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Experimental Evidence on the Properties of the California's Cap and Trade Price Containment Reserve

Rachel Bodsky

Domenic Donato

Kevin James

david porter
Chapman University, dporter@chapman.edu

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Rachel Bodsky

Domenic Donato

Kevin James

David Porter

Economic Science Institute

Chapman University

Abstract

We report on a series of experiments to examine the properties of California's Reserve Sale allocation mechanism to be implemented as part of the forthcoming cap and trade program and compare it with an alternative reserve sale mechanism. The proposed reserve sale mechanism allows covered entities to purchase allowances after the primary auction sale at fixed prices. If demand for units is greater the amount supplied in the reserve sale, a *Proportional Rationing* rule is used to distribute allowances based on submitted request for units. This rule is contrasted with to an alternative rule, *Equal Rationing* in which allowances are allocated one at a time until the quantity available is exhausted or the participants' requests are fulfilled. We find Equal Rationing outperforms Proportional Rationing allocating units with higher efficiency at a lower cost to participants. Additionally, we sorted subjects by quiz score, which yielded a significant impact on performance, suggesting that subjects with a better understanding of the environment outperformed their counterparts.

1. Introduction

California is instituting a state-wide cap and trade program to help reduce California's greenhouse gas emissions.[1] The program will require *covered entities*, businesses that are mandated by the law to participate, to surrender *allowances* that correspond to their level of produced emissions. An allowance permits a covered entity to emit one metric ton of carbon dioxide equivalent ($MTCO_2e$)[2] and may be purchased in a quarterly multi-unit uniform price auction.[3]

[1] Assembly Bill 32 mandates that California institute a cap and trade program. More details can be found on California Climate Change Portal, http://www.climatechange.ca.gov/cap_and_trade/index.html.

[2] Gases covered in the cap and trade program include carbon dioxide (CO_2), methane (CH_4), nitrous oxide (N_2O), hydrofluorocarbons (HFCs), perfluorocarbons (PFCs), sulfur hexafluoride (SF6), and nitrogen trifluoride (NF_3). Some gases, such as methane, are more effective at trapping heat than carbon dioxide. Thus, all emissions are measured in units relative to the heat-trapping potential of carbon dioxide. CARB Proposed Regulation to Implement the California Cap and Trade Program, Part I, Volume I. Pg II-2.
http://www.arb.ca.gov/regact/2010/capandtrade10/capisor.pdf

[3] A multi-unit uniform price auction is an auction were identical units of a commodity are sold to auction participants at the same price per unit. Participants submit bids in terms of quantity requested and the maximum willingness to pay per unit. Bids are sorted from highest to lowest by the bid's price element. The intersection of this demand curve and the supply determines the uniform price. Subjects whose price bid exceeds the uniform price receive the units they requested, subjects whose price bid is less than the uniform price do not receive units and subjects whose price bid is the same as the uniform price may receive all, some or none of the units they requested. For more information regarding the properties and procedures of a multi-unit uniform price auction, see: Kagel, John H. and Dan Levin. "Behavior in Multi-Unit Demand Auctions: Experiments with Uniform Price and Dynamic Vickery Auctions." *Econometrica*. Vol. 69. Issue 2. (2001): Pgs 413-454. Engelbrecht-Wiggans, Richard and Charles M. Kahn. "Multi-Unit auctions with uniform prices." *Economic Theory*. Vol. 12. Issue 2. (1997): Pgs. 227-258.

Entities participating in this auction raised concerns to California's Air Resources Board (CARB), the organization implementing the program, about the possibility of high auction clearing prices. The uncertainty of high market prices led CARB to develop a supplemental mechanism to help maintain low auction prices. There have been different ideas on how CARB should integrate such a mechanism, dubbed the Allowance Price Containment Reserve.[4] Initially, the Reserve was slated to simply be a means of releasing extra allowances into the auction when a price trigger was met. Covered entities, still fearing uncertainty regarding auction prices, requested a "storefront" where they could purchase allowances for fixed prices. However, a storefront would operate using a first-come, first-served mechanism which does not take an entity's value for allowances into account.[5]

CARB ultimately proposed an auction where entities submit bids only in terms of the quantity they wished to receive at the fixed prices. The new mechanism became known as the Reserve Sale and will be conducted three weeks following the quarterly allowance auction.[6] The Reserve Sale offers allowances at three different fixed prices, also known as tiers. Each tier contains 41.2 million allowances[7] and allowances are priced at $40 per metric ton for the first tier, $45 per metric ton for the second tier and $50 per metric ton for the third tier.[8] If the total number of allowances requested by covered entities participating in the Sale is less than the number of allowances in the tier, each entity receives what they requested. However, if the total number of allowances requested exceeds the number of allowances in the tier, each entity receives the proportion of the tier that they contributed to the total demand. For example, if only 50 allowances are available, but 100 allowances are requested, each entity receives one-half of their bid and they would pay the tier price.

Price containment mechanisms or "reserves" have been encouraged by academics and industry alike (Murray et al, 2009 and Lopomo et al, 2011) to ameliorate market uncertainty and moderate the price of allowances. However, the proportionality mechanism chosen by CARB to allocate Reserve allowances has not been studied. The most applicable research has found

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[4] CARB's Scoping Plan pg. 48, http://www.arb.ca.gov/cc/scopingplan/document/final_supplement_to_sp_fed.pdf.

[5] History of the development of the Allowance Price Containment Reserve was gathered from CARB's auction design expert, Raymond Olsson.

[6] The timing between the quarterly auction and the Reserve Sale is outlined in CARB's Request for Proposal for an Auction and Reserve Sale Operator. http://www.arb.ca.gov/cc/capandtrade/contracts/auction_operator_rfp.pdf

[7] The Reserve will be filled with 123.5 million allowances initially (out of the 2.7 billion issued for the years 2012 to 2020) and split equally among the three tiers. Any unsold auction allowances will be added to the Reserve. CARB Proposed Regulation to Implement the California Cap and Trade Program, Part I, Volume I. Pg. II-5. http://www.arb.ca.gov/regact/2010/capandtrade10/capisor.pdf

[8] These prices are for allowances in 2012. Prices will escalate by 5 percent plus the cost of inflation each year, such that the reserve prices for 2020 will be $60/metric ton for the first tier, $67/metric ton for the second tier, and $75/metric ton for the third tier. CARB Proposed Regulation to Implement the California Cap and Trade Program, Part I, Volume I. Pg. II-25. http://www.arb.ca.gov/regact/2010/capandtrade10/capisor.pdf

inefficiencies in proportional allocation mechanisms. Noussair and Porter (1992) considered two different rationing rules for allocating units in a uniform price auction: *priority service* and *proportional rationing*.[9] The priority service mechanism allocated available units to the highest bidders, while the proportional rationing mechanism allocated units to the highest bidders first, but then proportionally adjusted participants' allocations depending on how much supply was available. Noussair and Porter conclude that because subjects did not have consistent bidding strategies in the uniform price auction with proportional rationing, the same auction with priority service resulted in more efficient allocations.

As evidenced by the previous research, we hypothesize that the Proportional Rationing rule being used in the Reserve Sale could be problematic for California's cap and trade program. An initial thought experiment can illuminate the tough choices a business might face when participating in this environment. Consider the following: If entities believed demand in a particular tier would be high, it is in their best interest to inflate their quantity bid to ensure they get the allowances they need for compliance. However, overbidding exposes covered entities to purchasing more allowances than necessary, while underbidding or simply revealing what they truly need exposes entities to not acquiring enough allowances for compliance.

Ultimately, covered entities must decide if they believe prices in the allowance auction will be higher or lower than the Reserve Sale. When prices in the allowance auction are lower than the Reserve's fixed tier prices, no entity is expected to participate in the Reserve Sale. However, when prices are expected to be higher in the allowance auction, entities must decide whether to be competitive in the allowance auction or risk over or under-buying in the Reserve Sale for the prospect of a better price. This decision must come before placing a bid in the allowance auction due to the sequential nature of the two auctions. Due to the combination of both an unknown allowance auction clearing price and an unknown demand in the Reserve Sale, the Reserve Sale's allocation mechanism could introduce considerable uncertainty into the bidding process and may have an unintended feedback effect on the allowance auction.

As no data are available to help guide the design of CARB's proposed Reserve Sale, we turned to experimental methods. Additionally, we developed an alternative allocation rule that distributes allowances equally rather than proportionally. If cover entities' demand is greater than supply of allowances in a tier, an entity would receive allowances until either his demand is satisfied or the quantity available is exhausted. We call this the *Equal Rationing* mechanism and investigated it in comparison to the proportional allocation mechanism. We also examined both

[9] Harris and Raviv (1981) were the first to examine priority pricing in the case of uncertain demand. Chao and Wilson (1987) create the concept of priority classes to allow for efficient rationing when supply is curtailed. Spulber (1992) investigated the use of proportional rationing schemes in which individual demands are scaled up or down based on available supplies in electric power markets. Moulin (2000) uses and axiomatic approach to characterize proportional and priority rationing rules.

mechanisms to see if the existence of the Reserve Sale imposes an unintended effect on the allowance auction.

Our findings suggest that the quarterly allowance auction is greatly affected by the allocation mechanism chosen for the Reserve Sale. Unknown demand and proportional rationing greatly increases the uncertainty in how entities should participate in the Reserve Sale. This uncertainty drives entities to participate more heavily in the quarterly allowance auction, thus increasing the auction clearing price. Entities that are not willing to forgo the chance of low-priced allowances risk over or under-buying allowances, resulting in low overall efficiency for the system. These effects on the allowance auction are improved with the Equal Rationing mechanism. Relative efficiency[10] increases from 33% to 73% when Equal Rationing is used. We hypothesize this is because subjects are no longer tempted to over bid in the Reserve Sale as the Equal Rationing mechanism improves the ambiguity problem of the Proportional Rationing mechanism. These improvements are clearly reflected in the allowance auction prices and efficiency. Finally, we find that a comprehensive understanding of the environment is an important component of overall system effectiveness. Subjects in this experiment were grouped by quiz score. Groups in the experiment that demonstrated a better understanding of the environment as measured by the incentivized quiz achieved a relative efficiency of 67%.[11] Conversely, groups that performed worse on the same quiz achieved a relative efficiency of only 39%.

In the next section we supply a game-theoretic model of this rationing process and characterize the pure strategy equilibria. In Section 3, we provide a description of our experimental parameters followed by our experimental design. Section 5 provides the results of the experiments.

2. Formal Model

Participants compete with one another for a limited supply of allowances that are offered in two sequential periods: the allowance auction and the Reserve Sale. In order to bid optimally, participants must forecast demand for both the allowance auction and the Reserve Sale prior to participating. This requires participants to accurately forecast the actions of the other market participants and then use backward induction to determine their bidding strategy.

[10] *Relative efficiency* is a means of comparing the two allocation mechanisms' efficiency. It is scaled to be 0% for a realized efficiency that is equal to random allocation efficiency for the demand scenario and 100% for a realized efficiency equal to the maximum possible efficiency. Figure 6 in Section 5 shows the distribution of efficiencies realized in each demand scenario when units are allocated randomly.

[11] Subjects participated in this environment in groups of 8. Groupings were determined by quiz score unbeknownst to the subjects and the quiz was incentivized. More information on these groupings and the quiz are found in Section 3.

To aid in understanding the differences in our treatment mechanisms, we developed a general model given the structure of the mechanism being proposed by CARB. We find the equilibria of this model by examining participants' best responses to the strategy sets of the other participants. This process was applied to our experiment parameters to determine the equilibrium outcomes for each set of parameters. In what follows, we will use our experiment parameters, namely, two tiers in the Reserve Sale and eight participants who have elastic demand for a given number of units.

To reach a possible equilibrium, each participant maximizes their profit function subject to system constraints and the actions of the other participants. The profit function we use depends on the total number of units a participant obtains in both the Allowance Auction and the Reserve Sale. Let V_i denote the value per unit to participant i and H_i denote the total number of units for which participant i can receive value. If the total number of units that participant i obtains is less than or equal to H_i, then his profit is the sum of the profit he receives from units purchased in the auction and the units purchased in each tier. Participant i's per unit profit from z_i units purchased in the auction is his value V_i minus the auction price, P^a, so that total profit is $(V_i - P^a) * z_i$. The profit received from units purchased in the tiers is the participant's value for a unit minus the price of units in that tier, P_j^t, multiplied by the number of units obtained from that tier, q_{ij}. If the total number of units that participant i obtains is greater than the number of units that he has value for, H_i, then his profit is H_i multiplied by the value he has for each unit minus the cost of all units obtained. The maximization that must occur for each participant is represented in Equation (0):

$$Max: \begin{cases} (V_i - P^a) * z_i + \sum_{j=1}^{2} \left(V_i - P_j^t\right) * q_{ij}, & if\ z_i + \sum_{j=1}^{2} q_{ij} \leq H_i \\ V_i * H_i - \left(P^a * z_i + \sum_{j=1}^{2} P_j^t * q_{ij}\right), & otherwise \end{cases} \qquad (0)$$

While a participant is choosing a strategy set that maximizes their profit, the other participants' strategy sets are held constant. We iterate through the all of the participants until a solution is found.

2.1 Allowance Auction

Since our experimental environment consists of two tiers, the strategy vector for participant i in the Proportional Rationing experiment is $\{AP_i, AQ_i, T_{i1}, T_{i2}\}$, where AP_i is the price component of the auction bid, AQ_i is the quantity component of the auction bid, and the remaining elements of the strategy vector are the tier quantity bids, T_{ij} and i goes from 1 to 8.

The aggregate of the first two elements in each participant's strategy vector determines the number of units that each participant obtains in the auction and the price of the units sold in the auction. Units in the auction are allocated by sorting the auction bids so that the auction price bids are in descending order. The auction quantity bids are then fulfilled in that order until there are either no remaining auction quantity bids or all of the auction units have been allocated.

For example, assume that the allowance auction (with 60 units available and 8 participants) has concluded and the auctioneer received the following set of price bids and quantity bids:

Table 1: Allowance Auction Allocation Example

Participant	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Price Bid, AP_i	105	95	85	75	65	55	45	35
Quantity Bid, AQ_i	12	8	15	18	10	4	4	27

In this example, participant 2 submits a price bid of 95 and a quantity bid of 8. The total number of units requested is 98 (12+8+15+18+10+4+4+27), with the highest price bid at 105 and the lowest price bid at 35.

To determine how many units,z_i,a participant will obtain, the number of units remaining given participant *i*'s auction price bid, r_i, needs to be calculated. For example, to calculate r_5, or the number of units remaining give to participant 5, first sum all the auction quantity bids of the participants that had an auction price bid greater than participant 5's auction price bid. Participants 1 through 4 all have auction price bids greater than participant 5, so the sum would be 12 + 8 + 15 + 18 = 53. This means that 53 units have already been allocated to other participants so r_5 would equal 7, which is the number of units available in the auction, 60, minus the number of units that have already been allocated. z_5 denotes the number of units participant 5 receives and is the minimum of r_5 and AQ_5. In this example, $r_5 = 7$ and $\mathrm{AQ}_5 = 10$, which means that $z_5 = 7$ and participant 5 would be allocated 7 units.[12] The formulas used to calculate r_i and z_i are formally presented in Equations (1a) and (1b):

Units Remaining: $$r_i = \left(Z - \sum_k \begin{cases} AQ_k, & if\ AP_k > AP_i \\ 0, & otherwise \end{cases}\right), k \neq i \quad (1a)$$

Auction Allocation: $$z_i = \begin{cases} Min[r_i, AQ_i], & if\ r_i > 0 \\ 0, & otherwise \end{cases} \quad (1b)$$

After the auction units have been allocated, the auction price is set to the auction price bid of the participant that would have received a unit if there was one additional unit available in the auction. If all participants had their bids fulfilled, then the auction price would be zero. Extending the previous example, if there were 61 units rather than 60, participant 5 would have received that extra unit. This means that the resulting auction price would be 65.

[12] If there is a tie between two or more participants, then the units are distributed equally between the participants until there are no more units to allocate or all of the tied participants have had their quantity bids fulfilled.

Two dominant strategy constraints were placed on the domain of the auction bids in our experiments. The first is that participant i's auction price bid was limited to values between zero and the value that he had per unit, inclusive. This restriction is shown in Equation (2a):

Auction Price Bid Constraint: $$0 \leq AP_i \leq V_i \tag{2a}$$

The second constraint is that a participant's auction quantity bid was limited to integer values between zero and the number of units he had value for, inclusive. This restriction is shown in Equation (2b):

Auction Quantity Bid Constraint: $$0 \leq AQ_i \leq H_i \tag{2b}$$

2.2 Reserve Sale

We examine two different types of rationing mechanisms for the Reserve Sale units in the tiers, Proportional Rationing and Equal Rationing.

2.2.1 Proportional Rationing Mechanism

In the Proportional Rationing mechanism, each participant i submits a quantity bid T_{ij} for each tier j. If the total number of units requested in tier j, $\sum_{i=1}^{8} T_{ij}$, does not exceed the number of units available in that tier, Q_j, then each participant receives the number of units he requested in that tier. In this case, $T_{ij} = q_{ij}$. However, if the number of units requested in tier j is greater than the number of units available in that tier, then the units are allocated proportionally to the nearest whole unit and the remainder is allocated randomly.[13] To determine participant i's proportional allocation in tier j, we calculate pa_{ij} by summing all participants' tier j quantity bids (inclusive of i) and then dividing participant i's tier j quantity bid, T_{ij}, by the sum of the bids to obtain the proportion of units that participant i will be allocated. Participant i is then allocated this proportion of the units available in tier j and random units if available. Determining q_{ij} is formally written in Equation (3) and the formal representation of the proportional allocation, pa_{ij}, is shown in (3.1) :

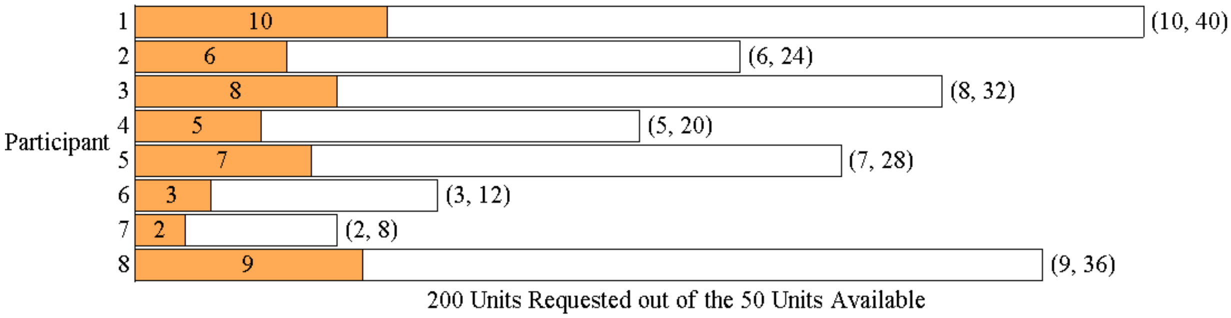
$$q_{ij} = \begin{cases} T_{ij} \quad , & if \sum_{i=1}^{8} T_{ij} \leq Q_j \\ \lfloor pa_{ij} \rfloor + \frac{Q_j - \sum_i^8 \lfloor pa_{ij} \rfloor}{8}, & Otherwise \end{cases} \tag{3}$$

[13] Agents who received a portion of the randomly allocated units actually received the *expected number* of units, or fractional units.

$$pa_{ij} = Q_j \left(\frac{T_{ij}}{\sum_{i=1}^{8} T_{ij}} \right) \tag{3.1}$$

Figure 1 shows example bids and the resulting allocations given the formal definition in Equation (3). In the example, 200 units were requested in the tier, but only 50 units were available, therefore participants receive only a quarter of what they requested. For example, Participant 3 requested 32 units but only receives 8 units (8, 32).

Figure 1: Proportional Rationing



The participant's bids total to 200 units. Since only 50 units are available, each participant receives a quarter of their requested amount (50, 200).

The only constraint placed on a participant's bid in the Proportional Rationing environment was that it must be an integer value between zero and the number of units available in that tier, inclusive. This restriction is shown in Equation (4):

Proportional Rationing Constraint: $$0 \leq T_{ij} \leq Q_j \tag{4}$$

2.2.2 Equal Rationing Mechanism

In the Equal Rationing mechanism, participants submit a single quantity bid, T_i, for the total number of units desired.[14] These bids are held in a bid vector, $\vec{B}$, where the i^{th} element, B_i, is the quantity bid of participant i. The system then allocates the units in each tier to the subjects equally until there are no more units available or every bid has been fulfilled. The process works by iteratively allocating units to each participant, and accumulating the allocations to each participant in an allocation vector, $\vec{A}$, where the i^{th} element, A_i, is the number of units allocated to participant i. This process continues as long as the sum of the units allocated to each participant is less than both the number of units in the tier, Q_j, and the sum of the units each participant requested, $\sum_{i=1}^{8} B_i$. When either of these conditions is violated, the allocation vector is returned, which provides the number of units each participant will receive in that tier.

[14] Every entity had a value for units greater than the tier prices in our parameter set, which is why only a single quantity bid was necessary.

Appendix A describes the formal process used to determine what each participant should be allocated in the Equal Rationing environment. The iterative process of used in Appendix A can best be explained using a numerical example. Consider the following: 50 units are available in Tier 1 and 50 units are available in Tier 2. Four subjects submit the following quantity bids: $\{20, 20, 30, 40\}$. This means that the bid vector, $\vec{B}$, is {20, 20, 30, 40} and the initial allocation vector, $\vec{A}$, is {0, 0, 0, 0}.

On the first iteration there are a total of four outstanding bids, therefore the maximum number of units that can be equally allocated to each participant is 50/4 or 12.5 units each. Since every participant has asked for a number greater than 12.5, each participant will be allocated 12.5 units from Tier 1 and the resulting allocation vector will be {12.5, 12.5, 12.5, 12.5}. All of the units available in Tier 1 have been allocated and there are still outstanding bids, so we will move on to allocate the units available in Tier 2.

The new bid vector is equal to the old bid vector minus the resulting allocation vector, {20, 20, 30, 40} - {12.5, 12.5, 12.5, 12.5} or {7.5, 7.5, 17.5, 27.5}. A new allocation vector is created to hold the resulting allocation of the units in Tier 2, {0, 0, 0, 0}.

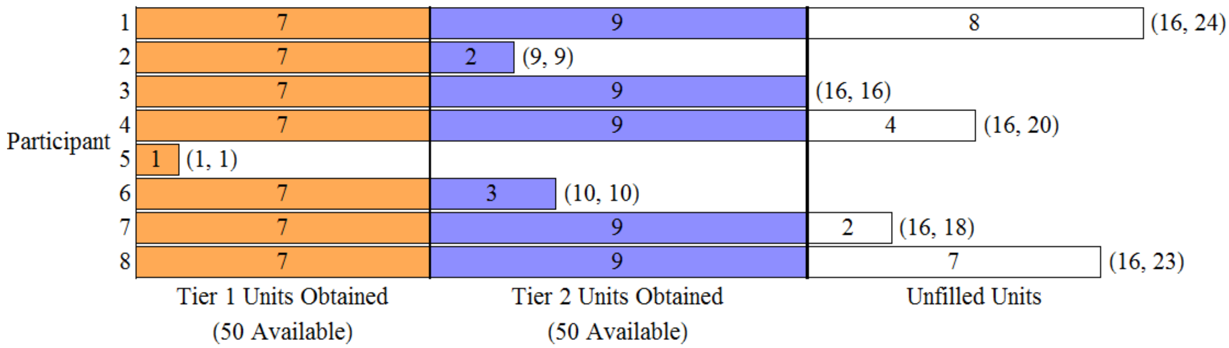
As there are four participants with outstanding bids, the allocation process above is repeated. The number of units that are offered to each participant is 50/4 or 12.5 units each. However, the first two participants only need 7.5 additional units and so they each receive 7.5 units. The last two participants need more than is being offered and so they receive the number of units being offered, 12.5 each.

The resulting allocation vector after the first iteration through Tier 2 units is {7.5, 7.5, 12.5, 12.5}. Since there are still outstanding bids and the units in Tier 2 have not been depleted, a second iteration is performed. Now there are only two participants with outstanding bids and the number of units remaining is 10, $50 - (7.5 + 7.5 + 12.5 + 12.5)$, so each of the participants with outstanding bids is offered 5 units, 10/2. The sum of the current offer and the amount that each participant has already been allocated is less than the number of units each one requested; thus, they each receive five additional units.

The resulting allocation vector after this second iteration is {7.5, 7.5, 17.5, 17.5} and the total number of units allocated at this point is 50, which means that the iteration will stop and return the allocation vectors {12.5, 12.5, 12.5, 12.5} from Tier 1 and {7.5, 7.5, 17.5, 17.5} from Tier 2.

Figure 2 diagrams a set of 8 sample bids, and the resulting unit allocations given the formal definition found in Equation 5.

Figure 2: Equal Rationing



With Equal Rationing, units are allocated so that participants have an equal number of units, or the amount requested. In the case of Tier 1, participant 5 received the one requested, and the remaining 7 participants split the 49 remaining units equally. Likewise, in Tier 2 participants 2 and 6 obtained their remaining bid, while the other participants each got a fifth of the other 45 units.

The constraint placed on a participant's bid in the Equal Rationing environment is similar to that of the Proportional Rationing constraint. A bid placed in the Equal Rationing environment must be an integer value between zero and the sum of the units available in both tiers, inclusive. This restriction is formalized in Equation (5).

Equal Rationing Constraint:
$$0 \leq T_i \leq \sum_{j=1}^{2} Q_j \tag{5}$$

Although Proportional Rationing and Equal Rationing have similar equilibrium auction prices, the strategy space and complexity that a participant faces when participating in one mechanism versus the other differs greatly. The number of strategy combinations in the Proportional Rationing mechanism is equal to a participant's value times the number of units she has value for multiplied by the product of the number of units available in each tier, $V_i * H_i * \prod_{j=1}^{2} Q_j$. While the number of strategy combinations in the Equal Rationing mechanism is equal to a participant's value times the square of the number of units she has value for, $V_i * {H_i}^2$.

In the next section we list the parameters used in our experiments and Nash equilibrium predictions based on the model structure defined above.

3. Experimental Environment and Parameters

Each experiment consisted of 32 independent rounds, with each round composed of two periods. The first period, which is referred to as Period A, was a uniform price sealed bid auction with 60 units available for purchase. The second period, which is referred to as Period B, was a sealed bid rationing sale with 100 units available. Unsold units did not carry over to other periods or rounds. Each subject received a unit value as well as a holding limit at the beginning of each round. The unit value represented the value of one unit to the subject and ranged from 35 to 105 experimental dollars per unit. The holding limit represented the total number of units for which the subject had value for that round and varied from 7 to 39 units.[15]

In Period A subjects placed a single bid which consisted of two elements: the quantity of units being requested (bid quantity) and the maximum willingness to pay per unit (bid price).[16] The computer program uses the multi-unit uniform price auction rules to find the allocation and auction-clearing price. After Period A, each subject was shown a summary page displaying the auction clearing price, the number of units he received, the number of units remaining in his holding limit, and his profit from the period. Figure 3 shows the screen that subjects interacted with when making their Period A decisions.

Figure 3: Period A Participant Screen

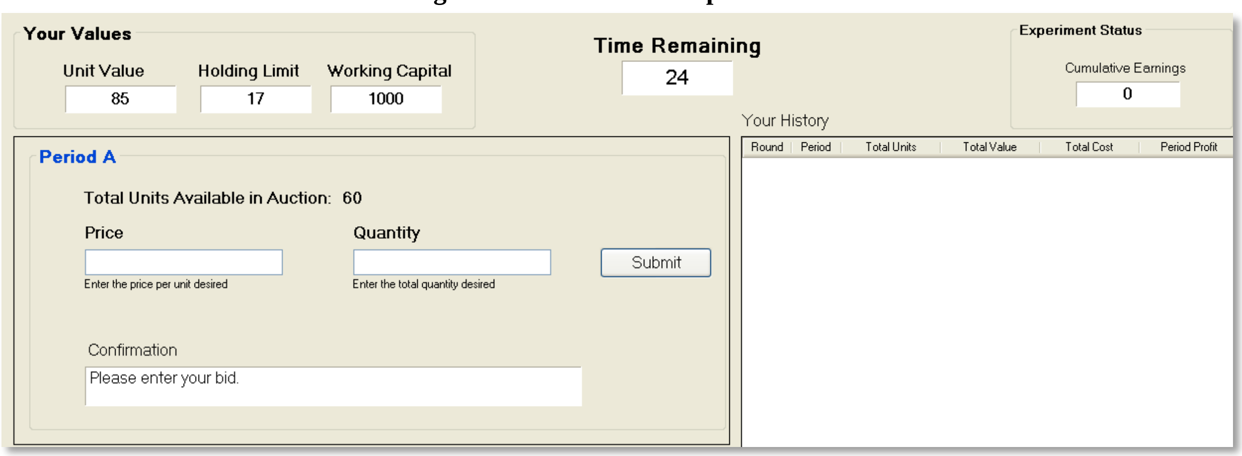


Figure 3 shows the screen participants interacted with during Period A. In this case, the participate had a unit value of 85 and a holding limit of 17. Participants entered their price and quantity bids in the text boxes in the middle of the screen. The confirmation box served as a means of communicating to the participant that their bid was accepted or not. After Period A concluded, a summary of what happed appeared on the right side of the screen under "Your History."

[15] For example, assume a subject had a value of $105 and a holding limit of 7 units. If this subject purchased a total of 8 units during the round, then each of the first seven units he bought during that round had a value of $105 to him, but the 8th unit he purchased had no value.

[16] Two restrictions were placed on the subjects' bid during the first period. A subject was not allowed to request a quantity greater than his holding limit and he could not submit a bid price greater than his value.

In Period B subjects could obtain units they did not receive in Period A. There were two price levels at which units were sold during Period B: Tier 1 priced at $20 and Tier 2 priced at $30. Each tier had 50 units available for purchase. The units sold during Period B were identical to those sold during Period A. The rationing mechanism used to allocate these units depended on the treatment (Proportional or Equal Rationing). Upon concluding Period B, a final summary page showed a subject how many units he received from each tier, the total cost of the units, and the net profit from the round. In addition, each subject was shown the total profit they earned that round and how many units they obtained. Subjects also had access to a history table that contained information regarding purchases, prices, and profits from previous rounds. Figure 4 shows the screens that subjects interacted with when making their Period B decisions. Notice that Period A results are displayed throughout the duration of Period B.

Figure 4: Period B Subject Screen

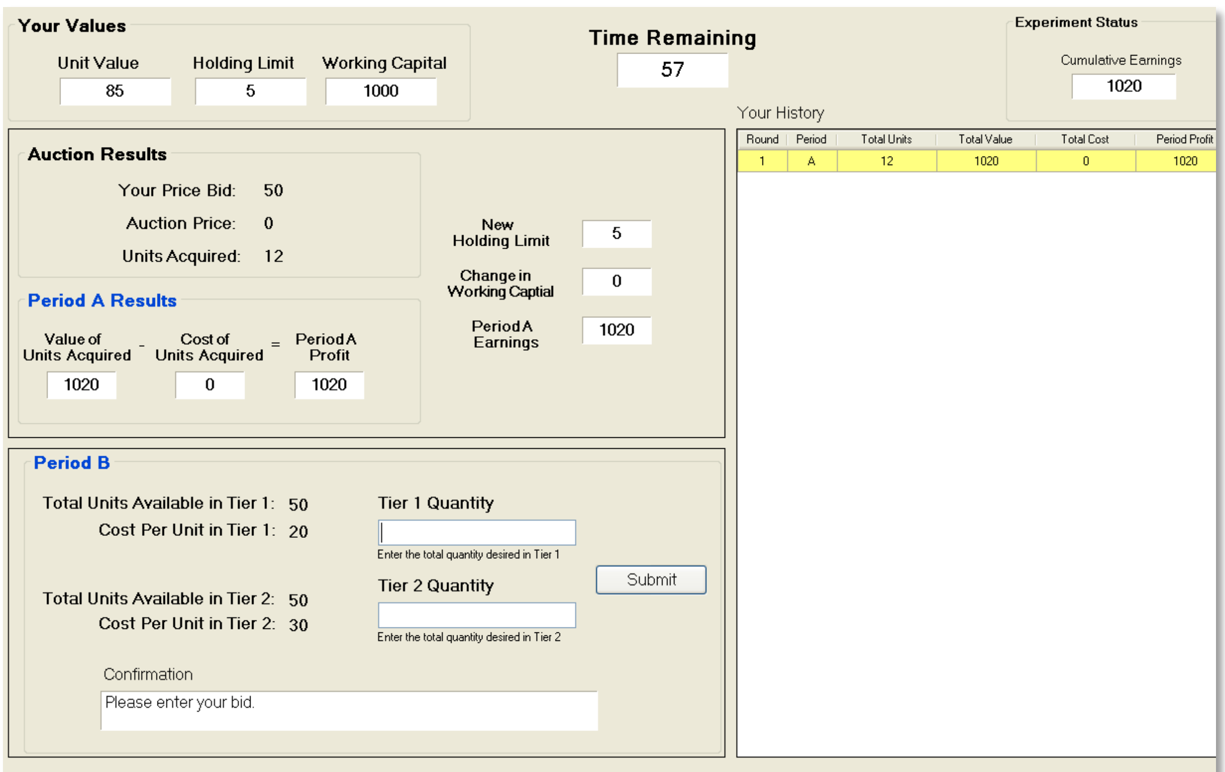


Figure 4 shows the screen participants interacted with during Period B. After completing Period A, the Period A results would appear where the participants had previously entered their bids. Additionally, Period B would open below Period A and participants could place bids for Period B units. Notice on the screen above, the participant had acquired 12 units in Period A and had paid a price of 0 for those units. Their Period A earnings were 1020. This information is also captured in the participant's history table.

Demand uncertainty was introduced by keeping each subject's holding limit and value private while changing the total demand each round. The sum of all subjects' holding limits always reflected one of four demand cases:

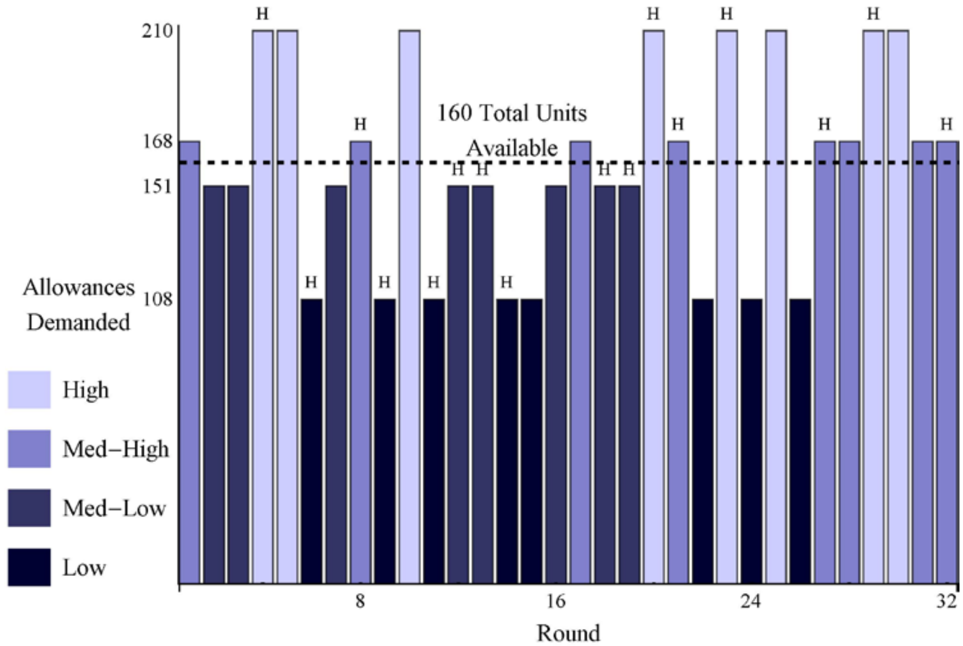
1. *Low Demand*: The sum of all subjects' holding limits equaled 108 units.[17] The low demand case was an environment where there was enough supply to satisfy demand with the units available in Period A and the first tier of Period B.
2. *Medium-Low Demand*: The sum of all subjects' holding limits equaled 151 units. The medium-low demand case simulated an environment where there were enough units between both periods to satisfy demand.
3. *Medium-High Demand*: The sum of all subjects' holding limits equaled 168 units. The Medium-High Demand case simulated an environment where there was small shortage in the supply of units.
4. *High Demand*: The sum of all subjects' holding limits equaled 210 units. The High Demand case simulated an environment where there was large shortage in the supply of units.

Within each demand case, subjects' holding limits were organized in two ways. The first method of organization, we call *High Value* and label with an "*H*". In this case, the subject with the highest value for allowances also had the highest holding limit. The subject with the second highest value had the second highest holding limit. This, in turn, meant that the subject with lowest value for allowances also had to lowest holding limits. In essence, we had an environment where holding limits descended with value for allowances. The second method of organization, we will call *Low Value*. In this case, the subject with the highest value for allowances had the lowest holding limit. Consequently, the subject with lowest value for allowances had the highest holding limit. In essence, we had an environment where holding limits ascended as value for allowances descended. With this added dimension to the four demand cases above, subjects experienced eight distinct environments throughout the course of the 32 rounds.

Each of the eight cases occurred the same number of times over the course of the experiment, however subjects never knew which demand state they would encounter. Figure 5 depicts the sequence of demand cases that the subjects faced over the course of an experiment.

[17] Another way to think about this is that the system had a demand for 108 units or there was a value for 108 units.

Figure 5: Total Demand over the 32 Rounds



The figure illustrates the different demand scenario subjects faced over 32 rounds each experiment. For example, subjects saw all four demand cases within the first six rounds. Period 1 had medium-high demand, periods 2 and 3 had medium-low demand, periods 4 and 5 had high demand and period 6 had low demand. Periods labeled with an H experienced the High Value environment where subjects' holding limits descended with value for allowances.

Given the demand conditions and units available, we used a Mathematica computer simulation to derive the frequencies of auction prices we could expect in our experiments. The simulation indicated that numerous Nash Equilibria exist in our given parameter set and a majority of them are the same regardless of the allocation mechanism. Multiple Nash Equilibria were found for the High (H), High (L) & Medium-High (H) demand cases. The other demand cases resulted in a single Nash Equilibrium with an auction price of zero. These features of the parameter set have motivated a number of our hypotheses discussed in the next section. An explanation of the Mathematica program can be found in Appendix C.

4. Experimental Design and Hypotheses

Each experiment consisted of sixteen subjects and lasted approximately 1.5 hours. The subjects interacted through their computer terminals. Before each experiment, subjects were given 35 minutes to read the instructions for the experiment and complete a 19-question quiz.[18] Subjects received $0.25 for each correct quiz response (for a maximum payoff of $4.75) and were able to refer back to the instructions while answering the quiz questions. The results of the quiz were used to sort the subjects. The eight highest-scoring subjects were placed into one group (Group 1), while the remaining eight formed the other group (Group 2). The subjects were informed that they would remain with the same group of eight participants throughout the experiment, but were not told how groups were formed. During the experiment, the two groups only interacted with subjects that were in their group; however, both groups participated in the same experimental treatment.

Although not methodologically common, using groups in this environment was a natural design choice. We believe that the environment and mechanisms were sufficiently complex that the use of "subject-filtering" would help provide a litmus test to the robustness of the mechanism. In particular, if one mechanism performs better regardless of the group, that mechanism has shown it can better handle a variety of bidding behaviors.

Subjects were paid based on their performance in the experiment and quiz. In the experiment, subjects earned money by purchasing units for a price below their value and only purchasing units they had value for, meaning they bought a number of units equal to or less than their holding limit. The uncertainty in the Reserve Sale makes it possible for subjects to buy more units than the holding limit and this could result in negative earnings.[19] Subjects earned an average of $21.30 in addition to a $7 show-up fee.

Experiments were conducted using a 2x2 design. The treatments were composed of the type of rationing mechanism used during the second period of a round and the group of subjects that participated in the mechanism. A total of sixteen experiments were conducted; four in each cell.

[18] For complete experiment instructions, quiz and quiz answers, please refer to Appendix B.

[19] To allow for an environment with negative earnings, the concept of "working capital" was introduced. Each subject maintained their own working capital account of 1000 experimental dollars. If a subjects profit from acquired units did not cover the cost of the units obtained, then the debt was taken from the working capital account. Before a subject could earn money again, he had to replenish his working capital fund. If a subject was consistently over buying units, his working capital fund could be completely depleted. In this case, a fine of $5.00 was imposed on the subject each time their working capital went below $0. After the fine was deducted from his earnings, his working capital was reset and he was able to continue participating. An earnings floor of -$5.00 was imposed. In the sessions conducted, none of the subjects' working capital accounts ever went below $0, therefore, this constraint was never violated.

Table 2 provides a succinct summary of the experiment treatments and the number of experiments conducted.

Table 2: Experiment Treatments

	Group 1	Group 2
Proportional Rationing	4	4
Equal Rationing	4	4

Each group of subjects faced one of two rationing mechanisms during the second period of each round. The rationing mechanism remained the same throughout the entire experiment. The two rationing mechanisms tested were the Proportional Rationing mechanism found in the current CARB proposal of the Reserve Sale, and an alternative, the Equal Rationing mechanism.

Given our experimental design, the model discussed in the preceding section and the results of our simulations, we have three hypotheses for our experimental results.

Hypothesis 1: Relative efficiency in the allowance auction will be the same in the Equal Rationing treatment and the Proportional Rationing treatment.

Hypothesis 2: The allowance auction price will be the same in the Equal Rationing treatment and the Proportional Rationing treatment.

Hypothesis 3: Group 1's performance will differ from Group 2's performance in terms of the Period A efficiency measure and the Period A auction price.

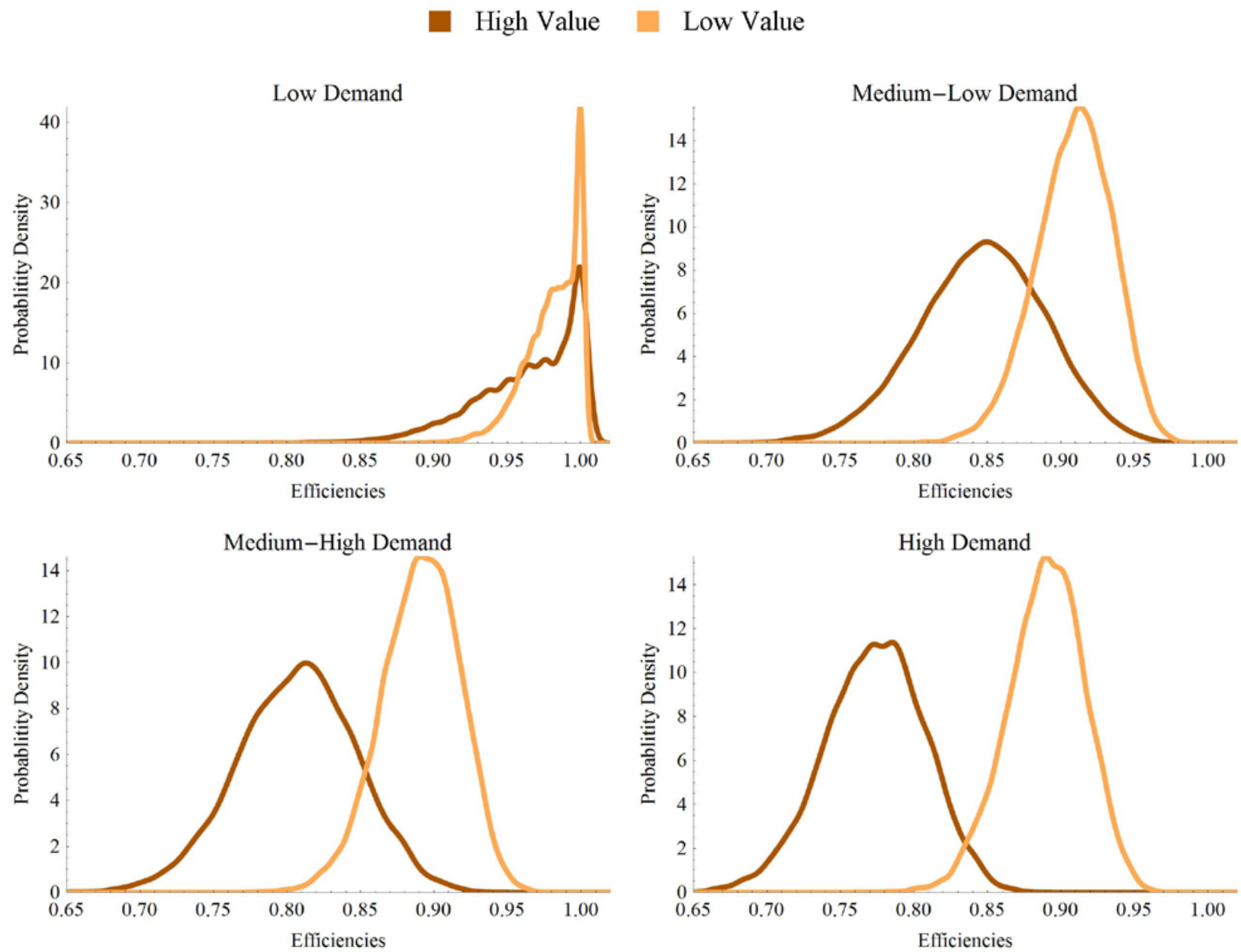
Group 1 and Group 2 are differentiated strictly on how well its members performed on the incentivized quiz before the start of the experiment. Taking this into account, it is plausible that Group 1's (who had the highest scoring members) performance in the experiment will differ from that of Group 2's performance. The relative efficiency between both groups will likely differ because of a disparity in understanding of the rationing allocation mechanism. If a high score on the quiz correlates to a better understanding of the environment, then Group 1 will be more likely to understand the relationship between a bid and their allocation in Period B. This means they are more likely to obtain an allocation of units closer to their remaining holding limit.

In the next section, we will address each of these hypotheses as we present the results of the experiments.

5. Results

We begin by examining the allocative efficiency of the treatments. In this system, surplus is defined as the value realized by participants. The total surplus of an allocation is therefore the sum of all value obtained by the participants. Units obtained beyond a participant's holding limit provided no additional surplus. Maximum surplus is achieved when all 160 units available are allocated to the participants with the highest value for the units. Absolute efficiency of an allocation was set to be the fraction of the maximum surplus that was realized by the allocation. However, it was found that randomly allocating units yields a median absolute efficiency of 88.9%, although it varies for the different demand scenarios. Therefore, comparing treatments across absolute efficiency is misleading, since random allocations generate high efficiencies. Therefore, we use *relative efficiency* to measure the differences between the two treatments. Relative efficiency was scaled to be 0% for a realized efficiency that was equal to the mean random allocation efficiency for the demand scenario and 100% for a realized efficiency equal to the maximum possible efficiency. Using relative efficiency rather than absolute Figure 6 shows the distribution of efficiencies realized in each demand scenario when units are allocated randomly.

Figure 6: Efficiency Distributions by Demand Scenario



Efficiencies varied between demand scenarios, with the higher demand resulting in lower efficiencies during random allocations. The lighter lines are for when the participants with the lowest value had the most demand, and the darker lines are for when the participants with the highest value had the most demand. The low demand environment has a large mass point at an efficiency of 1.

Aggregating the data across all sessions, Table 3 shows the median relative efficiency of the treatments over all periods.

Table 3: Median Relative Efficiency of Treatments over all Periods (N = 16)

	Group 1	Group 2	Marginal Means
Proportional	0.42	0.23	0.33
Equal	0.91	0.54	0.73
Marginal Means	0.67	0.39	

Result 1. *Overall, relative efficiency was higher in the Equal Rationing treatment than in the Proportional Rationing treatment, and Group 1 had higher relative efficiency than Group 2.*

We investigated round relative efficiencies using a ranked two-way ANOVA. This analysis ranks the round median relative efficiencies over the treatment variables and finds the mean ranking for each cell in our two-by-two design. These results are found in Table 4. The efficiency difference between allocation mechanisms was significant ($p < 0.01$). Figure 7 provides a chart of the median of the round relative efficiencies for each observation, and Table 4 displays the rank means for each treatment along with the marginal rank means.

Figure 7

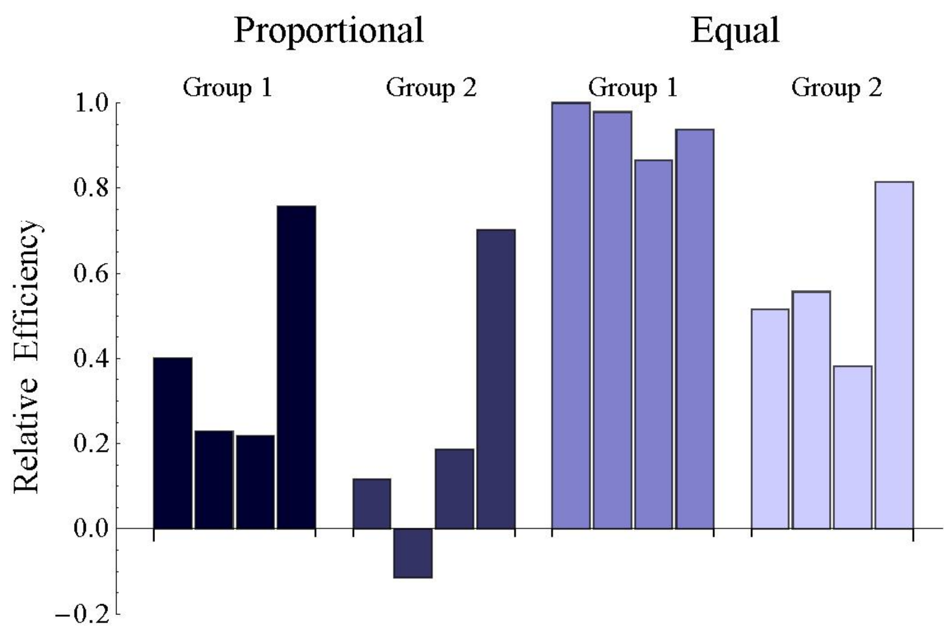


Figure 7 shows the median relative efficiency achieved in each session. There was a significant difference between both allocation mechanisms and groups. Group 1 outperformed Group 2, and the Equal Rationing mechanism on average had higher relative efficiencies than the Proportional Rationing mechanism.

Table 4: Ranked Median Relative Efficiency by Group and Treatment (N = 16)

	Group 1	Group 2	Marginal Rank Means
Proportional	10.25	13.0	11.625
Equal	2.5	8.25	5.375
Marginal Rank Means	6.375	10.625	

Each cell contains the mean relative efficiency rank for the four observations within each treatment. The marginal rank means are shown in the last row and column. Group 1 with Equal rationing had by far the highest efficiency, and the rationing mechanism appears to have such a large effect that Group 2 using the Equal Rationing mechanism managed higher efficiencies than the Group 1 using Proportional Rationing.

One source of the discrepancy between treatments is derived from *overbuying*, or purchasing more units than your holding limit, in Period B in the Proportional Rationing treatment (See

Result 3). Each unit overbought in a higher-demand environment provided no additional value for the participant who obtained it and at the same time prevented the unit from being allocated to the residual demand, diminishing total surplus in the system. By eliminating the incentive to request more units than a participant has value, the Equal Rationing treatment avoided most[20] of this inefficiency.

Another source of inefficiency came from high-valued participants under-buying units. Participants with the highest values would frequently under-reveal during Period A in order to obtain units at a potentially lower price during Period B. Often, however, they would fail to obtain all the units they had value for, driving down the efficiency of the system. Period B was much more reliable with the Equal Rationing mechanism. This encouraged participants with high values to under-reveal their value and holding limit in Period A to an even greater extent than with Proportional Rationing, decreasing prices in Period A (See Result 2) and increasing the demand from high-valued participants in Period B. Even with this shift towards Period B, high-valued participants were able to fulfill their demand more consistently with Equal Rationing, resulting in higher efficiency.

Significant group differences in efficiency were also identified ($p = 0.01$). Group 1, who performed better on the quiz, had significantly higher relative efficiency than that of the lower-scoring group, Group 2. The higher efficiency appears to stem from Group 1 participants with higher values opting for units in the auction, reducing the amount of severe under or over-buying in Period B.

Beyond efficiency, we compare the observed auction prices both between auction mechanism and to those that would have occurred had participants revealed their value and holding limit during the auction process. These comparisons can illuminate the extent of the feedback effect between the auction and the Reserve Sale mechanism in both treatments.

Result 2. *Auction prices were higher in the Proportional Rationing treatment relative to Equal Rationing, although Period A auction prices were significantly lower in both treatments than would have occurred through participant value revelation. While the auction price difference between mechanisms, there was no discernable difference in Group 1 and Group 2 auction prices.*

If participants simply bid their values and holding limits in the auction, the average price would have been 76.25 across all periods; actual prices tended to be much lower. Across all sessions, the average auction price was 35.2. The prospect of cheaper units in Period B drove participants to under-reveal both their value for units and their holding limit in the auction. This resulted in a

[20] Although the Equal Rationing mechanism eliminated overbuying in the Group 1 (the group who scored highest on the quiz), Group 2 occasionally overbought units.

greater risk of participants with higher value not obtaining all of their units, which often led to inefficiency in the system. Regardless of the mechanism used in the Reserve sale, the existence of it led to this inefficiency.

Figure 8 shows the median auction prices for each observation. By ranking the median auction prices (Table 5), a ranked two-way ANOVA was used to reveal a difference in median auction prices between allocation mechanisms ($p < 0.01$). This seems to indicate that although participants always under-revealed in the auction, the risk of either under or overbuying in Period B with Proportional Rationing drove participants to compete more vigorously within the auction, increasing prices. Although greater participation in the allowance auction should have resulted in units being allocated more efficiently in the Proportional Rationing treatment, Result 1 shows this was insufficient to offset the greater inefficiency that subjects created by purchasing more units that they had value for during Period B. Essentially, Period B's complexity using the Proportional Rationing treatment created so much efficiency loss that any gain in efficiency in Period A was dwarfed by the loss in Period B.

Figure 8

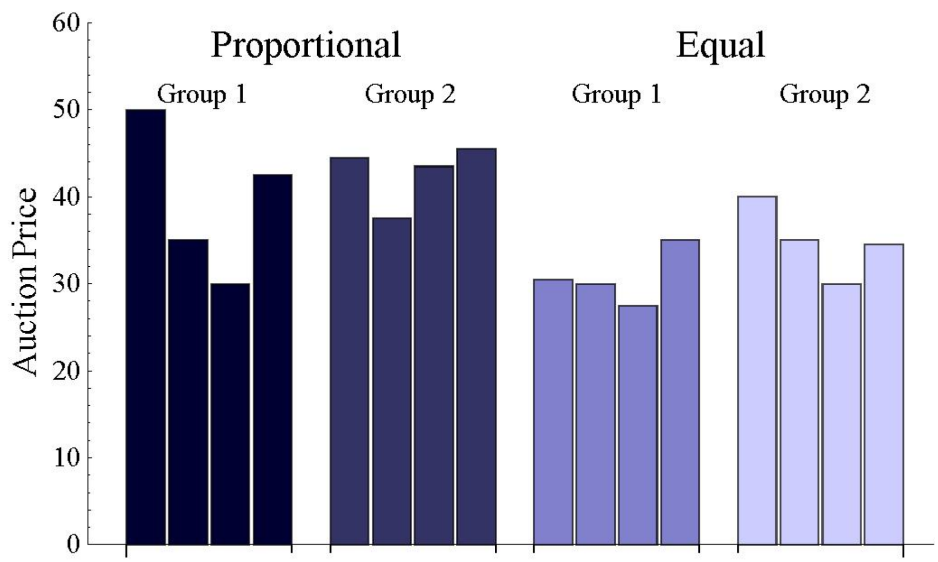


Figure 8 shows the median auction price in Period A for each session. Auction prices were significantly different between allocation mechanisms, with Equal Rationing tending to have lower prices than with Proportional Rationing. Meanwhile, there was no detectable difference in auction prices between groups.

Table 5: Ranked Median Round Auction Prices by Group and Treatment (N = 16)

	Group 1	Group 2	Marginal Rank Means
Proportional	7.25	4	5.625
Equal	12.75	10	11.375
Marginal Rank Means	10	7	

Each cell contains the mean of the ranks for the round auction prices for each observation within a treatment. The marginal rank means are shown in the last row & column. The Proportional Rationing mechanism tended to drive prices up in the auction, while groups showed no significant difference.

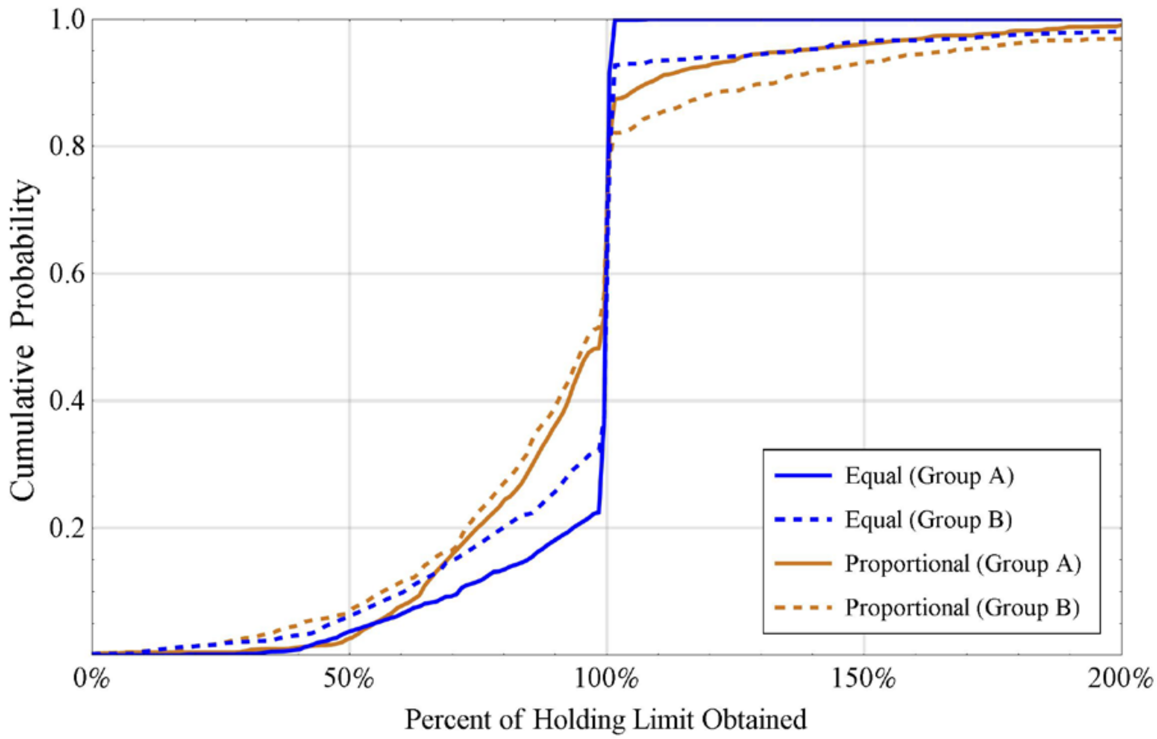
Meanwhile, despite significant differences in group efficiencies, there was no discernable difference in group auction clearing prices ($p = 0.12$).

Another serious concern with the Proportional Rationing treatment is the possibility of participants *overbuying units*, which is when participants receive more units than they have value. Due to uncertain demand, participants may request more units than necessary in order to compensate for predicted Proportional Rationing. Although overbuying by itself does not constitute decreased efficiency (money is transferred without a gain in surplus), it is likely there could have been greater surplus had the unit been allocated to another participant.

Result 3. *Overbuying was prevalent in the Proportional Rationing treatment for both Group 1 and Group 2.*

While our models predicted that in equilibrium no participant would receive more units than they have value for, overbuying occurred in both treatments. Inaccurate predictions of demand could lead to large overbuying in the Proportional Rationing treatment, and on average both groups overbought 10.9% of the time in this mechanism. The Equal Rationing treatment eliminated the incentives that lead to overbuying. Despite this, Group 2 still overbought, leading to an average overbuying rate of 3.5% with Equal Rationing. Using a one-tailed, two-sample t-test, this was found to be significantly lower than in the Proportional Rationing treatment ($p < 0.01$). Similarly, employing a two-sample t-test on groups showed that Group 1 overbought significantly less than Group 2 ($p < 0.01$) regardless of the rationing treatment. Figure 9 shows the frequency of over or under-buying in each group and treatment.

Figure 9: Buying Behavior by Group and Treatment



Above is the cumulative distribution of unit obtainment. Underbuying is defined as having remaining units with value at the end of a round. This must occur in the higher-level demand scenarios, as there are not enough units to meet demand. Overbuying is the opposite, where participants bought more than their holding limit. The Proportional Rationing groups tended to over-buy more often by both small and large amounts.

Although results 1 through 3 indicate that Equal Rationing is on average superior to Proportional Rationing, such a result may not hold up over a time. If participants in Proportional Rationing eventually learn to adapt, it is possible for it to remain the preferred mechanism. To test this, relative efficiency was measure over the course of ten early rounds and over the last 10 rounds. We defined the difference between the end round and the beginning rounds as *learning*. This is not a perfect metric however, as round demand was not the same for the early ten and final ten periods. Despite this, any differences between treatments would indicate differences in how participants learned between the environments.

Result 4. *Beginning and ending median relative efficiencies were not significantly different from one another, indicating that subjects were not learning in this environment.*

Figure 10 shows the differences in beginning and ending median efficiencies for each observation. A ranked two-way ANOVA (Table 6) identified no significant differences in learning between rationing mechanisms ($p = 0.56$). Any significant differences in group learning were also undetected ($p = 0.85$). Although learning may have occurred, there was no discernable difference is how participants learned between groups or allocation mechanisms.

Figure 10

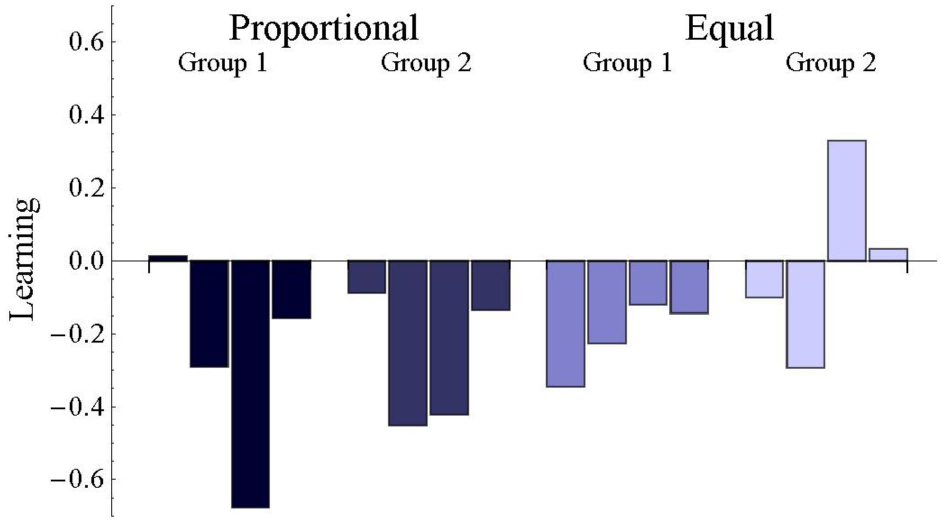


Figure 10 shows the learning (measured as the difference between beginning and ending relative efficiencies) by session. Learning was not found to be significantly different between either allocation mechanisms or groups. Most groups tended to perform worse over time, indicating that the ending demand scenarios were more difficult than the beginning ones.

Table 6: Ranked Difference in Median Relative Efficiencies by Group and Treatment (N = 16)

	Group 1	Group 2	Marginal Rank Means
Proportional	9.75	10.00	9.875
Equal	9.25	5.00	8.125
Marginal Rank Means	9.50	7.50	

Each cell contains the mean for the ranks of the difference in early and end session median efficiencies. The marginal rank means are shown in the last row/column. Despite the strong divergence in the efficiencies of each treatment, there is not enough data to support a conclusion of different efficiency trends over time.

6. Conclusions

Overall, we have found that the Equal Rationing mechanism outperforms the Proportional Rationing mechanism in the allocation of units. The Equal Rationing allocated units with higher efficiency at a lower cost to participants. There were very few cases of participants buying more units than necessary with Equal Rationing, compared to significant overbuying with Proportional Rationing. There was no difference found in efficiency progressed with time.

Meanwhile, separating participants by quiz score yielded significant changes. Those who performed best on the quiz had higher efficiencies and fewer cases of overbuying. However, they did not show any difference in auction prices.

California’s cap and trade program could benefit from redesigning the Reserve Sale allocation mechanism. By trying to keep quarterly allowance auction prices low, CARB and California’s cap and trade stakeholders inadvertently created a supplemental mechanism which could pose a risk to covered entities. Evidenced by the experiments and the computational modeling, it is clear that the Proportional Rationing mechanism currently slated to determine allowance allocations in the Reserve Sale leads to accidental over-buying and low efficiency as a result of demand uncertainty.

A possible alternative CARB can consider is the Equal Rationing mechanism which consistently outperforms the Proportional Rationing mechanism in experiments. This mechanism reduces the complexity of participating in the Reserve Sale because it eliminates the incentive to over demand allowances when subjects believed demand for allowances to be high. Additionally, the bureaucratic burden of switching to this mechanism would be minimal because it maintains the general structure of the Reserve Sale, but simply changes the method by which allowances are allocated. The Equal Rationing mechanism benefits the covered entities by minimizing their strategy space such that it is always a dominant strategy to reveal one’s true need in the Reserve Sale. Thus the Equal Rationing mechanism answers all of the stakeholder concerns: depresses prices in the quarterly allowance auction, reduces uncertainty and maintains an institution where covered entities can buy allowances at fixed prices.

While the goal of this paper was to investigate the Reserve Sale allocation mechanism CARB has currently designed and compare it to an alternative, the conclusions drawn throughout this paper question whether the Reserve Sale is needed at all. Another option CARB could employ is integrating the Reserve Sale supply into the quarterly allowance auction. Additional research on this topic could shed light on whether the Reserve, regardless of its allocation mechanism, is helpful or harmful in the long run by selling allowances at fixed prices. There are two ways that CARB could effectively eliminate the Reserve Sale. They could either add allowances from the Reserve into the quarterly allowance auction if the auction clearing price reached a certain level or they could simply add the entire supply of the Reserve to each allowance auction. While this paper cannot address those institutions directly, it is clearly a worthwhile research endeavor to fully understand the effects and impact of the Reserve Sale in California’s cap and trade program.

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APPENDIX A

Formal representation of the Equal Rationing Mechanism

Equation 5 defines the formal process used to determine what each participant should be allocated in the Equal Rationing environment. Equation 5 calls itself until the stopping condition describe above is met. Each time k is called, it updates the allocation vector and the number of units remaining, Q, which is the number of units in the tier minus the sum of all the elements in the allocation vector. When the stopping condition is met, the allocation vector is returned. The function k takes the following parameters:

- The bid vector, $\vec{B}$.
- The allocation vector, $\vec{A}$, which is initially a vector of zeros.
- The number of units remaining in the tier, Q, which is initially Q_j.
- The number of units available in the tier, Q_j.

$$k(\vec{B},\vec{A},Q,Q_j) = \begin{cases} k\left(\vec{B}, h(\vec{B},\vec{A},Q), Q_j - \sum_{i=1}^{8} h(\vec{B},\vec{A},Q)_i, Q_j\right), & \sum_{i=1}^{8} \vec{A}_i < Min\left[Q_j, \sum_{i=1}^{8} B_i\right] \\ \vec{A}, & otherwise \end{cases} \quad (5)5)$$

Equation 5.1 is used to determine how many units are allocated to each participant. This equation returns an allocation vector, $\vec{A}$.

$$h(\vec{B},\vec{A},Q) = \left\{ g\left(\vec{B}_1, \vec{A}_1, \frac{Q}{\sum_{i=1}^{8} f(\vec{B}_i,\vec{A}_i)}\right), \cdots, g\left(\vec{B}_8, \vec{A}_8, \frac{Q}{\sum_{i=1}^{8} f(\vec{B}_i,\vec{A}_i)}\right) \right\} \quad (5.1)$$

The actual allocation that an participant receives is given by the equation 5.2, which takes an participant's quantity bid, B_i, the number of units they have already been allocated, A_i, and the number of units being offered to each participant at this level in the recursion, Q^l. The maximum number of units that can safely be offered to each participant at a given recursion depth without taking their quantity bid and current allocation into account is the number of units still available, Q, divided by the number of participants with outstanding bids, $\sum_{i=1}^{8} f(B_i, A_i)$. If the number of units an participant requested minus the number he has already been allocated is greater than the number of units being offered, then the participant is given that quantity in addition to their current allocation. Otherwise, the participant is allocated an amount equal to the number of units he requested.

$$g(B_i, A_i, Q^l) = \begin{cases} Q^l + A_i, & B_i - A_i > Q^l \\ B_i, & otherwise \end{cases} \quad (5.2)$$

Equation 5.3 is used to determine whether a participant has an active bid given the current allocation vector. It returns 1 if an participant's bid minus the number of units he has currently been allocated is greater than zero; otherwise, 0 is returned.

$$f(B_i, A_i) = \begin{cases} 1, & B_i - A_i > 0 \\ 0, & otherwise \end{cases} \tag{5.3}$$

Since our environment has exactly two tiers, the number of units an participant receives in Tier 1, q_{i1}, is:

$$q_{i1} = k\left(\vec{B}, \vec{A}, Q_1, Q_1\right)_i \tag{5.4}$$

And the number of units an participant receives in Tier 2, q_{i2}, is:

$$q_{i2} = k\left(\vec{B} - k\left(\vec{B}, \vec{A}, Q_1, Q_1\right), \vec{A}, Q_2, Q_2\right)_i \tag{5.5}$$

APPENDIX B

Experiment Instructions, Quiz & Quiz Answers

This appendix provides all of the instructions, quizzes and quiz answers used in our experiments. Since our experiment had two treatments, the instructions, quizzes and quiz answers that differed between the two treatments are noted throughout.

Instructions

This is an experiment in market decision-making. There will be 32 rounds in the experiment. Each round is divided into two Periods, Period A and Period B.

You are one of several Participants who will buy units. You will have two opportunities to acquire units each round, first in Period A and then in Period B. Your earnings will depend on your decisions and the decisions of others in your group. Participants will be divided into 2 groups, each containing 8 participants. You will stay with your group for the duration of the experiment.

At all times during the experiment, your **Cumulative Earnings** are shown in the box at the top right of your screen. Your Cumulative Earnings are the sum of your earnings from all of the rounds in the experiment. At the end of the experiment, your Cumulative Earnings will be exchanged at a rate of 1 US dollar for every 1200 Experimental Dollars.

In this experiment you make money by buying units which have value to you. Each participant is given a **Unit Value**. The Unit Value represents your value for one unit. For example, this participant's value for one unit is 60.

Each participant is also given a **Holding Limit**. The Holding Limit represents the maximum number of units that give you value. For example, this participant's holding limit is 40. Any units beyond 40 will not return any value to the participant.

If you acquired units in Period A, the number of units you acquired will be deducted from your Holding Limit. Your Holding Limit for Period B will be your original Holding Limit minus any units acquired in Period A. If you acquired all the units which give you value in Period A, meaning your Period B Holding Limit is zero, your Holding Limit box will be highlighted in green and a bid for 0 units in Period B will be automatically submitted for you.

You will receive a new Unit Value and a new Holding Limit every round.

If you purchase more units than you have value for, you will have to pay for those units but you will not receive any value from them, resulting in potential losses. Losses in profit are deducted from your **Working Capital** account.

This participant has 1000 in Working Capital to cover any losses in profit.

If your Working Capital ever falls below the amount you started with, any future profits will first be paid back into the Working Capital account until it is fully replenished. Once your Working Capital is replenished, you will start earning money again. If your Working Capital ever falls below 0, you will be penalized by having 6000 E taken from your earnings ($5.00). After being penalized, your Working Capital will be reset to 1000 E.

Your first opportunity to buy units is in Period A. In Period A, there are a total of 60 units available for sale. In this period, every participant can submit the number of units they wish to buy and the price at which they wish to buy them. You will have 60 seconds to submit a bid for units.

There are three steps to place your bid in Period A:

1. Type the price at which you wish to buy each unit
2. Enter the number of units you would be willing to purchase at that price in the quantity box
3. Click the Submit Button

In the example on the screen, the participant has placed a bid to buy 15 units at a price of 10 each. You can see he has pressed the submit button because a confirmation message appeared in the Confirmation Box displaying his bid.

At the end of Period A, the **Auction Results** will appear on your screen. This will show you the price you bid for units in Period A, the **Auction Price** and how many units you acquired in Period A.

If your bid in Period A is **above** the Auction Price, you will receive the units you requested.

If your bid in Period A is **below** the Auction Price, you will receive none of the units you requested.

If your bid in Period A is **equal to** the Auction Price, you may receive some or none of the units you requested.

In the example on the screen, the participant paid 8 for the 15 units he requested. Notice, the Auction Price is below his bid price of 10.

Let's go through an example of how the Auction Price is determined.

5 participants submit bids for the number of units they wish to buy and the price they wish to pay for each unit. In this case, each participant's bid is highlighted a different color to illustrate the individual bids.

Bids are sorted by the price they bid from highest to lowest. The highest bid price in this case is 70 and the lowest is 30.

Starting with the highest bid price, units are allocated to the participants until there are no more units left. The red line marks the total amount of units available for allocation. In this case, it is 50. The Auction Price is equal to the price of the last bid to receive units (the yellow participant). They will only receive a portion of the 25 units they requested.

Below the Period A Auction Results, the results of Period A will appear.

The Period A Results box shows your **Period A Profit**. Your Period A Profit is calculated from the value of the units you acquired minus the cost of the units you acquired. The Period A Results will also show any changes in your Working Capital account. Finally, the Period A Results will report your earnings for that period as well as your **New Holding Limit**. Remember, your New Holding Limit is the difference between your Holding Limit and the units you acquired during Period A.

Your second opportunity to buy units is in Period B. In Period B, there are two Tiers, both of which have 50 units up for sale. In this period, participants can submit the number of units they wish to buy in each tier at the posted Tier prices. Tier 1 units are priced at 20 and Tier 2 units are priced at 30. Units in Tier 1 and Tier 2 are identical and have no differences other than their price. You will have 60 seconds to submit a bid for units.

Proportional Rationing Mechanism Instructions

There are three steps to place your bid in Period B:

1. Enter your quantity bid for Tier 1 in the Tier 1 quantity box
2. Enter your quantity bid for Tier 2 in the Tier 2 quantity box
3. Click the Submit Button

In the example on the screen, the participant has placed a quantity bid of 20 units in Tier 1 and 10 units in Tier 2. You can see he has pressed the submit button because a confirmation message appeared in the Confirmation Box displaying his bid.

At the end of Period B, the **Sales Results** will appear on your screen. This will show you how many units you acquired in both Tier 1 and Tier 2. In the example on the screen, the participant received 15 units from Tier 1 and 10 units from Tier 2. Remember, he had placed a bid for 20 Tier 1 units and 10 Tier 2 units.

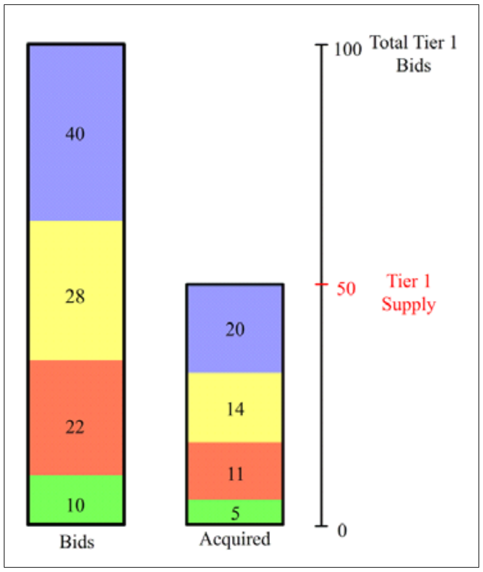
Determining how many units you will acquire from each Tier depends on how many units you and all of the other participants requested.

If the total quantity requested by all of the participants is **less than or equal to** the amount available in either Tier, participants will receive what they have requested.

If the total quantity requested by all of the participants is **greater than** the amount available in either Tier, participants will receive a portion of what they requested.

Let's go through an example of how units are allocated at the end of Period B.

Proportional Rationing Mechanism Example



4 participants submit bids for the number of units they wish to buy in Tier 1. Bids are summed to find the total quantity requested by all of the participants. In this case, the total quantity requested is 100 units. Notice that each participant's bid is highlighted a different color to illustrate the individual bids.

In this example, Tier 1 only has 50 units available to allocate to participants. Bids are reduced proportionally to the actual amount available. Notice that the total quantity available is half the amount of the total quantity requested. Each bid is then reduced by half. For example, the Green Participant submitted a bid for 10 units and only received 5 units.

Equal Rationing Mechanism Instructions

There are two steps to place your bid in Period B:

1. Type the total quantity you wish to buy in the Max Quantity box
2. Click the Submit Button

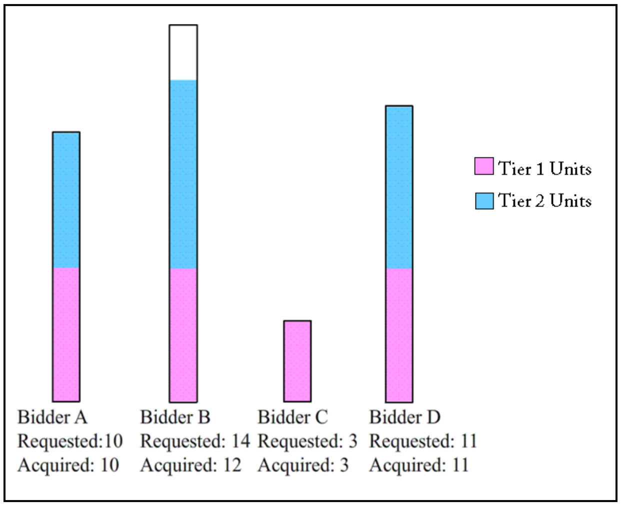
In the example on the screen, the participant has requested to buy 20 units. You can see he has pressed the submit button because a confirmation message appeared in the Confirmation Box displaying his bid.

At the end of Period B, the **Sales Results** will appear on your screen. This will show you how many units you acquired in both Tier 1 and Tier 2. Determining how many units you will acquire from each Tier depends on how many units all of the other participants requested.

Below is an example set of bids placed by 4 participants. Each participant's bid is shown as an empty bar.

Units are allocated equally among bidders until either a bidder's request has been satisfied or the Tier is exhausted.

Equal Rationing Mechanism Example



In the example below, Bidder C has only requested 3 units and is allocated 3 units. Bidders A, B and D requested more units than Tier 1 can satisfy.

This process is then repeated for Tier 2. If more units are requested from Tier 2 than there are units available, some participants will not receive all of the units they requested. No participant will ever receive more units than they requested.

Below the Period B Sale Results, the results of Period B will appear.

The Period B Results box shows your **Period B Profit**. Your Period B Profit is calculated from the value of the units you acquired in Tier 1 and Tier 2 minus the cost of the units. The Period B Results will also show any changes in your Working Capital account. Finally, the Period B Results will report your earnings for that period.

On the right side of your screen you will see the results of past rounds in your History Table.

The first two columns in the History Table indicate the Round Number and Period (either A or B). The 3rd column shows the **Total Units** you won during a given Period. The 4th column is the **Total Value** column. This shows the Total Value of the units bought in a given Period. The 5th column is the **Total Cost** column. This calculates the Total Cost of the units the participant bought in a given Period. Finally, the 6th column on the History Table shows your **Profit** for a given Period. Columns in the History Table are calculated the same way for Period B and allow easy comparison of your past performance between the two Periods.

To recap, here are the important things to remember:

Period A

- Your first opportunity to buy units is in Period A. All participants will submit a bid for how many units they wish to buy and at what price they wish to pay for those units.

- Units in Period A are allocated based on the Auction Price. Participants who are allocated units in Period A pay the Auction Price for the units they acquire.

Period B (Proportional Rationing)

- Your second opportunity to buy units is in Period B. The units are sold in two Tiers at two fixed prices, 20 and 30. To purchase units in Period B, enter the number of units you wish you buy at each price.

- Units are allocated based on the total quantity requested for each Tier. If the total quantity requested for a tier is **equal to or less** than the amount in the tier, every participant will get their requested amount. If the total quantity requested for a tier is **greater than** the amount in the tier, participants will get a portion of their requested amount.

Period B (Equal Rationing)

- Your second opportunity to buy units is in Period B. The units are sold in two Tiers at two fixed prices, 20 and 30. To purchase units in Period B, enter the number of units you wish you buy.

- Units are allocated based on the total quantity requested by all participants. Units will first be allocated from Tier 1 at a price of #Tier1Price# and then from Tier 2 at a price of #Tier2Price#.

Other Details

- You only have value for units up to your Holding Limit.

- If your Total Cost for the units you acquire in a period is greater than the Total Value you receive from them, the period losses will be subtracted from your Working Capital. Once your Working Capital is replenished, you can begin to start earning value from purchasing units again.

- Participants will be divided into 2 groups, each containing 8 participants. You will stay with your group for the duration of the experiment.

If you have any questions before or during the experiment, please raise your hand and a monitor will come by to answer your question privately.

We will now have a short quiz. You will be paid $0.25 for each correct answer. At the end of the quiz, you will be given an opportunity to check your answers before submitting.

Experiment Quiz

Use the following information to answer questions 1-3:
Suppose your Unit Value is 20 and your Holding Limit is 10. You purchase 5 units in Period A.

1) What is your Value of Units Acquired for Period A?

2) If you obtained a total of **10** units in Period A and B, what would your Value of Units Acquired be for the entire round (Period A and Period B)?

3) If you obtained a total of **20** units in Periods A and B, what would your Value of Units Acquired be for the entire round (Period A and Period B)?

Use the following information to answer questions 4-6:
Suppose there are 50 units available in Period A. The other 7 participants bids are as follows:

Price: 10, Units: 10
Price: 20, Units: 10
Price: 30, Units: 15
Price: 50, Units: 15
Price: 60, Units: 10
Price: 70, Units: 10
Price: 80, Units: 10

4) If you place a bid for 10 units at a price of **20**, what will be the Price? How many units will you receive?

5) If you place a bid for 10 units at a price of **55**, what will be the Price? How many units will you receive?

6) If you place a bid for 10 units at a price of **40**, what will be the Price? How many units will you receive?

Proportional Rationing Questions

Use the following information to answer questions 7-9:
Suppose there are 60 units available in Period B. The cost for each unit is 25. You have a Holding Limit of 15 and a Unit Value of 30. The other participants request a total of 60 units.

7) If you request **15** units, how many units will you receive? What is your profit?

8) If you request **20** units, how many units will you receive? What is your profit?

9) If you request **30** units, how many units will you receive? What is your profit?

Equal Rationing Questions

Use the following information to answer questions 7-9:
Suppose there are 100 units available in Period B. The cost for each unit is 25. You have a Holding Limit of 15 and a Unit Value of 30. The other participants request a total of 70 units, or 10 units for each of the other 7 participants.

7) If you request **10** units, how many units will you receive? What is your profit?

8) If you request **15** units, how many units will you receive? What is your profit?

9) If you request **20** units, how many units will you receive? What is your profit?

10) If your Working Capital is **1000** and your Period Profit is **-100**, what will be your New Working Capital? How much will your Earnings change by?

11) If your Working Capital is **900** and your Period Profit is **500**, what will be your New Working Capital? How much will your Earnings change by?

You may now submit your answers to the quiz. Before submitting, you may have your answers checked once. Feel free to go back and fix any questions you got wrong.

After submitting, please wait patiently until the experiment begins. You may review both the instructions and quiz while waiting.

Experiment Quiz Answers

1) Because the number of units you purchased did not exceed your holding limit, you received value for each of the 5 units. Since your Unit Value is 20, you therefore receive a value of 20 for each of the 5 units. 20 x 5 = 100.

2) You obtained a total of 10 units. Because this does not exceed your holding limit, you received value for each of the 10 units. Since your Unit Value is 20, you therefore receive a value of 20 for each of the 10 units. 20 x 10 = 200.

3) You obtained a total of 20 units. However, because your Holding Limit is only 10, you only receive value for 10 of the 20 units you obtained. Since your Unit Value is 20, you therefore receive a value of 20 for each of only 10 units. 20 x 10 = 200.

Please remember that any units obtained past your holding limit will be worthless to you.

4) The price is determined by giving out units in order from the highest priced offer to the lowest. Placing your bid in with the others and determining how many units each receives yields the following:

Price: 10,	Units: 10,	**Received: 0**	Price: 50,	Units: 15,	**Received: 15**
Price: 20,	Units: 10,	**Received: 0**	Price: 60,	Units: 10,	**Received: 10**
Price: 20,	**Units: 10,**	**Received: 0**	Price: 70,	Units: 10,	**Received: 10**
Price: 30,	Units: 15,	**Received: 5**	Price: 80,	Units: 10,	**Received: 10**

Since the last person to receive units offered to buy at a price of 30, the final price is 30. You received 0 units since your price was below 30.

5) The price is determined by giving out units in order from the highest priced offer to the lowest. Placing your bid in with the others and determining how many units each receives yields the following:

Price: 10,	Units: 10,	**Received: 0**	**Price: 55,**	**Units: 10,**	**Received: 10**
Price: 20,	Units: 10,	**Received: 0**	Price: 60,	Units: 10,	**Received: 10**
Price: 30,	Units: 15,	**Received: 0**	Price: 70,	Units: 10,	**Received: 10**
Price: 50,	Units: 15,	**Received: 10**	Price: 80,	Units: 10,	**Received: 10**

Since the last person to receive units offered to buy at a price of 50, the final price is 50. You received 10 units since your price was above 50.

6) The price is determined by giving out units in order from the highest priced offer to the lowest. Placing your bid in with the others and determining how many units each receives yields the following:

Price: 10,	Units: 10,	**Received: 0**	Price: 50,	Units: 15,	**Received: 15**
Price: 20,	Units: 10,	**Received: 0**	Price: 60,	Units: 10,	**Received: 10**
Price: 30,	Units: 15,	**Received: 0**	Price: 70,	Units: 10,	**Received: 10**
Price: 40,	Units: 10,	**Received: 5**	Price: 80,	Units: 10,	**Received: 10**

Since the last person to receive units offered to buy at a price of 40, the final price is 40. You were at the Price, so you received however many units were leftover, which was 5.

Proportional Rationing Question Answers

7) You requested 15 units. This brings the total requested number of units to 75. Since there are only 60 units available, each person will receive 60/75 = 4/5 of what they requested. Since you requested 15, you will receive 4/5 x 15 = 12 units.

Since your Holding Limit is 15, you will receive value for all 12 units. Each one gives you a value of 30, but costs 25. This means your total profit is (30 x 12) – (25 x 12) = (5 x 12) = 60.

8) You requested 20 units. This brings the total requested number of units to 80. Since there are only 60 units available, each person will receive 60/80 = 3/4 of what they requested. Since you requested 20, you will receive 3/4 x 20 = 15 units.

Since your Holding Limit is 15, you will receive value for all 15 units. Each one gives you a value of 30, but costs 25. This means your total profit is (30 x 15) – (25 x 15) = (5 x 15) = 75.

9) You requested 30 units. This brings the total requested number of units to 90. Since there are only 60 units available, each person will receive 60/90 = 2/3 of what they requested. Since you requested 30, you will receive 2/3 x 30 = 20 units.

Since your Holding Limit is 15, you will only receive value for 15 of your 20 units acquired. Each one gives you a value of 30, but costs 25. This means your total profit is (30 x 15) – (25 x 20) = (5 x 15) – (5 x 25) = -50.

Equal Rationing Question Answers

7) You requested 10 units. This brings the total requested number of units to 80. Since there are 100 units available, each person will receive what they requested. Since you requested 10, you will receive 10 units.

Since your Holding Limit is 15, you will receive value for all 10 units. Each one gives you a value of 30, but costs 25. This means your total profit is (30 x 10) – (25 x 10) = (5 x 10) = 50.

8) You requested 15 units. This brings the total requested number of units to 85. Since there are 100 units available, each person will receive what they requested. Since you requested 15, you will receive 15 units.

Since your Holding Limit is 15, you will receive value for all 15 units. Each one gives you a value of 30, but costs 25. This means your total profit is (30 x 15) – (25 x 15) = (5 x 15) = 75.

9) You requested 20 units. This brings the total requested number of units to 90. Since there are 100 units available, each person will receive what they requested. Since you requested 20, you will receive 20 units.

Since your Holding Limit is 15, you will only receive value for 15 of your 20 units acquired. Each one gives you a value of 30, but costs 25. This means your total profit is (30 x 15) – (25 x 20) = (5 x 15) – (5 x 25) = -50.

10) Any negative profits come out of your Working Capital rather than your earnings.
Since your Period Profit was -100, your Working Capital decreases from 1000 to 900. You neither gain nor lose Earnings, so your Earnings change by 0.

11) Any positive Period Profits first go into your Working Capital. If your Working Capital reaches 1000, any additional amount will instead go into your Earnings. Since your Working Capital is 900, the first 100 of your Period Profits goes into your Working Capital, raising it to 1000. The remaining 400 of the Period Profits go into your Earnings, increases them by 400.

APPENDIX C

We used Mathematica to create a program that searched for pure strategy equilibria in the eight parameter sets used in the experiment. Table 8 shows the holding limits and values that comprised each of the parameter sets. The program simulated the eight players best responding to one another in an environment that did not change (the same parameter set and mechanism was used for each simulation). Being that it was possible for multiple pure strategy equilibria to exist for a single parameter set; each parameter set was searched multiple times. Each simulation started with the players submitting random bid vectors that were drawn uniformly between the bounds provided by the constraints for each bid type. A simulation consisted of multiple iterations the number of which depended on how long it took to find a pure strategy equilibrium or cycle. Over the course of an iteration, each player was able to best respond to the other strategy sets of the other players. After a player chose its new strategy set, the next player was allowed to best respond. Once every player had a chance to update its strategy set, the next iteration would start. The order that the players best responded was chosen randomly at the start of each iteration. A simulation concluded when a pure strategy equilibrium was found or the players fell into a repeating cycle of best responding.

Table 7: Parameter Sets

Demand Cases H									
Values		105	95	85	75	65	55	45	35
Holding Limits	High (H)	39	35	31	28	25	21	17	14
	Med-High (H)	31	28	25	22	20	17	14	11
	Med-Low (H)	28	25	22	20	18	15	13	10
	Low (H)	20	18	16	14	13	11	9	7

Demand Cases L									
Values		105	95	85	75	65	55	45	35
Holding Limits	High (L)	14	17	21	25	28	31	35	39
	Med-High (L)	11	14	17	20	22	25	28	31
	Med-Low (L)	10	13	15	18	20	22	25	28
	Low (L)	7	9	11	13	14	16	18	20

Table 8 shows how holding limits changed depending on the values for units and the demand case. For example, in the Low (L) case, one player had a value of 105 per unit and a holding limit of 7 and another player had a value of 95 per unit and holding limit of 9. Players never knew the demand case they were in, they only their own value for a unit and their own holding limit.

After a pure strategy equilibrium was found, an output was produced, which contained the all of the bids for each of the players. To illustrate what is meant by a Nash Equilibrium strategy set, we have selected such a strategy set from each mechanism that were produced using the High (H) parameter set.

Proportional Rationing Mechanism Nash Equilibrium

The most common auction price given the High (H) parameter set is 75. The Mathematica program found a Nash Equilibrium strategy set that produces this auction price in the proportional rationing mechanism. The strategy set for all eight players is:

Table 2: Proportional Rationing Strategy Set Example

	Player							
Bid	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Auction Price	105	95	85	75	65	55	45	35
Auction Qty	26	16	18	28	25	21	17	14
Tier 1 Qty	50	50	50	50	50	50	50	50
Tier 2 Qty	50	50	50	50	50	50	50	50

In this example, notice that Player 1's strategy is to submit an auction price bid of 105 and auction quantity bid of 26 units for Period A. He also submits a bid for 50 units in Tier 1 and 50 units in Tier 2. Similarly, Player 8 will submit an auction price bid of 35 and an auction quantity bid of 14 units. He, like all of the other players, will also submit a bid for 50 units in Tier 1and 50 units in Tier 2.

Using this strategy set, the allocations the players will receive are found in Table 10:

Table 3: Proportional Rationing Allocation Example

	Player							
Allocation	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Auction Qty	26	16	18	0	0	0	0	0
Tier 1 Qty	6.25	6.25	6.25	6.25	6.25	6.25	6.25	6.25
Tier 2 Qty	6.25	6.25	6.25	6.25	6.25	6.25	6.25	6.25
Total	**38.5**	**28.5**	**30.5**	**12.5**	**12.5**	**12.5**	**12.5**	**12.5**

In this example, notice that Player 1 receives the 26 units requested in Period A and only 6.25 units from Tier 1 and 6.25 units from Tier 2. In total, Player 1 receives 38.5 units. In fact, all players receive the same number of units from Tier 1 and Tier 2.

This is a Nash Equilibrium because no player can make themselves better off by changing his bids. To examine this more closely, let's look at each player's strategy:

Player 1

	Auction	Tier 1	Tier 2
Bids	Price: 105 Qty: 26	50	50
Allocation	26	6.25	6.25

Total Acquired	38.5
Holding Limit	39
Difference	-0.5

This player's bids resulted in him obtaining 38.5 units. The player would like to obtain 0.5 additional units at a price below their value. As the units in the tiers currently cost less than units in the auction, this is where the player would like to obtain units first. However, he has already submitted the maximum bid in each of the tiers and is unable to increase the number of tier units obtained. The auction price is currently below the value that Player 1 has for the addition 0.5 units, but if he increases his bid to {105, 27} the auction price would increase to 85. This would not make the player better off as the player only has value for 0.5 of the unit, meaning that the value obtained would be 52.5 and the cost of the unit would be 85. In addition, raising the auction price would reduce the value obtained on the previous 26 units by 10 each. Thus, Player 1 would not want to change any of their bids.

Player 2

	Auction	Tier 1	Tier 2
Bids	Price: 95 Qty: 16	50	50
Allocation	16	6.25	6.25

Total Acquired	28.5
Holding Limit	35
Difference	-6.5

This player, like Player 1, would like additional units, and cannot obtain them in either of the tiers. The current profit obtained from the units purchase in the auction is 320, $(95 - 75) * 16$. If this player increase her auction quantity bid by even 1 additional unit, the auction price would increase to 85. Increasing the auction price by 10 would cut the profit obtained from the existing auction units obtained in half, a loss of 160. Each new unit purchased at the new auction price would produce a marginal profit of 10, so even if the player acquired all 6 units, she would still have 100 less profit. Thus, Player 2 would not want to change any of their bids.

Player 3

	Auction	Tier 1	Tier 2
Bids	Price: 85 Qty: 18	50	50
Allocation	18	6.25	6.25

Total Acquired	30.5
Holding Limit	31
Difference	-0.5

This player is in the same position as Players 1 and 2: he has value for additional units and cannot get any more tier units. For this player, increasing the auction quantity by even one would reduce the marginal profit of auction units from 10 to 0. Thus, Player 3 would not want to change any of their bids.

Players 4 – 8

Players 4 through 8 are all in the same position. They have all obtained the most units they can from each of the tiers and cannot obtain any units in the auction without submitting an auction

price bid that is higher than their value. Thus, Players 4 – 8 would not want to change any of their bids. Since none of the player's would like to change their bids given the bids of the other players, this is a Nash Equilibrium strategy set.

Equal Rationing Mechanism Nash Equilibrium

Now, we go through a similar example of a Nash Equilibrium strategy set produced by the Mathematica program for the same demand case using the Equal Rationing Mechanism. This strategy set, like the previous example, produces an auction price of 75. The strategy set for all eight players is:

Table 4: Equal Rationing Strategy Set Example

	Player							
Bid	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Auction Price	105	95	85	75	65	55	45	35
Auction Qty	25	22	13	28	25	21	17	14
Total Tier Qty	39	35	31	28	25	21	17	14

In this example, notice that Player 1's strategy is to submit an auction price bid of 105 and auction quantity bid of 25 units for Period A. He also submits a tier bid of 39.

Using this strategy set, the allocations the players will receive are found in Table 12:

Table 5: Equal Rationing Allocations Example

	Player							
Allocation	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Auction Qty	25	22	13	0	0	0	0	0
Tier 1 Qty	6.25	6.25	6.25	6.25	6.25	6.25	6.25	6.25
Tier 2 Qty	6.25	6.25	6.25	6.25	6.25	6.25	6.25	6.25
Total	**37.5**	**34.5**	**25.5**	**12.5**	**12.5**	**12.5**	**12.5**	**12.5**

In this example, notice that Player 1 receives the 25 units requested in Period A and only 6.25 units from Tier 1 and 6.25 units from Tier 2. In total, Player 1 receives 37.5 units.

Like the previous example, this is a Nash Equilibrium because no player can make themselves better off by changing her bids. To examine this more closely, let's look at each player's strategy:

Player 1

	Auction	Tier 1	Tier 2
Bids	Price: 105 Qty: 25	39	
Allocation	25	6.25	6.25

Total Acquired	37.5
Holding Limit	39
Difference	-1.5

This player's bids result in him obtaining 37.5 units. The player would like to obtain 1.5 additional units at a price below their value. The player will look to obtain units in the tiers first as the current auction price is greater than the tier prices. However, the player has already obtained the maximum number of units in the tiers that they can under the equal rationing mechanism, which is apparent by the fact that the number of tier units obtained is less than the player's tier bid. This means that the player can only obtain additional units from the auction. If the player increases his auction quantity bid by one unit, the auction price will increase from 75 to 85, which will lower the profit of the player by 250. Since the marginal increase in profit from the additional unit is only 10, the player is better off with his current bid set.

Player 2

	Auction	Tier 1	Tier 2
Bids	Price: 95 Qty: 22	35	
Allocation	22	6.25	6.25

Total Acquired	34.5
Holding Limit	35
Difference	-0.5

This player, like Player 1, would like additional units, and cannot obtain them in either of the tiers. Also, obtaining any additional units in the auction would cause the auction price to go up and the reduction in profit due to an auction price increase is greater than the profit obtained from the additional 0.5 units. Thus, this player cannot make herself better off by changing her bids.

Player 3

	Auction	Tier 1	Tier 2
Bids	Price: 85 Qty: 13	31	
Allocation	13	6.25	6.25

Total Acquired	25.5
Holding Limit	31
Difference	-0.5

This player is in the same position as Players 1 and 2: he has value for additional units and cannot get any more tier units. For this player, increasing the auction quantity by even one would reduce the marginal profit of auction units from 10 to 0. Thus, Player 3 would not want to change any of their bids.

Players 4 – 8

Players 4 through 8 are all in the same position. They have all obtained the most units they can from each of the tiers and cannot obtain any units in the auction without submitting an auction price bid that is higher than their value. Thus, Players 4 – 8 would not want to change any of their bids. Since none of the player's would like to change their bids given the bids of the other players, this is a Nash Equilibrium strategy set.

The frequencies of Nash Equilibria auction prices for each demand case can be found in Table 8. For example, consider the High (H) demand scenario. Table 8 shows that under the Proportional Rationing treatment, an auction price of 75 occurred 41.8% of the time during the simulation. Similarly, under the Equal Rationing treatment, an auction price of 75 occurred 56.4% of the time in the simulation. Additionally, for demand cases where demand is Low, Med-Low and Med-High (L), the auction price could always be equal to zero if participants are able to perfectly respond to one another's bids. This simulation shows that the auction prices which occur in the allowance auction are the same (despite different frequencies) regardless of the allocation mechanism.

Table 8: Frequency of Auction Prices

Proportional Rationing							
Auction Price		0	35	45	55	65	75
Frequency	High (H)	0.013	0.010	0.032	0.119	0.408	0.418
	High (L)	0.240	0.458	0.302			
	Med-High (H)	0.717	0.169	0.061	0.052		
	Med-High (L)	1.000					
	Med-Low (H)	1.000					
	Med-Low (L)	1.000					
	Low (H)	1.000					
	Low (L)	1.000					

Equal Rationing							
Auction Price		0	35	45	55	65	75
Frequency	High (H)	0.002	0.003	0.012	0.058	0.362	0.564
	High (L)	0.182	0.425	0.393			
	Med-High (H)	0.631	0.181	0.109	0.079		
	Med-High (L)	1.000					
	Med-Low (H)	1.000					
	Med-Low (L)	1.000					
	Low (H)	1.000					
	Low (L)	1.000					

APPENDIX D

Appendix D contains the Mathematica code used to run the simulations to test our model.

PROPORTIONAL RATIONING CODE:

```
AuctionPrice[allBids_,supply_]:=
 Module[{bids,units,price,index},
  bids = Sort[allBids,#1[[1]]>#2[[1]]&];
  units = supply;
  price=-1;
  If[Total[bids[[All,2]]]≤ units,
   price=0,
   index=1;
   While[units> 0,
    units-= bids[[index,2]];
    If[units≥0,
     index+=1]
    ];
   price=bids[[index,1]];
   ];
  Return[price];
  ]

AuctionUnits[yourBid_,allBids_,supply_]:=
 Module[{bidPosition,bids,bidTies,remainingUnits,unitsObtained,tiePosition},
  bids = Sort[allBids,#1[[1]]>#2[[1]]&];
  bidPosition = Extract[Flatten@Position[bids,yourBid],1];
  bidTies = Flatten@Position[bids[[All,1]],yourBid[[1]]];
  tiePosition = Flatten@Position[bidTies,bidPosition];
  remainingUnits = supply-Total[bids[[1;;bidTies[[1]]-1,2]]];
  unitsObtained=0;
  If[remainingUnits > 0,
   If[Length@bidTies≤1,
    If[remainingUnits ≥ yourBid[[2]],
     unitsObtained = yourBid[[2]],
     unitsObtained = remainingUnits],
    If[Total[bids[[bidTies[[1]];;bidTies[[-1]],2]]] ≤ remainingUnits,
     unitsObtained=yourBid[[2]],
     unitsObtained = TieAllocation[bids[[bidTies[[1]];;bidTies[[-
1]],2]],remainingUnits][[tiePosition[[1]]]];
     ]]];
  Return[unitsObtained];
  ]
```

```
TieAllocation[bids_,units_]:=
 Module[{bidsRemaining, unitsRemaining,count},
  bidsRemaining = bids;
  unitsRemaining = units;
  While[unitsRemaining>0,
   count = Length@bidsRemaining-Count[bidsRemaining,0];
   If[unitsRemaining ≥ count,
    unitsRemaining -= count;
    Do[
     If[bidsRemaining[[i]]>0,
      bidsRemaining[[i]] -= 1;]
     ,{i,1,Length@bidsRemaining}],
    Do[
     If[bidsRemaining[[i]]>0,
      bidsRemaining[[i]] -= unitsRemaining/count;]
     ,{i,1,Length@bidsRemaining}];
    unitsRemaining = 0;
    ]
   ];
  Return[bids-bidsRemaining];
  ]

TierUnits[yourBid_,yourTSwitch_,bids_,tSwitches_,units_]:=
 Module[{totalDemand,unitsObtained,wholeUnits,fractionalUnits},
  totalDemand=Total[bids];
  If[totalDemand>units,
   wholeUnits=Total[Table[Floor[(bids[[i]]/totalDemand)*units],{i,1,Length@bids}]];
   If[Total@tSwitches ==0,
    fractionalUnits=0,
    fractionalUnits=(units-wholeUnits)/Total@tSwitches
    ];
   If[yourTSwitch==1,
    unitsObtained=Floor[(yourBid/totalDemand)*units]+fractionalUnits,
    unitsObtained=Floor[(yourBid/totalDemand)*units]
    ];,
   unitsObtained=yourBid
   ];
  Return[unitsObtained];
  ]

Profit[entity_,strategySet_,auctionBids_,tierOneBids_,tierTwoBids_,tierWantsRandomUnits_,ho
ldingLimits_,values_,price_:-1]:=

Module[{aBids,t1Bids,t2Bids,auctionPrice,aUnits,t1Units,t2Units,tSwitches,profit,p,q1,q2,q3,r},
  {p,q1,q2,q3,r}=strategySet;
  t1Bids = Append[tierOneBids,q2];
```

```
  t2Bids = Append[tierTwoBids,q3];
  tSwitches=Append[tierWantsRandomUnits,r];
  aBids = Append[auctionBids,{p,q1}];
  aUnits = AuctionUnits[{p,q1},aBids,unitsInAuction];

  t1Units=TierUnits[q2,r,t1Bids,tSwitches,unitsInTier1];
  t2Units=TierUnits[q3,r,t2Bids,tSwitches,unitsInTier2];

 If[price==-1,
   auctionPrice=AuctionPrice[aBids,unitsInAuction],
   auctionPrice=price
   ];

  If[(aUnits+t1Units+t2Units)≤ holdingLimits[[entity]],profit=(values[[entity]]-
auctionPrice)*aUnits+(values[[entity]]-20)*t1Units+(values[[entity]]-30)*t2Units,
   profit=values[[entity]]*(holdingLimits[[entity]])-
(auctionPrice*aUnits+20*t1Units+30*t2Units)
   ];

  If[price≠-1,
   If[(Total[Select[aBids,#[[1]]≥ auctionPrice&][[All,2]]]< unitsInAuction)&&(values[[entity]]≥
auctionPrice),
    profit=0
    ]];

  Return[profit];
  ]

FindBestResponse[entity_,auctionBids_,tierOneBids_,tierTwoBids_,tierWantsRandomUnits_,ho
ldingLimits_,values_,auctionPrice_,q1_,searchSpace_]:=
 Module[{currentMax,newProfit,currentWinner},
  currentMax=0;
  currentWinner={0,{0,q1,0,0,0}};
  Do[

newProfit=Profit[entity,{p,q1,q2,q3,r},auctionBids,tierOneBids,tierTwoBids,tierWantsRandom
Units,holdingLimits,values,auctionPrice];
   If[newProfit≥ currentMax,
    currentMax=newProfit;
    currentWinner={newProfit,{p,q1,q2,q3,r}};]
   ,{p,searchSpace[[1,1]],searchSpace[[1,2]],searchSpace[[1,3]]}
   ,{q2,searchSpace[[2,1]],searchSpace[[2,2]],searchSpace[[2,3]]}
   ,{q3,searchSpace[[3,1]],searchSpace[[3,2]],searchSpace[[3,3]]}
   ,{r,searchSpace[[4,1]],searchSpace[[4,2]],searchSpace[[4,3]]}];

  Return[currentWinner];
```

```
  ]

BestResponseWithP[entity_,auctionBids_,tierOneBids_,tierTwoBids_,tierWantsRandomUnits_,
auctionPrice_,holdingLimits_,values_,iteration_,currentSet_]:=

Module[{response,firstSearch,secondSearch,firstCut,secondCut,p,areaSearch1,bestInArea1,area
Search2,bestInArea2,currentMax,newProfit,max,searchSpace},

  firstSearch=ParallelTable[
    searchSpace={{0,values[[entity]],5},{0,unitsInTier1,3},{0,unitsInTier2,3},{0,1,1}};

FindBestResponse[entity,auctionBids,tierOneBids,tierTwoBids,tierWantsRandomUnits,holding
Limits,values,auctionPrice,q1,searchSpace]
    ,{q1,0,holdingLimits[[entity]]}];
  max=Max[firstSearch[[All,1]]];
  (*SetSharedVariable[firstCut];*)
  firstCut = Select[firstSearch,#[[1]]==max&][[All,2]];
  firstCut=Sort[Sort[firstCut,#1[[2]]<#2[[2]]&],#1[[1]]>#2[[1]]&][[1]];

  secondSearch=ParallelTable[
    searchSpace={{Max[firstCut[[1]]-5,0],Min[firstCut[[1]]+5,values[[entity]]],1}
      ,{Max[firstCut[[3]]-6,0],Min[firstCut[[3]]+6,unitsInTier1],1}
      ,{Max[firstCut[[4]]-6,0],Min[firstCut[[4]]+6,unitsInTier2],1}
      ,{0,1,1}};

FindBestResponse[entity,auctionBids,tierOneBids,tierTwoBids,tierWantsRandomUnits,holding
Limits,values,auctionPrice,q1,searchSpace]
    ,{q1,Max[firstCut[[2]]-4,0],Min[firstCut[[2]]+4,holdingLimits[[entity]]],1}];
  max=Max[secondSearch[[All,1]]];
  secondCut = Select[secondSearch,#[[1]]==max&][[All,2]];
  secondCut=Sort[Sort[secondCut,#1[[2]]<#2[[2]]&],#1[[1]]>#2[[1]]&][[1]];
  Return[secondCut]
  ]

BestResponseOptimization[values_,holdingLimits_]:=

Module[{nPlayers,auctionBids,tierOneBids,tierTwoBids,tierWantsRandomUnits,response,newSt
rategy,isConverged,iteration,auctionPrice,randomOrder,pastStrategySets},
  nPlayers=Length@values;

auctionBids=Table[{RandomInteger[{0,values[[i]]}],RandomInteger[{0,holdingLimits[[i]]}]},{i
,1,nPlayers}];
  tierOneBids=Table[RandomInteger[{0,50}],{nPlayers}];
  tierTwoBids=Table[RandomInteger[{0,50}],{nPlayers}];
  tierWantsRandomUnits = Table[1,{nPlayers}];
  isConverged=False;
```

```
 iteration=1;
 pastStrategySets={};
 b=TableForm[{values,holdingLimits}];
 While[True,
  randomOrder=RandomSample[Range[nPlayers]];
  Do[
   player={iteration,randomOrder[[i]]};

response=BestResponseWithP[randomOrder[[i]],Drop[auctionBids,{randomOrder[[i]]}],Drop[ti
erOneBids,{randomOrder[[i]]}],Drop[tierTwoBids,{randomOrder[[i]]}],Drop[tierWantsRandom
Units,{randomOrder[[i]]}],-
1,holdingLimits,values,(*iteration*)1,Flatten@{auctionBids[[randomOrder[[i]]]],tierOneBids[[r
andomOrder[[i]]]],tierTwoBids[[randomOrder[[i]]]],tierWantsRandomUnits[[randomOrder[[i]]]]
}];
   newStrategy=response;
   auctionBids[[randomOrder[[i]]]]=newStrategy[[1;;2]];
   tierOneBids[[randomOrder[[i]]]]=newStrategy[[3]];
   tierTwoBids[[randomOrder[[i]]]]=newStrategy[[4]];
   tierWantsRandomUnits[[randomOrder[[i]]]]=newStrategy[[5]];
   x=TableForm[{auctionBids,tierOneBids,tierTwoBids,tierWantsRandomUnits}];
   a=AuctionPrice[auctionBids,unitsInAuction];
   ,{i,1,nPlayers}];
  AppendTo[pastStrategySets,{auctionBids,tierOneBids,tierTwoBids,tierWantsRandomUnits}];

  If[Length@pastStrategySets>1,
   If[pastStrategySets[[-2]]==pastStrategySets[[-1]],
     auctionPrice=AuctionPrice[auctionBids,unitsInAuction];
     Return[{auctionPrice,auctionBids,tierOneBids,tierTwoBids,tierWantsRandomUnits}],
     If[Length[Flatten[Position[pastStrategySets,pastStrategySets[[-1]]]]]>1,
      auctionPrice=AuctionPrice[auctionBids,unitsInAuction];

Return[{iteration,auctionPrice,auctionBids,tierOneBids,tierTwoBids,tierWantsRandomUnits}];,
      If[iteration≥ 30,
       auctionPrice=AuctionPrice[auctionBids,unitsInAuction];

Return[{iteration,auctionPrice,auctionBids,tierOneBids,tierTwoBids,tierWantsRandomUnits}]
       ];
      ];
     ];
   ];
  iteration+=1;
  ];
 auctionPrice=AuctionPrice[auctionBids,unitsInAuction];
 Return[{auctionPrice,auctionBids,tierOneBids,tierTwoBids,tierWantsRandomUnits}];
]
```

```
parameters =
{{{85,17},{45,28},{105,11},{95,14},{35,31},{65,22},{55,25},{75,20}},{{75,18},{65,20},{55,
22},{45,25},{105,10},{85,15},{95,13},{35,28}},{{55,22},{105,10},{65,20},{75,18},{85,15},{
95,13},{35,28},{45,25}},{{55,21},{45,17},{105,39},{75,28},{95,35},{85,31},{65,25},{35,14}
},{{65,28},{105,14},{55,31},{85,21},{75,25},{45,35},{35,39},{95,17}},{{85,16},{105,20},{6
5,13},{75,14},{55,11},{95,18},{35,7},{45,9}},{{105,10},{75,18},{55,22},{95,13},{85,15},{3
5,28},{65,20},{45,25}},{{35,11},{75,22},{65,20},{95,28},{105,31},{85,25},{55,17},{45,14}}
,{{85,16},{95,18},{45,9},{105,20},{55,11},{75,14},{35,7},{65,13}},{{105,14},{85,21},{55,3
1},{95,17},{65,28},{45,35},{35,39},{75,25}},{{45,9},{65,13},{85,16},{55,11},{35,7},{105,2
0},{75,14},{95,18}},{{85,22},{45,13},{75,20},{95,25},{105,28},{55,15},{65,18},{35,10}},{{
45,13},{65,18},{55,15},{105,28},{85,22},{95,25},{75,20},{35,10}},{{95,18},{65,13},{55,11}
,{105,20},{35,7},{85,16},{75,14},{45,9}},{{105,7},{75,13},{35,20},{45,18},{65,14},{55,16},
{95,9},{85,11}},{{35,28},{75,18},{105,10},{85,15},{55,22},{45,25},{65,20},{95,13}},{{65,2
2},{45,28},{55,25},{95,14},{105,11},{85,17},{35,31},{75,20}},{{55,15},{85,22},{105,28},{4
5,13},{35,10},{95,25},{65,18},{75,20}},{{35,10},{75,20},{105,28},{45,13},{85,22},{95,25},
{65,18},{55,15}},{{85,31},{75,28},{35,14},{95,35},{105,39},{55,21},{65,25},{45,17}},{{55,
17},{45,14},{105,31},{75,22},{35,11},{95,28},{85,25},{65,20}},{{75,13},{45,18},{85,11},{3
5,20},{95,9},{65,14},{105,7},{55,16}},{{35,14},{65,25},{55,21},{75,28},{85,31},{105,39},{
45,17},{95,35}},{{85,11},{55,16},{105,7},{95,9},{35,20},{45,18},{65,14},{75,13}},{{45,35},
{95,17},{65,28},{35,39},{85,21},{105,14},{75,25},{55,31}},{{105,7},{95,9},{85,11},{75,13}
,{65,14},{35,20},{45,18},{55,16}},{{35,11},{65,20},{55,17},{95,28},{45,14},{85,25},{75,22
},{105,31}},{{65,22},{95,14},{105,11},{35,31},{85,17},{45,28},{55,25},{75,20}},{{85,31},{
65,25},{95,35},{45,17},{35,14},{75,28},{105,39},{55,21}},{{75,25},{55,31},{95,17},{105,14
},{85,21},{35,39},{45,35},{65,28}},{{55,25},{75,20},{105,11},{95,14},{85,17},{45,28},{35,
31},{65,22}},{{95,28},{85,25},{105,31},{45,14},{35,11},{75,22},{55,17},{65,20}}};

demandCases=DeleteDuplicates@Table[Sort[parameters[[i]],#1[[1]]>#2[[1]]&],{i,1,Length@pa
rameters}];

excessDemandCases=Select[demandCases,Total[#[[All,2]]]>160&];

excessSupplyCases=Select[demandCases,Total[#[[All,2]]]<160&];

holdingLimits = parameters[[All,All,2]];

values = parameters[[All,All,1]];

unitsInAuction = 60;

unitsInTier1 = 50;

unitsInTier2 = 50;

nPlayers = Length@values;

Dynamic[z1]
```

```
Dynamic[z2]
Dynamic[b]
Dynamic[a]
Dynamic[x]
Dynamic[player]
(*Parallel*)Do[
 dateString = StringReplace[DateString[Date[]]," " → "_"];
 dateString = StringReplace[dateString,":" → "-"];
 z1="Cycle " <>ToString[j];
 equilibria=
  Table[
   z2="Case "<>ToString[i];

Prepend[BestResponseOptimization[excessDemandCases[[i,All,1]],excessDemandCases[[i,All,2
]]],i]
   ,{i,1,Length@excessDemandCases}];

Export[NotebookDirectory[]<>"ModelData\\Proportional\\"<>"Model_Data_Switch_Proportion
al_"<>dateString<>".txt",equilibria];
 ,{j,1,500}]
```

EQUAL RATIONING CODE:

```
AuctionPrice[allBids_,supply_]:=
 Module[{bids,units,price,index},
  bids = Sort[allBids,#1[[1]]>#2[[1]]&];
  units = supply;
  price=-1;
  If[Total[bids[[All,2]]]≤ units,
   price=0,
   index=1;
   While[units> 0,
    units-= bids[[index,2]];
    If[units≥0,
     index+=1]
    ];
   price=bids[[index,1]];
   ];
  Return[price];
  ]

AuctionUnits[yourBid_,allBids_,supply_]:=
 Module[{bidPosition,bids,bidTies,remainingUnits,unitsObtained,tiePosition},
  bids = Sort[allBids,#1[[1]]>#2[[1]]&];
  bidPosition = Extract[Flatten@Position[bids,yourBid],1];
```

```
  bidTies = Flatten@Position[bids[[All,1]],yourBid[[1]]];
  tiePosition = Flatten@Position[bidTies,bidPosition];
  remainingUnits = supply-Total[bids[[1;;bidTies[[1]]-1,2]]];
  unitsObtained=0;
  If[remainingUnits > 0,
   If[Length@bidTies≤1,
    If[remainingUnits ≥ yourBid[[2]],
     unitsObtained = yourBid[[2]],
     unitsObtained = remainingUnits],
    If[Total[bids[[bidTies[[1]];;bidTies[[-1]],2]]] ≤ remainingUnits,
     unitsObtained=yourBid[[2]],
     unitsObtained = TieAllocation[bids[[bidTies[[1]];;bidTies[[-
1]],2]],remainingUnits][[tiePosition[[1]]]];
     ]]];
  Return[unitsObtained];
  ]

TieAllocation[bids_,units_]:=
 Module[{bidsRemaining, unitsRemaining,count},
  bidsRemaining = bids;
  unitsRemaining = units;
  While[unitsRemaining>0,
   count = Length@bidsRemaining-Count[bidsRemaining,0];
   If[unitsRemaining ≥ count,
    unitsRemaining -= count;
    Do[
     If[bidsRemaining[[i]]>0,
      bidsRemaining[[i]] -= 1;]
     ,{i,1,Length@bidsRemaining}],
    Do[
     If[bidsRemaining[[i]]>0,
      bidsRemaining[[i]] -= unitsRemaining/count;]
     ,{i,1,Length@bidsRemaining}];
    unitsRemaining = 0;
    ]
   ];
  Return[bids-bidsRemaining];
  ]

g[b_,a_,Q_]:=Piecewise[{{b,b-a≤ Q},{Q+a,b-a>Q}}]
r[b_,a_]:=Piecewise[{{0,b-a≤0},{1,b-a>0}}]
q[bidVector_,acquiredVector_,Q_]:=Q/Sum[r[bidVector[[j]],acquiredVector[[j]]],{j,1,Length@b
idVector}]
h[bidVector_,acquiredVector_,Q_]:=Table[g[bidVector[[i]],acquiredVector[[i]],q[bidVector,acq
uiredVector,Q]],{i,1,Length@bidVector}]
k[bidVector_,acquiredVector_,Q_,Qt_]:=Piecewise[{{
```

```
    k[bidVector,h[bidVector,acquiredVector,Q],Qt-
Total[h[bidVector,acquiredVector,Q]],Qt],Total@acquiredVector<Min[Qt,Total@bidVector]},
   {Return[acquiredVector],True}}]


Profit[entity_,strategySet_,auctionBids_,tierBids_,holdingLimits_,values_,price_:-1]:=
 Module[{aBids,tBids,auctionPrice,aUnits,t1Units,t2Units,profit,p,q1,q2},
  {p,q1,q2}=strategySet;
  tBids = Insert[tierBids,q2,entity];
  aBids = Append[auctionBids,{p,q1}];
  aUnits = AuctionUnits[{p,q1},aBids,unitsInAuction];

  t1Units=k[tBids,Table[0,{Length@tBids}],unitsInTier1,unitsInTier1];
  t2Units=k[tBids-t1Units,Table[0,{Length@tBids}],unitsInTier2,unitsInTier2];

  If[price==-1,
   auctionPrice=AuctionPrice[aBids,unitsInAuction],
   auctionPrice=price
   ];

  If[(aUnits+t1Units[[entity]]+t2Units[[entity]])≤
holdingLimits[[entity]],profit=(values[[entity]]-auctionPrice)*aUnits+(values[[entity]]-
20)*t1Units[[entity]]+(values[[entity]]-30)*t2Units[[entity]],
   profit=values[[entity]]*(holdingLimits[[entity]])-
(auctionPrice*aUnits+20*t1Units[[entity]]+30*t2Units[[entity]])
   ];

  Return[profit];
  ]

FindBestResponse[entity_,auctionBids_,tierBids_,holdingLimits_,values_,auctionPrice_,q1_,sea
rchSpace_]:=
 Module[{currentMax,newProfit,currentWinner},
  currentMax=0;
  currentWinner={0,{0,q1,0}};
  Do[
   newProfit=Profit[entity,{p,q1,q2},auctionBids,tierBids,holdingLimits,values,auctionPrice];
   If[newProfit≥ currentMax,
    currentMax=newProfit;
    currentWinner={newProfit,{p,q1,q2}};]
   ,{p,searchSpace[[1,1]],searchSpace[[1,2]],searchSpace[[1,3]]}
   ,{q2,searchSpace[[2,1]],searchSpace[[2,2]],searchSpace[[2,3]]}
   ];

  Return[currentWinner];
  ]
```

```
BestResponseWithP[entity_,auctionBids_,tierBids_,auctionPrice_,holdingLimits_,values_,iterati
on_,currentSet_]:=

Module[{response,firstSearch,secondSearch,firstCut,secondCut,p,areaSearch1,bestInArea1,area
Search2,bestInArea2,currentMax,newProfit,max,searchSpace},

  firstSearch=ParallelTable(*Table*)[
    searchSpace={{0,values[[entity]],5},{0,holdingLimits[[entity]],2}};

FindBestResponse[entity,auctionBids,tierBids,holdingLimits,values,auctionPrice,q1,searchSpace
]
    ,{q1,0,holdingLimits[[entity]]}];
  max=Max[firstSearch[[All,1]]];
  SetSharedVariable[firstCut];
  firstCut = Select[firstSearch,#[[1]]==max&][[All,2]];
  firstCut=Sort[Sort[firstCut,#1[[2]]<#2[[2]]&],#1[[1]]>#2[[1]]&][[1]];

  secondSearch=ParallelTable(*Table*)[
    searchSpace={{Max[firstCut[[1]]-5,0],Min[firstCut[[1]]+5,values[[entity]]],1}
      ,{Max[firstCut[[3]]-3,0],Min[firstCut[[3]]+3,holdingLimits[[entity]]],1}};

FindBestResponse[entity,auctionBids,tierBids,holdingLimits,values,auctionPrice,q1,searchSpace
]
    ,{q1,Max[firstCut[[2]]-4,0],Min[firstCut[[2]]+4,holdingLimits[[entity]]],1}];
  max=Max[secondSearch[[All,1]]];
  secondCut = Select[secondSearch,#[[1]]==max&][[All,2]];
  secondCut=Sort[Sort[secondCut,#1[[2]]<#2[[2]]&],#1[[1]]>#2[[1]]&][[1]];
  Return[secondCut]
  ]

BestResponseOptimization[values_,holdingLimits_]:=

Module[{nPlayers,auctionBids,tierBids,lastAuctionBids,lastTierBids,response,newStrategy,isCo
nverged,iteration,auctionPrice,randomOrder},
  nPlayers=Length@values;

auctionBids=Table[{RandomInteger[{0,values[[i]]}],RandomInteger[{0,holdingLimits[[i]]}]},{i
,1,nPlayers}];
  tierBids=Table[RandomInteger[{0,holdingLimits[[i]]}],{i,1,nPlayers}];
  isConverged=False;
  iteration=1;
  b=TableForm[{values,holdingLimits}];
  While[!isConverged,
   randomOrder=RandomSample[Range[nPlayers]];
   Do[
```

```
    player={iteration,randomOrder[[i]]};

response=BestResponseWithP[randomOrder[[i]],Drop[auctionBids,{randomOrder[[i]]}],Drop[ti
erBids,{randomOrder[[i]]}],-
1,holdingLimits,values,(*iteration*)1,Flatten@{auctionBids[[randomOrder[[i]]]],tierBids[[rando
mOrder[[i]]]]}];
    newStrategy=response;
    auctionBids[[randomOrder[[i]]]]=newStrategy[[1;;2]];
    tierBids[[randomOrder[[i]]]]=newStrategy[[3]];
    x=TableForm[{auctionBids,tierBids}];
    a=AuctionPrice[auctionBids,unitsInAuction];
    ,{i,1,nPlayers}];
   If[(lastAuctionBids□auctionBids&&lastTierBids□tierBids),
    isConverged=True];
   iteration+=1;
   lastAuctionBids=auctionBids;
   lastTierBids=tierBids;
   If[iteration>30,
    Return[{iteration,auctionPrice,auctionBids,tierBids}]
    ];
   ];
  auctionPrice=AuctionPrice[auctionBids,unitsInAuction];
  Return[{auctionPrice,auctionBids,tierBids}];
  ]

Parameters =
{{{85,17},{45,28},{105,11},{95,14},{35,31},{65,22},{55,25},{75,20}},{{75,18},{65,20},{55,
22},{45,25},{105,10},{85,15},{95,13},{35,28}},{{55,22},{105,10},{65,20},{75,18},{85,15},{
95,13},{35,28},{45,25}},{{55,21},{45,17},{105,39},{75,28},{95,35},{85,31},{65,25},{35,14}
},{{65,28},{105,14},{55,31},{85,21},{75,25},{45,35},{35,39},{95,17}},{{85,16},{105,20},{6
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},{85,21},{35,39},{45,35},{65,28}},{{55,25},{75,20},{105,11},{95,14},{85,17},{45,28},{35,
31},{65,22}},{{95,28},{85,25},{105,31},{45,14},{35,11},{75,22},{55,17},{65,20}}};

demandCases=DeleteDuplicates@Table[Sort[parameters[[i]],#1[[1]]>#2[[1]]&],{i,1,Length@pa
rameters}];

excessDemandCases=Select[demandCases,Total[#[[All,2]]]>160&];

excessSupplyCases=Select[demandCases,Total[#[[All,2]]]<160&];

holdingLimits = parameters[[All,All,2]];

values = parameters[[All,All,1]];

unitsInAuction = 60;

unitsInTier1 = 50;

unitsInTier2 = 50;

nPlayers = Length@values;

Dynamic[z1]
Dynamic[z2]
Dynamic[b]
Dynamic[a]
Dynamic[x]
Dynamic[player]
Do[
 dateString = StringReplace[DateString[Date[]]," " → "_"];
 dateString = StringReplace[dateString,":" → "-"];
 z1="Cycle " <>ToString[j];
 equilibria=
  Table[
   z2="Case "<>ToString[i];

Prepend[BestResponseOptimization[excessDemandCases[[i,All,1]],excessDemandCases[[i,All,2
]]],i]
   ,{i,1,Length@excessDemandCases}];

Export[NotebookDirectory[]<>"ModelData\\Equal\\"<>"Model_Data_Equal_"<>dateString<>".t
xt",equilibria];
 ,{j,1,500}]
```

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