10-28-2013

# How Do Firms Become Different? A Dynamic Model

Matthew Selove
*Chapman University*, selove@chapman.edu



## Recommended Citation

Selove, M. (2014). "How Do Firms Become Different? A Dynamic Model," *Management Science*, 60, 4, p. 980-989. doi: 10.1287/mnsc.2013.1797

# How Do Firms Become Different? A Dynamic Model*

Matthew Selove
USC Marshall School of Business†

August 21, 2012

**Abstract**

This paper presents a dynamic investment game in which firms that are initially identical develop assets which are specialized to different market segments. The model assumes there are increasing returns to investment in a segment, for example, due to word-of-mouth or learning curve effects. I derive three key results: (1) Under certain conditions there is a unique equilibrium in which firms that are only slightly different focus all of their investment in different segments, causing small random differences to expand into large permanent differences. (2) On the other hand, if firms are sufficiently patient or if sufficiently large random shocks are possible, there is always an equilibrium in which the firm focused on the smaller segment changes its strategy, attacking its rival until it drives the rival out of the larger segment. (3) Surprisingly, a firm might sometimes want to reduce its own assets in a segment in order to entice its competitor to shift focus to this segment.

*I am grateful to my advisor, Birger Wernerfelt, for his support and guidance. Helpful comments were provided by Alessandro Bonatti, Shantanu Dutta, Florian Ederer, Nathan Fong, Bob Gibbons, Cristina Nistor, Stephen Ryan, Jiwoong Shin, Duncan Simester, Catherine Tucker, Ray Weaver, Zibin Xu, and Juanjuan Zhang; and also by attendants at the 2009 AMA conference and seminar participants at Chicago, Kansas, MIT, Rutgers, SKKU, U. Illinois, UCSD, USC, and Yale.

†email: selove@marshall.usc.edu

# 1 Introduction

Casual observation reveals that even firms in the same industry are different from each other. Each firm has unique assets such as its reputation, relationships, and production skills (Wernerfelt 1984; Prahalad and Hamel 1990; Teece et al. 1997; Dutta et al. 1999). Standard models of product design such as the Hotelling model imply that firms should differentiate to soften price competition (D'Aspremont et al. 1979). However, such models do not answer two important questions: What initially determines which customer segment each firm serves? And once firms start serving different segments, under what conditions will those differences persist over time, and under what conditions will firms eventually switch back-and-forth between different segments?

This paper develops a dynamic investment game that proposes answers to these questions. The model assumes that two competing firms each allocate investment across two customer segments of unequal size, and that there are increasing returns to investment in a segment, for example, due to word-of-mouth effects (Rob and Fishman 2005) or learning curves (Argote and Epple 1990). Small early differences arise between the the firms due to "random shocks" such as differences in founders' previous experiences or fortuitous discoveries of better production processes. I derive three key results, which are described in the three subsections of this introduction.

### 1.1 Conditions in which random shocks lead to permanent differences

I show that permanent differences are guaranteed to arise, with early random shocks determining where each firm ends up focusing its investment, if the following three conditions hold: (1) preemption effects are neither too weak nor too strong;[1] (2) the largest possible random shock is neither too small nor too large; and (3) firms are not too patient. When these conditions hold, the model does not depend (as many models of product design do) on an arbitrary selection among various possible equilibria to determine where each firm allocates investment.

[1]Preemption effects are defined as the extent to which an increase in one firm's assets in a segment makes it less profitable for the other firm to increase its assets in that same segment.

Rather, a rational firm must always follow the optimal path determined by its previous history.

One implication of this result is that small random events early in the life of a company can determine its eventual area of expertise. For example, because Ben and Jerry's co-founder Ben Cohen had sinus problems that make it hard for him to taste normal ice cream flavors, he liked to add texture to some of their homemade ice cream shop's flavors by including very large chunks of chocolate, cookies, or other "mix-ins" (Lager 1994; Dreifus 1994).[2] To the surprise of the founders, ice cream with large chunks turned out to be extremely popular with customers, so they starting packaging it into pint containers to sell in local grocery stores. Over the next five year they continuously modified the machines that dispensed their ice cream to help prevent the chunks from getting stuck and also experimented with different cookie dough mixes until they found one that would not clog the machinery. Partly as a result of this production expertise they developed, they eventually became the dominant producer of chunky ice cream flavors (Lager 1994; Collis and Conrad 2005).

Dell Computer is another example of a company that focused on an area of early, arguably random, success. Because college student Michael Dell only had $1,000 in capital when he started assembling and selling personal computers from his college dorm room, he could not maintain a large stock of inventory; instead, he and three employees custom assembled a computer for each order and then shipped it directly to the customer (Dell 1999). As the company grew, they "forced all of our people to focus 100 percent" on this build-to-order model by *not* selling through retailers, which helped Dell achieved a cost advantage through fast inventory turnover rates (Dell 1999).[3]

Thus, Ben and Jerry's initially created ice cream with large chunks due to Ben Cohen's sinus problems, and Dell initially learned to custom assemble computers due to a lack of funding. The model in this paper implies that increasing returns, for example, due to word-of-mouth

[2]Like other homemade ice cream shops, Ben and Jerry's initially sold both chunky ice cream flavors and traditional "smooth" flavors. Their key product innovation was to use bigger chunks (Lager 1994).

[3]As competition from manufacturing plants in China has eroded Dell's cost advantage, Dell has begun selling through retail stores and is reorganizing its supply chain to move away from the direct model (Shah 2008). We can think of the rise of computer manufacturing in China as an extremely unfavorable shock to Dell's core area of expertise. Consistent with the model in this paper, Dell has totally shifted its investment to developing a new area of expertise.

effects, learning curves, and the need for sets of complementary assets devoted to a particular segment, compelled each firm to invest in this area of early good fortune until it became an important source of competitive advantage for the firm.

### 1.2 Conditions in which major strategic changes can occur

On the other hand, I also show that, if sufficiently large random shocks can occur or if firms are sufficiently patient, then it is possible for firms to make major strategic changes, totally shifting focus from one segment to another. For example, Sony successfully drove Nintendo out of the segment of hard-core video game players partly because a large shock occurred due to the invention of CD-ROMs, an efficient information storage technology in which Sony had expertise and which they used in their PlayStation video game system (Edge 2009); and partly because Sony was patient enough to continue investing in the PlayStation for over a decade until Nintendo decided to stop competing for this segment and to pursue more casual gamers with its Wii system (Mossberg and Boehret 2006; Edge 2009). Thus, the model provides insight into factors that helped make this strategic attack successful.[4]

### 1.3 A firm can benefit from its own failure or its competitor's success

Finally, I show that a firm can benefit from its competitor's success or its own failure in a segment. For example, former Ben and Jerry's CEO Fred Lager speculated that their main competitor, Häagen-Dazs, did not invest in the modifications to production equipment needed to produce chunky flavors because Häagen-Dazs wanted to avoid distractions from their already successful business selling smooth flavors and was concerned that taking time to invest in chunky flavors would make their production process for smooth flavors less efficient (Lager 1994, p. 140). Thus, Ben and Jerry's arguably benefited from Häagen-Dazs' strength in

[4]It is interesting that both firms in this example are Japanese, as it is commonly believed that Japanese firms are more patient than American ones (e.g., Maskin 1995). The model in this paper predicts that patient firms are more likely to launch strategic attacks and are also more likely to shift their focus to a new area in response to an attack.

smooth flavors.[5]

The first two key results, which state conditions in which this model leads to either permanent differences or major strategic changes, are consistent with what we would expect given the model set-up. The main point of these results is to propose a new explanation for how firms develop (and potentially change) their marketing strategies, and to derive precise conditions in which this explanation holds. On the other hand, the result that a firm can benefit from its competitor's success or its own failure is more surprising, and directly contradicts previous theoretical results from dynamic investment games in which firms compete along a single dimension (Ericson and Pakes 1995); thus, this paper demonstrates an important new implication of allowing dynamic competition along multiple dimensions.

The rest of the paper is organized as follows. Section 2 discusses related literature. Section 3 presents the model set-up. Section 4 derives conditions that guarantee permanent differences arise. Section 5 derives conditions in which major strategic changes are always possible. Section 6 presents an extension in which each firm can destroy some of its assets. Section 7 concludes. Appendix A contains all proofs.

## 2 Related Literature

Product design models such as the Hotelling model imply that firms become different to soften price competition (e.g., D'Aspremont et al. 1979; Shaked and Sutton 1982; Goettler and Shachar 2001; Desai, Kekre, Radhakrishnan, and Srinivasan 2001; Orhun 2005; Shin 2007; Thomadsen 2007; Zhu and Singh 2009; Subramaniam and Gal-Or 2009). However, because these models are static, they are not well-suited to study whether each firm stays focused on a single segment or switches back-and-forth between segments, which is the key question studied in the current dynamic model. Furthermore, most previous product design models have multiple equilibria in which either firm could win a particular segment; by contrast, the

[5] Another possible explanation is that Häagen-Dazs did not realize how popular chunky flavors would become. However, from very early on Häagen-Dazs tried to pressure distributors not to carry Ben and Jerry's ice cream, suggesting that Häagen-Dazs did in fact realize chunky flavors could potentially appeal to a large customer segment (Lager 1994, p. 106).

current paper derives conditions in which there is a unique equilibrium with random shocks determining which segment each firm wins.

Some models predict that the first entrant will end up with the more attractive location (Prescott and Visscher 1977; Moorthy 1988) or with more assets (Sutton 1991).[6] However, while there is some empirical evidence for first-mover advantages (Urban et al. 1986), in many cases the most successful firm in a market was not the first entrant, suggesting that factors other than order-of-entry must play a role in determining firm performance (Golder and Tellis 1993). The current paper provides one possible explanation for such factors, showing how random historical events, along with firms' investment allocation decisions, can determine each firm's eventual area of expertise.

Previous models of dynamic investment games have also shown how random fluctuations can lead to differences in firms' asset levels (Budd et al. 1993; Villas-Boas 1993; Athey and Schmutzler 2001; Besanko and Doraszelski 2004; Hörner 2004). However, these earlier dynamic models assume each firm invests in a single type of asset. By contrast, the current paper allows for investment in two assets, which leads to the new result that a firm can sometimes benefit from reducing its own assets in a segment to entice its competitor to switch focus to this segment.

Another important limitation of previous dynamic investment games is that, to deal with the problem of multiple equilibria, they typically assume firms follow strategies that are Markov (so a firm's investment is a function only of current asset levels) and stationary (so investment functions do not change over time). However, in reality firms can and do change their strategies for reasons that are not just based on changes in their assets. For example, a firm that is currently focusing on a niche customer segment might hire a new chief executive officer who simply decides to adopt the more aggressive strategy of attacking the mainstream segment. To allow for such changes in strategy that are not related to underlying assets, the current paper does not restrict firms to Markov perfect equilibria, but instead uses the more general concept of subgame perfect equilibrium. This leads to the new result that, under some conditions,

[6] Other theoretical research has explored why later movers might have an advantage (e.g., Chen and Xie 2007).

regardless of the history of the game, there is an equilibrium in which the less profitable firm launches a successful attack and eventually becomes more profitable than its rival.

My model bears some resemblance to the model by Levinthal (1997) in which firms search across a "rugged landscape" for an optimal organizational form. One major difference is that in this earlier model each firm tries to optimize its own fitness, without regard for how its decisions will affect subsequent decisions by rivals. By contrast, the current paper uses a game-theoretic approach in which each firm accounts for potential competitive reactions, which allows for the possibility of strategic attacks that attempt to drive a rival out of the segment.

Other papers attribute differences among firms to their inability to optimize, for example, due to principal-agent problems (Gibbons 2006; Ellison and Holden 2008) or boundedly rational managers (Barney 1986; Goldfarb and Xiao 2009). By contrast, the current paper shows that, even if firms behave optimally, differences in firms' investment incentives can lead to persistent differences in their performance along various dimensions.

# 3 Model Set-up

Assume firms $i$ and $j$ compete in two segments, possibly of different sizes. For each time $t \in \{1, 2, \ldots\}$, let $M_{i,t}$ and $N_{i,t}$ denote firm $i$'s assets devoted to the mainstream and niche segment, respectively, and let $m_{i,t}$ and $n_{i,t}$ denote its investment in these segments.

Firm $i$'s profits in the mainstream segment in period $t$ are:

$$\pi_{i,M,t} = F(M_{i,t}) - G(M_{j,t}) - \psi M_{i,t} M_{j,t} - C m_{i,t} \tag{1}$$

Its profits in the niche segment are:

$$\pi_{i,N,t} = \alpha \Big[ F(N_{i,t}) - G(N_{j,t}) - \psi N_{i,t} N_{j,t} \Big] - C n_{i,t} \tag{2}$$

where $F' > 0$, $F'' > 0$, $G' > 0$, $\alpha \in [0, 1]$, $\psi \geq 0$. The second derivative of $G$ can be either positive or negative, but we do need $G'' \geq -\psi$.[7] The parameter $C$ is a scaling factor that represents the unit cost of investment. Firm $j$'s profit functions are the same as those of Firm $i$, except with the firm indexes reversed.[8]

Note that the model assumes increasing returns to a firm's own assets in a segment ($F'' > 0$). There are many possible sources of increasing returns, including demand-side effects, such as word-of-mouth incentives (Rob and Fishman 2005), reputation effects (Kreps and Wilson 1982), and network effects (Arthur 1989); as well as supply-side effects, such as division of labor benefits (Smith 1776) and learning curves (Argote and Epple 1990). More generally, having an existing set of assets devoted to a segment often makes it easier or more valuable for a firm to acquire additional assets devoted to this segment. For example, if a firm is already good at designing products that appeal to a particular type of customer, it makes sense for this firm to invest in the production skills needed to satisfy this customer segment's needs, and to invest in distribution agreements that enable them to reach this segment. Such asset complementarity can generate increasing returns to a firm's overall level of assets devoted to a particular segment. All of my results would also hold for S-shaped (increasing-then-decreasing) returns as long as marginal returns do not decrease to the point that firms wants to diversify.[9]

The negative interaction terms $-\psi M_{i,t} M_{j,t}$ and $-\psi N_{i,t} N_{j,t}$ imply that it is more profitable for a firm to invest in a segment where its competitor is weak. The parameter $\psi$ represents the strength of these preemption effects. For example, $\psi$ will tend to be high if consumers' search costs are low (Kuksov 2004), or if firms are very similar on dimensions other than the ones represented by the assets in which firms are investing (Bronnenberg 2008).

Both firms start at time $t = 0$ with zero assets in each segment, and assets evolve as follows:

$$M_{i,t} = \gamma M_{i,t-1} + m_{i,t} + \epsilon_{m,i,t} \tag{3}$$

[7]This restriction on the second-derivative of G ensures that a firm that is leading in the mainstream segment and trailing in the niche segment would prefer its competitor to focus in the niche segment; this condition is needed for the result about asset destruction in section 6.

[8]A supplemental appendix available on the author's website (Supplemental Appendix B) presents an example of a model of competition that is consistent with these profit functions.

[9]A supplemental appendix available on the author's website (Supplemental Appendix C) discusses this and other reasons for diversification.

$$N_{i,t} = \gamma N_{i,t-1} + n_{i,t} + \epsilon_{n,i,t} \tag{4}$$

The term $\gamma \in [0, 1)$ reflects depreciation of assets, for example, due to employees leaving the firm or loss of organization knowledge. The constant depreciation rate guarantees that assets are bounded below some finite value, and also implies that a firm must constantly reinvest in a segment where it is strong in order to maintain its asset level in that segment.[10]

I assume there is a constraint on a firm's total investment in each period:

$$m_{i,t} + n_{i,t} \leq 1 \tag{5}$$

This constraint could hold, for example, due to capital market imperfections (Myers and Majluf 1984) or employees' limited time and attention. Alternatively, a convex cost of investment would lead to similar results.[11]

The $\epsilon_{.,i,t}$ are iid random variables that follow a distribution with no mass points and with support on a finite range [0,$\epsilon_{max}$]. These variables represent random shocks to firms assets, for example, due to fortunate discoveries of better production processes, random experimentation with new product designs that happens to lead to a better understanding of customer preferences, or unpaid endorsements by celebrities. Note that, while these random shocks are assumed non-negative, the model does allow for a reduction in a firm's assets due to the depreciation term. Asset depreciation combined with an $\epsilon$ term that is close to zero for the period effectively results in a negative shock to the firm's assets in a segment.

A pure strategy $S_i = \{S_{i,1}, S_{i,2}, \ldots\}$ for firm $i$ includes a mapping for each time $t$ of current asset levels and the history of both firms' investment decisions into firm $i$'s current investment decisions:

$$S_{i,t}(A_{t-1}, h_{t-1}) \rightarrow \{m_{i,t}, n_{i,t}\} \tag{6}$$

[10]For example, even today Ben and Jerry's must continuously reinvest in training its employees and maintaining its equipment to retain its production expertise at creating chunky ice cream pints. Despite these efforts, they still occasionally produce "bad batches" of ice cream, in which all of the chunks float to the bottom. This information was provided to me during a tour of the Ben and Jerry's factory in Vermont.

[11]Even if each firm's capacity for investment grew over time, results similar to those in this paper would still hold, as along as firms do not reach a point of diminishing returns in their core expertise that causes them to diversify investment.

where $A_{i,t} \equiv \{M_{i,t}, N_{i,t}, M_{j,t}, N_{j,t}\}$ and $h_t$ is the history of all previous investment decisions for both firms through time $t$.[12] Typical dynamic investment games impose the restriction that firms follow stationary Markov strategies, which would imply that strategies do not change over time and are only a function of the current state variables. Note that the current model is more general in that it allows strategies to change over time and also allows strategies to depend on previous investment decisions, which are assumed to be publicly observable.

Finally, both firms have discount factor $\delta$, and Firm $i$'s objective at each time $t$ is to maximize its expected discounted profits:

$$V_{i,t}(A_{t-1}, h_{t-1}, S_i, S_j) = E\left[\sum_{u=t}^{\infty} \delta^{u-t}\Big(\pi_{i,M,t} + \pi_{i,N,t}\Big)\Big| A_{t-1}, h_{t-1}, S_i, S_j\right] \tag{7}$$

A pair of pure strategies $\{S_i, S_j\}$ is a subgame perfect equilibrium if for every $t$, $i$, $A_{t-1}$, $h_{t-1}$, and alternative strategy $\tilde{S}_i$ we have $V_{i,t}(A_{t-1}, h_{t-1}, S_i, S_j) \geq V_{i,t}(A_{t-1}, h_{t-1}, \tilde{S}_i, S_j)$. In other words, given the current asset levels and history of the game, each player always maximizes the expected discounted value of its remaining profits given its competitor's strategy.

I would like to focus on *where* firms invest, as opposed to *whether* they invest at all. Therefore, I assume that $C$ is small enough that, for all possible asset values, each firm can immediately increase its net profits by investing in at least one segment. A sufficient condition for this to be true is:

$$C < F'(0) - \psi\left[\frac{1+\epsilon_{max}}{1-\gamma}\right] \tag{8}$$

Without this assumption, a firm might want to diversify its investment to preempt is competitor from investing in either segment.

To summarize, two competing firms allocate investment across two segments; the profit function involves increasing returns to a firm's own assets in a segment and a negative interaction between the focal firm's assets and its competitor's assets in a segment; assets change due to depreciation, investment, and random shocks; there is a constraint to a firm's

[12]Formally, $h_0 = \emptyset$ and for $t \geq 1$, $h_t \equiv \{m_{i,1}, n_{i,1}, m_{j,1}, n_{j,1}, \ldots, m_{i,t}, n_{i,t}, m_{j,t}, n_{j,t}\}$.

total investment in each period; and firms play a subgame perfect equilibrium in which each firm maximizes expected discounted profits over an infinite number of periods.

Because this game has an uncountable state space, compact action sets, a countably infinite number of time periods, and an error term distribution with no mass points, the model conforms to the assumptions in Chakrabarti (1999), which guarantees a subgame perfect equilibrium exists. In principle, the game might only have mixed strategy equilibria, although I will derive conditions in which a pure strategy equilibrium exists.

# 4 Permanent Differences

Given this model set-up, I now derive conditions in which permanent differences are guaranteed to arise, with the random shocks determining which firm wins each segment.

To help ensure analytical tractability, this section focuses on the case in which firms are "myopic," meaning each firm maximizes its expected profits in the current period without regard for the future ($\delta = 0$). Starting by analyzing the myopic case is common in the dynamic investment games literature (e.g., Athey and Schmutzler 2001). One justification for this approach is that strategies followed by myopic firms approximate those followed by impatient firms that place small positive weight on future profits; such firms allocate investment primarily based on where their marginal returns are currently the highest, and they are generally unwilling to sacrifice short-term profits for long-term gain. Technically, because expected discounted profits are a continuous function of the discount factor $\delta$, any strategy that is strictly dominated for $\delta = 0$ is also strictly dominated for any $\delta$ sufficiently small, so firms' equilibrium policies become arbitrarily close to those described in this section as their discount factors approach zero. However, section 5 relaxes the assumption that firms are myopic and explores how my results change if firms are very patient ($\delta$ is close to 1).

Given $\delta = 0$, in each period firm $i$ makes investment choice $S_{i,t}$ to maximize $E[\pi_{i,M,t} + \pi_{i,N,t}]$ conditional on firm $j$'s investment choice $S_{j,t}$. A *pure strategy* equilibrium occurs if neither firm randomizes its investment choice, and at each time $t$ each firm's strategy is always optimal given its competitor's strategy. Appendix A proves the following preliminary result.

**Lemma 1.** *If $\delta = 0$, a pure strategy equilibrium exists, and in any pure strategy equilibrium, for each $i$ and $t$, $S_{i,t} = \{1, 0\}$ or $S_{i,t} = \{0, 1\}$.*

Intuitively, given that each firm's profit function is convex in its own assets, in any given period a firm's optimal strategy is always to focus all of its investment in a single segment. I will use the terminology "Firm $i$ focuses in the mainstream segment at time $t$" to indicate that $S_{i,t} = \{1, 0\}$ and "Firm $i$ focuses in the niche segment at time $t$" to indicate that $S_{i,t} = \{0, 1\}$. Also, recall that strategies can depend on the asset state and game history, but for notational simplicity I often drop these arguments and simply write $S_{i,t}$ instead of $S_{i,t}(A_{t-1}, h_{t-1})$.

In principle, the game could have multiple equilibria. In periods where neither firm has a dominant strategy, there is one equilibrium in which Firm $i$ focuses in the mainstream segment while Firm $j$ focuses in the niche segment, and another equilibrium in which these roles are reversed. For example, if the two segments are the same size ($\alpha = 1$) or if preemption effects are very strong ($\psi$ is large enough), then in the first period (in which firms start with zero assets at state $A_0 = \{0, 0, 0, 0\}$), either firm might focus in the mainstream segment, while its competitor focuses in the niche segment.

I would like to derive conditions that rule out such cases, so investment decisions do not depend on an arbitrary selection of equilibrium. In particular, I derive a set of conditions that ensure there is a *unique* equilibrium in which firms initially focus in the mainstream segment ($S_{i,1} = S_{j,1} = \{1, 0\}$), but they later become permanently focused on different segments, with the random shocks determining which firm focuses on each segment.

There are three conditions required for this result to hold. Intuitively, these conditions place restrictions on the size of the niche segment, the strength of competitive preemption effects, and the size of the largest possible random shocks. There is tension among the three conditions in that each of these parameters must be neither too high nor too low.

Condition 1 requires that the niche segment ($\alpha$) is *small* enough and preemption effects ($\psi$) are *weak* enough that firms initially race for the mainstream segment; whereas condition 2 requires that the niche segment is *large* enough and preemption effects are *strong* enough that one firm will eventually concede the battle for this segment and switch its focus. Similarly, condition 2 requires that the upper bound on random shocks ($\epsilon_{max}$) is *large* enough that they eventually cause enough of a gap between the firms that one firm switches focus, but condition 3 requires that this bound is *small* enough that firms do not continue making strategic changes due to random fluctuations over time. When all of these conditions hold, random fluctuations in assets are guaranteed to lead to large permanent differences.

In order to specify these conditions more precisely, I first define the following operators. For simplicity of notation, I have suppressed the subscripts on the $\epsilon$ terms; recall that these terms are assumed to be iid.

$$B(X) \equiv E\Big[F(X+1+\epsilon) - F(X+\epsilon)\Big] \tag{9}$$

$$L(X) \equiv \psi E\big[X+\epsilon\big] \tag{10}$$

Intuitively, $B(X)$ gives the expected profit increase from investing one unit in the mainstream segment if a firm's asset level in this segment is $X$, not accounting for competitive preemption effects. $L(X)$ gives the marginal expected loss due to preemption effects from investing one unit in the mainstream segment if the competitor has asset level of $X$ in this segment, accounting for the competitor's expected random shock in the segment. For the niche segment, these values need to be scaled by $\alpha$. These operators make it possible to express the following conditions more clearly.

The first key condition states for all $X \in \big[0, \frac{1}{1-\gamma}\big]$ and all $Y \leq (X\epsilon_{max})$ the following holds:

**Condition 1.**

$$B(\gamma X) - L\Big(\gamma X(1+\epsilon_{max}) + 1\Big) > \alpha\big[B(\gamma Y) - L(\gamma Y)\big] \tag{11}$$

Because this condition applies when $X$ and $Y$ both equal zero, it ensures that both firms initially have a dominant strategy of investing in the mainstream segment. This condition also ensures that firms continue racing for the mainstream segment as long as the relative asset differences between them is not too large.

On a more technical level, I show that Condition 1 guarantees that at least one firm always has a dominant strategy of investing in the mainstream segment. This implies there is a unique equilibrium in each period in which the firm with a dominant strategy focuses in the mainstream segment and its competitor plays its best response to this strategy. (See the proof of Proposition 1 in Appendix A for more details.) Thus, the choice of where each firm focuses its investment is driven by random shocks and not by equilibrium selection.

The second key condition needed for random shocks to cause permanent differences is:

**Condition 2.**

$$B(\gamma X_M) - L(\gamma X_H + 1) < \alpha\big[B(\gamma X_L) - L(0)\big] \tag{12}$$

where $X_L = \epsilon_{max}\left(\frac{1}{1-\gamma}\right)$, $X_M = \left(\frac{1}{1-\gamma}\right)$, and $X_H = (1+\epsilon_{max})\left(\frac{1}{1-\gamma}\right)$. This condition ensures that a large enough gap eventually opens between the firms that one switches focus to the niche segment.

The third key condition is:

**Condition 3.**

$$B(\gamma X_L) - L(\gamma X_M + 1) < \alpha\big[B(\gamma X_M) - L(\gamma X_L)\big] \tag{13}$$

where $X_L$ and $X_M$ are defined as above. This condition ensures that once firms have focused on different segments long enough, random shocks will never cause them to switch their focus.

Figures 1, 2, and 3 illustrate the implications of these condition graphically.[13] Together these three conditions ensure that firms initially race for the mainstream segment (condition 1), that eventually random shocks lead to a large enough gap between them that one firm changes its strategy and repeatedly focuses on the niche segment (condition 2), and that this leads to permanent differences between the firms with no further strategic changes (condition 3). These observations are formalized in the following result.

[13]To provide additional intuition for these conditions, a supplemental appendix available on the author's website (Supplemental Appendix B) presents an example of a model of per-period competition, and derives parameter values for which these conditions hold for that particular model of competition.

*Figure 1: Condition 1 ensures that firms initially race for the mainstream segment (this figure illustrates the condition for one particular value of X).*

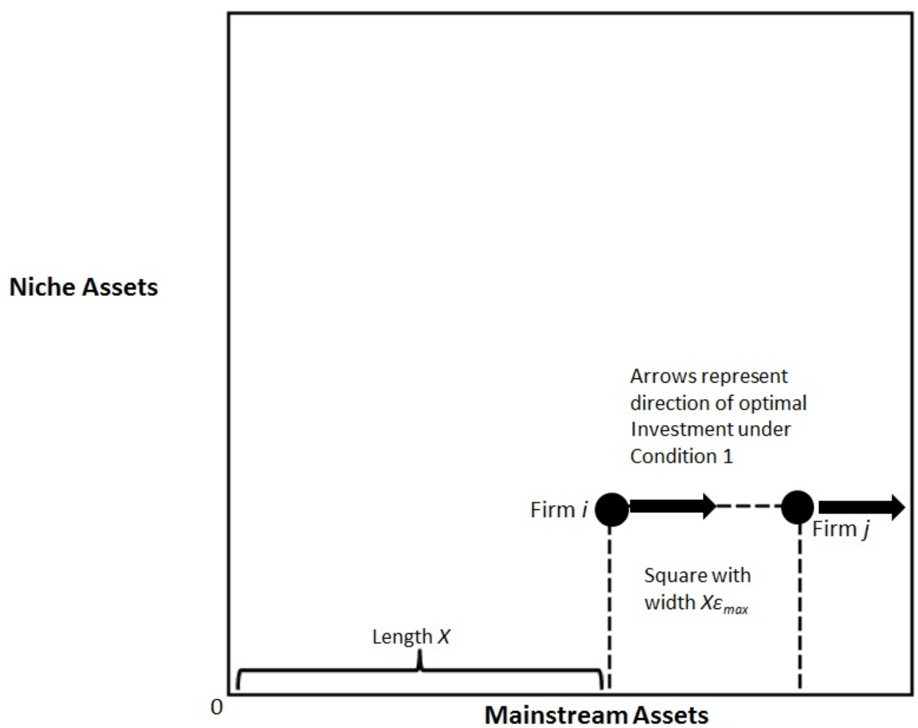



*Figure 2: Condition 2 guarantees that a sufficient "gap" will eventually open between the two firms that one firm will start investing in the niche segment.*

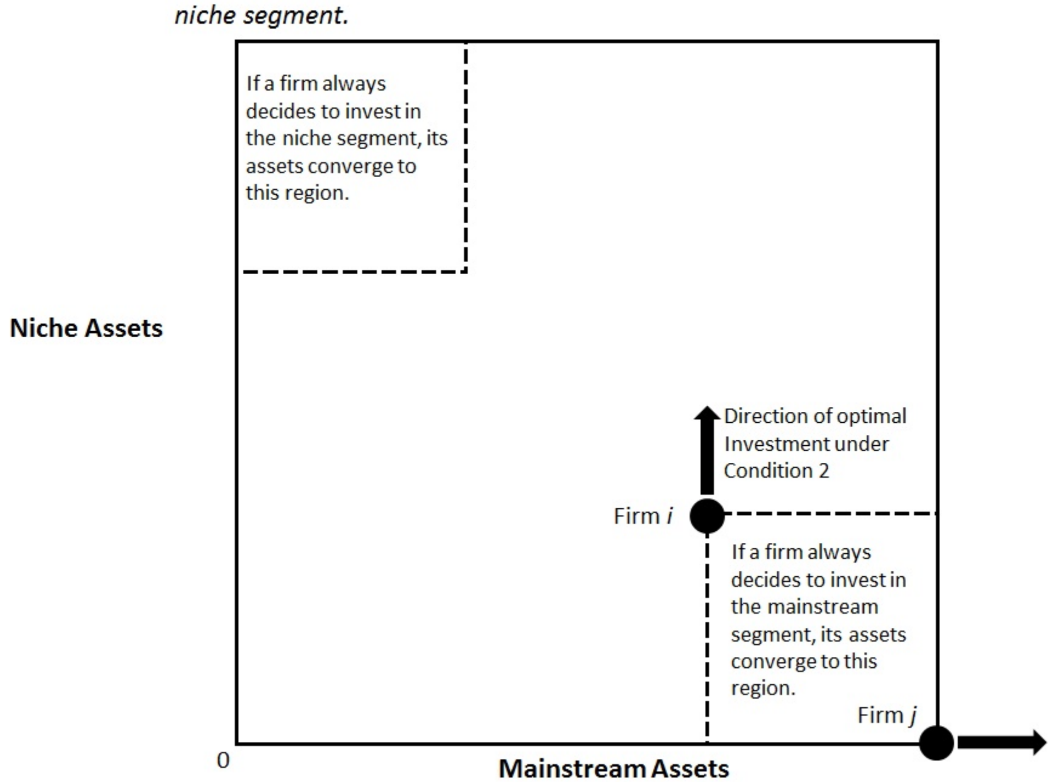
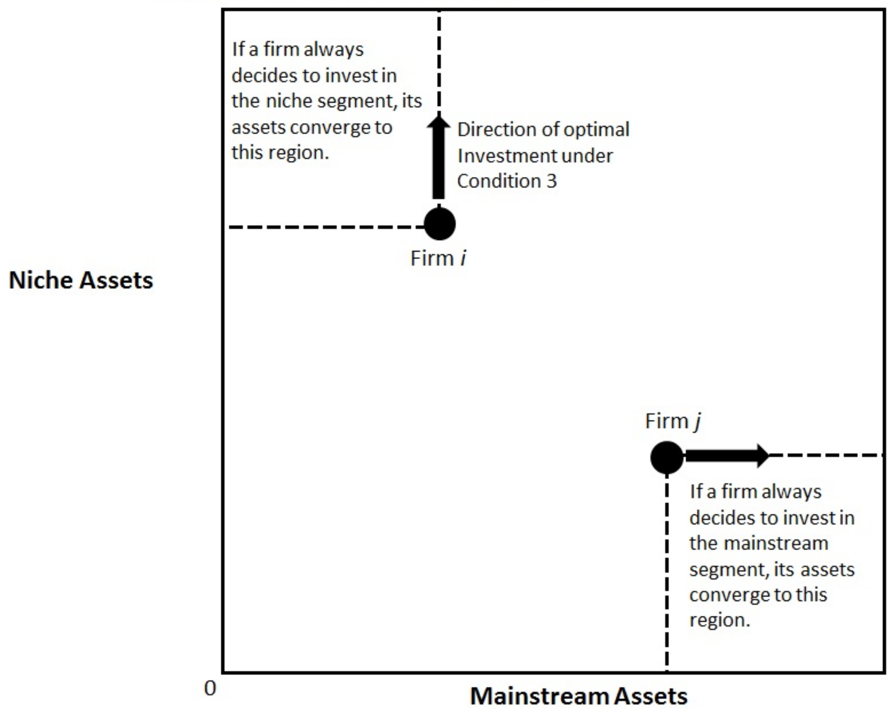


*Figure 3: Condition 3 guarantees that, once firms have focused in different segments long enough, the firm in the niche segment will never want to attack the mainstream segment.*




**Proposition 1.** *If $\delta = 0$ and Conditions 1, 2, and 3 hold, then with probability one firms have unique equilibrium strategies at every time $t$ given asset levels $A_{t-1}$. In equilibrium, both firms initially focus their investment in the mainstream segment, but the firms eventually become permanently focused on different segments. Formally, $S_{i,1}(A_0) = S_{j,1}(A_0) = \{1, 0\}$, and there exists a $T$ for which one of the following is true:*

*(1) $S_{i,t}(A_{t-1}) = \{1, 0\}$ and $S_{j,t}(A_{t-1}) = \{0, 1\}$ for all $t > T$ or*

*(2) $S_{i,t}(A_{t-1}) = \{0, 1\}$ and $S_{j,t}(A_{t-1}) = \{1, 0\}$ for all $t > T$*

The proof of this proposition in Appendix A follows the intuition outlined above, while formally confirming that the game will, with probability one, evolve as described.[14]

Note that Conditions 1 and 2 can only hold simultaneously if there are increasing returns. In fact, increasing returns play a key role in this result. Intuitively, under the conditions of this proposition, a firm only switches focus to the niche segment after its recent shocks have been

[14]In the equilibrium of Proposition 1, strategies depend on the asset state ($A_{t-1}$), but not on investment histories ($h_{t-1}$), so I have only included the former argument in the strategies. By contrast, equilibrium strategies in Proposition 3 will depend on both $A_{t-1}$ and $h_{t-1}$.

stronger in the niche segment (moving it up the increasing returns curve in that segment), and its competitor's recent shocks have been stronger in the mainstream segment (generating strong preemption effects in that segment). Only one firm can be in such a position at a time, and so there is always at most one firm that is willing to switch to the niche segment, and there can never be multiple equilibria.

# 5 Major Strategic Changes

The previous section derived conditions that guarantee permanent differences arise. By contrast, the current section shows that if we allow sufficiently large random shocks or if firms are sufficiently patient, firms can make major strategic changes over time.

As the size of the largest random shock grows, the boxed regions in Figure 3 grow until they become arbitrarily close to intersecting. This ensures that a firm focusing on the niche segment will eventually come close enough to its rival that its best strategy is to attack the mainstream segment.

**Proposition 2.** *If $\delta = 0$ and Conditions 1, 2, and 3 hold, then with probability one firms have unique equilibrium strategies at every time $t$ given asset levels $A_{t-1}$. In equilibrium, both firms initially focus their investment in the mainstream segment for at least one period, and one firm eventually switches focus to the niche segment; however, this firm later switches focus back again to the mainstream segment.*

When the conditions of this proposition hold, over time both firms continually shift their focus due to random fluctuations.

Results so far have focused on the case when each firm is myopic. Such a firm bases its investment decisions entirely on where its current marginal returns are highest, given its own state and given its competitor's state. These previous results also approximate the behavior of impatient firms that place *small* positive weight on future profits. On the other hand, I now show how the results change when firms are patient and place *large* weight on future profits.

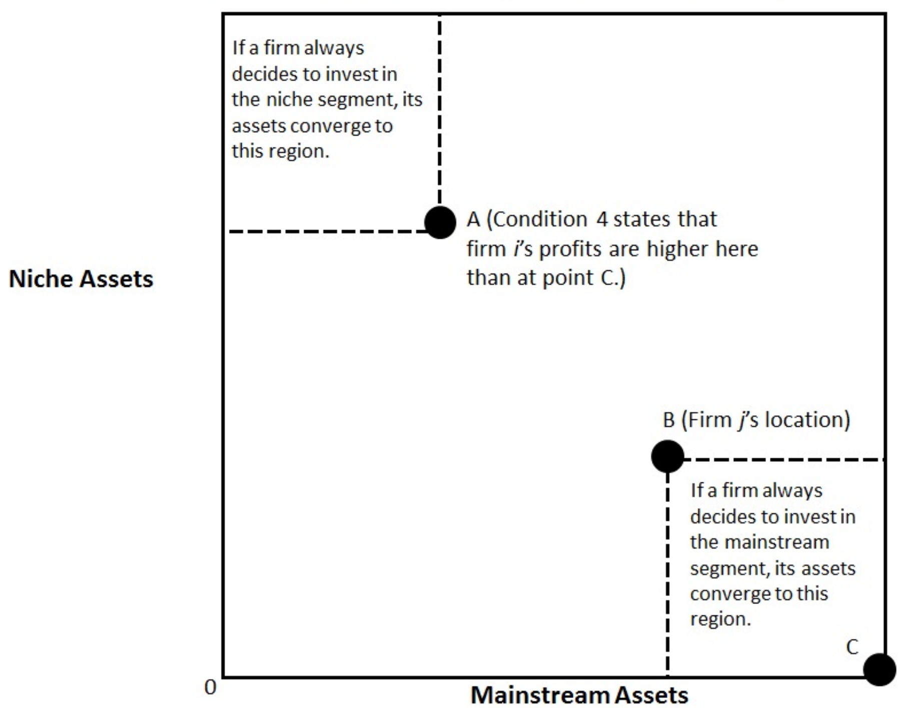
A forward-looking firm must take into account how its decision in any given period affects the future state of the game and the future behavior of its competitor. Also, as firms become more patient, they become willing to battle for the mainstream segment for a longer period of time in the hope of eventually winning this segment. In fact, I show that if firms become patient enough, then regardless of their current assets there is always an equilibrium in which a firm focusing on the niche segment changes its strategy and attacks the mainstream segment. Formally, assume the following condition holds:

**Condition 4.**

$$F(X_H) + \alpha F(0) - \psi\Big(\frac{1}{1-\gamma}\Big)X_M < F(X_L) + \alpha F(X_M) - \psi\alpha\Big(\frac{1}{1-\gamma}\Big)X_L \tag{14}$$

where $X_H$, $X_M$, and $X_L$ are defined as in the previous section.

*Figure 4: Condition 4 guarantees that, if a firm's competitor repeatedly focuses on the mainstream segment, the focal firm's long-run profits are always higher if it repeatedly focuses on the niche (rather than mainstream) segment.*




Intuitively, Condition 4 is a stronger version of Condition 3. This new condition implies that preemption effects are strong enough that, if the competitor's strategy is always to focus on the mainstream segment, then the focal firm's long-run profits from attacking this segment are lower than its profits from staying focused on the niche segment, even if random shocks turn out to be most favorable to such an attack.

We might expect that this condition would ensure permanent differences arise, and in fact it does ensure there is an equilibrium in which firms stay permanently focused on different segments. However, condition 4 also guarantees that if both firms are sufficiently patient, and if the firm focused on the niche segment adopts an aggressive strategy of always attacking the mainstream segment, then its competitor's best response is eventually to retreat from this segment. As a result, when both firms are sufficiently patient, major strategic changes are always possible regardless of the history of the game.

**Proposition 3.** *If Condition 4 holds, there exists a $\widehat{\delta} \in (0, 1)$ such that if $\delta \geq \widehat{\delta}$, then for any possible history of the game, there is a subgame perfect equilibrium in which from this period forward Firm $i$ always focuses its investment in the mainstream segment, and Firm $j$ eventually becomes permanently focused in the niche segment. Formally, for any time $T$, feasible asset state $A_{T-1}$, and game history $h_{T-1}$, there is a subgame perfect equilibrium in which $S_{i,t}(A_{t-1}, h_{t-1}) = \{1, 0\}$ for all $t \geq T$, and $S_{j,t}(A_{t-1}, h_{t-1})$ converges to $\{0, 1\}$ as $t \to \infty$. There is also another subgame perfect equilibrium in which these roles are reversed.*

The formal proof in Appendix A shows that, under the conditions of the proposition, neither firm has an incentive to deviate from the proposed equilibrium. Intuitively, a very patient firm is willing to sacrifice short-term profits, shift its investment out of its existing area of expertise in the small segment, and start building assets in the larger segment. If the competitor is also sufficiently patient, then its best response to this strategy is eventually to retreat from the larger segment and shift focus to the smaller segment.

# 6 Model Extension: Asset Destruction

One counterintuitive implication of this model is that a firm can sometimes benefit from a decrease in its own assets. For example, imagine a firm has a lead in the mainstream segment, but not large enough a lead to cause its competitor to switch focus to the niche segment. The firm might benefit from a *reduction* in its own niche assets, which could entice the competitor to focus on the niche segment.

To formalize this intuition, I now extend the model to allow each firm to destroy some of its own assets. At the beginning of each period $t$, Firm $i$ chooses a mainstream asset level anywhere in the range $[0, M_{i,t-1}]$, and chooses a niche asset level in the range $[0, N_{i,t-1}]$, while Firm $j$ makes analogous choices. Choosing an asset level below the top of these ranges represents partial asset destruction.

The game timing at each period is now: (1) Firms simultaneously make asset destruction decisions in both segments. (2) Firms simultaneously make investment decisions. (3) Random shocks occur, and profits are realized.

For analytical tractability, I again focus on the case of myopic firms. Note that a myopic firm may want to destroy some of its assets at the start of the period to affect the current period's investment choices. Appendix A contains a formal proof of the following result.

**Proposition 4.** *If $\delta = 0$ and Conditions 1, 2, and 3 hold, there exist asset levels for which there is a subgame perfect equilibrium in which one firm chooses to destroy some of its niche assets.*

For such a strategy to work, it is important for asset destruction to be publicly observable and irreversible. Otherwise, a firm might want to temporarily reduce its niche assets at the start of the period in order to influence its competitor's investment decision, but then restore those "destroyed" assets before the end of the period.

# 7 Conclusion

This paper has developed a model in which firms dynamically compete for different market segments. Under certain conditions, each firm becomes permanently focused on the segment where it has the best initial luck.

This model could help explain why regional differences in the market shares of consumer packaged goods (CPG) makers tend to persist over such long time periods — manufacturers founded on the West Coast continue to have higher market share there, whereas those founded on the East Coast continue to have higher market share there, in some cases even after 100 years of competition (Bronnenberg, Dhar, and Dubé 2007, 2009). This model implies that in CPG markets, where key assets like brands undergo relatively small random shocks in any given year, there should be persistent differences between firms. On the other hand, in markets for high tech products, in which assets like up-to-date technical expertise are subject to large shocks as new technologies arrive and displace old ones, large changes in strategy should be more common.

A surprising insight from the model is that a firm might want to deliberately reduce its assets in a particular segment. For example, a company might want to spin off a division that focuses on a small niche segment as a way of committing to limit the resources available to this division, thus enticing its competitor to invest in the niche segment.

Future research could extend the model to incorporate several important phenomena that were not addressed in the current paper. For example, firms often face uncertainty over exogenous factors such as demand for a product (Hitsch 2006), the firm's underlying efficiency (Jovanovic 1982); these factors can lead firms to exit the market or reposition a product as new information arrives. In addition, while the current paper has focused on dynamic investment allocation decisions, firms can also dynamically adjust their prices, for example, by offering a different price to their current customers, which then influences the size of their loyal customer base in future periods (Villas-Boas 1999; Shin and Sudhir 2010). It would be interesting to extend the model to explore how these phenomena affect optimal investment behavior.

# References


[1] Argote, L., and D. Epple (1990). "Learning Curves in Manufacturing," *Science*, 247, 4945, p. 920-924.

[2] Arthur, W. B. (1989). "Competing Technologies, Increasing Returns, and Lock-In by Historical Events," *Economic Journal*, 99, 394, p. 116-131.

[3] Athey, S, and A. Schmutzler (2001). "Investment and Market Dominance," *RAND Journal of Economics*, 32, 1, p. 1-26.

[4] Barney, J. (1986). "Strategic Factor Markets: Expectations, Luck, and Business Strategy," *Management Science*, 32, 10, p. 1231-1241.

[5] Bertsekas, D. P. (2007). *Dynamic Programming and Optimal Control, Volume 2*. Athena Scientific, Belmont, MA.

[6] Besanko, D. and U. Doraszelski (2004). "Capacity Dynamics and Endogenous Asymmetries in Firm Size," *RAND Journal of Economics*, 35, 1, p. 23-49.

[7] Bronnenberg, B. (2008). "Competition in CPG Industries: Sustaining Large Local Advantages with Little Product Differentiation," *Quantitative Marketing and Economics*, 6, p. 79-107.

[8] Bronnenberg, B. J., S. K. Dhar, and J. P. Dubé (2007). "Consumer Packaged Goods in the United States: National Brands, Local Branding," *Journal of Marketing Research*, 44, 1, p. 4-13.

[9] Bronnenberg, B. J., S. K. Dhar, and J. P. Dubé (2009). "Brand History, Geography, and the Persistence of Brand Shares," *Journal of Political Economy*, 117, 1, p. 87-115.

[10] Budd, C., C. Harris, and J. Vickers (1993). "A Model of the Evolution of Duopoly: Does the Asymmetry Between Firms Tend to Increase or Decrease?" *Review of Economic Studies*, 60, 3, p. 543-573.

[11] Chakrabarti, S. K. (1999). "Markov Equilibria in Discounted Stochastic Games," *Journal of Economic Theory*, 85, p. 294-327.

[12] Chen, Y. and J. Xie (2007). "Cross-market Network Effect with Asymmetric Customer Loyalty: Implications for Competitive Advantage," *Marketing Science*, 26, 1, p. 52-66.

[13] D'Aspremont, C., J. J. Gabszewicz, and J.-F. Thisse (1979). "On Hotelling's Stability in Competition," *Econometrica*, 47, 5, p. 1145-1150.

[14] Dell, M. (1999). *Direct From Dell*. HarperCollins, New York.

[15] Desai, P., S. Kekre, S. Radhakrishnan, and K. Srinivasan (2001). "Product Differentiation and Commonality in Design: Balancing Revenue and Cost Drivers," *Management Science*, 47, 1, p. 37-51.

[16] Dreifus, C. (1994). "Passing the Scoop; Ben & Jerry," *New York Times*, December 18, 1994.

[17] Dutta, S., O. Narasimhan, and S. Rajiv (1999). "Success in High-Technology Markets: Is Marketing Capability Critical?" *Marketing Science*, 18, 4, p. 547-568.

[18] *Edge* (2009). "The Making Of PlayStation," http://www.edge-online.com/ magazine/the-making-of-playstation, April 24, 2009.

[19] Ellison, G., and R. Holden (2008). "A Theory of Rule Development," Mimeo.

[20] Ericson, R. and A. Pakes (1995). "Markov-Perfect Industry Dynamics: A Framework for Empirical Work," *The Review of Economic Studies*, 62, 1, p. 53-82.

[21] Gibbons, R. (2006). "What the Folk Theorem Doesn't Tell Us," *Industrial and Corporate Change*, 15, p. 381-386.

[22] Goettler, R. L. and R. Shachar (2001), "Spatial Competition in the Network Television Industry," *RAND Journal of Economics*, 32, 4, p. 624-656.

[23] Golder, P. and G. Tellis (1993). "Pioneer Advantage: Marketing Logic or Marketing Legend?" *Journal of Marketing Research*, 30, 2, p. 158-170.

[24] Goldfarb, A. and M. Xiao (2009). "Who Thinks About the Competition? Managerial Ability and Strategic Entry in US Local Telephone Markets," Mimeo.

[25] Hitsch, G. J. (2006). "An Empirical Model of Optimal Dynamic Product Launch and Exit Under Demand Uncertainty," *Marketing Science*, 25, 1, p. 25-50.

[26] Hörner, J. (2004). "A Perpetual Race to Stay Ahead," *Review of Economic Studies*, 71, 4, p. 1065-1088.

[27] Jovanovic, B. (1982). "Selection and the Evolution of Industry," *Econometrica*, 50, 3, p. 649-670.

[28] Kreps, D. M. and R. Wilson (1982). "Reputation and Imperfect Information," *Journal of Economic Theory*, 27, 2, p. 253-279.

[29] Kuksov, D. (2004)."Buyer Search Costs and Endogenous Product Design," *Marketing Science*, 23, 4, p. 490-499.

[30] Lager, F. (1994). *Ben & Jerry's, the Inside Scoop*. Crown, New York.

[31] Levinthal, D. (1997). "Adaptation on Rugged Landscapes." *Management Science*, 43, 7, p. 934-950.

[32] Maskin, E. (1995). "Long-term Investment by Japanese and American Firms," *Japan and the World Economy*, 7, p 249-254.

[33] Mintzberg, H. and J. Waters (1985). "Of Strategies, Deliberate and Emergent," *Strategic Management Journal*, 6, 3, p. 257-272.

[34] Moorthy, K. S. (1988). "Product and Price Competition in a Duopoly," *Marketing Science*, 7, 2, p. 141-168.

[35] Mossberg, W. and K. Boehret (2006). "Battle of the Boxes: PlayStation 3 vs. Wii," *Wall Street Journal*, November 29, 2006, p. D1.

[36] Myers, S. C., and N. Majluf (1984). "Corporate Financing and Investment Decisions when Firms Have Information that Investors Do Not Have," *Journal of Financial Economics*, 1, 2, p. 187-221.

[37] Orhun (2005). "Spatial Differentiation in the Supermarket Industry," Mimeo.

[38] Prahalad, C. K. and G. Hamel (1990). "The Core Competence of the Corporation," *Harvard Business Review*, 68, 3, p. 79-91.

[39] Prescott, E. C. and M. Visscher (1977). "Sequential Location among Firms with Foresight," *Bell Journal of Economics*, 8, 2, p. 378-393.

[40] Rob, R. and A. Fishman (2005). "Is Bigger Better? Customer Base Expansion through Word-of-Mouth Reputation," *Journal of Political Economy*, 113, 5, p. 1146-1162.

[41] Shah, Agam (2008). "Dell Ironing out Kinks in Supply Chain, Executive Says," *InfoWorld*, April 03, 2008.

[42] Shaked, A. and J. Sutton (1982). "Relaxing Price Competition through Product Differentiation," Review of Economic Studies, 49, p. 3-13.

[43] Shin, J. (2007). "How Does Free Riding on Customer Service Affect Competition?," *Marketing Science*, 26, 4, p. 488-503.

[44] Shin, J. and K. Sudhir (2010). "A Customer Management Dilemma: When Is It Profitable to Reward Your Own Customers?", *Marketing Science*, forthcoming.

[45] Smith, A. (1776). *An Inquiry into the Nature and Causes of the Wealth of Nations.* Bantam, New York, 2003.

[46] Subramaniam, R. and E. Gal-Or (2009). "Quantity Discounts in Differentiated Consumer Product Markets," *Marketing Science*, 28, 1, p. 180-192.

[47] Sutton, J. (1991). *Sunk Costs and Market Structure: Price Competition, Advertising, and the Evolution of Concentration.* MIT Press, Cambridge, MA.

[48] Teece, D. J., G. Pisano, and A. Shuen (1997). "Dynamic Capabilities and Strategic Management," *Strategic Management Journal*, 18, p. 509-533.

[49] Thomadsen, R. (2007). "Product Positioning and Competition: The Role of Location in the Fast Food Industry," *Marketing Science*, 26, 6, p. 792-804.

[50] Urban, G. L, T. Carter, S. Gaskin, Z. Mucha (1986). "Market Share Rewards to Pioneering Brands: An Empirical Analysis and Strategic Implications," *Management Science*, 32, 6, p. 645-659.

[51] Villas-Boas (1993). "Predicting Advertising Pulsing Policies in an Oligopoly: A Model and Empirical Test," *Marketing Science*, 12, 1, p. 88-102.

[52] Villas-Boas (1999). "Dynamic Competition with Customer Recognition," *RAND Journal of Economics*, 30, 4, p. 604-631.

[53] Wernerfelt, B. (1984). "A Resource-based View of the Firm," *Strategic Management Journal*, 5, 2, p. 171-80.

[54] Zhu, T. and V. Singh (2009). "Spatial Competition with Endogenous Location Choices: An Application to Discount Retailing," *Quantitative Marketing and Economics*, 7, p. 1-35.

# Appendix A: Proofs

**Proof of Lemma 1:** Given $\delta = 0$, at each time $t$ Firm $i$ chooses investment levels $\{m_{i,t}, n_{i,t}\}$ to maximize $E\big[\pi_{i,M,t}(m_{i,t}, m_{j,t}) + \pi_{i,N,t}(n_{i,t}, n_{j,t})\big]$ subject to $m_{i,t} + n_{i,t} \leq 1$, conditional on Firm $j$'s investment choices $\{m_{j,t}, n_{j,t}\}$. I will first show it can never be an optimal for Firm $i$ to choose any investment levels other than $\{1, 0\}$ or $\{0, 1\}$. By differentiating firm $i$'s profit function in each segment, we have:

$$\frac{dE[\pi_{i,M,t}]}{dm_{i,t}} = E\Big[F'(\gamma M_{i,t-1} + m_{i,t} + \epsilon_{m,i,t})\Big] - \psi E\Big[\gamma M_{j,t-1} + m_{j,t} + \epsilon_{m,j,t}\Big] - C \tag{A1}$$

$$\frac{dE[\pi_{i,N,t}]}{dn_{i,t}} = \alpha\Bigg[E\Big[F'(\gamma N_{i,t-1} + n_{i,t} + \epsilon_{n,i,t})\Big] - \psi E\Big[\gamma N_{j,t-1} + n_{j,t} + \epsilon_{n,j,t}\Big]\Bigg] - C \tag{A2}$$

Inequality (8) ensures that (A1) is positive for any feasible asset values. This implies that Firm $i$ would never set $m_{i,t} + n_{i,t} < 1$ because it could always increase its expected profits by increasing $m_{i,t}$ until its investment constraint is binding, that is, until $m_{i,t} + n_{i,t} = 1$. We can therefore substitute $(1 - m_{i,t})$ for $n_{i,t}$ and rewrite Firm $i$'s objective as maximizing: $E\big[\pi_{i,M,t}(m_{i,t}, m_{j,t}) + \pi_{i,N,t}(1 - m_{i,t}, n_{j,t})\big]$ subject to $m_{i,t} \in [0, 1]$. The assumption that $F'' > 0$ implies that this expected profit function is convex in $m_{i,t}$, and the maximum must occur at one of the extreme points $m_{i,t} = 0$ or $m_{i,t} = 1$. Thus, the only possible optimal levels for $\{m_{i,t}, n_{i,t}\}$ are $\{1, 0\}$ or $\{0, 1\}$. Although there could be mixed strategy equilibria in which firms randomize between these two investment choices, I will show that a pure strategy equilibrium must exist.

Considering that $\{1, 0\}$ or $\{0, 1\}$ are the only possible optimal strategies, Firm $i$'s best response to $\{m_{j,t}, n_{j,t}\}$ is to choose $\{1, 0\}$ if the following inequality holds:

$$E\big[\pi_{i,M,t}(1, m_{j,t}) - \pi_{i,M,t}(0, m_{j,t})\big] \geq E\big[\pi_{i,N,t}(1, n_{j,t}) - \pi_{i,N,t}(0, n_{j,t})\big] \tag{A3}$$

If this inequality does not hold, then Firm $i$'s best response is $\{0, 1\}$.

If either firm has a dominant strategy, there is a pure strategy equilibrium in which this firm plays its dominant strategy and its competitor plays a best response to this dominant strategy.

On the other hand, suppose neither firm has a dominant strategy, so that each firm might have a best response of either $\{1, 0\}$ or $\{0, 1\}$ depending on what its competitor does. Note that Firm $i$'s marginal returns to investing in the mainstream segment, given by (A1), are decreasing in $m_{j,t}$, while its marginal returns to investing in the niche segment, given by (A2), are decreasing in $n_{j,t}$. In other words, a firm's marginal investment returns in a segment become lower if its competitor invests in that segment. As a result, if Firm $i$ does not have a dominant strategy, its best response to $\{1, 0\}$ must be $\{0, 1\}$ and its best response to $\{0, 1\}$ must be $\{1, 0\}$. Analogous results hold for Firm $j$. Thus, if neither firm has a dominant strategy, there is a pure strategy equilibrium in which Firm $i$ plays $\{1, 0\}$ while Firm $j$ plays $\{0, 1\}$, and another pure strategy equilibrium in which these roles are reversed.

Together these results guarantee that there is a pure strategy equilibrium of each possible subgame at every time $t$. QED

**Proof of Proposition 1:** The proof proceeds in three steps. I will show that under the conditions of this proposition: (1) With probability one, at each time $t \geq 1$ there is a unique pure strategy equilibrium in which at least one firm (which we label Firm $i$) has a dominant strategy of setting $S_{i,t} = \{1, 0\}$. (2) With probability one, there exists a $T > 0$ such that one firm (which we label Firm $i$) has assets $M_{i,T} \in [X_M, X_H]$ and $N_{i,T} \in [0, X_L]$, while the other firm (which we label Firm $j$) has assets $M_{j,T} \in [0, X_L]$ and $N_{j,T} \in [X_M, X_H]$. (3) For all $t > T$, there is a unique pure strategy equilibrium in which $S_{i,t} = \{1, 0\}$ and $S_{j,t} = \{0, 1\}$.

*Step 1:*

Note that if firm $i$ has always invested in the mainstream segment for $t$ periods, its assets then lie in a region that is a square with width determined by the largest possible random shock:

$$\sum_{u=1}^{t} \gamma^{t-1} \leq M_{i,t} \leq \sum_{u=1}^{t} \gamma^{t-1}(1 + \epsilon_{max}) \tag{A4}$$

$$0 \leq N_{i,t} \leq \sum_{u=1}^{t} \gamma^{t-1} \epsilon_{max} \tag{A5}$$

The box enclosed by the dashed lines in Figure 1 provides an example of how this region looks after a finite number of periods.

I will show that Condition 1 guarantees that for all $t$ at least one firm always lies in the regions defined by (A4) and (A5). Condition 1 immediately implies that both firms have a dominant strategy of investing in the mainstream segment in period one, so both firms' assets lie in this region in the first period. I will show that if at least one firm lies in this region at time $t$, then at least one firm will also lie in the region at time $t + 1$.

Suppose *both* firms lie in this region at time $t$. Without loss of generality, label the firms such that $N_{i,t} \leq N_{j,t}$. Inequalities (A4) and (A5) imply $N_i < M_i \epsilon_{max}$. Therefore, Condition 1 implies:

$$B(\gamma M_i) - L\Big(\gamma M_i(1 + \epsilon_{max}) + 1\Big) > \alpha\big[B(\gamma N_i) - L(\gamma N_i)\big] \tag{A6}$$

Both firms assets satisfying (A4) and (A5) implies that $M_j < M_i(1 + \epsilon_{max})$, and the firms were labeled such that $N_j > N_i$. Therefore, because $B' > 0$ and $L' > 0$, (A6) implies:

$$B(\gamma M_i) - L(\gamma M_j + 1) > \alpha\big[B(\gamma N_i) - L(\gamma N_j)\big] \tag{A7}$$

Thus, firm $i$ has a dominant strategy of investing in the mainstream segment, which guarantees that it will focus in the mainstream segment again in period $t+1$.

Now suppose only Firm $i$'s assets satisfy (A4) and (A5) at time $t$. If $N_i \leq N_j$, the same argument as above holds. On the other hand, even if $N_i > N_j$, I will show that Firm $i$ still must have a dominant strategy of investing in the mainstream region. In this case, we must have $M_i > M_j$, or Firm $j$'s assets would satisfy (A4) and (A5) too. Also, because Firm $i$'s assets satisfy (A4) and (A5), we have $N_i \leq (M_i \epsilon_{max})$. These two facts imply that:

$$M_i(1 + \epsilon_{max}) > M_j + N_i \tag{A8}$$

By substituting the right side of (A8) into (A6) we have:

$$B(\gamma M_i) - L(\gamma M_j + \gamma N_i + 1) > \alpha\big[B(\gamma N_i) - L(\gamma N_i)\big] \tag{A9}$$

Because $L$ is a linear function, this is equivalent to:

$$B(\gamma M_i) - L(\gamma M_j + 1) - L(\gamma N_i) > \alpha\big[B(\gamma N_i) - L(\gamma N_i)\big] \tag{A10}$$

Because $\alpha < 1$, this implies that:

$$B(\gamma M_i) - L(\gamma M_j + 1) > \alpha\big[B(\gamma N_i)\big] \tag{A11}$$

Because $L$ is non-negative, this implies that:

$$B(\gamma M_i) - L(\gamma M_j + 1) > \alpha\big[B(\gamma N_i) - L(\gamma N_j)\big] \tag{A12}$$

Thus, Condition 1 still implies Firm $i$ has a dominant strategy of investing in the mainstream segment, and it will stay in the mainstream focus region in the next period.

To summarize, I have shown that at least one firm's assets satisfy (A4) and (A5) in period 1; and that if at least one firm is in this region in period $t$ then the same will be true in period $t+1$; this guarantees that at least one firm will always be in this region.

We have established that in each period one firm (call if Firm $i$) has a dominant strategy of focusing in the mainstream segment. Thus, in each period there is a unique equilibrium in which this firm sets $S_{i,t} = \{1, 0\}$ and its competitor plays its best response to this strategy. (Technically, there are *two* equilibria (not a unique equilibrium) if Firm $j$ is exactly indifferent between setting $S_{j,t} = \{1, 0\}$ and $S_{j,t} = \{0, 1\}$ in response to $S_{i,t} = \{1, 0\}$; however, the strict convexity of the profit functions and the assumption that error terms have no mass points ensure that assets have zero probability of landing precisely at such a state, so the equilibrium is unique with probability one).

*Step 2:*

I will now show that one firm (let us call it Firm $j$) eventually switches its focus to the niche segment and its assets then enter region defined by: $M_j \in [0, X_L]$ and $N_j \in [X_M, X_H]$, where $X_L$, $X_M$, and $X_H$ are defined as in section 4.

By the results shown in Step 1, at any time $t$ at least one firm's assets are in the region defined by (A4) and (A5). If only one firm is in this region, label this firm Firm $i$. If both firms are in the region, label the one with lower niche assets as Firm $i$. The results from step one of the proof imply Firm $i$ focuses in the mainstream segment in the current period. Now suppose Firm $i$ receives shocks $\{\epsilon_{max}, 0\}$, while firm $j$ receives shocks $\{0, \epsilon_{max}\}$. (Technically, the argument holds if firms receive shocks sufficiently close to these values.) These shocks ensure that Firm $i$'s assets remain in the region defined by (A4) and (A5); if Firm $j$'s assets were not in this region, they do not enter this region in the next period; and if Firm $i$ had lower niche assets than Firm $j$, this continues to be the case in the next time period. Therefore, the results from Step 1 guarantee that Firm $i$ focuses in the mainstream segment again in the next time period.

If the firms repeatedly continue receiving such shocks, then Firm $i$ always focuses in the mainstream segment and its assets converge to point $\{X_H, 0\}$, and regardless of where Firm $j$ focuses its assets reach a point where $M_j < X_M + d$ and $N_j > X_L - d$ for any positive $d$. If we choose $d > 0$ sufficiently small, then Condition 2 guarantees that Firm $j$ focuses in the niche segment if $M_{j,t} < X_M + d$, $N_{j,t} > X_L - d$, $M_{i,t} > X_H - d$, and $N_{i,t} < d$. Thus, given this series of shocks, Firm $j$ eventually focuses in the niche segment. If the firms still continue receiving such shocks, Firm $j$'s niche assets grow while its mainstream assets shrink, so it continues focusing in the niche segment, and eventually its assets enter the region where $M_j \in [0, X_L]$ and $N_j \in [X_M, X_H]$.

The law of large numbers guarantees that such a series of shocks eventually occurs with probability one. To see why, let $Z$ be the minimum number of such shocks needed to ensure that, from any feasible starting point, firms reach a state where $M_j \in [0, X_L]$ and $N_j \in [X_M, X_H]$. Now define a sequence of random variables $z_1$, $z_2$, $\ldots$, where $z_1$ equals 1 if such shocks occur in each of the first $Z$ time periods of the game and equals 0 otherwise, $z_2$ is defined equivalently for the next $Z$ time periods, and so on. Note that the expectation of each variable is positive because there is always positive probability of this sequence of shocks occurring. The law of large numbers guarantees that the average of these variables converges to their expectation, which can only occur if such a sequence of shocks eventually does occur. Thus, Firm $j$'s assets eventually enter the proposed region. Intuitively, because it is always possible for firms to have a string of good luck in different segments, if the game continues long enough such a string of shocks will eventually occur. Also, note that the labeling of Firm $j$ was based on the state before this series of shocks occurred; depending on the early shocks in the game, either firm could be the one we label "Firm $j$," so either could end up in the niche segment.

*Step 3:*

We have established that for some $T > 0$, one firm (call it Firm $i$) has assets that satisfy $M_{i,T} \in [X_M, X_H]$ and $N_{i,T} \in [0, X_L]$, while the other firm (call it Firm $j$) has assets that satisfy $M_{j,T} \in [0, X_L]$ and $N_{j,T} \in [X_M, X_H]$.

Condition 3, along with the assumptions of increasing returns and a negative interaction with the competitor's assets, imply that whenever each firm's assets are in the regions given above, Firm $i$ focuses in the mainstream segment and Firm $j$ focuses in the niche segment. Therefore, each firm stays in its respective region in the next period. Thus, they will always remain in these regions. QED

**Proof of Proposition 2:** Steps 1 and 2 from the proof of Proposition 1 still hold. The only difference is we are now assuming Condition 3 does not hold, so Step 3 of Proposition 1 no longer holds.

Label the firm with larger niche assets as Firm $j$. Now suppose Firm $j$ receives a series of shocks with values $\{\epsilon_{max}, 0\}$, while firm $i$ receives a series of shocks with values $\{0, \epsilon_{max}\}$. (As in the previous proof, the argument still holds if firms receive shocks sufficiently close to these values.) Given this series of shocks, Firm $i$'s assets will converge to point $\{X_M, X_L\}$, while Firm $j$'s assets will converge to point $\{X_L, X_M\}$. Because Condition 3 no longer holds, once both firms are within a small enough distance $d$ from these points, Firm $j$ switches its investment focus to the mainstream segment, and if the firms continue receiving such shocks, Firm $j$ continues focusing in the mainstream segment and eventually enters the region given by (A4) and (A5).

As in the previous proof, because this entire sequence of events has positive probability of occurring in finite amount of time, the law of large numbers guarantees that it does eventually occur. QED

**Proof of Proposition 3:** I will show that, under the conditions of the proposition, the subgame starting at any time $T$ has an equilibrium in which Firm $i$ plays the "aggressive" strategy of always investing in the mainstream segment and Firm $j$ plays a best response to this strategy. (By symmetry, this subgame also has another equilibrium in which these roles are reversed.)

I first define a new variable $a_t \in \{i, j\}$ which is a function of the firms' investment history since time $T$ and indicates which firm is currently playing an "aggressive" strategy. Regardless of the starting asset state, $A_{T-1}$, I show that there is an equilibrium in which $a_T = i$, and for all $t > T$ the game proceeds as follows: (1) the firm designated as aggressive by the variable $a_t$ focuses in the mainstream segment at time $t$; (2) if the aggressive firm stays with this equilibrium strategy at time $t$, then $a_{t+1} = a_t$ (the same firm stays aggressive in the next period); however, if the aggressive firm deviates from this equilibrium strategy, then $a_{t+1} \neq a_t$ (the other firm then becomes the aggressive one); (3) the firm that is not aggressive eventually becomes permanently focused on the niche segment.

The proof proceeds in two steps. The first step shows that the non-aggressive firm does not want to deviate from this equilibrium, and the second step shows that the aggressive firm also does not want to deviate.

*Step 1:*

Let $a_T = i$, so Firm $j$ is the non-aggressive firm. Given the proposed equilibrium, Firm $i$ will always invest in the mainstream segment regardless of what Firm $j$ does, so Firm $j$ simply plays a best response to this strategy.

Suppose we fix Firm $i$'s assets at the point $\{X_M, X_L\}$, and Firm $j$ knows that its shocks will always be $\{\epsilon_{max}, 0\}$. Note that Firm $j$'s assets converge to the point $\{X_L, X_M\}$ if and only if its investment strategy converges to setting $m_{j,t} = 0$ and $n_{j,t} = 1$. Given such a strategy, as $\delta \to 1$, Firm $j$'s value function satisfies:

$$(1-\delta)V_{j,t} = (1-\delta)E\left[\sum_{u=t}^{\infty} \delta^{u-t}\Big(\pi_{j,M,t} + \pi_{j,N,t}\Big)\right] \xrightarrow[\delta \to 1]{} \Big[\pi_{j,M}(X_M, X_L) + \pi_{j,N}(X_L, X_M) - C\Big] \tag{A13}$$

where we define:

$$\pi_{j,M}(M_{j,t}, M_{i,t}) = F(M_{j,t}) - G(M_{i,t}) - \psi M_{j,t} M_{i,t} \tag{A14}$$

$$\pi_{j,N}(N_{j,t}, N_{i,t}) = \alpha\Big[F(N_{j,t}) - G(N_{i,t}) - \psi N_{j,t} N_{i,t}\Big] \tag{A15}$$

Intuitively, as firms become very patient, the discounted value of their profits depends only on the long-run value to which their profits converge. By Condition 4, we have:

$$\pi_{j,M}(X_M, X_L) + \pi_{i,N}(X_L, X_M) > \pi_{j,M}(X_M, X_H) + \pi_{j,N}(X_L, 0) \tag{A16}$$

Because each firm's profits are a convex function of its own assets, for any $\theta \in (0,1)$:

$$\pi_{j,M}(X_M, X_L) + \pi_{i,N}(X_L, X_M) > \pi_{j,M}\Big(X_M, \theta X_L + (1-\theta)X_H\Big) + \pi_{j,N}\Big(X_L, \theta X_M\Big) \tag{A17}$$

Thus, Firm $j$'s long-run profits are maximized when its assets converge to $\{X_L, X_M\}$, and any strategy for which this does not occur implies that as $\delta \to 1$, then $(1-\delta)V_{j,t}$ converges to a value strictly less than the right side of (A13). Thus, as long as $\delta$ is sufficiently high, so Firm $j$ places enough weight on future profits, its best response is eventually to become permanently focused in the niche segment.

Note that the above derivation assumed Firm $j$'s shocks were most favorable to it investing in the mainstream segment. Any other set of shocks would cause Firm $j$ to favor investing in the niche segment even more strongly. Similarly, if Firm $i$'s mainstream assets exceed $X_M$, or its niche assets are less than $X_L$, this also makes investing in the niche segment relatively more profitable for Firm $j$. In fact, given Firm $i$'s strategy of always focusing in the mainstream segment, its assets are guaranteed to permanently cross the threshold at which $M_i > X_M$ and $N_i < X_L$ within finite amount of time. Thus, as long as $\delta$ is sufficiently large, Firm $j$'s best response requires that it eventually becomes permanently focuses in the niche segment. We have showed that Firm $j$ does not deviate from the proposed equilibrium. The next step is to show that Firm $i$ also does not deviate from this equilibrium.

*Step 2:*

Given $a_T = i$ at time $T$, as long as Firm $i$ always focuses in the mainstream segment from this point forward, then $a_t = i$ for all $t > T$, and Firm $j$ starts focusing in the niche segment within a finite amount of time (as shown in Step 1). In this case, as $\delta \to 1$, Firm $i$'s value function satisfies:

$$(1-\delta)V_{i,t} \underset{\delta\to 1}{\longrightarrow} E\Big[\pi_{i,M}\Big(X_M + \widetilde{\epsilon}_{i,m}, \widetilde{\epsilon}_{j,m}\Big) + \pi_{i,N}\Big(\widetilde{\epsilon}_{i,n}, X_M + \widetilde{\epsilon}_{j,n}\Big) - C\Big] \tag{A18}$$

where we define $\widetilde{\epsilon}_{i,m} \equiv \sum_{t=0}^{\infty} \gamma^t \epsilon_{i,m,t}$, and the other $\widetilde{\epsilon}$ variables are defined similarly.

On the other hand, if Firm $i$ ever deviates and does not focus in the mainstream segment at any time $t$, then $a_{t+1} = j$. Firm $j$ then focuses on the mainstream segment from that point forward, and Firm $i$ begins focusing on the niche segment within finite amount of time. In this case, as $\delta \to 1$, Firm $i$'s value function satisfies:

$$(1-\delta)V_{i,t} \underset{\delta\to 1}{\longrightarrow} E\Big[\pi_{i,M}\Big(\widetilde{\epsilon}_{i,m}, X_M + \widetilde{\epsilon}_{j,m}\Big) + \pi_{i,N}\Big(X_M + \widetilde{\epsilon}_{i,n}, \widetilde{\epsilon}_{j,n}\Big) - C\Big] \tag{A19}$$

It follows immediately from the fact that the profit functions are increasing in a firm's own assets and decreasing in the competitor's assets, and from the fact that the niche profit function is simply the mainstream profit function scaled by $\alpha < 1$, that the right side of (A18) is greater than the right side of (A19). Therefore, even if Firm $i$'s short-run profits are higher if it focuses in the niche segment, as long as it places sufficient weight on future profits, it will not deviate from the proposed equilibrium.

Let $\widehat{\delta}$ be the minimum value needed for both Steps 1 and 2 to hold. As long as $\delta$ exceeds this value, and Condition 4 holds, we have shown that, regardless of the history of the game, there is an equilibrium in which Firm $i$ always focuses on the mainstream segment from the current period forward, and Firm $j$ eventually starts focusing in the niche segment. By symmetry, there is also an equilibrium in which these roles are reversed. Thus, the game always has multiple equilibria regardless of its current state. QED

**Proof of Proposition 4:** I will show there exist asset values such that there is an equilibrium in which Firm $i$ destroys some of in its own niche assets and Firm $j$ does not destroy any assets. Suppose firm $j$ has assets $\{X_M, X_L\}$, while Firm $i$ has assets $\{X_H, N_i\}$, where $N_i \in [0, X_L]$. Condition 1 guarantees that Firm $i$ invests in the mainstream segment at any such state. On the other hand, Condition 1 guarantees that Firm $j$ invests in the *mainstream* segment if $N_i = X_L$, while Condition 2 guarantees that Firm $j$ invests in the *niche* segment if $N_i = 0$. Intuitively, given the negative interaction term in each firm's profit function, a reduction in Firm $i$'s niche assets makes it more attractive for Firm $j$ to invest in the niche rather than mainstream segment. Let $\widehat{N_i}$ denote the cut-off value such that there is an equilibrium in which Firm $j$ invests in the niche segment for $N_i = \widehat{N_i}$ but invests in the mainstream segment for $N_i > \widehat{N_i}$.

Consider the case when Firm $i$'s niche assets are $\widehat{N_i} + d$, where $d > 0$. If neither firm destroys any assets, then both firms invest in the mainstream segment, and Firm $i$'s expected profits are:

$$E\Big[\pi_{i,M}\Big(\gamma X_H + 1 + \epsilon_{i,m,t}, \gamma X_M + 1 + \epsilon_{j,m,t}\Big) + \pi_{i,N}\Big(\gamma(\widehat{N_i} + d) + \epsilon_{i,n,t}, \gamma X_L + \epsilon_{j,n,t}\Big)\Big] - C \quad \text{(A20)}$$

On the other hand, if Firm $i$ chooses to reduce its niche assets to $\widehat{N_i}$, then Firm $i$ invests in the mainstream segment while Firm $j$ invests in the niche segment, and Firm $i$'s expected profits are:

$$E\Big[\pi_{i,M}\Big(\gamma X_H + 1 + \epsilon_{i,m,t}, \gamma X_M + \epsilon_{j,m,t}\Big) + \pi_{i,N}\Big(\gamma\widehat{N_i} + \epsilon_{i,n,t}, \gamma X_L + 1 + \epsilon_{j,n,t}\Big)\Big] - C \quad \text{(A21)}$$

As $d$ approaches zero, then (A21) minus (A20) approaches:

$$\begin{aligned} E\Big[G\Big(\gamma X_M + 1 + \epsilon_{j,m,t}\Big) - G\Big(\gamma X_M + \epsilon_{j,m,t}\Big) + \alpha G\Big(\gamma X_L + \epsilon_{j,n,t}\Big) - \alpha G\Big(\gamma X_L + 1 + \epsilon_{j,n,t}\Big)\Big] \\ + L(\gamma X_H + 1) - \alpha L(\gamma\widehat{N_i}) \end{aligned} \quad \text{(A22)}$$

By the assumption that $G'' \geq -\psi$, the sum of the four terms in the expectation operator is greater than $-\psi\gamma(X_M - X_L)$. And because $L(X) - L(Y) = \psi(X - Y)$ for any $X$ and $Y$, the sum of the last two terms is greater than $\psi(\gamma X_H + 1 - \gamma\widehat{N_i})$. This implies that expression (A22) is greater than:

$$-\psi\gamma(X_M - X_L) + \psi\gamma(X_H - \widehat{N_i}) + \psi \quad \text{(A23)}$$

Because $X_H > X_M$ and $\widehat{N_i} < X_L$, this expression is always strictly greater than zero. Thus, Firm $i$'s expected profits increase when it reduces its assets from $\widehat{N_i} + d$ to $\widehat{N_i}$. Intuitively, as $d$ becomes becomes sufficiently small, the direct impact on Firm $i$'s profits from reducing its own niche assets approaches zero. However, the indirect effect from enticing Firm $j$ to change its investment stays strictly positive, resulting in an overall increase in Firm $i$'s profits.

We have shown that if Firm $j$ has asset levels $\{X_M, X_L\}$, Firm $i$ has asset levels $\{X_H, \widehat{N_i}+d\}$ with $d$ sufficiently small, and Firm $j$ does not destroy any assets, then Firm $i$'s best response is to reduce its niche assets to $\widehat{N_i}$.

We now need to show that Firm $j$'s best response is not destroy any of its assets. To demonstrate this, I will show that Firm $j$'s asset destruction cannot influence Firm $i$'s investment behavior. Condition 1 implies that:

$$B(\gamma X_H) - L(\gamma X_H + 1) > \alpha\big[B(\gamma\widehat{N_i}) - L(\gamma\widehat{N_i})\big] \tag{A24}$$

Because $X_H - X_M = X_L > \widehat{N_i}$, this implies that:

$$B(\gamma X_H) - L(\gamma X_M + 1) > \alpha\big[B(\gamma\widehat{N_i}) - L(0)\big] \tag{A25}$$

This implies that even if Firm $j$ reduces its niche assets to zero, Firm $i$ still has a dominant strategy of investing in the mainstream segment. Thus, Firm $j$ reducing its assets would directly reduce its own profits without affecting Firm $i$'s investment behavior in the current period, and so Firm $j$ does not engage in asset destruction, and the proposed equilibrium holds. QED

# Supplemental Appendices

**Abstract**

These supplemental appendices will not appear in the paper, but will be available on the author's website. Appendix B presents a model of competition consistent with the profit functions used in the paper. Appendix C studies diversification. Appendix D presents results for the special case in which there are no interactions between firms' investment decisions.

# Appendix B. Microfoundation for Profit Functions

I now present a simply theoretical example of competition that is consistent with the per-period profit functions used in this model.

Assume firms $i$ and $j$ compete in two segments. Each firm sells a separate product in each segment, and customers will only purchase a product targeted toward their particular segment. In each segment, firms are (exogenously) located at opposite ends of a Hotelling line of length 1. In the mainstream segment, customers have mass 1, and in the niche segment they have mass $\alpha$, where $\alpha \leq 1$.

Firms invest in assets that improve their quality on a vertical dimension, which is orthogonal to the Hotelling line. Recall that $M_{i,t}$ and $N_{i,t}$ are firm $i$'s assets devoted to the mainstream and niche segment, respectively, at time $t$. A mainstream customer located a distance $d$ from firm $i$ receives utility $U - kd + M_{i,t} - P_{i,m,t}$ from purchasing firm $i$'s product, where $k$ is customers' transportation cost and $P_{i,m,t}$ is firm $i$'s price in the mainstream segment. Without loss of generality set marginal production cost equal to zero.

We focus on parameter values are in a range where the market is covered and both firms have positive demand in equilibrium. Formally, we need the constant $U > \frac{3k}{2}$, and we also need for each firm's assets in each segment to lie in the range $[0, 3k]$, which will always be true if the depreciation factor satisfies $\gamma < 1 - \frac{1+\epsilon_{max}}{3k}$. At any time $t$, demand for firm $i$'s product in each segment is given by:

$$D_{i,m,t} = \frac{1}{2} + \frac{M_{i,t} - M_{j,t} - P_{i,m,t} + P_{j,m,t}}{2k} \tag{B1}$$

$$D_{i,n,t} = \alpha \left[ \frac{1}{2} + \frac{N_{i,t} - N_{j,t} - P_{i,n,t} + P_{j,n,t}}{2k} \right] \tag{B2}$$

Solving firm $i$'s first-order condition in the mainstream segment we have:

$$P^*_{i,M,t} = \frac{k}{2} + \frac{M_{i,t} - M_{j,t}}{2} + \frac{P^*_{j,M,t}}{2} \tag{B3}$$

If we then solve for firm $j$'s analogous first order condition and plug the resulting expression for $P^*_{j,M,t}$ into the above equation, we derive the following equilibrium price and demand for firm $i$ in the mainstream segment:

$$P^*_{i,M,t} = k + \frac{M_{i,t} - M_{j,t}}{3} \tag{B4}$$

$$D^*_{i,M,t} = \frac{1}{2} + \frac{M_{i,t} - M_{j,t}}{6k} \tag{B5}$$

Multiplying these two together, firm $i$'s equilibrium mainstream profits are:

$$
\begin{aligned}
\pi^*_{i,M,t} &= \frac{k}{2} + \frac{M_{i,t} - M_{j,t}}{3} + \frac{\left[M_{i,t} - M_{j,t}\right]^2}{18k} \\
&= \frac{k}{2} + \frac{M_{i,t}}{3} + \frac{M_{i,t}^2}{18k} - \frac{M_{j,t}}{3} + \frac{M_{j,t}^2}{18k} - \frac{M_{i,t}M_{j,t}}{9k}
\end{aligned}
\tag{B6}
$$

Equilibrium profits in the niche segment are analogous, except that they are scaled by $\alpha$ because the segment is smaller.

Note this profit function is consistent with the profit functions (1) and (2), including increasing returns to a firm's own assets in a segment and a negative interaction between the focal firm's assets and the competitor's assets in a segment. Intuitively, when a firm has higher equilibrium demand (either because its own assets are high or its competitor's assets are low in a particular segment), incremental improvements in its product quality allow it to raise its price and collect incremental profits from a larger number of customers, which implies marginal returns to quality are higher.[15]

### Parameter Values for Conditions 1 to 3

I now derive parameter values for which the model of competition developed in this appendix is consistent with Conditions 1 to 3 from the main body of the paper. Given the equilibrium profits (B6), the operator (9) is equivalent to:

$$
\begin{aligned}
B(X) &= \frac{1}{3} + \frac{1}{18k} E\Big[(X+1+\epsilon)^2 - (X+\epsilon)^2\Big] \\
&= \frac{1}{3} + \frac{1}{18k} + \frac{X + E\big[\epsilon\big]}{9k}
\end{aligned}
\tag{B7}
$$

Also, given (B6), the operator (10) is equivalent to:

$$
L(Y) = \frac{Y + E\big[\epsilon\big]}{9k} \tag{B8}
$$

This implies that:

$$
B(X) - L(Y) = \frac{6k + 1 + 2(X - Y)}{18k} \tag{B9}
$$

Plugging (B9) into Conditions 1 to 3, we have:

[15] This is just one example of how increasing returns can arise from strategic interactions. In principle, we could also generate microfoundations with increasing returns based on word-of-mouth effects or learning curves, for example. The specific sources of increasing returns will depend on the details of the particular industry in question. In order to focus on the dynamic interactions between firms, the rest of the paper uses the general profit functions given by (1) and (2), rather than focusing on a particular model of per-period competition.

$$\textbf{Condition 1:} \qquad \alpha < \frac{6k - 1 - 2\left(\frac{\gamma}{1-\gamma}\right)\epsilon_{max}}{6k+1}$$

$$\textbf{Condition 2:} \qquad \alpha > \frac{6k - 1 - 2\left(\frac{\gamma}{1-\gamma}\right)\epsilon_{max}}{6k+1+2\left(\frac{\gamma}{1-\gamma}\right)\epsilon_{max}}$$

$$\textbf{Condition 3:} \qquad \alpha > \frac{6k - 1 - 2\left(\frac{\gamma}{1-\gamma}\right)(1-\epsilon_{max})}{6k+1+2\left(\frac{\gamma}{1-\gamma}\right)(1-\epsilon_{max})}$$

Intuitively, Condition 1, which ensures at least one firm always has a dominant strategy of focusing in the mainstream segment, is more likely to hold when: the niche segment is small ($a$ small), preemption effects are weak (which is true in this model when $k$ is large), and when the potential gap that can open between firms focusing in the same segment is small (which is true when $\gamma$ and $\epsilon_{max}$ are small). Condition 2, which ensures one firm eventually switches focus to the niche segment, is more likely to hold in the opposite of these cases. Thus, there is tension between these first two conditions, but there is a range of parameter values in which both hold. Finally, Condition 3, which ensures firms do not keep switching back and forth between segments, is similar to Condition 2 except that Condition 2 requires that $\epsilon_{max}$ be sufficiently large, while Condition 3 requires that $\epsilon_{max}$ be sufficiently small.

As an example of parameter values that satisfy these conditions, if we set $k = 4$, $\alpha = 0.6$, $\gamma = 0.85$, $\epsilon_{max} = 0.5$, all three conditions hold, which means Proposition 1 applies. If we keep the same parameter values but increase $\epsilon_{max}$ to 0.6, Conditions 1 and 2 still hold, but Condition 3 does not, which means Proposition 2 applies.

# Appendix C. Diversification

The main body of the paper has presented conditions in which increasing returns compel each firm to stay permanently focused on a segment where it has early good fortune. On the other hand, I have also showed that if the random shocks are large enough, or if firms are patient enough, then it is always possible for firms to make major strategic changes, totally shifting investment focus from one segment to another.

I now derive conditions in which each firm adopts an intermediate strategy, diversifying investment into both segments simultaneously. For example, this could happen if: (1) the firm reaches a point of decreasing returns in its core expertise, and returns in this segment become low enough that investing in the other segment is more profitable; (2) assets in different segments are complementary, and the effect of this complementarity (which favors diversification) outweighs the effect of increasing returns (which favors specialization); (3) the firm wishes to deter competitive entry by making both segments unattractive to a potential entrant; or (4) small exploratory investments in a segment lead to a small chance of a large "breakthrough discovery." For illustrative purposes, this section will focus on the last of these reasons (possible breakthrough discoveries), but similar results could be shown for any of the other three reasons for diversification.

Until now I have assumed that the distribution of random shocks does not depend on how much a firm invests. To be more realistic, we might expect that in order to have a favorable shock in a segment, a firm must make some positive investment there. In particular, assume that the equations for how assets change over time, (3) and (4), are modified as follows:

$$M_{i,t} = \gamma M_{i,t-1} + m_{i,t} + \beta(m_{i,t}) \tag{C1}$$

$$N_{i,t} = \gamma N_{i,t-1} + n_{i,t} + \beta(n_{i,t}) \tag{C2}$$

The random variable $\beta$ represents the results of a potential breakthrough discovery. As an illustrative example, assume that the breakthrough term takes on values of either 0 or $D$ (where $D > 0$) and that:

$$P\Big[\beta(m) = D\Big] = \begin{cases} 0 & \text{if } m < b, \\ p & \text{if } m \geq b \end{cases} \tag{C3}$$

where the minimum investment threshold required to make a possible breakthrough in a segment is denoted by $b$, and the probability of a breakthrough given that this threshold is met is denoted by $p \in (0, 1]$.

**Proposition 5.** *Assume that investment of at least $b$ units in a segment makes a "breakthrough discovery" possible, so that assets change according to equations* (C1)*,* (C2)*, and* (C3)*. If the investment threshold (denoted by $b$) is sufficiently small, and firms are not too patient ($\delta$ is not too large), in any equilibrium each firm always invests at least $b$ units in each segment.*

**Proof:** Suppose at time $t$ Firm $i$ sets its investment in at least one segment less than $b$. I will show that, given the conditions of the proposition, the firm would be better off increasing its investment in this segment to $b$. By increasing its investment in this segment to $b$, the firm gains a probability $p$ of having a favorable shock of size $D$ in this segment. The possibility of this "breakthrough discovery" increases the firm's expected profits in the current period by at least $\alpha p\big[F(D) - F(0) - \psi D X_H\big]$. Recall we are assuming $F'(0)$ is sufficiently large that this term is guaranteed to be positive.

This additional investment must come at the expense either of increased total investment or reduced investment in the other segment. Therefore, the cost of this shift in investment is at most the maximum of $Cb$ and $\left[F(X_H) - F(X_H - b)\right]$. Note these terms both approach zero as $b$ approaches zero, whereas the benefit of the possibly breakthrough discovery remains positive and does not approach zero.

Next we must consider the impact on future profits of this shift in investment. Let $\Delta_{max}$ denote the largest possible change in Firm $i$'s value function that can result form a one-period change in its investment. Note that because profits are bounded, this value has a finite upper bound. The discounted value of this future impact is no greater than $\delta\Delta_{max}$, which approaches zero as the discount factor $\delta$ approaches zero. Therefore, as long as $b$ and $\delta$ are small enough, the impact on its expected discounted profits when a firm puts at least $b$ units of investment into each segment must be positive, and so each firm always invests at least this amount in each segment in equilibrium.

QED

This result is made possible because (C3) implies that the probability of a breakthrough is a non-convex function of investment. In other words, Proposition 5 only applies if the probability of a breakthrough generates *decreasing* returns to investment over a certain range of investment levels, for example, if expected returns to investment are very high to the point that the minimum threshold for a breakthrough is achieved, but then marginal returns become lower for levels of investment in excess of this threshold.

# Appendix D. Independent Investment Decisions

The version of the model in this appendix assumes that equilibrium profits have an additive functional form, so that there are no interactions in competing firms' investment decisions, and each firm's investments depend only on its own assets. Technically, I assume $\psi = 0$. We could think of this version of the model as applying to a monopoly or to firms that are selling very different products. This version of the model simplifies the analysis and helps convey intuition for how the model works. It also illustrates that, even if firms do not explicitly care about being different from each other, they can still become permanently focused on different segments if they happen to experience different early shocks (although they could also end up focusing in the same segment if they experience similar early shocks). By contrast, section 4 allows for interactions in firms' investment decisions, and derives conditions that *guarantee* firms become different.

Given $\psi = 0$, the two firms' assets do not interact in the profit functions, so it is possible to formulate each firm's maximization problem as an infinite period dynamic program.[16] For convenience of notation, the following expression suppresses the subscripts on the investment variables, $m_{i,t+1}$ and $n_{i,t+1}$:

$$V(M_{i,t}, N_{i,t}) = F(M_{i,t}) + \alpha F(N_{i,t}) + \max_{m,n:m\geq 0,n\geq 0,(m+n)\leq 1} \delta\big\{E[V(M_{i,t+1}, N_{i,t+1})] - Cm - Cn\big\} \tag{D1}$$

The following proposition characterizes optimal investment behavior for each firm.

**Proposition 6.** *If $\psi = 0$, then for any given state, the only possible optimal policies are to invest nothing, to invest as much as possible in the mainstream segment, or to invest as much as possible in the niche segment. If a firm invests in the mainstream segment when its assets levels are $(M, N)$, then it also invests in the mainstream segment for all states $(M_H, N_L)$, where $M_H \geq M$ and $N_L \leq N$. An analogous result holds for investment in the niche segment.*

**Proof:** I will first show that the value function $V(M, N)$ is jointly convex in its two arguments. For any feasible asset values $\{M_1, N_1\}$ and $\{M_2, N_2\}$, and any $\theta \in (0, 1)$, define $M_3 \equiv \theta M_1 + (1-\theta)M_2$ and $N_3 \equiv \theta N_1 + (1-\theta)N_2$ . I will show that:

$$\theta V(M_1, N_1) + (1-\theta)V(M_2, N_2) > V(M_3, N_3) \tag{D2}$$

Define $\widehat{V}(M, N)$ as the expected discounted profits generated by starting at state $\{M, N\}$ but following the investment policy that would be optimal when starting at state $\{M_3, N_3\}$. Because this policy may not be optimal when starting at state $\{M, N\}$, it must be true that $V(M, N) \geq \widehat{V}(M, N)$. Therefore, inequality (D2) is guaranteed to hold if:

$$\theta \widehat{V}(M_1, N_1) + (1-\theta)\widehat{V}(M_2, N_2) > V(M_3, N_3) \tag{D3}$$

The first term on the left side of this inequality represents $\theta$ times the expected discounted profits for a firm that starts at state $\{M_1, N_1\}$ and follows the policy that would be optimal starting at state $\{M_3, N_3\}$. The second term on the left side represents $(1-\theta)$ times the profits to a firm starting at state $\{M_2, N_2\}$, while the term on the right side represents the profits to firm starting at state $\{M_3, N_3\}$, each following this same policy. Even though the same policy is followed in all three cases, because firms start at different points they will always lead to different asset levels. After $t$ periods of following this policy, mainstream assets will be of the

[16] There could be other equilibria (supported by mutual punishment strategies) in which each firm bases its investment decisions partly on the other's assets, but for now I focus on the equilibrium in which they make independent decisions. Also, to simplify notation, I have omitted from the value function the terms reflecting firm $j$'s impact on firm $i$'s profits. In the equilibrium in which each firm's investment depends only on its own assets, this simplified value function leads to the same optimal policy as the original value function.

form $(\gamma^t M_1 + x)$ for the first firm, $(\gamma^t M_2 + x)$ for the second firm, and $(\gamma^t M_3 + x)$ for the third firm, where $x$ represents the cumulative effect of each firm's mainstream investments and random shocks.

By the convexity of the profit function $F$, we know that:

$$\theta F(\gamma^t M_1 + x) + (1-\theta) F(\gamma^t M_2 + x) > F(\gamma^t M_3 + x) \tag{D4}$$

Similar analysis holds for the niche segment. This implies that $\theta$ times the first firm's profits plus $(1-\theta)$ times the second firm's profits are always grater than the third firm's profits. In other words, inequality (D3) holds, which implies that (D2) holds.

I have established that each firm's value function is strictly convex. Given that the set of feasible actions is a convex set with linear constraints, this implies that only corner solutions can be optimal. In other words, the only possible optimal policies are to invest as much as possible in the mainstream segment, to invest as much as possible in the niche segment, or to invest nothing.

It also immediately follows from the convexity of the value function that an increase in mainstream assets makes investing in the mainstream segment more attractive, and an increase in niche assets makes investing in the niche segment more attractive.

It still remains to be shown that an increase in *niche* assets cannot make investing in the *mainstream* segment more attractive. Technically, I need to show that for any asset values $\{M, N\}$ and any $x > 0$:

$$V(M+1, N) - V(M, N) \geq V(M+1, N+x) - V(M, N+x) \tag{D5}$$

This is equivalent to:

$$V(M+1, N) + V(M, N+x) \geq V(M, N) + V(M+1, N+x) \tag{D6}$$

This inequality can be proven using a similar approach to the proof of convexity of the value function shown above. In particular, define $\widetilde{V}(M+1, N)$ is the expected discounted profits from starting at state $\{M+1, N\}$ and adopting the following policy. In each period, compute the optimal policy for firms that started at sates $\{M, N\}$ and $\{M+1, N+x\}$, and mimic whichever one invests more in the *mainstream* segment. Similarly, define $\overline{V}(M, N+x)$ as the expected discounted profits from starting at state $\{M, N+x\}$ and mimicking whichever of the two firms invests more in the *niche* segment.

Inequality (D6) is guaranteed to hold as long as:

$$\widetilde{V}(M+1, N) + \overline{V}(M, N+x) \geq V(M, N) + V(M+1, N+x) \tag{D7}$$

The sum of the mainstream assets of the two firms on the left of this inequality is always the same as the sum for the last two, but the variance is at least as great for the first two. Therefore, the convexity of the value function guarantees that total mainstream profits for the first two firms are weakly higher than for the last two. A similar argument holds for niche assets. In other words, (D6) holds, which implies that an increase in niche assets cannot make investing in the mainstream segment more attractive, and vice versa.

These result imply that, for a given value function $V$, it is possible to compute the optimal policy function as follows. If firm $i$'s current asset levels are $(M, N)$, compute the expected return to investing one unit in the mainstream segment:

$$\Big(E[V(\gamma M + 1 + \epsilon_m, \gamma N + \epsilon_n)] - E[V(\gamma M + \epsilon_m, \gamma N + \epsilon_n)]\Big) - C \tag{D8}$$

Then compute the expected return of investing one unit in the niche segment.

$$\Big(E[V(\gamma M + \epsilon_m, \gamma N + 1 + \epsilon_n)] - E[V(\gamma M + \epsilon_m, \gamma N + \epsilon_n)]\Big) - C \tag{D9}$$

If both expressions are negative, do not invest in either segment. If at least one expression is positive, invest one unit in the segment with the higher expected return and zero units in the other segment.

Each firm can have multiple optimal strategies only if these two expressions are exactly equal or if the larger of them exactly equals zero. Suppose assets are at a point where the two expressions are equal. Because each firm's value function is strictly convex, as shown above, any positive perturbation of the firm's assets in either segment implies that (D8) and (D9) are no longer equal. Thus, the set of points at which a firm has more than one optimal strategy has measure zero. Given that the error terms have no mass points, the probability of arriving at such a point is zero, and with probability one each firm always has a pure strategy that is its unique optimal strategy.

QED

I now show how the investment behavior described in this proposition determines the evolution of a firm's assets. In particular, the following numerical example illustrates how a firm's asset growth can exhibit "path dependence," meaning that small random events have large permanent consequences.

Figure D1 presents the optimal policy function for one particular numerical example.[17] The x-axis gives the firm's mainstream assets; the y-axis its niche assets. When a firm has low assets in both segments, it makes no investment. In practical terms, the "do not invest" region indicates that a firm should not enter the market unless it already has assets that provide a small advantage in serving one segment. These assets might have been developed through the

[17] I calculate the optimal policy function over a 60x60 grid of values in Matlab, using modified policy iteration, with ten value iterations for each policy iteration (see Bertsekas 2007, p. 43).

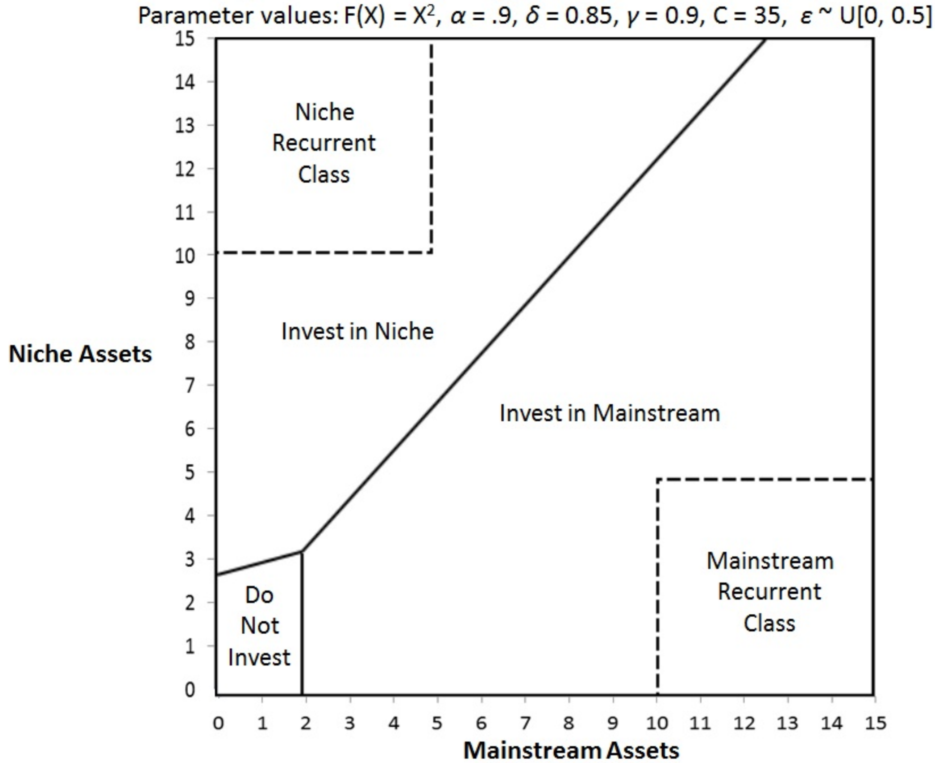
firm's activities in another market, or they might result from the founder's prior experience or useful discovery.

Once a firm builds up enough assets in a segment, it starts to focus all of its investment in that segment. It then continues to focus in that same segment unless it has a large enough positive shock in the other segment that it switches its investment focus. However, once a firm begins investing in the niche segment (for example), the most likely outcome is that it will move up the y-axis until it reaches the "niche recurrent class." Once this occurs, the firm always focuses its investment in the niche segment; its assets fluctuate along both dimensions due to random chance, but never leave the recurrent class.

Note that each recurrent class is a perfect square. This is a result of the assumption of iid error terms in each segment. For example, imagine a firm always invests as much as possible in the niche segment. If it repeatedly receives the *worst* possible random shock in each segment, its assets converge to the bottom-left corner of the niche recurrent class. However, if the firm repeatedly receives the *best* possible random shock in the mainstream segment, given the constant depreciation rate, the furthest it can move along the horizontal dimension is to the right boundary of the niche recurrent class. Similarly, if it receives the best possible random shock in the niche segment, the furthest it can move along the vertical dimension is to the top of this recurrent class.

*Figure D1: Optimal policy function and recurrent classes*

Parameter values: $F(X) = X^2$, $\alpha = .9$, $\delta = 0.85$, $\gamma = 0.9$, $C = 35$, $\varepsilon \sim U[0, 0.5]$




For this numerical example, a firm that starts with no assets could end up permanently trapped in either recurrent class, depending on random changes early in the firm's life. Thus, firms that are initially identical can become permanently different based on small differences in the early random shocks they experience.

To summarize, this section has shown that permanent differences between firms can arise even on dimensions for which firms do not explicitly care about being different from competitors. Under increasing returns, each firm has an incentive to focus on the dimension where it has the most early success, and so firms that are only slightly different might make investments that take then in completely different directions. However, in this version of the model firms could also end up focusing in the same segment if they experience similar early shocks. By contrast, Section 4 in the main body of the paper derives conditions that guarantee firms become permanently focused on *different* segments.

**Example: Word-of-mouth effects for ice cream.** I now present a simply theoretical example illustrating one possible source of increasing returns, with independent investment decisions. Assume two ice cream shops each sell smooth and chunky flavors. Each shop's mainstream assets represent its ability to create high quality smooth flavors, and niche assets represent its ability to create high quality chunky flavors. The random shocks represent fortunate discoveries of better recipes, such as Ben and Jerry's discovering that people like large chunks, and investments represent efforts to improve the production process for a particular type of flavor.

It is convenient (but not essential for the intuition of this example to hold) to assume prices for each shop and each flavor are fixed at the same level and have the same constant marginal production cost, so profits are a linear function of demand at the fixed price level. In each period, a group of customers of mass 1 who only eat smooth ice cream, and a group of mass $\alpha$ who only eat chunky ice cream, each randomly choose a shop and try a sample of its ice cream to determine its quality. Each customer then either makes a purchase from that shop or makes no purchase at all. Of the customers who visit shop $i$, the probability of each smooth flavor customer making a purchase is $\frac{M_{i,t}}{Q_{max}}$, and the probability of each chunky flavor customer making a purchase is $\frac{N_{i,t}}{Q_{max}}$, where $Q_{max}$ denotes maximum possible quality level. All customers remain in the market for one period. At this point, a shop's profits in each period are a linear function of its quality in each segment, so there are constant returns to investment.

I now incorporate word-of-mouth effects into the model and show how this can generate increasing returns. Assume each customer who makes a purchase leaves the shop with his ice cream and, with probability $w$, meets a friend who asks where he purchased the ice cream and how good it is. If the ice cream is a smooth flavor, this friend then visits the shop and purchases ice cream with probability $\frac{M_{i,t}}{Q_{max}}$, and if the ice cream is a chunky flavor, the friend visits the shop and purchases with probability $\frac{N_{i,t}}{Q_{max}}$. If customers' friends do not overlap, so no one hears about a product from two different sources, then demand does not depend on the competitor's quality, and shop $i$'s mainstream (smooth flavor) profits are proportional to

$\frac{M_{i,t}}{Q_{max}} + w\left(\frac{M_{i,t}}{Q_{max}}\right)^2$, and niche (chunky flavor) profits are proportional to $\alpha\left[\frac{N_{i,t}}{Q_{max}} + w\left(\frac{N_{i,t}}{Q_{max}}\right)^2\right]$. Now there are increasing returns, and the example fits the assumptions of the model.

The intuition is that, when customers learn about products through word-of-mouth, improvements to product quality make more people aware that the product exists, in addition to increasing the fraction of people who purchase it conditional on awareness. The multiplication of these effects leads to increasing returns. This theoretical example bears resemblance to the model by Rob and Fishman (2005), in which word-of-mouth effects also generate increasing returns. One important difference is that in their model firms invest along a single dimension, whereas in the current model firms invest along two dimensions, which can lead only slightly different firms to focus entirely on different segments.