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Can A Future Choice Affect A Past Measurement's Outcome?

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Can a Future Choice Affect a Past Measurement's Outcome?

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An EPR experiment is studied where each particle undergoes a few weak measurements of different spin-orientations, whose outcomes are individually recorded. Then the particle undergoes a strong measurement along a spin orientation freely chosen at the last moment. Bell-inequality violation is expected between the two strong measurements. At the same time, agreement is expected between all same-spin measurements, whether weak or strong. A contradiction thereby ensues: i) A weak measurement cannot determine the outcome of a successive strong one; ii) Bell's theorem forbids spin values to exist prior to the final choice of the spin-orientation to be measured; and iii) Indeed no disentanglement is inflicted by the weak measurements; yet iv) The weak measurements' outcome agrees with those of the strong ones. The only reasonable resolution seems to be that of the Two-State-Vector Formalism, namely that the weak measurement's outcomes anticipate the experimenter's future choice, even before the experimenter themselves knows what their choice is going to be. Causal loops are avoided by this anticipation remaining encrypted until the final outcomes enable to decipher it.

Introduction

Bell's theorem [1] has dealt the final blow on all attempts to explain the EPR correlations [2] by invoking previously existing local hidden variables. While the EPR spin outcomes vary in accordance with the particular combination of spin-orientations chosen for each pair of measurements, Bell proved that the correlations between them are cosine-like and nonlinear (Eq. (1) hence these combinations cannot all co-exist in advance. Consequently, nonlocal effects between the two particles have been commonly accepted as the only remaining explanation.

It is possible, however, to explain the results without appeal to nonlocality, by allowing hidden variables to operate within the Two-State Vector Formalism (TSVF). The hidden variable would then be the *future* state-vector affecting weak measurements at present. Then, what appears to be nonlocal in *space* turns out to be perfectly local in *spacetime*.

Following is a proof for this account, of which a schematic example is given in Fig. 1. As this proof is bound to elicit searches for loopholes within it, we describe it elsewhere in greater detail and with several control experiments [12]. Here we describe its essential core.

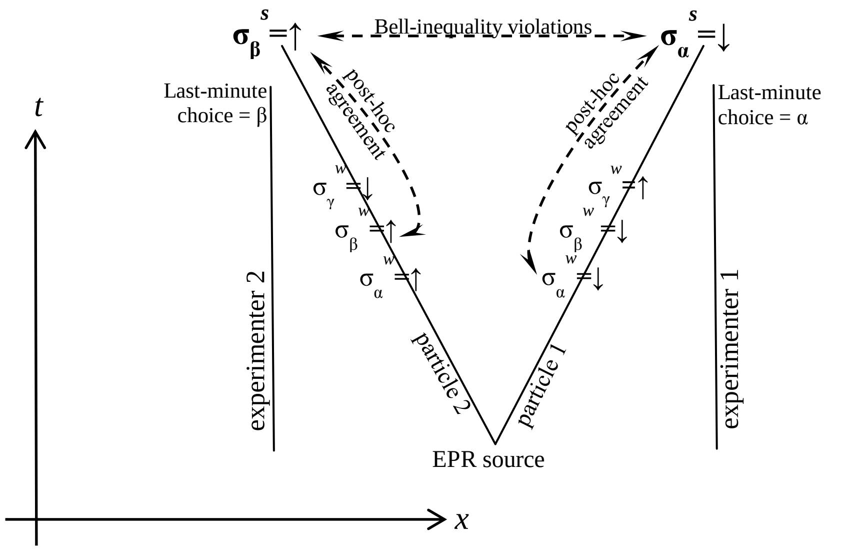

Fig. 1. Spacetime diagram of an EPR experiment where each particle undergoes three pre-set weak measurements and one freely-chosen strong one. The weak measurements seem to give early records of the strong measurements' results which, by Bell's proof, could not have existed prior to the strong measurements' choice.

This paper's outline is as follows. Sec. 1 introduces the foundations of TSVF and 2 introduces weak measurement. 3 describes a combination of strong and weak measurements on a single particle illustrating a prediction of TSVF. In 4 we proceed to the EPR-Bell version of this experiment. Secs.5-7 discuss and summarize the predicted outcomes' bearings.

1. The TSVF Formulation of a Particle's State between Two Noncommuting Spin Measurements

Consider a large ensemble of *N* particles, each undergoing two consecutive strong measurements, along the co-planar spin orientations α and β. The correlation between their outcomes depends on their relative angle $\theta_{\alpha\beta}$:

(1) $<\sigma_\alpha\sigma_\beta>=\cos\theta_{\alpha\beta}$.

Also, by the uncertainty relations between spin operators, these two measurements disturb each other's outcomes: If, *e.g.*, the *α* measurement is repeated after the β, with β being orthogonal to *α*, then *α* has an equal probability to give the opposite value.

ABL [4] argued that, at any time between the two measurements, the particle's state is equally determined by *both* of them. The probability for measuring the eigenvalue c_j of the observable *c*, given the initial and final states $|\psi(t')\rangle$and $\langle\Phi(t'')|$, respectively, is described by the symmetric formula:

(2) $$P(c_j)=\frac{\langle\Phi(t'')|c_j\rangle\langle c_j|\psi(t')\rangle}{\sum_i\langle\Phi(t'')|c_i\rangle\langle c_i|\psi(t')\rangle}.$$

The probability thus seems to have a definite value which agrees with both measurement outcomes, due to two state-vectors [3], one evolving from the past,

(3) $$|\psi(t')\rangle=\exp(\int_t^{t'}-iH/\hbar dt)|\psi(t)\rangle \; (t< t'),$$

and the other from the future:

(4) $$\langle\Phi(t'')|=\langle\Phi(t)|\exp(\int_t^{t''}iH/\hbar dt) \; (t''<t).$$

creating the two-vector

(5) $\langle \Phi(t'')| \ |\psi(t')\rangle$.

2. Weak Measurement

It is for the detection of such delicate intermediate states that weak measurement [5] has been conceived. Weak measurement couples each spin to a device whose pointer moves $\lambda/\sqrt{N}$ or $-\lambda/\sqrt{N}$ units upon measuring, respectively, ↑ or ↓ (Eq. (3). Let the pointer value have a Gaussian noise with 0 expectation and $\delta >> \frac{\lambda}{\sqrt{N}}$ standard deviation. When measuring a single spin, we get most of the results within the wide $\frac{\lambda}{\sqrt{N}} \pm \delta$ range, but when summing up the *N/2* results, we find most of them within the much narrower $\lambda\sqrt{N}/2 \pm \delta\sqrt{N}/\sqrt{2}$ range, thereby agreeing with the strong result when $\lambda >> \delta$.

Let an ensemble of *N* particles undergo an interaction Hamiltonian of the form

(6) $H_{\text{int}}(t) = \frac{\lambda}{\sqrt{N}} g(t) A_s P_d$,

where A_s denotes the measured observable and P_d is canonically conjugated momentum to Q_d, representing the measuring device's pointer position. The coupling *g(t)* is nonzero only for the time interval $0 \le t \le T$ and normalized according to

(7) $\int_0^T g(t) dt = 1$.

The measurement's weakness (and consequently strength) is due to the small factor $1/\sqrt{N}$ [1], inversely proportional to the ensemble's size.

When the N particles have different states, *e.g.*, spins, the weak measurement correctly gives their *average*. When all particles share the same ↑ or ↓ spin value along the same orientation, weak measurement correctly indicates that its outcome gives the *entire ensemble's* state. As pointed out in [4]:

(8) $\langle \bar{A} \rangle_w = \frac{1}{N} \sum_{i=1}^{N} \langle A^{(i)} \rangle_w = \langle \psi | A | \psi \rangle$,

i.e., $\bar{A}$'s weak value approaches the expectation value of A operating on $|\psi\rangle$. The weak measurement's operation thus guarantees its agreement with the strong measurement.

3. Combining Strong and Weak Measurements

We are now in a position to give a thought-experimental demonstration of the claim made in Sec. 12: A particle's state between two strong measurements carries both the past and future outcomes. Consider an ensemble of N particles. Then,

3.1. Procedure

1. On morning Bob strongly measures all particles' spins along the α-orientation. He measures them one by one and assigns them serial numbers.

2. On noon Alice weakly measures all particles' spins along the α and β orientations as well as a third coplanar orientation γ. Her measurements are similarly individual, each numbered particle measured in its turn, and the measuring device is calibrated before the next measurement. For reasons explained below, she repeats this series 3 times, total 9

[1]Weakness of 1/N is sufficient in this case where one measuring apparatus is used, but for the cases considered in the next sections we chose $1/\sqrt{N}$ interaction strength. See also [4] and [5].

weak measurements per each particle. All lists of outcomes are then publically recorded, *e.g.*, engraved on stone (Fig. 2), along 9 rows. Summing up her α-measurements (whether $\alpha^{(1)}$, $\alpha^{(2)}$,$\alpha^{(3)}$separately or all 3*N* together) she finds the spin distribution 50%↑-50%↓. Similarly for her β and γ measurements.

3. On evening Bob, *oblivious of Alice's noon outcomes*, again strongly measures all *N* particles, this time along the β orientation. He then draws a ‾‾|__|‾‾ binary line along his row of outcomes such that all ↑ outcomes are above the line and all ↓ outcomes below it.

4. Bob then gives Alice the two lists of his morning and evening outcomes. The lists are coded, such that *x/y/z* stand for α/β/γ and "*above line*"/"*below line*" for ↑/↓.

5. Based on Bob's lists, Alice slices her data, recorded since morning, according to Bob's divisions. In terms of Fig. 2, she merely shifts each of the ‾‾|__|‾‾ lines from Bob's lists to each of her 9 rows of outcomes carved on stone. Each of the 9 *N* rows is thereby split into two *N/2* sub-rows, one above and the other below the binary line, which she re-sums separately. This is done twice, first for the morning strong measurements' list and then for the evening one.

3.2. Predictions

Upon Alice's re-summing up her each of sliced lists, QM obliges the following:

1. Out of the 9 sliced rows of the weak measurements' outcomes, 3 immediately stand out with maximal correlation with Bob's *above/below*$_x$ list, indicating that x=α, *above*=↑ *below*=↓. Similarly for Bob's evening *above/below*$_y$ list: 3 other rows show that y=β, *above*=↑ *below*=↓. In short, all weak measurements agree with the strong ones, *whether performed before or after them*, to the extent that enables

Alice to know exactly which particle was subjected to which spin measurement by Bob, and what was the outcome.

2. Hence, all same-spin weak measurements confirm one another.

3. Even the third spin orientation weakly measured by Alice, γ, is correlated with *α* and β according to the same probabilistic relations (Eq. (1).

4. Even in case Bob's measurement is along an orientation other than α, β, or γ, Alice’s data can precisely reveal this orientations, as well as all the individual spin values, by employing the (1) relations.

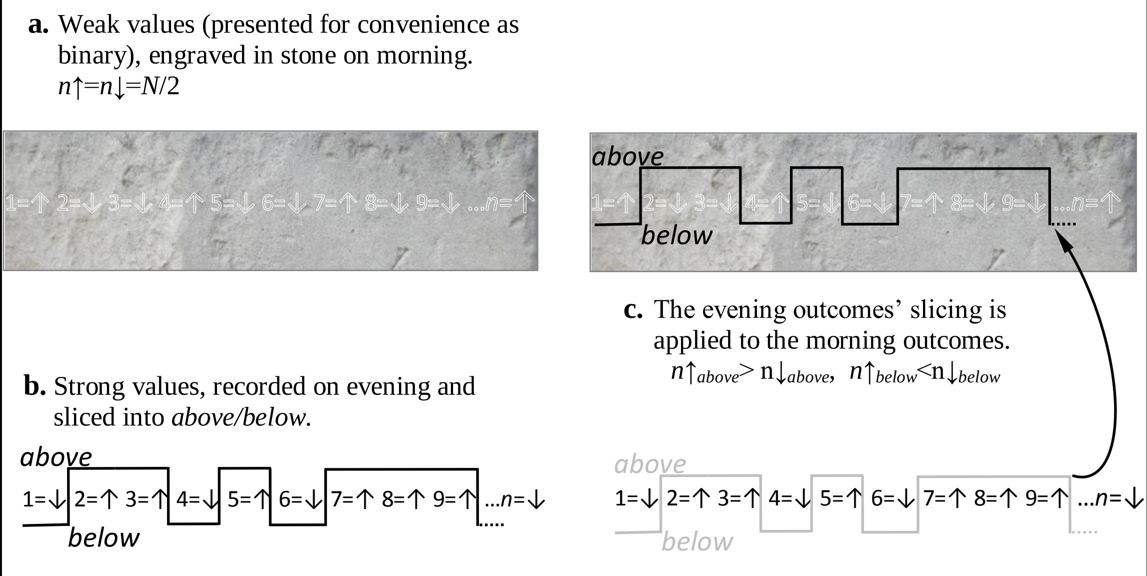

Fig. 2. The time-order of weak and strong measurements, indicating the latter’s outcomes presence within the former.

These predictions are unique in two respects. The weak measurements results precisely repeat themselves despite the fact that, for each pair of same-spin weak measurements, two noncommuting measurements were made between them. For example, the spin along the α-orientation remains the same upon the next weak spin α measurement despite the intermediate β and γ spin measurements.

Even more striking is the fact that all weak measurements equally agree with the past and future strong measurements. While it is not surprising that the noon weak measurements *confirm* the morning strong outcomes, it is certainly odd that they *anticipate* the evening ones.

This fully accords with the TSVF. Mainstream physics, however, would prefer a simpler explanation. Perhaps, *e.g.*, the weak measurements introduce some subtle kind of β collapse, hence the later strong β measurements' outcomes simply reaffirms it, despite the intermediate α and β weak measurements.

We have carefully considered this possibility elsewhere [6] and proved its inadequacy. Moreover, our next experiment would be much harder to account for along these one-vector lines.

4. Combining Strong and Weak Measurements in the EPR-Bell Experiment

We can now demonstrate the weak outcomes' anticipation of a future choice. Consider an EPR-Bell experiment [1,2] on an ensemble of *N* particle pairs.

4.1. Procedure

a. On morning, Alice carries out 9 weak measurements on each particle, 3 along each orientation, *α,* β and γ (with the coupling strength appropriately weakened). Every result is recorded, alongside with the pair's serial number among the *N*, the particle's identity (Right/Left) within the pair, and the weak measurement's number among the 9 (Fig. 3). The entire list is then engraved on stone (Fig. 2) along 9 rows.

b. On evening, Bob, oblivious of Alice's data, performs one strong spin measurement on each particle. For simplicity, he chooses only one spin-orientation for all right-hand particles and one for all left-hand ones. With sufficiently large *N*, he can choose a pair of measurements anew for each pair of particles. The crucial fact is this: *The spin orientations are chosen at the last moment by Bob's free choice*.

c. Bob sends Alice a list of his outcomes in which the spin orientations and values are coded: *x/y/z* for α/β/γ and *above/below* for ↑/↓.

d. Based on Bob's lists, Alice slices her data, carved on stone since morning, according to Bob's divisions, again shifting the binary line from each of Bob's lists to her rows, as in Sec 3.

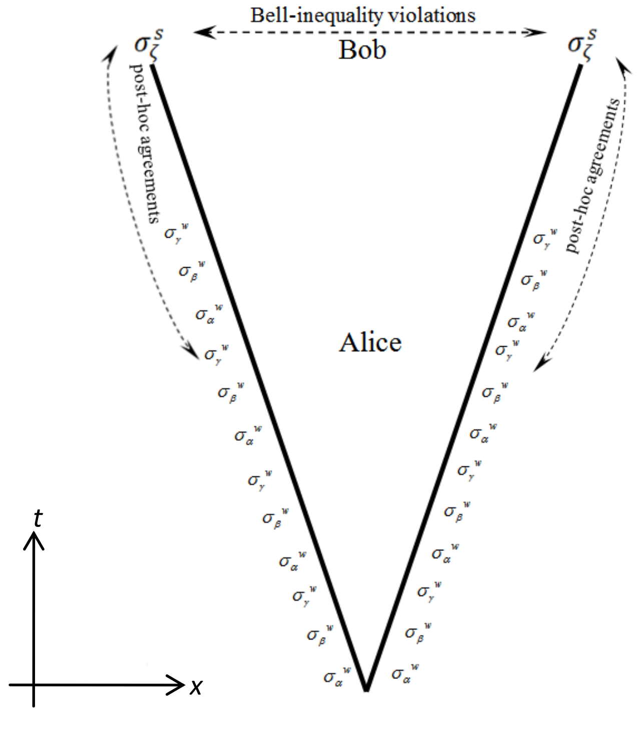

Fig. 3. An EPR setting with several weak measurements followed by strong ones**.**

4.2. Predictions

Calculating the new separate averages of each sub-ensemble, QM obliges the following (a statement about a weak measurement refers to its *overall* outcome):

1. Bob's strong measurements' outcomes exhibit the familiar Bell-inequality violations [1], indicating that their correlations could not be

formed locally and hence that the particles were superposed prior to his measurements.

2. Alice's weak outcomes strictly agree with those of the strong measurements, exhibiting similar Bell-correlations;
3. with the following addition: For each particle, all the strong measurements carried out on the other particle determine its spin as if they occurred *in its own past*, with the ↑/↓ sign inverted, regardless of the measurements' actual timing.

5. Will One Vector Do?

Naturally, more conservative interpretations ought to be considered before concluding that measurements' results anticipate a future event. By normal causality, it must be Alice's results which affected Bob's, rather than *vice versa*. It might be, for example, some subtle bias induced by her weak measurements later to affect his strong ones. In what follows we give normal causality due hearing and show its inadequacy.

A past-to-future effect can be straightforwardly ruled out by posing the following question: How robust is the alleged bias introduced by the weak measurements? *i*) If it is robust enough to *oblige* the strong measurements, then it is equivalent to full collapse, which is ruled out by the fact that the particles remain entangled. *ii*) On the other hand, a partial bias is equally ruled out by the predicted robust correlation between all same spin measurements, whether weak or strong.

Another way to disprove the one-vector account is by the following question: Can Alice predict Bob's outcomes on the basis of her own data? To do that, she must feed all her rows of outcomes into a computer that searches for a possible series of spin-orientation choices plus measurement outcomes, such that, when she slices her rows accordingly, she will get the complex pattern of correlations described above. The number of such possible sequences that she gets from her computation is $\binom{N}{N/2} \propto \frac{2^N}{\sqrt{N}}$. Each such sequence enables her to slice each of her rows into two $N/2$ halves and get the above correlations between her weak measurements and the predicted strong measurements. Notice that, according to Sec. 2, the results' distribution is a Gaussian with $\lambda\sqrt{N}/2$ expectation and $\delta\sqrt{N}/\sqrt{2}$ standard deviation, so a δ shift in one of the results, or even $\sqrt{N}$ of them, is very probable. Hence, even if Alice guesses right Bob's choices, she still cannot tell which results he would get because there are many similar subsets giving roughly the same value. Also, as Aharonov *et al.* pointed out in [3], when Alice finds a subset with a significant deviation, its origin is probably a measurement error rather than a specific physical value. Obviously, then, *present data is insufficient to predict the future*.

For Bob to make a genuine choice, in contrast, things are entirely different. *He needs not know anything about Alice's data, so his choice is not affected by it*. To see that, let us reverse the above guessing task and suppose that Bob does not make any measurement but misinforms Alice that he has done that. He thus fabricates a list of *x/y/z* choices well as *above*/*below* outcomes.

Can he do that? The probability goes to zero as long as he does not know Alice's data. Only if he has full access to it, and only with enormous computation, the fraud is possible. Even then, Bob gets many such possible sequences as was pointed out above. Moreover, even after such a fabricated sequence is given, Alice can expose it. For example, she can carry her own strong measurements on a few particle pairs. Then,

1. When her spin-orientation choices repeat the real measurements carried out by Bob, her outcomes must strictly be identical to his. Otherwise his list would turn out to be fraudulent.
2. When her choices differ from Bob's, her outcomes must deviate from his in accordance with (1), yet, as the particles must be disentangled after being measured, her outcomes *must not violate Bells inequality*. Otherwise, again, Bob's list would turn out to be fraudulent.

To summarize, any one-vector interpretation must deny Bob's choices of spin-orientation any real freedom, and moreover must ascribe the results of his measurements to the influence of Alice's outcomes. As we have shown, such convoluted effects can be easily ruled out. In contrast, the two-vector interpretation invokes only one direct effect, namely that of Bob's choice, *choice actually taken*, on Alice's myriad outcomes.

6. What Kind of Causality?

Regardless, therefore, of the above result's oddity from mainstream QM's view, they fully accord with the TSVF. Recall first the Bell proof:

For an entangled pair, no set of spin values can exist beforehand so as to give the predicted correlations for all possible choices of spin orientations to be measured.

Applied to our setting, this prohibition seems to allow only the following account:

1. On morning, several weak spin measurements were performed on N particles, resulting in an even ↑/↓ distribution. These outcomes were recorded, thereby becoming definite and irreversible.
2. Then on evening, all the particles were subjected to strong measurements, on spin orientations chosen randomly, hence *unknown beforehand,* even to the experimenter himself.

3. All these evening measurements exhibited Bell inequality violation within each pair.
4. Next, all the morning lists were sliced in accordance with the evening outcomes.
5. Unequivocal correlations emerged between all the morning and evening outcomes.
6. By Bell's theorem, the particle pairs could not have been correlated on morning for *whatever* possible spin-orientations that *may* be chosen to be measured on evening.
7. Neither could the strong measurements' outcomes have been determined by the weak measurements, for, in that case, the particles would be *disentangled* already on morning, failing to violate Bell's locality on evening.
8. Ergo, the weak measurements' agreement with the strong measurements could have been obtained only by the former *anticipating* the spin orientation *to be* chosen for the latter. This result indicates the existence of a hidden variable of a very subtle type, namely the future state-vector.

7. Summary

Our proof rests on two well-established findings: *i*) Bell's nonlocality theorem and *ii*) The causal asymmetry between weak and strong measurements.

The EPR-Bell experiment proves that one particle's spin outcome depends on the choice of the spin-orientation to be measured on the other particle, and its outcome thereof. Relativistic locality is not necessarily violated in this experiment, as it allows that it was either Alice whose choices affected Bob's, or *vice versa*.

This reciprocity, however, does not hold for a combination of measurements of which one is weak and the other strong. *The latter affects the former, never vice versa*. Therefore, when a weak measurement precedes a strong one, the only possible direction for the causal effect is from future to past.

We stress again that attempt to dismiss the weak measurement's peculiar outcomes by invoking some subtle collapse due to the weak measurement, or any other form of contaminating the initial superposed states, have been thoroughly considered and ruled out [1] [6] [7].

Also, while earlier predictions derived from the TSVF were sometimes dismissed as counterfactuals, *there is nothing counterfactual in the experiments proposed in this paper*. Our predictions refer to actual measurements whose outcomes are objectively recorded. Moreover, our experiment turns even the counterfactual part of the EPR experiment into an actual physical result: Prediction (3) in subsection 3.2 refers to a spin-orientation *not* eventually chosen for strong measurements, thereby being a mere "if" in the ordinary EPR experiment. In our setting, even this unperformed choice yields actual and even repeatable results through the weak measurements.

Finally, this experiment sheds a new light on the age-old question of free will. Apparently, a measurement's anticipation of a human choice made much later renders the choice fully deterministic, bound by earlier causes. One profound result, however, shows that this is not the case. The choice anticipated by the weak outcomes *can become known only after that choice is actually made*. This inaccessibility, which prevents all causal paradoxes like “killing one's grandfather,” secures human choice full freedom from both past and future constraints. A rigorous proof for this compatibility between TSVF and free choice is given elsewhere in detail [6].

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