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A Dynamic Model of Competitive Entry Response

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Abstract

I develop a dynamic investment game with a "memoryless" R&D process in which an incumbent and an entrant can invest in a new technology, and the entrant can also invest in the old technology. I show that an increase in the probability of successfully implementing a technology can cause the incumbent to *reduce* its investment. Under certain conditions, if the success probability is high, the incumbent allows the entrant to win the new technology so that firms reach an equilibrium in which they use different technologies, and threats of retaliation prevent attacks; but if the success probability is low, such an equilibrium cannot be sustained, and both firms eventually implement both technologies.

1 Introduction

When a new entrant attacks an incumbent, the incumbent typically has many advantages such as established retail locations and expertise in current technology. The entrant might then try to develop its own advantages, for example, by investing in a new distribution channel such as the Internet, a new technology, or a lower-cost business model. The incumbent must then decide how to respond to this threat.

Firms facing this problem have used a variety of entry-response strategies. For example, following E-trade's early success with online trading technology, Charles Schwab invested in its own online trading platform. Now both firms offer online trading. By contrast, when BestBridalPrices.com launched an online wedding dress store, many traditional wedding shops such as Priscilla of Boston decided to stay focused on their traditional business and *not* to sell dresses online. Other approaches include delayed response or responding only if the entrant directly attacks the incumbent's traditional business. For example, after EasyJet's entry as a "no frills" airline, British Airways initially continued to focus on the traditional full-service format; however, when EasyJet began moving upscale and serving more business passengers, British Airways retaliated by adopting a "no frills" model for some of its short-haul European routes.

This paper develops a dynamic investment model that derives conditions in which these various entry-response strategies are optimal. The model assumes an incumbent with expertise in an old business "format" faces competition from a new entrant. Both firms can invest in a new business "format," which might represent a new technology or, more generally, a new business approach, which is now possible due to exogenous technological progress or changes in customer preferences. Either firm can potentially use both formats; for example, they can distribute a product through traditional retail stores and over the Internet.

A firm has some random probability of successfully implementing a format in which it makes positive investment. If a firm fails to implement a format in a given period, it can try again in the following period. For example, an airline trying to adopt a lower-cost business model might need to renegotiate contracts with its union and train employees to spend less time handling each customer complaint. If these negotiations and organizational changes fail one year, the airline can try them again the next year.

Although many factors could affect firms' investment decisions, I focus on three key parameters. The first is the strength of preemption effects, which tend to prevent a firm from investing in a format its competitor is already using. I operationalize this parameter by allowing customers to have uniformly distributed brand preferences; when brand preferences are weak, if two firms use the same format, intense price competition ensues and profits generated from that format are low. Therefore, preemption effects are strong; that is, once one firm has implemented a format, the other firm has little incentive to do so.

The second key parameter is the strength of cannibalization effects across formats. I operationalize this parameter by allowing for three groups of customers: those who consider only the old format, those who consider only the new format, and those who consider both formats. If the number of customers who will consider both formats is large, a firm that is already using one format has little incentive to implement a second format, because doing so would mostly cannibalize sales from its existing format rather than attracting new customers.

The third key parameter is the probability that a firm that invests in a format will successfully implement this format at an operational or organizational level. In some cases, implementing a new format is fairly straightforward, and firms that make the necessary investments are almost certain to implement the format successfully;

in other cases, even a firm that makes substantial investments in a format might fail to implement it. I use a general functional form for success probability that makes exploring both cases possible.

I show that interesting interactions occur among these three parameters. In particular, when preemption and cannibalization effects are strong, an increase in success probability causes the incumbent to invest *more* in the new format in an attempt to deter the entrant from investing. This result is consistent with previous theoretical research on innovation (Gilbert and Newbery 1982; Reinganum 1983). More surprisingly, when preemption and cannibalization effects are weak, an increase in success probability causes the incumbent to invest *less* in the new format to avoid the threat that the entrant will retaliate by investing in the old format. Thus, I show that an increase in the ease (or expected speed) with which firms can implement formats might *discourage* the incumbent's investment in the new format.

As an illustrative example, consider the contrast between the airline industry and the package-shipping industry. For a traditional full-service airline, implementing a no-frills format (or for a no-frills airline, implementing a full-service format) is a major organizational challenge, requiring firms to invest in retraining employees, renegotiating union contracts, and developing new pricing skills, all of which have a fairly high chance of failure (Sanchez 1994; Dutta et al. 2003). By contrast, in the package-shipping industry, for primarily ground-based carrier UPS to acquire more airplanes (or for primarily air-based carrier FedEx to acquire more trucks) is a more straightforward investment, because both firms have the logistical expertise to manage both ground and air shipping (Composit 2004).

We might expect the difficulty of implementing new formats would compel airlines to stay focused, whereas the ease of implementing new formats would compel package-shipping firms to diversify, but in fact the opposite has occurred. Over time, tradi-

tional airlines have improved aircraft turnaround times, renegotiated labor contracts, and stopped offering free meals and free checked bags, while the no-frills airlines have added additional routes, improved customer service, and started offering optional services such as early boarding to attract business passengers. The two types of airlines have become similar, with both offering an efficient "no-frills" level of service, and better service at a higher price (Cowell 2002; McCartney 2011; Jacobs 2013). By contrast, UPS and FedEx remain more differentiated, with UPS focusing heavily on ground transportation and FedEx focusing heavily on air transportation (Darell 2011).

Of course, competitive outcomes in these industries depend on many complex forces beyond those in this paper's theoretical model. Nonetheless, this model provides one potential explanation for why traditional airline incumbents (faced with the threat from no-frills airlines) have invested continuously in the difficult task of developing low-cost no-frills expertise, whereas package-shipping incumbent UPS (faced with the threat from FedEx) has largely avoided the easier task of expanding its air-shipping service.

Section 2 discusses related literature. Section 3 presents the formal model and results. Section 4 presents two model extensions that study asymmetric formats. Section 5 concludes. A supplemental online appendix contains all proofs.

2   Related Literature

Previous theoretical literature in economics and marketing has studied optimal defensive strategies (Schmalensee 1978; Hauser and Shugan 1983; Reinganum 1983; Fudenberg and Tirole 1984; Katz and Shapiro 1987; Purohit 1994; Kalra, Rajiv, and Srinivasan 1998; Balasubramanian 1998) and entry strategies (Gelman and Salop

1983; Carpenter and Nakamoto 1990; Narasimhan and Zhang 2000; Joshi, Reibstein, and Zhang 2011). The current paper contributes to this literature by incorporating competition between an entrant and an incumbent into a dynamic investment game in which firms make repeated investments over a theoretically infinite number of time periods. Because successful investment can lead to a series of reactions and counter-reactions, this model generates new insights into how threats of strategic retaliation influence investment behavior.

Previous theoretical research has also shown that multi-market contact in a repeated game can help firms sustain high prices (Bernheim and Whinston 1990) or sustain an arrangement in which they focus on different markets (Karnani and Wernerfelt 1985; Bronnenberg 2008). The current paper differs in two key respects. First, I show how preemption effects, cannibalization effects, and the difficulty of implementing a format interact to determine whether firms can sustain an equilibrium in which they stay focused on different formats. Second, I show that multi-format contact can create an asymmetry in the investment incentives of an incumbent and an entrant; for example, in some cases, the entrant invests heavily in the new format, whereas the incumbent invests nothing.

Another related stream of research has developed dynamic investment models involving increasing returns (Athey and Schmutzler 2001; Rob and Fishman 2005), which implies that firms invest more in areas of current strength than in areas of current weakness (Selove 2010). By contrast, the current paper does not involve increasing returns. Instead, concerns over cannibalization and competitive retaliation sometimes compel firms to stay focused.

Empirical literature has studied factors that determine whether incumbents invest in new technologies (e.g., Christensen 1997; Chandy and Tellis 1998, 2000; Debruyne and Reibstein 2005) or lower-cost business formats (e.g., Ritson 2009). These papers

have identified concerns over cannibalization and preemption as key factors that determine whether firms adopt new technologies, and whether defensive strategies are successful. This paper uses a formal game-theoretic model to clarify how these factors determine firms' optimal investment strategies.

3 Model

Assume two firms, indexed by $i \in \{A, B\}$, can compete using two possible business formats, indexed by $j \in \{1, 2\}$. At any time t, the formats used by each firm are given by $X_t = (X_{A,1,t}, X_{A,2,t}; X_{B,1,t}, X_{B,2,t})$, where

$$X_{i,j,t} = \begin{cases} 1 & \text{if firm } i \text{ uses format } j \text{ at time } t \\ 0 & \text{otherwise} \end{cases}$$

The game begins in state $(1, 0; 0, 0)$, with firm A (the incumbent) using only format 1, and firm B (the entrant) not using either format. Assume firms cannot exit a format (or equivalently, exit costs are sufficiently high), so that once a firm starts using a format, it always continues to do so. If firm i did *not* use format j at time $t-1$, the probability that it will successfully implement (and begin using) this format at time t is the following (as long as this function is not greater than one):

$$F(e_{i,j,t}) = \frac{z}{d} \ln(de_{i,j,t} + 1) \tag{1}$$

where $z > 0$, $d > 0$, and $e_{i,j,t}$ is the amount firm i invests in format j at time t. This success function is memoryless in the sense that past failed investments have no effect on the current state.

Taking the first derivative, we have:

$$F'(e_{i,j,t}) = \frac{z}{de_{i,j,t} + 1} \tag{2}$$

Note that $F'(0) = z$, and the parameter d determines how rapidly the marginal value of investment decreases.

Figure 1. Investment Success Function Examples
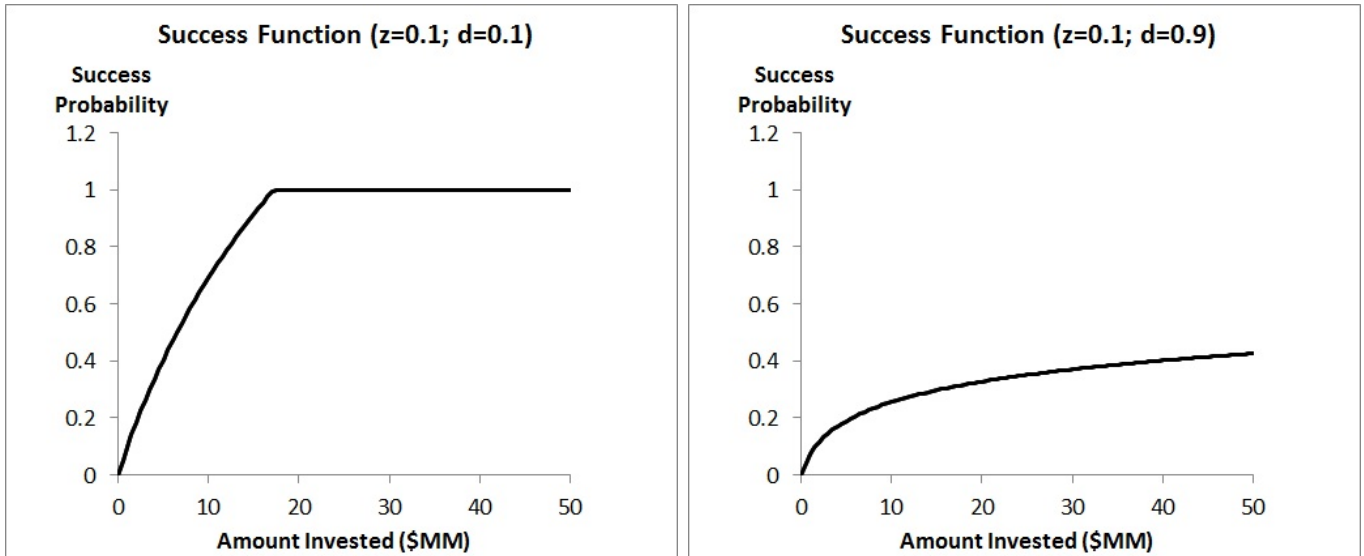
Figure 1 gives examples of this investment success function for two different sets of parameter values. For both examples, $z = 0.1$, so the marginal impact of the first dollar invested is the same. However, for the first example, $d = 0.1$, meaning each additional dollar invested continues to have a large impact on the probability of success, whereas for the second example, $d = 0.9$, meaning the marginal impact of each additional dollar rapidly decreases. A key point of this paper is to explore how these two different types of success functions affect the equilibrium outcome of the game.

I assume a firm cannot simultaneously invest in both formats in a given period.[1] I also assume firms alternate their investments, so firm A invests only in odd-numbered periods and firm B invests only in even-numbered periods.[2] These assumptions simplify the analysis by ensuring only one state variable can change in any given period.

Let $\pi^A_{(X_t)}$ and $\pi^B_{(X_t)}$ represent firm A's and firm B's profits, respectively, as a function of the current state X_t. Each firm has discount factor δ and maximizes expected discounted profits. Firm A's objective is to maximize:

$$E\left[\sum_{t=0}^{\infty} \delta^t \left(\pi^A_{(X_t)} - e_{A,1,t} - e_{A,2,t}\right)\right] \tag{3}$$

Firm B has an analogous objective function. I assume firms play a Markov perfect equilibrium (MPE) of this dynamic investment game.

In principle, we could make the model more realistic by relaxing some of these assumptions. For example, we could allow firms to exit formats, and similar results would still hold if we also restricted the profit functions in such a way that exit is never optimal.[3] We could also allow for multiple discrete states of success (and "partially-successful" investment) or allow for simultaneous investment by both firms in both formats. Such changes would complicate the analysis technically, but the same basic forces described in section 3.3 would still determine whether firms can sustain an equilibrium in which they stay focused on different formats.

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[1]Intuitively, due to limitations on managerial time and attention or other internal resource constraints, there are often diseconomies of scope to investment during a given time period.

[2]For another example of a dynamic model in which firms alternate moves, see Maskin and Tirole (1988).

[3]For the profit functions used in this paper, if there are no exit costs, and if firms reach state (1,1;1,1), an equilibrium could exist in which they both retreat, so the game goes back to state (1,0;0,1). After these "retreats" occur, if we assume firms can immediately re-enter any format they have exited in the past, then neither firm would resume using both formats, because its competitor could immediately retaliate by doing the same. The assumption that firms cannot exit a format is a simple way to rule out this type of equilibrium. Alternatively, we could modify the profit functions so mutual retreat *decreases* profits, or we could assume firms face exist costs.

However, one key assumption cannot be relaxed. The restriction to Markov strategies implies that a firm can retaliate only in response to a change in the game's state; that is, a firm can react to its competitor's successful investment, but *cannot* react to failed investment. In fact, Proposition 1 depends on the possibility that each firm could have a long string of investment failures without facing retaliation for these failed investments. This assumption is reasonable if companies can keep their investments secret until they reach some level of technical or operational success. For example, the technology firm Apple is famous for keeping its new product strategies secret, preventing competitive reaction, until it is ready for product launch (Lashinsky 2012).

If each firm *can* observe and react to its competitor's failed investments, Proposition 1 does not hold. However, Proposition 2 still holds; and the equilibria described in Propositions 3, 4, and 5 still exist. More generally, allowing firms to react to failed investments would not rule out any of the equilibria identified in this paper; it would, however, permit additional equilibrium outcomes.[4]

3.1 Product market competition

I now introduce a model of product market competition that gives rise to a profit function for each firm at each possible state.

Assume a unit mass of customers vary along two dimensions. First, their brand preferences are represented by a Hotelling line with length 1 and per-unit transportation cost β. Firm A is fixed at the left side of the line and firm B is fixed at the right side.[5] Customers also vary in their format preferences. A fraction α will buy only

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[4]Allowing firms to react to failed investments implies permitting all subgame perfect equilibria, which contains the set of Markov perfect equilibria considered here (see Maskin and Tirole 1988). Note that Proposition 1 states conditions in which both firms implement both formats in *all* equilibria; allowing for additional equilibria can overturn this result. By contrast, Proposition 2 states only that a particular equilibrium exists.

[5]Other theoretical papers have also assumed firms are exogenously located at opposite sides of a Hotelling line in order to focus on other firm decisions (e.g., Simester 1995; Ellison 2005).

using format 1, another α will buy only using format 2, and the remaining $1 - 2\alpha$ are indifferent between the two formats, where $\alpha \in [0, \frac{1}{2}]$. One can think of customers who will use only the new format as having a latent preference they do not realize until at least one firm implements the new format, at which point these customers enter the market.

In each period, a customer buys at most one product. Suppose a customer is located a distance ψ from the left side of the line. If this customer has either a preference for format 1 or no format preference, he derives utility $V - \beta\psi - P_{A,1}$ if he purchases from firm A using format 1 at price $P_{A,1}$. However, if he has a preference for format 2, he derives utility $-\infty$ from any transaction using format 1. The utility of purchasing from firm B or with format 2 can be computed in a similar manner.

Without loss of generality, assume marginal production costs are zero. Throughout the paper, I also assume:

Assumption 1. $\qquad 2\beta < V < 2\beta\left(\frac{1-\alpha}{1-2\alpha}\right)$

This assumption ensures the market is covered in equilibrium and that, when firms use different formats (at state $(1, 0; 0, 1)$), they each set the monopoly price.[6]

Given this set-up, the online appendix proves that an equilibrium exists in which prices are as follows. If a firm is the only one that uses a format, it sets price $V - \beta$ in that format; if both firms use a format, they each set price β in that format. Intuitively, Assumption 1 guarantees each format has enough loyal customers that a firm sets the monopoly price whenever it is the only one to use a format; on the other hand, firms set the standard competitive price from the Hotelling model in any format used by both firms. Table 1 reports equilibrium profits for each firm in each possible state.

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[6]If the second inequality in Assumption 1 did not hold, in some cases, state $(1, 0; 0, 1)$ would have a pure strategy price equilibrium in which firms set prices *below* the monopoly level, and in other cases, this state would have a mixed strategy price equilibrium in which firms randomize over prices.

Table 1. Equilibrium Profits at Each State

State	Firm A's profits	Firm B's profits
$(1,0;0,0)$	$(V-\beta)(1-\alpha)$	0
$(1,1;0,0)$	$(V-\beta)$	0
$(1,0;0,1)$	$(V-\beta)\frac{1}{2}$	$(V-\beta)\frac{1}{2}$
$(1,0;1,0)$	$\beta(1-\alpha)\frac{1}{2}$	$\beta(1-\alpha)\frac{1}{2}$
$(1,1;0,1)$	$(V-\beta)\alpha+\beta(1-\alpha)\frac{1}{2}$	$\beta(1-\alpha)\frac{1}{2}$
$(1,1;1,0)$	$(V-\beta)\alpha+\beta(1-\alpha)\frac{1}{2}$	$\beta(1-\alpha)\frac{1}{2}$
$(1,0;1,1)$	$\beta(1-\alpha)\frac{1}{2}$	$(V-\beta)\alpha+\beta(1-\alpha)\frac{1}{2}$
$(1,1;1,1)$	$\beta\frac{1}{2}$	$\beta\frac{1}{2}$

The profits in Table 1 arise from the one-shot equilibrium of the pricing game. Note this equilibrium *does* allow for competitive price reaction when a firm implements a new format. For example, at state $(1,0;0,1)$, the firms are using different formats, and both firms set the monopoly price. If the incumbent then implements the new format, so the state moves to $(1,1;0,1)$, the entrant immediately cuts its price. This threat of immediate price retaliation helps discourage the incumbent from making this investment. On the other hand, if we allowed collusive pricing at all states, this could remove the threat of price retaliation and *encourage* firms to attack each other's format. More generally, we could allow firms to price collusively at some states and competitively at others, which would encourage investment behavior that leads to the states with collusive pricing. However, I leave the topic of collusive pricing for future research; the current paper focuses on the one-shot price equilibrium.

To summarize, this model captures two key aspects of multi-format competition. First, new formats vary in the degree to which they expand the market as opposed to cannibalizing from the old format (which is determined in this model by α). Second, formats vary in the degree to which they can support multiple profitable firms (which is determined in this model by β). One could also use more realistic and complicated

models of product market competition, for example, with asymmetries between firms and formats. Section 4 gives examples of how such asymmetries can affect dynamic investment competition.

Table 2. Variables in the Model

$i \in \{A, B\}$	Index of firms
$j \in \{1, 2\}$	Index of formats
$t \in \{0, 1, 2, \ldots\}$	Index of time
$X_{i,j,t} \in \{0, 1\}$	Indicator of whether firm i uses format j at time t
$e_{i,j,t}$	Firm i's investment in format j at time t
F	Function that maps investment into success probability
z	Rate of increase in success probability for first dollar invested
d	Determines how rapidly the marginal impact of investment decreases
δ	Each firm's discount factor
$\pi^A_{(X_t)}, \pi^B_{(X_t)}$	General profit functions for each firm
V	Value of the product to a customer who is zero distance from the firm
α	Fraction of customers loyal to each format
β	Length of Hotelling line

3.2   Equilibrium existence

The online appendix proves the following lemma.

Lemma 1. *A pure strategy Markov perfect equilibrium exists.*

This result holds for any $z > 0$, $d > 0$, and any V, α, and β satisfying Assumption 1. Lemma 1 does *not* guarantee equilibrium uniqueness. For example, in some cases, there is an MPE in which both firms invest nothing at state $(1, 0; 0, 1)$, and another MPE in which both firms make positive investment at this state. The following section derives conditions in which firms can (or cannot) sustain an equilibrium with no investment at this state.

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3.3 Weak preemption and cannibalization effects

I first explore the case in which preemption and cannibalization effects are relatively weak (α and β are large). I show that when the success probability is high, there is an equilibrium in which the incumbent allows the entrant to win the new format, and firms then perpetually use different formats. However, if the success probability is low, such an equilibrium does not exist, and both firms eventually implement both formats. The results in this section are the key new insights of the paper.

Define a state as "absorbing" if neither firm invests once that state is reached. Based on the set-up of the game, state (1,1;1,1) is obviously absorbing. We can then use backward induction to find equilibrium investment levels for all other states.

Consider what happens at state $(1, 0; 1, 1)$. At this state, as the incumbent decides whether to invest in the new format, it faces both the problem of cannibalizing its existing sales and the problem of potentially entering a format that the entrant is already using. If the following condition holds, neither of these effects is strong enough to stop the incumbent from investing at this state:

$$\left(\frac{1}{1-\delta}\right)\frac{\alpha\beta}{2} > \frac{1}{z} \tag{4}$$

This condition ensures the discounted gains from entering the new format are enough to justify investing the first marginal dollar. In this case, state $(1, 0; 1, 1)$ is not absorbing, because the incumbent makes positive investment at this state, and therefore the game will eventually move to state $(1, 1; 1, 1)$.

Table 1 implies that the smallest incremental profits from adding a new format occur when Firm A (for example) implements format 2 and moves the state from $(1, 0; 0, 1)$ to $(1, 1; 0, 1)$. Generating these incremental profits over an infinite number of periods would be enough to justify the first marginal dollar of investment if the

following condition holds:[7]

Condition 1.

$$\left(\frac{1}{1-\delta}\right)\left[\frac{\alpha\beta}{2} - \left(V - 2\beta\right)\left(\frac{1}{2} - \alpha\right)\right] > \frac{1}{z}$$

When this condition holds, any state at which one firm (but not the other) is using both formats cannot be absorbing, because the other firm would always make positive investment until it also implements both formats.

On the other hand, state $(1, 0; 0, 1)$ could be absorbing. At this state, each firm must worry that implementing another format will lead to retaliation by its competitor. For example, if firm A implements the new format, its profits will temporarily increase; however, once firm B successfully retaliates, the game moves to state $(1, 1; 1, 1)$, at which point increased competition in each format causes firm A's profits to drop below their initial level. This threat of retaliation can prevent investment at state $(1, 0; 0, 1)$ if and only if the following inequality holds (for notational convenience, this condition is stated in terms of the general profit function π^A):

$$\left(\frac{\pi^A_{(1,1;0,1)}\left[1 + \delta(1 - F(e^*))\right] + [\delta/(1-\delta)]F(e^*)\pi^A_{(1,1;1,1)}}{1 - \delta^2(1 - F(e^*))}\right) - \left(\frac{1}{1-\delta}\right)\pi^A_{(1,0;0,1)} \leq \frac{1}{z} \quad (5)$$

where e^* denotes firm B's optimal investment level in the old format at state $(1, 1; 0, 1)$. The first term on the left side of this inequality represents the expected discounted profits to firm A just after reaching state $(1, 1; 0, 1)$, accounting for firm B's eventual retaliation; the second term represents the expected discounted profits to firm A of staying permanently at state $(1, 0; 0, 1)$.

Whether this inequality holds depends on how quickly Firm B is expected to

[7]Moving from state $(1, 0; 0, 1)$ to state $(1, 1; 0, 1)$ allows the incumbent to capture an additional $\frac{\alpha}{2}$ customers, who each pay price β. However, it also causes a fraction $\left(\frac{1}{2} - \alpha\right)$ of the firm's customers to move from format 1, at which they paid price $V - \beta$, to format 2, at which they pay the lower price β.

retaliate following a successful attack by Firm A. If the parameter d is very large, each additional dollar spent has a rapidly decreasing effect on the probability of success, so once state $(1, 1; 0, 1)$ is reached, in each period Firm B will simply make a small "exploratory" investment that gives it a small chance of success. On the other hand, if d is small, each incremental dollar continues to have a large effect on the success probability, so Firm B will invest enough to give it a large chance of success in any given period.

Formally, as $d \to \infty$, $F(e^*) \to 0$, which implies that the expected time required for successful retaliation grows without bound, and the left side of inequality (5) approaches $\frac{1}{1-\delta}\left(\pi^A_{(1,1;0,1)} - \pi^A_{(1,0;0,1)}\right)$. When we insert the values from Table 1, this expression is the same as the left side of Condition 1, which implies (5) does not hold, and the threat of retaliation *cannot* prevent investment at state $(1, 0; 0, 1)$.

Intuitively, when implementing a format is sufficiently difficult (d is sufficiently large), the expected time required to retaliate becomes so long that firms do not worry about retaliation. Rather, they each invest a small amount in the other's format because Condition 1 guarantees that the expected discounted profits of a successful attack are enough to justify investing the first marginal dollar. Although each firm's expected success probability in any given period is low, one firm eventually succeeds in its attack, and its competitor eventually succeeds in retaliating, and so both firms end up using both formats. The online appendix proves this result formally.

Proposition 1. *If Condition 1 holds and d is sufficiently large, then in any equilibrium, both firms implement both formats in the long run (with probability one).*

Figure 2. When implementing a format is difficult (d is large), both firms implement both formats. (Arrows indicate possible paths the game state can follow.)
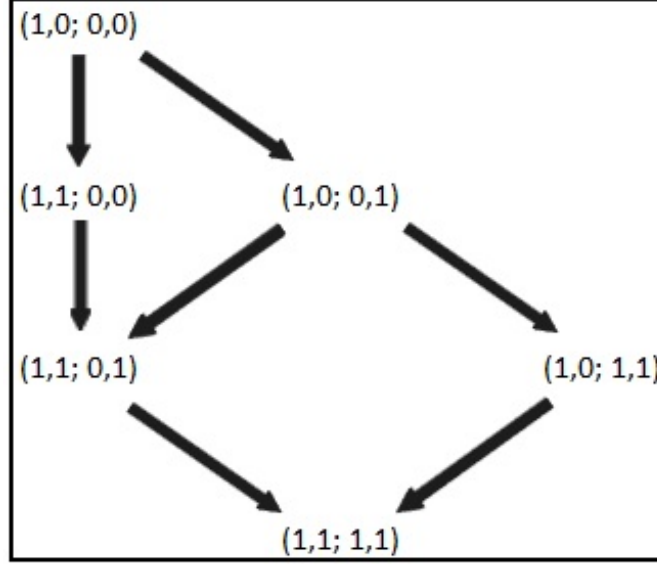
I now consider the case in which implementing a format is easy. When d is sufficiently small, $F(e^*) = 1$, and retaliatory investments are guaranteed to succeed in the next period after an attack. Inequality (5) then becomes

$$\left(\pi^A_{(1,1;0,1)} - \pi^A_{(1,0;0,1)}\right) - \left(\frac{\delta}{1-\delta}\right)\left(\pi^A_{(1,0;0,1)} - \pi^A_{(1,1;1,1)}\right) \leq \frac{1}{z} \tag{6}$$

Thus, unless firms have very low discount factors, inequality (5) holds when d is sufficiently small, in which case the threat of immediate retaliation can prevent investment at state $(1, 0; 0, 1)$.

The following condition is sufficient to ensure (6) holds. This condition also ensures that if d is small enough, an equilibrium exists in which the incumbent invests nothing at the initial state $(1, 0; 0, 0)$, guaranteeing that the industry reaches the absorbing state $(1, 0; 0, 1)$.

Condition 2.

$$\alpha V - \left(\frac{\delta^3}{1-\delta}\right)\left(\frac{V-2\beta}{2}\right) < \frac{1}{z}$$

The first term on the left side of this condition is an upper bound on the short-term benefits the incumbent gains from implementing the new format (moving the game from state $(1,0;0,0)$ to state $(1,1;0,0)$), and the second term is the cost of provoking the entrant to implement both formats (given that d is small enough that the entrant will immediately implement any format in which it invests). Condition 2 ensures that, at the initial state, the sum of these two effects is not great enough to justify the incumbent investing the first marginal dollar in the new format.[8] Note that Conditions 1 and 2 are both more likely to hold when δ is large. By choosing δ sufficiently close to one, it is straightforward to find parameter values for which both conditions hold.

The online appendix proves the following proposition.

Proposition 2. *If Conditions 1 and 2 hold, and d is sufficiently small, there is an equilibrium in which firm B is the only one that invests in the new format; once it successfully implements this format, neither firm makes further investment.*

Note that if either firm ever implements both formats, the other firm keeps investing until it also implements both formats. For example, at state $(1,1;0,1)$, the entrant invests in the old format, knowing the incumbent has no way to retaliate. On the other hand, at state $(1,0;0,1)$, the entrant does *not* invest in the old format because the incumbent would then retaliate by investing in the new format. Thus, the game progresses as follows. At the initial state $(1,0;0,0)$ the incumbent avoids investing in the new format so that it retains a credible way to retaliate against the entrant. Once the entrant implements the new format, and the game reaches state

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[8]This condition is somewhat stronger than necessary, but it is more notationally succinct than the weakest possible condition would be.

$(1, 0; 0, 1)$, neither firm encroaches on the other's format, because neither firm wants to end up at state $(1, 1; 1, 1)$.

Figure 3. When implementing a format is easy (d is small), a firm will not implement a second format because doing so would lead to swift retaliation. (Arrows indicate possible paths the game state can follow.)
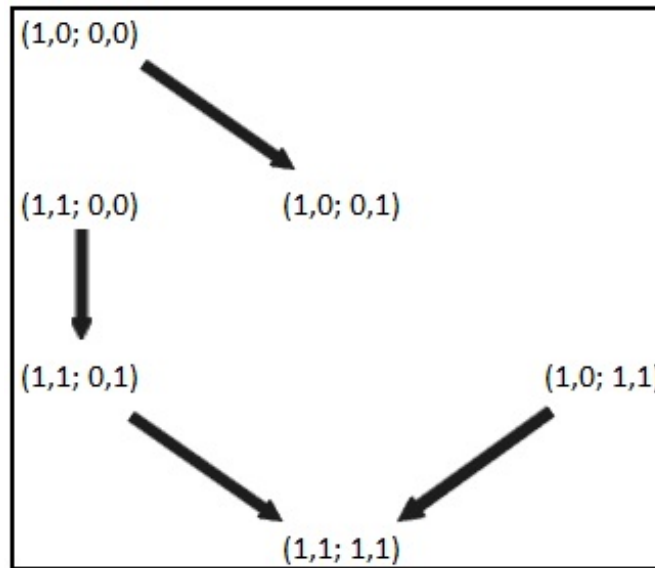
To summarize, Proposition 1 states that when the success probability is *low*, both firms must eventually implement both formats, whereas Proposition 2 states that when the success probability is *high*, there is an equilibrium in which firms reach an absorbing state where they use different formats.

3.4   Strong preemption and cannibalization effects

The previous section showed that an increase in success probability can make the incumbent less willing to invest in the new format. The current section shows that this effect can be reversed.

Intuitively, the previous section assumed preemption and cannibalization effects were weak, and so the incumbent primarily faced a trade-off between the short-term gains from adopting the new format and the long-term loss due to competitive

retaliation. In that case, an increase in success probability made the threat of retaliation more immediate, which made the incumbent *less* willing to invest in the new format to increase its short-term profits. By contrast, the current section assumes preemption and cannibalization effects are strong. In this case, an increase in success probability makes the relative benefits of preempting the potential entrant more immediate, which makes the incumbent *more* willing to cannibalize its existing sales by investing in the new format. Because the results in this section are similar to results from previous theoretical research on innovation (Gilbert and Newbery 1982; Reinganum 1983), I keep the exposition of these results relatively brief.

The current section assumes the following conditions hold:

Condition 3.

$$\left(\frac{1}{1-\delta}\right)(V-\beta)\alpha < \frac{1}{z}$$

Condition 4.

$$\left(\frac{1}{1-\delta}\right)\frac{\beta(1-\alpha)}{2} < \frac{1}{z}$$

Condition 3 implies that cannibalization effects are strong (α is small), whereas Condition 4 implies that preemption effects are strong (β is small). The online appendix shows that these conditions guarantee all states except the initial state are absorbing (see proof of Proposition 3).

I also assume the following condition holds:

Condition 5.

$$(V-\beta)\alpha + \left(\frac{\delta}{1-\delta}\right)\frac{V-\beta}{2} > \frac{1}{z}$$

This condition is sufficient to ensure the entrant invests in the new format at the initial state. As $d \to \infty$, the expected time for the entrant to successfully implement the new format grows without bound, and Condition 3 ensures the incumbent invests nothing at the initial state due to concerns over cannibalizing its existing sales.

On the other hand, as $d \to 0$, the entrant is guaranteed to successfully implement the new format the first time it moves. In this case, Condition 5 guarantees that the incumbent also invests in the new format at the initial state. The first term in that condition represents the incumbent's immediate profit impact from implementing the new format, whereas the second term reflects the long-term benefit of preventing the entrant from implementing the new format.

The online appendix formally proves the following.

Proposition 3. *If Conditions 3, 4, and 5 all hold, then all states except the initial state are absorbing. If d is sufficiently large, then at the initial state, the incumbent makes zero investment and the entrant is guaranteed to win the new format in the long run. However, if d is sufficiently small, then at the initial state, both firms make positive investment in the new format.*

Figure 4. When cannibalization and preemption effects are strong and implementing a format is *difficult* (*d* is large), the incumbent allows the entrant to win the new format.
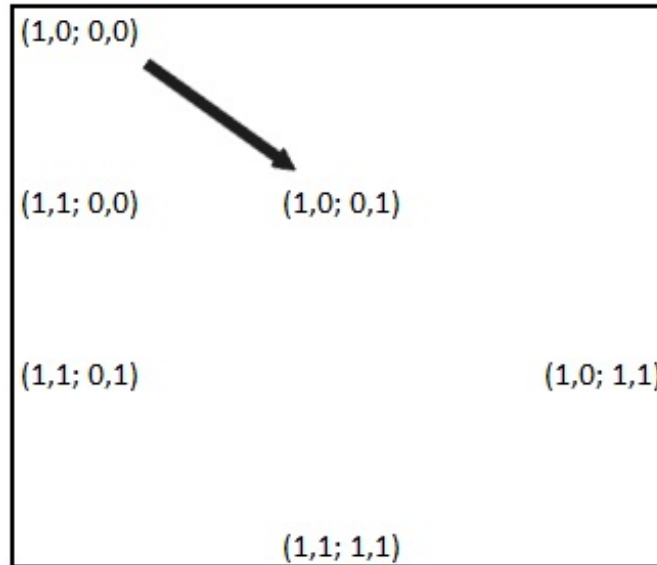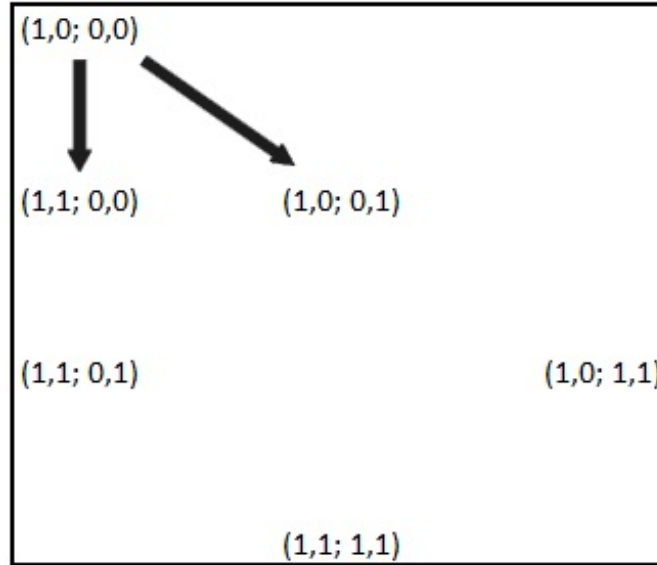
Figure 5. When cannibalization and preemption effects are strong and implementing a format is _easy_ (d is small), the incumbent is willing to cannibalize its sales to try to preempt investment by the entrant.

Survey data show that high-tech firms vary substantially in how much their managers say they are willing to cannibalizes existing sales (Chandy and Tellis 1998). When a new technology has a winner-take-all property (preemption effects are strong), Proposition 3 implies that incumbents should be willing to cannibalize existing sales if the probability that an entrant can quickly implement the new technology is high, but incumbents should not be willing to cannibalize existing sales if this probability is low.

3.5 Other regions of the parameter space

Previous sections have studied cases in which preemption and cannibalization effects are either both weak or both strong. I do not present detailed results for other combinations of these parameter values, because the results are more straightforward and less interesting than those in the previous sections. If preemption effects are weak and cannibalization effects are strong, the incumbent allows the entrant to win

the new format; and if preemption effects are strong and cannibalization effects are weak, the two firms race to enter the new format. These results do not depend on the parameter d that determines how quickly the marginal impact of investment diminishes.

4 Extensions: Asymmetric formats

Until now, I have assumed profit functions are symmetric across formats. To illustrate how large asymmetries can change investment incentives, I now present two model extensions. The first assumes formats differ in terms of fixed costs, and the second assumes either firm's use of the new format eliminates profits from the old format.

4.1 Fixed expense in the old format

Many Internet-based business models allow firms to avoid fixed expenses, such as physical retail locations, that are associated with traditional business models. I now show that such fixed expenses in the old format can allow the incumbent to attack the new format without fear of retaliation.

Assume there is a recurring fixed expense f to operating the old format, which is avoided in the new format. For example, at state $(1, 0; 0, 1)$, firm A's profits are now $\left((V - \beta)\frac{1}{2} - f\right)$, whereas firm B's profits are still $\left((V - \beta)\frac{1}{2}\right)$. Also assume Condition 1 holds, so cannibalization and preemption effects alone are too weak to prevent investment. If the fixed expense is high enough that the following condition holds, the entrant will never invest in the old format:

Condition 6.
$$\left(\frac{1}{1 - \delta}\right)\left[(V - \beta)\alpha - f\right] < \frac{1}{z}$$

The online appendix proves the following result formally.

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Proposition 4. *If Conditions 1 and 6 hold, in equilibrium both firms eventually implement the new format, but the entrant never invests in the old format.*

Figure 6. Large fixed expenses in the old format allow the incumbent to invest in the new format without fear of retaliation. (Arrows indicate possible paths the game state can follow.)
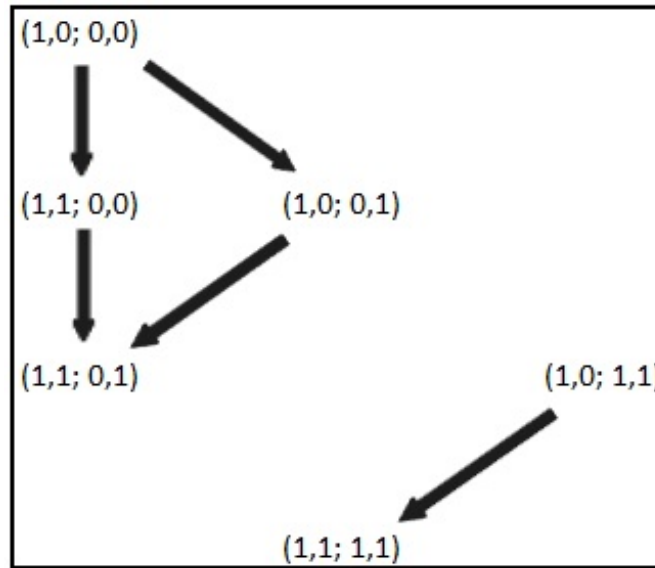
As an example in which the incumbent could attack the new format without fear of retaliation, when incumbent Charles Schwab invested in its online trading platform, it would have been unprofitable for E-trade to retaliate by building a large network bricks-and-mortar locations. Schwab now offers both online and in-person investment formats, whereas E-trade focuses on the online format.[9]

4.2   Delayed entry response

Online stores often free-ride on customer service provided by traditional stores, making traditional stores much less profitable (Anderson et al. 2009). To illustrate how this type of asymmetric channel conflict can affect investment decisions, I make

---

[9]According to their company websites, Schwab has over 300 physical branches, whereas E-trade has only 30.

the extreme assumptions that either firm's use of the new format eliminates profits from the old format, but either firm's use of the old format does *not* affect the profits from using the new format.[10]

Under these new assumptions, if implementing a format is difficult enough, at first the incumbent avoids investing in the new format, because implementing this format would entirely cannibalize its existing profits. However, once the entrant finally succeeds in implementing the new format, destroying all profits in the old format, the incumbent starts investing in the new format as well. This result is similar in spirit to previous results by Katz and Shapiro (1987), who study whether an incumbent or an entrant innovates first, if firms can imitate each other's innovations.[11]

The online appendix proves the following proposition formally.

Proposition 5. *If either firm's use of the new format eliminates profits in the old format, Condition 1 holds, and d is sufficiently large, then the incumbent initially makes no investment; after the entrant successfully implements the new format, the incumbent begins investing in the new format until it also implements this format.*

As an example of delayed entry response, wedding dress shops face the potential threat that a bride could spend several hours trying on dresses in their store and using their customer service until she finds a dress she likes, and then buy a similar dress online at a much lower price.[12] Currently most new brides are reluctant to buy

---

[10]In the case of free-riding, the new format might benefit from the existence of the old format. To allow this benefit to occur, we could make the alternative assumption that use of the new format leaves the old format just profitable enough that the incumbent will continue to use the old format.

[11]Using the terminology of Katz and Shapiro (1987), the conditions of Proposition 5 imply the entrant has stronger "stand-alone incentives" (defined as the increase in a firm's profits if it innovates and its competitor does nothing) than the incumbent, and firms have equal "preemption incentives" (defined as the difference in profits from innovating first rather than second). As a result, the entrant innovates first.

[12]For example, see the blog post at http://henjofilms.com/?p=586 ("My Online Wedding Dress Buying Experience!"), in which a recently married woman says she visited several traditional wedding stores and found a dress she liked priced at $2,800, but instead bought a similar dress online for $350.

a dress online (Bertagnoli 2011), so for now the best strategy for the traditional shops may be to stick with their current format; however, if an online retailer ever builds a strong-enough reputation that free-riding on customer service becomes a major problem in this market, traditional wedding shops might want to start investing in the online channel as well.

Figure 7. If either firm's use of the new format eliminates profits in the old format, and implementing a format is difficult, the incumbent does not invest in the new format until the entrant has successfully adopted this format. (Arrows indicate possible paths the game state can follow.)
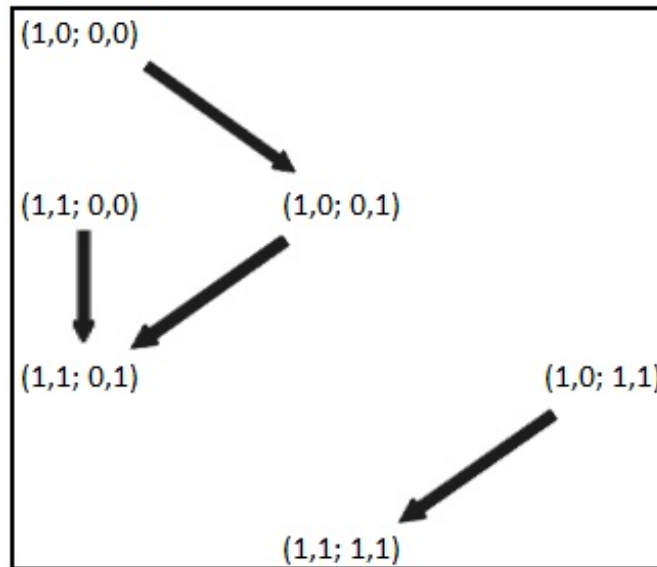
4.3   Other possible asymmetries

This section has introduced two possible types of asymmetry into the model. One could imagine other possible asymmetries as well. For example, if the new format is more difficult to implement than the old, the entrant might start by investing in the old format. On the other hand, if the entrant has a weaker brand than the incumbent, then the entrant might no longer be able to make a credible threat of attacking the old format (undermining the equilibrium of Proposition 2), or the entrant might avoid entering the market altogether. I leave such model extensions for future research.

5   Conclusion

This paper has developed a model in which an incumbent and an entrant compete in a market where a new business format has become available. If implementing a format is difficult, firms can attack each other without worrying about swift retaliation, so each firm continually invests a small amount in the other's format until they both implement both formats. By contrast, if implementing a format is easy, the incumbent allows the entrant to win the new format, and firms can then sustain an equilibrium in which they stay focused on different formats.

These results hold when preemption and cannibalization effects are weak. On the other hand, when both of these effects are strong, an increase in the ease of implementing formats makes the incumbent more willing to cannibalize its sales by investing in the new format.

The paper includes two model extensions that illustrate how asymmetric formats can affect investment incentives. Future research could further extend the model, for example, to allow firms to differ in how easily they can implement new formats, or to allow customer preferences to change over time.

Future research could also empirically investigate the predictions of the model, looking for evidence that incumbents tend to avoid investing in new formats in industries where all of the following hold: (1) entry barriers keep the number of firms small; (2) conditional on entry, implementing a format is relatively easy; and (3) multiple firms could profitably implement both formats (preemption and cannibalization effects are weak). In such industries, this model implies incumbents should retain the threat of implementing the new format later as a form of retaliation.

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Supplemental Online Appendix:
Proofs of All Results

Price Equilibrium for the Product Market Competition Model: At state $(1, 0; 0, 0)$, if Firm A sets price $P_{A,1} \leq V - \beta$, it captures all $(1 - \alpha)$ customers who will purchase using format 1. If it sets its price in the range $P_{A,1} \in (V - \beta, V)$, it captures some positive fraction of these customers. If it sets $P_{A,1} \geq V$, it captures no customers. To be precise, Firm A's profits are:

$$
\pi_A = \begin{cases}
P_{A,1}(1 - \alpha) & \text{if } P_{A,1} \leq V - \beta \\
P_{A,1}(1 - \alpha)\left[\frac{V - P_{A,1}}{\beta}\right] & \text{if } P_{A,1} \in (V - \beta, V) \\
0 & \text{if } P_{A,1} \geq V
\end{cases}
\tag{7}
$$

Note that setting price strictly below $V - \beta$ results in strictly lower profits than setting price equal to $V - \beta$. Also, setting price above V results in zero profits. Therefore, any optimal solution must lie in the interval $[V - \beta, V]$. At the interior of this interval:

$$
\begin{aligned}
\frac{d\pi_A}{dP_{A,1}} &= (1 - \alpha)\left[\frac{V - 2P_{A,1}}{\beta}\right] \\
&< (1 - \alpha)\left[\frac{2\beta - V}{\beta}\right] \\
&< 0
\end{aligned}
\tag{8}
$$

where the first inequality holds because we are focusing on a price interval where $P_{A,1} > V - \beta$, and the second inequality follows because Assumption 1 states $V > 2\beta$. This derivative being negative implies that Firm A's profits increase as its lowers its price to the point $P_{A,1} = V - \beta$.

At state $(1, 1; 0, 0)$, we first focus on the case for which $P_{A,1} < P_{A,2}$. As long as both prices lie in the interval $[V - \beta, V]$, Firm A's profits are:

$$
\pi_A = (1 - \alpha)P_{A,1}\left[\frac{V - P_{A,1}}{\beta}\right] + \alpha P_{A,2}\left[\frac{V - P_{A,2}}{\beta}\right]
\tag{9}
$$

Taking first derivatives, we have:

$$
\frac{\partial \pi_A}{\partial P_{A,1}} = (1 - \alpha)\left[\frac{V - 2P_{A,1}}{\beta}\right]
\tag{10}
$$

$$
\frac{\partial \pi_A}{\partial P_{A,2}} = \alpha\left[\frac{V - 2P_{A,2}}{\beta}\right]
\tag{11}
$$

Similar derivations to those in (8) show that both of the above terms are negative. By symmetry, if $P_{A,1} > P_{A,2}$, both partial derivatives are also negative. Finally,

if $P_{A,1} = P_{A,2}$, the *left* partial derivative (with respect to either price) is given by (10), and the *right* partial derivative is given by (11). Note these derivatives are also negative.

All of these derivatives being negative implies that, as long as prices are in the interval $[V-\beta, V]$, the firm's profits increase as it decreases both prices. Setting either price strictly below $V - \beta$ would decrease profit margins without increasing demand, and setting either price above V would result in zero demand for that format, either of which would result in lower profits. Therefore, the optimal solution is to set both prices exactly at $V - \beta$ and capture all potential customers in the market.

For state $(1, 0; 0, 1)$, I will show there is an equilibrium in which both firms set the monopoly price: $P_{A,1} = P_{B,2} = V - \beta$. I will show that if Firm B sets this price, then Firm A's best response is to set this price, and vice versa. If Firm B sets $P_{B,2} = V - \beta$, then Firm A's demand function can be broken into four intervals. In the lowest price interval, Firm A captures all $(1 - \alpha)$ customers who will purchase using format 1; in the second price interval, it captures all α of the customers who are loyal to format 1 and some positive fraction of the $(1 - 2\alpha)$ customers who will purchase with either format; in the third price interval it captures some positive fraction of both of these customer types; and in the fourth price interval it captures no customers. To be precise, Firm A's profits are:

$$
\pi_A = \begin{cases}
P_{A,1}(1 - \alpha) & \text{if } P_{A,1} \leq V - 2\beta \\
P_{A,1}\left[\alpha + (1 - 2\alpha)\left(\frac{V - P_{A,1}}{2\beta}\right)\right] & \text{if } P_{A,1} \in (V - 2\beta, V - \beta) \\
P_{A,1}\left[\alpha\left(\frac{V - P_{A,1}}{\beta}\right) + (1 - 2\alpha)\left(\frac{V - P_{A,1}}{2\beta}\right)\right] & \text{if } P_{A,1} \in (V - \beta, V) \\
0 & \text{if } P_{A,1} \geq V
\end{cases}
\tag{12}
$$

At the point $P_{A,1} = V - \beta$, the first derivative as Firm A *increases* its prices is:

$$
\begin{aligned}
\frac{d_{+}\pi_A}{dP_{A,1}} &= \alpha\left[\frac{V - 2P_{A,1}}{\beta}\right] + (1 - 2\alpha)\left[\frac{V - 2P_{A,1}}{2\beta}\right] \\
&= \alpha\left[\frac{2\beta - V}{\beta}\right] + (1 - 2\alpha)\left[\frac{2\beta - V}{2\beta}\right] \\
&< 0
\end{aligned}
\tag{13}
$$

where the inequality follows because Assumption 1 states $V > 2\beta$. This derivative being negative implies that raising prices reduces profits. Also at the point $P_{A,1} =$

$V - \beta$, the first derivative as Firm A *decreases* its prices is:

$$
\begin{aligned}
\frac{d_{-}\pi_A}{dP_{A,1}} &= \alpha + (1 - 2\alpha)\left[\frac{V - 2P_{A,1}}{2\beta}\right] \\
&= \alpha + (1 - 2\alpha)\left[\frac{2\beta - V}{2\beta}\right] \\
&= \alpha + (1 - 2\alpha) - (1 - 2\alpha)\left[\frac{V}{2\beta}\right] \qquad (14) \\
&= (1 - \alpha) - (1 - 2\alpha)\left[\frac{V}{2\beta}\right] \\
&> 0
\end{aligned}
$$

where the inequality holds because Assumption 1 implies $\frac{V}{2\beta} < \frac{1-\alpha}{1-2\alpha}$. This derivative being positive implies that lowering prices also reduces profits.

Thus, we have shown that there is a local maximum at $P_{A,1} = V - \beta$. By differentiating (13) and (14) with respect to $P_{A,1}$, we find that the second derivative of π_A is negative (for both increases and decreases in price), so this local optimum is also a global optimum. By symmetry, Firm B's best response is also to set $P_{B,2} = V - \beta$. Intuitively, Assumption 1 guarantees there is an equilibrium in which both firms set prices at the kink in the demand curve occurring at the highest price for which a firm captures all customers loyal to its own format.

At state $(1, 0; 1, 0)$, both firms compete for the $(1 - \alpha)$ mass of customers who will purchase using format 1. Firm A's demand is $(1 - \alpha)\left(\frac{1}{2} + \frac{P_{B,1} - P_{A,1}}{2\beta}\right)$, and Firm B has analogous demand. These are simply the demand functions from the standard Hotelling model, and the standard derivations for this model show that in equilibrium firms set prices $P_{A,1} = P_{B,1} = \beta$, and each firm captures one-half of the available customers.

At state $(1, 1; 0, 1)$, Firm A uses both formats, whereas Firm B uses only format two. I will show there is an equilibrium in which $P_{A,1} = V - \beta$ and $P_{A,2} = P_{B,2} = \beta$. Given Firm A's prices in the proposed equilibrium, Firm B's profits are $P_{B,2}(1-\alpha)\left(\frac{1}{2} + \frac{\beta - P_{B,2}}{2\beta}\right)$. By differentiating with respect to price and finding first-order conditions, it is straightforward to show that Firm B's best response is $P_{B,2} = \beta$.

Given Firm B's price in the proposed equilibrium, Firm A's profits are:

$$
\pi_A = \alpha P_{A,1}\left[\frac{V - P_{A,1}}{\beta}\right] + \alpha P_{A,2}\left[\frac{1}{2} + \frac{\beta - P_{A,2}}{2\beta}\right] + (1 - 2\alpha)\widehat{P_A}\left[\frac{1}{2} + \frac{\beta - \widehat{P_A}}{2\beta}\right] \qquad (15)
$$

where $\widehat{P_A} \equiv \min\{P_{A,1}, P_{A,2}\}$. The three terms on the right side of this equation represent Firm A's profits from customers who will only use format 1, those who will only use format 2, and those who will use either format, respectively. For notational simplicity, I have not explicitly included bounds on the prices, but this function applies as long as the three terms in brackets each lie in the interval $[0, 1]$.

By the same logic as the derivation for the monopoly case $(1, 0; 0, 0)$, Firm A's profits from the first group are maximized when it sets $P_{A,1} = V - \beta$, and by the same logic as the derivation for the standard Hotelling model, Firm A's profits from the latter two groups are maximized when it sets $P_{A,2} = \widehat{P_A} = \beta$. All three objectives can be accomplished simply by setting $P_{A,1} = V - \beta$ and $P_{A,2} = \beta$.

Intuitively, the model set-up in which customers have either extreme format preferences or no preference between formats allows Firm A to perfectly price discriminate by using its format 1 price to extract monopoly profits from those loyal to format 1, while using its format 2 price to set the optimal competitive price for all other customers.

Because the model is symmetric across firms and formats, states $(1, 1; 1, 0)$ and $(1, 0; 1, 1)$ are analogous to state $(1, 1; 0, 1)$.

Finally, at state $(1, 1; 1, 1)$, I will show there is an equilibrium in which both firms set prices in both formats equal to β. Given Firm B's price in the proposed equilibrium, Firm A's profits are:

$$\pi_A = \alpha P_{A,1}\left[\frac{1}{2} + \frac{\beta - P_{A,1}}{2\beta}\right] + \alpha P_{A,2}\left[\frac{1}{2} + \frac{\beta - P_{A,2}}{2\beta}\right] + (1 - 2\alpha)\widehat{P_A}\left[\frac{1}{2} + \frac{\beta - \widehat{P_A}}{2\beta}\right] \quad (16)$$

where $\widehat{P_A} \equiv \min\{P_{A,1}, P_{A,2}\}$. By the same logic as the derivations for the standard Hotelling model, Firm A's profits for all three groups of customers are maximized if it sets $P_{A,1} = P_{A,2} = \widehat{P_A} = \beta$. This can be accomplished simply by setting $P_{A,1} = P_{A,2} = \beta$. Firm B's best response is analogous to that of Firm A, so the proposed equilibrium holds. QED

Proof of Lemma 1: A pure Markov strategy for Firm A is a mapping $S^A(X_t) \to (e_{A,1}, e_{A,2})$ of the current state X_t into investment levels for each format. A pure Markov strategy S^B for Firm B is defined similarly. A pair of strategies (S^A, S^B) is an MPE if at each state each firm's strategy is optimal given its competitor's strategy. In principle I allow for mixed strategies, but I will show that a pure strategy MPE

exists.

The proof starts by deriving investment levels and value functions for state $(1, 1; 1, 1)$, and then works backwards, showing that for every state there exist investment levels such that each firm is behaving optimally given its competitor's strategy at the current state and given continuation values at subsequent states.

Let $\Lambda^A_{X_t}$ denote Firm A's value function at state X_t in an odd period (in which Firm A invests), and $\overline{\Lambda}^A_{X_t}$ denote its value function in an even period (in which Firm B invests). Similarly, $\Lambda^B_{X_t}$ denotes Firm B's value function in an *even* period, and $\overline{\Lambda}^B_{X_t}$ denotes Firm B's value function in an *odd* period.

At state $(1, 1; 1, 1)$, both firms have a dominant strategy of investing nothing because investment has no effect on the state, which implies

$$\Lambda^A_{1,1;1,1} = \overline{\Lambda}^A_{1,1;1,1} = \left(\frac{1}{1-\delta}\right)\pi^A_{1,1;1,1} \tag{17}$$

$$\Lambda^B_{1,1;1,1} = \overline{\Lambda}^B_{1,1;1,1} = \left(\frac{1}{1-\delta}\right)\pi^B_{1,1;1,1} \tag{18}$$

At state $(1, 0; 1, 1)$, Firm B invests nothing because its investment has no effect on the state, whereas Firm A sets $e_{A,1} = 0$ and chooses $e_{A,2}$ to maximize its expected discounted profits. We can write Firm A's optimization problem at this state as:

$$\Lambda^A_{1,0;1,1} = \max_{e_{A,2}} \left\{ \phi + \sum_{u=0}^{\infty} \left[1 - F(e_{A,2})\right]^{u+1} \left[(\delta^{2u} + \delta^{2u+1})\pi^A_{(1,0;1,1)} + \delta^{2u+2}\phi\right] \right\} \tag{19}$$

where we define:

$$\phi \equiv -e_{A,2} + F(e_{A,2})\left[\pi^A_{1,1;1,1} + \delta\overline{\Lambda}^A_{1,1;1,1}\right] \tag{20}$$

Intuitively, ϕ represents the cost of investment plus the probability of success in a given period times the expected discounted profits that result from success. The first term after the summation sign in (19) represents the probability of $u + 1$ failures in a row. Each failure guarantees that the firm continues to earn profits $\pi^A_{(1,0;1,1)}$ in the current period and in the next period, and that in two periods the firm will invest again and have another chance of success.

If we define $e_{max} \equiv \left[exp(d/z) - 1\right]/d$, by inserting this expression in (1), we see that $F(e_{max}) = 1$. Therefore, no firm will ever choose investment above e_{max}. Because (19) is a continuous function maximized over a closed and bounded interval, $[0, e_{max}]$, the Extreme Value Theorem implies that this function obtains its maximum at some

point $e^*_{A,2}$. Note the proposition does not claim equilibrium uniqueness, so there is no need to show that this optimal point is unique, only that an optimum exists.

Similarly, at states $(1, 1; 0, 1)$ and $(1, 1; 1, 0)$, Firm A invests nothing while Firm B chooses an investment level to maximize a function analogous to (19).

At state $(1, 1; 0, 0)$, Firm A invests nothing, and Firm B chooses whether to invest in Format 1 or 2. Given that the formats are symmetric, Firm B is indifferent between these formats, and similar derivations to those above show there is an equilibrium in which it invests in Format 2 at state $(1, 1; 0, 0)$.

We now consider state $(1, 0; 0, 1)$. At this state, each firm decides how much to invest in the format it is not yet using; that is, Firm A chooses $e_{A,2}$, and Firm B chooses $e_{B,1}$. In equilibrium, each firm's strategy must be optimal given its competitor's strategy. In fact, I will show that each firm's optimal investment level at this state is weakly increasing in its competitor's investment level (investments are strategic complements), and therefore results from Vives (1990) guarantee existence of a pure strategy equilibrium in investments at this state.

Define $\widetilde{\Lambda}^A_{1,0;0,1}(e^*_{A,2}(e_{B,1}), e_{B,1})$ as Firm A's expected discounted profits starting at state $(1, 0; 0, 1)$ in an even period, if Firm B always invests $e_{B,1}$ at this state, and if Firm A always chooses the investment level $e^*_{A,2}$ that is optimal given $e_{B,1}$. The proof that $e^*_{A,2}$ is weakly increasing in $e_{B,1}$ proceeds in three steps.

Step one is to show that $\widetilde{\Lambda}^A_{1,0;0,1} \geq \overline{\Lambda}^A_{1,0;1,1}$ for all $e_{B,1}$. This can be proved as follows. Because $\pi^A_{1,0;0,1} > \pi^A_{1,0;1,1}$, Firm A could always generate greater total discounted profits by investing nothing at state $(1, 0; 0, 1)$ until the state changes to $(1, 0; 1, 1)$ than it can if the game has already moved to state $(1, 0; 1, 1)$. This implies that, when Firm A optimizes, it must be better off at state $(1, 0; 0, 1)$ than at state $(1, 0; 1, 1)$.

Step two is to show that $\widetilde{\Lambda}^A_{1,0;0,1}$ is weakly decreasing in $e_{B,1}$. This can be proved as follows. Suppose, at state $(1, 0; 0, 1)$, Firm B always invests $e^H_{B,1}$ and Firm A always chooses the investment level $e^*_{A,2}(e^H_{B,1})$. Now suppose, during a single period t, Firm B reduces its investment to some level $e^L_{B,1} < e^H_{B,1}$. This lower investment level decreases the probability of the game moving to state $(1, 0; 1, 1)$ at time t. If both firms then proceed with their original strategies, Firm B's one-time investment reduction increases Firm A's expected discounted profits because, as noted above, Firm A is better off at state $(1, 0; 0, 1)$ than at state $(1, 0; 1, 1)$. If the game is still at state $(1, 0; 0, 1)$ at time $t + 2$, and Firm B again invests $e^L_{B,1}$ instead of $e^H_{B,1}$, this again increases Firm A's expected discounted profits. By induction, if Firm B permanently decreases its investment level at state to $(1, 0; 0, 1)$ to $e^L_{B,1}$, this must also increase

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Firm A's expected discounted profits. Thus, Firm A's optimized value of $\widetilde{\Lambda}^A_{1,0;0,1}$ must be greater for $e^L_{B,1}$ than for $e^H_{B,1}$.

Step three is to show that $e^*_{A,2}$ is weakly increasing in $e_{B,1}$. This can be proved as follows. If Firm A chooses an optimal investment level $e^*_{A,2}$ at the interior of the interval $[0, e_{max}]$, the following first-order condition must hold:

$$-1 + F'(e^*_{A,2})\left[\left(\pi^A_{1,1;0,1} - \pi^A_{1,0;0,1}\right) + \delta\left(\overline{\Lambda}^A_{1,1;0,1} - \widetilde{\Lambda}^A_{1,0;0,1}(e^*_{A,2}(e_{B,1}), e_{B,1})\right)\right] = 0 \quad (21)$$

We have shown that an increase in $e_{B,1}$ causes $\widetilde{\Lambda}^A_{1,0;0,1}$ to decrease, which implies that the term in brackets in (21) increases. Given that F' is a decreasing function (F is concave), $e^*_{A,2}$ must also increase for (21) to continue to hold.

Intuitively, the more Firm B invests, the less attractive staying at state $(1,0;0,1)$ is for Firm A, and therefore the more Firm A would like to invest in an effort to leave this state and move to state $(1,1;0,1)$. By symmetry, analogous results hold for Firm B. Because each firm chooses investment from the compact set $[0, e_{max}]$ and each firm's optimal investment level at state $(1,0;0,1)$ is weakly increasing in its competitor's investment level, Tarski's fixed point theorem (see Vives 1990, page 310) guarantees that a pair of strategies $(e^*_{A,2}, e^*_{B,1})$ exist at this state, such that each firm is behaving optimally given its competitor's strategy.

At state $(1,0;1,0)$, both firms choose investment levels in the new format. Derivations similar to those for state $(1,0;0,1)$ show that each firm's optimal investment level is weakly increasing in its competitor's investment level, which guarantees existence of a pure strategy equilibrium.

Finally, at state $(1,0;0,0)$, Firm A chooses how much to invest in Format 2, while Firm B decides *where* and how much to invest. Given the simplifying assumption that a firm can only invest in one format at a time, Firm B will prefer to invest in Format 1 if $\Lambda^B_{1,0;1,0} > \Lambda^B_{1,0;0,1}$ and Format 2 otherwise. Once Firm B decides where to invest, the same approach described above (for state $(1,0;0,1)$) can be used to show that investment levels are strategic complements, which guarantees existence of a pure strategy equilibrium. QED

Proof of Proposition 1: I will show that as $d \to \infty$, each firm's investment level converges to zero at every state; nonetheless, investment levels remain strictly positive (even as they approach zero), guaranteeing that both firms eventually implement both formats.

In any Markov perfect equilibrium, at any time t, each firm's investment decision must be optimal given its value function in that equilibrium. For example, if Firm A invests at time t at state $(1, 0; 1, 1)$, then $e_{A,2,t}$ must maximize:

$$-e_{A,2,t} + F(e_{A,2,t})\left[\pi^A_{1,1;1,1} + \delta\overline{\Lambda}^A_{1,1;1,1}\right] + \left[1 - F(e_{A,2,t})\right]\left[\pi^A_{(1,0;1,1)} + \delta\overline{\Lambda}^A_{1,0;1,1}\right] \quad (22)$$

The first derivative of (22) with respect $e_{A,2,t}$ to is:

$$-1 + F'(e_{A,2,t})\left[\pi^A_{1,1;1,1} + \delta\overline{\Lambda}^A_{1,1;1,1} - \pi^A_{(1,0;1,1)} - \delta\overline{\Lambda}^A_{1,0;1,1}\right] \quad (23)$$

From (2), we can see that, as $d \to \infty$, $F'(e_{i,j,t}) \to 0$ for and any $e_{i,j,t} > 0$, which implies that (23) converges to -1. In other words, for any given positive investment level, once d becomes large enough, reducing investment increases a firm's expected discounted profits. Thus, optimal investment levels must approach zero as $d \to \infty$. Similar analysis implies that investment levels also approach zero at all other states.

As investment levels approach zero, the expected time spent at the current state grows without bound, and the value for Firm i of being at any given state converges to its expected discounted profits from remaining at this state forever:

$$\Lambda^i_{X_t} \xrightarrow[d \to \infty]{} \left(\frac{1}{1 - \delta}\right)\pi^i_{X_t} \quad (24)$$

$$\overline{\Lambda}^i_{X_t} \xrightarrow[d \to \infty]{} \left(\frac{1}{1 - \delta}\right)\pi^i_{X_t} \quad (25)$$

I now consider the marginal impact of investment at the point $e_{i,j,t} = 0$. From (2), we can see that

$$F'(0) = z \quad (26)$$

Inserting (24), (25), and (26) into (23) yields

$$-1 + z\left(\frac{1}{1 - \delta}\right)\left(\pi^A_{1,1;1,1} - \pi^A_{1,0;1,1}\right) \quad (27)$$

Condition 1 guarantees that (27) is greater than zero. Thus, at state $(1, 0; 1, 1)$, for d sufficiently large, investing the first marginal dollar in Format 2 increases Firm A's expected discounted profits.

Similar analysis shows that Condition 1 guarantees, for any state at which a firm is

not already using both formats, investing the first marginal dollar leads to an increase in a firm's expected discounted profits. This ensures that firms always make strictly positive investment at any such state, which guarantees that in the long run they both implement both formats. QED

Proof of Proposition 2: I will show that, under the conditions of the proposition, there exists an equilibrium in which firms behave as follows: At states $(1,0;1,1)$, $(1,1;0,1)$, and $(1,1;1,0)$, the firm that is not yet using both formats invests $e_{max} \equiv \left[exp(d/z)-1\right]/d$ in the format it is not yet using, guaranteeing that it implements this format immediately. At state $(1,1;0,0)$, Firm B invests e_{max} in Format 2. At state $(1,0;0,1)$, both firms invest nothing. At state $(1,0;0,0)$, Firm A invests nothing and Firm B invests e_{max} in Format 2. By the one-stage deviation principle, it is sufficient to show that, if both firms follow these strategies, no firm can profitably deviate from these strategies at any single time period t.

Given the proposed strategies, if Firm A invests at time t at state $(1,0;1,1)$, it chooses $e_{A,2,t}$ to maximize:

$$-e_{A,2,t}+F(e_{A,2,t})\Lambda^A_{1,1;1,1}+ \left(1-F(e_{A,2,t})\right)\left[(1+\delta)\pi^A_{1,0;1,1}+\delta^2\left(-e_{max}+\Lambda^A_{1,1;1,1}\right)\right] \quad (28)$$

where $\Lambda^A_{1,1;1,1} = \frac{\pi^A_{1,1;1,1}}{1-\delta}$. Differentiating with respect to $e_{A,2,t}$ yields:

$$-1 + F'(e_{A,2,t})\left[(1+\delta)\left(\pi^A_{1,1;1,1} - \pi^A_{1,0;1,1}\right) + \delta^2 e_{max}\right] \quad (29)$$

As $d \to 0$, $e_{max} \to \frac{1}{z}$ and $F'(e) \to z$ for all $e \in [0, e_{max}]$, which implies that expression (29) converges to:

$$-1 + \delta^2 + z(1+\delta)\left(\pi^A_{1,1;1,1} - \pi^A_{1,0;1,1}\right) \quad (30)$$

Rearranging terms, this expression is greater than zero if:

$$\left(\pi^A_{1,1;1,1} - \pi^A_{1,0;1,1}\right) > \frac{1-\delta}{z} \quad (31)$$

Condition 1 ensures that this inequality holds.   Thus, for d sufficiently small, expression (29) is positive over the interval $[0, e_{max}]$, and so it is optimal for Firm A to invest e_{max} in Format 2 at state $(1,0;1,1)$. Similarly, it is optimal for Firm B to invest e_{max} in Format 1 at state $(1,1;0,1)$, and in Format 2 at state $(1,1;1,0)$. Similar derivations also show that it is optimal for Firm B to invest e_{max} in Format

2 at state $(1, 1; 0, 0)$.

We now consider state $(1, 0; 0, 1)$. Given the proposed equilibrium, at this state Firm A chooses $e_{A,2,t}$ to maximize:

$$-e_{A,2,t} + F(e_{A,2,t})\left[\pi^A_{1,1;0,1} + \left(\frac{\delta}{1-\delta}\right)\pi^A_{1,1;1,1}\right] + \left(1 - F(e_{A,2,t})\right)\left(\frac{1}{1-\delta}\right)\pi^A_{1,0;0,1} \quad (32)$$

Differentiating with respect to $e_{A,2,t}$, and noting that as $d \to 0$ then $F'(e) \to z$, yields:

$$-1 + z\left[\left(\pi^A_{(1,1;0,1)} - \pi^A_{(1,0;0,1)}\right) - \left(\frac{\delta}{1-\delta}\right)\left(\pi^A_{(1,0;0,1)} - \pi^A_{(1,1;1,1)}\right)\right] \quad (33)$$

Condition 2 is sufficient to ensure that this expression is less than zero, so Firm A's optimal strategy is to invest nothing at state $(1, 0; 0, 1)$. Similarly, Firm B invests nothing at this state.

The proof that Firm A invests nothing at state $(1, 0; 0, 0)$ is similar. Given the proposed equilibrium, implementing the new format would increase Firm A's profits in the short run, but would lead to lower long-run profits for Firm A because the game would end up in state $(1, 1; 1, 1)$ instead of state $(1, 0; 0, 1)$. Considering both effects, Condition 2 is sufficient to ensure that Firm A invests nothing at the initial state.

The only remaining question is what Firm B does at the initial state. Given that $\pi^B_{(1,0;0,1)} > \pi^B_{(1,0;1,0)}$, and given that Firm A invests nothing at state $(1, 0; 0, 1)$, regardless of how much Firm A invests at state $(1, 0; 1, 0)$, it can be shown that $\Lambda^B_{1,0;0,1} > \Lambda^B_{1,0;1,0}$. Therefore, Firm B prefers to invest in Format 2 at the initial state. Derivations similar to those in expressions (28) through (31) show that it is optimal for Firm B to invest e_{max} in this format. QED

Proof of Proposition 3: I will first show that both firms make zero investment at all states except the initial state. If Firm A invests at time t at state $(1, 0; 1, 1)$, it chooses $e_{A,2,t}$ to maximize (22). The derivative of this expression is

$$-1 + F'(e_{A,2,t})\left[\pi^A_{1,1;1,1} + \delta\overline{\Lambda}^A_{1,1;1,1} - \pi^A_{(1,0;1,1)} - \delta\overline{\Lambda}^A_{1,0;1,1}\right] \quad (34)$$

Note that $F' \leq z$, and $\overline{\Lambda}^A_{1,1;1,1} = \frac{\pi^A_{1,1;1,1}}{1-\delta}$. Also, when Firm A is optimizing, we must have $\overline{\Lambda}^A_{1,0;1,1} \geq \frac{\pi^A_{1,0;1,1}}{1-\delta}$ because Firm A could invest nothing at this state and generate profits $\pi^A_{1,0;1,1}$ forever. Substituting in these values, we see that (34) is less than or

equal to:

$$-1 + z\left[\left(\frac{1}{1-\delta}\right)\left(\pi^A_{1,1;1,1} - \pi^A_{1,0;1,1}\right)\right] \tag{35}$$

Condition 4 guarantees that this expression is less than zero, which implies that (34) is also negative for all $e_{A,2,t} \geq 0$, and it is optimal for Firm A to invest nothing at state $(1,0;1,1)$. Similar derivations show that, under Condition 4, Firm B invests nothing at states $(1,1;0,1)$, $(1,1;1,0)$, and $(1,1;0,0)$.

At states $(1,0;0,1)$ and $(1,0;1,0)$, each firm's optimal investment level could theoretically depend on its competitor's investment level. However, slight modifications to the above derivations show that, at state $(1,0;0,1)$, Condition 4 guarantees each firm's optimal strategy is to invest nothing regardless of its competitor's investment choice at this state. Similarly, Condition 3 guarantees it is optimal to invest nothing at state $(1,0;1,0)$.

The only remaining question is what happens at the initial state $(1,0;0,0)$. Because $\pi^B_{1,0;0,1} > \pi^B_{1,0;1,0}$, Firm B clearly prefers to invest in Format 2. The question is how much it invests, and how much (if anything) Firm A also invests in Format 2.

When d is sufficiently large, I will show that only Firm B makes positive investment in the new format. The proof is similar to the proof of Proposition 1. As $d \to \infty$, optimal investment levels approach zero. This implies that Firm i's value function at state X_t approaches $\left(\frac{1}{1-\delta}\right)\pi^i_{X_t}$, and so a firm will only invest in a format if successfully implementing the format increases its profits by at least $\frac{1-\delta}{z}$. Condition 5 guarantees that the profit increase when Firm B moves from state $(1,0;0,0)$ to state $(1,0;0,1)$ exceeds this value, while Condition 3 guarantees that the profit increase for Firm A when it moves from state $(1,0;0,0)$ to state $(1,1;0,0)$ is less than this value. Therefore, the entrant makes positive investment in the new format until it eventually implements this format, while the incumbent makes no investment.

When d is sufficiently small, I will first show that Firm B has a dominant strategy of investing e_{max} in Format 2.

If Firm B invests at time t at state $(1,0;0,0)$, it chooses $e_{B,2,t}$ to maximize

$$-e_{B,2,t} + F(e_{B,2,t})\left[\pi^B_{1,0;0,1} + \delta\overline{\Lambda}^B_{1,0;0,1}\right] + \left[1 - F(e_{B,2,t})\right]\left[\pi^B_{(1,0;0,0)} + \delta\overline{\Lambda}^B_{1,0;0,0}\right] \tag{36}$$

The first derivative of this expression is

$$-1 + F'(e_{B,2,t})\left[\pi^B_{1,0;0,1} + \delta\overline{\Lambda}^B_{1,0;0,1} - \pi^B_{(1,0;0,0)} - \delta\overline{\Lambda}^B_{1,0;0,0}\right] \tag{37}$$

Recall that Firm B generates zero profits at states $(1,0;0,0)$ and $(1,1;0,0)$, and given the conditions of this proposition, it makes no investment at state $(1,1;0,0)$. Therefore, Firm B earns zero profits if Firm A implements the new format and moves the state to $(1,1;0,0)$, and regardless of Firm A's investment level at state $(1,0;0,0)$, it must be true that $\overline{\Lambda}^B_{1,0;0,0} \leq \delta \Lambda^B_{1,0;0,0}$. Therefore, (37) is greater than or equal to

$$-1 + F'(e_{B,2,t}) \left[\pi^B_{1,0;0,1} + \delta \pi^B_{1,0;0,1} + \delta^2 \left(\Lambda^B_{1,0;0,1} - \Lambda^B_{1,0;0,0} \right) \right] \tag{38}$$

I will show that, for d sufficiently small, Firm B invests e_{max}. It suffices to show there is no profitable deviation at any time t. For the proposed investment level, Firm B is guaranteed to implement the new format immediately the first time it invests, which implies $\Lambda^B_{1,0;0,1} - \Lambda^B_{1,0;0,0} = e_{max}$. As $d \to 0$, $e_{max} \to \frac{1}{z}$ and $F'(e_{B,2,t}) \to z$ for $e_{B,2,t} \in [0, e_{max}]$. Inserting these values into (38) yields:

$$-1 + \delta^2 + z \pi^B_{1,0;0,1}(1 + \delta) \tag{39}$$

Condition 5 guarantees that this expression is greater than zero, which implies that (37) must be greater than zero over the interval $[0, e_{max}]$, and it is optimal for Firm B to invest e_{max} at time t. Note that (37) being strictly positive implies investing e_{max} is the unique optimal solution that achieves the value $\Lambda^B_{1,0;0,0} = -e_{max} + \left(\frac{1}{1-\delta} \right) \pi^B_{1,0;0,1}$.

We have shown that, at state $(1,0;0,0)$, Firm B has a dominant strategy of investing e_{max} in the new format, regardless of how much Firm A invests. I now show that Firm A's best response at this state is also to invest e_{max} in the new format. Given Firm B's strategy, if Firm A invests at time t at state $(1,0;0,0)$, it maximizes:

$$-e_{A,2,t} + F(e_{A,2,t}) \left(\frac{1}{1-\delta} \right) \pi^A_{1,1;0,0} + \left(1 - F(e_{A,2,t}) \right) \left[\pi^A_{1,0;0,0} + \left(\frac{\delta}{1-\delta} \right) \pi^A_{1,0;0,1} \right] \tag{40}$$

Differentiating with respect to $e_{A,2,t}$, and noting that as $d \to 0$, $F'(e_{A,2,t}) \to z$ yields the following for $e_{A,2,t} \in [0, e_{max}]$:

$$-1 + z \left[\pi^A_{1,1;0,0} - \pi^A_{1,0;0,0} + \left(\frac{\delta}{1-\delta} \right) \left(\pi^A_{1,1;0,0} - \pi^A_{1,0;0,1} \right) \right] \tag{41}$$

Condition 5 ensures that this expression is positive, so that Firm A's optimal strategy is to set $e_{A,2,t} = e_{max}$. Thus, when d is sufficiently small, each firm invests e_{max} in the new format, and whichever firm moves first implements this format. Firms then make no further investment. QED

Proof of Proposition 4: The proofs that Firm B does not invest in Format 1 at states $(1, 1; 0, 1)$ and $(1, 0; 0, 1)$ are the same as the analogous proofs from Proposition 3. In particular, under Condition 6, the incremental profits to Firm B if it moves from state $(1, 1; 0, 1)$ to state $(1, 1; 1, 1)$ are less than $\frac{1-\delta}{z}$, so it makes no investment at this state. Also, under Condition 6 the incremental profits to Firm B from being at state $(1, 0; 1, 1)$ instead of state $(1, 1; 0, 1)$ are less than $\frac{1-\delta}{z}$, so Firm B invests nothing in Format 1 at state $(1, 0; 0, 1)$, even if it knows Firm A will then immediately implement Format 2.

On the other hand, at state $(1, 0; 0, 1)$, Condition 1 guarantees that the incremental profits to Firm A from implementing Format 2 are greater than $\frac{1-\delta}{z}$, and so Firm A makes positive investment at this state. Similarly, Condition 1 guarantees that Firm A invests in Format 2 at state $(1, 0; 1, 0)$, and that Firm B invests in Format 2 at state $(1, 1; 0, 0)$.

Finally, at least one firm must make positive investment in Format 2 at state $(1, 0; 0, 0)$ because, given the equilibrium investment behavior at subsequent states described above, either firm can guarantee a permanent profit increase of at least $\frac{1-\delta}{z}$ by implementing this format. Also note that Condition 6 guarantees Firm B would never invest in Format 1 (the old format with the fixed expense) at this state because the incremental profits from doing so are less than $\frac{1-\delta}{z}$. QED

Proof of Proposition 5: As shown in the following table, this proposition assumes a different equilibrium profit function than the previous results.

Table 3. Equilibrium Profits When the New Format Eliminates Profits in the Old Format

State	Firm A's profits	Firm B's profits
$(1,0;0,0)$	$(V-\beta)(1-\alpha)$	0
$(1,1;0,0)$	$(V-\beta)(1-\alpha)$	0
$(1,0;0,1)$	0	$(V-\beta)(1-\alpha)$
$(1,0;1,0)$	$\beta(1-\alpha)\frac{1}{2}$	$\beta(1-\alpha)\frac{1}{2}$
$(1,1;0,1)$	$\beta(1-\alpha)\frac{1}{2}$	$\beta(1-\alpha)\frac{1}{2}$
$(1,1;1,0)$	$(V-\beta)(1-\alpha)$	0
$(1,0;1,1)$	0	$(V-\beta)(1-\alpha)$
$(1,1;1,1)$	$\beta(1-\alpha)\frac{1}{2}$	$\beta(1-\alpha)\frac{1}{2}$

The first part of this proof is similar to the proof of Proposition 1. As $d \to \infty$, optimal investment levels approach zero. This implies that Firm i's value function at state X_t approaches $\left(\frac{1}{1-\delta}\right)\pi^i_{X_t}$, and so Firm i makes positive investment in Format j if and only if successfully implementing Format j increases its profits by at least $\frac{1-\delta}{z}$.

Under Condition 1, and using the modified profit functions in Table 3, the profit increase from implementing a new format exceeds $\frac{1-\delta}{z}$ when Firm B invests in Format 2 to move from state $(1,0;0,0)$ to state $(1,0;0,1)$ or from state $(1,1;0,0)$ to state $(1,1;0,1)$, and when Firm A invests in Form 2 to move from state $(1,0;0,1)$ to state $(1,1;0,1)$, but not in any other case. Therefore, the entrant makes positive investment in the new format at the initial state, and with probability one it eventually implements this format; the incumbent then begins investing in the new format until it too eventually implements this format. QED