The Poincaré Duality Theorem and its Applications

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The Poincaré Duality Theorem and its Applications

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Outline

In this talk I will explain the duality between the deRham cohomology of a manifold M and the compactly supported cohomology on the same space. This phenomenon is entitled “Poincaré duality” and it describes a general occurrence in differential topology: a duality between spaces of closed, exact differentiable forms on a manifold and their compactly supported counterparts. In order to define and prove this duality, I will start with the definition of the dual space of a vector space, with the definition of a positive definite inner product on a vector space, then define the concept of a manifold. I will continue with the definition of differentials on a differentiable manifold and their corresponding spaces necessary to this analysis. I will then introduce the concepts of a good cover of a manifold, manifolds of finite type, and orientation, all necessary concepts towards the goal of defining and proving Poincaré duality. I will finish with the proof of the Poincaré duality in the case of M orientable and admits a finite good cover, with examples.

Other forms of the Poincaré duality

The Poincaré duality describes a general occurrence in differential topology: a duality between spaces of closed, exact differentiable forms on a manifold and their compactly supported counterparts. In order to define and prove this duality, I will start with the definition of the dual space of a vector space, with the definition of a positive definite inner product on a vector space, then define the concept of a manifold. I will continue with the definition of differentials on a differentiable manifold and their corresponding spaces necessary to this analysis. I will then introduce the concepts of a good cover of a manifold, manifolds of finite type, and orientation, all necessary concepts towards the goal of defining and proving Poincaré duality. I will finish with the proof of the Poincaré duality in the case of M orientable and admits a finite good cover, with examples.

References