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Coalgebraic Semantics of Reflexive Economics (Dagstuhl Seminar 15042)

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Report from Dagstuhl Seminar 15042

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Abstract

This report documents the program and the outcomes of Dagstuhl Seminar 15042 "Coalgebraic Semantics of Reflexive Economics".

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1 Executive Summary

Samson Abramsky
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Alexander Kurz
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Viktor Winschel

A growing number of researchers have been discovering analogies in the foundations of both computer science and economics. The goal of this seminar is to interface computer science with economics and game theory and to take advantage of the programming language semantics methods in theoretical computer science based on lambda calculus, coalgebras, modal logic and category theory.

The theoretical thread of interest to this seminar and common to both computer science and economics is the phenomenon that may be circumscribed by notions such as reflexivity, self-reference, impredicativity, infinite regress, recursion, or fixed points.

In computer science, the phenomena of self-reference, self-application and recursion played a crucial role in the foundational work of Gödel, Church, Turing and Kleene in the 1930s. Nevertheless, powerful mathematical models of the semantics of recursion became available only with the work of Scott on models of the untyped lambda calculus and subsequent research in domain theory. The combination of domain theory with the theory of types in programming languages and their categorical semantics has led to the development of a

powerful tool box. More recently, this tool box has been further strengthened by advances in coalgebra. It provides for a wide variety of dynamic systems the mathematical tools of (bi)simulation and coinduction as well as a variety of techniques from category theory.

In economics, and the social sciences more generally, reflexivity arises from the obvious fact that cognitive agents reason about themselves, others and the society they live in. This leads to self-reference and recursion in, for example, theories of belief formation as beliefs of beliefs (Harsanyi type spaces) or theories of institutions as rules to change rules. More generally, a social system consists of individuals who are learning about a process in which others are learning as well. Learning the state of an interactive system is therefore rather different compared to learning the parameter values that govern a physical process. When the observer is a part of the system, the act of learning changes the thing to be learned. The traditional mathematical tools in economics are hardly suited to solve these problems in a sufficiently general way and they make it difficult for computer scientists, once they need to solve similar or common problems, to understand the problem formulation and the solutions already achieved by economists.

The specific subfields of computer science and economics discussed above suggest to explore methods of program semantics and category theory in general and, in particular, of bisimulation and coinduction in economics. Furthermore, coalgebra gained prominence as providing models for concurrency, a topic that has hardly been touched upon in economics explicitly, even so it underlies the most general kind of issues in economics, namely those regarding centralization versus decentralization in theories of economic systems, administration, firms and markets.

Particular topics in which we see scope for methods from the semantics of programming languages include infinitely repeated games, econometrics and system theory, epistemic game theory and interactive learning in multi-agent systems.

More generally, research in program semantics and logics in computer science is typically motivated by problems arising in programming languages and software engineering. In one direction, economic modeling will become more important in software engineering. In the other direction, computational economics may as well profit from a modern approach to language design not only in terms of reflexivity at the theoretical level but also at the practical level of modeling software.

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3 Abstracts of the Workshops

3.1 Category Theory 2

Neil Ghani (University of Strathclyde, GB)

In Category Theory 2, students will have learned about basic category theory and in particular its core definitions of category, functor, natural transformation, limit and colimit. This will have given them a feel for what category theory is all about. However, the key technical tool that category theory has contributed to mathematics and the every day practice of the working category theorist is the notion of an adjunction. In this lecture we will introduce the concept of an adjunction, cover the different presentations of this concept and cover examples, both elementary and more significant.

3.2 Rationality

Pierre Lescanne (ENS – Lyon, FR)

This session was different in format to the others, being a free-form group discussion rather than a presentation by a speaker. It was spontaneously organised after an intriguing comment by Pierre Lescanne in a previous session. The session was based on the question of which logics are suitable for modelling the reasoning processes of economic agents, and in particular whether intuitionistic logic (in which every proof of an existential statement must include an explicit 'witness' as justification) could be better suited than the more usual classical logic (which allows 'pure existence proofs'). Part of the discussion focussed on operational questions of proof search, and whether logics for reasoning agents should be chosen for the efficiency of reasoning rather than expressive or deductive power. This is linked to discussions in the economics literature on models of bounded rationality as well as models in behavioral economics, but with the modern focus on proof search as a model of computation.

3.3 Coalgebraic Games

Achim Blumensath (TU Darmstadt, DE)

This workshop presented a new abstract framework designed to compose economic games. It is sufficiently general in order to represent all types of games encountered in economics. The framework is based on category theoretical techniques and coalgebras. Coalgebras have been successfully used to model the observable behavior of systems with an unobservable state space. The fundamental notion of a process is represented as a coalgebra of a certain type. It is shown how to use them in order to describe stage games, finitely, infinitely and potentially infinitely repeated games with imperfect and incomplete information based on deterministic,

non-deterministic or probabilistic decisions of observing agents in endogenous networks. The framework allows to compose games sequentially, in parallel and hierarchically and it provides a formal account of the behavior of the aggregated system. The games are directly implementable in high-level functional programing languages. The abstract mathematics of this approach links economics to the latest developments in mathematical game theory and theoretical computer science. Coalgebras arise as functorial fixed points that open the door to self-referential and reflexive structures that underly the Lucas critique, institutional economics, belief formation and many other social modeling issues.

3.4 Harsanyi Type Spaces as Coalgebras

Elias Tsakas (Maastricht University, NL)

A belief hierarchy is a description of an agent's beliefs about some fundamental space of uncertainty, her beliefs about everybody else's beliefs, and so on. During the past few decades, belief hierarchies have become an integral tool of modern economic theory, often used to analyze games with incomplete information (Harsanyi, 1967-68), as well as in order to provide epistemic characterizations for several solution concepts, such as rationalizability (Brandenburger and Dekel, 1987; Tan and Werlang, 1988), Nash equilibrium (Aumann and Brandenburger, 1995), and correlated equilibrium (Aumann, 1987), just to mention a few. Belief hierarchies are in general very complex objects, consisting of infinite sequences of probability measures. This makes them in principle very hard to handle and sometimes even to describe, especially when it comes to high order beliefs. Having recognized this difficulty, Harsanyi (1967-68) proposed an indirect Bayesian representation of belief hierarchies, known as the type space model. Formally, Harsanyi's model consists of a set of types for each agent and a continuous mapping from each type to the corresponding conditional beliefs over the product of the fundamental space of uncertainty and the opponent's type space. This structure induces a belief hierarchy for every type, thus reducing the infinite-dimensional regression of beliefs to a single-dimensional type. Mertens and Zamir (1985) and Brandenburger and Dekel (1993) completed the analysis by showing the existence of the universal type space, which represents all belief hierarchies satisfying some standard coherency properties. When the fundamental space of uncertainty is a Polish space, the universal type space is terminal in the category of type spaces. Recently it was shown that the infinite hierarchies of beliefs can be represented as coalgebras.

3.5 Coalgebras and Algebras in Scientific Modelling

Baltasar Trancon y Widemann (TU Ilmenau, DE)

The coalgebraic approach to the semantics and behavior of composed systems provides a complementary view on complex systems. The traditional scientific method is based on an algebraic approach where a system is decomposed into and composed from parts. This constitutes a point of view which is based on a constructive rather than a behavioral or

observational specification as in the coalgebraic methodology. The algebraic point of view is emphasizing the "doing" while the coalgebraic point of view is emphasizing the "seeing" aspect of a scientific explanation. Accordingly, it is only within the algebraic approach where the notion of emerging properties is needed in order to capture properties at the system level. The behavioral, coalgebraic approach does capture the system properties by instead deriving the behaviour of the parts from the behaviour of the system. The interaction of the parts is captured by the syntax or their allowed composition. However, it is important to see that coalgebras dualise and not supersede the traditional algebraic approach to modelling.

3.6 The Cohomology of Non-Locality and Contextuality

Samson Abramsky (University of Oxford, GB)

In a paper of Samson Abramsky and Adam Brandenburger, sheaf theory was used in order to analyze the structure of non-locality and contextuality. Moreover, on the basis of this formulation, it can be shown that the phenomena of non-locality and contextuality can be characterized precisely in terms of obstructions to the existence of global sections. The aim in the presented work is to build on these results, and to use the powerful tools of sheaf cohomology to study the structure of non-locality and contextuality. The Cech cohomology is used on an abelian presheaf derived from the support of a probabilistic model, viewed as a compatible family of distributions, in order to define a cohomological obstruction for the family as a certain cohomology class. This class vanishes if the family has a global section. Thus the non-vanishing of the obstruction provides a sufficient (but not necessary) condition for the model to be contextual. It is shown that for a number of salient examples, including PR boxes, GHZ states, the PeresMermin magic square, and the 18-vector configuration due to Cabello et al. giving a proof of the Kochen-Specker theorem in four dimensions, the obstruction does not vanish, thus yielding cohomological witnesses for contextuality.

3.7 Higher Order Games

Jules Hedges (Queen Mary University of London, GB)

The study of higher order games comes out of the proof-theoretic work on selection functions by Martin Escardó and Paulo Oliva. This talk focussed on a single worked example, namely that replacing utility maximisation with a fixpoint operator models agents who attempt to coordinate with the majority (proving a simple model of the so-called Keynes beauty contest), and similarly an anti-fixpoint operator models agents who attempt to differentiate from the majority. However this is a quickly evolving area with several directions being investigated. For example higher order games allow the use of monads, a concept from category theory used to model side-effects in programming languages, to also model economic 'side-effects' such as probabilistic choices, learning and external effects. Another direction, using ideas from the theory of control operators in programming languages, leads to a fully compositional game theory that should be very useful in complex practical examples (using a category

whose morphisms are suitably generalised games) and a graphical notation based on string diagrams in quantum information theory. Another idea is to use ideas based on Escardó's 'seemingly impossible functional programs' together with computable analysis to compute solutions of games. Finally there is also a hope that higher order games could serve as a common structure between several other theories, including some described in this report, allowing them to be used together.

4 Outlook

There have been various discussions about the possible paths for the research to be done in the future. The mathematical research for the economic applications is to be done mainly in the fields of category theory, type theory and coalgebras. The applications can be characterized as those with a high model complexity, either by being large models or irregular by being composed from various parts that can not be glued easily by traditional methods. A first effort is to give the current state of research in game theory a semantic and a composable representation.

4.1 Work on Reflexive Economics

A semantical approach to equilibria and rationality, 2009, by Dusko Pavlovic: This is the first paper that is taking advantage of the semantical approach to programming languages and uses it within the realm of economics and game theory.

On the Rationality of Escalation, 2010, by Pierre Lescanne, Perrinel Matthieu: Here it is argued that speculative bubbles are rational if viewed from a coalgebraic point of view.

Coalgebraic Analysis of Subgame-perfect Equilibria in Infinite Games without Discounting, 2013, by Samson Abramsky and Viktor Winschel: In this paper we prove coinductively a property of an infinite game, namely subgame perfectness of a strategy profile.

A Coalgebraic Framework for Games in Economics, 2013, Achim Blumensath, Viktor Winschel: This paper presents a complete coalgebraic framework for composing complex games from simple ones. The glueing is done by natural transformations on deterministic, nondeterministic or probabilistic choice functors.

A Higher-order Framework for Decision Problems and Games, 2014, by Jules Hedges, Paulo Oliva, Evguenia Winschel, Viktor Winschel, Philipp Zahn: This framework, again fully compositional, is build on higher-order functions and provides a high-level language to formulate goals or preferences that are context dependent and that traditionally need to be modelled by transforming the outcome space for representing behavior as utility maximization.

4.2 Research

An important extension and unification of current work is to unite the higher-order games and the coalgebraic games. The coalgebraic games so far lack a generalized or higher-order approach to preferences as the higher-order games do. And the higher-order game lack so far a coalgebraic treatment of infinite games that are essential to many economic applications.

Both approaches provide compositionality and it is interesting how both approaches unite within a category of games.

Having defined coalgebraic structures within game theory it is interesting to extend it to modal logic. This could serve as a general approach to define all kinds of predicates on games that are of interest during the modelling process, with equilibrium predicates being one of many other possible ones. The main modalities of interest in economics are epistemic and temporal modalities. However, other could be useful as well like deontic ones.

Another interesting path of research is to include inductive reasoning within a game theoretical context. Either as learning agents within the very game itself or as tools for the modeller to expose the game to data for an empirical evaluation of the model.

Both applications have to rely on some form of statistical analysis. The classical one would take the axiomatic approach, which can also be seen from a categorical point of view using the Giry monad and convex sets. The alternative algorithmic approach could take advantage of basing induction on the same footing as the approach in the higher-order games. Namely representing the units of modelling, like games, players, strategies or equilibria, as an algorithm or higher-order function. In an algorithmic statistical approach the units of induction are as well algorithms with the advantage that induction takes place at the language level of the modeling tools and not as usual as in classical statistics at the level of parameters of some function representing some economic concept.

An important path for research is finally the refinement of equilibrium solvers. So far there has been some progress to check for equilibria for some given strategy profiles in arbitrarily composed games. But there is hardly a universal approach to be expected to be able to find automatically equilibria in all kinds of games.

Within the quest for game solvers it might be useful to use some traditional numerical approaches like function approximation or some modern variants of it like sparse grids. But also there might be scope for an application of the seemingly impossible functional algorithms of Martin Escardo and in general infinite precision algorithms.

4.3 Industrial Applications

There are various industrial applications possible for the research on the semantics of games.

A large application of a compositional game theory is the field of smart energy grids where large and heterogeneous networks of decentralized reasoner are to be modelled and implemented.

The application of behavioral economics that can be unified under the higher-order games approach are marketing applications or online recommendation systems that need to be based on non-optimizing or heuristic reasoning, learning approaches and software implementation issues for online surveys.

The tools of compositionality that lend themselves into suitable approaches to network theory do also allow for a unified approach to the theory of optimal currency areas and money theory.

Another highly profitable and important area of application for compositional games is industrial organization. Here, questions of the utility of merging companies is of interest to the companies themselves but also to courts and institutions surveying the competition or monopoly developments in markets. Within this application a formal modelling approach is already established.

Finally, a compositional and implementable approach to game theory can be used to build

Table 1 Workshops.

Tutorials	**Monday**		**Tuesday**	**Wednesday**
09:00–10:30	Category Theory (Samson Abramsky)		Coalgebras and Semantics (Alexander Kurz)	Theory of Science: Algebraic and Coalgebraic Point of View (Michael Hauhs)
11:00–12:30	Microeconomics and Game Theory (Philipp Zahn)		Macroeconomics (Viktor Winschel)	
Workshops 13:30–14:30	**Monday** Category Theory 2 (Neil Ghani)	Rationality (Pierre Lescanne)	**Tuesday** Coalgebras and Algebras in Scientific Modelling (Baltasar Trancon y Widemann)	
15:00–16:30	Coalgebraic Games (Achim Blumensath)		Contextuality and Locality by Sheafs and Cohomology (Samson Abramsky)	
17:00–18:00	Harsanyi Type Spaces as Coalgebras (Elias Tsakas)		Higher Order Games (Jules Hedges)	

education software for universities, massive online courses or MBA schools where students of strategic decision making are educated. Here a software where actual players or computer players interact in order to train decision makers can be of much interest.

An essential goal for the emerging field of the semantics of economics is the provision of software. The software will be a compiler for some high-level language that is tailored to the domain of economics such that modelling and programming can take place as close as possible to the high-level concepts in narrative and natural language economics.

5 Schedule

We have organized this interdisciplinary workshop to encompass two parts, the morning and the afternoon sessions as shown in Table 1.

In the morning there have been presentations from both researchers of computer science and economics. Each group gave lectures in order to introduce the other group to the basic methodological tools in the two sciences and the basic problems that are addressed within. The economics talks were divided in microeconomics and macroeconomics. The computer scientific presentations introduced the audience to category theory and to coalgebras and their use in the semantics of programming languages. Also, there was a methodological talk on the scientific method from an algebraic and a coalgebraic point of view.

In the afternoons, the second part of the seminar, we had several workshops that introduced already existing work within our emerging field of reflexivity and semantics of economics and game theory.

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