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Sectoral Changes and the Increase in Women's Labor Force Participation[*]

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December 2007

Abstract

Throughout the second half of the 20th century, women in the U.S. decided to move increasingly into the labor market. This paper investigates the growth of the service sector as an explanation for the increase in women's employment. It develops an economic model that can account for the increase in women's employment and the growth of the service sector at the same time. A growth model with three sectors and a home production technology is constructed in order to quantitatively assess the contribution of sectoral productivity differences to the change in women's employment decision. The model parameters are calibrated to match time allocations in 1950, and sectoral productivities are taken from the data. This model demonstrates that a higher rate of productivity growth in market services compared to home services can account for a large fraction of the observed increase in women's employment from 1950 to 1990.

1   Introduction

The significant takeoff in the growth rate of women's employment throughout the second half of the 20th century has received much attention in economic and other social research. In the United Sates, the percent of women employed in the market increased from around 19% in 1950 to 39% in 1990.[1] This increase was mainly due to the rising participation rate of married women, causing a dramatic change in women's role in the household.

Another well known development during this period was the expansion of the service sector. Total service sector employment increased from 58% in 1950 to 75% in 1990. Both the labor force participation of women and the service sector's share of employment took off in 1950s. The growth of the service sector is one of many competing explanations for the growth in women's labor force participation, but the relationship has not been carefully modeled.[2] This paper explores the extent to which increases in the productivity of service provision in the market sector relative to home production can account for the magnitude of the increase in women's labor force participation.

The hypothesis here is that the type of work done at home is similar to a lot of services provided in the market. Some examples are child care, care for the elderly, care for the sick, filling out IRS forms, carrying out finances, teaching etc.[3] This paper conjectures that, as the production of these services in the market becomes relatively more efficient, a shift from home production to market production is observed. Since it is the women who work more at home, an increase in their market employment is observed due to the movement from unmeasured to measured work.

Several papers have examined the economic mechanism that moved women into the labor market. Caucutt et al. (2002) find that increasing the returns to labor-market experience for women results in higher labor supply for young women, and a delay in the timing of fertility. According to Jones et al. (2003), small decreases in the wage gap between men and women can explain the significant increase in average hours worked by married women and the relative constancy of hours worked by single women, and by single and married men for the time period between 1950 and 1990. Olivetti (2006) finds that a shift in women's wage level cannot account for the increase in labor supply of married women, while an increase in the return to experience can.

Others have examined changes in preferences and attitudes as driving forces of the increasing market employment of women. Fernández et al. (2004) point to changes in male preferences as a possible explanation. In their model, sons of working mothers prefer

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[1] Goldin (1990) documents that the number of women in the paid labor force has advanced steadily in the U.S. since the early 1800's but that the increase has been rapid from World War II on.

[2] Earlier studies by Fuchs (1978, 1980) talk about the correlation between the growth of the service sector and the increasing labor force participation of women. Becker (1985) also mentions the rapid growth of the services sector as one of the major causes of the increased participation of married women. Goldin (1990) points to the expansion of the service sector, and with it the increase in white collar jobs, as a possible explanation for the higher participation of women in market work.

[3] The idea that services produced at home have their counterparts in the market was also expressed by Eisner (1988).

working wives, hence more and more women decide to work. Fogli et al. (2007) argue that as uncertainty regarding the effect of maternal employment on children is reduced, more and more women enter the labor force. In their framework, each generation updates their parents' beliefs by observing the children of employed women.

This paper provides an alternative explanation that is not inconsistent with most of the explanations in the literature. There is no doubt that women's decision to work will be effected by more than one factor, but this paper relies on only one particular exogenous change, namely the relative change in sectoral productivities, and explores to what extend it can explain changes in women's labor force participation. In particular, it is argued that the main factor moving women from their homes into the market is not very different from those that moved men from their homes into the market but much earlier, namely changes in productivity. Men formerly worked on their own farms or in the agricultural sector as workers but with industrialization most of them moved into the manufacturing sector. In the 20th century, for similar reasons, women "quit" their jobs at home and start working in the service sector.[4] This paper does not attempt to explain why productivities of different sectors changed at different rates.

There are other studies that focus on technological change as a factor influencing women's decision to participate in market work. Greenwood et al. (2005) argue that it is the decrease in the price of household durables that liberates women from the home and results in a substantial increase in married women's labor force participation. So, it is the increase in home production productivity due to higher availability of home appliances that moves women out of their homes. Although I do not argue that productivity at home did not increase, I do argue that it is the higher productivity growth in market services compared to home services that move women into the labor market. In their theoretical work, Galor and Weil (1996) argue that with economic development the nature of jobs change in favor of women. Skill-biased technological advances in market production move women from their homes into the market.[5] My paper complements that literature. The argument here is that relative productivity changes lead to the increase of the service sector, a sector where women have a comparative advantage.

Although more and more women as well as men enter service occupations, the data clearly documents women's higher tendency to enter this sector. In the U. S., roughly 5 out of 6 working women are employed in the service sector. This ratio is almost stable from 1950 to 1990. Data on occupational choice gives us a better idea about what type of jobs women mostly take. Table A1 in the Appendix shows the 20 leading occupations of women in 2006. 44% of working women ages 16 and older are employed in these 20 categories. A conservative classification shows that at least 72% of these occupations are

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[4]Locay (1990) argues that "the reason why men made the move toward market production before women is that presumably women have a comparative advantage in the later stages of production..," and service industries are likely to involve these stages of production.

[5]In their model, men and women have equal endowments of mental input (brains), but men have more physical strength (brawn) than women. Capital and brains are complements in the market production function. As the capital stock rises with economic development, so does the demand for brains. Tam (2003) uses a similar model.

in the service sector.[6] This paper takes no stand on why women work more in one sector compared to another, or why they work more at home.

A growth model with three sectors (agriculture, industry, services) and a home production technology is developed in order to quantitatively assess the contribution of sectoral productivity differences to the change in women's employment decision. This model extends Rogerson's (2005) model of structural transformation by incorporating gender specific labor decisions, to allow for gender specific changes in economic activity. It is assumed that women can work at home and in market services. Men on the other hand are allowed to work in agriculture, industry (manufacturing), and market services. These are extreme but reasonable assumptions. First, throughout the period considered in this paper, the fraction of women in agriculture and in industry is low and constant, and the main interest of this paper is the change in women's market employment, not its level.[7] Second, the data shows that although men do spend time for home production, that time is considerably less compared to women.

The framework of the model is such that non-homotheticities are central to the reduction of employment in agriculture.[8] The movement of resources from industry into services, and from home produced to market produced services, on the other hand, are due to differences in sectoral labor productivities. Given that manufacturing and service goods are poor substitutes, the relatively higher productivity growth in industry leads to a shift in employment from industry into services. In the presence of a relatively higher productivity growth in market services compared to home services, the high substitutability of home and market produced services allows for a movement of labor from the home into the service sector.[9] In this model, it is this latter mechanism that explains the increase in women's market employment.

The parameters of the model are calibrated to match time allocations to the different sectors in 1950, and the manufacturing sector in 1990. One of the parameters in the model is chosen such that it implies a high substitutability of services produced at home with those produced in the market. All sectoral productivities in the model are exogenous. Productivities, namely labor productivities, for the market sectors are constructed using sectoral GDP data from the NIPA tables, and sectoral employment data. As a proxy for productivity in the home sector, activities in the market that are similar to those at home

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[6]Secretaries and administrative assistants, bookkeeping, accounting and auditing clerks, customer service representatives, first-line supervisory/managers of office and administrative support, and office clerks are not included in the 72%, since these occupations could be in any sector. But notice that a big part of these occupations might be in the service sector.

[7]Historically, women used to work in agriculture too. This is not inconsistent with the story in this paper. The model here could be modified to capture a longer period. The way to incorporate this into the current framework would be to assume that market produced agricultural goods and home produced agricultural goods are substitutable.

[8]So, when the economy becomes richer, the share of agriculture declines. For a more detailed discussion of this approach see Kongsamut et al. (2001).

[9]An earlier example of this type of model is provided by Baumol (1967). For a more recent application see Ngai and Pissarides (2004). Ngai and Pissarides (2007) quantify the changing allocation of time between the home, agriculture, manufacturing, and service sectors along this line, but they do not explicitly model gender.

are identified. In particular, the labor productivity of a subcategory of the aggregate service sector called "services" is used as the productivity series for non-market services. Given the data on relative changes in sectoral productivities, the paper examines how gender specific time allocations change over time.

The results indicate that given a higher rate of productivity growth in market services compared to non-market (home) services, and a high degree of substitutability between these two, the model can account for a large fraction of the observed increase in women's service sector employment from 1950 to 1990. Although this model restricts women's market activity to the service sector, it is successful in accounting for the increase in women's total employment rate observed in the data. The model underpredicts the increase in men's service sector employment, but can account for most of the increase in the rate of total employment in market services over the whole period. Consistent with the data, the model predicts a decrease in the time a household allocates to home production.[10]

The rest of the paper is organized as follows. Facts in the U.S. as well as international data are documented in section 2. Section 3 outlines the model. Calibration of the model is discussed in section 4. Quantitative results of the model are presented in section 5. A sensitivity analysis is conducted in section 6. Section 7 concludes the paper. Appendix A includes additional tables and figures. Appendix B provides some computational details.

2   Data Analysis

This section documents the structural changes that took place in the U.S. economy from 1950 to 1990, namely changes in sectoral shares in GDP as well as in employment. Throughout the paper, sectoral data are aggregated into three main categories: agriculture, industry, and services.[11] Further, this section provides data on the changes in gender specific employment rates. To show the link between the service sector and female employment, and hence to motivate the paper, data on gender specific sectoral changes in employment rates are also presented.[12]

A summary of sectoral GDP in constant 1977 prices is presented in Table 1. The data for this table are taken from the National Income and Product Accounts provided by the Bureau of Economic Analysis (BEA).[13] The share of agriculture decreased from

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[10]For data on this, see Juster and Stafford (1991).

[11]Agriculture includes agriculture, forestry, and fishing. Industry includes mining, construction, manufacturing, electric, gas, and sanitary services. Services include transportation, communications, wholesale trade, retail trade, finance, insurance, and real estate, services, and government.

[12]The reason I am not conducting this analysis for a longer period is the lack of consistent gender and sector specific data.

[13]Nominal GDP data and two different price index series for two different parts of the period covered in this paper were used to compute Real GDP by sector. Chained type price indexes were available starting from 1977. The NIPA Statistical Tables provide enough information to compute GDP deflator starting from 1950 and going up to 1977.

7.3% in 1950 to 3.5% in 1990. During the same period the share of industry declined from 41.8% to 34%. The share of services on the other hand increased by almost 23%. In 1950 roughly 51% of GDP originated from services. In 1990 this number increased to 62.5%.[14]

Table 1: Sectoral Shares of GDP (constant 1977)

Year	Agriculture	Industry	Services
1950	7.32	41.80	50.88
1960	5.40	41.91	52.70
1970	3.37	40.21	56.01
1980	2.81	36.32	60.88
1990	3.52	33.98	62.50
% Δ 1950-90	-51.96	-18.71	22.85

Table 2 summarizes the share of the three sectors in total employment. The data used for this table are also taken from the BEA. Data on full-time and part-time employees by detailed sectors are available. In this paper, I define employment as full-time employees + 0.5 * part-time employees. The share of service sector employment increased at the expense of employment in industry and agriculture. The employment share of agriculture decreased by 65% from 1950 to 1990. For industry it decreased by 39%. In 1950, 58% of the employed population was in the service sector. This number increased to 75% in 1990.

Table 2: Sectoral Shares in Total Employment

Year	Agriculture	Industry	Services
1950	4.52	37.79	57.69
1960	3.22	34.59	62.19
1970	1.87	31.86	66.27
1980	1.86	28.36	69.78
1990	1.57	23.11	75.32
% Δ 1950-90	-65.27	-38.85	30.57

The BEA does not provide gender specific, and more importantly gender and sector specific data on employment. Data on the economically active population for the years 1950, 1960, 1970, 1980, and 1990 are taken from LABORSTA, a Labor Statistics Database provided by the International Labor Organization (ILO).[15] Sector and gender specific employment data are then constructed by applying the ratios of economically active men

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[14]The numbers in this table are for private industries only. So, government is not included in services.

[15]The economically active population comprises all persons of either sex who furnish the supply of labor for the production of goods and services during a specified time-reference period. We can think of it as the labor force.

and women in agriculture, industry, and services to the sectoral employment data from the BEA.[16]

Table 3 provides information on the total and gender specific rate of employment from 1950 to 1990.[17] During this period, the rate of total employment increased by 37%. Changes were very different for men and women. Male employment increased by only 9% from 1950 to 1990, whereas women's employment more than doubled during these 40 years. In 1990, almost 40% of all women were employed in the market. Although this number is still lower than that for men, the gap is closing rapidly.[18]

Table 3: Rate of Employment by Gender

Employment - to - Population Ratio			
Year	Total	Men	Women
1950	0.32	0.45	0.19
1960	0.32	0.43	0.21
1970	0.36	0.46	0.26
1980	0.40	0.47	0.33
1990	0.44	0.49	0.39
% Δ 1950-90	36.88	8.62	102.83

Figure 1 shows the fraction of all women and all men who are employed in services, industry, or agriculture. The first thing we observe is that both men and women increase their participation in the service sector relative to the other sectors. For women the rate of increase is higher. We can see that the increase for men is mainly coming from their decreasing share of employment in industry and agriculture. On the other hand both the percentage of women who work in industry and the already low share of women in agriculture, stay almost constant throughout the period. These observations suggest that the increase in women's service sector employment might be driven by a move from home

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[16]The Integrated Public Use Microdata Series (IPUMS) provides gender and occupation specific data, but in terms of comparability occupation is a problematic variable. Classifications change considerably over time. Although certain data sets are adjusted for these differences I decided not to use those with the belief that the numbers in this paper might be used for international comparisons. Adjustments would be even more problematic with cross-country data.

[17]The results in this paper would not change much, if I would use working age population (ages 15-64) instead of total population. If working population is used, the increase in women's employment rate becomes 100.84% instead of 102.83%. This would mean that the model needs to explain a slightly lower rise in female employment. I decided not to use working age population because different sectors might be different in terms of the number of people who continue to work past their retirement age. Also, there might be international differences in defining working age population. If so, the numbers in this paper, as they are, are better in terms of comparability.

[18]McGrattan and Rogerson (1998) provide a unique look at how the lifetime pattern of work hours has changed since 1950 for different demographic groups. Based on data collected by the U.S. Census Bureau during the 1950-1990 decennial censuses, weekly hours worked have decreased by 14.7 percent for males and increased by 74.3 percent for females. The numbers we find are somewhat different, since McGrattan and Rogerson use weekly hours of work per person, and we use employment data, and hours worked are higher in industry and lower in services.
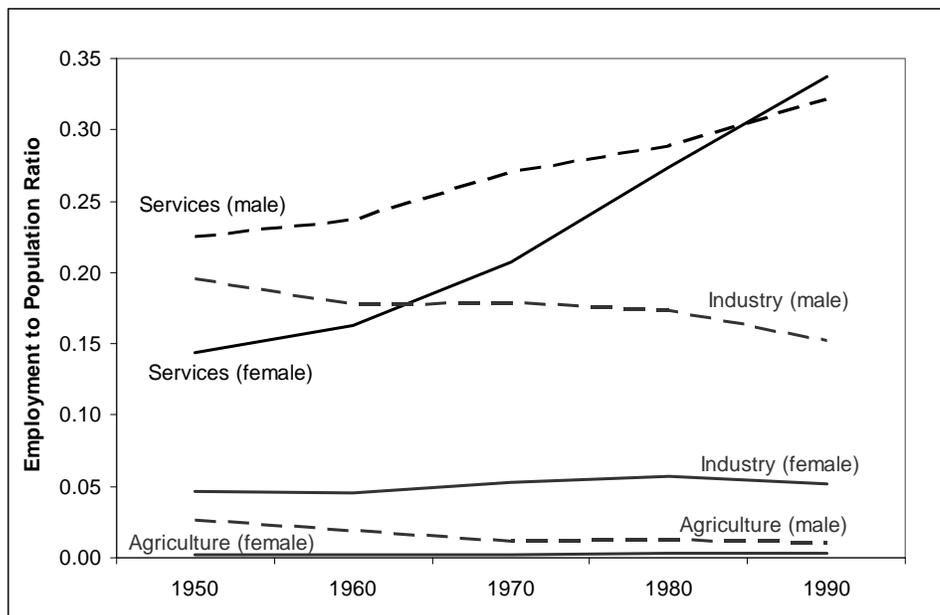
Figure 1: Gender Specific Sectoral Change in Employment

to market production. This conjecture is supported by time use surveys that show that the share of discretionary time used for home work is declining over time. This will be the main motivation for the model choice in this paper.

The increase in the service sector's share in employment and women's labor force participation is not unique to the United States. Figure 2 displays the correlation between the increases in these two variables over the 1950-1990 period for 23 countries. The figure reveals that in 20 of the 23 countries being analyzed, the service sector's share in employment and women's labor force participation rate have a very high positive correlation, higher than 0.80, for the period between 1950 and 1990. One of the exceptions is Chile with a somewhat lower correlation, namely 0.53. The other two are Austria and Turkey with correlations of -0.22 and -0.94 respectively. So the general picture is that from 1950 to 1990 the service sector is expanding and more and more women are moving into the labor market in many countries.[19]

The correlation between the service sector's share in employment and women's labor force participation rate for the whole sample is 0.39. If Austria and Turkey are excluded it increases to 0.63. Figure 3 is a scatter diagram for the whole sample. [20]

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[19] For a more detailed cross-country discussion of sector and gender specific labor force data see Akbulut (2006).

[20] Some of the 23 countries in this sample are still highly agrarian in 1950. The data seems to indicate that manufacturing is more male intensive than agriculture, and women's market employment is higher in agrarian countries compared to countries where manufacturing's share in employment is high. This in turn suggests that in countries where the service sector's share in employment increases at the expense of the agricultural sector, women move from one measured type of work to another and hence no jump is observed in their market employment. In countries where the expansion of the service sector comes
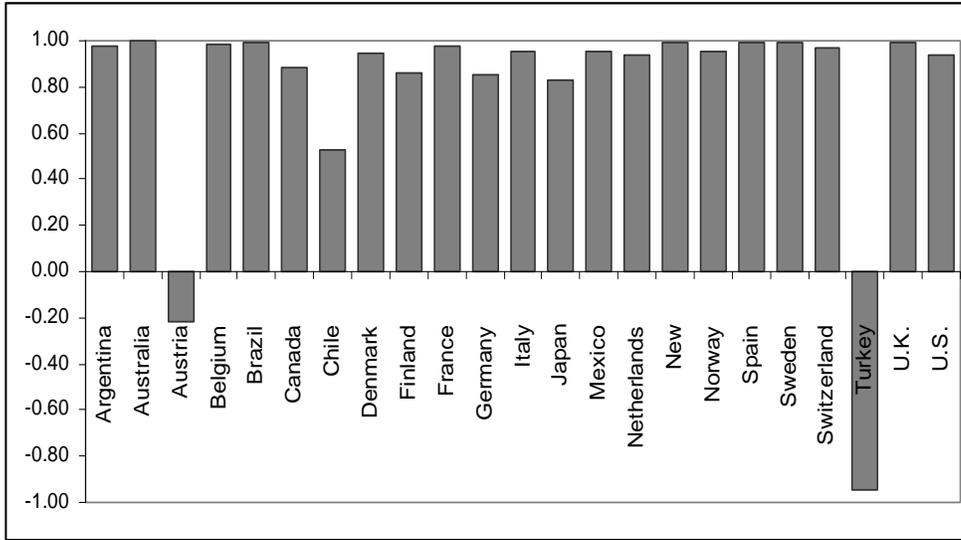
Figure 2: Correlation - Share of Services and Women's Labor Force Participation
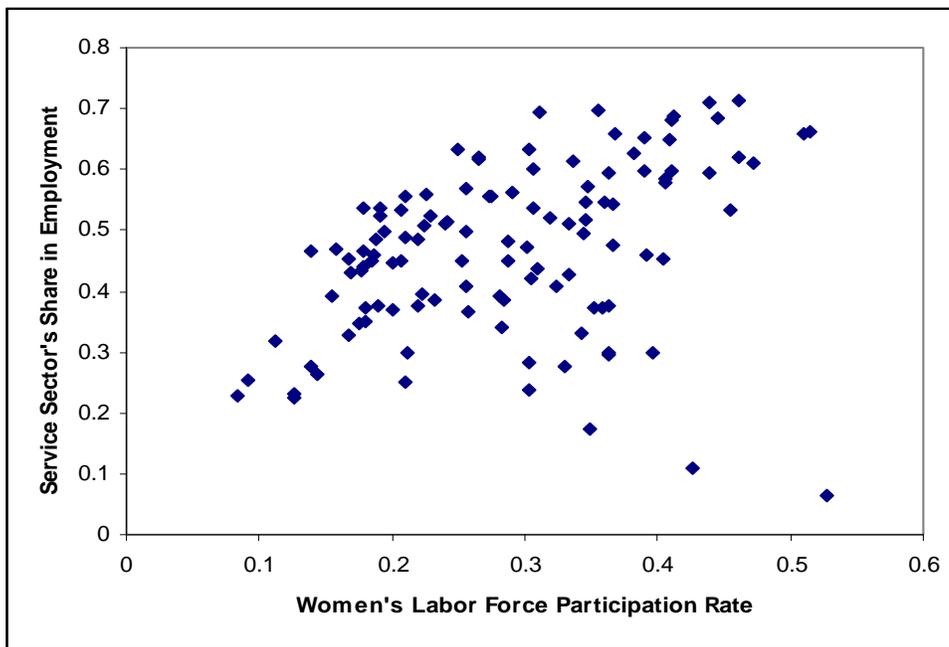

Figure 3: Scatter Plot - Share of Services and Women's Labor Force Participation

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with a fall in the manufacturing sector's share in employment, a higher increase in women's labor force participation rate is observed.

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The international evidence strengthens the motivation of the paper to investigate if the growth of the service sector may lead to a growth in women's labor force participation that is quantitatively consistent with the data.

3 A Model of Sectoral Change

The model used here is one of sectoral reallocation of economic activity, or "structural change" as it is also called. It has four sectors of activity; agriculture, manufacturing, market services and non-market (home) services. The model is closely related to that studied by Rogerson (2005). As mentioned in the introduction, Rogerson's model is transformed into one with gender specific time allocation decisions. The model is static, so all structural changes will be solely due to the interaction of productivity increases with the type of preferences adopted in the model. The model is kept very simple to focus attention on the following question: "Can differences between market and non-market productivities account for the increasing labor force participation of women?"

3.1 Preferences

There is a representative household, which is made up of a male and a female. The household lives for an infinite number of periods, and derives utility from aggregate consumption and leisure. The preferences are given by:

$$\sum_{t=0}^{\infty} \beta^t \{U(C_t, L_t) + G(A_t)\} \tag{1}$$

where C_t is the household consumption of a composite good (industrial goods and services) in period t, L_t is an aggregator for household leisure in period t, and A_t is the consumption of agricultural goods in period t. The utility function U is given by:

$$U(C_t, L_t) = \alpha_C \log(C_t) + (1 - \alpha_C) \log(L_t) \tag{2}$$

where α_C is the weight of the consumption good in the utility function.

The aggregator for leisure L_t, and the composite good C_t, are assumed to take the following forms:

$$L_t = L(L_{mt}, L_{ft}) = L_{mt}^{\alpha_L} L_{ft}^{(1-\alpha_L)} \tag{3}$$
$$C_t = C(M_t, S_t) = [\alpha_M M_t^{\varepsilon} + (1 - \alpha_M) S_t^{\varepsilon}]^{1/\varepsilon}$$

where L_{mt} is leisure of the male and L_{ft} is leisure of the female. Here, α_L is the share of the male's leisure in total household leisure, M_t and S_t are household consumption of manufacturing (industrial) goods and services respectively, α_M is the share of manufacturing goods in the composite consumption good, and $\frac{1}{1-\varepsilon}$ is the elasticity of substitution

between manufacturing goods and services. As long as this elasticity of substitution is not unity, uneven technological progress will generate structural changes.[21]

It is assumed that the service good can be produced both in the market and at home. The aggregate consumption of services S_t is given by the following CES aggregator:

$$S_t = [\alpha_S(S_{mt})^\eta + (1-\alpha_S)(S_{nt})^\eta]^{1/\eta} \tag{4}$$

where S_{mt} is household consumption of services produced in the market at time t, S_{nt} is household consumption of services produced at home in period t, and $\frac{1}{1-\eta}$ is the elasticity of substitution between these two types of service goods. Again, this specification will lead to a reallocation of labor in the presence of different productivity growth rates.

The following specification is used for $G(A_t)$, the utility derived from the consumption of agricultural products:

$$\begin{aligned}
G(A_t) &= \min\{A_t, \bar{A}\} \qquad if \qquad A_t \geqslant \bar{A} \\
G(A_t) &= -\infty \qquad\qquad if \qquad A_t < \bar{A}
\end{aligned} \tag{5}$$

This simple form implies that the representative household consumes $\bar{A}$ units of the agricultural good at all times. Improvements in agricultural productivity will lead to a decline in employment in this sector. Kongsamut et al. (2001) mention that facts documented in studies of consumption patterns are in parallel with findings that the growth in per capita income tends to be accompanied by a rise in services and a decline in the agricultural sector.[22] Note that in our specification, not only the share of agriculture is decreasing, but agricultural production is constant over time. This assumption is not at odds with the data. Agricultural output per person is not constant from 1950 to 1990, but there is no upward or downward trend, and fluctuations from the mean are not high.

3.2 Time allocation constraints

In this economy, men and women are each endowed with one unit of time. For simplicity it is assumed that only men can work in agriculture and manufacturing, and only women can work in non-market services. Both men and women can work in market services. These assumptions may seem very strong, but as we saw in the data analysis section, the fraction of women in agriculture and industry is very low, and more importantly it is not changing over time. Also, the main focus of this paper is the increasing labor force participation of women, and women seem to be moving into the service sector. As mentioned in the introduction, five out of six working women are employed in the service sector. Considering these factors, the assumptions in the model seem reasonable.

The time allocation constraint for the representative male is given by:

$$1 = H_{mAt} + H_{mMt} + H_{mSmt} + L_{mt} \tag{6}$$

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[21] The class of models with homothetic preferences has been used by Baumol (1967), and Ngai and Pissarides (2004) to generate structural transformations.

[22] For facts on consumptions patterns see Houthakker and Taylor (1970), and also Bils and Klenow (1998).

where H_{mAt} is the time allocated by the male to production in the agricultural sector, H_{mMt} is the time allocated by the male to production in manufacturing, H_{mSmt} is the time allocated by the male to production in market services, and L_{mt} is the male's leisure. The time allocation constraint for the representative female is given by:

$$1 = H_{fSmt} + H_{Snt} + L_{ft} \tag{7}$$

where H_{fSmt} is the time allocated by the female to production in market services, H_{fSnt} is the time allocated by the female to production in non-market (home) services, and L_{ft} is the female's leisure.

3.3 Production technology

For simplicity it is assumed that the production technologies in all sectors are linear in labor, and labor is the only input. The production technologies for agriculture, manufacturing, market services, and non-market (home) services are respectively:

$$
\begin{aligned}
A_t &= \theta_{At} H_{At} &\quad where \quad& H_{At} = H_{mAt} \\
M_t &= \theta_{Mt} H_{Mt} &\quad where \quad& H_{Mt} = H_{mMt} \\
S_{mt} &= \theta_{Smt} H_{Smt} &\quad where \quad& H_{Smt} = H_{mSmt} + H_{fSmt} \\
S_{nt} &= \theta_{Snt} H_{Snt} &\quad where \quad& H_{Snt} = H_{fSnt}
\end{aligned}
\tag{8}
$$

For $j = A$, M, Sm, and Sn, the θ_{jt}'s in the production functions are sector specific productivities that change over time. Productivities in the model are exogenous and will be taken from the data. A more detailed discussion of this will be provided in the calibration section. Notice that in this model productivity in a sector is equivalent to labor productivity, and that we can interpret it as the private return to working in a particular activity.

3.4 Firm's Problem

The representative firms in agriculture, industry and services respectively solve the following problems:

$$
\begin{aligned}
&\max(P_{At} \theta_{At} H_{At} - w_{At} H_{At}) \\
&\max(P_{Mt} \theta_{Mt} H_{Mt} - w_{Mt} H_{Mt}) \\
&\max(P_{Smt} \theta_{Smt} H_{Smt} - w_{Smt} H_{Smt})
\end{aligned}
\tag{9}
$$

P_{At}, P_{Mt}, and P_{Smt} are the prices of the agricultural good, the industrial good and (market) services respectively. w_{At}, w_{Mt}, and w_{Smt} are the wages in these sectors. Notice that since labor is freely mobile in this economy, in equilibrium wages in all sectors will be equal; $w_{At} = w_{Mt} = w_{Smt} = w_t$.

3.5 Household's Problem

Since the model is static in nature, in each period t the representative household will maximize the following objective function:

$$\alpha_C \log([\alpha_M M_t^\varepsilon + (1-\alpha_M)([\alpha_S(S_{mt})^\eta + (1-\alpha_S)(S_{nt})^\eta]^{1/\eta})^\varepsilon]^{1/\varepsilon}) \tag{10}$$
$$+ (1-\alpha_C)\log([1 - H_{mAt} + H_{mMt} + H_{mSmt}]^{\alpha_L}[1 - H_{fSmt} + H_{Snt}]^{(1-\alpha_L)}) + \bar{A}$$

subject to the budget constraint and the home production technology:

$$P_{At}\bar{A} + P_{Mt}M_t + P_{Smt}S_{mt} \leq w_t(H_{mAt} + H_{mMt} + H_{mSmt} + H_{fSmt}) \tag{11}$$
$$S_{nt} \leq \theta_{Snt} H_{fSnt}$$

3.6 Equilibrium

A competitive equilibrium in this economy in period t is a collection of allocations and prices $(M, S_m, S_n, H_A, H_M, H_{mSm}, H_{fSm}, H_{fSn}, P_A, P_M, P_{Sm}, w)$ such that, given values of the state variables $(\theta_A, \theta_M, \theta_{Sm}, \theta_{Sn})$ and given prices

(i) the allocations maximize firms' profit (maximize Eq.9)

(ii) the allocations solve the representative household's maximization problem

(maximize Eq.10 subject to Eq.11)

(iii) all markets clear

Since all markets are competitive, Welfare Theorem 1 applies in this economy. So, a social planner's problem will give us the same allocations as the above problem. Prices can then be obtained from the productivities. Computational details on the social planner's problem are provided in Appendix B.

4 Calibration

The model economy is calibrated to the 1950 U.S. economy. In particular, the model is parameterized so that it matches time allocations to the different sectors in 1950, and the manufacturing sector in 1990. All sectoral productivities in the model are exogenous.

4.1 Productivities

Productivities for agriculture, industry, and market services are all taken from the data. These are labor productivity (output per worker) series that are constructed by aggregating the detailed sectoral labor productivities under each main category using output

as weights. The labor series used for these calculations are the employment data described in the data analysis section. For output, real GDP from the NIPA tables is used. Productivity calculations are based on private industries only.

Measuring productivity in non-market services (home production), is a problem. The home production literature, where a single good can be produced in the market and at home, usually uses the same productivity growth rate for market and non-market technologies. Parente, Rogerson, and Wright (2000) use a 2% growth rate for the market and the home production sectors. McGrattan, Rogerson and Wright (1995) use a common gross trend growth rate of 1.0063%, but different technology shocks. Market in their analysis includes nondurable goods and services. Rogerson (2005), on the other hand, pins down productivities by matching the observed change in sectoral time allocations for the U.S. from 1956 to 2000. The implied annual production technology growth rates are 0.4% for the home, and 1.7% for market services.

In this paper, I identify activities in the market that are similar to those at home, and assume that the measured productivity of these activities reflects the productivity in home services. In particular, I take labor productivity of the category called "Services" from the data and use it as the productivity series for non-market services. It is a subcategory of the aggregate service sector, and seems to be the closest category to home services that can be found in the data.[23] Although a closer substitute can be found if one goes to more detailed classifications, it is not possible to calculate productivities, since employment data is not available in such detail. The productivity series used in this paper imply annual growth rates of 0.6% and 1.2%, for non-market and market services respectively. As in Rogerson's (2005) model, it is the relatively lower productivity growth in non-market services compared to market services that leads to the shift from home to market services.[24]

Productivity levels for the initial period, namely 1950, are normalized to one, i.e., $\theta_{A1} = \theta_{M1} = \theta_{Sm1} = \theta_{Sn1} = 1$. A comparison of all productivities is provided in Figure 4. Productivity growth in aggregate services is lower than in agriculture and manufacturing. The lowest growth rate is in the subcategory "services" that we use as a measure for non-market (home) services.

4.2 Parameters

The key parameter in the model is η, where $1/(1 - \eta)$ is the elasticity of substitution between market and non-market services. The value of η is set to 0.75. Numbers for the elasticity of substitution between market and non-market goods reported in the literature

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[23]This category includes the following classifications: Hotels and other lodging places, Personal services, Business services, Auto repair, services, and parking, Miscellaneous repair services, Motion pictures, Amusement and recreation services, Health services, Legal services, Educational services, Social services, Membership organizations, Miscellaneous professional services, Private households

[24]If one would consider gains to specialization or economies of scale in the market, productivity at home could be even smaller. Hence, the productivity series that is used could be interpreted as an upper bound for the non-market technology.
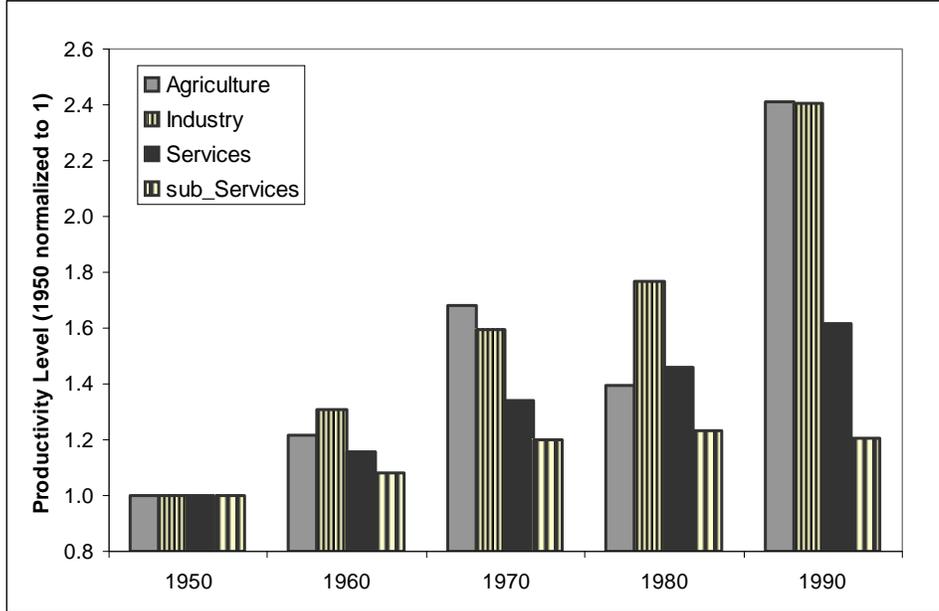
Figure 4: Sector Specific Labor Productivities

are smaller. McGrattan et al. (1995) estimate a value of 0.429 for this parameter. However in the model studied here, the elasticity of substitution is between market and non-market services. Hence, industrial goods which have relatively less non-market produced substitutes are not included. This specification is consistent with a higher elasticity of substitution. With the same motivation, Olivetti (2001) calibrates a value of 0.75 for this parameter, where the elasticity of substitution is between maternal time and market goods and services such as day care. It is the substitutability of market and home services, which will allow for the reallocation of labor in the presence of different productivity growth rates. Due to the importance of this parameter, a sensitivity analysis will be provided below.[25]

Another important parameter is ε, where $1/(1 - \varepsilon)$ is the elasticity of substitution between manufactured goods and services. ε is chosen so that the model matches the time allocation in the industry sector in 1990. The implied value is $\varepsilon = -0.75$. Rogerson (2005) uses a value of -1.5. Ngai and Pissarides (2004) use a value of -2.33 and also evaluate -9. Notice that this parameter will be the driving force of the reallocation of labor from industry to services, and only a negative value for ε will result in a reallocation from the sector with higher productivity growth to one with lower productivity growth. A sensitivity analysis will be provided for the values -0.2 and -1.5.

Given η, and ε, the share parameters α_C, α_M, α_L, α_S, and agricultural output $\bar{A}$,

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[25]Notice that I am holding the elasticity of substitution between service goods and home goods constant. It might have actually increased, since more and more substitutable services became available over time.

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are calibrated to match the initial labor allocation of total population in agriculture and industry, initial labor allocation of men and women in market services, and a 25% time allocation to home production by the household. The fraction of time devoted to work at home is based on time use surveys provided by Juster and Stafford (1991).[26]

In our model only the woman works at home, so we target $H_{fSn1} = 0.5$ to get the 0.25 average allocation of time to non-market work for the household. Similarly, since only the man works in agriculture and industry, we target $H_{mA1} = 0.0289$ and $H_{mM1} = 0.2418$ to match the 0.0145 and 0.1209 averages found in the data for the whole population. Targets for the time allocation to market services are $H_{mSm1} = 0.2250$ and $H_{fSm1} = 0.1442$, for male and female respectively, again to match the data. Finally, $\bar{A} = H_{A1} = 0.0289$ since $\theta_{A1} = 1$.

The following table lists the calibrated parameters:

Table 4: Summary of Calibrated Parameters

Parameter	Definition	Value
α_C	share of consumption in the utility function	0.5636
α_M	share of manufacturing in composite good	0.1517
α_L	share of male's leisure in household leisure	0.5863
α_S	share of market services in total services	0.4810
η	elasticity of substitution market & non-market services $\left(\frac{1}{1-\eta}\right)$	0.75
ε	elasticity of substitution manufactured goods & services $\left(\frac{1}{1-\varepsilon}\right)$	-0.75
$\bar{A}$	agricultural output	0.0289

Section 6 provides a sensitivity analysis of the model to the key parameters. It also discusses alternative productivity series for the non-market production. Before that we turn to the quantitative results of our benchmark calibrated model.

5   Quantitative Results

This section presents the results for the calibrated model. It is assumed that the calibrated preference parameters do not change over time. In the presence of sector specific differences in productivity growth rates, the model will generate changes in time allocations to the different sectors and to home production. The question is, what fraction of the observed change in employment rate, and especially what fraction of the observed increase in women's employment rate, can be accounted for by this model?

Notice that the model was parameterized so that it matches time allocations to the different sectors in 1950, and the manufacturing sector in 1990. This means that the

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[26]They present that the Michigan Time Use survey indicates that a married couple, on average, devotes 25% of discretionary time to unpaid home production and 33% of discretionary time to work in the market place for pay. These numbers are based on averages of time use surveys for 1965 and 1981. No time use surveys are available for 1950.
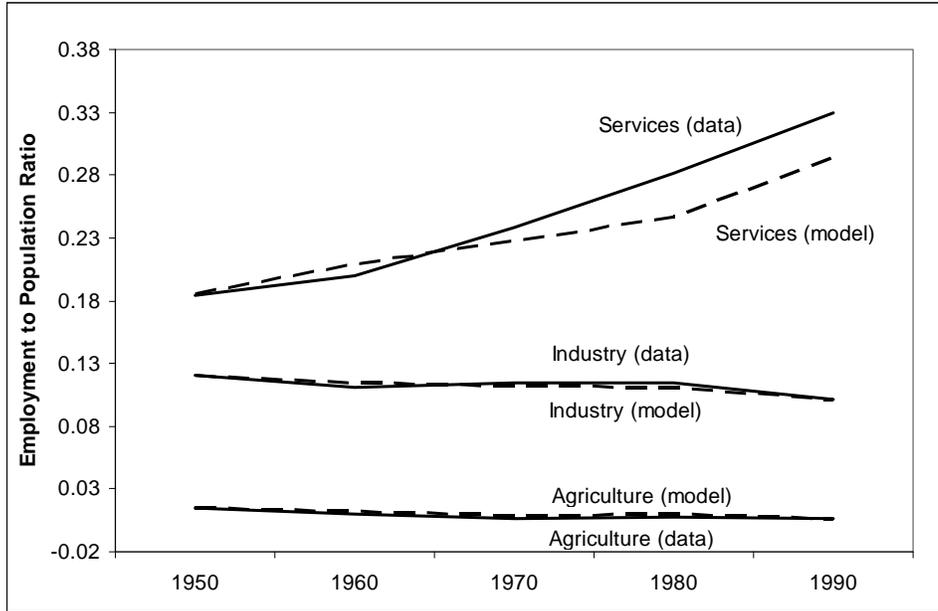
Figure 5: Sectoral Change in Total Employment

exact fit of the sectoral time allocations in the model to that in the data in 1950, as well as the perfect fit in the manufacturing sector in 1990 will be by construction. Although the presence of productivity growth in the agricultural sector will by construction lead to a fall in agricultural employment, the magnitude of the fall is not forced at all.

Figure 5 presents the sectoral (structural) change in total employment. In 1950, 18.45% of the population was working in the service sector, 12% in industry (manufacturing), and 1.45% in agriculture. Over the following 40 years, employment in market services increased to almost 33%, that in manufacturing and agriculture dropped to 10.1% and 0.7% respectively. The model does a good job in capturing the increase in the rate of total employment in market services. It captures 75% of the 14.55 percentage point increase. The model generates a slight drop in the rate of total agricultural employment, which is consistent with the data.

Changes in gender specific agricultural employment are provided in Figure 6. The model can account for all of the decrease in male's agricultural employment. Both, in the model and in the data, men's agricultural employment decreases from 3% to 1%. Notice that women in the model do not work in the agricultural sector. Since, in the data, the share of women in agricultural employment is almost constant from 1950 to 1990, most of the decrease in total agricultural employment is due to the changes in men's participation.

The change in service sector employment by gender is provided in Figure 7. The model is successful in replicating the increasing participation of women in market services from 1950 to 1990. In the data, women's service sector employment rate increases from around
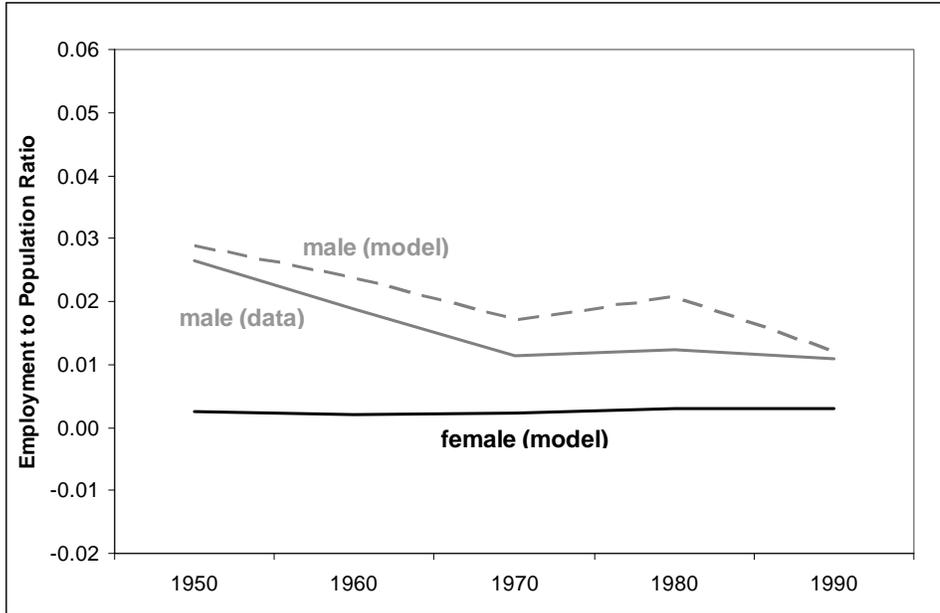
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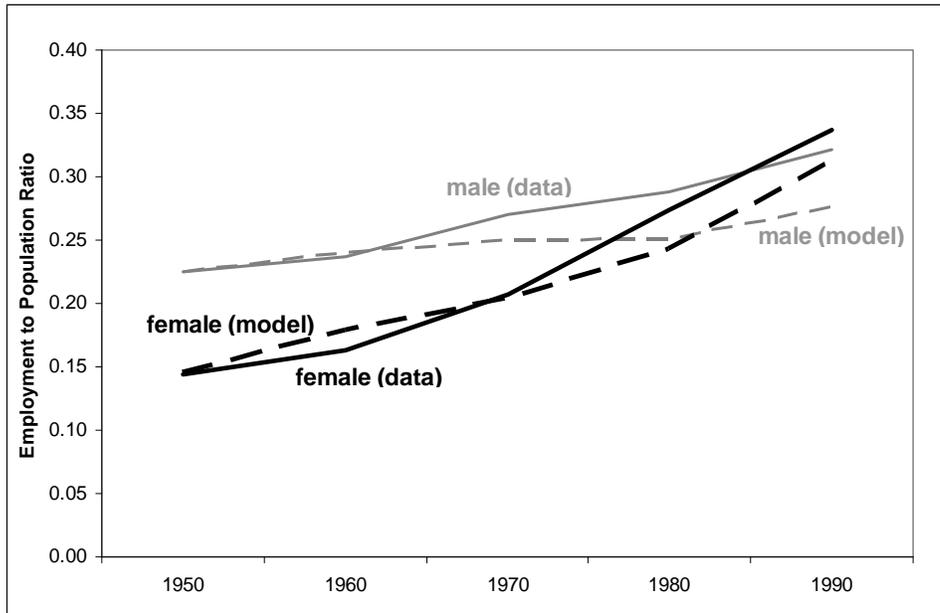Figure 6: Gender Specific Change in Agricultural Employment

Figure 7: Gender Specific Change in Service Employment

15% to 34% during this period. The increase in the model is from 15% to 31%. So, the model can account for 86% of the 19 percentage point increase. The data shows that 22.5% of men were working in the service sector in 1950; 40 years later 32% of men were employed in this sector. The model can account for 53% of the almost 10 percentage
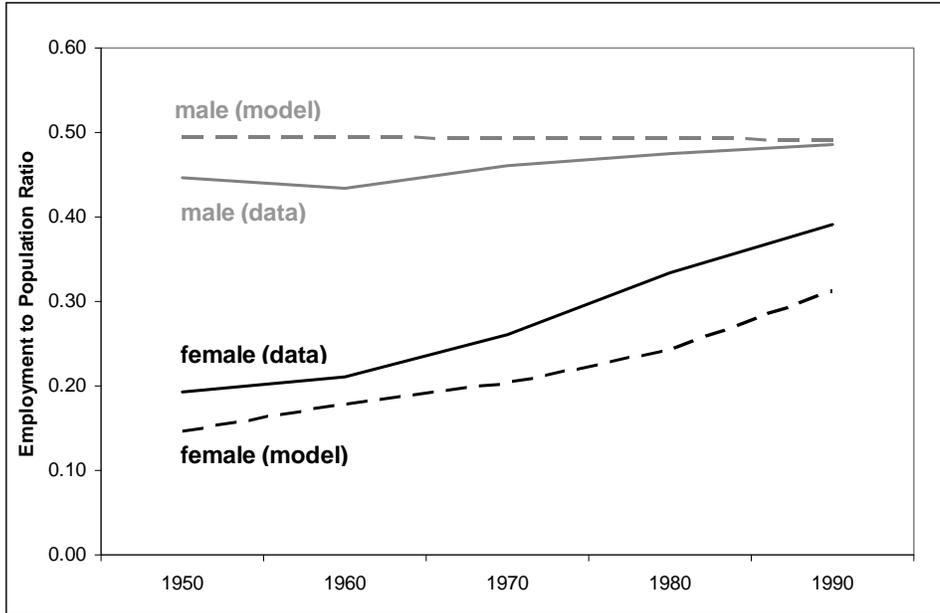
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Figure 8: Gender Specific Change in Total Employment

point increase for men.

Figure 8 compares the results of the model with the data for gender specific total employment. Men's employment to population ratio in the model stays almost constant from 1950 to 1990. In the data, we observe a 5 percentage point increase. The reason for this is that the model underpredicts the increase in men's service sector employment. In the data, the employment rate of women is 19% in 1950, and increases by 20 percentage points over the next 40 years. In the model, it increases from 15% to 31%. Although this model restricts women's market activity to the service sector, it can account for most of their increasing labor force participation observed in the data. This is due to the fact that most women work in the service sector anyway.

Figure 9 is an evaluation of the cumulative performance of the model. Since the main focus of this paper is women's increasing market participation, it compares growth rates of the rate of female employment in the data with that in the model for the periods 1950-1960, 1950-1970, 1950-1980, and 1950-1990. The data shows a 9.21% increase in women's employment rate over the 10 years after 1950. The model overestimates the growth rate for this period by 13.6 percentage points. It performs much better for the other periods. The model overpredicts the growth rate from 1950 to 1970 by 4.5 percentage points, underpredicts the growth rate over the thirty years after 1950 by 6 percentage points, and again overpredicts the growth rate for the entire period (1950-1990) by 11 percentage points.

Deviations from the data are higher if the performance of the model is evaluated by decade. The model overpredicts the increase in women's employment rate in the first
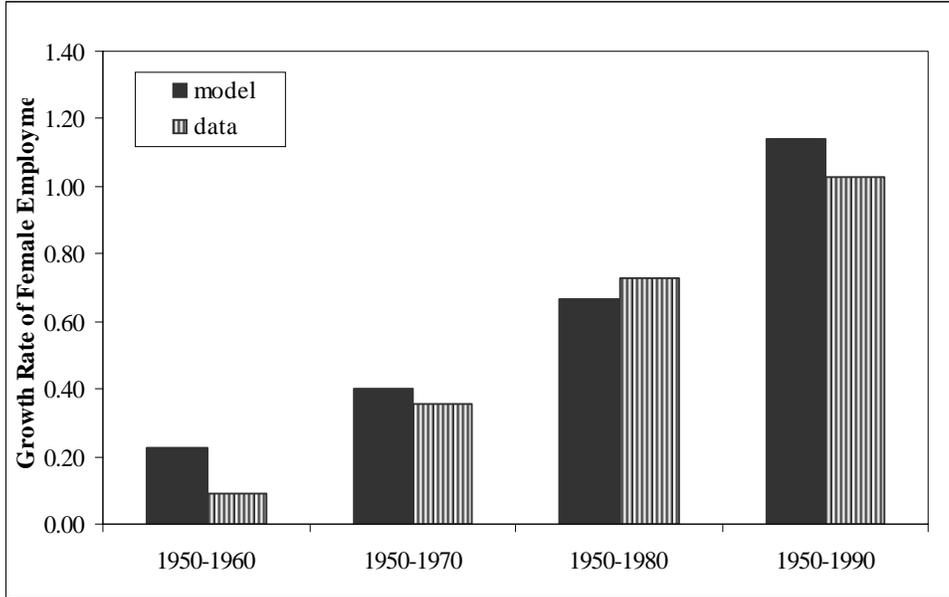
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Figure 9: Cumulative Performance of the Model

and last decades of the period analyzed in this paper. It underpredicts the growth from 1960 to 1980. These deviations might be partly explained by changes in fertility. Figure A1 in Appendix A compares the performance of the model by decade and changes in the fertility rate.[27] These preliminary results look promising. The model is underpredicting the increased participation of women in the market during periods of major decline in the fertility rate.

Although the model is less successful in capturing the growth rates of women's employment rate for the individual decades, its overall performance is quite good. It is particularly successful in replicating the increase in women's total rate of employment and their increasing participation in the service sector over the 40 year period. The model also predicts a decrease in the time a household allocates to home production, which is also consistent with the data.

Given the high correlation between the service sector's share in employment and women's labor force participation rate in many countries which was discussed in section 2, the model constructed in this paper seems to have the potential of explaining developments in women's labor force participation in other countries as well.[28] This is left for

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[27]Fertility rate here is the crude birth rate, namely the birth rate per 1000 per year.

[28]One way to improve the model would be to relax the assumption that women do not work in agriculture. If agricultural goods could be produced at home (in their own garden) and in the market, and if the production of these goods in the market would become relatively more efficient in the market, women would first move into the agricultural sector. Later, with the productivity in the agricultural sector increasing and agricultural consumption staying constant, a decline in this sector would follow. If at the same time the service sector increases, instead of moving back to their homes, women would move into the service sector. Hence, no increase in their market employment would be observed.

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future work.

6 Sensitivity Analysis

As mentioned in the calibration section, η, the parameter for elasticity of substitution between market and non-market services, is a key parameter in our model. In this paper, I pick a value of 0.75, which corresponds to an elasticity of substitution of 4. The performance of the model in terms of mimicking the increase in women's rate of employment in market services drops, once η is decreased. If a value of 0.5 is used for this parameter, the model can capture only 18% of the 19 percentage point increase in women's service sector employment rate. With a value of 0.6 for η, slightly more but still only 35% of the increase can be captured. Using a value of 0.65 and 0.7, the model can respectively account for 48% and 64% of the 19 percentage point increase in women's service sector employment rate. The performance of the model increases as the elasticity of substitution between market and home services increases. So, women's time allocation decision is highly sensitive to this parameter.

A reduction in the value of ε, the parameter for elasticity of substitution between manufactured goods and services, reduces the rate of decline in industry employment. This leads to an increase in women's employment rate in market services, since men are entering this sector at a lower rate now. An increase in this parameter leads to overestimation of the decrease in industry employment. Men can now move into the market service sector at a higher rate, and hence fewer women are needed in this sector. If the value of this parameter is decreased from -0.75 to -1.5, the model accounts for 82% of the 19 percentage point increase in women's service sector employment rate instead of 86%. But it accounts for 69% of the 10 percentage point increase in men's service sector employment rate instead of 53%. If a value of -0.2 is used for this parameter, the model accounts for 89% of the increase in female employment rate in the service sector but only for 28% of the increase for men. Notice that women's time allocation decision is not very sensitive to this parameter, but men's is.

The next exercise conducted in this section is of an exploratory nature. I ask the following question: What level of home production productivity (productivity of non-market services) do we need, so that the model perfectly matches the observed increase in women's participation rate in market services? The necessary productivity series is computed. This new series is then compared to the benchmark used in this paper, namely the home production technology that was computed using actual data for the subcategory "services".

Figure 10 compares the values of these home production productivities in 1950, 1960, 1970, 1980, and 1990. The values in 1950 are normalized to 1. Since the model used in this paper is static, time allocation decisions in a certain year are based on the productivity level of that year only. The two series are surprisingly similar. A 2% higher level of productivity would be needed in 1960 for women's service sector employment in the
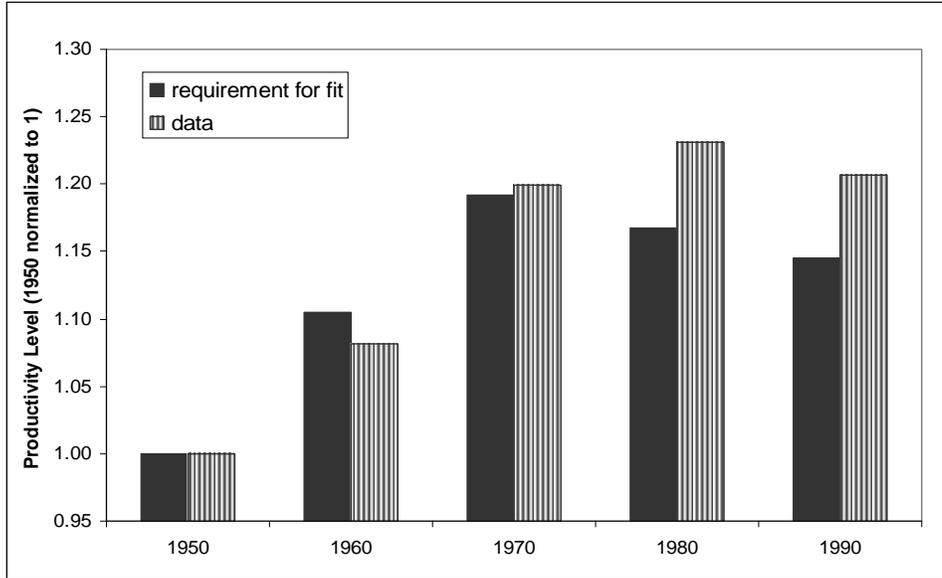
Figure 10: Comparison of Productivities for Home Production

model to perfectly fit that in the data. Lower productivity levels are required for the remaining decades. With a 1% lower level in 1970, and 5% lower levels in 1980 and 1990, the increasing service sector employment of women in the model would mimic that in the data.

A comparison of annual technology growth rates of home production from one decade to the other shows that, with a growth rate of 1% instead of 0.8% during the period from 1950 to 1960, and a growth rate of 0.8% instead of 1% during the period from 1960 to 1970, the model perfectly matches the increase in women's service sector employment during these two decades. From 1970 to 1980, productivity for the home technology in this paper grows at an annual rate of 0.3%. To mimic the increase in the rate of women in services during the same period, an annual growth rate of -0.2% is needed. During the last decade (1980-1990), the productivity levels taken from the data correspond to an annual growth rate of -0.2%. This is exactly the required rate for a perfect fit.

It is important to stress once again that the results presented in this paper are sensitive to the productivity data we are using. Figure 11 shows how the performance of the model in terms of capturing the growing participation rate of women in the service sector changes for different productivity series for the home technology. In particular, I evaluate how the model performs when annual productivity growth rates at home are 30% and 50% lower than in the aggregate service sector.[29] Remember that with the productivities of the subcategory "services", which we use as a benchmark, the model is capturing 86% of the 19 percentage point rise. With a 30% lower productivity growth rate at home

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[29]Note that if the same productivity series is used for market and non-market services, the model cannot capture the increase in women's service sector employment at all.
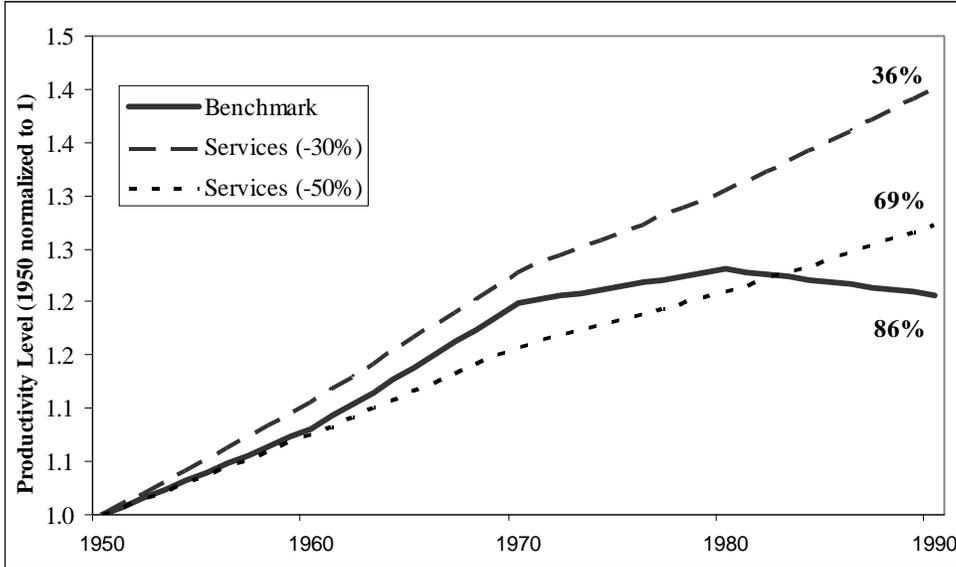
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Figure 11: Performance of the Model with different Home Productivities

compared to the service sector, the model can account for only 36% of the 19 percentage point increase in the female employment rate in market services. It captures 69% of the increase if home production productivity increases half as fast as the productivity of market services.

7 Conclusion

This paper explores the extent to which increases in the productivity of service provision in the market sector relative to home production can account for the magnitude of the increase in women's labor force participation. The idea here is based on two main observations. First, both the share of service sector employment and the female employment rate in the U.S. experienced a takeoff in their growth rates after 1950. Second, sectoral as well as occupation specific data indicate that women mostly work in the service sector.

A structural transition model with 3 sectors and a home production technology is developed in order to quantitatively assess the contribution of relative changes in sectoral productivities to changes in gender specific time allocation decisions. This model demonstrates that given a higher rate of productivity growth in market services compared to non-market (home) services, and a high degree of substitutability between these two, it can account for a large fraction of the observed increase in the female employment rate over the period 1950-1990.

This simple model of structural change is helpful in understanding the forces that can affect time allocation decisions of households. The results presented in this paper are

mainly driven by two forces: the parameter for elasticity of substitution between market and non-market services, and the exogenous differences in labor productivities. If the productivity in market services grows faster than the productivity in home production, there will be a shift from non-market to market services, as long as the two services provided are substitutes.

Although the high substitutability between market services and home services in this paper is by assumption, I believe that there is enough evidence to support it. Data shows that individuals working in the market spend much less time working at home than unemployed individuals. Employed agents with higher wages substitute out of home and into market production. This suggests that there is a good deal of substitutability between market and home goods. This is especially true for market services and home goods, since the movement from home is mainly into the service sector. How high the elasticity of substitution really is, that of course is another question. No measure is available for home production productivity, hence it is hard to find evidence supporting the lower productivity at home compared to that in the market. Gains to specialization or economies of scale in market production could be reasons for the relatively higher productivity growth rates in the market.

This paper draws attention to two economically important research topics: estimating the substitution between market services and home services, and measuring home production productivity. Once we get a better idea about the elasticity of substitution between market and home services, and the production technology at home, we will better understand why women decided to move into market employment at such an increasing rate. This paper is a first attempt to link structural changes and changes in female labor force participation. It focuses on the U.S. but provides a framework that has the potential to address cross-country changes in women's market employment. Applying this model to other countries, as well as looking at a longer time horizon is left for future work.

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Appendix A: Additional Tables and Figures

Table A1: 20 Leading Occupations of Employed Women in 2006 (employment in thousands)

Occupation		Total Employed Women	% Women in Total Employed
All occupations		66,870	46
Total of 20 leading occupations		29,210	
% of 20 leading occupations in total occupations		44	
Occupations that are in the service sector	Registered Nurses	2,309	91
	Cashiers	2,291	75
	Elementary and middle school teachers	2,220	82
	Retail salespersons	1,740	51
	Nursing, psychiatric, and home health aides	1,694	89
	First-line supervisors/managers of retail sales workers	1,436	42
	Waiters and waitresses	1,401	72
	Childcare workers	1,320	94
	Receptionists and information clerks	1,301	93
	Maids and housekeeping cleaners	1,285	90
	Accountants and auditors	1,071	60
	Teacher assistants	869	92
	Cooks	811	43
	Hairdressers, hairstylists, and cosmetologists	716	93
	Preschool and kindergarten teachers	674	98
Occupations that could be in any sector	Secretaries and admin. assistants	3,348	97
	Bookkeeping, accounting, and auditing clerks	1,364	90
	Customer service representatives	1,349	70
	First-line supervisors/managers of office and administrative support	1,163	75
	Office clerks, general	848	82
% of 20 leading occupations that are in services		72	
% of 20 leading occupations that could be in any sector		28	

Source: U.S. Department of Labor, Bureau of Labor Statistics, Annual Averages 2006.

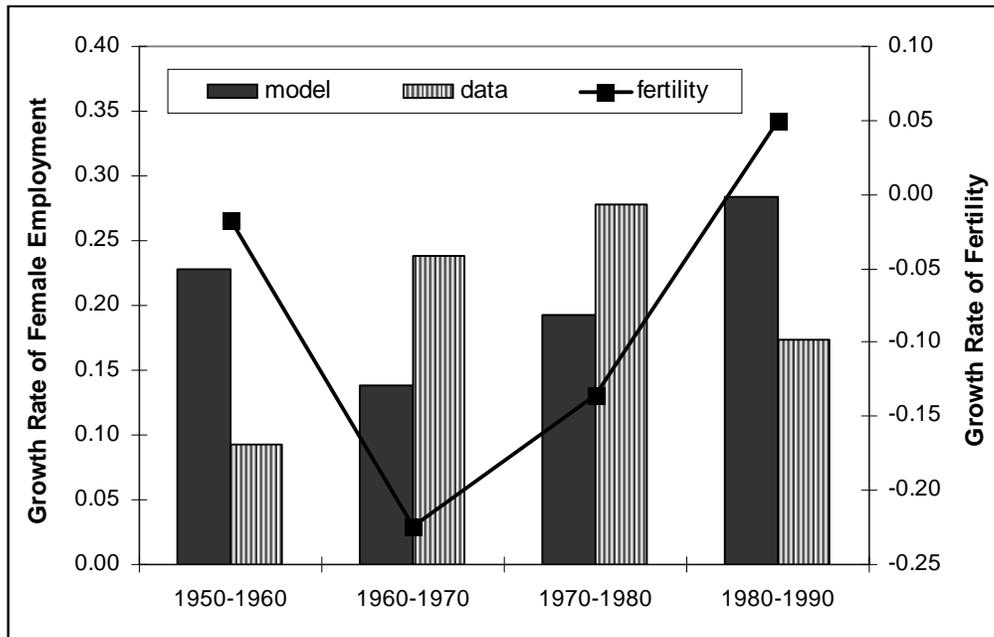Figure A1: Performance of the Model by Decade and Changes in Fertility

Figure 12:

Appendix B: Computational Details

I solve the social planner's problem. Since the model is static, I can solve it separately for all periods.

The household will always consume $\bar{A}$ units of the agricultural good. We get the following law of motion for time spent in the agricultural sector.

$$H_{mAt} = \frac{\bar{A}}{\theta_{At}}$$

The optimization is then given by:

$$\max \alpha_C \log([\alpha_M M_t^{\varepsilon} + (1 - \alpha_M)([\alpha_S(S_{mt})^{\eta} + (1 - \alpha_S)(S_{nt})^{\eta}]^{1/\eta})^{\varepsilon}]^{1/\varepsilon})$$
$$+ (1 - \alpha_C) \log(L_{mt}{}^{\alpha_L} L_{ft}{}^{(1-\alpha_L)}) + \bar{A}$$

subject to:

$$M_t \leq \theta_{Mt} H_{mMt}, \qquad S_{mt} \leq \theta_{Smt}(H_{mSmt} + H_{fSmt}), \qquad S_{nt} \leq \theta_{Snt} H_{fSnt}$$
$$H_{mMt} + H_{mSmt} + L_{mt} = 1 - H_{mAt}, \qquad H_{fSmt} + H_{fSnt} + L_{ft} = 1$$

$$M_t \geq 0, \qquad S_{mt} \geq 0, \qquad S_{nt} \geq 0,$$
$$L_{mt} \in [0,1], \qquad H_{mMt} \in (0,1], \qquad H_{mSmt} \in [0,1]$$
$$L_{ft} \in [0,1], \qquad H_{fSmt} \in [0,1], \qquad H_{fSnt} \in (0,1]$$

Forming the Lagrangian, we have

$$\pounds_0 = \alpha_C \log[a_M M^{\varepsilon} + (1 - a_M)[\alpha_S(S_{mt})^{\eta} + (1 - \alpha_S)(S_{nt})^{\eta}]^{\varepsilon/\eta}]^{1/\varepsilon}$$
$$+ (1 - \alpha_C)[\alpha_L \log(1 - H_{mAt} - H_{mMt} - H_{mSmt})$$
$$+ (1 - \alpha_L) \log(1 - H_{fSmt} - H_{fSnt})] + \bar{A}$$
$$+ \lambda_{Mt}[\theta_{Mt} H_{mMt} - M_t] + \lambda_{Smt}[\theta_{Smt}(H_{mSmt} + H_{fSmt}) - S_{mt}]$$
$$+ \lambda_{Snt}[\theta_{Snt} H_{fSnt} - S_{nt}]$$

First-order necessary conditions for this problem are:

$$[M_t]: \frac{\alpha_C \alpha_M M^{\varepsilon-1}}{[a_M M^\varepsilon + (1-a_M)[\alpha_S(S_{mt})^\eta + (1-\alpha_S)(S_{nt})^\eta]^{\varepsilon/\eta}]} = \lambda_{Mt}$$

$$[S_{mt}]: \frac{\alpha_C(1-\alpha_M)}{[a_M M^\varepsilon + (1-a_M)[\alpha_S(S_{mt})^\eta + (1-\alpha_S)(S_{nt})^\eta]^{\varepsilon/\eta}]}\alpha_S S_{mt}^{\eta-1}$$

$$: [\alpha_S S_{mt}^\eta + (1-\alpha_S)S_{nt}{}^\eta]^{(\varepsilon/\eta)-1} = \lambda_{Smt}$$

$$[S_{nt}]: \frac{\alpha_C(1-\alpha_M)}{[a_M M^\varepsilon + (1-a_M)[\alpha_S(S_{mt})^\eta + (1-\alpha_S)(S_{nt})^\eta]^{\varepsilon/\eta}]}(1-\alpha_S) S_{nt}^{\eta-1}$$

$$: [\alpha_S S_{mt}^\eta + (1-\alpha_S)S_{nt}{}^\eta]^{(\varepsilon/\eta)-1} = \lambda_{Snt}$$

$$[H_{mMt}]: \lambda_{Mt}\theta_{Mt} = \frac{(1-\alpha_C)\alpha_L}{(1 - H_{mAt} - H_{mMt} - H_{mSmt})}$$

$$[H_{mSmt}]: \lambda_{Smt}\theta_{Smt} \le \frac{(1-\alpha_C)\alpha_L}{(1 - H_{mAt} - H_{mMt} - H_{mSmt})}$$

$$[H_{fSmt}]: \lambda_{Smt}\theta_{Smt} \le \frac{(1-\alpha_C)(1-\alpha_L)}{(1 - H_{fSmt} - H_{fSnt})}$$

$$[H_{fSnt}]: \lambda_{Snt}\theta_{Snt} = \frac{(1-\alpha_C)(1-\alpha_L)}{(1 - H_{fSmt} - H_{fSnt})}$$

The first-order necessary conditions simplify to:

$$\frac{\alpha_C \alpha_M M^{\varepsilon-1}}{[a_M M^\varepsilon + (1-a_M)[\alpha_S(S_{mt})^\eta + (1-\alpha_S)(S_{nt})^\eta]^{\varepsilon/\eta}]}\theta_{Mt}$$

$$= \frac{(1-\alpha_C)\alpha_L}{(1 - H_{mAt} - H_{mMt} - H_{mSmt})}$$

$$\frac{\alpha_C(1-\alpha_M)}{[a_M M^\varepsilon + (1-a_M)[\alpha_S(S_{mt})^\eta + (1-\alpha_S)(S_{nt})^\eta]^{\varepsilon/\eta}]}\alpha_S S_{mt}^{\eta-1}[\alpha_S S_{mt}^\eta + (1-\alpha_S)S_{nt}{}^\eta]^{(\varepsilon/\eta)-1}\theta_{Smt}$$

$$\le \frac{(1-\alpha_C)\alpha_L}{(1 - H_{mAt} - H_{mMt} - H_{mSmt})}$$

$$\frac{\alpha_C(1-\alpha_M)}{[a_M M^\varepsilon + (1-a_M)[\alpha_S(S_{mt})^\eta + (1-\alpha_S)(S_{nt})^\eta]^{\varepsilon/\eta}]}\alpha_S S_{mt}^{\eta-1}[\alpha_S S_{mt}^\eta + (1-\alpha_S)S_{nt}{}^\eta]^{(\varepsilon/\eta)-1}\theta_{Smt}$$

$$\le \frac{(1-\alpha_C)(1-\alpha_L)}{(1 - H_{fSmt} - H_{fSnt})}$$

$$\frac{\alpha_C(1-\alpha_M)}{[a_M M^\varepsilon + (1-a_M)[\alpha_S(S_{mt})^\eta + (1-\alpha_S)(S_{nt})^\eta]^{\varepsilon/\eta}]}(1-\alpha_S) S_{nt}^{\eta-1}[\alpha_S S_{mt}^\eta + (1-\alpha_S)S_{nt}{}^\eta]^{(\varepsilon/\eta)-1}\theta_{Snt}$$

$$= \frac{(1-\alpha_C)(1-\alpha_L)}{(1 - H_{fSmt} - H_{fSnt})}$$

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The following problem is solved:

$$\arg\max(Case1, Case2, Case3)$$

$Case1:$	$H_{mSmt} > 0$	$H_{fSmt} = 0$	corner solution
$Case2:$	$H_{mSmt} = 0$	$H_{fSmt} > 0$	corner solution
$Case3:$	$H_{mSmt} > 0$	$H_{fSmt} > 0$	interior solution