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Free Riding in Noncooperative Entry Deterrence with Differentiated Products

Dan Kovenock* and Suddhasatwa Roy†

We examine free riding and underinvestment in noncooperative entry deterrence in the Gilbert and Vives (1986) model with differentiated products. Our analysis proves that for products that are differentiated enough, when both entry allowing and entry deterring equilibria coexist, the symmetric entry deterring equilibrium may Pareto dominate the entry equilibrium. Hence, "coordination failure" underinvestment in entry prevention can occur. However, as claimed, the overinvestment result of Gilbert and Vives remains robust to moderate amounts of product differentiation. We also show that coordination failure underinvestment arises in a wide variety of entry deterrence models and does not rely on assumptions regarding strategic substitutability or complementarity of precommitments.

JEL Classification: L13

1. Introduction

This paper reexamines the phenomenon of free riding in entry deterrence when established firms in an oligopoly are unable to coordinate their entry preventing activities. Previous authors (e.g., Bernheim 1984; Gilbert and Vives 1986; Waldman 1987, 1991; Appelbaum and Weber 1992) have highlighted the public good aspect of noncooperative entry prevention—if costly entry deterring actions are successfully undertaken by a proper subset of the incumbents, then incumbents outside of that subset cannot be excluded from the benefits of deterred entry. It is in this sense that entry deterrence acquires the nature of a public good. This observation has prompted previous researchers to raise the "free rider" question (with its associated welfare implications): Can there occur underinvestment in entry deterrence due to the incentive for each firm to free ride on the others' (costly) entry preventing activities?

The free rider problem in entry deterrence is first mentioned in the sequential entry model of Bernheim (1984). However, though Bernheim discusses the possibility of free riding in his model, the free rider problem is not the main focus of his paper. In fact, as pointed out by Waldman (1987, p. 309, footnote 2), the author's discussion of the role of the free rider problem is "... somewhat vague as regards whether Bernheim feels the free rider problem would ever be important in a noncooperative entry deterrence setting."[1]

Gilbert and Vives (1986) is the first paper in which the underinvestment issue is explicitly addressed. The authors define underinvestment in entry deterrence to be associated with one or more of the following:

> "(a) Incumbents' total profits are higher preventing than allowing entry, but the (unique) industry equilibrium allows entry.

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[1] We thank an anonymous referee for pointing this out.

(b) Either entry prevention or entry may be an industry equilibrium, but incumbents' profits are higher when entry is prevented.
(c) An established monopoly (or colluding incumbents) prevents entry in more situations than an established, noncooperating oligopoly." (p. 77)

Label (a), (b), and (c) as underinvestment of type 1, 2, and 3, respectively. Gilbert and Vives (G&V) go on to show that in none of these respects is there underinvestment in their quantity setting homogeneous product model. On the contrary, they demonstrate a strong possibility of overinvestment.

G&V consider a situation where symmetric noncooperative incumbents with constant marginal costs of production make credible commitments to outputs in the preentry stage.[2] The entrant incurs a fixed entry cost if it enters the industry and makes its entry and output decision after observing the incumbents' output choices. Consequently, there exists a "limit output," which if jointly produced by the oligopoly, deters entry.[3] G&V find that entry is prevented when limit outputs are small, while entry is allowed for larger limit outputs. For limit outputs in an intermediate range, both allowing entry and preventing entry are equilibria. They prove that in this intermediate range where both entry accommodating and entry preventing equilibria coexist, the unique entry equilibrium Pareto dominates every deterrence equilibrium. In other words, compared with any deterrence equilibrium (there is typically a continuum of such equilibria) the accommodation equilibrium yields strictly higher profits to every incumbent. Hence, if the industry settles on an entry preventing equilibrium, the implication is that overinvestment exists because, by jointly reducing incumbents' outputs and allowing entry, every incumbent can be made better off.

This paper introduces product differentiation into the G&V model and shows that sufficiently large amounts of product differentiation can generate underinvestment in entry prevention. The intuition is straightforward. Consider an incumbent's profit in any entry deterring equilibrium where exactly the limit output is produced by the oligopoly. In the homogeneous good model, the price of the product is always the (constant) price that clears the limit output, regardless of how the limit output is distributed among the incumbents. This, coupled with constant marginal costs, implies that each incumbent's profit is continuously increasing in its own output or, equivalently, decreasing in rival's output.[4] Consequently, each firm wants the largest share in the limit output—an incentive that rules out the possibility of underinvestment. However, with differentiated products, the larger an incumbent's share of the limit output is, the smaller the price is. There are, then, two opposing forces at work: An increase in an incumbent's share of the limit output has a positive influence on its profit, but the resultant fall in price has a negative effect on profit. When the second effect outweighs the first, the incumbent's deterrence profit need not be continuously increasing in own output, that is, it can be increasing in the other's quantity over some range and decreasing over other ranges.[5] This could weaken an incumbent's incentive to have the largest share in the entry deterring output and could generate underinvestment. Note that, starting from the homogenous good benchmark, increasing the degree of product differentiation strengthens the second effect, that is, for sufficiently

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[2] See Allen et al. (2000) for an interpretation of output precommitment as a reduced form for capacity in a three-stage game of sequential capacity choice followed by simultaneous price setting.

[3] The limit output Z is determined by the level of the fixed entry cost F, for example, $Z = \max(0,1 - c - 2\sqrt{F})$ in the case of a symmetric duopoly with a constant unit cost of production c and a linear inverse demand curve $P = \max(0,1 - Q)$.

[4] Incumbent i's profit at an entry deterring equilibrium is given by $[P(Z) - c]\, q_i$, where $P(.)$ is the inverse demand function, c is the constant marginal cost, and $[P(Z) - c]$ is a fixed number.

[5] We shall henceforth use the term "deterrence profit" to refer to an incumbent's maximum profit from deterring entry for given levels of its rivals' choice variable.

differentiated products, an increase in rival output (and a consequent decrease in own output) raises own price to such an extent that its positive effect on profit more than compensates for the negative effect on profit of a lower own share of the limit output, thereby resulting in a deterrence profit that is increasing in rival output over the relevant range.[6]

We formalize this intuition by incorporating the differentiated products demand structure of Vives (1985) into the G&V model. Using G&V's methodology, the equilibria of the game are characterized in terms of the limit output. Focusing on the region where both entry allowing and entry deterring equilibria exist, we show that underinvestment of type 2 is a distinct possibility, namely, the entry allowing equilibrium may be Pareto dominated by an entry deterring equilibrium. We call this type of underinvestment "coordination failure" underinvestment—there exists an equilibrium where entry is deterred, but coordination failure may lead to one that allows entry and yields lower payoffs to all incumbents. This sort of underinvestment can arise even when type 1 and type 3 underinvestment are absent and is more likely with greater amounts of product differentiation. However, moderate amounts of product differentiation preserve the G&V overinvestment result. We demonstrate that coordination failure underinvestment, in our model, can occur only if each incumbent's entry deterring profit is increasing in its rival's output at the point at which it is indifferent between allowing entry and preventing entry. A numerical example of underinvestment is also provided.

Bernheim (1984) is the first paper to recognize the possibility of coordination failure underinvestment by pointing out the existence of one type of equilibrium in which each incumbent firm makes a zero investment in entry deterrence and equilibria of a second type where investments are just sufficient to deter entry. However, since the second kind of equilibria exist if and only if entry deterrence is jointly profitable, Bernheim chooses to focus on the symmetric entry deterring equilibrium on the grounds that the possibility of "informal communication" would lead to the industry settling on the equilibrium that is not Pareto dominated by the entry allowing equilibrium.

In similar vein, Waldman (1987, 1991) adopts Bernheim's equilibrium choice rule and sets out to investigate whether or not, given this equilibrium choice rule, the free rider problem is important. While the possibility of coordination failure underinvestment is clearly understood, as evidenced by the discussion in Waldman (1987) of Bernheim's paper, the author's adoption of Bernheim's equilibrium choice rule necessarily results in coordination failure underinvestment being ruled out. More specifically, Waldman (1987) considers uncertainty regarding the exact investment needed to deter entry and demonstrates that while such uncertainty causes underinvestment in the Bernheim framework, introducing uncertainty in the G&V model does not change their original conclusions and free riding remains nonexistent. However, he defines underinvestment as the situation where the aggregate investment at the noncooperative equilibrium is less than that which maximizes expected joint profits, that is, his results refer to underinvestment of type 3 only.

On the other hand, in the sequential entry model of Waldman (1991), underinvestment is regarded as the situation where allowing entry is the unique noncooperative equilibrium even though deterring entry is mutually more profitable for all incumbents. Thus, his conclusion that the presence of multiple potential entrants is crucial for underinvestment in entry deterrence is valid for type 1 underinvestment only. In their externalities model, Appelbaum and Weber (1992, p. 474) consider precommitments that can be unambiguously classified as "public goods" or "public bads" and predict that "if precommitments constitute 'public goods' but make incumbents 'tough', we have under-investment." They use the same definition of underinvestment as Waldman (1987), and so their results must be

[6] See section 3 and footnote 15 for further discussion.

similarly qualified. In other words, because of their approach, none of these later studies explore the possibility of type 2 underinvestment.

This may be reconciled by pointing out that, owing to the methodology used, much of the later literature has overlooked an important feature of lumpy public goods models. Lumpy public goods such as entry deterrence, by their very nature, give rise to discontinuous payoffs and multiple equilibria along with the associated coordination failure problems.[7] While first recognized by Bernheim (1984) and explicitly investigated by G&V, multiple Nash equilibria are ruled out in Waldman (1987) by the introduction of uncertainty, while the use of multiple potential entrants in Waldman (1991) performs a similar role. This allows the author to completely ignore situations (explicitly characterized by G&V) where both allowing entry and deterring entry can be equilibrium outcomes. Appelbaum and Weber assume concavity of each incumbent's expected profit function, an assumption that clearly does not hold in our model (and will not hold for cases with uncertainty that is not too large). Consequently, their model does not generate multiple equilibria, enabling them to sidestep the coordination failure problem entirely. Furthermore, their predictions on underinvestment relative to the collusive outcome depend crucially on their definition of a "public good" ("public bad")—both entry deterring profit and entry allowing profit for each incumbent must be increasing (decreasing) in other incumbents' precommitments. Since our differentiated product framework can have opposite signs for these marginal profits, Appelbaum and Weber's model cannot shed any light on free riding in our model. In fact, their analysis cannot even say anything conclusively about the G&V environment, where entry prevention is unambiguously a "public bad."

We demonstrate that coordination failure underinvestment in entry prevention can arise in the G&V framework when products are differentiated enough. Our analysis suggests that how an incumbent's maximum entry deterring profit changes with respect to its rival's output holds the key to the free riding issue. We may conjecture that for strategic substitutes, any entry deterrence model characterized by a deterrence profit that is increasing in rivals' precommitments over the relevant range is a likely candidate for coordination failure underinvestment. Two such examples of precommitment equilibria models with strategic substitutes are provided at the end of the paper. The first model introduces increasing marginal costs into the G&V homogeneous product model and, by showing the equivalence with the differentiated products model, demonstrates the possibility of type 2 underinvestment in this scenario.[8] Coordination failure underinvestment is also shown to emerge in the second model, where incumbents precommit to cost reducing research and development (R&D) (prior to the entry decision) before engaging in Cournot quantity setting.

The following section sets up our differentiated products version of the G&V model and characterizes the equilibria of the game. Section 3 examines the underinvestment phenomenon, while section 4 outlines the increasing marginal costs model and the R&D model. The last section concludes.

2. The Differentiated Products Model

We consider a two-stage noncooperative game with complete information played by two incumbents (firms 1 and 2) and a potential entrant (firm 3). In stage 1, firms 1 and 2 precommit to quantity levels x_1 and x_2, respectively. In stage 2, firm 3 makes its entry decision after observing

[7] Henceforth, all references to "public goods" will be in the context of lumpy public goods.

[8] We are grateful to Xavier Vives for educating us on this issue.

x_1 and x_2. If entry occurs a fixed cost F is incurred by the entrant. All firms are assumed to have identical constant marginal costs of production. We focus on subgame perfect equilibria.

We use the Vives (1985) differentiated products demand structure where an active firm i's inverse demand is given by

$$p_i = a - bx_i - g\sum_{j\neq i} x_j; \qquad i,\ j = 1,\ \ldots\ n; \quad b\ \geq\ g > 0; \quad a > 0. \tag{1}$$

in the region of quantity space where prices are positive. The number of active firms is n and g/b is a measure of the substitutability between products, ranging from zero when the goods are independent, that is, when $g = 0$, to one when the goods are perfect substitutes, that is, when $g = b$. Without loss of generality the intercept term a is interpreted as the net of the constant marginal cost.

Using backward induction, we first derive the entrant's decision rule. Let $x_3 = x_3^*\ (x_1,\ x_2)$ be the entrant's optimal output if it enters. Then, assuming the entrant enters if and only if it makes a positive profit, we can easily derive its optimal decision rule as

$$\begin{array}{rl} \text{Enter (and set } x_3 = x_3^*(x_1,x_2)) & \text{if } x_1 + x_2 < Z, \\ \text{Stay out} & \text{if } x_1 + x_2 \geq Z, \end{array} \tag{2}$$

where $Z \equiv (a - 2\sqrt{bF})/g$ is the limit output that just deters entry. Not surprisingly, the limit output is decreasing in the fixed cost F that the entrant must incur to enter the industry. More interestingly, Z is decreasing in the substitutability parameter g, reflecting that, *ceteris paribus,* the greater the substitutability between products the less the incumbents need to produce to keep the entrant out.

Now, consider firm 1's optimization problem.[9] Let $x_2 < Z$. (If $x_2 \geq Z$, firm 1 ignores the entry threat.) If incumbent 1 produces $x_1 < Z - x_2$, firm 3 enters with $x_3 = x_3^*\ (x_1,x_2)$ and firm 1 earns $\Pi_1^E(x_1;x_2) = \{a - bx_1 - g[x_2 + x_3^*(x_1,x_2)]\}x_1$. Otherwise, if $x_1 \geq Z - x_2$ entry does not occur and firm 1 gets a profit of $\Pi_1^{\mathrm{NE}}(x_1;x_2) = (a - bx_1 - gx_2)x_1$. Firm 1's profit functions are as shown in Figure 1.[10] These profit functions (the positions of which are fixed by arbitrary x_2) are functions of firm 1's own quantity. The higher curve $\Pi_1^{\mathrm{NE}}(x_1;x_2)$ shows firm 1's profit if the entrant stays out. The other curve depicts net profit of firm 1 for various x_1 when firm 3 enters and produces $x_3^*\ (x_1,\ x_2)$. For x_1 less than $Z - x_2$, firm 1's profit moves along $\Pi_1^E(x_1;x_2)$. At $x_1 = Z - x_2$ profit jumps up to $\Pi_1^{\mathrm{NE}}(x_1;x_2)$ and stays on this higher curve for larger x_1.

Let $r_1^E(x_2)$ and $r_1^{\mathrm{NE}}(x_2)$ be the (unique) maximizers of $\Pi_1^E(x_1;x_2)$ and $\Pi_1^{\mathrm{NE}}(x_1;x_2)$, respectively. Whenever $r_1^{\mathrm{NE}}(x_2) + x_2 \geq Z$, firm 1's unconstrained profit maximizing output $r_1^{\mathrm{NE}}(x_2)$ (ignoring the entry threat) automatically blockades entry and yields the greatest profit. Since $r_1^{\mathrm{NE}}(x_2) + x_2$ is increasing in x_2, there exists a unique $x_2^B(Z)$ solving $r_1^{\mathrm{NE}}(x_2) + x_2 = Z$ (see Appendix). For all $x_2 \geq x_2^B(Z)$, firm 1 produces $r_1^{\mathrm{NE}}(x_2)$ and entry is blockaded.

However, for $x_2 < x_2^B(Z)$, entry is no longer blockaded since $r_1^{\mathrm{NE}}(x_2) + x_2$ falls short of the limit output Z. Incumbent 1 can prevent entry by choosing $Z - x_2$ or can allow entry by producing $r_1^E(x_2)$. We define $\Pi_1^{\mathrm{NE}*}(x_2) = \Pi_1^{\mathrm{NE}}(Z - x_2;x_2)$ and $\Pi_1^{E*}(x_2) = \Pi_1^E(r_1^E(x_2);x_2)$ as the maximum profit that firm 1 can earn from deterring entry and allowing entry, respectively. As shown in the Appendix, similar to G&V,

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[9] We shall, henceforth, use incumbent 1 as the representative firm, keeping in mind that the case for firm 2 is exactly symmetric.

[10] It may be readily verified that for positive x_1, the Π_1^{NE} curve lies everywhere above the Π_1^E curve and has a larger horizontal intercept than Π_1^E.

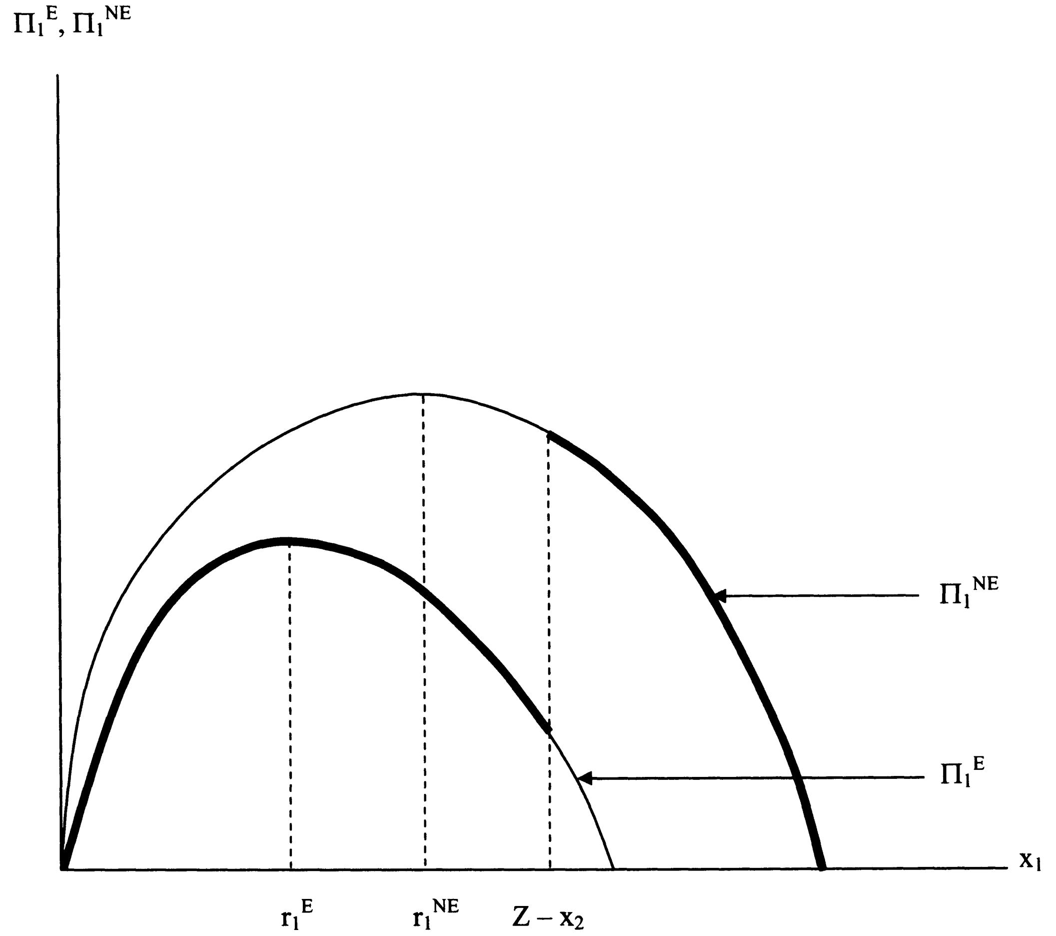

Figure 1. Incumbent 1's "Entry" and "No Entry" Profit as a Function of Own Quantity, for Given x_2

$\Pi_1^{E*}(x_2)$ is decreasing and strictly convex in x_2. However, the shape of $\Pi_1^{NE*}(x_2)$ can be very different from that in G&V. The explicit expression for firm 1's maximum profit from deterring entry is

$$\Pi_1^{NE^*}(x_2) = (Z - x_2)(a - bZ + [b - g]x_2). \tag{3}$$

When the goods are perfect substitutes ($g = b$), from Equation 3 it is clear that $\Pi_1^{NE*}(x_2)$ is linear and decreasing in x_2, that is, we are in the G&V case. However, for smaller g, $\Pi_1^{NE*}(x_2)$ is strictly concave, increasing in the rival's output until some point $x_2^{\max} = ([2b - g]Z - a)/(2[b - g])$ and then decreasing.[11] These profit functions are depicted in Figure 2. Note that at $x_2 = x_2^B(Z)$, firm 1's profit from deterring entry must dominate that from allowing entry. This ensures a unique intersection $x_2^I(Z)$

[11] Note that $Z > a/(2b - g)$ is required for the entry deterring profit to be increasing over some positive range. Otherwise, we get $x_2^{\max} < 0$ and firm 1's maximum entry deterring profit decreases continuously for all nonnegative x_2 so that the Gilbert and Vives result remains valid. Hence, as claimed, their overinvestment prediction is robust to moderate amounts of product differentiation.

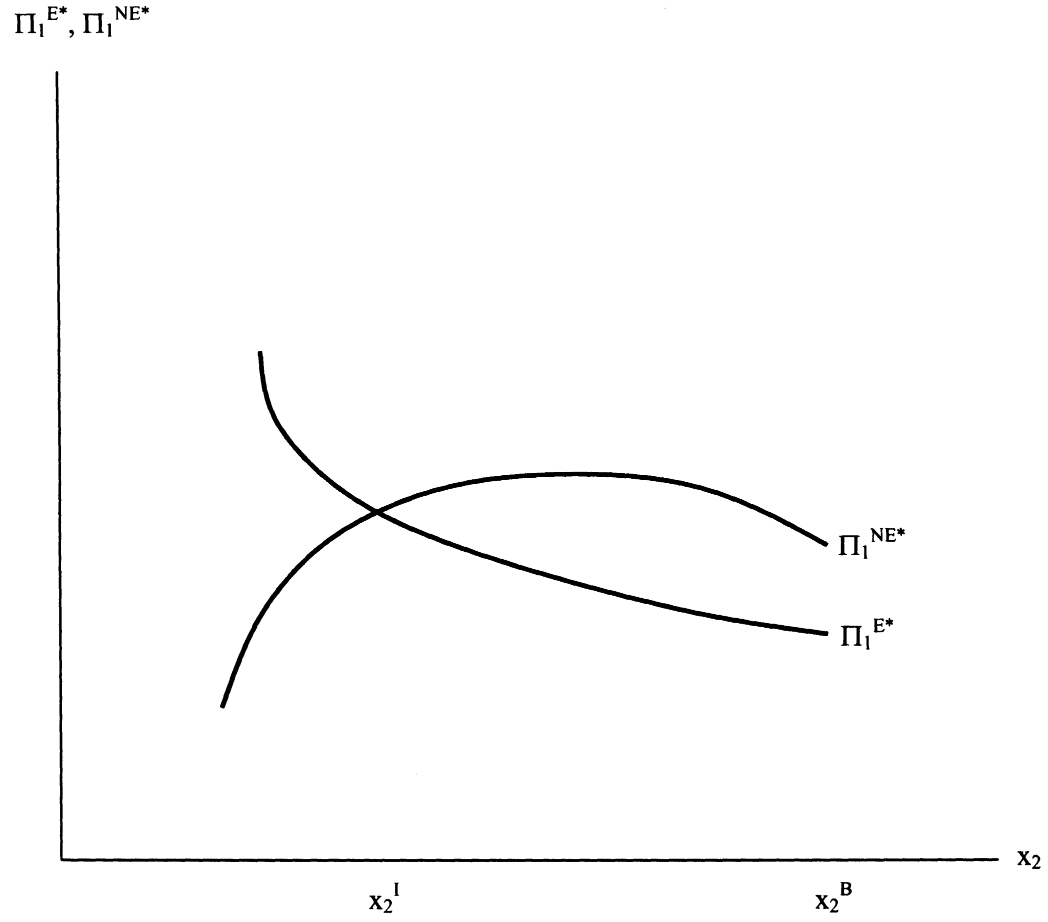

Figure 2. Incumbent 1's Maximum Profit from Allowing Entry and from Deterring Entry as a Function of the Other's Quantity

where $\Pi_1^{NE*}(x_2) = \Pi_1^{E*}(x_2)$, that is, at this rival output, 1 is indifferent between allowing entry (by producing $r_1^E(x_2^I(Z))$ and deterring entry (by producing $Z - x_2^I(Z)$).[12]

Firm 1's overall best response $\sigma_1(x_2)$ is, then (see Figure 2):

$$\sigma_1(x_2) = \begin{cases} r_1^E(x_2) & \text{when } x_2 \in [0, x_2^I(Z)], & \text{i.e., allow entry.} \\ Z - x_2 & \text{when } x_2 \in [x_2^I(Z), x_2^B(Z)], & \text{i.e., deter entry.} \\ r_1^{NE}(x_2) & \text{when } x_2 \geq x_2^B(Z), & \text{i.e., blockade entry.} \end{cases} \tag{4}$$

Firm 1's best response is shown in Figure 3. The straight line *ZZ* is the locus of points for which firm 1 and firm 2's outputs add up to exactly the limit output. Points below *ZZ* represent combinations of x_1 and x_2 that allow entry, while points above this line denote individual outputs that deter entry. For low values of x_2 firm 1 chooses the quantity $r_1^E(x_2)$ that maximizes its profit from allowing entry, that is, entry is allowed. This is true for all x_2 less than $x_2^I(Z)$. At $x_2 = x_2^I(Z)$, firm 1's best response jumps up to make up the difference between the limit output and firm 2's quantity, that is, entry is

[12] We define x_2^I (Z) to be zero if the intersection occurs at negative x_2.

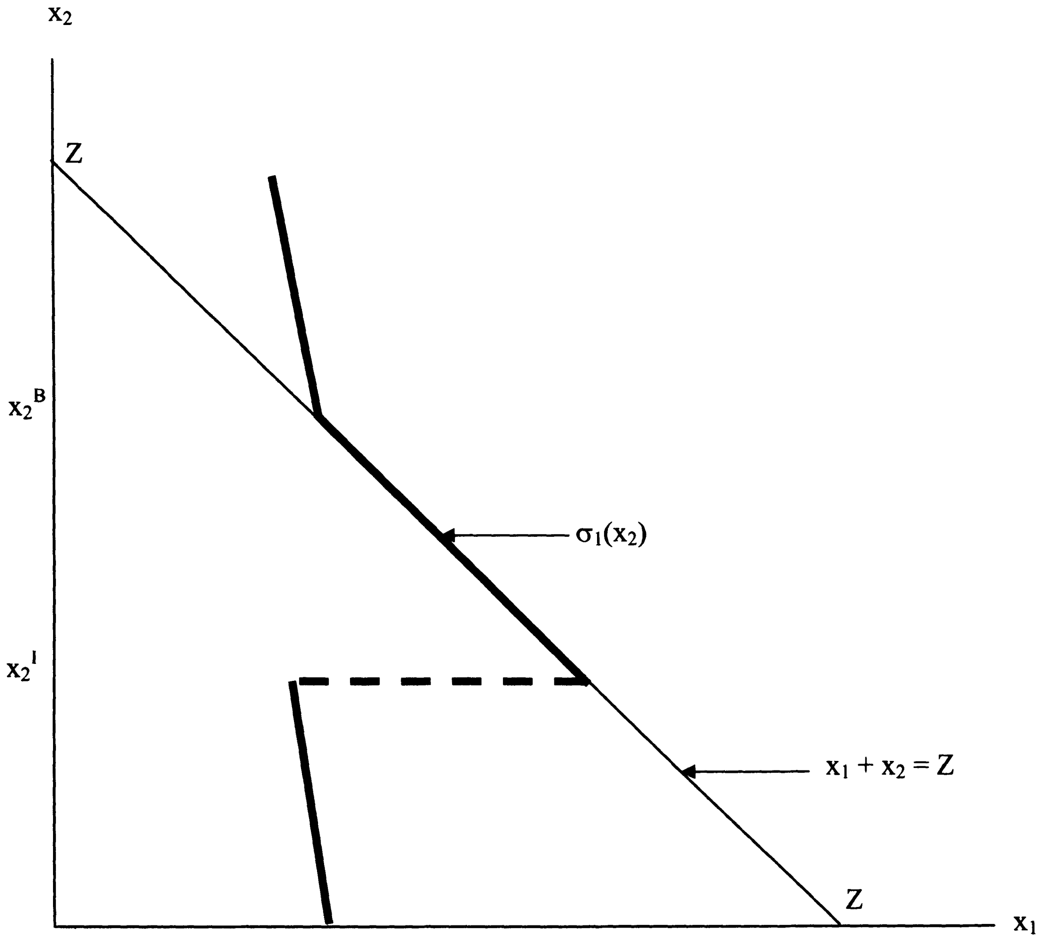

Figure 3. Incumbent 1's Overall Best Response

deterred. For very high values of x_2 firm 1 ignores the entry threat and produces $r_1^{NE}(x_2)$, that is, entry is blockaded.

Since the incumbent firms are symmetric, firm 2's best response may be derived in analogous fashion and the equilibria of this game can be identified using G&V's methodology. The characterization of the set of equilibria is critically dependent on $x_2^I(Z)$, the properties of which were characterized by G&V. Lemma 1 shows that these properties are robust to the introduction of product differentiation (see Figure 4).

Lemma 1. Let Z_m be the limit output for which a monopolist is indifferent between allowing entry and deterring entry. Then $Z_m > 0$ and $x_2^I(Z)$ is zero on $[0, Z_m]$ and for $Z > Z_m$, $x_2^I(Z)$ is increasing in Z with constant slope greater than unity up to a/g.

Proof. See Appendix.

The best response of incumbent 1 given in Equation 4 along with Lemma 1 can now be used to derive the equilibria in terms of the limit output Z. The (unique) entry allowing equilibrium can be calculated from the intersection of the incumbents' best responses when the intersection occurs at a point before the one at which an incumbent becomes indifferent between allowing and deterring

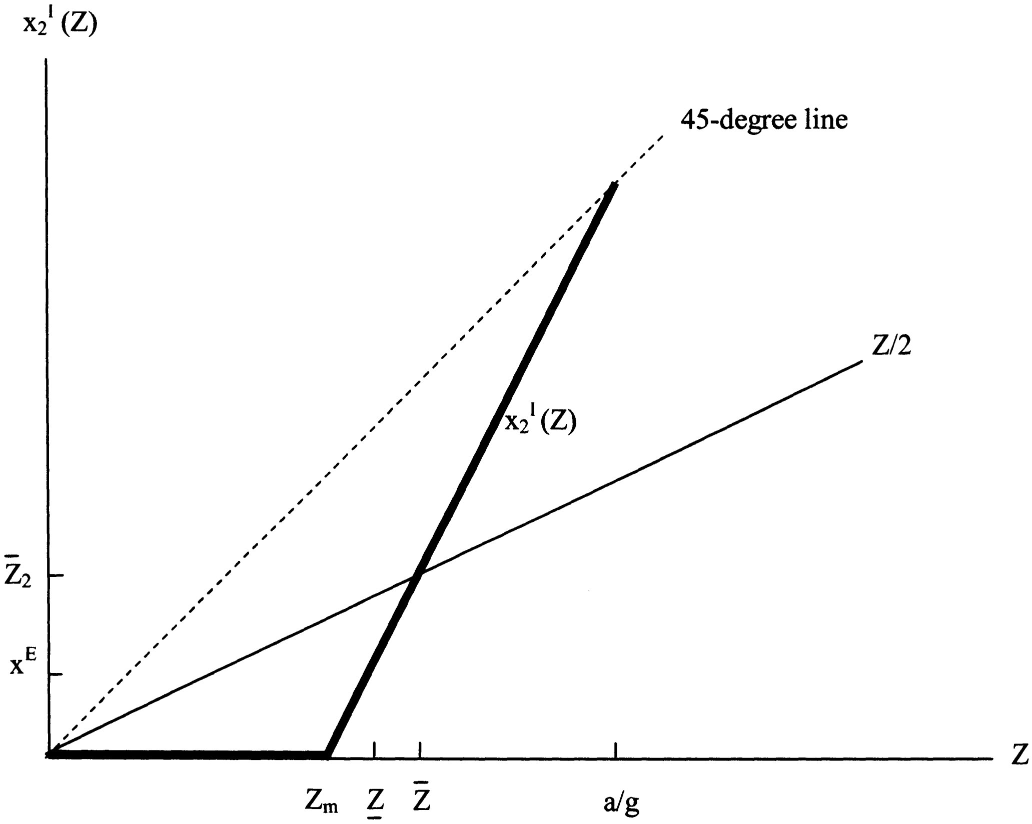

Figure 4. The Rival Output x_2^I (Z) That Makes Incumbent 1 Indifferent between Allowing and Deterring Entry, as a Function of the Limit Output

entry. This entry equilibrium has both firms producing equal amounts given by $x_1 = x_2 = x^E$. If there is no entry threat, both incumbents behave as unconstrained duopolists and in equilibrium produce $x_1 = x_2 = x^{NE}$. Denote the aggregate output in the unconstrained case as x^U(with $x^U = 2x^{NE}$). If the limit output is less than x^U, entry is automatically blockaded. Hence, consider $Z > x^U$.

Let $\underline{Z}$ be the smallest limit output for which allowing entry is an equilibrium, that is, $\underline{Z}$ solves $x_2^I(Z) = x^E$. This means that when its rival produces x^E each incumbent is indifferent between allowing entry (by producing x^E itself) and deterring entry (by choosing an output level of $\underline{Z} - x^E$). Further, denote by $\bar{Z}$ the largest limit output for which preventing entry is an equilibrium, that is, $\bar{Z}$ solves $x_2^I(Z) = Z/2$. Since $Z/2$ is greater than x^E and $x_2^I(Z)$ is increasing in Z, we know that $\bar{Z} > \underline{Z}$. We can now characterize the equilibria of this game in terms of the limit output Z as described in Proposition 1.[13]

Proposition 1.

(i) When $Z \leq x^U$, each incumbent produces x^{NE} and the entrant stays out in equilibrium. Entry is blockaded by the oligopoly. The equilibrium is unique and symmetric.

(ii) When $x^U < Z < \underline{Z}$, any incumbent outputs in the set $D = \{(x_1, x_2) \in R_+^2: x_1 + x_2 = Z, x_2^I(Z) \leq x_i \leq Z - x_2^I(Z), i, j = 1, 2; i \neq j\}$, and the entrant staying out is an equilibrium. The oligopoly prevents entry by producing exactly Z.

[13] The interested reader should refer to the Appendix for the technical details.

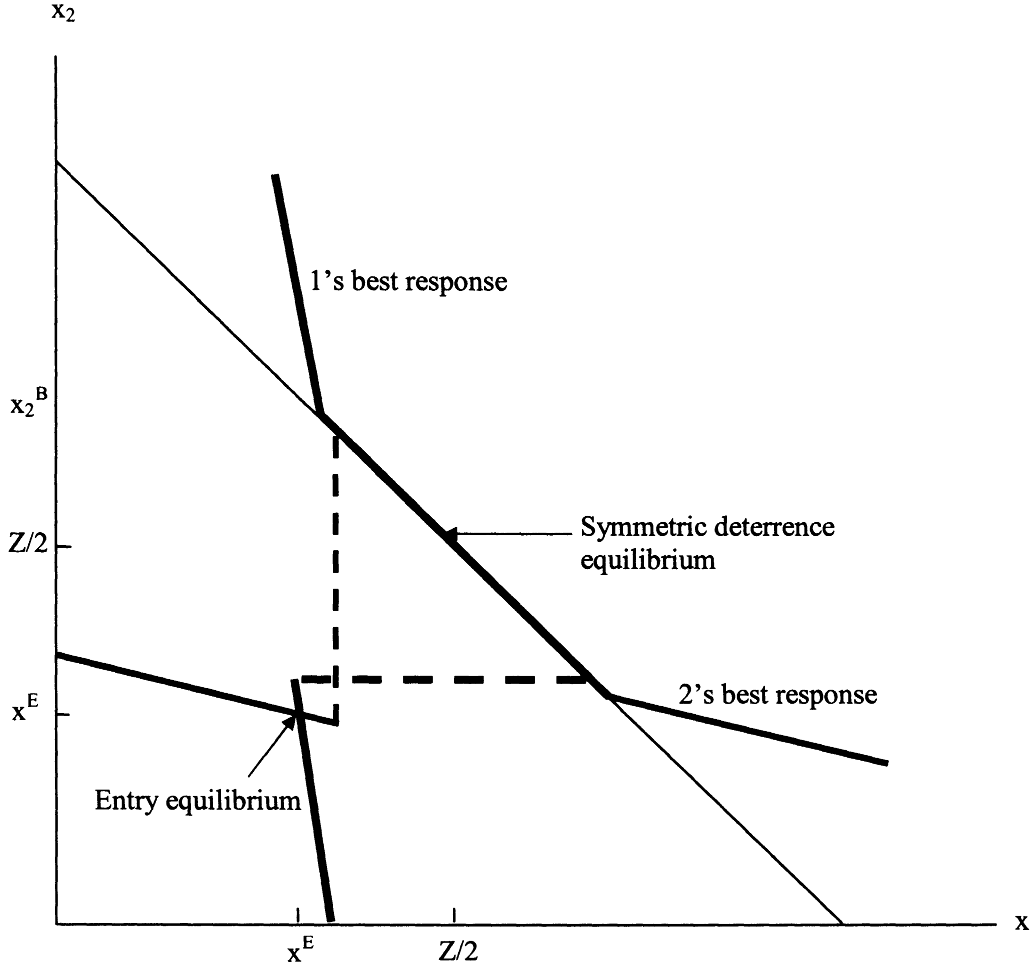

Figure 5. Incumbents' best responses for $\underline{Z} \leq Z \leq \bar{Z}$. That is, when both entry allowing and entry deterring equilibria coexist.

(iii) When $\underline{Z} \leq Z \leq \bar{Z}$, either allowing entry or preventing entry is an equilibrium. If entry is deterred, any incumbent outputs in the set D, and the entrant staying out is an equilibrium. If entry is accommodated, each incumbent producing x^E and the entrant entering with an output of $x_3^*(x^E, x^E)$ is an equilibrium.

(iv) When $Z > \bar{Z}$, the unique equilibrium has each incumbent producing x^E and the entrant entering the industry with an output of $x_3^*(x^E, x^E)$.

Case i of Proposition 1 tells us that when the limit output is very small, each incumbent ignores the threat of entry and the unconstrained equilibrium aggregate output automatically blockades entry. However, for larger limit outputs we get the entry deterring regime of case ii where the incumbents produce exactly the limit output in equilibrium. As the limit output increases further, both entry allowing and entry deterring equilibria are possible (case iii), while for even greater levels of limit output (case iv), the unique equilibrium is to allow entry. Figure 5 illustrates case iii where both allowing entry and deterring entry are equilibria.

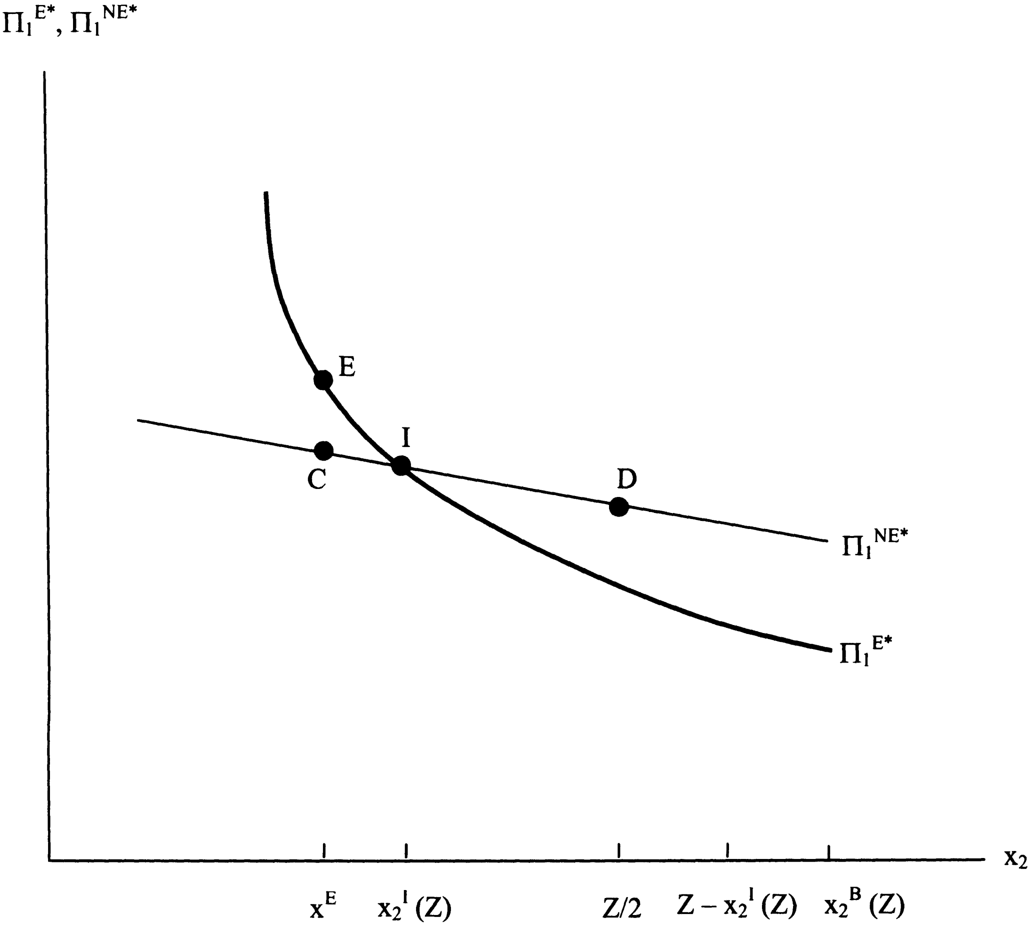

Figure 6. Incumbent 1's Maximum Profit from Allowing Entry and from Deterring Entry as a Function of the Other's Quantity in the Gilbert and Vives Homogeneous Product Model

3. Underinvestment with Differentiated Products

In this section we show that underinvestment of type 2 can arise in the differentiated products model even though type 1 and type 3 underinvestment are absent. We state this latter fact as a proposition. The proof is analogous to Gilbert and Vives (1986).

PROPOSITION 2. There is no underinvestment of type 1 or type 3 in the differentiated products model.

Let us now focus on coordination failure underinvestment. In the G&V homogeneous product setting, incumbent 1's maximum profit from deterring entry, $\Pi_1^{NE*}(x_2)$, is continuously decreasing in incumbent 2's output, as shown in Figure 6. Let $\underline{Z} \leq Z \leq \bar{Z}$, so that both allowing entry and deterring entry are equilibria. Then, from Proposition 1 we know that the quantity x^E that each produces at the entry equilibrium is less than the rival output $x_2^I(Z)$, for which an incumbent is indifferent between allowing and preventing entry. Further, in any entry deterring equilibrium, the rival output (including that corresponding to the symmetric deterrence equilibrium, $Z/2$) is greater than $x_2^I(Z)$.

Now, consider firm 1's profits. If incumbent 2 produces x^E then incumbent 1, by itself choosing x^E and allowing entry, earns a profit at the entry equilibrium corresponding to point E on the $\Pi_1^{E*}(x_2)$ curve. We know that at any entry deterring equilibrium, $x_2^I(Z) \leq x_2 \leq Z - x_2^I(Z)$ must hold. It should now be easy to see that for any $x_2^I(Z) \leq x_2 \leq Z - x_2^I(Z)$, firm 1 by producing $Z - x_2$ earns a lower profit at the corresponding entry deterring equilibrium compared with the entry equilibrium profit at E since $\Pi_1^{\mathrm{NE}*}(x_2)$ is continuously decreasing. In particular, incumbent 1's profit at the symmetric entry deterring equilibrium, corresponding to point D on the $\Pi_1^{\mathrm{NE}*}(x_2)$ curve, is always unambiguously lower than its profit at the entry equilibrium. By symmetric considerations firm 2 prefers the entry equilibrium to any of the deterrence equilibria, and, hence, the entry equilibrium always Pareto dominates every deterrence equilibrium in the G&V framework. This forms the gist of G&V's overinvestment argument. If the actual equilibrium realized in the industry deters entry, then each incumbent has overinvested in the sense that each would have been better off by producing less and allowing entry.

Dropping the homogeneous good assumption can affect these conclusions. For sufficiently large amounts of product differentiation, it is now no longer necessary for an incumbent's entry deterring profit to be continuously decreasing in the other's output over the range of deterrence equilibria. This feature makes it impossible to unequivocally claim that, when both types of equilibria coexist, the entry equilibrium Pareto dominates every deterrence equilibrium.

Starting from the homogeneous good case with $g = b$ shown in Figure 6, an increase in product differentiation affects the $\Pi_1^{E*}(x_2)$ and $\Pi_1^{\mathrm{NE}*}(x_2)$ curves in the following manner. First, a decrease in g shifts $\Pi_1^{E*}(x_2)$ upward, but the curve continues to retain its decreasing convex shape. On the other hand, increasing the degree of product differentiation introduces concavity into the $\Pi_1^{\mathrm{NE}*}(x_2)$ curve but keeps the vertical intercept unchanged.[14] More specifically, starting from $g = b$, as g decreases, the deterrence profit function becomes concave with the new curve lying everywhere above the original one except at zero rival output. As the degree of differentiation keeps increasing, the $\Pi_1^{\mathrm{NE}*}(x_2)$ curve becomes more and more concave until, after exceeding a certain critical level, it becomes upward sloping at $x_2 = 0$.[15]

Hence, for sufficiently differentiated products, we get the situation depicted in Figure 7 where both entry allowing and entry preventing equilibria coexist and incumbent 1's maximum entry deterring profit is increasing at $x_2^I(Z)$, that is, the rival output at which it is indifferent between accommodating entry and deterring entry. Recall that $x_2^{\max}$ is the rival output for which firm 1's profit from deterring entry reaches a maximum, and assume that this maximum entry deterring profit $\Pi_1^{\mathrm{NE}*}(x_2^{\max})$ is greater than firm 1's profit at the entry equilibrium $\Pi_1^{E*}(x^E)$.[16]

Here again, in any entry deterring equilibrium $x_2^I(Z) \leq x_2 \leq Z - x_2^I(Z)$ must hold. However, now, in contrast to the G&V scenario, at any entry deterring equilibrium where x_2 is sufficiently large incumbent 1 is strictly better off relative to the entry equilibrium; for instance, if incumbent 2 produces half the limit output, $Z/2$, then incumbent 1's equilibrium profit at D is strictly greater than that at E. This implies that the entry equilibrium does not Pareto dominate the symmetric entry deterring equilibrium. In fact, in this case, at the symmetric entry deterring equilibrium, both

[14] We thank an anonymous referee for helping us sharpen this discussion.

[15] The first partial of maximum entry allowing profit with respect to g is $-[(2b-g)(a-gx_2)][(2b-g)(2b^2-g^2)x_2 + 2b(a-gx_2)(b-g)]/[4b(2b^2-g^2)^2]$, which is negative for all rival outputs. As for the deterrence profit function, the first partial with respect to g is $-(Z-x_2)x_2$, which is negative for all $x_2 > 0$. The first derivative of deterrence profit with respect to x_2 is $(2b-g)Z - a - 2(b-g)x_2$, and this term is positive at $x_2 = 0$ when $g < 2b - a/Z$. Note that the fixed cost is being adjusted accordingly to keep the limit output constant.

[16] If this condition is not satisfied, the entry equilibrium will, obviously, Pareto dominate every entry deterring equilibrium.

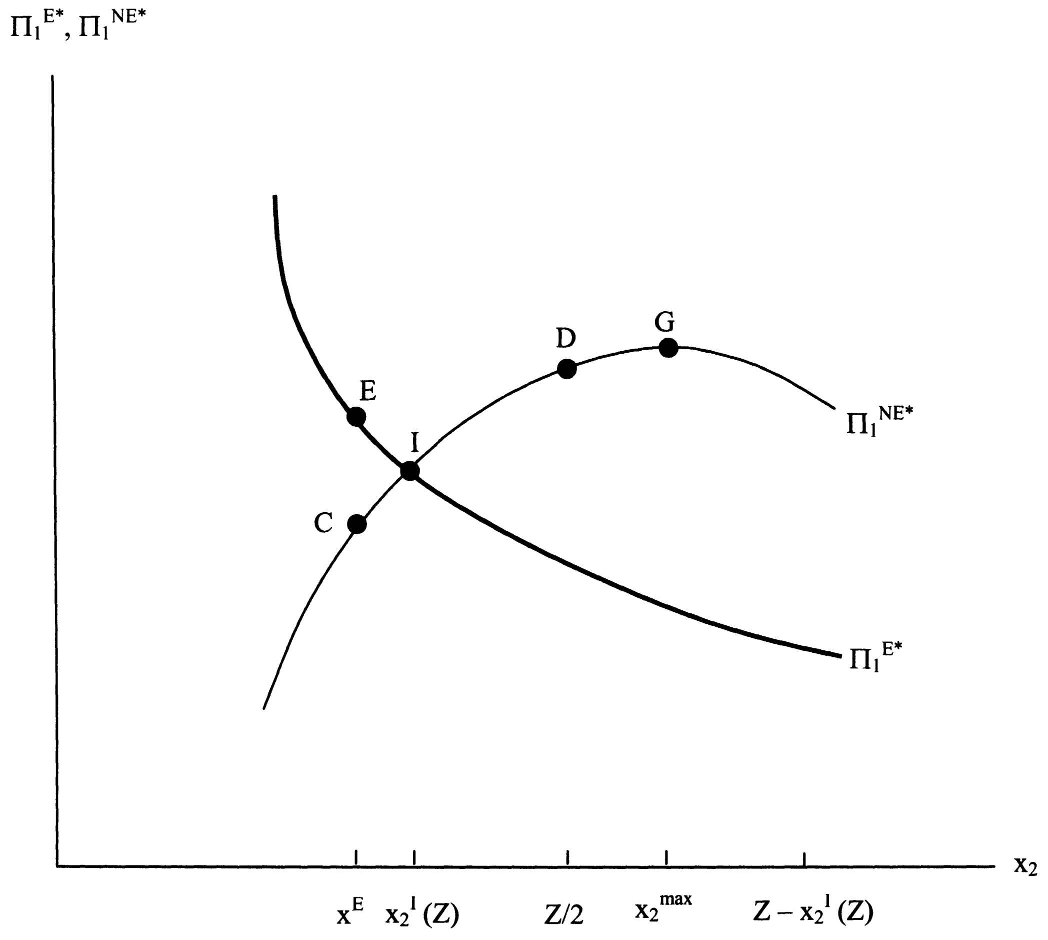

Figure 7. Incumbent 1's Maximum Profit from Allowing Entry and from Deterring Entry as a Function of the Other's Quantity When Its Entry Deterring Profit Is Increasing at $x_2^I(Z)$

incumbents earn a profit represented by *D* and, hence, the symmetric entry deterring equilibrium Pareto dominates the entry equilibrium. Underinvestment of type 2, or coordination failure underinvestment, arises.

Note that the rising portion of firm 1's entry deterring profit function $\Pi_1^{NE*}(x_2)$ plays a crucial role in the emergence of coordination failure underinvestment in the differentiated products model. However, if the entry deterring profit curve $\Pi_1^{NE*}(x_2)$ intersects the entry allowing profit curve $\Pi_1^{E*}(x_2)$ after it has started decreasing then this type of underinvestment cannot arise, as illustrated in Figure 8. The reasoning is similar to that given above for the nonexistence of type 2 free riding in the G&V model. This is formally stated in Proposition 3 below.

PROPOSITION 3. Let $\underline{Z} \leq Z \leq \bar{Z}$, so that both entry allowing and entry deterring equilibria exist. Then, $\forall\ i, j = 1, 2;\ i \neq j$, a necessary condition for coordination failure underinvestment is that an incumbent's maximum profit from deterring entry $\Pi_i^{NE*}(x_j)$ be increasing in the other's output at the point where it is indifferent between allowing entry and deterring entry.

PROOF. Suppose $\underline{Z} \leq Z \leq \bar{Z}$ but $x_2^{max} \leq x_2^I(Z)$. From Proposition 1(iii), $x^E \leq x_2^I(Z)$, and since $\Pi_i^{E*}(x_j)$ is decreasing in x_j, $\Pi_i^{E*}(x^E) \geq \Pi_i^{E*}(x_2^I(Z))$. Since $\Pi_i^{NE*}(x_j)$ is decreasing in x_j for $x_j > x_2^I(Z)$,

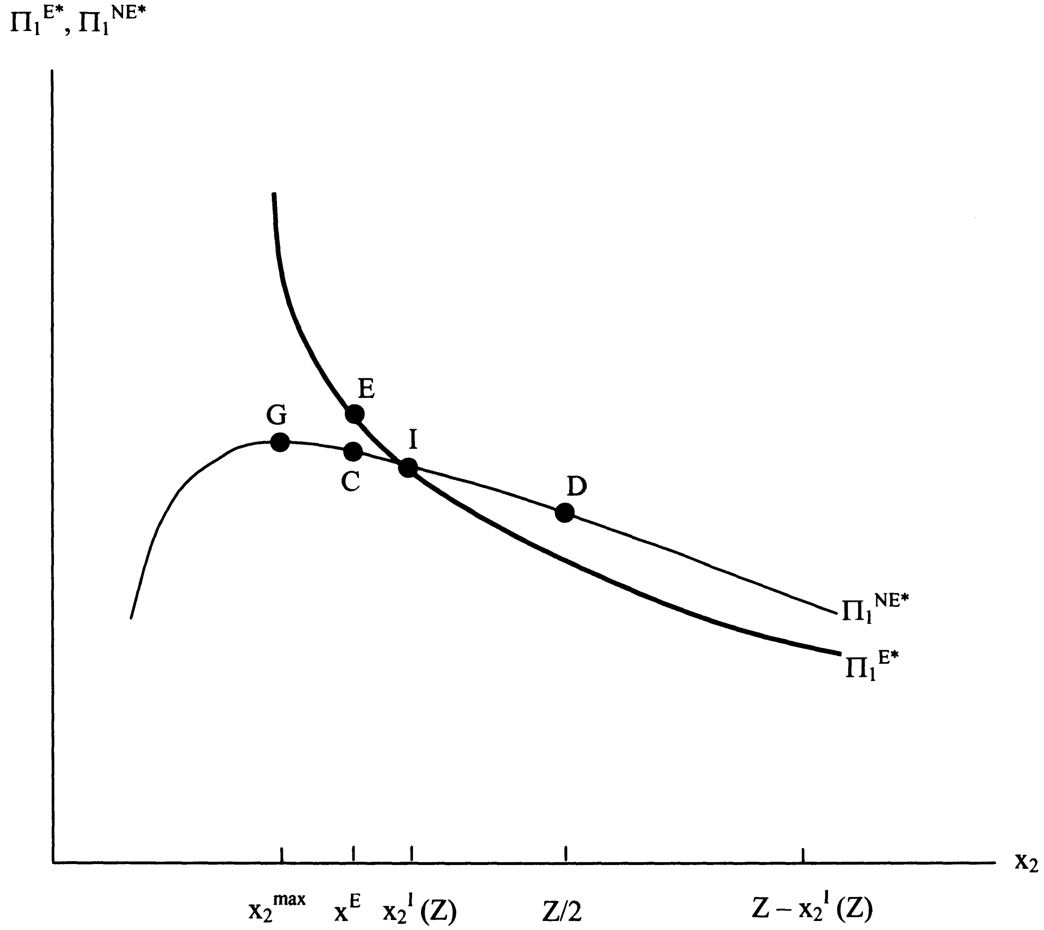

Figure 8. Incumbent 1's Maximum Profit from Allowing Entry and from Deterring Entry as a Function of the Other's Quantity When Its Entry Deterring Profit Is Decreasing at $x_2^I(Z)$

$\Pi_i^{NE*}(x_2^I(Z)) \geq \Pi_i^{NE*}(x_j)\ \forall\ x_j \in [x_2^I(Z), x_2^B(Z)]$. Now, using the fact that $\Pi_i^{E*}(x_2^I(Z)) \geq \Pi_i^{NE*}(x_2^I(Z))$ and that $x_j \geq x_2^I(Z)$, we get $\Pi_i^{E*}(x^E) \geq \Pi_i^{NE*}(x_j)$. Hence, the entry equilibrium Pareto dominates every deterrence equilibrium and there can never be any underinvestment of type 2. *QED.*

A numerical example of type 2 underinvestment is presented in Table 1. Here, the demand parameter a, is normalized to unity and b is set equal to 2. Choosing a limit output of $Z = 0.52$, the degree of substitutability, g, is allowed to vary, and for every g, the fixed cost, F, is chosen so as to maintain the limit output at 0.52. The chosen parameters ensure that both entry allowing and entry deterring equilibria coexist ($\underline{Z} \leq Z \leq \bar{Z}$) and that each incumbent's maximum profit from deterring entry is increasing in the other's output at the point where it is indifferent between allowing entry and deterring entry ($x_2^I(Z) < x_2^{max}$). When $g = 1.1$, the entry equilibrium Pareto dominates the symmetric entry deterring equilibrium. However, unlike G&V, the entry equilibrium does not Pareto dominate *every* entry deterring equilibrium. Each incumbent would prefer the deterrence equilibrium where it produces $Z - x_2^{max}$ and the other produces x_2^{max}. As g decreases, starting from $g = 1.05$, this gets reversed and it is clear that each incumbent's profit in the symmetric entry deterring equilibrium is greater than its profit at the entry allowing equilibrium. Thus, the symmetric entry deterring

Table 1. Comparison of Incumbent 1's Profit at the Entry Allowing Equilibrium, the Symmetric Entry Deterring Equilibrium, and the Profit from Deterring Entry When Rival Produces $x_2^{\max}$, as the Degree of Product Differentiation, g, Varies from 0.8 to 1.1, for $a = 1$, $b = 2$, and $z = 0.52$

g	F	$\underline{Z}$	$\bar{Z}$	x^E	$x_2^I(Z)$	$Z/2$	$x_2^{\max}$	$Z - x_2^I(Z)$	$\Pi_1^{E*}(x^E)$	$\Pi_1^{NE*}(Z/2)$	$\Pi_1^{NE*}(x_2^{\max})$
0.8	0.0426	0.5156	0.5921	0.1852	0.1915	0.26	0.2767	0.3285	0.0631	0.0707	0.0711
0.85	0.0389	0.5131	0.5870	0.1828	0.1932	0.26	0.2774	0.3268	0.0608	0.0673	0.0677
0.9	0.0354	0.5105	0.5817	0.1805	0.1952	0.26	0.2782	0.3248	0.0586	0.0640	0.0643
0.95	0.0320	0.5080	0.5763	0.1784	0.1978	0.26	0.2790	0.3222	0.0565	0.0606	0.0610
1.0	0.0288	0.5054	0.5708	0.1765	0.2008	0.26	0.28	0.3192	0.0545	0.0572	0.0576
1.05	0.0258	0.5028	0.5653	0.1746	0.2043	0.26	0.2811	0.3157	0.0526	0.0538	0.0542
1.1	0.0229	0.5003	0.5596	0.1729	0.2084	0.26	0.2822	0.3116	0.0508	0.0504	0.0509

equilibrium Pareto dominates the entry allowing equilibrium. Hence, there is coordination failure underprovision of public goods.

4. Other Examples of Coordination Failure Underinvestment

This section briefly discusses two other models that exhibit coordination failure underinvestment. Our purpose is to show that this type of underinvestment can be prevalent in a variety of models with precommitment equilibria and should, hence, be studied in greater depth.

The Increasing Marginal Costs Model

Our first model (to be called the increasing marginal costs model) retains the homogeneous product structure of the G&V model but relaxes the constant marginal costs assumption. We consider an industry characterized by linear demand for a homogeneous good and quadratic costs of production where the incumbents precommit to quantities before the entrant makes its entry decision. The intuition for underinvestment in this framework is as follows. Here, as in the G&V model, whenever entry is deterred the price is constant and invariant to how the limit output is distributed among the incumbents. However, unlike G&V, as an incumbent's share of the limit output expands it incurs increasingly larger costs of production. Consequently, when the additional revenue from a unit increase in output is more than offset by the additional cost of producing that unit, an incumbent's entry deterring profit decreases in own output or, equivalently, its deterrence profit increases in rival's output. When this happens, producing the entire limit output is too costly for any single incumbent and the incentive to have the largest share in the limit output is attenuated—a setting that is conducive to the emergence of underinvestment.

Formally demonstrating the existence of coordination failure underinvestment in the increasing marginal costs model is simply a matter of invoking the equivalence under quantity competition, observed by Vives (1990), between differentiated products with constant marginal costs and homogeneous products with increasing marginal costs. Let inverse demand for the homogeneous good be

$$p = \alpha - \gamma \sum_j x_j; \qquad j = 1, \ldots n; \quad \gamma > 0, \quad \alpha > 0, \tag{5}$$

where n is the number of active firms, and let $C(x_i) = \delta x_i^2$ be each firm's total cost function. Then, firm i's profit is given by

$$
\begin{aligned}
\Pi_i(x_i, x_{-i}) &= \left(\alpha - \gamma \sum_j x_j\right) x_i - \delta x_i^2, \qquad \text{or} \\
\Pi_i(x_i, x_{-i}) &= \left[\alpha - (\gamma + \delta) x_i - \gamma \sum_{j \neq i} x_j\right] x_i.
\end{aligned}
\tag{6}
$$

Note that Equation 6 corresponds to quantity competition with differentiated products and constant marginal costs if we interpret the intercept term as net of the marginal cost. In fact, for $\alpha = a$, $\gamma = g$, and $\delta = b - g$, Equation 6 is identical to a firm's profit in our differentiated products model. With the same type of analysis, it can be verified that for sufficiently steep marginal costs, underinvestment of type 2 can also arise in the homogeneous good case. Here, too, this type of underinvestment can occur only if an incumbent's entry deterring profit is increasing in the other's output at the point where it is indifferent between allowing entry and preventing entry. However, the G&V overinvestment result remains robust to moderately increasing marginal costs.

The dampening effect of increasing marginal costs on entry preventing incentives is also found, in a different but related context, in the sequential entry model of Vives (1988). He considers a model with a single incumbent and a pool of potential entrants sequentially choosing outputs in a homogenous good industry before the market clears. With constant marginal cost of production, the incumbent never allows entry of any firms that will subsequently deter further entry, since it prefers producing the entire limit output itself. However, with increasing marginal costs, if the pool of potential entrants is large and marginal costs are increasing, producing the limit output may be too costly and so the incumbent may not want to be the sole entry preventer.

The R&D Model

The second model (to be called the R&D model) is adapted from D'Aspremont and Jacquemin (1988) and considers a simple two-stage game where two symmetric incumbents precommit to production cost reducing R&D levels before the entry decision takes place. Though the cost of investment, assumed quadratic, is incurred in the first stage, the R&D choice determines production costs for the subsequent period. The second stage is characterized by simultaneous (Cournot) quantity setting with the entrant facing an avoidable fixed entry cost. The strategic variable here is similar to the strategic variable, quantity, of the G&V model, and the differentiated products model in that R&D, too, is a strategic substitute. That is, in the absence of any entry threat, the best responses are downward sloping. Now, instead of a limit output there exists a critical aggregate amount of cost reduction that if undertaken by the industry deters entry.

More specifically, we assume an inverse linear demand curve given by $P = \mathrm{Max}\{0, 1 - Q\}$ where Q is the aggregate output produced in the second stage. The incumbents' production costs are represented by $C_i\,(q_i, x_i) = [a - x_i]\, q_i$, $i = 1, 2$; where $a \in (0, 1)$ and $x_i \in [0, a]$. This formulation yields constant unit costs for the incumbents of $c_i (= a - x_i)$, $i = 1, 2$, for any given prior investment choice. The entrant is assumed to have a constant unit cost of $c_3 = a$. The cost of investment is assumed to be quadratic of the form bx_i^2 with $b > 0$, reflecting the existence of diminishing returns to R&D expenditures. These assumptions allow us to explicitly derive the entry deterring critical investment as a sum of the incumbents' investments.

The analysis of the cost reducing game is entirely analogous to the differentiated products model and delivers the same conclusions. We again get coordination failure underinvestment for the following reason. If an incumbent's R&D decreases (and the other's R&D increases) in moving from one entry deterring equilibrium to another, its production costs rise and its gross (Cournot) profit is reduced relative to the initial deterrence equilibrium. However, for large investment cost coefficients, the fall in investment costs may be so significant as to actually increase its net profit. Consequently, an incumbent's equilibrium deterrence profit can be initially increasing in its rival's R&D before tapering off and decreasing over the rest of the domain. When this happens, similar to the differentiated products framework, the incumbent may no longer want the largest possible share in the critical aggregate investment, and type 2 underinvestment may occur—the entry equilibrium may not Pareto dominate every deterrence equilibrium. More specifically, we may demonstrate that this type of underinvestment can occur only if each incumbent's entry deterring profit is increasing in its rival's R&D at the point where it is indifferent between allowing entry and preventing entry.

While this paper has focused on entry deterring variables that are strategic substitutes, coordination failure may also occur with strategic complements. For instance, we may conjecture that underinvestment is possible in a differentiated products industry where incumbents can credibly precommit to prices in the preentry period and the entrant stays out if incumbent prices are low enough.[17] The intuition is that when entry is prevented, a decrease in an incumbent's price enhances demand for its product but reduces the price received per unit sold such that its deterrence profit may decrease in the other's price and may reduce its incentive to have the smallest possible "share" in the limit price—a setting that may be conducive to underinvestment. While the realism of this scenario may be open to debate given the questionable commitment value of prices as entry deterring variables, it does serve to buttress our conviction that coordination failure underinvestment in entry deterrence is prevalent in a wide variety of settings and deserves a closer study.[18]

5. Conclusion

This paper demonstrates that sufficiently large amounts of product differentiation can generate coordination failure underinvestment in entry deterrence, that is, even though there exists an entry deterring equilibrium, imperfect coordination of incumbents' actions may result in an equilibrium where entry is allowed and all incumbents earn lower profits. This type of underinvestment, first discussed by Bernheim (1984) and explicitly investigated by Gilbert and Vives (1986), has been ignored by the later literature. We show that with sufficiently differentiated products, an incumbent's entry deterring profit increases in the other's quantity (in the relevant region) and weakens its incentive to have the largest share in the limit output, thus generating underinvestment. Gilbert and Vives' type 2 overinvestment result rests critically on the dual assumptions of a homogeneous product and a constant marginal cost. Our analysis shows that relaxing the homogenous product assumption can generate coordination failure underinvestment. Also, our increasing marginal cost model shows that sufficiently

[17] This conjecture may be verified by using the demand structure of Shubik (1980) $q_i = \{a - b[p_i + g(p_i - \bar{p})]\}/n$, where $\bar{p} = (\sum p_i)/n$ is the average of all prices p_i, $i = 1, \dots n$, n is the total number of active firms in the industry, and g is a measure of the substitutability between products ($g > 0$). This demand structure yields the entry preventing price combination as an aggregate of the incumbents' prices and is consequently amenable to the methodology employed throughout this paper.

[18] We have also looked at an advertising model by Shubik (1980), which examines a three-stage game where two symmetric incumbents in a differentiated products industry commit to market share enhancing advertising outlays in the first stage. In the second stage, the entrant chooses its advertising level and pays an entry cost if it decides to enter. Simultaneous price competition occurs in the final stage. Each incumbent's advertising outlay turns out to be a strategic complement (upward sloping best response) over smaller values of the other's advertising and a strategic substitute over larger values of the rival's advertising level. Our analysis reveals that there is never any type 2 underinvestment in the advertising model since each incumbent's equilibrium entry preventing profit is monotonically decreasing in the other's advertising.

steep marginal costs can also result in type 2 underinvestment. However, for moderate amounts of product differentiation and moderately steep marginal costs, each incumbent still wants the largest share in the limit output and G&V's overinvestment result remains valid.

We believe that coordination failure equilibria are common in settings with lumpy public goods and should receive more attention in the entry deterrence context. In keeping with this view, we outline two other models of entry deterrence—an increasing marginal costs model and an R&D model—where, again, underinvestment is caused by weakening an incumbent's incentive to produce the entire limit investment. Our analysis suggests that coordination failure underinvestment can occur for strategic substitutes only if the private benefit from the public good—deterred entry—is increasing in the rival incumbent's precommitment variable over the relevant region.

Finally, it should be pointed out that introducing uncertainty into our model may have interesting implications for the free rider problem. Waldman (1987) shows that though the introduction of uncertainty into the G&V framework does not result in underinvestment, adding uncertainty to the Bernheim model causes every equilibrium to be characterized by the free rider problem with its resulting underinvestment. Since introducing product differentiation into the G&V model produces type 2 underinvestment, it may be conjectured that differentiated products in conjunction with uncertainty would result in the possibility of a unique underinvestment equilibrium.[19] The investigation of this issue shall be taken up in future research on the free rider problem in noncooperative entry deterrence.

Appendix

1. We show there exists a unique $x_2^B(Z)$ such that entry is blockaded for all $x_2 \geq x_2^B(Z)$.

Maximization of $\Pi_1^{\mathrm{NE}}(x_1;x_2)$ yields $r_1^{\mathrm{NE}}(x_2) = (a - gx_2)/(2b)$ with a slope of $-g/2b$, where $-1 < -g/2b < 0$. Thus, $r_1^{\mathrm{NE}}(x_2) + x_2$ is increasing in x_2 and there is a unique solution $x_2^B(Z) = (2bZ - a)/(2b - g)$ solving the equation $r_1^{\mathrm{NE}}(x_2) + x_2 = Z$, such that 1 ignores the entry threat and produces $r_1^{\mathrm{NE}}(x_2)$ for all $x_2 \geq x_2^B(Z)$.

2. We show $\Pi_1^{E*}(x_2)$ is decreasing and convex in x_2, $\Pi_1^{\mathrm{NE}*}(x_2)$ is strictly concave in x_2, and we derive their intersection point $x_2^I(Z)$. We also derive analytic expressions for x^E, x^{NE}, $\underline{Z}$, and $\bar{Z}$.

For $x_1 < Z - x_2$, the entrant enters with an output of $x_3^*\ (x_1, x_2) = [a - g(x_1 + x_2)]/(2b)$ and we get $\Pi_1^E(x_1;x_2) = [(2b - g)(a - gx_2)x_1 - (2b^2 - g^2)x_1^2]/(2b)$. Maximizing this with respect to x_1 yields $r_1^E(x_2) = [(2b - g)/(4b^2 - 2g^2)](a - gx_2)$, which is downward sloping with a slope greater than -1. Substituting for $r_1^E(x_2)$, we get $\Pi_1^{E*}(x_2) = [(2b - g)^2(a - gx_2)^2]/[8b(2b^2 - g^2)]$, which is clearly decreasing and convex in x_2. On the other hand, the first and second partials of $\Pi_1^{\mathrm{NE}*}(x_2) = (Z - x_2)[a - bZ + (b - g)x_2]$ show that it is concave in x_2 and attains a maximum at $x_2^{\max} = ([2b - g]Z - a)/(2[b - g])$. Since $\Pi_1^{\mathrm{NE}*}(x_2^B(Z)) > \Pi_1^{E*}(x_2^B(Z))$, we get $x_2^I(Z) = \max(0, [E_1 - 2aE_3 + (E_2 + 2gE_3)Z]/D)$ as the unique intersection of $\Pi_1^{\mathrm{NE}*}$ and Π_1^{E*} such that $\Pi_1^{\mathrm{NE}*} > \Pi_1^{E*}$ for $x_2 > x_2^I(Z)$ and $\Pi_1^{\mathrm{NE}*} < \Pi_1^{E*}$, otherwise. ($D$, E_1, E_2, and E_3 are all functions of a, b, and g given by $D = (-4b^2 + 2bg + g^2)^2$, $E_1 = -8ab^3 + 4ab^2g + ag^3$, $E_2 = 16b^4 - 8b^3g - 8b^2g^2 + 4bg^3$, and $E_3 = \sqrt{2b^2g(8b^3 - 6b^2g - 4bg^2 + 3g^3)}$).

Since $r_1^E(x_2)$ and $r_2^E(x_1)$ are downward sloping with a slope greater than -1, their intersection is unique and yields $x^E = a(2b - g)/[2(2b^2 - g^2) + g(2b - g)]$, the output produced by each incumbent in the entry equilibrium. Similarly, the intersection of $r_1^{\mathrm{NE}}(x_2)$ and $r_2^{\mathrm{NE}}(x_1)$ is also unique and is given by $x^{\mathrm{NE}} = a/(2b + g)$. Under blockaded entry, both incumbents behave as unconstrained duopolists and in equilibrium produce $x_1 = x_2 = x^{\mathrm{NE}}$. Recall that $\underline{Z}$ solves $x_2^I(Z) = x^E$ and $\bar{Z}$ solves $x_2^I(Z) = Z/2$. Explicit calculations yield $\underline{Z} = [Dx^E - (E_1 - 2aE_3)]/(E_2 + 2gE_3)$ and $\bar{Z} = [-2(E_1 - 2aE_3)]/[2(E_2 + 2gE_3) - D]$.

PROOF OF LEMMA 1. The proof follows Gilbert and Vives (1986). First, we show that Z_m is positive. If x_m is the monopoly output then $Z_m > x_m$ must hold because for any $Z \leq x_m$ a monopolist would always prefer to deter entry. So Z_m must be positive. Here, $Z_m = (a/2b)(1 + \sqrt{1 - [(2b - g)^2/2(2b^2 - g^2)]})$ and $x_m = a/2b$. Second, to show that $x_2^I(Z)$ is zero for all Z on the interval [0, Z_m], consider any $Z \leq Z_m$. If $x_2^I(Z) > 0$ then, since firm 1's entry allowing profit is decreasing in x_2, $\Pi_1^{\mathrm{NE}*} < \Pi_1^{E*}$ at a zero rival output. This implies that a monopolist would prefer to allow entry, which contradicts the fact that $Z \leq Z_m$. So, $x_2^I(Z)$ must be zero for all Z on [0, Z_m]. Third, we show that $x_2^I(a/g) = a/g$. When $Z = a/g$ and $x_2 < a/g$, then deterring entry makes incumbent 1's

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[19] We thank an anonymous referee for pointing this out.

price zero and so it always prefers allowing entry to preventing entry. Hence, it is indifferent between accommodation and deterrence only when its rival produces a/g, which implies that $x_2^l(a/g) = a/g$.

Finally, we prove that $x_2^l(Z)$ is increasing with constant slope greater than unity on $[Z_m, a/g]$. For $Z_m < Z < a/g$, $x_2^l(Z) = [E_1 - 2aE_3 + (E_2 + 2gE_3)Z]/D$ and $dx_2^l/dZ = (E_2 + 2gE_3)/D$, or, equivalently, $dx_2^l/dZ = (16b^4 - 8b^3g - 8b^2g^2 + 4bg^3 + 2gE_3)/[16b^4 - 8b^3g - 8b^2g^2 + 4bg^3 + (g^4 - 8b^3g + 4b^2g^2)]$. Now, E_3 can be rewritten as $\sqrt{2b^2g(2b^2 - g^2)(4b - 3g)} > 0$, while $16b^4 - 8b^3g - 8b^2g^2 + 4bg^3 = 8b^3(b - g) + 8b^2(b^2 - g^2) + 4bg^3 > 0$. So, the numerator of dx_2^l/dZ is positive. Recall that the denominator D is positive. Further, $(g^4 - 8b^3g + 4b^2g^2) = -g(4b^3 - g^3 + 4b^2(b - g)) < 0$ and, hence, $(16b^4 - 8b^3g - 8b^2g^2 + 4bg^3 + 2gE_3)/[16b^4 - 8b^3g - 8b^2g^2 + 4bg^3 + (g^4 - 8b^3g + 4b^2g^2)] > 1$. *QED.*

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