4-28-2016

Salience, Framing, and Decisions under Risk, Uncertainty, and Time

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Comments
Working Paper 16-08

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Abstract

We propose a salience-based model of framing effects for decisions under risk, uncertainty, and time. The model formalizes a class of matrix-based frames and identifies two important types of frames: *minimal frames* which provide the simplest representation of choices, and *transparent frames* which make the appeal of classical rationality axioms more transparent. The model predicts violations of rational choice theory (e.g., the Allais paradox, Ellsberg paradox, present bias) when options are represented in minimal frames, but predicts behavior to be more consistent with the classical axioms when the same choices are presented in transparent frames, consistent with recent experimental evidence.

Keywords: *Salience; Allais Paradox; Ellsberg Paradox; Present Bias; Framing Effects; Rationality*

[1] M. Birnbaum, E. Brandstatter, A. Glockner, E. Johnson, K. Katsikopoulos, M. Nunez, M. Regenwetter, and N. Wilcox provided helpful comments on earlier drafts. Research for this manuscript was conducted while Jonathan Leland was on detail as Senior Fellow, Office of Research, Consumer Financial Protection Bureau. The views expressed are those of the authors and do not necessarily reflect those of the National Science Foundation, the Consumer Financial Protection Bureau or the United States.

1. Introduction

In 1937 Paul Samuelson proposed that individuals might choose between consumption plans such that they maximize their discounted utility (DU) from future consumption. In 1947, von Neumann and Morgenstern proposed that people choose between risky prospects *as if* they maximize their expected utility (EU). In 1954, Savage proposed the theory of subjective expected utility (SEU) for decisions under uncertainty where probabilities are not objectively known. To this day, these models constitute the standard theories of rational choice over time, under risk, and under uncertainty. Nevertheless, almost from their inception, persistent questions have been raised regarding their descriptive accuracy.

Strotz (1955) critiqued Discounted Utility theory, expressing concern that people would be unable to commit themselves to future plans resulting in *time-inconsistent* choices. Allais (1953) posed a challenge to Expected Utility theory, providing intuitive situations where people systematically violate the independence axiom in decisions involving a sure-thing and a lottery. Ellsberg (1961) presented evidence on choices between risky and uncertain prospects indicating that people may not have coherent beliefs representable by subjective probabilities but rather exhibit *ambiguity aversion.* These and subsequent challenges to the descriptive adequacy of EU, SEU, and DU have defined the research agenda on individual decision making over the past five decades. Researchers have proposed a number of alternative models of risky choice, other models to accommodate anomalies in intertemporal choice, and still others to explain ambiguity aversion. In this paper, we provide a more unified model of decision making across domains that simultaneously bridges the gap between the EU, SEU and DU models and their respective critiques by Allais, Ellsberg, Strotz, and others.

We begin by considering risky and intertemporal choices between a dominating and a dominated alternative in Section 2. Evidence demonstrates that when choices are framed so as to make it possible to identify the dominance relationship through simple cross-prospect comparisons, people choose the dominating alternative. However, for choices framed so as to obscure the dominance relationship, salient attribute differences across alternatives that favor the dominated alternative can lead people to systematically violate dominance.

To account for this type of behavior, in Section 3, we introduce comparative models of expected utility, subjective expected utility, and discounted utility in which agents choose based on an evaluation procedure that involves cross-lottery comparisons of payoffs and probabilities or payoffs and time periods, respectively. We generalize these models under the assumption that large differences in payoffs, probabilities and times delays are perceived as particularly salient and systematically overweighted in the evaluation process[2]. We draw on empirical evidence regarding properties characterizing human perception of numerical differences to specify properties of salience functions.

The predictions of a model of choice that involves comparisons of magnitudes – payoffs, probabilities, dates of receipt or payment – will be sensitive to which attributes are being compared.[3] This, in turn, depends on how the choices are framed. As such, we supplement our choice model with a theory of the framing of alternatives. Here we define two approaches to representing alternatives that each apply across decision domains. One approach, which we term a *minimal frame*, provides the simplest representation of choices and formalizes the 'prospect' presentation of lotteries pioneered in experiments by Kahneman and Tversky (1979). A second approach to framing alternatives, which we term a *transparent frame*, makes the normative appeal of the classical rationality axioms more transparent, and formalizes Savage's (1954) state matrix representation of lotteries, but without assuming statistical independence or correlation between payoffs. Taken together, our model of the choice process, characterization of the perceptual system and treatment of frames constitute the model that we refer to as Salience Weighted Utility over Presentations (SWUP).

The remainder of the paper examines implications of the model. In Section 4, we discuss framing effects predicted by SWUP between minimal and transparent frames. Specifically, we show that under general conditions, the SWUP model predicts that people will exhibit the classical anomalies for choices presented in minimal frames, but obey the axioms of rational choice when the options are presented in transparent frames. In particular, the same transparent frames shown to reduce or eliminate the violations of dominance discussed in Section 2 are predicted to reduce common consequence and common ratio violations of expected utility, the Ellsberg paradox, and present bias. We review experimental evidence supporting these predictions. Section 5 provides a general analysis of rational behavior predicted in transparent

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[2] This approach to comparative, context-dependent evaluation is related to work by Gonzalez-Vallejo (2002), and Scholten and Read (2010), among others in psychology, and Loomes and Sugden (1982), Rubinstein (1988, 2003), Leland (1994, 1998 and 2002), Loomes (2010), Bordalo et al (2012) and Koszegi and Szeidl (2013), among others, in economics. Models of risky choice by Reyna and Brainerd (1991) and research on heuristics by Gigerenzer et al (1999) are also related.

[3] For discussions and demonstrations of this point, see Leland (1994, 1998, 2010).

frames. Section 6 considers the most famous example of a framing effect in the literature. Section 7 discusses the related literature. Section 8 concludes. Proofs are provided in the appendix.

2. A Motivating Example

Tversky and Kahneman (1986) presented experimental subjects with the following decision:

Consider the following two lotteries, described by the percentage of marbles of different colors in each box and the amount of money you win or lose depending on the color of a randomly drawn marble. Which lottery do you prefer?

Option	*90% white*	*6% red*	*1% green*	*1% blue*	*2% yellow*
A	*$0*	*win $45*	*win $30*	*lose $15*	*lose $15*

Option	*90% white*	*6% red*	*1% green*	*1% blue*	*2% yellow*
B	*$0*	*win $45*	*win $45*	*lose $10*	*lose $15*

Tversky and Kahneman reported that given the choice between Option A and Option B shown above, all 88 subjects in their experiment chose B. This is unsurprising since B differs from A only in offering a 1% chance of a larger gain ($45 versus $30) and a 1% chance of a smaller loss (-$10 versus -$15), and thus stochastically dominates A. However, Tversky and Kahneman also observed that given the choice between C and D, below, a majority (58%) of subjects chose C.

Which lottery do you prefer?

Option C	*90% white*	*6% red*	*1% green*	*3% yellow*
	$0	*win $45*	*win $30*	*lose $15*

Option D	*90% white*	*7% red*	*1% green*	*2% yellow*
	$0	*win $45*	*lose $10*	*lose $15*

At first glance, this choice may also seem reasonable. Both options offer nearly equal probabilities of a $45 gain and a $15 loss but C offers a 1% chance of a much better outcome ($30 versus -$10). Despite the intuitive appeal of Option C, the choice between C and D is a simple reformulation of the choice between A and B, in which the probabilities of identical outcomes are combined or 'coalesced' (Birnbaum, 1999; Birnbaum and Navarrette, 1998). These manipulations hide the differences essential to detecting the dominance relation, and introduce a comparison (a gain of $30 versus a loss of $10) that appears salient and favors the dominated alternative.

Scholten and Read (2014) present evidence of similar behavior in intertemporal choice. Given a choice between consumption plans A and B below, they find most subjects chose B, suggesting the difference between $100 versus $5 in a year is more important than receiving $75 versus nothing today:

A) Receive $75 today and receive $5 in one year

B) Receive $100 in one year (56%)

When given a choice between C and D, below, they found most subjects chose C.

C) Receive $75 today (73%)

D) Receive $100 in one year.

In this choice, the difference in when one gets paid (now versus one year) may be more salient than the difference in what one gets paid ($75 versus $100). Taken together the majority choices of B over A and C over D suggest that C should be preferred to A. However, as shown below, A clearly dominates C.

A) { $75, today ; $5, one year }

C) { $75, today ; $0, one year }

These examples demonstrate that people obey dominance when the dominance relationship is transparent but may violate it when the differences in payoffs critical to detecting dominance are hidden. To the extent people do not stand by their choice of dominated alternatives once the dominance relationship is revealed, these anomalies are examples of decision errors. They do not contradict the assumption that people have coherent preferences. But they draw into question the standard assumption that people choose as if they computed an expected or discounted utility index for each option separately and then chose the option with the larger value. Instead, these behaviors result from a context-dependent evaluation process that involves comparisons across attributes, in which case the framing of alternatives may matter.

3. Salience Weighted Utility over Presentations

Given the effects of salient comparisons on behavior illustrated in Section 2, we proceed to model choices in three steps, first specifying the evaluation process or computational decision algorithm agents employ to choose between alternatives, then characterizing the nature of salience perceptions that drive the evaluation, and then defining frames over which the evaluation process operates.

3.1 A Model of Salience Weighted Evaluation

For the purposes of developing a comparative model of risky and intertemporal prospect evaluation we begin by representing the options in a *presentation* or *frame* as shown in Figure 1 (to be defined more formally in Section 3.3). The word "frame" was initially used in the context of gain-loss framing effects following Kahneman and Tversky (1979) and Tversky and Kahneman (1981), but today is used more broadly to refer to any violation of description invariance and we use it in this broader sense.

Figure 1. Presentations or "Frames" for Decisions under Risk and Over Time

Choice Frame for Lotteries

	(x_1,y_1)	(p_1,q_1)	(x_2,y_2)	(p_2,q_2)		(x_i,y_i)	(p_i,q_i)		(x_n,y_n)	(p_n,q_n)
p	x_1	p_1	x_2	p_2	…	x_i	p_i	…	x_n	p_n
q	y_1	q_1	y_2	q_2	…	y_i	q_i	…	y_n	q_n

Choice Frame for Consumption Plans

	(x_1,y_1)	(r_1,t_1)	(x_2,y_2)	(r_2,t_2)		(x_i,y_i)	(r_i,t_i)		(x_n,y_n)	(r_n,t_n)
a	x_1	r_1	x_2	r_2	…	x_i	r_i	…	x_n	r_n
b	y_1	t_1	y_2	t_2	…	y_i	t_i	…	y_n	t_n

We consider one-dimensional arrays $\mathbf{p}$ and $\mathbf{q}$ which represent lotteries p and q (in a well-defined sense) that offer a finite and equal number of outcomes denoted $\mathbf{x_i}$ and $\mathbf{y_i}$, $i = 1,2, \dots, n$, where each $\mathbf{x_i}$ occurs with probability, $\mathbf{p_i}$ and each $\mathbf{y_i}$ occurs with probability $\mathbf{q_i}$. Likewise, we consider one-dimensional arrays $\mathbf{a}$ and $\mathbf{b}$ which represent consumption plans a and b that offer a finite and equal number of outcomes, where each outcome $\mathbf{x_i}$ occurs in time period, $\mathbf{r_i}$ and each $\mathbf{y_i}$ occurs in period $\mathbf{t_i}$, for all i. We use bold font for attributes in an array and italicized font for attributes in the support of a lottery or consumption plan.

In our setup, there is a finite set, X, of outcomes. A *lottery* is a mapping $p: X \rightarrow [0,1]$ such that $\sum_{x \in X} p(x) = 1$. Denote the set of all lotteries by $\Delta(X)$. Let there be a preference relation, $\succ$, over $\Delta(X)$ reflecting the decision maker's true preferences. The decision maker is assumed to have standard preferences (conforming to expected utility theory). That is, for all lotteries $p, q \in \Delta(X)$,

(1) $$p \succ q \text{ if and only if } \sum_{x \in X} p(x)U(x) > \sum_{y \in X} q(y)U(y).$$

This is the conventional approach. Next, we ask how might an individual evaluate *representations* of lotteries like the ones shown in Figure 1? An extensive body of experimental work in both economics and psychology has demonstrated that even small changes in representations can have consequential effects on behavior. To investigate this possibility, let *i* = 1, 2, …,*n* index the location of the i^{th} attribute (payoff, probability) in a frame.

The decision maker is given two arrays and is asked to choose one. Here we consider a second relation $\gtrsim$ over representations. The relation $\gtrsim$ may be viewed as a 'perceptual relation' (rather than a preference relation) which assigns higher rankings to arrays that "look better" given the frame. For the generic frame in Figure 1, given (1), an unbiased perceptual relation can be expressed as:

(2) $$\mathbf{p} \gtrsim \mathbf{q} \text{ if and only if } \sum_{i=1}^{n} \mathbf{p_i} U(\mathbf{x_i}) > \sum_{i=1}^{n} \mathbf{q_i} U(\mathbf{y_i}),$$

for all $\mathbf{p}, \mathbf{q}$, such that $\mathbf{p}$ is a *representation* of p and $\mathbf{q}$ is a *representation* of q and for all $p, q \in \Delta(X)$.

To account for the role of salience perception in decision making, we let the decision maker use a "salient" evaluation defined over frames when deciding between alternatives. In particular, salient comparisons between alternatives may frustrate the expression of the decision maker's true preferences. The decision maker may instead be systematically swayed by changes in the arrangement of attributes in a frame which systematically make some comparisons focal and which make others inconsequential, as was the case in the stochastic dominance framing example in Section 2. To the extent the decision maker does not prefer stochastically dominated alternatives, the choice of Option C over D in Section 2 is a case where the representation that 'looks better' is not consistent with the decision maker's preferences. Rather than assuming that choices always 'reveal preferences,' our approach admits two possibilities whenever the relations $\succ$ and $\gtrsim$ are not equivalent: In any given situation (i) choices may reveal preferences (consistent with $\succ$) or (ii) they may reveal systematic decision errors or deviations from preference maximization due to biases in salience perception (consistent with $\hat{\succ}$).

To further motivate the possibility that frames may frustrate the expression of preference, note that equations (1) and (2) provide an *alternative-based evaluation* - one lottery is strictly preferred to another, if and only if it yields a greater expected payoff to the decision maker. The examples in Section 2 suggest instead that agents choose by making across-lottery comparisons of payoffs and their associated probabilities of occurrence. Building on Leland and Sileo (1998), note that the alternative-based evaluation in (2) may be written equivalently as an *attribute-based evaluation* such that $\mathbf{p} \hat{\succ} \mathbf{q}$ if and only if (3) holds:

(3) $$\sum_{i=1}^{n}[(\mathbf{p_i}-\mathbf{q_i})(U(\mathbf{x_i})+U(\mathbf{y_i}))/2+(U(\mathbf{x_i})-U(\mathbf{y_i}))(\mathbf{p_i}+\mathbf{q_i})/2]>0.$$

Note that the evaluation procedures in (2) and (3) operate over frames rather than over lotteries directly. Nevertheless, (2) and (3) both characterize frame-invariant preferences (they are not sensitive to changes in frames). The attribute-based evaluation in (3) computes probability differences associated with outcomes weighted by the average utility of those outcomes plus utility differences of outcomes weighted by their average probability of occurrence. A decision maker who chooses among representations according to the attribute-based evaluation in (3) will be indistinguishable from one who chooses according to the alternative-based evaluation in (2). But now suppose that in the process of comparing risky alternatives an agent notices when the payoff in one alternative is "a lot more money" than the payoff in another and when one alternative offers "a much better chance" of receiving an outcome than the other. In these cases, we will assume that large differences in attribute values across different alternatives are perceived as particularly salient or attract disproportionate attention and are overweighted in the evaluation process. To capture this intuition that larger differences in attributes are over-weighted or attract disproportionate attention, we place salience weights $\phi(\mathbf{p_i},\mathbf{q_i})$ on probability differences and $\mu(\mathbf{x_i},\mathbf{y_i})$ on payoff differences, yielding the following model of *salience-weighted evaluation,* in which $\mathbf{p} \hat{\succ} \mathbf{q}$ if and only if (4) holds:

(4) $$\sum_{i=1}^{n}[\phi(\mathbf{p_i},\mathbf{q_i})(\mathbf{p_i}-\mathbf{q_i})(U(\mathbf{x_i})+U(\mathbf{y_i}))/2+\mu(\mathbf{x_i},\mathbf{y_i})(U(\mathbf{x_i})-U(\mathbf{y_i}))(\mathbf{p_i}+\mathbf{q_i})/2]>0.$$

We refer to model (4) as Salience-Weighted Utility over Presentations (SWUP). Note that SWUP has a dual interpretation as a model of *salience-based choice* that overweights large differences or as a model of *similarity-based choice* that underweights small differences. Note also that (3) is clearly a special case of (4) in which all salience weights are equal.

The model extends more generally to the domain of uncertainty[4]. Suppose there is a finite set of possible states of nature, where a lottery is assigned to be played in each state. Index the states by s = {1,2,…,S}. Denote uncertain prospects by f and g, where f assigns lottery $f(s)$ to each state s = 1,2,…S. Likewise, g assigns lottery $g(s)$ to each state. In the classic alternative-based evaluation, there is assumed to be a unique subjective probability distribution, π_s, over states (Anscombe and Aumann, 1963) such that f is preferred over g if and only if (5) holds (where $f_s(x)$ is the probability of outcome x in state s):

(5) $$\sum_{s\in S}\sum_{x\in X}\pi_s[f_s(x)U(x)] > \sum_{s\in S}\sum_{y\in X}\pi_s[g_s(y)U(y)].$$

Let there be a frame for each state, where $\mathbf{f^s}$ represents $f(s)$, $\mathbf{g^s}$ represents $g(s)$, and the i[th] payoff and probability for $\mathbf{f^s}$ are denoted $\mathbf{x_{is}}$ and $\mathbf{f_{is}}$, respectively (with analogous notiation for $\mathbf{g^s}$), for all $i = 1, \dots, n(s)$. Given two multi-dimensional arrays $\mathbf{f} = \{\mathbf{f^1}, \dots, \mathbf{f^S}\}$ and $\mathbf{g} = \{\mathbf{g^1}, \dots, \mathbf{g^S}\}$, the analogous formula to (5) is such that **f** "looks better" than **g** if and only if (6) holds:

(6) $$\sum_s^S\sum_i^{n(s)}\pi_s[\mathbf{f_{is}}U(\mathbf{x_{is}})] > \sum_s^S\sum_i^{n(s)}\pi_s[\mathbf{g_{is}}U(\mathbf{y_{is}})].$$

Next, we introduce the corresponding model of salience-weighted evaluation in which **f** "looks better" than **g** if and only if (7) holds:

(7) $$\sum_s^S\sum_{i=1}^{n}\pi_s[\phi(\mathbf{f_{is}},\mathbf{g_{is}})(\mathbf{f_{is}}-\mathbf{g_{is}})(U(\mathbf{x_{is}})+U(\mathbf{y_{is}}))/2$$
$$+\mu(\mathbf{x_{is}},\mathbf{y_{is}})(U(\mathbf{x_{is}})-U(\mathbf{y_{is}}))(\mathbf{f_{is}}+\mathbf{g_{is}})/2] > 0.$$

The model extends analogously to choices over time such as those depicted in the lower panel of Figure 1. As before, let i = 1,2,…,n denote the position of the i[th] attribute (payoff, time period) in a frame. For decisions over discrete time periods t ∈ {0, 1, 2, …, T}, a *consumption plan, a,* is a sequence of dated outcomes in X. Denote the set of consumption plans by C. We study choices between consumption plans a and b where $a := (x_0, x_1, \dots, x_T)$ and $b :=$

[4] The classic distinction between risk and uncertainty was noted by Knight (1921) and Ellsberg (1961): Under risk, the decision maker knows the probabilities of all possible outcomes whereas some probabilistic information is missing for decisions under uncertainty.

$(y_0, y_1, \ldots, y_T)$. As in the standard discounted utility model, a decision maker has a preference relation, $\succ_t$, over consumption plans such that for all $a, b \in C$,

(8) $$a \succ_t b \text{ if and only if } \sum_{x \in X} \delta^r U(x) > \sum_{y \in X} \delta^t U(y),$$

where δ is a constant discount factor.

The decision maker is now given two arrays and is asked to choose one. Here we consider the relation $\hat{\succ}_t$ over representations of consumption plans. The relation $\hat{\succ}_t$ may be viewed as a 'perceptual relation' which assigns higher rankings to arrays that "look better" given the frame. For the generic frame in Figure 1, given (8), a perceptual relation can be expressed as follows:

(9) $$\mathbf{a} \hat{\succ}_t \mathbf{b} \text{ if and only if } \sum_{i=1}^{m} \delta^{\mathbf{r_i}}[U(\mathbf{x_i})] > \sum_{i=1}^{m} \delta^{\mathbf{t_i}}[U(\mathbf{y_i})],$$

for all $\mathbf{a}, \mathbf{b}$, such that $\mathbf{a}$ is a *representation* of a and $\mathbf{b}$ is a *representation* of b and for all $a, b \in C$.

The alternative-based evaluation in (9) is equivalent to an attribute-based evaluation in which $\mathbf{a} \hat{\succ}_t \mathbf{b}$ if and only if (10) holds:

(10) $$\sum_{i}^{m}[(\delta^{\mathbf{r_i}} - \delta^{\mathbf{t_i}})(U(\mathbf{x_i}) + U(\mathbf{y_i}))/2 + (U(\mathbf{x_i}) - U(\mathbf{y_i}))(\delta^{\mathbf{r_i}} + \delta^{\mathbf{t_i}})/2] > 0.$$

Placing salience weights $\theta(\mathbf{r_i}, \mathbf{t_i})$ on time differences and $\mu(\mathbf{x_i}, \mathbf{y_i})$ on payoff differences, yields a salience-weighted evaluation in which $\mathbf{a} \hat{\succ}_t \mathbf{b}$ if and only if:

(11) $$\sum_{i}^{m}[\theta(\mathbf{r_i}, \mathbf{t_i})(\delta^{\mathbf{r_i}} - \delta^{\mathbf{t_i}})(U(\mathbf{x_i}) + U(\mathbf{y_i}))/2 + \mu(\mathbf{x_i}, \mathbf{y_i})\,(U(\mathbf{x_i}) - U(\mathbf{y_i}))(\delta^{\mathbf{r_i}} + \delta^{\mathbf{t_i}})/2] > 0.$$

We refer to an agent who chooses according to salience-based evaluation models (formulas 4, 7, and 11) as a *focal thinker* since such an agent focuses on salient differences in attributes. Such an agent chooses the alternative which 'looks better', with respect to that agent's perceptual system. Note that whether one alternative 'looks better' than another to an agent, according to $\hat{\succ}$, depends on both the agent's risk preferences, time preferences and subjective beliefs (as measured by $U, \delta,$ and π, respectively) *and* the salience agents ascribe to large versus small differences in payoffs, time delays, and probabilities (as measured by $\mu, \theta,$ and ϕ, respectively). In this respect, SWUP provides a bridge between economic and psychological approaches to decision making by modeling choice as dependent on both properties of preferences and properties of the perceptual system, rather than attributing all behavior to preferences.

3.2 The Nature of Salience Perceptions

The salience functions μ, ϕ and θ determine the only ways in which the behavior of a focal thinker differs from a rational agent who chooses over arrays according to formulas 2, 6, and 9. We assume a salience function exhibits the properties of the perceptual system in Definition 1.

Definition 1 (Salience Function; Bordalo et al., 2013): A *salience function* $\sigma(\mathbf{x}, \mathbf{y})$ is any (non-negative), symmetric[5] and continuous function that satisfies the following three properties:

1. **Ordering:** If $[\mathbf{x}', \mathbf{y}'] \subset [\mathbf{x}, \mathbf{y}]$ then $\sigma(\mathbf{x}', \mathbf{y}') < \sigma(\mathbf{x}, \mathbf{y})$.
2. **Diminishing Sensitivity :** σ exhibits diminishing sensitivity if for any $\mathbf{x}, \mathbf{y}, \boldsymbol{\epsilon} > 0$, $\sigma(\mathbf{x} + \boldsymbol{\epsilon}, \mathbf{y} + \boldsymbol{\epsilon}) < \sigma(\mathbf{x}, \mathbf{y})$.

Ordering suggests that the perceptual system is more sensitive to attributes that have both a larger absolute difference and a larger ratio (e.g., the difference between 100 and 1 is more salient than the difference between 75 and 25). Diminishing sensitivity extends this property such that that for a fixed absolute difference, the perceptual system is more sensitive to larger ratios (e.g., the difference between 100 and 1 is more salient than that between 200 and 101).

In addition to being assumed by Bordalo et al. (2012, 2013), Properties 1 and 2 are well-supported in the psychology literature. One source is research in psychophysics. The ordering property is consistent with the "symbolic distance" effect - it takes adults longer to correctly respond to questions regarding which of two numbers is larger, the smaller their arithmetic difference.[6]

A long tradition in psychology has studied the sensitivity of the perceptual system to changes in the magnitude of a stimulus. Since the Weber-Fechner law was introduced in the 19th century, it has been widely recognized that *diminishing sensitivity* is a fundamental property of the perceptual system that applies across a range of sensory modalities including tone, brightness, and distance (Stevens, 1957). Schley and Peters (2014) provide experimental support indicating that diminishing sensitivity also characterizes how the brain maps symbolic numbers onto mental magnitudes.

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[5] A function $f(\mathbf{x}, \mathbf{y})$ is symmetric if $f(\mathbf{x}, \mathbf{y}) = f(\mathbf{y}, \mathbf{x})$.

[6] For example, Moyer and Landauer (1967) found that it takes adults longer to answer the question "Which number is larger, 2 or 3?" than to answer the question "Which number is larger, 2 or 7?"

There is also precedent for assuming these or related properties in the decision making literature. Diminishing sensitivity is assumed as a property of preferences in both original and cumulative prospect theory (Kahneman & Tversky, 1979; Tversky and Kahneman, 1992). In addition, diminishing sensitivity is an analog to the property of *increasing absolute similarity* in Leland (2002) in the context of similarity judgments, and is closely related to Scholten and Read's (2010) *diminishing absolute sensitivity* assumption for delays and outcomes in intertemporal choice. Bordalo et al. (2012) explicitly assume salience perceptions regarding payoff differences obey ordering and diminishing sensitivity while Rosa et al. (1993) considered the implications of diminishing sensitivity for decisions regarding the introduction of new products. Finally, diminishing absolute sensitivity is assumed in Prelec and Loewenstein's (1991) model of decision making over time although as a property of preferences. In contrast, our approach does not modify the basic ingredients of the rational economic models – the utility function, discount factor, or subjective probability distribution retain their economic interpretation as measuring risk preference, time preference, and subjective beliefs. Instead, these dimensions are weighted by functions that account for the perception of differences in risk, time, and money. In this respect, SWUP provides a unification of models based on similarity and salience perceptions (e.g., Rubinstein 1988, 2003; Leland 1994, 2002; Bordalo et al. 2012) with the standard economic models of preference-based choice.

3.3 Frames

To represent a lottery, p, we employ a one-dimensional array, $\mathbf{p}$, consisting of $n(\mathbf{p})$ outcomes and $n(\mathbf{p})$ corresponding probabilities. Denote the ith outcome in $\mathbf{p}$ and the ith corresponding probability by $\mathbf{x_i}$ and $\mathbf{p_i}$, respectively. Notice that outcome-probability pairs appear in no particular order in the array. Also, some outcomes could be repeated and some probabilities can be zero in the array. Hence, there could be many arrays representing the same lottery.

Definition 2 (Representation of a lottery): We say that an array $\mathbf{p}$ is a *representation* of lottery p if the following two properties hold:

(i) For i =1,2,...,$n(\mathbf{p})$, $\sum_{\mathbf{i}} \mathbf{p_i} = 1$

(ii) For all i such that $\mathbf{x_i} = x$, $\sum_{\mathbf{i}} \mathbf{p_i} = p(x)$.

Note that a representation $\mathbf{p}$ of lottery p differs from the lottery itself since it permits the same outcome to appear more than once in the array provided that the corresponding probabilities sum to the overall probability of that outcome. Two representations of different lotteries presented jointly, constitute a frame.

Definition 3 (Frame for lotteries): A *presentation* or *frame*, $\mathbf{F}\{\mathbf{p},\mathbf{q}\}$ of lotteries, p and q, is a matrix containing a representation **p** of p and a representation **q** of q.

In our analysis, we will consider cases where both representations in a frame have the same dimension, although this dimension can vary across frames.

To represent a consumption plan, a, we employ a one-dimensional array, **a**, consisting of $m(\mathbf{a})$ outcomes and $m(\mathbf{a})$ corresponding time periods. Denote the i^th^ outcome in **a** and the i^th^ corresponding period by $\mathbf{x}_i$ and $\mathbf{r}_i$, respectively.

Definition 4 (Representation of a consumption plan): We say that an array **a** is a *representation* of consumption plan a if the following hold:

(i) For any dated outcome, $x_t \neq 0$, the pair $(\mathbf{x},\mathbf{t})$ is in **a** if and only if x_t is in a.

(ii) For any dated outcome, $x_t = 0$, if the pair $(\mathbf{x},\mathbf{t})$ is in **a** then x_t is in a.

(iii) If there are T dated outcomes in a, then $\dim(\mathbf{a}) \leq 2\mathrm{T}$.

A representation **a** of consumption plan a differs from the consumption plan itself since it permits periods of zero consumption to be 'compressed' (an outcome of zero and its corresponding time period might not be in **a**). In addition, there is no restriction on the order in which the outcomes in a appear in **a**. Finally, property (iii) implies that **a** contains at most all T outcomes and the corresponding T periods. When two representations of different consumption plans are presented together, they constitute a frame.

Definition 5 (Frame for consumption plans): A *presentation* or *frame*, $\mathbf{F}\{\mathbf{a},\mathbf{b}\}$ of two consumption plans, a and b, is a matrix containing a representation, **a** of a and a representation, **b** of b.

3.4 Minimal Frames

We next formalize two important types of frames, which we term minimal frames and transparent frames. A *minimal* or *efficient frame* contains the minimum number of cells in a frame of the form in Figure 1 necessary to represent the choices between lotteries or consumption plans. That is, in a minimal frame, $i = 1,2,\dots,n$ where $n = \max\{|supp(p)|,|supp(q)|\}$ for lotteries and, $i = 1,2,\dots,m$, where $m = \max\{|supp(a)|,|supp(b)|\}$ for consumption plans (where we define the support of a consumption plan to be the set of outcomes corresponding to non-zero consumption in that plan). For minimal frames, the representation of alternatives in Figure 1 is related to Birnbaum's (2004a) tree-representation of lotteries. In particular, a pair of lotteries in Birnbaum's 'coalesced' form generate a minimal frame. Minimal frames may also be viewed as a standard way of presenting choice alternatives and they formalize the 'prospect form' in which lotteries have traditionally been presented to experimental subjects since Kahneman and Tversky (1979).

We consider minimal frames that are *monotonic in outcomes* (for decisions under risk) in the sense that all outcomes are ordered to be weakly increasing ($\mathbf{x_1} \leq \mathbf{x_2} \leq \cdots \leq \mathbf{x_n}$ and $\mathbf{y_1} \leq \mathbf{y_2} \leq \cdots \leq \mathbf{y_n}$) or weakly decreasing, as well as minimal frames that are *monotonic in time periods* (for intertemporal choices) in which all time periods are ordered to be weakly increasing.[7] While it is an empirical question regarding how people mentally frame lotteries and consumption plans, it seems plausible that people naturally think in minimal frames since they are arguably the simplest representation of a decision problem. We next show that for lotteries or consumption plans with a common support size, monotonic minimal frames are essentially unique[8].

Proposition 1 (Uniqueness of Monotone Minimal Frames: Risk): *For a choice between two lotteries p and q, with* $|supp(p)| = |supp(q)|$, *there exists a frame that is minimal and monotonic in outcomes which is unique up to the operations of row-switching and reversal of column order.*

Proposition 2 (Uniqueness of Monotone Minimal Frames: Time): *For a choice between two consumption plans a and b, with* $|supp(a)| = |supp(b)|$, *there exists a frame that is minimal and monotonic in time periods which is unique up to the operation of row-switching.*

A focal thinker compares the ith outcome, probability, or delay in the representation of one alternative in a frame with the ith outcome, probability, or delay, respectively, in the representation of the other alternative. Note that such a decision maker will not be sensitive to whether a monotonic frame is increasing or decreasing, in which case the behavior of this agent is uniquely determined for any monotone minimal frame.

[7] Time has a natural forward direction that makes it implausible that time periods are framed in a decreasing monotonic presentation. One may also consider a presentation that is monotonic in probabilities, but such a presentation is not needed in our analysis.

[8] For non-degenerate lotteries and for consumption plans with different support sizes, monotone minimal frames can be uniquely defined by filling in the attributes of the lottery or consumption plan that has the smaller support size with all zeros in the frame after all the outcomes in the support of the lottery or consumption plan have been displayed. These zeros capture the notion that the extra outcomes in the option with the larger support size are compared with 'nothing'.

We next consider the framing of degenerate lotteries. A lottery is said to be *degenerate* if it yields a single outcome with probability 1. Consider a choice between a degenerate lottery p yielding x with certainty and a non-degenerate lottery q. Given these options, it seems almost unavoidable that one compares each outcome in q to the unique outcome in p. We thus adopt the convention that choices involving a degenerate lottery are framed as in Figure 2.

Figure 2. Choice Frame with a Degenerate Lottery

	(x_1, y_1)	(p_1, q_1)	(x_2, y_2)	(p_2, q_2)	…	(x_i, y_i)	(p_i, q_i)	…	(x_n, y_n)	(p_n, q_n)
p	x	p_1	x	p_2	…	x	p_i	…	x	p_n
q	y_1	p_1	y_2	p_2	…	y_i	p_i	…	y_n	p_n

3.5 Transparent Frames

In this section we define a transparent frame which formalizes and generalizes Savage's state representation of lotteries without necessarily implying correlation between lotteries. Given two lotteries, p and q, a pair (x, r), consisting of an outcome x with probability r, is a *common consequence* if $x \in supp(p) \cap supp(q)$ and $p(x) \geq r,\ q(x) \geq r$. All other pairs of outcomes and corresponding probabilities are *distinct consequences*. That is, a common consequence between two lotteries is one with the same outcome occurring with the same probability in both lotteries. We say (x, r) is a *maximal common consequence* if (x, r) is a common consequence for which $p(x) = r$ or $q(x) = r$ (or both). We next define a transparent frame. Our definition is constructive: it uniquely specifies how to construct a transparent frame given any pair of lotteries, even if the two lotteries have different support sizes.

Definition 6 (Transparent frame for lotteries): Given any pair of lotteries, a *transparent frame* is a frame of the form in Figure 1 with the following properties:

1) **Presentation of outcome-probability pairs (Common Consequence Separation):** All (maximal) common consequences are separated from all distinct consequences such that all maximal common consequences are adjacent, and all distinct consequences are adjacent as shown in Figure 3[9].

[9] In Figure 3, there are k common consequences (with corresponding outcomes $z_1,\ldots,z_k$) where $k \geq 0$. The remaining pairs of payoff column vectors and corresponding probability column vectors are distinct consequences.

Figure 3. A Transparent Frame of Lotteries p and q

$\mathbf{p}$	$\mathbf{x_n}$	$\mathbf{p_{k+n}}$	…	$\mathbf{x_i}$	$\mathbf{p_i}$	…	$\mathbf{x_1}$	$\mathbf{p_{k+1}}$	$\mathbf{z_k}$	$\mathbf{p_k}$	…	$\mathbf{z_1}$	$\mathbf{p_1}$
$\mathbf{q}$	$\mathbf{y_n}$	$\mathbf{p_{k+n}}$	…	$\mathbf{y_i}$	$\mathbf{p_i}$	…	$\mathbf{y_1}$	$\mathbf{p_{k+1}}$	$\mathbf{z_k}$	$\mathbf{p_k}$	…	$\mathbf{z_1}$	$\mathbf{p_1}$

2) **Presentation of outcomes (monotonicity):** Outcomes are ordered such that $\mathbf{x_n} \geq \cdots \geq \mathbf{x_1}$; $\mathbf{y_n} \geq \cdots \geq \mathbf{y_1}$; $\mathbf{z_k} \geq \cdots \geq \mathbf{z_1}$.

3) **Presentation of probabilities (alignment):** Probabilities are presented so that

   (i) $\mathbf{p_i} = \mathbf{q_i}$ for all i.

   (ii) Given $\mathbf{x_i} = x$ and $\mathbf{y_i} = y$,

   $\mathbf{p_i} = min\left(p(x) - \sum_{j<i:\mathbf{x_j}=x} \mathbf{p_j},\ q(y) - \sum_{j<i:\mathbf{y_j}=y} \mathbf{q_j}\right)$ for all i.

In (ii), $p(x)$ is the overall probability of outcome x in lottery p, and $\mathbf{p_j}$ is the probability of outcome x_j in the jth payoff column vector in the frame. That is, the specific probabilities are set equal to the minimum remaining probability mass for each subsequent ranked pair of outcomes. The algorithm for computing the probabilities in the frame (property 3 (ii)) ensures that the frame has the minimum number of cells subject to satisfying properties 1), 2), and 3) (i), and that the frame is uniquely defined even for lotteries with different support sizes.

We next offer a definition of transparent frames for consumption plans. Given two consumption plans, a and b, a pair (x, t), consisting of an outcome x in period t, is a *common consequence* if the decision maker receives x in period t from either consumption plan.

Definition 7 (Transparent frame for consumption plans): Given any pair of consumption plans, a *transparent frame* is a frame of the form in Figure 1 with the following properties:

1) **Presentation of outcome-delay pairs (Common Consequence Separation):** All common consequences are separated from all distinct consequences such that all common consequences are adjacent in the frame, and all distinct consequences are adjacent, as shown in Figure 4.

Figure 4. A Transparent Frame of Consumption Plans a and b

a	$\mathbf{x_1}$	$\mathbf{t_1}$	…	$\mathbf{x_i}$	$\mathbf{t_i}$	…	$\mathbf{x_k}$	$\mathbf{t_k}$	$\mathbf{z_{k+1}}$	$\mathbf{t_{k+1}}$	…	$\mathbf{z_n}$	$\mathbf{t_n}$
b	$\mathbf{y_1}$	$\mathbf{t_1}$	…	$\mathbf{y_i}$	$\mathbf{t_i}$	…	$\mathbf{y_k}$	$\mathbf{t_k}$	$\mathbf{z_{k+1}}$	$\mathbf{t_{k+1}}$	…	$\mathbf{z_n}$	$\mathbf{t_n}$

2) **Monotonicity:** Time periods are ordered as $\mathbf{t_1} \leq \cdots \leq \mathbf{t_k}$ and $\mathbf{t_{k+1}} \leq \cdots \leq \mathbf{t_n}$.

3) **Alignment:** $\mathbf{t_i} = \mathbf{r_i}$ for all i.

4. Framing Effects between Minimal and Transparent Frames

We now consider the implications of SWUP regarding violations of rational choice theory including violations of stochastic dominance discussed in Section 2, the Allais paradox, the common ratio effect, the Ellsberg paradox, present bias, and violations of cancellation. We will show that even a 'parameter-free' version of SWUP predicts that each of these anomalies will occur in minimal frames, but that behavior consistent with models of rational choice will obtain when the anomalies are presented in transparent frames. We illustrate the model under the simplest possible specification of SWUP in which $U(x) = x$, and salience perceptions for both payoffs and probabilities are computed by a parameter-free salience function of the form:

$$\sigma(\mathbf{x}, \mathbf{y}) = \frac{|\mathbf{x} - \mathbf{y}|}{|\mathbf{x}| + |\mathbf{y}|}, \tag{12}$$

when it is not the case that $\mathbf{x} = \mathbf{y} = 0$, and $\sigma(\mathbf{0}, \mathbf{0}) = 0$. This salience function was introduced by Bordalo et al. (2013) in the context of consumer choice and is applied here to the salience of payoffs, probabilities, and time delays. In the following, we highlight the predicted choices of a focal thinker in bold font.

4.1 Stochastic Dominance in Minimal and Transparent Frames

One of the most basic axioms of rational choice under risk is consistency with first-order stochastic dominance: If a lottery p offers at least as good an outcome at every probability increment as a lottery q and p offers a strictly better outcome at some probability increment, then p (first-order) stochastically dominates q. Consider again the example due to Tversky and Kahneman (1986) from Section 2, shown in transparent and minimal frames in Figure 5.

Figure 5. Stochastic Dominance in Minimal and Transparent Frames

Stochastic Dominance in Transparent Frames

	x_1,y_1	p_1,q_1	x_2,y_2	p_2,q_2	x_3,y_3	p_3,q_3	x_4,y_4	p_4,q_4	x_5,y_5	p_5,q_5
p	**45**	**0.01**	**-10**	**0.01**	**45**	**0.06**	**0**	**0.90**	**-15**	**0.02**
q	30	0.01	-15	0.01	45	0.06	0	0.90	-15	0.02

Stochastic Dominance in Minimal Frames

	x_1,y_1	p_1,q_1	x_2,y_2	p_2,q_2	x_3,y_3	p_3,q_3	x_4,y_4	p_4,q_4
p′	0	0.9	45	0.07	-10	0.01	-15	0.02
q′	**0**	**0.9**	**45**	**0.06**	**30**	**0.01**	**-15**	**0.03**

Given a transparent presentation of lotteries p and q, all subjects chose the stochastically dominant alternative, p (Tversky and Kahneman, 1986). However, when represented in a minimal frame as p' and q' many subjects violated dominance. Under SWUP, q' is chosen over p' in a minimal frame if

(13) $$\phi(0.07,0.06)(0.45) + \mu(-10,30)(-0.40) + \phi(0.02,0.03)(0.15) < 0.$$

Under the simplest version of SWUP (letting $U(x) = x$, and employing the parameter-free salience function in (12) to compute the salience of both payoff and probability differences), a direct calculation confirms that (13) holds. In the transparent frame, p is chosen over q under SWUP for any salience function and any increasing utility function. Thus SWUP readily explains the framing effect. In contrast, leading models of decisions under risk either always satisfy stochastic dominance or always violate it and thus cannot explain the dependence of stochastic dominance on the framing of alternatives.

4.2 The Allais Paradox in Minimal and Transparent Frames

The Allais paradox (Allais, 1953) is among the best known and most robust violations of EU. It involves choice problems like those shown in the left panel of Figure 6 where the independence axiom of EU is often violated. A decision maker first chooses between option q, offering \$2400 with certainty and a lottery p, offering a 33% chance of \$2500, a 66% chance of \$2400, and a 1% chance of \$0. The decision maker is also asked to choose between lottery $\tilde{q}$ offering 34% chance of \$2400 (and \$0 otherwise) and lottery $\tilde{p}$ offering a 33% chance of \$2500 (and \$0 otherwise). When confronted with such choices, Kahneman and Tversky (1979) observed that most subjects chose q over p and chose $\tilde{p}$ over $\tilde{q}$. This preference pattern is inconsistent with EU which predicts that a decision maker with strict preferences will choose either p and $\tilde{p}$ or q and $\tilde{q}$.

In the choice between p and q in minimal frames, the comparison of 2400 and 0 is more salient than the comparison of 2500 and 2400, prompting a decision maker to choose the certain option, q. However, in the choice between $\tilde{p}$ and $\tilde{q}$, the comparison between 2400 and 0 is not cued. Instead, the decision maker compares the upside of winning 2500 instead of 2400 with the downside of forfeiting a 1% chance in the probability of winning. To the extent that this \$100 difference is more salient than the 0.01 difference in probabilities, the decision maker chooses $\tilde{p}$

over $\tilde{q}$, thereby violating EU. More formally, under SWUP, q is chosen over p in the minimal frame if

(14) $$\mu(2500{,}2400)(33) + \mu(0{,}2400)(-24) < 0.$$

In addition, $\tilde{p}$ is chosen over $\tilde{q}$ in the minimal frame if

(15) $$\mu(2500{,}2400)(33.5) + \phi(0.33{,}0.34)(-24.5) > 0.$$

Under the parameter-free specification of SWUP (with $U(x) = x$ and the salience function from (12)), inequalities (14) and (15) both hold, generating the Allais paradox in minimal frames.

Figure 6. The Allais Paradox in Minimal and Transparent Frames

Allais Paradox in Minimal Frames

	x_1,y_1	p_1,q_1	x_2,y_2	p_2,q_2	x_3,y_3	p_3,q_3
p	2500	0.33	2400	0.66	0	0.01
q	**2400**	**0.33**	**2400**	**0.66**	**2400**	**0.01**

	x_1,y_1	p_1,q_1	x_2,y_2	p_2,q_2
p̃	**2500**	**0.33**	**0**	**0.67**
q̃	2400	0.34	0	0.66

Allais Paradox in Transparent Frames

	x_1,y_1	p_1,q_1	x_2,y_2	p_2,q_2	x_3,y_3	p_3,q_3
p	2500	0.33	0	0.01	2400	0.66
q	**2400**	**0.33**	**2400**	**0.01**	**2400**	**0.66**

	x_1,y_1	p_1,q_1	x_2,y_2	p_2,q_2	x_3,y_3	p_3,q_3
p̃	2500	0.33	0	0.01	0	0.66
q̃	**2400**	**0.33**	**2400**	**0.01**	**0**	**0.66**

Now consider how an individual choosing between these options would act if presented with the transparent frames in the right-hand panel of Figure 6. Here, the components common to each decision (i.e., (\$2400, 0.66) in the choice between p and q and (\$0, 0.66) in the choice between $\tilde{p}$ and $\tilde{q}$) are isolated and the decision in both cases depends on comparisons between 2500 and 2400 and between 2400 and 0. As a result, the choice between p and q and the choice between $\tilde{p}$ and $\tilde{q}$ will each depend on the sign of (16):

(16) $$\mu(2500{,}2400)\big(U(2500) - U(2400)\big)(0.33) + \mu(0{,}2400)(U(0) - U(2400))(0.01).$$

Thus, SWUP predicts choices between p and q and between $\tilde{p}$ and $\tilde{q}$ to be consistent. Given the additional assumptions that $U(x) = x$ and μ is given by (12), SWUP predicts preferences of q and $\tilde{q}$, consistent with empirical observations in this format (Leland, 2010; Bordalo et al., 2012).

The possibility that the Allais paradox might be sensitive to framing was first suggested just a year after Allais published his paper, by Savage (1954). He proposed presenting the choices in a state-dependent matrix with prizes determined by draws from a bag of tickets numbered 1 through 100 (e.g., in the choice between p and q, a draw of a ticket numbered 1 through 33 yields a prize of \$2500 if p is chosen and \$2400 if q is chosen). Indeed, our definition of a transparent frame formalizes Savage's (1954) state matrix representation of lotteries, although in transparent frames there are no assumptions or implications regarding correlation between lotteries. A number of recent studies (Leland, 2010; Bordalo et al., 2012; Incekara-Hafalir and Stecher, 2012; Birnbaum and Schmidt, 2015; Harman and Gonzalez, 2015) examining the impact of transparent frames on the incidence of Allais violations of independence consistently report fewer violations in transparent frames than observed when traditional prospect (i.e., minimal) presentations are used. As Incekara-Hafalir and Stecher (2012) comment, "We find that given a transparent presentation, expected utility theory performs surprisingly well, and that the alternative theories perform poorly except inasmuch as they make the same predictions as expected utility theory."

4.3 The Common Ratio Effect in Minimal and Transparent Frames

A second well-known violation of the independence axiom is the common ratio effect (Allais, 1953). Figure 7 displays a classic version due to Kahneman and Tversky (1979). Consider the minimal frames (left panel), which display a choice between lotteries p and q, offering an 80% chance of \$4000 versus \$3000 with certainty, and a choice between $\tilde{p}$ and $\tilde{q}$, offering a 20% chance of \$4000 versus a 25% chance of \$3000. In this example, a majority of subjects chose q over p and chose $\tilde{p}$ over $\tilde{q}$, when the choices were presented in minimal frames. This response pattern violates EU which predicts choices of either p and $\tilde{p}$ or q and $\tilde{q}$. In prospect theory (Kahneman and Tversky, 1979) this choice pattern is attributed to a *certainty effect* in which outcomes that occur with certainty are overweighted. Under SWUP, q is chosen over p in minimal frames if (17) holds:

(17) $$\mu(4000,3000)(800) + \mu(0,3000)(-600) < 0.$$

In addition, $\tilde{p}$ is chosen over $\tilde{q}$ in the minimal frame if

(18) $$\mu(4000,3000)(225) + \phi(0.20, 0.25)(-175) > 0.$$

That is, $\tilde{p}$ is chosen over $\tilde{q}$ if the salience of the \$1000 difference in payoffs is greater than the salience of the 0.05 difference in probabilities. Under the parameter-free specification of SWUP, inequalities (17) and (18) both hold and the decision maker chooses q and $\tilde{p}$, as observed. Note that in the choice between $\tilde{p}$ and $\tilde{q}$, the comparison between 3000 and 0 is not cued in minimal frames. However, if the same choices are presented in transparent frames (right panel of Figure 7), SWUP predicts that subjects will satisfy the independence axiom and choose

q and $\tilde{q}$ since the comparison between 3000 and 0 is salient in both choices. More formally, the choice between p and q and the choice between $\tilde{p}$ and $\tilde{q}$ can each be shown to depend on the sign of (19):

(19) $\mu(4000{,}3000)\big(U(4000) - U(3000)\big)(0.20) + \mu(0{,}3000)(U(0) - U(3000))(0.05).$

Thus, SWUP predicts choices between p and q and between $\tilde{p}$ and $\tilde{q}$ to be consistent. Given the additional assumptions that $U(x) = x$ and μ is given by (12), SWUP predicts preferences of q and $\tilde{q}$, consistent with empirical observations in this format (Leland, 2010; Bordalo et al., 2012; and Harman and Gonzalez, 2015).

This prediction is also supported by the 'isolation effect' (Kahneman and Tversky, 1979) for a choice between a pair of two-stage gambles where the first stage, common to both gambles, involves a 75% chance of $0, and a 25% chance of advancing to the second stage, in which one gamble yields p and the other yields q. The majority of subjects preferred the gamble that yielded q in the second stage (Kahneman and Tversky, 1979). This prediction follows from SWUP if isolating the 75% chance of $0 from both options in the first stage makes this common consequence transparent and thus focuses attention on the difference between 3000 and 0 in the second stage.

Figure 7. The Common Ratio Effect in Minimal and Transparent Frames

Common Ratio Effect in Minimal Frames

	(x_1,y_1)	(p_1,q_1)	(x_2,y_2)	(p_2,q_2)
p	4000	0.80	0	0.20
q	**3000**	**0.80**	**3000**	**0.20**

	(x_1,y_1)	(p_1,q_1)	(x_2,y_2)	(p_2,q_2)
p′	**4000**	**0.20**	**0**	**0.80**
q′	3000	0.25	0	0.75

Common Ratio Effect in Transparent Frames

	(x_1,y_1)	(p_1,q_1)	(x_2,y_2)	(p_2,q_2)
p	4000	0.80	0	0.20
q	**3000**	**0.80**	**3000**	**0.20**

	(x_1,y_1)	(p_1,q_1)	(x_2,y_2)	(p_2,q_2)	(x_3,y_3)	(p_3,q_3)
p′	4000	0.20	0	0.05	0	0.75
q′	**3000**	**0.20**	**3000**	**0.05**	**0**	**0.75**

4.4 The Ellsberg Paradox in Minimal and Transparent Frames

A novel prediction of SWUP is that ambiguity aversion should be reduced when alternatives are presented in transparent frames. Schneider, Leland, and Wilcox (2016) tested this prediction in a controlled laboratory experiment. The upper panel in Figure 8 depicts a choice between alternatives A and B in minimal frames. After making a series of such choices, each subject will draw a ticket from a bag. If the ticket is red and the subject has chosen option B, that subject plays a lottery in which there is a 75% chance of winning $25 and a 25% chance of winning nothing. If the ticket is blue, the subject instead plays a lottery in which there is a 25% chance of winning \$25 and a 75% chance of winning nothing. If the subject chooses option A, the subject plays a lottery offering a 50% chance of winning \$25 and a 50% chance of winning nothing irrespective of the ticket color. In these types of choices, most subjects chose A suggesting they believe the probability of a blue ticket draw is *greater* than 0.5.

The lower panel in Figure 8 depicts a choice between alternatives A' and B' in minimal frames. A', like A, offers a 50% chance of winning \$25 and a 50% chance of winning nothing irrespective of the ticket color. For Lottery B', the subject has a 25% chance of winning \$25 and a 75% chance of winning nothing if a red ticket is drawn, and a 75% chance of winning \$25 and a 25% chance of winning nothing if a blue ticket is drawn. Here, the same people who chose A frequently choose A' suggesting that they believe the probability of a blue ticket is *less* than 0.5. This is an example of the Ellsberg Paradox which suggests that people do not assign well-defined subjective probabilities to states, but rather prefer alternatives with known probabilities over unknown probabilities, a behavior termed *ambiguity aversion.*

Under SWUP, with a uniform prior over states[10] and normalizing $U(25) = 1$ and $U(0) = 0$, A is chosen over B in the minimal frame in Figure 8 if

(20) $$0.5\phi(0.5,0.75)(-0.25) + 0.5\phi(0.5,0.25)(0.25) > 0.$$

By symmetry and diminishing sensitivity of ϕ, we have $\phi(0.5,0.25) > \phi(0.5,0.75)$, and inequality (20) holds for any salience function and any utility function. Similarly, A′ is chosen over B′, giving rise to ambiguity aversion.

Figure 8. The Ellsberg Paradox in Minimal Frames

	Red Ticket				Blue Ticket			
A	**\$25**	**0.50**	**\$0**	**0.50**	**\$25**	**0.50**	**\$0**	**0.50**
B	\$25	0.75	\$0	0.25	\$25	0.25	\$0	0.75

	Red Ticket				Blue Ticket			
A′	**\$25**	**0.50**	**\$0**	**0.50**	**\$25**	**0.50**	**\$0**	**0.50**
B′	\$25	0.25	\$0	0.75	\$25	0.75	\$0	0.25

[10] In estimating a mean-dispersion model of ambiguity preference to explain their data, Schneider, Leland, and Wilcox (2016) estimate the subjective prior assigned to red and blue ticket states to be uniform.

More generally, we can demonstrate the relationship between the probability salience function ϕ and ambiguity aversion in the classical paradoxes of Ellsberg (1961). Consider Ellsberg's two-color paradox. Suppose there are two urns. Urn 1 contains 50 red balls and 50 black balls. Urn 2 contains an unknown mixture of 100 red and black balls. The decision maker is given two choices:

Choice 1: Choose between A and B	**Choice 2: Choose between C and D**
A. Win $100 if red is drawn from Urn 1	C. Win $100 if black is drawn from Urn 1
B. Win $100 if red is drawn from Urn 2	D. Win $100 if black is drawn from Urn 2

Subjective expected utility (SEU) requires choices of either A and D or B and C in order for there to be a well-defined subjective probability distribution. However, Ellsberg found most people choose A and C with objective probabilities over options B and D, with ambiguous probabilities, thereby exhibiting ambiguity aversion. The minimal frame for these choices (for a given state s) is displayed in Figure 9, where $q(s)$ is the probability of drawing a red ball from Urn 2 in state s.

Figure 9. Minimal Frame for the Two-Color Ellsberg Paradox

A(s)	**$100**	**0.5**	**$0**	**0.5**
B(s)	$100	$q(s)$	$0	$1 - q(s)$

C(s)	**$100**	**0.5**	**$0**	**0.5**
D(s)	$100	$1 - q(s)$	$0	$q(s)$

Under SWUP, focusing on salient probabilities, yields a resolution to Ellsberg's paradox. We state our result under the assumption of a uniform prior which is a very intuitive and plausible prior in the context of Ellsberg's paradoxes. For the two-color paradox we prove the following result:

Proposition 3 (Diminishing Sensitivity and Ambiguity Aversion): *Under a uniform prior, a focal thinker exhibits ambiguity aversion in monotone minimal frames if ϕ satisfies diminishing sensitivity.*

By a similar analysis it can be shown that diminishing sensitivity of ϕ also implies ambiguity aversion in Ellsberg's (1961) three-color paradox.

Now consider Figure 10 in which Ellsberg-style choices are presented in a transparent frame. If the decision maker has a uniform prior over the red and blue ticket states, then SWUP predicts ambiguity aversion in minimal frames, and ambiguity-neutrality (indifference between A and B) in transparent frames. Ambiguity neutrality is predicted in transparent frames since the common consequences in each state-contingent lottery are obvious. In addition, the diminishing sensitivity relationship between probabilities that was salient in the minimal frame is not cued in the transparent frame which instead focuses comparisons on the differences between $0 and $25.

Schneider, Leland, and Wilcox (2016) considered the simple setting of a world with two types of agents – those who are ambiguity-averse (agents who always choose A), and those who are ambiguity-neutral (agents who randomize between A and B with equal probability). They computed the unique proportion of ambiguity neutral agents which exactly fits the distribution of ambiguity-averse and ambiguity-seeking choices observed in their data, for both minimal and transparent frames. This approach estimated there to be about 43% ambiguity-neutral agents in minimal frames but 63% ambiguity neutral agents in transparent frames. While this framing effect is predicted by SWUP, we are not aware of an alternative model which predicts this frame-dependence of ambiguity aversion.

Figure 10. The Ellsberg Paradox in a Transparent Frame

	Red Ticket						Blue Ticket					
A	$0	0.25	$25	0.50	$0	0.25	$25	0.25	$25	0.25	$0	0.50
B	$25	0.25	$25	0.50	$0	0.25	$0	0.25	$25	0.25	$0	0.50

	Red Ticket						Blue Ticket					
A′	$25	0.25	$25	0.25	$0	0.50	$0	0.25	$25	0.50	$0	0.25
B′	$0	0.25	$25	0.25	$0	0.50	$25	0.25	$25	0.50	$0	0.25

4.5 Present Bias in Minimal and Transparent Frames

An analogous framing effect between minimal and transparent frames is predicted for choices over time. Consider the minimal frames in Figure 11. The stationarity axiom of Discounted Utility theory (Koopmans 1960), implies that people should choose either A and A' or B and B'. However, experiments show that people frequently choose A and B', a result termed *present bias* (Laibson, 1997). This behavior has been most frequently explained in terms of people exhibiting hyperbolic discounting (Loewenstein and Prelec, 1992) or quasi-hyperbolic discounting (Laibson, 1997). Under SWUP, present bias occurs for choices in the minimal frame because the upside of receiving a payoff now versus in two months is more salient than the downside of receiving 510 instead of 530. However, this downside becomes relatively more salient when the payoffs are each delayed two years, as in the choice between A' and B'.

Figure 11. Present Bias in Minimal and Transparent Frames

Present Bias in Minimal Frames

	(x_1,y_1)	Months
A	**510**	**0**
B	530	2

	(x_1,y_1)	Months
A′	510	24
B′	**530**	**26**

Present Bias in Transparent Frames

	(x_1,y_1)	Months	(x_2,y_2)	Months
A	510	0	0	2
B	**0**	**0**	**530**	**2**

	(x_1,y_1)	Months	(x_2,y_2)	Months
A′	510	24	0	26
B′	**0**	**24**	**530**	**26**

Formally, under SWUP, present bias occurs in the minimal frames in Figure 11 if (21) and (22) hold:

(21) $\theta(0,2)\left(1-\delta^{(2/12)}\right)(520)+\mu(510,530)(-10)\left(1+\delta^{(2/12)}\right)>0,$

(22) $\theta(24,26)\left(\delta^{2}-\delta^{(26/12)}\right)(520)+\mu(510,530)(-10)\left(\delta^{2}+\delta^{(26/12)}\right)<0,$

where δ is the annual discount factor. Under SWUP, with the parameter-free salience function and linear utility, (21) and (22) both hold for all $\delta\in[0.895,0.995]$.

Next, compare the choice between A and B in the minimal frame (left hand side of Figure 11) and in the transparent frame (right hand side of Figure 11). In the minimal frame, the difference between receiving a payoff immediately or after a two-month delay is more salient than the difference between a payoff of 510 or 530, resulting in the choice of the smaller sooner reward (option A). However, in the transparent frame, the difference between 530 and 0 is more salient than the difference between 510 and 0, resulting in the choice of the larger later reward (option B). This finding is an example of the 'hidden zero effect' originally observed by Magen et al. (2008) in a between-subjects design. This example is due to Read and Scholten (2012) who observed highly significant choice reversals in a within-subject design. Under the same specification of SWUP, B is chosen in the transparent frame for all $\delta\in[0.794,0.995]$. The hidden zero effect occurs under SWUP because the transparent frame shifts attention from comparing differences in payoffs versus differences in time delays to focusing on larger versus smaller payoff differences, producing more patient behavior. Consistent with this implication of

SWUP, Fisher and Rangel (2014) found that explicitly shifting attention from the time dimension to monetary amounts makes subjects more patient.

More generally, we can formalize present bias in Figure 12, where, $\mathbf{y} > \mathbf{x} > 0, \mathbf{t} > \mathbf{r} \geq 0$. The figure displays a generic choice between a smaller sooner (*SS*) and a larger later (*LL*) reward (represented by SS and LL, respectively, in Frame (i)), and an arithmetic operation applied to this base frame (Frame (ii)), which adds a constant to each time period. First, we provide a definition of hyperbolic discounting in the context of choices between smaller sooner versus larger later rewards. When $r = 0$ in Frames (i) and (ii), the behavior is referred to as *present bias.* It characterizes pairs of choices such as $80 now versus $100 in one year and $80 in five years vs. $100 in six years, where people frequently choose the smaller payoff when it is immediate, but select the larger payoff when both are delayed by the same amount of time.

Definition 8 (Hyperbolic Discounting): Consider minimal frames (i) and (ii) in Figure 12, for any $\mathbf{y} > \mathbf{x} > 0$, $\mathbf{t} > \mathbf{r} \geq 0$, $\boldsymbol{\Delta} > 0$. *Hyperbolic discounting* holds if $\mathrm{SS} \precsim \mathrm{LL}$ implies $\mathrm{LL}' \succsim \mathrm{SS}'$.

Figure 12. Hyperbolic Discounting

(i)	(x_1,y_1)	Period
SS	**x**	**r**
LL	y	t

(ii)	(x_1,y_1)	Period
SS′	x	$r + \Delta$
LL′	**y**	$\mathbf{t} + \Delta$

Proposition 4 (Diminishing Sensitivity and Hyperbolic Discounting): *A focal thinker exhibits hyperbolic discounting in minimal frames if and only if* θ *satisfies diminishing sensitivity.*

4.6 Violations of Cancellation in Minimal and Transparent Frames

Returning to the example by Read and Scholten (2012) from Section 4.5, suppose instead of adding a constant delay to alternatives A and B, that we instead added a common consequence to both options. This provides a test of the cancellation axiom, an independence condition in intertemporal choice, which states that preferences between two dated outcomes should not change if the same dated outcome is added to both alternatives. Despite its intuitive appeal, there are also intuitive cases where it is systematically violated. To illustrate, consider choices between A and B and between A' and B' in minimal and transparent frames in Figure 13 where the latter pair are obtained from the former by adding a common consequence of losing $520 in 1 month to each option. In such cases, SWUP predicts a shift in preference toward the delayed option B'.

Indeed, similar preference reversals have been observed by Rao and Li (2011). We say a violation of cancellation holds if A is chosen over B, but B' is chosen over A'. Under the simple specification of SWUP with linear utility and the parameter-free salience function, a violation of cancellation holds in the minimal frame for all $\delta \in [0.794, 0.995]$, the same parameter values that resolve the hidden zero effect. This 'cancellation effect' is incompatible with virtually all major models of time preference in the literature including the discounted utility model, Loewenstein and Prelec's (1992) model of hyperbolic discounting, and Laibson's (1997) model of quasi-hyperbolic discounting. Note that the cancellation axiom is satisfied in the transparent frame since in that case, B' is chosen over A' under SWUP if and only if B is chosen over A.

Figure 13. Violations of Cancellation in Minimal and Transparent Frames

The Cancellation Effect in Minimal Frames

	(x_1,y_1)	Month
$\boldsymbol{A}$	**510**	**0**
B	530	2

	(x_1,y_1)	Month	(x_2,y_2)	Month
A'	510	0	-520	1
$\boldsymbol{B'}$	**-520**	**1**	**530**	**2**

The Cancellation Effect in Transparent Frames

	(x_1,y_1)	Month	(x_2,y_2)	Month
A	510	0	0	2
$\boldsymbol{B}$	**0**	**0**	**530**	**2**

	(x_1,y_1)	Month	(x_2,y_2)	Month	(x_3,y_3)	Month
A'	510	0	0	2	-520	1
$\boldsymbol{B'}$	**0**	**0**	**530**	**2**	**-520**	**1**

5. Predictions for Transparent Frames

In this section we consider five prominent rationality principles: Stochastic dominance, the independence axiom for decisions under risk, ambiguity-neutrality for decisions under uncertainty, and the stationarity and cancellation axioms for decisions over time. We show that any focal thinker satisfies stochastic dominance, independence, ambiguity-neutrality, stationarity, and cancellation, when choices are presented in transparent frames[11]. Our results in this section are general in that they hold for any salience functions and any utility functions.

[11] One might wish to consider the weaker prediction that a decision maker is simply 'more likely' to satisfy the axioms in transparent frames. This can be accomplished by allowing for a probability, γ, that the decision maker mentally re-frames the problem as a minimal frame when given a transparent frame of a decision. This parameter can be viewed as an individual characteristic of the decision maker which measures the 'strength' of the framing effect for that agent. The value $1 - \gamma$ then provides the probability that the decision maker satisfies the classical axioms when choice alternatives are presented in transparent frames.

5.1 Stochastic Dominance in Transparent Frames

Consider one of the most basic tenets of rational choice under uncertainty – respect for stochastic dominance. SWUP predicts that for transparent frames, a focal thinker always prefer probabilistically more to probabilistically less. Since payoffs across alternatives are ordered monotonically, any differences in the evaluation process in (4) will always favor the stochastically dominant option:

Definition 9 (Stochastic Dominance): Lottery p (first-order) stochastically dominates q if $P(x) \leq Q(x)$ for all $x \in$ X, with at least one strict inequality, where $P(x)$ and $Q(x)$ are the cumulative distribution functions corresponding to p and q, respectively. Given a frame $\mathbf{F}\{\mathbf{p}, \mathbf{q}\}$, we say that a focal thinker *satisfies stochastic dominance* if p (first-order) stochastically dominates q implies $\mathbf{p} \mathrel{\widehat{\gtrsim}} \mathbf{q}$.

Proposition 5: *A focal thinker satisfies stochastic dominance for all transparent frames.*

5.2 Independence in Transparent Frames

An agent choosing based on (4) will not exhibit the common consequence or the common ratio effect in a transparent frame, and thus the independence axiom of EUT is predicted to be observed to hold in such cases. Recall that the independence axiom states, for all $p, q, p'' \in \Delta(X)$, and for all $\alpha \in (0,1)$,

$$p \succ q \text{ if and only if } \alpha p + (1-\alpha)p'' \succ \alpha q + (1-\alpha)p''.$$

An analogous condition over representations can be defined as follows:

Definition 10 (Independence over representations): A focal thinker satisfies *independence over representations* if for all representations $\mathbf{p}, \mathbf{q}$, and $\mathbf{p}''$ of lotteries $p, q, p'' \in \Delta(X)$, and for all $\boldsymbol{\alpha} \in (0,1)$,

$$\mathbf{p} \mathrel{\widehat{\gtrsim}} \mathbf{q} \text{ if and only if } \boldsymbol{\alpha}\mathbf{p} + (\mathbf{1}-\boldsymbol{\alpha})\mathbf{p}'' \mathrel{\widehat{\gtrsim}} \boldsymbol{\alpha}\mathbf{q} + (\mathbf{1}-\boldsymbol{\alpha})\mathbf{p}''.$$

Just as the independence axiom of EUT requires that preferences over lotteries p and q are independent of whether or not they are mixed with the common lottery, p'' in the same proportions, Definition 9 implies that perceptions over representations $\mathbf{p}$ and $\mathbf{q}$ are independent of whether or not they are mixed with the common representation, $\mathbf{p}''$, in the same proportions. Independence over representations does not hold in general for a focal thinker (for arbitrary frames). However, it *does* hold for all pairs of representations in transparent frames.

Proposition 6: *A focal thinker satisfies independence over representations for all transparent frames.*

5.3 Ambiguity Neutrality in Transparent Frames

Here we reconsider Ellsberg's two-color paradox from Section 4.4, for which we will show that SWUP implies ambiguity neutrality in transparent frames. The transparent frames for Choices 1 and 2 in Section 4.4 are shown in Figure 14, where state $s \in \{0,1,\dots,100\}$ indexes the number of red balls in Urn 2, and $p(s) = |50 - s|/100$. Note that when $p(s) = 0.25$, Figure 14 is essentially equivalent to the frame in Figure 10.

Figure 14. The Ellsberg Paradox in Transparent Frames

	States favoring A: $s \in \{0,1,\dots,50\}$						States favoring B: $s \in \{51,52,\dots,100\}$					
A	\$100	$p(s)$	\$100	$0.5 - p(s)$	\$0	0.5	\$0	$p(s)$	\$100	0.5	\$0	$0.5 - p(s)$
B	\$0	$p(s)$	\$100	$0.5 - p(s)$	\$0	0.5	\$100	$p(s)$	\$100	0.5	\$0	$0.5 - p(s)$

	States favoring C: $s \in \{51,52,\dots,100\}$						States favoring D: $s \in \{0,1,\dots,50\}$					
C	\$100	$p(s)$	\$100	$0.5 - p(s)$	\$0	0.5	\$0	$p(s)$	\$100	0.5	\$0	$0.5 - p(s)$
D	\$0	$p(s)$	\$100	$0.5 - p(s)$	\$0	0.5	\$100	$p(s)$	\$100	0.5	\$0	$0.5 - p(s)$

Without loss of generality, set $U(100) = 1$ and $U(0) = 0$. Denote the set of states favoring A by $\overline{S}$ and the set of states favoring B by $\underline{S}$. The SWUP evaluation for the choice between A and B is:

(23) $$\sum_{s\in\overline{S}} \pi_s\, \mu(100,0)\big(p(s)\big) + \sum_{s\in\underline{S}} \pi_s\, \mu(0,100)\big(-p(s)\big)$$

where π_s is the subjective probability that the true state is s. All other differences within each column vector in the frame cancel. Under a uniform prior, the decision maker is necessarily indifferent between A and B (by symmetry of the salience function μ) in which case the evaluation in (23) equals zero. Moreover, even if the distribution is not uniform, (23) implies ambiguity neutrality since if (23) is positive, the decision maker would prefer A and D and if (23) is negative, the decision maker prefers B and C. This argument also extends to Ellsberg's (1961) classic three-color paradox.

5.4 Stationarity in Transparent Frames

Transparent frames are also predicted to induce rational behavior in intertemporal choice consistent with stationarity. Recall that the stationarity axiom states, given $a, b, a', b' \in C$, where $a \coloneqq (x_r), b \coloneqq (y_{r+\Delta}), a' \coloneqq (x_t)$,
$b' \coloneqq (y_{t+\Delta}),\ t, r \geq 0, x, y \in \mathbb{R}$ and $\Delta \geq 0$, that if $a \sim_t b$ then $a' \sim_t b'$, where $\sim$ denotes indifference. Next, consider an analogous condition for representations.

Definition 11 (Stationarity over representations)[12]: Given any representations $\mathbf{a}, \mathbf{b}, \mathbf{a}', \mathbf{b}'$ of $a, b, a', b' \in C$, where $\mathbf{a} \coloneqq (\mathbf{x}, \mathbf{r}), \mathbf{b} \coloneqq (\mathbf{y}, \mathbf{r} + \Delta), \mathbf{a}' \coloneqq (\mathbf{x}, \mathbf{t}), \mathbf{b}' \coloneqq (\mathbf{y}, \mathbf{t} + \Delta)$, and $\mathbf{t}, \mathbf{r} \geq 0, \mathbf{x}, \mathbf{y} \in \mathbb{R}$ and $\Delta \geq 0$, a focal thinker satisfies *stationarity over representations* if $\mathbf{a} \hat{\sim}_t \mathbf{b}$ implies $\mathbf{a}' \hat{\sim}_t \mathbf{b}'$.

Proposition 7: *A focal thinker satisfies stationarity over representations for all transparent frames.*

5.5 Cancellation in Transparent Frames

We next consider the cancellation axiom satisfied by discounted utility theory. The cancellation axiom is an intertemporal analog to the independence axiom in EUT and implies that preferences do not depend on outcomes both consumption plans have in common. The axiom states, given consumption plans $a, b, a', b' \in C$, where $a \coloneqq (x_r), b \coloneqq (y_t), a' \coloneqq (x_r, z_\Delta), b' \coloneqq (y_t, z_\Delta)$, with $t, r, \Delta \geq 0, x, y, z \in \mathbb{R}$, that $a \succ_t b$ if and only if $a' \succ_t b'$. An analogous condition over representations is defined as follows:

Definition 12 (Cancellation over representations): Given any representations $\mathbf{a}, \mathbf{b}, \mathbf{a}', \mathbf{b}'$ of consumption plans $a, b, a', b' \in C$, where $\mathbf{a} \coloneqq (\mathbf{x}, \mathbf{r}), \mathbf{b} \coloneqq (\mathbf{y}, \mathbf{t}), \mathbf{a}' \coloneqq (\mathbf{x}, \mathbf{r}; \mathbf{z}, \Delta), \mathbf{b}' \coloneqq (\mathbf{y}, \mathbf{t}; \mathbf{z}, \Delta)$, with $\mathbf{t}, \mathbf{r}, \Delta \geq 0, \mathbf{x}, \mathbf{y}, \mathbf{z} \in \mathbb{R}$, a focal thinker satisfies *cancellation over representations* if $\mathbf{a} \hat{\succ}_t \mathbf{b}$ if and only if $\mathbf{a}' \hat{\succ}_t \mathbf{b}'$.

Proposition 8: *A focal thinker satisfies cancellation over representations for all transparent frames.*

[12] The relation $\hat{\sim}$ can be interpreted as "looks as good as," just as $\hat{\succ}$ has the interpretation "looks better than."

6. Framing Effects between Positive and Negative Frames

SWUP also provides a new perspective for studying other framing effects, such as those between positive and negative frames. Consider one of the most famous examples of a framing effect, due to Tversky and Kahneman (1981), in which respondents are told that the U.S. is preparing for the outbreak of an epidemic which is expected to kill 600 people. Policy makers must choose between two prevention strategies: Program A saves 200 lives. Program B has a 1/3 chance of saving 600 people and a 2/3 chance of saving no one. A different group of respondents was given a choice between Programs C and D, and were told that if Program C is taken, then 400 people will die. If Program D is taken, then there is a 1/3 chance that no people will die and a 2/3 chance that all 600 people will die. The frames of both decisions are given in Figure 15.

Programs A and C differ only in how the outcomes are labeled (as lives saved or lives lost) and are thus logically equivalent. This observation also holds for Programs B and D. However, most people chose A over B and chose D over C, thereby exhibiting a framing effect.

The traditional explanation for this framing effect is due to a value function that is concave for gains and convex for losses. SWUP provides a different perspective. Under SWUP, Program A is chosen over Program B if saving no lives versus saving 200 lives is more salient than saving 200 lives versus 600 lives. In addition, D is chosen over C if having 0 deaths versus 400 deaths is more salient than having 400 deaths versus 600 deaths. Formally, for $U(x) = x$, a focal thinker chooses Program A over Program B if $\mu(200,0) > \mu(200,600)$. Analogously, the focal thinker chooses Program D over Program C if $\mu(-400,-600) < \mu(-400,0)$. Both of these inequalities hold for the parameter-free salience function from (12). SWUP thus yields the choice for A over B in the 'lives saved' frame and the choice for D over C in the 'lives lost' frame, as observed by Tversky and Kahneman.

Figure 15. Framing Effect between Positive and Negative Frames

	(x_1,y_1)	(p_1,q_1)	(x_2,y_2)	(p_2,q_2)
Program A	**200 lives saved**	**1/3**	**200 lives saved**	**2/3**
Program B	600 lives saved	1/3	0 lives saved	2/3

	(x_1,y_1)	(p_1,q_1)	(x_2,y_2)	(p_2,q_2)
Program C	400 people will die	1/3	400 people will die	2/3
Program D	**0 people will die**	**1/3**	**600 people will die**	**2/3**

7. Related Literature

A leading descriptive model for decisions under risk is widely recognized to be Tversky and Kahneman's (1992) cumulative prospect theory (CPT). While CPT can explain most phenomena for decisions under risk, it does not explain any of the framing effects in Section 4 such as those for the Allais paradox, the common ratio effect, stochastic dominance, the Ellsberg paradox, present bias, or violations of cancellation.

The model of Loewenstein and Prelec (1992) can account for hyperbolic discounting, but cannot explain violations of cancellation identified by Rao and Li (2011) or the hidden zero effect identified by Magen et al. (2008). Models of quasi-hyperbolic discounting (Laibson 1997; O'Donoghue and Rabin, 1999) explain present bias, but also do not explain these other features of time preferences.

The role of diminishing sensitivity in explaining hyperbolic discounting also arises in other models such as those of Prelec and Loewenstein (1991), Kim and Zauberman (2009), and Scholten and Read (2010). SWUP identifies an additional implication of this property (as in Proposition 3 on ambiguity aversion) and embeds the 'salience' functions satisfying diminishing sensitivity in a form that is otherwise equivalent to the standard models of rational choice under risk, uncertainty, and time. Under this approach, diminishing sensitivity is generally linked to systematic deviations from rational behavior across all three domains.

Rubinstein (1988), and Leland (1994, 1998) among others, have proposed models of risky choice that involve ignoring small or similar differences between attributes across alternatives and attending to large, dissimilar ones. Like SWUP, these models imply that choices will be sensitive to the way alternatives are framed to the extent framing determines what is being compared. Like SWUP, they also imply that theoretically inconsequential arithmetic manipulations of attribute values may influence choices to the extent these manipulations influence which attribute values are perceived as similar and which appear different. Leland (2002) and Rubinstein (2003) demonstrated that similarity reasoning extends naturally to intertemporal choice. However, while this class of models provides a plausible explanation for anomalous behaviors observed in risky and intertemporal choice, the models are too imprecise in that they do not clearly specify when two attributes are similar or dissimilar, and are too non-compensatory to provide an adequate depiction of behavior in general.

Recent models by Bordalo et al. (2012) and Koszegi and Szeidl (2013) are closely related to SWUP in that they assume that the salience of differences across alternatives influences choices through their impact on expected and discounted utility, respectively. However, neither model considers the possibility that decisions under risk or over time might be swayed by the perceived salience of differences in probabilities or dates of receipt. As a result, they cannot explain robust behaviors such as the Ellsberg paradox and present bias between smaller sooner and larger later rewards that arise from salience perceptions on these dimensions. Bordalo et al. (2012) also assume that a decision maker forms a salience ranking over all possible payoff combinations that can occur between two independent lotteries. For instance, in a choice between two independent lotteries, each with five distinct outcomes, their salience model assumes the decision maker forms a salience ranking over the 25 possible binary payoff combinations. In contrast, under SWUP, the decision maker need only compare the 10 column vectors (five payoff vectors and five probability vectors) in the frame. In addition, the model in Bordalo et al. (2012) assumes that it is correlation between lotteries, rather than framing, that turns the Allais paradox on or off. However, Incekara-Hafalir and Stecher (2012) employed the same correlation structure for both prospect and Savage presentations and observed highly significant framing effects. Similar framing effects for the common ratio effect were observed by Harless (1992). Allais paradox studies of Birnbaum (2004b, 2007) also found framing to be responsible for mediating the effect. Leland (1998) demonstrated that choices under risk are more susceptible to changes in framing than to the correlation structure between lotteries.

The comparative nature of SWUP is somewhat reminiscent of regret theory (Loomes and Sugden, 1982; Bell, 1982) and the perceived relative argument model (PRAM) due to Loomes (2010). However regret theory predicts sensitivity to correlation, not framing effects, and does not apply across decision domains, while PRAM applies to choices under risk involving at most three outcomes.

The transfer-of-attention-exchange (TAX) model (Birnbaum and Chavez, 1997; Birnbaum and Navarrete, 1998) provides an alternative explanation for framing effects in decisions under risk (such as stochastic dominance violations and the Allais paradox) as arising from ‘coalescing’ and ‘branch-splitting’ effects in a tree representation of lotteries. However, the TAX model does not apply to decisions under ambiguity or over time and thus does not explain the Ellsberg paradox or its dependence on framing, nor does it explain the anomalies in intertemporal choice.

Salant and Rubinstein (2008) formalize frames in the context of consumer theory and relate their model to the revealed preference framework. Here, we provide a formalization of frames for decisions under risk, uncertainty, and time. For decisions under risk, minimal frames are related to Birnbaum's (1999) tree representation of lotteries in 'coalesced form'. In particular, a choice set in which all lotteries are in coalesced form generates a minimal frame. Transparent frames are similar to Birnbaum and Schmidt's (2015) tree representation of lotteries in 'canonical split form,' but are distinct in that the canonical split form does not separate common consequences from distinct consequences. In addition, minimal and transparent frames each apply more broadly across decision domains.

8. Conclusion

We have formalized two ways of presenting choice alternatives - minimal frames (the simplest, most efficient representation of choices), and transparent frames (that make the normative appeal of the classical axioms more transparent). Familiar anomalies are predicted to occur in minimal frames, whereas more normative behavior is predicted in transparent frames. The predictions for both minimal and transparent frames are consistent with experimental evidence and provide a unified approach to decision making across the domains of risk, uncertainty, and time while relying on minimal behavioral assumptions. In particular, the *same* comparative model of salience-based choice and the same properties of salience perception predict behavior across each of these three domains. The predictions rely primarily on (i) the property of diminishing sensitivity, (ii) the distinction between minimal and transparent frames, and (iii) the salience-based decision algorithm that operates over frames. Diminishing sensitivity thus emerges as a general principle that produces some of the major observed behaviors under risk, uncertainty and time (risk aversion, ambiguity aversion, and hyperbolic discounting). Moreover, a simple sufficient condition that generates all framing effects in the experimental examples from Section 4 is to consider a focal thinker with linear utility and salience perceptions given by the parameter-free salience function.

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Appendix: Proofs of Propositions

Proposition 1 (Uniqueness of Monotone Minimal Frames: Risk): *For a choice between two lotteries p and q, with* |*supp*(p)| = |*supp*(q)|, *there exists a frame that is minimal and monotonic in outcomes which is unique up to the operations of row-switching and reversal of column order.*

Proof: Let |*supp*(p)| = |*supp*(q)| = *n.* Then a minimal frame has dimension *n,* and the outcomes in *supp*(p) and *supp*(q) coincide exactly with the outcomes in the frame. Since a minimal frame that weakly monotonic in outcomes must be strictly monotonic, outcomes are presented such that the ith largest outcome in the row vector of p is in the same column as the ith largest outcome in the row vector of q, where we recall that $i = 1, 2, \ldots, n$ indexes the *position* of the ith column vector in the frame.■ The proof of Proposition 2 is analogous to the proof of Proposition 1.

Proposition 3 (Diminishing Sensitivity and Ambiguity Aversion): *Under a uniform prior, a focal thinker exhibits ambiguity aversion in monotone minimal frames if ϕ satisfies diminishing sensitivity.*

Proof: As indicated in the text, the proof here is for Ellsberg's two-color paradox. An analogous argument demonstrates that diminishing sensitivity of ϕ also resolves Ellsberg's three-color paradox. Let s denote the number of red balls in Urn 2. Since the number of black balls is given by $100 - s$, the state of the urn is fully characterized by $s.$ For each state, the presentation for Choice 1 is given by Figure 9, where $\mathbf{q(s)}$ denotes the probability of drawing a red ball from Urn 2 in state s. Without loss of generality, we normalize the payoffs such that $U(\mathbf{100}) = 1$, and $U(\mathbf{0}) = 0$. For a focal thinker, A is chosen over B if and only if inequality (25) holds:

(25) $\sum_{s=1}^{m} \pi_s \, [\phi(\mathbf{0.5}, \mathbf{q(s)})(\mathbf{0.5} - \mathbf{q(s)})] > 0.$

Under a uniform prior, inequality (25) becomes

(26) $\frac{1}{101}[\sum_{s=0}^{s=50} \phi(\mathbf{0.5}, \mathbf{q(s)})(\mathbf{0.5} - \mathbf{q(s)}) + \sum_{s=51}^{s=100} \phi(\mathbf{0.5}, \mathbf{q(s)})(\mathbf{0.5} - \mathbf{q(s)})] > 0,$

which implies

(27) $\sum_{s=0}^{s=50} \phi(\mathbf{0.5}, \mathbf{q(s)})(\mathbf{0.5} - \mathbf{q(s)}) + \sum_{s=0}^{s=50} \phi(\mathbf{0.5}, \mathbf{1} - \mathbf{q(s)})(\mathbf{q(s)} - \mathbf{0.5}) > 0.$

To see that (27) holds, note that for each $\mathbf{q(s)} \in [0,0.5)$ diminishing sensitivity and symmetry of ϕ imply $\phi(\mathbf{0.5}, \mathbf{1} - \boldsymbol{q(s)}) < \phi(\mathbf{0.5}, \mathbf{q(s)})$. In particular, by symmetry, $\phi(\mathbf{0.5}, \mathbf{1} - \mathbf{q(s)}) = \phi(\mathbf{0.5} + \mathbf{0.5} - \mathbf{q(s)}, \mathbf{q(s)} + \mathbf{0.5} - \mathbf{q(s)}) = \phi(\mathbf{0.5} + \boldsymbol{\epsilon}, \mathbf{q(s)} + \boldsymbol{\epsilon})$. Thus, by diminishing sensitivity, (27) holds, yielding a choice for the risky over the ambiguous urn. The argument follows analogously for the choice between C and D, resulting in ambiguity aversion. ■

Proposition 4 (Diminishing Sensitivity and Hyperbolic Discounting): *A focal thinker exhibits hyperbolic discounting in minimal frames if and only if* θ *satisfies diminishing sensitivity.*

Proof: Note that a focal thinker views **SS** and **LL** to look equally good if and only if (28) holds:

(28) $\mu(\mathbf{x},\mathbf{y})\big(U(\mathbf{y})-U(\mathbf{x})\big)(\delta^{\mathbf{r}}+\delta^{\mathbf{t}})=\theta(\mathbf{r},\mathbf{t})(\delta^{\mathbf{r}}-\delta^{\mathbf{t}})(U(\mathbf{y})+U(\mathbf{x}))$.

A focal thinker always chooses $\mathbf{LL}'$ over $\mathbf{SS}'$ if and only if inequality (29) holds:

(29) $\mu(\mathbf{x},\mathbf{y})\big(U(\mathbf{y})-U(\mathbf{x})\big)\big(\delta^{\mathbf{r}+\Delta}+\delta^{\mathbf{t}+\Delta}\big)>\ \theta(\boldsymbol{r}+\boldsymbol{\Delta},\boldsymbol{t}+\boldsymbol{\Delta})\big(\delta^{\mathbf{r}+\Delta}-\delta^{\mathbf{t}+\Delta}\big)\big(U(\mathbf{y})+U(\mathbf{x})\big)$.

By factoring out δ^{Δ} and by substitution, we see that hyperbolic discounting holds if and only if we have $\theta(\mathbf{r},\mathbf{t})>\theta(\mathbf{r}+\boldsymbol{\Delta},\mathbf{t}+\boldsymbol{\Delta})$, which holds for any $\Delta>0$ if and only if θ satisfies diminishing sensitivity. ■

Proposition 5 (Stochastic Dominance): *A focal thinker satisfies stochastic dominance for all transparent frames.*

Proof: If p first-order stochastically dominates q, then in any transparent frame $\mathbf{x_i}\geq\mathbf{y_i}$ for all i, and all probability differences are zero. Thus, the salience weights in (4) favor **p** over **q** in each binary comparison for which the differences are not zero. ■

Proposition 6 (Independence): *A focal thinker satisfies independence over representations for all transparent frames.*

Proof: If p and q have no common consequences, then in transparent frames the lotteries are represented as in Figure A.1, where $p'':=(z_1,s_1;\dots;z_m,s_m)$.

Figure A.1. Independence in Transparent Frames

p	$\mathbf{x_1}$	$\mathbf{p_1}$	…	$\mathbf{x_n}$	$\mathbf{p_n}$
q	$\mathbf{y_1}$	$\mathbf{p_1}$	…	$\mathbf{y_n}$	$\mathbf{p_n}$

$\mathbf{p}'$	$\mathbf{x_1}$	$\boldsymbol{\alpha}\mathbf{p_1}$	…	$\mathbf{x_n}$	$\boldsymbol{\alpha}\mathbf{p_n}$	$\mathbf{z_1}$	$(\mathbf{1}-\boldsymbol{\alpha})\mathbf{s_1}$	…	$\mathbf{z_m}$	$(\mathbf{1}-\boldsymbol{\alpha})\mathbf{s_m}$
$\mathbf{q}'$	$\mathbf{y_1}$	$\boldsymbol{\alpha}\mathbf{p_1}$	…	$\mathbf{y_n}$	$\boldsymbol{\alpha}\mathbf{p_n}$	$\mathbf{z_1}$	$(\mathbf{1}-\boldsymbol{\alpha})\mathbf{s_1}$	…	$\mathbf{z_m}$	$(\mathbf{1}-\boldsymbol{\alpha})\mathbf{s_m}$

In Figure A.1, $\mathbf{p}'\equiv\boldsymbol{\alpha}\mathbf{p}+(\mathbf{1}-\boldsymbol{\alpha})\mathbf{p}''$ and $\mathbf{q}'\equiv\boldsymbol{\alpha}\mathbf{q}+(\mathbf{1}-\boldsymbol{\alpha})\mathbf{p}''$. For a focal thinker, the common consequences cancel. Since the distinct consequences are ordered monotonically in outcomes, the salience-weighted evaluation under SWUP (given by (4)) for the choice between **p** and **q** is the same as for the choice between $\mathbf{p}'$ and $\mathbf{q}'$. In particular, we have

$$\mathbf{p}\gtrless\mathbf{q}\ \text{ if and only if }\ \textstyle\sum_{i=1}^{n}\mu(\mathbf{x_i},\mathbf{y_i})\big(U(\mathbf{x_i})-U(\mathbf{y_i})\big)\mathbf{p_i}>0 \text{ if and only if } \mathbf{p}'\gtrless\mathbf{q}'.$$

Now suppose that p and q have one or more common consequences. Consider an arbitrary (maximal) common consequence between p and q that occurs with probability r. If the corresponding outcome is not in the support of p'', then this common consequence occupies a column vector separated from the distinct consequences, in which case the outcome differences and probability differences cancel in both choices. If the common consequence is in the support of p'', and occurs in p'' with probability s, then the common consequences are merged into a single outcome column vector and a corresponding probability column vector in the choice between p' and q', such that the overall probability of the common outcome is $r + (1 - \alpha)s$. Again, the outcome and probability differences cancel and do not affect the evaluation. ■

Proposition 7 (Stationarity): *A focal thinker satisfies stationarity over representations for all transparent frames.*

Proof: Consider the frames between consumption plans a and b and between a' and b' in Figure A.2. Note that $\mathbf{a} \precsim_t \mathbf{b}$ implies $\mu(\mathbf{x}, \mathbf{0})\mathbf{x}\delta^{\mathbf{r}} = \mu(\mathbf{0}, \mathbf{y})(\mathbf{y})\delta^{\mathbf{r}+\Delta}$. In addition, $\mathbf{a}' \precsim_t \mathbf{b}'$ implies that $\mu(\mathbf{x}, \mathbf{0})\mathbf{x}\delta^{\mathbf{t}} = \mu(\mathbf{0}, \mathbf{y})(\mathbf{y})\delta^{\mathbf{t}+\Delta}$. Note that we can write $\mathbf{t} \equiv \mathbf{r} + \boldsymbol{s}$, for some constant s. Then δ^{s} can be factored out and a focal thinker satisfies stationarity over representations in transparent frames. ■

Figure A.2. Stationarity in Transparent Frames

a	**x**	**r**	**0**	**r + Δ**	**a′**	**x**	**t**	**0**	**t + Δ**
b	**0**	**r**	**y**	**r + Δ**	**b′**	**0**	**t**	**y**	**t + Δ**

Proposition 8 (Cancellation): *A focal thinker satisfies cancellation over representations for all transparent frames.*

Proof: Consider the transparent frames between consumption plans a and b and between a' and b' in Figure A.3. Under SWUP, the common consequence (z, Δ) cancels in transparent frames since the differences in payoffs and delays are each zero, in which case the choice between a' and b' is evaluated by a focal thinker in the same way as the choice between a and b.■

Figure A.3. Cancellation in Transparent Frames

a	**x**	**r**	**0**	**t**	**a′**	**x**	**r**	**0**	**t**	**z**	**Δ**
b	**0**	**r**	**y**	**t**	**b′**	**0**	**r**	**y**	**t**	**z**	**Δ**