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Another Example of a Credit System that Coexists with Money*

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Abstract

We study an economy in which exchange occurs pairwise, there is no commitment, and anonymous agents choose between random monetary trade or deterministic credit trade. To accomplish the latter, agents can exploit a costly technology that allows limited record-keeping and enforcement. An equilibrium with money and credit is shown to exist if the cost of using the technology is sufficiently small. Anonymity, record-keeping and enforcement limitations also permit some incidence of default, in equilibrium.

Key words: Credit, Default, Money, Search

JEL classification: E40, E50, E51, G20

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1 Introduction

This paper considers a variant of the monetary search model in Shi (1995) and Trejos and Wright (1995), in order to study the coexistence between money and credit in a decentralized trading environment. The variations considered partially relax some of the frictions that are assumed in the typical model. Specifically, we maintain the assumption of no commitment and pairwise matches, but we introduce a costly technology as in Camera (2000), which allows deterministic matches, some enforcement, and an improvement in record-keeping. So, 'credit-like' trades become possible among anonymous partners, and these trades can coexist with monetary exchange in equilibrium.

The model is as follows. If agents can produce during a period, then they can opt to exploit the costly technology instead of trading as in the typical monetary search model. The former option generates disutility for the period, but also allows these agents to be anonymously paired as a potential consumer or producer, in each of two consecutive periods. An agent's initial role is determined by a coin flip and is reversed in the second period. In a credit trade, initial producers (creditors) transfer consumption to their partners (debtors), and these promise to reciprocate with transfers to whomever will be their next-period partner. Debtors may not wish to meet their obligations, and default may arise due to limitations in record-keeping and some enforcement. Agents who did not keep promises in their last credit trade are assigned a *bad* credit record, as opposed to *good*. If they attempt another credit trade, then the technology may recognize them; this triggers a one-time utility sanction and resets their record to good.

The analysis shows that for sufficiently small costs of the technology, equilibrium outcomes arise in which credit trades coexist with monetary exchange. These outcomes, however, display some incidence of default. The intuition is as follows. In the model, transfers are assumed to satisfy take-it-or-leave-it offers from consumers to producers. So, transfers reflect producers' continuation payoffs, which depend not only on the enforcement parameters but also on the cost of credit trading. This cost must be sufficiently low or the expected return from credit would not be sufficiently attractive relative to monetary trade. The return from credit, however, cannot be too high or no one would sell for money. Thus, there must also be some default, which means that enforcement cannot be perfect. Indeed, we characterize the trade-off between the technology's cost and enforcement capabilities in sustaining equilibria with money and credit.

This analysis contributes to a monetary literature concerned with how the availability of credit affects allocations, and the role of money. In one strand of this literature, credit is sustained thanks to financial intermediaries that are introduced in otherwise competitive economies with frictions (e.g., see Azariadis et al., 2001, Jafarey and Rupert, 2001, or Bullard and Smith, 2003). In another strand, random meetings, anonymity, no commitment and enforcement limitations provide explicit microfoundations for money. Here, if individual trading histories are public, then any monetary allocation could be replicated without money, while there can be no credit if histories are private (Kocherlakota, 1998). To open the door to credit and money, some of this work introduces imperfect or partial knowledge of individual histories. For example, in Kocherlakota and Wallace (1998) individual histories are made public only with a lag, which lessens the threat of punishment for defectors and sustains equilibria with money and credit. In the mechanism design analysis of Cavalcanti and Wallace (1999a,b), instead, money (inside or outside) coexists with credit because only a subset of agents has public histories. Yet other examples on the coexistence of money and credit introduce what basically amounts to a limited participation friction in a prototypical banking sector (e.g., Cavalcanti et al. 1999, Williamson 1999, 2004), or alternatively consider various possibilities of long-term partnerships (e.g., Shi 1996, Li 2001, Corbae and Ritter 2004).

Our paper adds to this literature by providing a further example of coexistence of money and credit, though we do not employ a mechanism design analysis. Our framework is in the tradition of the microfoundations of money literature, and displays pairwise exchange, anonymity, private histories, no commitment, and enforcement limitations. Anonymity and no commitment open the door to default, and money is used in trade only by some agents, as in some of the existing studies that have introduced some knowledge of individual histories, limited participation, or long-term partnerships. Unlike those studies, credit-like trades in our model are made possible thanks to the introduction of a costly technology that improves upon the random meeting process and also permits some limited record-keeping and enforcement. It is these limitations, as well as anonymity, that let money coexist with credit in our model.

2 The Model

The basic layout combines the models in Shi (1995) and Trejos and Wright (1995) with the variation in Camera (2000). Time is discrete and continues forever and there is a unit-mass continuum of non-storable commodities and infinitely-lived individuals, who are anonymous and specialize in consumption/production. Agents and commodities are uniformly distributed among $N \geq 3$ different sets denoted $i = 1, ..., N$ and we refer to any agent from set i as agent i. Agent i consumes commodity i and can produce commodity $i+1$ (modulo N), has period utility $u(q)$ from $q \geq 0$ consumption and suffers disutility q from producing $q \geq 0$. Assume $u(q)$ is strictly increasing, concave, twice differentiable, $u(0) = 0$, $u'(0) = \infty$, and $u(q) \geq q$ for $q \in [0, \hat{q}]$. The common discount rate is $r > 0$.

Initially, a population fraction $m \in (0, 1)$ has one indivisible unit of fiat money, while the remaining agents can produce $q \geq 0$ units of their specific commodity. In order to avoid multiple asset holdings, we make the standard assumption that an agent can produce only if he has no money and can hold at most one unit of money. So, we call *producer* someone without money and *consumer* everyone else. At the end of the initial date, agents with money are free to discard it, in order to become producers.

There are two spatially separated trading sectors, denoted *spot* and *credit* market. Only producers can choose to trade in the credit market, while everyone else must trade in the spot market. The spot market is a standard search economy in which trade histories are unobservable, and there is neither commitment nor enforcement (e.g. Shi 1995 or Trejos and Wright 1995). In particular, meetings are random and such that the probability of a meeting is simply the sum of the population fraction of agents present in the spot market. For simplicity it is assumed that in every meeting a type i agent is matched with probability $\frac{1}{2}$ to (a randomly selected) agent of type $i+1$ and of type $i-1$ (mod. N) otherwise. So, even if all matches are single coincidence, meetings are difficult if few people trade in the spot market.

The credit market makes use of a technology that allows some partial record-keeping, better matching and limited enforcement. Producers who access it on date t suffer $\phi > 0$ disutility and must remain in it until the end of date $t+1$. The disutility ϕ is assumed to accrue upon entrance, because we interpret it as the cost of operating a technology that lessens some of the frictions present in the spot market. Specifically, the technology's features are as follows.

Record keeping. The technology keeps only a partial record of every agent's credit market history; it includes the last dates on which he entered and exited the credit market, his match on those dates and the actions taken. Given their record, agents can have one of four possible labels on date $t \geq 1$. Those who last entered the credit market on $t-1$ and are still present at the start of t, can be labeled either debtors or creditors. Agent i is a *debtor* if on $t-1$ he met some agent $i-1$, and is a *creditor* if he met some agent $i+1$. Agents who last exited the credit market at the end of date $\tau \leq t-1$ can be labeled as having either a bad or a good credit record, denoted $j = b, g$. This depends on their actions and labels on date τ. Agent i has a *good* record ($j = g$) if on date τ he was either a creditor, a debtor who produced for some creditor $i+1$, or was sanctioned for having a bad record (more below). He has a *bad* record ($j = b$) if on date τ he was a debtor who defaulted, i.e., did not produce for some creditor $i+1$. Thus, creditors who suffer a default do not consume in their second date on the credit market and start the following date as producers with credit record g.

Enforcement and matching. Consider credit market participants on date t. Some entered on date $t-1$ and some are new entrants. The former are debtors or creditors and the technology pairs each debtor of type i to a randomly selected creditor of type $i+1$. Those who entered now, on date t, are all producers and, upon entrance, the technology checks their credit record. Good records are correctly identified but a bad record is identified as good with probability $1-\theta \in (0,1)$. Those recognized as having a good record must stay in the market for two periods. On the first (date t) they are matched among themselves as in the spot market, so producer i meets a (randomly selected producer) type $i+1$ or $i-1$, according to a coin flip. Those recognized as having a bad record are imposed a one-time utility loss, their record is reset to g and must exit the credit market at the end of the period (they can return in the future).

Summing up, producers or money holders can trade in the spot market, as in the typical search monetary model. Producers can opt to engage in a two-period sequence of unilateral transfers on a credit market where a costly technology allows better matching as well as imperfect record-keeping and enforcement. A transfer received (given) in the first period represents a loan and makes the producer a debtor (creditor). Debtor i's repayment obligation is discharged in the second period via a transfer to any creditor $i+1$. Debtors may skip repayment but risk a future one-period utility sanction. Creditors who suffer a default do not consume for the period.

The assumed restrictions on record-keeping reduce the set of possible histories, the state space, hence complexity. Anonymity, enforcement and record-keeping limitations, instead, open the door to default and to the coexistence of money and credit.

3 Stationary Equilibria

To discuss the coexistence of money and credit we will restrict attention to subgame perfect equilibria in which (i) strategies are time-invariant and symmetric across agent types, (ii) both trading sectors are active, and (iii) money circulates on the spot market.[1]

At the beginning of each date an agent can be in one of six possible states. He can be in the credit market, as a creditor or a debtor. Or he can be outside the credit market as a producer or a money holder with (credit) record j. We say that the agent is a 'defaulter' if $j = b$. Let $G_p$and G_m denote the beginning-of-period stationary population fractions (for any agent type i) of producers and money holders with record $j = g$; for defaulters we use B_p, B_m. On the credit market, the population fractions of creditors and debtors are denoted P_c and P_d. Letting $\alpha \in [0, 1]$ be the equilibrium probability that the representative debtor repays the debt, we have that the population fraction αP_c represents creditors who get repayment, and $(1 - \alpha) P_c$ are creditors who suffer a default.

In (monetary) equilibrium we must have

$$\begin{aligned} m &= G_m + B_m, \\ 1 - m &= G_p + B_p + P_d + P_c. \end{aligned} \tag{1}$$

After the start of a period, producers choose a market. Let $\sigma_j \in [0, 1]$ be the probability that the representative producer with record j selects the credit market. So, the population fraction

$$P_p \equiv G_p(1 - \sigma_g) + B_p(1 - \sigma_b),$$

comprises spot market producers. The fraction $G_p\sigma_g + B_p\sigma_b(1 - \theta)$ includes producers who enter the credit market and are recognized as having a good record. During each period, these agents are equally likely to become creditors or debtors, so

$$P_c = P_d = P \equiv \frac{G_p\sigma_g + B_p\sigma_b(1 - \theta)}{2}.$$

[1]Previous research suggests that several types of stationary outcomes can arise in the model, with one or both markets active, depending on the size of ϕ (see Camera, 2000).

The population fraction $B_p \sigma_b \theta$ is producers with record b who enter the credit market and are sanctioned. Clearly, if both sectors are active and money is valued, then we must have $P \in (0, 1-m)$.

3.1 Value Function

In the credit market, let q_c and q_d denote the first and second period equilibrium transfers between producers, i.e., the loan and the repayment; let q_p denote the utility penalty imposed on producers found to have a bad record. In the spot market, let q_m denote the commodities that trade for money in equilibrium.

Denote the stationary end-of-period expected lifetime utility as $V_{j,k}$ for an agent who starts next period in state (j, k) with $k = p, m$ (producer or money holder); use V_d and V_c for those starting next period as debtors or creditors. Also, let Π_j^S and Π_j denote (expected) trade surpluses, in the spot and credit market, to producers with record j. In equilibrium

$$\begin{aligned}
rV_{j,p} &= \sigma_j(\Pi_j - \Pi_j^S) + \Pi_j^S \\
rV_d &= \max(-q_d + V_{g,p} - V_d, V_{b,p} - V_d) \\
rV_c &= V_{g,p} - V_c + \alpha u(q_d) \\
rV_{j,m} &= \tfrac{P_p}{2}[u(q_m) + V_{j,p} - V_{j,m}]
\end{aligned} \tag{2}$$

The right hand sides of these functional equations display expected flow returns from trade. The first line shows that, at the start of any date, a producer with record j earns Π_j^S surplus on the spot market. Entering the credit market (with probability σ_j) gives $\Pi_j - \Pi_j^S$ surplus. The next two lines show that a debtor can repay q_d or can default, and then leaves the credit market; a creditor gets repayment q_d with probability α and starts next date as a producer with record g. The last line is the value to having a money to those with record j; a spot trade consumption opportunity arises with probability $\frac{P_p}{2}$, and the agent buys q_m consumption.

In the expressions above

$$\begin{aligned}
\Pi_j^S &= \tfrac{m}{2}(V_{j,m} - q_m - V_{j,p}), \\
\Pi_g &= -\phi + \tfrac{1}{2}(V_d + u(q_c) - V_{g,p}) + \tfrac{1}{2}(V_c - q_c - V_{g,p}), \\
\Pi_b &= -\phi + (1-\theta)[\tfrac{1}{2}(V_d + u(q_c) - V_{b,p}) + \tfrac{1}{2}(V_c - q_c - V_{b,p})] + \theta(V_{g,p} - q_p - V_{b,p}).
\end{aligned} \tag{3}$$

The expected surplus to a spot market producer, Π_j^S, depends on his record j, the strategies of others, and the distribution of agents. With probability $\frac{m}{2}$ he meets a buyer with money;

voluntary selling generates net payoff $V_{j,m} - q_m - V_{j,p} \geq 0$. In all other encounters, the producer's net payoff is zero. The second and third lines describe the expected surplus to producers with record g and b who enter the credit market, given $\sigma_j > 0$ for some j. After suffering disutility ϕ their credit records are checked. With probability θ a defaulter is discovered, sanctioned q_p and leaves the market. Unrecognized defaulters are free to trade as producers g, and are equally likely to consume or produce.

3.2 Terms of trade, best responses and distribution of agents

Terms of trade are determined via bilateral negotiations assumed to satisfy take-it-or-leave-it (TOL) offers from consumers to producers. On the spot market trading histories are private information so the equilibrium offer q_m cannot depend on producers' records, unless their distribution across markets is degenerate. Hence, offers may leave unequal surpluses to producers with different records. To see why, a buyer with money selects $q_m \in \{q_{g,m}, q_{b,m}\}$ where

$$q_{j,m} = V_{j,m} - V_{j,p}, \quad \text{for } j = b, g$$

i.e., the optimal offer leaves no surplus to at least some producers. No other offer can increase the probability of a purchase, without decreasing the buyer's expected gain. Since all buyers face the same matching probabilities, q_m is independent of the buyer's record.

The optimal offer q_m of a buyer with credit record j' is unique and must maximize his expected surplus, contingent on a random match with a producer:

$$q_m = \arg\max\{[V_{j',p} + u(x_m) - V_{j',m}](B_p \mathbf{1}_b + G_p \mathbf{1}_g) : x_m = q_{g,m}, q_{b,m}\} \tag{4}$$

where $\mathbf{1}_j = 1$ if $V_{j,m} - x_m \geq V_{j,p}$ and 0 otherwise.

Now consider the credit market. Only producers recognized as g can trade, so

$$q_c = V_c - V_{g,p} \quad \text{and} \quad q_d = V_{g,p} - V_d, \tag{5}$$

because of TOL offers. Thus, if $V_{g,p} > V_{b,p}$, then the undetected defaulters may earn surplus from lending. We also define the utility sanction by q_p where

$$q_p = V_{g,p} - V_{b,p}. \tag{6}$$

So, discovered defaulters simply earn no surplus for the period.

Now let $\{\alpha', \sigma_j'\}$ denote the representative agent's best responses given that everybody else selects $\{\alpha, \sigma_j\}$ and the terms of trade are as above. Individual optimality requires

$$\begin{aligned} &\alpha' = \arg\max\{x(V_{g,p} - q_d - V_{b,p}) : x \in [0,1]\}, \\ &\sigma_j' = \arg\max\{x(\Pi_j - \Pi_j^S) : x \in [0,1]\}, \\ &u(q_m) + V_{j,p} \geq V_{j,m}. \end{aligned} \tag{7}$$

The last inequality ensures that it is optimal to spend money, instead of holding on to it.

The population fraction G_p is time-invariant if

$$B_p\sigma_b\theta + \alpha P_d + P_c + G_m\frac{P_p}{2} - G_p\sigma_g - G_p(1-\sigma_g)\frac{m}{2} = 0 \tag{8}$$

The inflows include four terms. The term $B_p\sigma_b\theta$ is the sanctioned producers b (who become g); $\alpha P_d + P_c$ accounts for debtors who do not default and creditors since they both become producers g; G_m accounts for money holders g who spend money in the spot market. The outflows are producers g who choose credit and those who sell for money. Time-invariance of B_p requires

$$(1-\alpha)\,P_d + B_m\frac{P_p}{2} - B_p\sigma_b - B_p(1-\sigma_b)\frac{m}{2} = 0, \tag{9}$$

where $(1-\alpha)\,P_d$ are debtors who default and become producers b and $B_p\sigma_b$ accounts for the fact that every producer b who enters the credit market changes state, either due to sanctioning or confusion over his record. Finally, G_m and B_m are time invariant if

$$\begin{aligned} G_p(1-\sigma_g)\frac{m}{2} - G_m\frac{P_p}{2} = 0, \\ B_p(1-\sigma_b)\frac{m}{2} - B_m\frac{P_p}{2} = 0. \end{aligned} \tag{10}$$

We are now ready to present a definition of equilibrium.

Definition 1 *Given (θ, ϕ), a symmetric stationary equilibrium with coexistence of money and credit (an equilibrium, for short) is a list of strategies $\{\alpha, \sigma_b, \sigma_g\}$, quantities $\{q_c, q_d, q_m, q_p\}$, value functions $\{V_c, V_d, V_{j,m}, V_{j,p}\}_{j=b,g}$, and distribution of agents $\{P_c, P_d, G_k, B_k\}_{k=m,p}$ that satisfy (1)-(10), $P \in (0, 1-m)$, and $\{\alpha', \sigma_b', \sigma_g'\} = \{\alpha, \sigma_b, \sigma_g\}$.*

4 The coexistence of money and credit

In equilibrium money and credit coexist, so we must have $q_c > 0$ (there must be credit) and $0 < P_p < 1-m$ (there must be producers on both markets). Several equilibrium strategy vectors

$\{\alpha, \sigma_b, \sigma_g\}$ are possible. The next result narrows down the set of strategies compatible with this equilibrium (all proofs are in the appendix).

Lemma 1 *If an equilibrium exists, then it must have the following characteristics: (i)* $\alpha \in (0,1)$ *and* $0 < \sigma_b < \sigma_g = 1$*; (ii)* $V_c > V_{g,p} > V_{b,p} = V_d = 0$ *and* $V_{g,m} > V_{b,m} > 0$*; and (iii)* $q_m = V_{b,m}$.

Two features of the equilibrium stand out. First, if money and credit coexist, then there must be some default. Intuitively, deterministic credit trade is preferable to random spot trade. This implies that a good credit record is valuable, as it ensures unfettered access to deterministic trading. It also implies that $\alpha < 1$. If, in fact, debt-repayment is always individually optimal, then the resulting absence of default removes any incentive to trade on the spot market. Of course $\alpha > 0$, or lending would be suboptimal due to the absence of any future repayment. To see why, notice that certain default cannot be an equilibrium because discounting would imply $V_c < V_{g,p}$, hence $q_c < 0$.

Second, only producers with credit record b trade in the spot market and hold all the money. Due to take-it-or-leave-it offers, the value to being a producer b must thus be zero, so the value of money must satisfy $q_m = V_{b,m} > 0$. The latter is uniquely defined by the solution to

$$q_m - \tfrac{P_p}{2r+P_p} u(q_m) = 0, \tag{11}$$

which satisfies the individual optimality condition $u(q_m) + V_{j,p} \geq V_{j,m}$. Note also that in equilibrium producers g would not sell for money on the spot market because $V_{b,m} < V_{g,m}$. We are now in a position to discuss existence of equilibrium.

Proposition 1 *If* ϕ *is sufficiently small, then an equilibrium with money and credit exists. In equilibrium the loan and repayment amounts* q_c *and* q_d *must satisfy*

$$u(q_c) = \tfrac{\phi[2(1+r)-\theta]}{(1+r)(1-\theta)} \quad \textit{and} \quad q_d = \tfrac{\phi\theta}{(1+r)(1-\theta)}. \tag{12}$$

The central result is that an equilibrium with money and credit exists as long as the technology used to sustain credit trading is sufficiently inexpensive. Credit, in this case, is granted despite the fact that there is some default in equilibrium and bad loans have no residual value. The intuition is simple. Producers trade off the direct cost generated by credit trades with the indirect cost associated to random monetary spot exchange. If credit market transactions are cheap, then there is a threshold level of default below which those who have a good record strictly prefer to avoid random monetary trade. Those who have defaulted, instead, are simply indifferent to trading locations; their continuation payoff is zero whether they sell for money (due to TOL offers) or if they attempt to get credit (due to enforcement). This last feature, explains why there is default in equilibrium. The continuation payoff to those who repay debts is zero (due to TOL offers), so they are indifferent to defaulting.

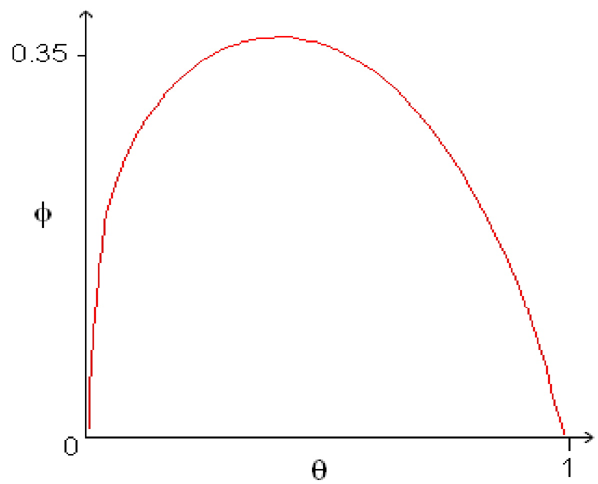

Figure 1

We illustrate our findings with the help of Figure 1 drawn for $u(q) = \sqrt{q}$ and $r = .01$. The area under the curve indicates regions of the parameter space (ϕ, θ) under which an equilibrium is possible. Note that ϕ must be sufficiently small, and that, given ϕ, we must have $\theta \in (\underline{\theta}, \overline{\theta}) \subset (0, 1)$. The reason for this latter finding is that as $\theta \to 0$ then there is no sanctioning for default, so $\alpha > 0$ cannot be individually optimal. The opposite is true when $\theta \to 1$, since defaulters would never access the credit market. Notice also that, for this parameterization, there can be two values of θ that are consistent with the same equilibrium default rate α. Intuitively, all else equal, the more difficult is to sanction defaulters, i.e., the lower is θ, the greater is the incentive for debtors to not reciprocate for the original transfer (loan). This, in turn reduces the incentive to lend. We see from (12) that both q_c and q_d fall as θ falls. It follows that if θ is low, a given α can be sustained in equilibrium only if the transfers are also low. If θ is high, instead, defaulters are sanctioned sufficiently often that the amount of credit granted can be much higher.

5 Final remarks

This article provides a further example of the coexistence of money and credit in an economy with frictions and pairwise exchange. The model is in the tradition of the microfoundations of money literature, and it assumes pairwise exchange among anonymous agents who face commitment, enforcement and informational limitations. Previous work has explored the coexistence of money and credit in similar frameworks by either introducing imperfect public knowledge of individual histories, or limited participation, or the possibility of long-term partnerships. In our model, instead, credit-like trades are made possible thanks to the introduction of a costly technology that can be freely selected by any agent who can produce. This technology improves upon the random meeting process and also permits some limited record-keeping and enforcement. It is these limitations, as well as anonymity, that allow money to coexist with credit in our model.

We conclude with three comments. First, the model admits multiple equilibria; for instance, there can be outcomes in which there is only credit or only money. Second, changes in the quantity of money do not affect equilibrium consumption on the credit market (see (12)), for two reasons. Changes in money do not meaningfully affect the outside options of credit market participants since producers g never sell for money and producers b never earn surplus by selling for money (due to TOL bargaining). Changes in money also do not affect credit meetings, as these do not depend on the proportion of market participants. Finally, we conjecture that if bad loans had some residual value, then repayment could be unnecessary to sustain credit. Suppose, for instance, that creditors who suffer a default could receive some small consumption $q < q_d$ at a later date. This could be accomplished by forcing discovered defaulters to produce or by taxing all credit market participants. If agents are patient enough, then even the certainty of default could sustain an equilibrium with money and credit. In addition, bad loans could conceivably circulate as a form of valuable inside money, even if their residual worth is minimal.

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Appendix

Proof of Lemma 1

Consider outcomes as in Definition 1 and use the expressions in (2)-(3).

1. Clearly, $q_c > 0$, or no one would enter the credit market because $\phi > 0$. Since $q_c = V_c - V_{g,p}$, then we need $V_c > V_{g,p}$. We must also have $\alpha > 0$. If in fact $\alpha = 0$, then $V_c = \frac{V_{g,p}}{1+r}$ and so $q_c = V_c - V_{g,p} < 0$, a contradiction. For $\alpha > 0$ to be individually optimal we need $-q_d + V_{g,p} \geq V_{b,p}$; since $q_d = V_{g,p} - V_d$, then it follows that $V_d = 0$ and so $V_{b,p} = 0$. The latter also implies $\Pi_b^S = \Pi_b = 0$. Clearly, $\Pi_b^S = 0$ requires $q_m = V_{b,m}$ (since $V_{b,p} = 0$).

2. Proving that $\alpha < 1$. Suppose that, in fact, $\alpha = 1$. Here all producers have record g. In order for money to have value some producers g must sell on the spot market, so we need $\sigma_g \in (0,1)$, which requires $\Pi_g = \Pi_g^S = rV_{g,p}$. Take-it-or-leave-it offers then imply $q_m = V_{g,m} - V_{g,p}$; so, $\Pi_g^S = 0 = \Pi_g = V_{g,p}$. But then, since $q_d = V_{g,p} - V_d = V_{g,p}$, we would have $q_d = 0$, which implies $V_c = \frac{V_{g,p}}{1+r} = 0$ and so $q_c = 0$, a contradiction. Therefore we must have $\alpha \in (0,1)$.

3. Proving that $\sigma_j > 0$ for $j = b, g$. When $\alpha \in (0,1)$ we have $B_p, G_p > 0$ and, clearly, we cannot have $\sigma_b = \sigma_g = 0$. Suppose that $\sigma_g > \sigma_b = 0$. This contradicts the stationarity condition (9), since the fraction of producers b would increase over time (due to default and absence of sanctioning). A similar contradiction arises if $\sigma_b > \sigma_g = 0$. Hence, we must have $\sigma_j > 0$ for $j = b, g$. The latter implies $rV_{g,p} = \Pi_g$. Of course, $V_{g,p} > 0$ or else we would have $q_d = 0$, and so $V_c = q_c = 0$, which is not an equilibrium.

4. Proving that $\sigma_g = 1$ and $\sigma_b \in (0,1)$. Recall that $\alpha \in (0,1)$ and $V_{b,p} = \Pi_b = \Pi_b^S = 0$. Suppose that, in fact, $\sigma_g \in (0,1)$. In this case we must have $\Pi_g^S = \Pi_g = rV_{g,p}$, by individual optimality. Using (2), we get $V_{g,m} - V_{b,m} \leq V_{g,p}$ whenever $V_{g,p} \geq 0 = V_{b,p}$. Hence, $\Pi_g^S = \frac{m}{2}(V_{g,m} - V_{b,m} - V_{g,p}) \leq 0$, which implies $V_{g,p} \leq 0$, a contradiction. To prove $\sigma_b \in (0,1)$ suppose that, in fact, $\sigma_b = 1$. Then $P_p = 0$ because $\sigma_g = 1$. So $V_{j,m} = 0$ for $j = b, g$, which is not an equilibrium. Hence, $P_p = B_p(1 - \sigma_b)$, $G_m = 0$ and $B_m = m$. It is immediate from (2) that $V_{g,m} > V_{b,m} > 0$ for $q_m > 0$ satisfying (11).

Proof of Proposition 1

Use Lemma 1 and note that in equilibrium $V_{g,p} = \frac{u(q_c)-2\phi}{1+2r} < u(q_c)$. The last line in (3) indicates that $\Pi_b = 0$ if $\frac{2\phi}{1-\theta} - V_{g,p} = u(q_c)$. Substitute for $V_{g,p}$ to get

$$u(q_c) = \tfrac{\phi[2(1+r)-\theta]}{(1+r)(1-\theta)},$$

so $u(q_c) > 2\phi$ for all r, θ, and $V_{g,p} > 0$. We conclude that the equilibrium q_c solving (12) is unique, positive, and increases in θ, ϕ, r. Given (12) we obtain

$$V_{g,p} = \tfrac{\phi\theta}{(1+r)(1-\theta)}. \tag{13}$$

From the third line in (2) we must have $\alpha u(q_d) > V_c - V_{g,p} = q_c$, or else $V_c < 0$. Since $q_d = V_{g,p}$, we need $\alpha u(V_{g,p}) > q_c$. Given $\alpha \in (0,1)$, note that $\alpha u(V_{g,p}) - q_c$ is hump-shaped in ϕ and vanishes for $\phi = 0$, which is when $V_{g,p} = q_c = 0$. Note also that q_c and $V_{g,p}$ increase in ϕ. By concavity of u and $u'(0) = \infty$, we have $\alpha u(V_{g,p}) > q_c$ if ϕ is sufficiently small.

In equilibrium $\sigma_g = 1$ is individually optimal if $\Pi_g \geq \Pi_g^S$. Suppose a producer with record g deviates and enters the spot market. Then, $\Pi_g^S = 0$ since he would not sell for money. To see it, notice that from (2) and (11) we have

$$V_{g,m} - V_{g,p} \quad = \tfrac{P_p}{2r+P_p}u(q_m) - \tfrac{2r}{2r+P_p}V_{g,p} \quad = q_m - \tfrac{2r}{2r+P_p}V_{g,p},$$

which implies $V_{g,m} - q_m < V_{g,p}$. Since $rV_{g,p} = \Pi_g > 0$ in equilibrium, then $\Pi_g > \Pi_g^S$.

To complete the proof, note that $\alpha \in (0,1)$ must satisfy $q_c = V_c - V_{g,p} > 0$. Using $q_d = V_{g,p}$ and V_c from (2) we get

$$\begin{aligned} V_c - V_{g,p} &= \tfrac{1}{1+r}[V_{g,p} + \alpha u(V_{g,p})] - V_{g,p} \\ \Rightarrow \qquad q_c &= \tfrac{1}{1+r}[\alpha u(V_{g,p}) - rV_{g,p}]. \end{aligned} \tag{14}$$

In equilibrium $\alpha \in (0,1)$ must satisfy (14) given that q_c satisfies (12) and $V_{g,p}$ satisfies (13). The unique q_c and $V_{g,p}$ that satisfy (12) and (13), are positive and constant in α. Define the RHS of (14) by the continuous function $f(\alpha; \theta, \phi)$, increasing in α. Clearly $f(0; \theta, \phi) < 0 < f(1; \theta, \phi)$. Also, $f(1; \theta, \phi)$ is hump-shaped in $V_{g,p}$, vanishing at $V_{g,p} = 0$ and at some other value $V_{g,p} > 0$. Recall that q_c and $V_{g,p}$ increase in ϕ as well as θ and that $q_c, V_{g,p} \to 0$ as $\phi \to 0$ for any given $\theta \in (0,1)$. Since u is concave and $u'(0) = \infty$, it follows that, given $\theta \in (0,1)$, we have $q_c < f(1; \theta, \phi)$ for $\phi > 0$ sufficiently small and $q_c \geq f(1; \theta, \phi)$ for ϕ sufficiently large. So, there are sufficiently small positive values ϕ, which depend on θ, such that $f(0;\theta,\phi) < q_c < f(1;\theta,\phi)$. In that case, the intermediate value theorem implies that there exists a unique value of $\alpha \in (0,1)$ such that $q_c = f(\alpha;\theta,\phi)$.

Now observe that $V_{g,p} \to 0$ as $\theta \to 0$ and $\frac{\partial V_{g,p}}{\partial \theta} > 0$; so, given $\phi > 0$ we have that $f(\alpha;\theta,\phi)$ is hump-shaped in θ, vanishing at $\theta = 0$ and at some other value $\theta < 1$. Hence, given $\phi > 0$, there are two values of $\theta \in (0,1)$ that solve $q_c = f(\alpha;\theta,\phi)$ because $f(\alpha;0,\phi) = 0 < q_c|_{\theta=0}$ while, for θ sufficiently close to one, we have $q_c > f(\alpha;\theta,\phi) = 0$. Using the implicit function theorem and (14), we have that $\frac{d\alpha}{d\theta} < 0$, for the low value of θ, and $\frac{d\alpha}{d\theta} > 0$ otherwise.

Once we have α, we obtain σ_b by means of the laws of motion. Clearly, $P = P^* = \frac{G_p + B_p\sigma_b(1-\theta)}{2}$, $P_p = B_p(1-\sigma_b)$, $G_m = 0$ and $B_m = m$. From (9) we get $P(1-\alpha) = B_p\sigma_b$ and so, using P, we get $G_p = G_p^*$ with

$$G_p^* = \frac{B_p\sigma_b[2-(1-\alpha)(1-\theta)]}{1-\alpha}.$$

It is easy to see that (8) is always satisfied by $P = P^*$ and $G_p = G_p^*$. Finally, the constraint (1) with $P = P^*$ and $G_p = G_p^*$ gives us

$$1 - m = B_p\{1 + \sigma_b[3 + \theta + \alpha(1-3\theta)]\}.$$

There is a continuum of pairs $(B_p, \sigma_b) \in (0, 1-m) \times (0,1)$ that satisfies the above. As (B_p, σ_b) change, then q_m changes since P_p changes. Thus the equilibrium pairs (B_p, σ_b) must satisfy $V_{g,m} \geq V_{g,p}$, i.e., those with money at the end of the initial date (who have never defaulted in the past) keep it instead of becoming a producer g. The inequality gives

$$\frac{P_p}{2r+P_p}[u(q_m) + V_{g,p}] \geq V_{g,p},$$

which is satisfied if ϕ is sufficiently small, i.e., if $V_{g,p}$ is sufficiently small.