e-Research: A Journal of Undergraduate Work

Volume 2
Number 2 *Vol 2, No 2 (2011)*

Article 6

September 2014

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The Bernoulli Family: their massive contributions to mathematics and hostility toward each other

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Abstract

Throughout the history of mathematics, there are several individuals with significant contributions. However, if we look at the contribution of a single family in this field, the Bernoulli probably outshines others in terms of both the number of mathematicians it produced and their influence on the development of mathematics. The most outstanding three Bernoulli mathematicians are Jacob I Bernoulli (1654-1705), Johann I Bernoulli (1667-1748), and Daniel Bernoulli (1700-1782), all three of whom were the most influential math experts in the academic community yet very hostile to each other. Their family structure and jealousy toward each other might have fueled their massive contributions to mathematics.

Keywords: Bernoulli family, history of mathematics, Jacob Bernoulli, Johann Bernoulli, Daniel Bernoulli

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Introduction

The Bernoulli family was originally from Holland with strong Calvinism religion. In 1567, Philip, the King of Spain, sent a large army to punish those who were opposed to Spanish rules, enforced adherence to Roman Catholicism, and re-established his authority. To avoid Spanish religious persecution, the Bernoulli fled to Basel in Switzerland, which at that time was a great commercial hub of central Europe (1-2).

Initially, members of the Bernoulli family were successful traders and prospered in business. All of the Bernoulli mathematicians were descendants of Nicholas Bernoulli, a prominent merchant in the spice business in Basel. Despite the non-mathematic family traditions, later members of the Bernoulli turned out to be the most influential math experts in the academic community (5).

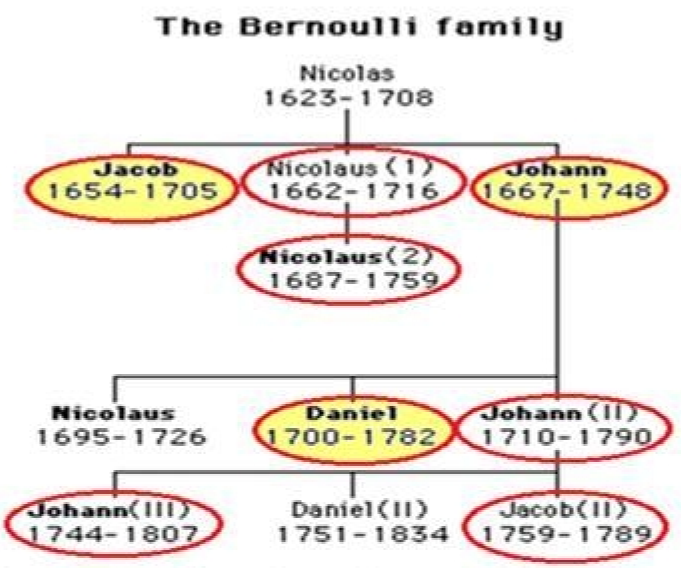


Fig 1. The Bernoulli Family Tree (the ones circled in red are mathematician members)

Within three successive generations, this family produced as many as eight mathematicians who contributed vastly to the mathematic world. Together with Isaac Newton, Gottfried Leibniz, Leonhard Euler, and Joseph Lagrange, the Bernoulli family dominated mathematics and physics in the 17th and 18th centuries, making critical contributions to differential calculus, geometry, mechanics, ballistics, thermodynamics, hydrodynamics, optics, elasticity, magnetism, astronomy, and probability theory. Unfortunately, the elder Bernoullis were as conceited and arrogant as they were brilliant. They engaged in bitter rivalries with one another (7).

Among the eight Bernoulli mathematicians, the most famous and outstanding three were Jacob I Bernoulli (1654-1705), Johann I Bernoulli (1667-1748), and Daniel Bernoulli (1700-1782). In this paper, we will focus our discussion on these three mathematicians (5-8).

Jacob Bernoulli (1654-1705)

Jacob Bernoulli (1654-1705) - Nicholas Bernoulli's oldest son- was the first in the family to be recognized as a strong influential mathematician although there was no family tradition before him. His parents compelled him to study philosophy and theology, which he greatly resented but he did acquire a theology degree in 1676. After taking his theology degree, Bernoulli moved to Geneva where he worked as a tutor. He then travelled to France, spending two years studying with the followers of Descartes. In 1681 Bernoulli travelled to the Netherlands and then to England where, he met several mathematicians, including the famous scientists Robert Hooke (1635-1703) and Robert Boyle (1627-1691). As a result of his travels, Bernoulli developed correspondence with many mathematicians which he carried on over many years. He was intrigued with mathematics and astronomy so much he grew interest in these fields regardless of his father's opposition. Jacob Bernoulli returned to Switzerland and taught mechanics at the University of Basel since 1683. Although his degree was in theology and he was offered an appointment in the Church, he turned it down. Bernoulli's real love was for mathematics and theoretical physics and it was in these topics that he taught and researched (2-5)

Jacob Bernoulli published five treatises on infinite series between 1682 and 1704. The first two of these contained many results, such as fundamental result that S(1/n) diverges, which Bernoulli believed was new but they had actually been proved by Mengoli 40 years earlier.

In May 1690, in a paper published in Acta Eruditorum (the oldest scientific journal that began appearing since 1682 under the supervision of Gottfried Leibniz (1646-1716)), Jacob Bernoulli showed that the problem of determining the isochrone is equivalent to solving a first-order nonlinear differential equation. The isochrone, also called curve of constant descent, is the curve along which a particle will descend under gravity from any point to the bottom in exactly the same time no matter where it starts on the curve. It had been studied by Huygens in 1687 and Leibniz in 1689. Also in this paper, Jacob Bernoulli was the first mathematician to use the term "integral calculus". Earlier, Leibniz had designated it by "Calculus summatorium" (8).

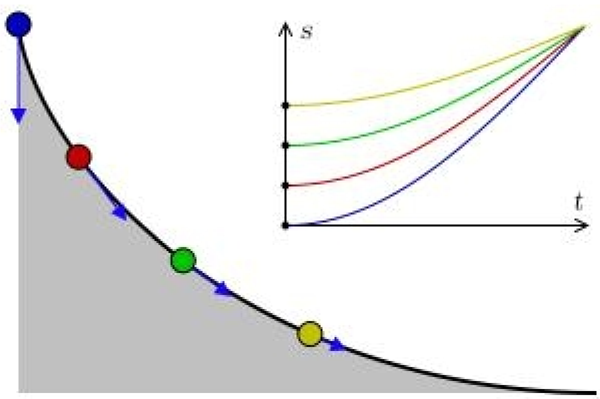


Fig 2. The isochrone

After finding the differential equation, Bernoulli then solved it by what we now call separation of variables. Jacob Bernoulli's paper of 1690 is important for the history of calculus, since that is where the term "integral" appears for the first time with its integration meaning. In 1696, Jacob Bernoulli solved the equation now called "the Bernoulli equation":

$$y' = p(x)y + q(x)y^n$$

Jacob used differential equations in tackling various geometrical and mechanical problems (4).

The Catenary problem was also one of the problems which generated a lot of enthusiasm among mathematicians after the invention of calculus. The word "catenary" comes from the Latin word "catena," which means chain. In the May 1690 issue of "Acta Eruditorum", Jacob wrote "And now let this problem be proposed: To find the curve assumed by a loose string hung freely from two fixed points". Earlier, Galileo (1564-1642) had dealt with this problem and wrongly concluded that the curve would be a parabola. In 1646, Christian Huygens (1629-1695), at the age of sixteen, proved that the curve cannot be a parabola. One year after the appearance of the problem, in the June 1691 issue of "Acta Eruditorum," three correct solutions were published. The three solutions came from Huygens, Leibniz, and Johannn Bernoulli (Jacob's younger brother). Although they approached the problem from three different points of view, all of them concluded that the curve would be a catenary. Jacob himself failed to solve the problem. The Cartesian equation of a catenary is:

$$y = \frac{e^{ax} + e^{-ax}}{2a}$$

Where, *a* is a constant whose value depends on the mass per unit length of the chain and the tension of suspension (9).

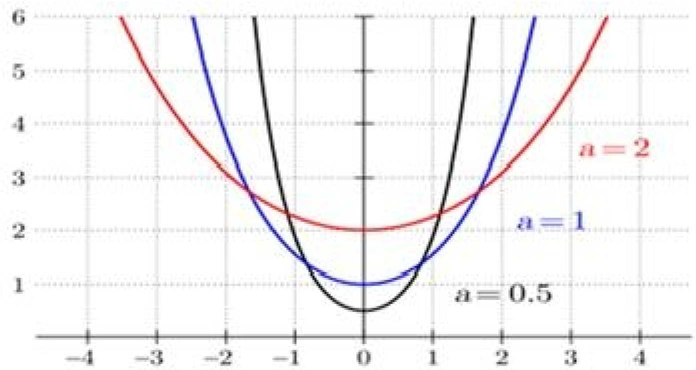


Fig 3. The catenary

Hung chain Spider web Gateway Arch

Fig 4. Examples of the catenary in real life

The study of the catenary problem was soon applied in the building and designing of suspension bridges.

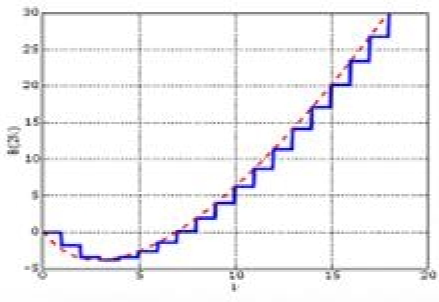
Jacob Bernoulli was one of the pioneers of the mathematical theory of probability. His first paper on probability theory was published in 1685. The greatest contribution of Jacob Bernoulli is *Ars Conjectandi* (published posthumously, 6 years after his death in 1713; also called "The Art of Conjecturing") contained many of his finest concepts: his theory of permutations and combinations; the so-called Bernoulli numbers, by which he derived the exponential series; his treatment of mathematical and moral predictability; and the subject of probabilitycontaining what is now called the Bernoulli law of large numbers, basic to all modern sampling theory (6-8).

$$\textbf{Bernoulli numbers: } B_m(n) = \sum_{k=0}^{m} \sum_{v=0}^{k} (-1)^v \binom{k}{v} \frac{(n+v)^m}{k+1},$$

The first few are:

B_0	=	1
B_1	=	$-\frac{1}{2}$
B_2	=	$\frac{1}{6}$
B_4	=	$-\frac{1}{30}$
B_6	=	$\frac{1}{42}$
B_8	=	$-\frac{1}{30}$
B_{10}	=	$\frac{5}{66}$
B_{12}	=	$-\frac{691}{2730}$
B_{14}	=	$\frac{7}{6}$
B_{16}	=	$-\frac{3617}{510}$
B_{18}	=	$\frac{43867}{798}$
B_{20}	=	$-\frac{174611}{330}$
B_{22}	=	$\frac{854513}{138}$

When graphed, they create a distinctive curve:



The numbers were discovered around the same time at the start of the 18th Century by the Swiss mathematician Jakob Bernoulli and the Japanese mathematician Seki Kowa

Fig 5. The Bernoulli numbers are a sequence of rational numbers with deep connections to number theory.

In 1689, Jacob Bernoulli published many research papers on the theory of infinite series. He was the first to think about the convergence of an infinite series and proved that the series

$$\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots$$

is convergent. He was also the first to propose continuously compounded interest, which led him to investigate:

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n.$$

Using the binomial expansion of

$$\left(1+\frac{1}{n}\right)^n,$$

he showed that this limit lies between 2 and 3, although the value of the limit would need to wait until Euler and it is equal to e.

Jacob also showed special interest in the logarithmic spiral, which was introduced in 1637 by Rene Descartes (1596-1650). At that time, the polar equation of the curve was written as

$$r = e^{aq}$$

Where *q* is measured in radians and *a* denotes the rate of increase of the spiral.

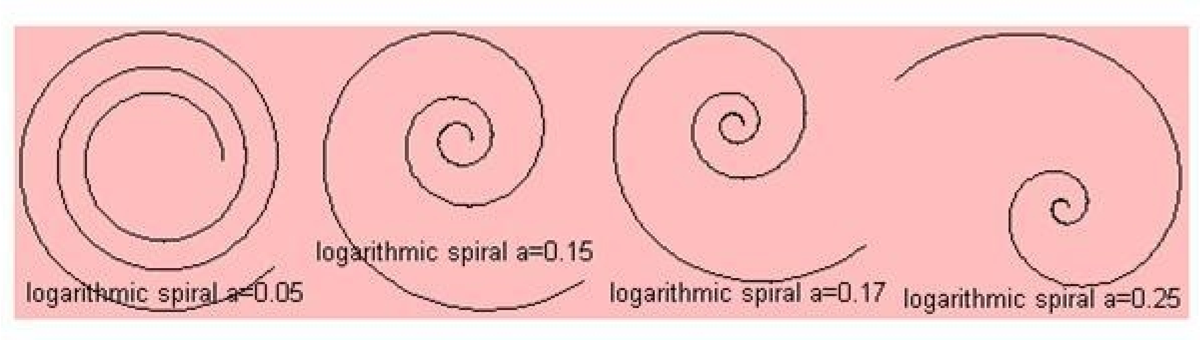


Fig 6. The logarithmic spiral

The property of the logarithmic spiral which surprised Jacob Bernoulli the most is that it remained unaltered under many geometrical transformations. For example, typically a curve suffers a drastic change under inversion but a logarithmic spiral generates another logarithmic spiral under inversion, which is a mirror image of the original spiral. The evolute, pedal, and caustic of a logarithmic spiral are the very same spiral. Jacob Bernoulli was so astonished by the "magic" of the logarithmic spiral that he requested to have a figure of a logarithmic spiral engraved on his tombstone with the words *Eadem mutata resurgo ("Changed and yet the same, I rise again")* written under it. He wrote that the logarithmic spiral 'may be used as a symbol, either of fortitude and constancy in adversity, or of the human body, which after all its changes, even after death, will be restored to its exact and perfect self." However, unfortunately, possibly from ignorance, the artisan engraved an Archimedian spiral instead (7-8).

Fig 7. Jacob Bernoulli's gravestone

Jacob Bernoulli became the chair of mathematics at Basel in 1695 and held the position until his death in 1705 when the position was taken over by his younger brother, Johann I Bernoulli.

Johann I Bernoulli (1667-1748):

Johann Bernoulli was Jacob Bernoulli's younger brother. He was a brilliant mathematician who made important discoveries in the field of calculus. His father, Nicholas, again provided Johann with a well-rounded education so that he could eventually enter the family business. However, Johann had no desire to work in the spice industry, and enrolled in the University of Basel in 1683 to study medicine.

He asked his brother, Jacob, to teach him mathematics. By this time, Jacob, who was twelve years older than Johann, was a professor of mathematics at the University of Basel. With Jacob's help, Johann studied Leibniz's papers on calculus until he became proficient, and his mathematical prowess soon matched that of his brother's (2-5*)*.

At the time, calculus was a difficult subject to understand. The Bernoulli brothers were among the few mathematicians in Europe to understand calculus. At first, Jacob had no problem teaching his little brother. He realized his brother's talents and quick-study of mathematics so he offered to work with Johann. The cooperation between the two brothers soon degenerated, however, into vitriolic arguments. As time went on, Johann's ego was getting larger. He began to brag about his work and at the same time belittled his brother. Jacob was so angry he made crude comments about Johann's abilities, referring to him as a student repeating what the teacher taught him, in other words, a parrot. Jacob and Johann went back and forth with insulting comments about each other in the academic community which developed a notorious reputation of their family togetherness (2-7).

Despite family problems, Johann was an excellent mathematician. Johann was perhaps even more productive as a scientist than was Jacob. He studied the theory of differential equations, discovered fundamental principles of mechanics, and the laws of optics. He used calculus to solve problems which Newton failed to solve regarding the laws of gravitation. Johann also did lots of important work on the study of the equation

$$y = x^x.$$

He discovered the Bernoulli series and made advances in theory of navigation and ship sailing. In addition, he is famous for his tremendous letter writing (over 2500 messages) and his tutoring of another great mathematician, Leonhard Euler (9).

Johann proposed the so-called brachistrochrone problem, the curve of most rapid descent of a particle sliding from one point to another under the influence of gravity. Johann invited the "shrewdest mathematicians all over the world" to solve this problem and allowed six months time for it. Five correct solutions were received. Of these, four came from renowned mathematicians, L'Hôpital, Leibniz, Jacob and Johann himself and one from an anonymous mathematician. However, seeing the style of tackling the problem, experts could recognize that the anonymous solver was no one but Sir Isaac Newton and Johann commented on it "Ex ungue Leonem" (Tell a lion by its claw). Newton later admitted that he solved the problem by thinking relentlessly for 12 hours over it. The answer to this problem is a fragment of an inverted cycloid. A cycloid is the curve traced by a fixed point on the perimeter of a wheel rolling along a straight line (2-3).

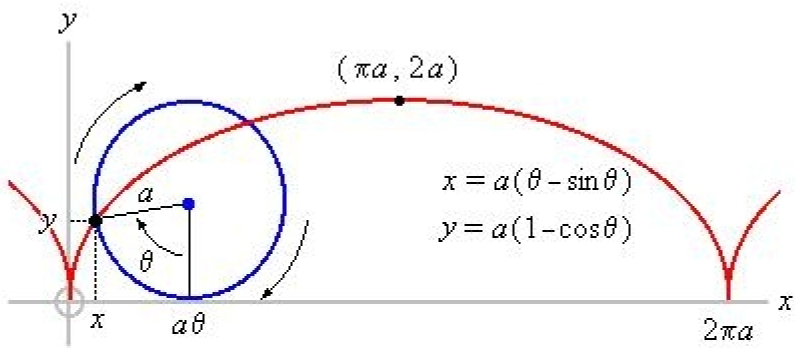


Fig 8. A cycloid is the locus of a point P on a circle as the circle rolls along a line

Johann's method of solving the brachistochrone problem was revolutionary in that it formed the basis for a new branch of calculus, calculus of variations. Calculus of variations, a generalization of calculus, deals with extremizing functionals, as opposed to ordinary calculus which deals with functions. In 1673, twenty-three years after the brachistotone problem, Huygens discovered that the cycloid is also the solution of the "tautochrone problem". In the "tautochrone problem," it is required to find a curve so that a particle, moving under gravity, will reach a given point on the curve in equal time irrespective of its initial position on the curve.

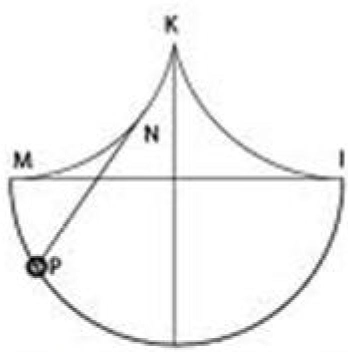


Fig 9. The tautochrone curve

Knowing that the cycloid is the solution of both the "brachistotone problem" and the "tautochrone problem," Johann was moved to write: "But you will be petrified with astonishment when I say that exactly this same cycloid, the tautochrone of Huygens, is the brachistotone we are seeking" (2).

In 1691, Johann was introduced to Marquis de L'Hôpital, a young mathematician with a keen interest in calculus, who asked Johann if he would give him lectures on the new integral calculus and the differential calculus of which he had heard very little about. L'Hôpital had heard little about the calculus due to the war going on in Europe at the time, which slowed the transmission of information dramatically.

While tutoring L'Hôpital, Johann signed an arrangement saying that he would send all of his discoveries to L'Hôpital to do what he wished, in return for a regular salary. This arrangement resulted in one of Johann's biggest contributions to the calculus being known as L'Hôpital's rule. Hence, although Johann "discovered" this relationship, it was L'Hôpital who got the credit for it (*7).*

Johann Bernoulli also used his position of authority to act as an advocate for Leibniz, who was embroiled in a battle with Isaac Newton over ownership of the credit for inventing calculus. There was a debate in mathematical circles about who the true inventor was, and whether Leibniz plagiarized Newton's work, and vice-versa. Bernoulli publicly criticized Newton's calculus in scientific journals, and demonstrated that Leibniz's version of calculus, which used differential notation, was superior by using it to solve problems that Newton could not. Bernoulli's influence was so great that Leibniz's methods became the first choice in Europe (5-6).

Johann wanted the chair of mathematics at University of Basel which was held by his brother but he was unable to get it so Johann accepted a chair at the University of Groningen in Netherland. He vowed not to come back to Basel but in 1705, Johann's father-in-law was dying and asking for his daughter and grandchildren, so Johann came back to Basel. While traveling, he did not know his brother, Jacob, died of tuberculosis. Once he realized his brother's death, Johann took over his position. During his life, Johann was awarded many honors including membership at the Academies of Science at Paris, Berlin, St. Petersburg, and many others. He passed away in Basel on the first day of the year 1748 (8-9).

Daniel Bernoulli (1700-1782):

Daniel Bernoulli was Johann Bernoulli's second son. He was born in Groningen, Netherlands, on February 8, 1700. As his own father did to him, Johann Bernoulli did not want his son to enter the field of mathematics. Johann Bernoulli tried to convince Daniel that there is no money in mathematics and tried to place Daniel in an apprenticeship with a merchant, but Daniel resisted. Daniel began studying logic and philosophy at the University of Basel when he was thirteen and received his bachelor's degree two years later. In the following year, he obtained his master's degree. His father eventually allowed Daniel to study medicine, and for the next few years, Daniel studied in Heidelberg, Germany, and Strasbourg, France (1-4).

In the early 1720s, Bernoulli traveled to Venice to practice medicine. He also hoped to further his studies at the University of Padua, but he became too ill to follow this plan. Instead, he worked on mathematics, producing his first publication, "Mathematical Exercises," in 1724. Within a year, Bernoulli had also designed an hourglass for use at sea. The trickle of sand had to be constant, no matter how violently the ship might move, even in a storm. At the age of twenty-five, he submitted this work to the annual contest of the Paris Academy and won the grand prize.

As a result of this prize and the fame resulting from "Mathematical Exercises," Bernoulli was invited by Catherine I of Russia, widow of Peter the Great, to accept the chair of mathematics at the Imperial Academy in St. Petersburg. His brother, Nicolaus, was also offered a chair in mathematics at St. Petersburg at the same time (6).

Here, the brothers began to consider a game in which a coin is tossed "n" times until the first "heads" turns up. If a head occurs for the first time on the *n*th toss then you will be paid 2^n dollars. How much would you be willing to pay to play this game? Although the expected value of the payoff is infinite as:

$$\begin{aligned} E &= \frac{1}{2}\cdot 2 + \frac{1}{4}\cdot 4 + \frac{1}{8}\cdot 8 + \cdots \\ &= 1 + 1 + 1 + \cdots \\ &= \infty . \end{aligned}$$

Of course, no one seems to be willing to pay this sum to play this game. So, Daniel Bernoulli's resolution of the paradox was to suppose that utility is not linear in the payoff but instead, strictly concave, e.g., ln(*x*). What certain payoff, *c*, would an individual with such a utility function regard as equivalent to the St. Petersburg lottery? This *c* would have to satisfy:

$$\begin{aligned}\ln(c) &= \sum_{i=1}^{\infty} (1/2)^i \ln(2^i) \\ &= \left(\sum_{i=1}^{\infty} (i/2^i)\right)\ln(2) \\ &= 2\ln(2) \\ &= \ln(4)\end{aligned}$$

And thus, $c = 4$ a rather modest value to place upon the infinite expectation. This is became known as the St. Petersburg paradox.

Tragically, Nicolaus died within a few months of the brothers' arrival in St. Petersburg so Johann Bernoulli arranged for his best student and also Daniel Bernoulli's childhood friend, Leonhard Euler to come help him. The next few years would be the most productive period of Bernoulli's life. While studying vibrating systems, he defined the simple nodes and frequencies of a system's oscillation, and demonstrated that the strings of musical instruments move in an infinite number of harmonic vibrations all along the string (3-7).

Between 1725 and 1749 Daniel won 10 prizes from the Paris Academy of Sciences for work on astronomy, gravity, tides, magnetism, ocean currents, and the behavior of ships at sea. He also made substantial contributions in probability.

The relationship between Johann and Daniel became strained when, in 1734, they both competed for the Grand Prize of the Paris Academy, a school of mathematical and physical sciences. They both won, and the thought of sharing the prize with his own son sent Johann into a rage. He banned Daniel from his house.

The relationship was damaged further when Johann Bernoulli published a piece on hydrodynamics in 1739 entitled, "Hydraulica" whereas one year earlier, Daniel published "Hydrodynamica," a very similar work. Johann blatantly plagiarized "Hydrodynamica" in his publication of "Hydraulica," and even made it seem as if his work was published first by backdating it to 1732. The plagiarism, however, was soon uncovered.

In his later years, Daniel Bernoulli continued to work in calculus, mechanics, conservation of energy, oscillation, and the movement of bodies in a resisting medium. He was a member of the London Royal Society; the Paris Academy of Sciences, the Berlin Academy, and the St. Petersburg Academy of Sciences. Bernoulli died in Basel on March 17, 1782, just a few weeks before his eighty-second birthday (8-9).

Conclusion:

The Bernoulli's competitive and combative family relationship is usually said to curtail their collaboration and hold back potential success which could have happened had they not been so ruthless to each other. However, perhaps their resentment and enviousness also challenged and inspired each other, resulting in their outstanding mathematic work. Had Jacob not been so excellent in math and at the same time, so envious of his brother, Johann would not have the same pressure and motivation to thrive and surpass Jacob and vice versa. The same argument also applies for Johann and his son, Daniel. Despite their jealousy and hostility toward each other, the Bernoulli family is undoubtedly the best mathematician family in terms of the number of mathematical geniuses it produced and the significant impact of their contributions to the academic community.

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