $\mathbf{p} = \mathbf{p}_n$ may coexists with $\mathbf{p}_f = \hat{\mathbf{p}}$, while $\mathbf{p}_f = \mathbf{p}^L$ does not coexist with $\mathbf{p}_f = \hat{\mathbf{p}}$). Furthermore, in a buyers' market the possibility to commit to negotiations can help allocate the gains from trade more efficiently, compared to trading mechanisms based on non-negotiable offers.

7 Conclusion

We have endogenized the trading mechanism in an economy with random short-lived matches between heterogeneous buyers and homogeneous sellers. We have studied how heterogeneity and the traders' continuation values–which are endogenous–influence the sellers' choice of trade mechanism.

Sellers trade off the probability of an immediate sale against the surplus expected from it, choosing whether to trade with everyone and how quickly. In equilibrium sellers may simply target one buyer type via non-negotiable offers (price posting), or may price discriminate (haggling). When sellers expect 'small' gains from sales to some buyer type, they target only the other type, via a non-negotiable price that extracts his *entire* surplus. This market resembles one in which sellers 'post prices.' Else, the seller will trade with both buyer types, but at different prices–as if they 'haggled.' This can be done by making a sequence of non-negotiable offers or–to avoid trade delays–by committing to sharing the surplus with buyers who reveal their valuation.

A price setting externality arises because of a strategic complementarity in the sellers' pricing choices. Since individual pricing selections must take into account option values that reflect the prices expected to prevail on the market, equilibrium multiplicity and coordination failures may result.

Extensions could include mixed strategies, directed search, or more general trading mechanisms, to study links between price dispersion and trade mechanism heterogeneity.

Appendix

Proof of Lemma 1

Consider a bilateral bargaining match with a buyer of known valuation i. Suppose the offers generate surplus in the second stage of the match, $u_i - V_i - V \geq 0$. In this case the seller is expected to participate in a second stage, should he refuse the initial offer. Then the optimal first stage offer made by the buyer, $p^i_{n,1}$, must solve

$$p^i_{n,1} - \delta V = \delta \left[\theta \left(p^{i'} - V \right) + (1-\theta)(p^i_{n,2} - V) \right].$$

Note that $p^{i'}$ is the seller's optimal counter-offer in the second stage. It takes away the buyer's surplus. In the second stage the buyer will offer $p^i_{n,2} = V$, i.e. the seller's reservation value. The solution is $p^i_{n,1} = \delta V + \delta\theta \left(u_i - V_i - V \right).$

Now suppose surplus cannot be generated in the second stage of the match, $u_i - V_i - V \leq 0$. Then clearly $p^i_{n,1} = \delta V$. Since $p^i_{n,1} \geq \delta V$, the seller always trades. The worse case scenario for a buyer is $\theta = 1$, when he cannot make a counter-offer. Here $p^i_{n,1} = \delta u_i - \delta V_i < u_i - \delta V_i$. This implies that $p^i_{n,1} < p^i$, so buyer i has also some surplus when he negotiates. Thus both buyer and seller are willing to trade at price $p^i_{n,1}$. When negotiations take place

$$\delta V_i = \begin{cases} \frac{\alpha(1-\theta)\left[\frac{(1-\delta\theta)u_i}{1-\theta} - \delta V\right]}{r+\alpha(1-\theta)} & \text{if } p^i_{n,1} > \delta V \\ \frac{\alpha}{r+\alpha}\left(u_i - \delta V\right) & \text{if } p^i_{n,1} = \delta V \end{cases} \tag{10}$$

Notice that $\delta\left(r+\alpha\right) > \alpha$ always (note that $r\delta = (1-\delta^2)/\delta$). Since $V \geq 0$, then $u_H - V_H > u_L - V_L > 0$. This implies that $p^H_{n,1} > p^L_{n,1}$.

The lifetime utility to a seller, under negotiations is $V = V_n$ where

$$V_n = \begin{cases} \frac{\theta\sigma}{r+\theta\sigma}\left[\lambda\left(u_L - V_L\right) + (1-\lambda)\left(u_H - V_H\right)\right] & \text{if} \quad p^i_{n,1} > \delta V \ \forall i \\ \frac{\theta\sigma(1-\lambda)}{r+\theta\sigma(1-\lambda)}\left(u_H - V_H\right) & \text{if} \quad p^H_{n,1} > p^L_{n,1} = \delta V \end{cases} \tag{11}$$

Here $u_H - V_H > V$ when $p^H_{n,1} > p^L_{n,1} = \delta V$ and $p^i_{n,1} > \delta V \ \forall i$. Notice that (i) $u_H - V_H > \frac{\theta\sigma(1-\lambda)}{r+\theta\sigma(1-\lambda)}$ and (ii) $u_H - V_H > \frac{\theta\sigma}{r+\theta\sigma}\left[\lambda\left(u_L - V_L\right) + (1-\lambda)\left(u_H - V_H\right)\right]$ since $u_H - V_H > u_L - V_L$. ■

Proof of Lemma 2

With no loss in generality let $p_{f,1} = p_1$ and $p_{f,2} = p_2$. Let $0 < \lambda < 1$. Conjecture that

$$u_H - \delta V_H \geq u_L - \delta V_L > \delta V \tag{12}$$

holds.[9] Because we are focusing only on pure strategies, and because future utility is discounted, we also conjecture that if in equilibrium the seller targets high-valuation buyers, then p_1 is chosen such that $\beta^H_1 = 1$, i.e. a high-valuation buyer trades with certainty in the first stage.[10] We verify these conjectures below.

Suppose p_1 has been rejected. Clearly it is not optimal for the seller to offer a p_2 that leaves surplus to *every* buyer type, hence $p_2 \in \left\{p^{L'}, p^{H'}\right\}$.

[9] The other cases do not generate asymmetric information, in equilibrium. If $\max\left\{u_L - \delta V_L, u_H - \delta V_H\right\} < \delta V$ no trade ever takes place as no match creates surplus. If surplus exists only in one type of match, only one buyer type buys, doing so in the first stage since the seller knows exactly what type to target, i.e. $\mathbf{p} = \mathbf{p}^i$. The reason is simple. In the stage game the price has to be greater than δV (from (4)). If $u_L - \delta V_L < \delta V < u_H - \delta V_H$ then $p_2 = p^{H'}$ and $p_1 = p^H$. In this case, type H accepts p_1, while type L never accepts. For a similar reason type L accepts the offer $p_1 = p^L$, while type H never accepts if $u_H - \delta V_H < \delta V < u_L - \delta V_L$.

[10] Given that the seller targets high-valuation buyers, it is clearly suboptimal to pick p_1 such that the high valuation buyer refuses the first-stage offer, but accepts the second stage offer.

Suppose $p_2 = p^{L'}$. Clearly, only $p_2 \leq p^{i'}$ is accepted by i. Now think about p_1. Only $p_1 \leq p^L$ is accepted in the first stage by buyer L. Hence, the key is to understand what an H buyer will do in the first stage. If $p_1 = p^L$, then both types accept the first-stage offer. The problem is that the high-valuation buyer earns surplus, at the seller's expense.

Thus, suppose the seller selects $p_1 > p^L$. If he is facing buyer L, he rejects it. Given this the seller cannot be certain if it is a low- or a high-valuation buyer. The reason is that the H buyer might also prefer to refuse p_1 if p_2 is more advantageous. Thus, the seller updates his belief that the buyer is L, call this updated probability λ', using Bayes' rule:

$$\lambda' = \frac{\Pr[p_1 \text{ refused}|i=L]\cdot\Pr[i=L]}{\Pr[p_1 \text{ refused}|i=L]\cdot\Pr[i=L]+\Pr[p_1 \text{ refused}|i=H]\cdot\Pr[i=H]} = \frac{\lambda}{1-(1-\lambda)\beta_1^H} \geq \lambda$$

Obviously, $\lambda' - \lambda > 0$ since a rejection has occurred.[11]

What should the seller do? Recall that $p_2 = p^{L'}$ or $p_2 = p^{H'}$ are the two possible choices. Thus, suppose she offers $p^{H'}$, to leave the high valuation buyer indifferent. The associated second-stage *undiscounted* seller's expected payoff is then $\lambda' V + \left(1 - \lambda'\right) p^{H'}$. Thus, there are two cases to consider based on whether $p^{L'} > \lambda' V + \left(1 - \lambda'\right) p^{H'}$ holds or not.

Given our conjecture that in equilibrium p_1 is chosen to induce $\beta_H = 1$, then $\lambda' = 1$. That is, if p_1 is refused the seller knows for sure that the buyer is a low type. Thus, $\lambda' V + \left(1 - \lambda'\right) p^{H'} = V$, the seller's her reservation value. It is obvious that offering $p_2 = p^{L'}$ is better than choosing $p^{H'}$ iff $p^{L'} \geq V$ as in this case $p_2 = p^{L'}$ gives the seller a larger payoff. Otherwise, the seller optimally offers $p^{H'}$. We discuss the two cases below.

1. $p^{L'} \geq V \Rightarrow u_L - V_L \geq V$

   Here the seller prefers to offer $p_2 = p^{L'}$. Moving backward, let $\widehat{p}$ denote the highest p_1 that is accepted by type H, when he expects $p_2 = p^{L'}$. Thus, $\hat{p}$ must satisfy $u_H - \widehat{p} - \delta V_H = \delta(u_H - p^{L'} - V_H)$, implying $\widehat{p} = u_H - \delta(u_H - u_L + V_L)$. It is evident that $\widehat{p} > p^L$.

   What is then the optimal p_1? There can be two cases: $p_1 = p^L$, accepted by every buyer, or

[11] Note that we use $\Pr[i = L] = \lambda$ without loss in generality. This would not be possible if buyer's i willingness to enter the trading game, say the probability $\eta_i(\mathbf{p})$, hinged on the offers $\mathbf{p}$ expected in the match. This could be the case if buyers could search once more during the period, or if there were an implicit cost to entering the trading match, for instance. In these cases the seller's 'prior' that the buyer is of low type, call it Λ, must be consistent with the (known) distribution of types and the buyers' equilibrium strategies. Using Bayes' rule $\Pr[i = L] \equiv \Lambda = \frac{\lambda\eta_L(\mathbf{p})}{\lambda\eta_L\mathbf{p}+(1-\lambda)\eta_H(\mathbf{p})}$. Therefore $\Lambda = \lambda$ in our case, since $\eta_i(\mathbf{p}) = 1$ $\forall i$.

$p_1 = \widehat{p}$, accepted only by buyer H. In this case the seller's beginning-of-trade payoffs are:

$$s(\mathbf{p}) = \begin{cases} p^L & = u_L - \delta V_L & \text{if} \quad \mathbf{p} = \mathbf{p}^L \\ (1-\lambda)\,\widehat{p} + \lambda\delta p^{L'} & = \delta u_L + (1-\lambda)\,(1-\delta)\,u_H - \delta V_L & \text{if} \quad \mathbf{p} = \widehat{\mathbf{p}} \end{cases} \tag{13}$$

Comparing the payoffs from the two strategies, $\mathbf{p} = \widehat{\mathbf{p}}$ if $(1-\lambda)\,u_H > u_L$, and $\mathbf{p} = \mathbf{p}^L$ otherwise.

2. $p^{L'} < V \Rightarrow u_L - V_L < V$

   Here the seller prefers to offer $p_2 = p^{H'}$ because selling to a low type would generate a surplus loss. What should be p_1? Focus on $p_1 > p^L$, since $p_1 = p^L$ implies there is never a second stage (all buyers accept it). The seller should *not* offer $p^L < p_1 < \widehat{p}$, as this leaves buyer H some surplus (reason: if $p_1 = \widehat{p}$ makes buyer H indifferent when $p_2 = p^{L'}$, then he will strictly prefer it when $p_2 > p^{L'}$). But then, if it is optimal to target the buyer H in the second stage, it must be optimal to target him in the first stage, i.e. $p_1 = p^H$.[12] The buyer accepts it equilibrium (the seller can lower the price a bit, otherwise). Hence $\mathbf{p} = \mathbf{p}^H$ is optimal and

$$s(\mathbf{p^H}) = \lambda\delta V + (1-\lambda)\,p^H = \lambda\delta V + (1-\lambda)\,(u_H - \delta V_H) \tag{14}$$

From the discussion above it follows that

$$\begin{aligned} b_L(\mathbf{p}) = & \begin{cases} u_L - p^L & = \delta V_L & \text{if} \quad \mathbf{p} = \mathbf{p}^L \\ \delta(u_L - p^{L'}) & = \delta V_L & \text{if} \quad \mathbf{p} = \widehat{\mathbf{p}} \\ \delta V_L & & \text{if} \quad \mathbf{p} = \mathbf{p}^H \end{cases} \\ b_H(\mathbf{p}) = & \begin{cases} u_H - p^L & = u_H - u_L + \delta V_L & \text{if} \quad \mathbf{p} = \mathbf{p}^L \\ u_H - \widehat{p} & = \delta(u_H - u_L) + \delta V_L & \text{if} \quad \mathbf{p} = \widehat{\mathbf{p}} \\ u_H - p^H & = \delta V_H & \text{if} \quad \mathbf{p} = \mathbf{p}^H \end{cases} \end{aligned} \tag{15}$$

are the buyer's net period utilities in each possible equilibrium. Notice that (13)-(15) verify the validity of (12), and that the seller always offers a p_1 such that $\beta_H = 1$, as conjectured.■

Proof of Proposition 1 and Corollary 1

[12] Reason: we have seen that a rejection of p_1 implies an *increase* in the probability the buyer is L. If it is optimal to target the high buyer in the second stage, despite it being now *less* likely that the partner is H, it must be optimal to target H in the first stage, when it is *more* likley the buyer is H.

Here $\pi^* = 0$ hence $V = V_f$. Consider an equilibrium $\mathbf{p}^* = \mathbf{p}_f$, calculate the value functions, and check (2) and (8) (all other conditions are satisfied). Using $\mathbf{p}^i$, $s(\mathbf{p})$ and $b_i(\mathbf{p})$, then $V_L = 0$ and

$$\delta V_H = \begin{cases} \frac{\alpha(u_H - u_L)}{r+\alpha} \\ \delta\frac{\alpha(u_H-u_L)}{r+\alpha} \\ 0 \end{cases} \quad and \quad \delta V = \begin{cases} \frac{\sigma u_L}{r+\sigma} & \text{if } \mathbf{p}_f = \mathbf{p}^L \\ \frac{\sigma}{r+\sigma}\left[\delta u_L + (1-\lambda)(1-\delta)\, u_H\right] & \text{if } \mathbf{p}_f = \widehat{\mathbf{p}} \\ \frac{\sigma(1-\lambda)u_H}{r+\sigma(1-\lambda)} & \text{if } \mathbf{p}_f = \mathbf{p}^H \end{cases} \tag{16}$$

Consider each possible equilibrium $\mathbf{p}^H$, $\widehat{\mathbf{p}}$ and $\mathbf{p}^L$, separately.

Let $\mathbf{p}_f = \mathbf{p}^H$. From (8), $\mathbf{p}^H$ is individually optimal if $u_L - V_L < V$. This amounts to

$$\frac{u_L}{u_H} < \bar{u}(\lambda) \equiv \frac{\sigma(1-\lambda)}{r\delta + \delta\sigma(1-\lambda)}.$$

Here (i) $\bar{u}(\lambda)$ is decreasing in λ, $0 < \bar{u}(\lambda) < \frac{\sigma}{\delta(r+\sigma)}$ and $\lim_{\lambda\to 1} \bar{u}(\lambda) = 0$; (ii) $\bar{u}(\lambda)$ decreases in r with $\lim_{r\to 0} \bar{u}(\lambda) = 1$, since $r\delta \to 0$ as $r \to 0$; (iii) $\bar{u}(\lambda) > 1-\lambda$ if $\lambda > \underline{\lambda} = \frac{r+\sigma}{\sigma} - \frac{1}{\delta}$.

Let $\mathbf{p}_f = \widehat{\mathbf{p}}$. From (8), $\widehat{\mathbf{p}}$ is individually optimal if $u_L - V_L \geq V$ and $1-\lambda > \frac{u_L}{u_H}$. These imply

$$\underline{u}(\lambda) \equiv \frac{\sigma(1-\delta)(1-\lambda)}{r\delta} \leq \frac{u_L}{u_H} < 1-\lambda$$

where $\underline{u}(\lambda)$ is decreasing in λ, $0 < \bar{u}(\lambda) < \frac{\sigma(1-\delta)}{r\delta}$ and $\lim_{\lambda\to 1} \bar{u}(\lambda) = 0$. Note $\underline{u}(\lambda) < 1-\lambda$ iff $\sigma < \frac{r\delta}{1-\delta} = \frac{1+\delta}{\delta}$, always satisfied since $\frac{1+\delta}{\delta} > 1$ and $\sigma < 1$. Also, $\bar{u}(\lambda) > \underline{u}(\lambda)$, since $r\delta^2 = 1-\delta^2$, and $\bar{u}(\lambda) < 1-\lambda$ if $\lambda < \underline{\lambda}$. Thus $\mathbf{p}_f = \mathbf{p}^H$ and $\mathbf{p}_f = \widehat{\mathbf{p}}$ coexist if $\underline{u}(\lambda) \leq \frac{u_L}{u_H} \leq \bar{u}(\lambda)$ and $\lambda < \underline{\lambda}$.

Let $\mathbf{p}_f = \mathbf{p}^L$. From (8), $\mathbf{p}^L$ is individually optimal if $1-\lambda \leq \frac{u_L}{u_H}$ and $u_L - V_L \geq V$, rearranged as $\sigma < \frac{r\delta}{1-\delta} = \frac{1+\delta}{\delta}$, always holding. Thus, we need $\frac{u_L}{u_H} \geq 1-\lambda$. This implies $\mathbf{p}_f = \mathbf{p}^L$ and $\mathbf{p}_f = \widehat{\mathbf{p}}$ cannot coexist. Since $\bar{u}(\lambda) \geq 1-\lambda$ when $\lambda \geq \underline{\lambda}$, then $\mathbf{p}_f = \mathbf{p}^L$ and $\mathbf{p}_f = \mathbf{p}^H$ coexist when $\lambda \geq \underline{\lambda}$.■

Proof of Proposition 2 and Corollary 2

We first prove (3) holds, given commitment. From (8), $p_{f,1} \geq p^L$ and $p_{f,2} \geq p^{L'}$. Since $p^L_{n,1} < p^L$, (3) holds strictly for $i = L$. Suppose (3) does not hold for $i = H$. Using the definition of $b^i(\mathbf{p})$, it must be that $p^H_{n,1} > \min\{p_{f,1}, \delta p_{f,2} + u_H(1-\delta)\}$. This is impossible: a refusal to negotiate tells the seller that the buyer's type is $i = H$. Then, the seller would optimally set $\mathbf{p}_f = \mathbf{p}^H$, a contradiction.

If $\pi^* = 0$ the value functions must satisfy $V_L = 0$ and (16).

If $\pi^* = 1$, then (11) implies $p^H_{n,1} > p^L_{n,1} = \delta V$ only if $u_L - V_L \leq V$. Using (10)-(11) this is

$$\frac{u_L}{u_H} \leq \underline{\underline{u}}(\lambda) = \frac{\theta\sigma(1-\lambda)}{r + \alpha(1-\theta) + \theta\sigma(1-\lambda)}.$$

By continuity, if $\frac{u_L}{u_H} > \underline{\underline{u}}(\lambda)$ then $u_L - V_L > V$ so that $p^i_{n,1} > \delta V$ $\forall i$. Notice $0 < \underline{\underline{u}}(\lambda) < 1$ and $\underline{\underline{u}}(\lambda) > 1 - \lambda$. if $\lambda > \bar{\lambda} = \frac{r+\alpha(1-\theta)}{\theta\sigma}$, where $\bar{\lambda} < 1$ if $\theta > \frac{r+\alpha}{\sigma+\alpha}$ and $r < \sigma$.

Given $p^i_{n,1} = \max\{\delta V, \delta V + \delta\theta(u_i - V_i - V)\}$ and $p^H_{n,1} > p^L_{n,1}$ we must consider three cases: 1) $V \geq u_H - V_H > u_L - V_L$ in which case $p^i_{n,1} = \delta V$ $\forall i$; 2) $u_H - V_H > V \geq u_L - V_L$, in which case $p^L_{n,1} = \delta V < p^H_{n,1}$, and 3) $u_i - V_i > V$, in which case $p^i_{n,1} > \delta V$ $\forall i$.

$p^i_{n,1} = \delta V$ $\forall i$, cannot be an equilibrium: $\pi = 0$ since $s(\mathbf{p}_n) = \delta V$ implies $V_n = 0$. $p^L_{n,1} = \delta V < p^H_{n,1}$, cannot be an equilibrium either. It requires $u_L - V_L \leq V$, thus the seller would set $\pi = 0$ and $\mathbf{p}_f = \mathbf{p}^H$ (see Lemma 2) since (i) the seller receives no surplus in negotiations with a buyer $i = L$ as $p^L_{n,1} = \delta V$ and (ii) the seller earns less by choosing $\pi = 1$ since $p^H_{n,1} = \delta V + \delta\theta(u_H - V_H - V) < p^H = u_H - \delta V_H$ (from Lemma 1, $u_H - V_H > V$).

Only $p^i_{n,1} = \delta V + \delta\theta(u_i - V_i - V)$ can be an equilibrium. Here $V = V_n$ where

$$\delta V_n = \frac{\theta\sigma[\delta(r+\alpha) - \alpha]}{r[r + \alpha(1-\theta) + \theta\sigma]}[\lambda u_L + (1-\lambda)u_H].$$

To prove $\pi = 1$ is individually optimal, we must verify that, given $\mathbf{p}^* = \mathbf{p}_n$, then $s(\mathbf{p}_n) \geq s(\mathbf{p}_f)$.

Consider only $\mathbf{p}_f = \mathbf{p}^L, \hat{\mathbf{p}}$ since $\mathbf{p}_f = \mathbf{p}^H$ is not a possible deviation when $\mathbf{p}^* = \mathbf{p}_n$. The reason is $p^i_{n,1} > \delta V$ requires $u_L - V_L > V$, hence $\mathbf{p}_f = \mathbf{p}^H$ is suboptimal. Thus compare

$$\begin{aligned} s(\mathbf{p}_n) = &\quad \delta V(1-\theta) + \delta\theta[\lambda(u_L - V_L) + (1-\lambda)(u_H - V_H)] \\ s(\mathbf{p}^L) = &\quad u_L - \delta V_L \\ s(\hat{\mathbf{p}}) = &\quad \delta u_L + (1-\lambda)(1-\delta)u_H - \delta V_L \end{aligned} \tag{17}$$

where $V_L > 0$ since $\mathbf{p}^* = \mathbf{p}_n$. Specifically, from (10), $\delta V_i = \frac{\alpha(1-\theta)\left[\frac{(1-\delta\theta)u_i}{1-\theta} - \delta V\right]}{r+\alpha(1-\theta)}$.

If $\theta = 0$ then $s(\mathbf{p}_n) < s(\mathbf{p}_f)$, since $p^i_{n,1} = \delta V$ $\forall i \Rightarrow V_n = 0$. One can provide expressions for $\frac{u_L}{u_H}$ that satisfy $s(\mathbf{p}_n) = s(\mathbf{p}_f)$. These expressions are cumbersome, so we follow an alternative route. Since $\frac{\partial V_n}{\partial\theta} > 0$ and $\frac{\partial V_L}{\partial\theta} < 0$, then $\frac{\partial s(\mathbf{p}_n)}{\partial\theta} > 0$ and $\frac{\partial s(\mathbf{p}_f)}{\partial\theta} > 0$. Therefore consider the case $\theta = 1$. Using continuity in θ and δ, we prove existence of $\mathbf{p}^* = \mathbf{p}_n$ using the intermediate value theorem.

If $\theta = 1$, use V_i and V from (17) to get $s(\mathbf{p}_n)|_{\theta=1}$ and $s(\mathbf{p}_f)|_{\theta=1}$ for the mutually exclusive cases $\mathbf{p}_f = \mathbf{p}^L$ and $\mathbf{p}_f = \hat{\mathbf{p}}$.

1) $\mathbf{p}_f = \mathbf{p}^L$ is a possible deviation if $u_L - V_L \geq V$ and $1 - \lambda \leq \frac{u_L}{u_H}$ (from Lemma 2). Then $s(\mathbf{p}_n)|_{\theta=1} \geq s(\mathbf{p}^L)|_{\theta=1}$ if $\frac{u_L}{u_H} \leq u_L(\lambda) = \frac{\alpha(1-\lambda)(1-\delta)+r(1-\lambda)\delta}{\alpha(1-\lambda)(1-\delta)+r(1-\lambda\delta)} < 1$. Since $\frac{u_L}{u_H} > \underline{\underline{u}}(\lambda)$ is necessary for $\mathbf{p}^* = \mathbf{p}_n$, then $\pi = 1$ is individually optimal when $\underline{\underline{u}}(\lambda) < \frac{u_L}{u_H} < u_L(\lambda)$ (from Lemma 2). Since

prices are linear in θ, $\frac{\partial s(\mathbf{p}_n)}{\partial\theta} > 0$ and $\frac{\partial s(\mathbf{p}_f)}{\partial\theta} > 0$, and both functions are continuous in θ, u_L and u_H, then by the intermediate value theorem there exists a $0 < \theta^* < 1$ and a $\frac{u_L}{u_H} = u^*(\lambda) \in (0,1)$ such that $s(\mathbf{p}_n) \geq s(\mathbf{p}^L)$ for all $\theta > \theta^*$ and $\underline{\underline{u}}(\lambda) < \frac{u_L}{u_H} \leq u^*(\lambda)$.

2) $\mathbf{p}_f = \hat{\mathbf{p}}$ is a possible deviation if $u_L - V_L \geq V$ and $1-\lambda > \frac{u_L}{u_H}$. It is easy to show that for $\theta = 1$ then $s(\mathbf{p}_n) \geq s(\hat{\mathbf{p}})$ if $\frac{u_L}{u_H} \leq u_{LL}(\lambda) = \frac{\delta r - (1-\delta)(r+\alpha)}{\delta r - (1-\delta)\alpha}$. It is obvious that $u_{LL}(\lambda) > 0$ only if δ is sufficiently close to one, and it is negative otherwise. Once again, by the intermediate value theorem we conclude that there exist θ and δ sufficiently large and an intermediate $\frac{u_L}{u_H}$ such that $s(\mathbf{p}_n) \geq s(\hat{\mathbf{p}})$ hence $\pi = 1$ is individually optimal.

Coexistence. Suppose $\mathbf{p}^* = \mathbf{p}_n$. From Proposition 1, $\underline{\underline{u}}(\lambda) < \bar{u}(\lambda)$ and $\underline{\underline{u}}(\lambda) \leq \underline{u}(\lambda)$ if $\theta \leq \bar{\theta} = \frac{\delta(r+\alpha)}{1+\delta(1+\alpha)} < 1$. Hence, if $\theta \leq \bar{\theta}$ then $\underline{\underline{u}}(\lambda) \leq \underline{u}(\lambda) < \bar{u}(\lambda)$ so that $\mathbf{p}^* = \mathbf{p}_n$ may coexist with $\mathbf{p}^* = \mathbf{p}_f$ where $\mathbf{p}_f = \mathbf{p}^H, \hat{\mathbf{p}}$. If $\theta > \bar{\theta}$ then (i) $\underline{u}(\lambda) < \underline{\underline{u}}(\lambda) \leq 1-\lambda$ if $\lambda \leq \bar{\lambda}$ so that $\mathbf{p}^* = \mathbf{p}_n$ may coexist with $\mathbf{p}^* = \mathbf{p}_f$ where $\mathbf{p}_f = \mathbf{p}^H, \hat{\mathbf{p}}$ and (ii) $\underline{u}(\lambda) < 1-\lambda < \underline{\underline{u}}(\lambda)$ if $\lambda > \bar{\lambda}$, so that $\mathbf{p}^* = \mathbf{p}_n$ may coexist with $\mathbf{p}^* = \mathbf{p}_f$ where $\mathbf{p}_f = \mathbf{p}^H, \mathbf{p}^L$.■

Welfare

Given the definition of welfare

$$\begin{aligned}
\delta W(\mathbf{p}^H) &= \frac{\sigma(1-\lambda)}{r+\sigma(1-\lambda)} u_H \\
\delta W(\hat{\mathbf{p}}) &= \frac{\sigma}{r+\sigma}[\delta\lambda u_L + (1-\lambda)u_H] - \frac{(1-\lambda)\delta(u_H-u_L)r(\sigma-\alpha)}{(r+\alpha)(r+\sigma)} \\
\delta W(\mathbf{p}_n) &= [\lambda u_L + (1-\lambda)u_H]\frac{\alpha+\theta\delta(\sigma-\alpha)}{r+\alpha(1-\theta)+\theta\sigma} \\
\delta W(\mathbf{p}^L) &= [\lambda u_L + (1-\lambda)u_H]\frac{\alpha}{r+\alpha} + u_L\frac{r(\sigma-\alpha)}{(r+\alpha)(r+\sigma)}
\end{aligned}$$

Let $\alpha = \sigma$. Then $W(\hat{\mathbf{p}}) < W(\mathbf{p}_n) = W(\mathbf{p}^L)$. Also, $W(\mathbf{p}^H) > W(\mathbf{p}^L)$ when $\frac{u_L}{u_H} < \frac{\alpha(1-\lambda)}{r+\alpha(1-\lambda)}$, while $W(\mathbf{p}^H) < W(\hat{\mathbf{p}})$ if $\frac{u_L}{u_H} > \frac{\sigma(1-\lambda)}{\delta[r+\sigma(1-\lambda)]}$. Notice that $\frac{\sigma(1-\lambda)}{\delta[r+\sigma(1-\lambda)]} > \frac{\alpha(1-\lambda)}{r+\alpha(1-\lambda)}$ (when $\alpha = \sigma$). Therefore $W(\hat{\mathbf{p}}) < W(\mathbf{p}^H) < W(\mathbf{p}^L)$ when $\frac{\alpha(1-\lambda)}{r+\alpha(1-\lambda)} < \frac{u_L}{u_H} < \frac{\sigma(1-\lambda)}{\delta[r+\sigma(1-\lambda)]}$. Therefore:

(i) $W(\mathbf{p}^H) < W(\hat{\mathbf{p}}) < W(\mathbf{p}_n) = W(\mathbf{p}^L)$ when $\frac{u_L}{u_H}$ is large

(ii) $W(\hat{\mathbf{p}}) < W(\mathbf{p}^H) < W(\mathbf{p}_n) = W(\mathbf{p}^L)$ when $\frac{u_L}{u_H}$ is neither too large nor too small

(iii) $W(\hat{\mathbf{p}}) < W(\mathbf{p}_n) = W(\mathbf{p}^L) < W(\mathbf{p}^H)$ when $\frac{u_L}{u_H}$ is small.

It is easy to show that if $\frac{u_L}{u_H} \geq \frac{(1-\lambda)\theta(1+\delta^{-1}-\alpha)}{(1+\delta^{-1})(\delta^{-1}-\theta\lambda)+\lambda\theta\alpha}$ then $\left.\frac{\partial W(\mathbf{p}^L)}{\partial\sigma}\right|_{\sigma=\alpha} \geq \left.\frac{\partial W(\mathbf{p}_n)}{\partial\sigma}\right|_{\sigma=\alpha} > 0$. Therefore let σ be in a neighborhood around α. By continuity, in general we can say that:

(i) If $\frac{u_L}{u_H}$ is large

- $W(\mathbf{p}^H) < W(\hat{\mathbf{p}}) < W(\mathbf{p}_n) < W(\mathbf{p}^L)$ if $\sigma > \alpha$
- $W(\mathbf{p}^H) < W(\hat{\mathbf{p}}) < W(\mathbf{p}^L) < W(\mathbf{p}_n)$ if $\sigma < \alpha$.

(ii) if $\frac{u_L}{u_H}$ is small

- $W(\hat{\mathbf{p}}) < W(\mathbf{p}^L) < W(\mathbf{p}_n) < W(\mathbf{p}^H)$ if $\sigma > \alpha$
- $W(\hat{\mathbf{p}}) < W(\mathbf{p}_n) < W(\mathbf{p}^L) < W(\mathbf{p}^H)$ if $\sigma < \alpha$.

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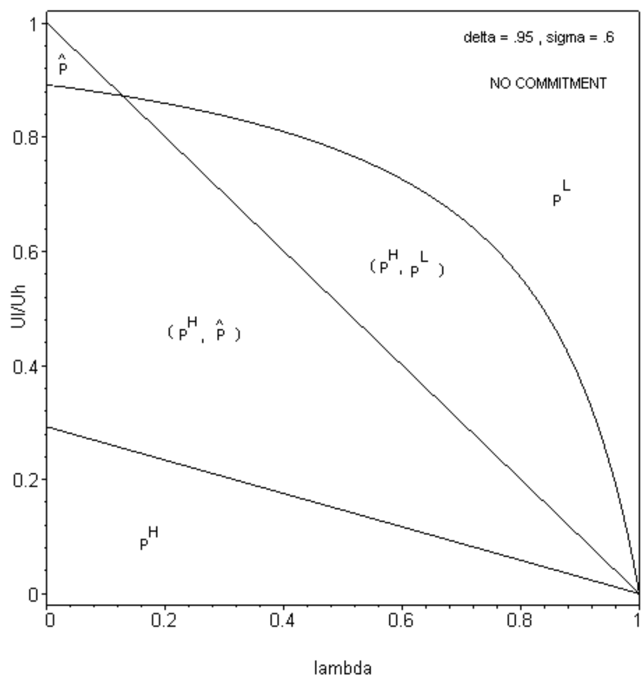

Figure 1

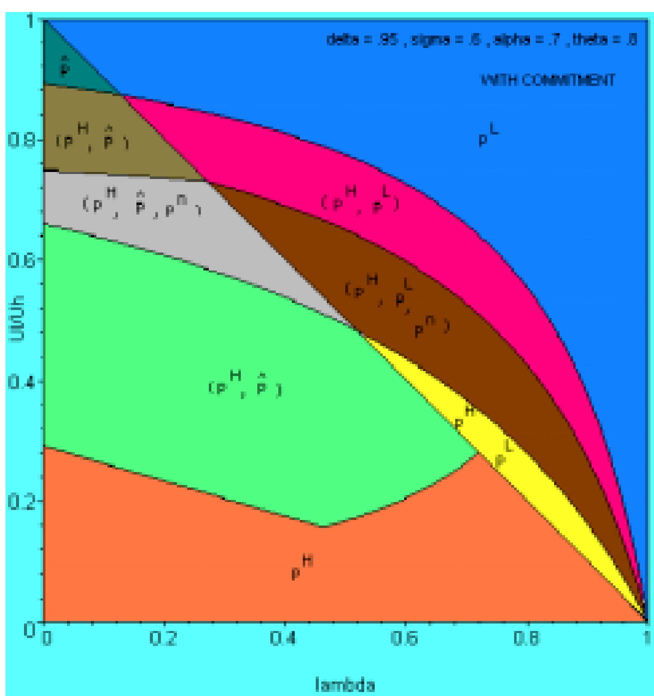

Figure 2